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Equilibrium interest rates and the anchoring of inflation expectations

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Abstract

In Chapter 1, based on a paper cohautored by Adriana Grasso (LUISS University), I propose a consumption-based model that allows for an inverted term structure of real and nominal risk-free rates. In equilibrium, real interest rates depend not only on shocks to consumption growth but also on expectations about future consumption growth volatility. In bad states, a high uncertainty makes agents more willing to accumulate precautionary savings and to rebalance their bond portfolios towards longer maturities, pushing the equilibrium short-term yields above long-term ones. Pricing time-varying volatility risk is essential to obtain the inversion of the real curve and allows to price the average level and slope of the nominal one.

In Chapter 2, based on a paper cohautored by Laura Sigalotti (Bank of Italy), I propose a new indicator of the anchoring of inflation expectations based on a logistic model. By inspecting the comovement of daily changes in short and long-term inflation swap rates, it measures the odds that strong variations in short-term inflation compensation are channelled to large movements of the same sign in long-term one. The indicator is able to capture the transition from anchored to unanchored expectations, implying a non-linear pass-through of shocks to long-term expectations. Since 2014, the asymmetric pass-through from short- to long-term inflation compensation suggests that the degree of anchoring of euro area inflation expectations have diverged from the US's and UK's one.

DISCLAIMER - LIBERATORIA

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Chapter 1

Consumption volatility risk and the inversion of the yield curve

1.1 Introduction

The inversion of the term structures of interest rates, which happens when short-term yields are above long-term ones, is an occasional, yet not rare event. Looking at postwar US data on the 10-to-1 year term spread, ten relevant episodes of inversion are observed, accounting for about 10% of total daily observations (see Figure 1.1). The dynamics of the term spread gives insights on the transmission of monetary policy, on possible recession signals (Estrella and Hardouvelis, 1991) and on optimal portfolio strategies, therefore many different literatures have been investigating their determinants and implications.

Empirical evidence supports the role of the real component of the term structure during inversions. Data on US TIPS (i.e., inflation-protected securities) and nominal bonds from Gurkaynak, Sack and Wright (2007, 2010) suggest that the real term spread fluctuates substantially over time, and that it has inverted during the last 10 years; moreover, fluctuations in the real component contribute significantly to the volatility of nominal yields at both short and long maturities. Still, there is no widely accepted theory explaining the basic mechanics of an inversion.

We propose a parsimonious consumption-based model of the term structure of interest rates that allows the inversion of the real component. We build on the classic frameworks of Campbell and Cochrane (1999) and Wachter (2006), which have been successful in reproducing a wide variety of asset pricing phenomena such as the procyclicality of stock prices, the size of equity premia and the long-run predictability of excess returns, among others. In these models, a representative agent has consumption preferences with respect to a habit level, and variations in the surplus over habit drive both the desire to smooth consumption over time and to accumulate precautionary savings, the latter depending on changes in risk aversion. These two forces have opposite effects on the implied equilibrium risk-free rate, and, potentially, on the slope of the real term structure via both the level and volatility of consumption growth. Assuming log-normal consumption growth, Campbell and Cochrane (1999) offset them to produce a constant risk free rate, while Wachter (2006) makes consumption smoothing motive always prevail such that reasonable estimates of consumption growth volatility do not allow the implied yield curve to invert.

Our framework features time-varying volatility of consumption growth and learning. Consumption growth is a Markov switching process in which unobservable volatility switches between two regimes; agents update risk perception only gradually, and, in equilibrium, real interest rates depend not only on a series of shocks to consumption growth, but also on expected volatility. The perceived macroeconomic risk can be high such that the precautionary saving motive prevails, with saving propensity shifting from the short to the long-run. In terms of bond pricing, a high perceived rollover risk makes investors incline to lock-in bond portfolios, allowing equilibrium prices of long-term bonds to be higher with respect to short-term ones, i.e. an inversion of the yield curve.

Our model is mainly inspired by three studies. The key feature of consumption growth volatility being unobservable and time-varying is taken from Boguth and Kuehn (2013), who explored the connection between macroeconomic uncertainty and asset prices finding consumption growth volatility predicting returns for risk-exposed firms; the emphasis on long- vs. short-run risk is in the spirit of Bansal and Yaron (2004), that propose plausible solutions to asset pricing puzzles based on a persistent component in expected growth and on fluctuating uncertainty; lastly, our point of the importance of expected volatility in the long-run with respect to that in the short-run is in line with the intuition that the entire volatility term structure is relevant in the pricing of the yield curve (Breeden, Litzenberger and Jia, 2015). The latter paper notes that, in 2005-2006, the US yield curve inverted amid no expected decline in growth and, at the same time, the term structure of volatilites (proxied by the 2-year/3-month spread of S&P 500 implied volatilities) was substantially positive indicating risk tilted to the long-run; the authors claim that an upward-sloping volatility term structure can help explain cases of a downward sloping yield curve that are not necessarily related to expected declines in growth.

The heteroskedasticity of consumption growth has been first documented by Ferson and Merrick (1987), Whitelaw (1990), and Bekaert and Liu (2004); more generally, economic uncertainty has been introduced in the analysis of stock prices and risk premia by Kandel and Stambaugh (1990), among others. Our work is in the spirit of Bekaert, Engstrom and Xing (2009), who explicitly model the stand-alone importance of investor's uncertainty about fundamentals with respect to risk aversion in a five-factor asset pricing framework. From a different perspective, Kurmann and Otrok (2013) analyze movements in the slope of the term structure in the sample 1959-2005, claiming that news about future total factor productivity (TFP) are the main factors behind the inversion of the curve; as suggested by the authors, time-varying consumption growth volatility is "an additional important ingredient in the long-run risk story", and that the investigation of the correspondence between TFP news shocks and volatility shocks is a "promising avenue of future research".

This chapter is organized as follows. Section 1.2 describes the benchmark model and lays out some empirical findings on the relation between real rates and consumption. Section 1.3 presents the model of the real short rate with regime switches in the volatility of the surplus-consumption ratio and explains the mechanics of the inversion of the real and nominal term structures. Section 1.4 describes the empirical analysis and Section 1.5 concludes.

1.2 Benchmark model and stylized facts

Throughout this chapter, we explain the main arguments that motivate our research. First, we describe the features of the model proposed by Campbell and Cochrane (1999) (CC henceforth) that we take as a benchmark, focusing on the the equilibrium risk-free rate; then. we make the point of the instability of the relationship between real short rates and consumption.

1.2.1 Benchmark model

Representative agents have preferences over consumption with respect to a slow-moving reference level X_t, that is an exogenous habit level:

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma} \tag{1.1}$$

where β is the subjective time discount factor and γ the utility curvature. The surplus-consumption ratio is defined as the excess consumption over the consumption level C_t:

$$S_t = \frac{C_t - X_t}{C_t} \tag{1.2}$$

Consumers' relative risk aversion is time-varying and countercyclical:

$$\xi_t = \frac{\gamma}{S_t} \tag{1.3}$$

Assuming a lognormal i.i.d. consumption growth, the lognormal stochastic discount factor allows to derive the equilibrium risk-free rate in closed form. Denoting with $\{g, \sigma, \psi\}$ mean consumption growth, standard deviation of consumption growth and habit persistence, and being $\bar{S}$ the average level (i.e., steady state value) of surplus-consumption ratio, $s_t = \log(S_t)$ and $\bar{s} = \log(\bar{S})$, one can prove that the real one-period risk-free rate is proportional to deviations of s_t from $\bar{s}$:

$$r_{t,t+1} = \bar{r} - b(s_t - \bar{s}) \tag{1.4}$$

where

$$\bar{r} = -\ln\delta + \gamma g - \frac{\gamma^2\sigma^2}{2\bar{S}^2} \tag{1.5}$$

and

$$b = \gamma(1-\psi) - \frac{\gamma^2\sigma^2}{\bar{S}^2} \tag{1.6}$$

Substituting Equations 1.5 and 1.6 into 1.4 we get

$$r_{t,t+1} = -\ln\delta + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[1 + \lambda(s_t)\right]^2 \tag{1.7}$$

For a more detailed description of the benchmark model, see Appendix A.1. Importantly, being $\{\beta, \gamma, g, \sigma, \psi\}$ all constant parameters, it follows that b is constant over time. The sign of the latter is crucial in order to determine the response of the real rate to surplus consumption. If $b > 0$, then the intertemporal substitution effect dominates: in good times (i.e., high surplus consumption over habits), agents' willingness to save to smooth consumption over time drives the equilibrium risk-free rate down. On the contrary, if $b < 0$, then the precautionary saving effect dominates: in good times, a less-risk averse agent wants to borrow to consume more today driving up the equilibrium interest rate.[1]

In CC's framework, b is 0 to completely offset these two effects. Instead, Wachter (2006) parameterizes b as a positive constant, so that the inter-temporal substitution effect always wins out: positive consumption shocks increasing the surplus drive the equilibrium risk-free rate down. Note that the term b determines not only the level, but also the slope of the equilibrium term structure of risk free rates: if $b > 0$, then the dominance of the intertemporal substitution motive is such that, in bad times, agents value consumption today more than consumption tomorrow and the equilibrium term structure is always upward sloping.

In the next Section we empirically test the importance of consumers' expectations and uncertainty in determining the level of interest rates; we now complete a preliminary analysis by having a closer look at the relationship between s_t and r_t.

1.2.2 Real rates and surplus consumption

We have previously shown that, in standard consumption-based models featuring habit, the equilibrium real risk-free rate is either constant or a negative function of the surplus-consumption ratio. Assuming Government bond rates in the United States as risk free, we investigate this issue empirically by comparing the historical dynamics of the real rate to that of the surplus-consumption ratio. Real rates – that cannot be proxied by TIPS in this analysis due to data availability – are estimated as the difference between the 3-month T-Bill rate and 3-month expected inflation, with the latter proxied by inflation forecasts made from an estimated autoregressive process (see Appendix A.2 for details); the surplus-consumption ratio is instead constructed as the weighted average of past consumption growth with decreasing weights, as in Wachter (2006).[2] Figure 1.2 displays the two series on a quarterly frequency from 1962 to 2014.

A quick graphical inspection suggests that the co-movement between the two is not stable over time: correlation seems positive between late 60's and late 70's, then negative during the 80's and 90's, unclear on the rest of the sample. To analyze this relationship more formally, we estimate a time-varying b by making rolling regressions of the real 3-month rate on a constant and on our surplus-consumption proxy on 10-year windows. The equation is

$$r_{t,t+1} = a_t + b_t \sum_{j=1}^{40} \phi^j \Delta c_{t-j} + \epsilon_{t+1} \tag{1.8}$$

The estimated coefficients $\hat{a}_t$ and $\hat{b}_t$ are displayed in Figure 1.3.

Two things are worth to be mentioned: first of all, both the slope and the intercept exhibit large time variations, ranging from negative to positive values; secondly, the two rolling estimates are strongly

[1] In bad times, on the contrary, the consumption smoothing propensity drives the equilibrium interest rate up, while precautionary saving motive drives it down.

[2] While surplus-consumption is theoretically influenced by all its own past values, we choose 40 quarters as the cut-off point.

negatively correlated: a high positive intercept is coupled with a highly negative load on surplus consumption. This entails two thoughts: (a) real rates depends positively by the surplus-consumption in some part of the sample, negatively in some others; (b) a specific, time-varying, component seems to be embedded in both coefficients with opposite signs.

1.3 Model

Throughout this chapter we explain our entire framework. First, we introduce a Markov switching process for the consumption growth and derive the new stochastic discount factor (Subsection 1.3.1); second, we discuss the behaviour of the equilibrium risk-free rate and the equilibrium term structure (Subsection 1.3.2); third, we include inflation to explain the implication of the model for the nominal yield curve (Subsection 1.3.3).

1.3.1 Markov switching consumption growth and equilibrium risk-free rate

We adopt the same set of preferences as CC and keep the same notation throughout the Section. We assume that, instead of being lognormal, consumption growth is a Markov switching process, in which volatility switches between two regimes.[3] Denoting with g the non-switching drift, the process of log consumption growth Δc_{t+1} is

$$\Delta c_{t+1} = g + \sigma_{\zeta_{t+1}} \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0,1) \tag{1.9}$$

with σ_{ζ_t} being either σ_h (high) or σ_l (low), with $\sigma_h > \sigma_l$. Volatility is unobservable, depending on a latent variable ζ_t indicating the state of the economy. Agents infer the state of the economy from observable consumption data. Denote by $\mathbf{P}$ the transition probability of being in state $j = h, l$ coming from state $i = h, l$

$$\mathbf{P} = \begin{bmatrix} p_{hh} & p_{hl} \\ p_{lh} & p_{ll} \end{bmatrix}, \tag{1.10}$$

which is given and known to the agents at each point in time; new incoming information updates the likelihood of each state

$$\eta_t = \begin{bmatrix} f(\Delta c_t | s_t = 1, \mathbf{X}_{t-1}) \\ f(\Delta c_t | s_t = 2, \mathbf{X}_{t-1}) \end{bmatrix},$$

where $\mathbf{X}_{t-1}$ represents all information at time $t-1$. Then, updated likelihoods and transition probabilities are used to form the *posterior probability* of being in each state based on the available data: call $\xi_{t|t-1} \in \mathbb{R}^2$ the posterior belief vector at time $t-1$, Bayes' Law implies that

$$\xi_{t+1|t} = \mathbf{P}' \frac{\xi_{t|t-1} \odot \eta_t}{\mathbf{1}'(\xi_{t|t-1} \odot \eta_t)}$$

where $\odot$ denotes element-by-element product and $\mathbf{1}$ is a 2-by-1 vector of ones.

[3]Given that the trade off between intertemporal substitution and precautionary saving does not depend on the drift of consumption growth, to keep the model as parsimonious as possible we do not impose latent states for it.

As consumption growth, autoregressive surplus consumption is also Markov switching:

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)\sigma_{\zeta_{t+1}}\epsilon_{t+1} \tag{1.11}$$

where ϕ is the AR coefficient. $\lambda(s_t)$, called *sensitivity function*, is a negative function of s_t: the higher the surplus consumption, the lower the sensitivity of s to innovations in consumption growth; moreover, $\lambda(s_t)$ is inversely proportional to the long run steady state level $\bar{S}$.

The stochastic discount factor (SDF) is a function of the surplus consumption:

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\frac{S_{t+1}}{S_t}\right)^{-\gamma} = \delta\exp\left\{-\gamma[g + (1-\phi)(\bar{s}-s_t) + (\lambda(s_t)+1)\sigma_{\zeta_{t+1}}\epsilon_{t+1}]\right\} \tag{1.12}$$

Solving for the equilibrium risk-free rate involves the computation of the expectation of the SDF as a function of the two stochastic components of s_t, i.e. $\{\epsilon,\zeta\}$. After some algebra, we get

$$r_{t+1} = \ln\frac{1}{E_t^{(\epsilon,\zeta)}(M_{t+1})} = -\ln\delta + \gamma g - \gamma(1-\phi)(s_t-\bar{s}) - \ln E_t^{(\epsilon,\zeta)}\left(e^{-\gamma[\lambda(s_t)+1]\sigma_{\zeta_{t+1}}\epsilon_{t+1}}\right) \tag{1.13}$$

where the last term on the right hand side is

$$-\ln E_t^{(\epsilon,\zeta)}\left(e^{-\gamma[\lambda(s_t)+1]\sigma_{\zeta_{t+1}}\epsilon_{t+1}}\right) = -\ln\sum_{j\in\{h,l\}}\xi_{t+1|t}(j)E_t^{(\epsilon)}\left(e^{-\gamma[\lambda(s_t)+1]\sigma_j\epsilon_{t+1}}|\sigma_{\zeta_{t+1}}=\sigma_j,\xi_{t+1|t}\right) \tag{1.14}$$

Equation 1.14 tells that, in a Markov switching world, agents have expectations about the future states – that can be characterized by high or low volatility – and weight them by the posterior probability (i.e., the belief they have at time t) that such state will be a high or a low volatility state. We interpret it as a *precationary saving effect*, provided that Equation 1.13 differs from 1.7 only for that. In the extreme cases in which $\xi_{t+1|t}(\sigma_h) = 0$ or $\xi_{t+1|t}(\sigma_h) = 1$, the formula for the equilibrium risk free rate collapses to CC's one.

The key result of our model is that the intensity of the precautionary saving effect depends not only on the current state, but also on agent's beliefs and, precisely, on the posterior probability attached to the two states. Assume that σ_l is low enough to let the intertemporal substitution effect dominate on precautionary saving, and let σ_h high enough to allow the opposite. Provided that $\xi_{t+1|t}$ weights the two conditional expectations, a high $\xi_{t+1|t}(\sigma_h)$ can made the σ_h scenario dominate: in that case, the precautionary saving term overcomes the intertemporal substitution.

To summarize, the equilibrium one-period interest rate depends on the combination of the current state and beliefs over next period. Indeed, states in which s_t is high might no longer be perceived as good states if σ is also expected to be high: taken s_t as given, when $\xi_{t+1|t}(\sigma_h)$ is higher than $\xi_{t+1|t}(\sigma_l)$, the equilibrium risk-free rate is driven up. Therefore, the combination of high s_t and low $\xi_{t+1|t}(\sigma_h)$ defines good states, while bad states are those with low s_t and high $\xi_{t+1|t}(\sigma_h)$. $\xi_{t+1|t}$ evolves based on the updated likelihood of the two states. Intuitively, agents follow a learning process: a sequence of large shocks to consumption growth slowly induce agents to weight more the high volatility state, while a sequence of small shocks slowly push them towards the low volatility state.

By introducing Markov switching consumption growth, we make the trade-off between intertemporal substitution and precautionary saving motives endogenous. The flexibility of this specification allows to match the fact that the correlation between real short rates and surplus consumption is time-varying, and provides a rationale for the periods of positive correlations that appear from the empirical estimation of Equation 1.8.

1.3.2 The term structure of real risk-free rates

In the previous subsection, we have highlighted the key features underlying this model: time-varying posterior beliefs allow both the inter-temporal and precautionary saving motives to dominate in different times, making the correlation of r_t with s_t also time-varying. Let's now turn to the pricing of real risk-free bonds with maturities beyond one period to infer the behaviour of the entire term structure of interest rates.

The price at time t of a real bond maturing after n periods ($P_{n,t}$) is computed as the expectation of the future compounded SDFs until maturity. From the Euler equation:

$$\begin{aligned} P_{n,t} &= E_t\left[M_{t+1}P_{n-1,t+1}\right] \\ &= E_t[e^{\ln\delta-\gamma g+\gamma(1-\phi)(s_t-\bar{s})-\gamma[\lambda(s_t)+1]\sigma_{\zeta_{t+1}}\epsilon_{t+1}}P_{n-1,t+1} \\ &= \sum_{j\in\{h,l\}}\xi_{t+1|t}(j)E_t[e^{\ln\delta-\gamma g+\gamma(1-\phi)(s_t-\bar{s})-\gamma[\lambda(s_t)+1]\sigma_j\epsilon_{t+1}}P_{n-1,t+1}|\sigma_{\zeta_{t+1}}=\sigma_j,\xi_{t+1|t}] \end{aligned} \tag{1.15}$$

with boundary condition $P_{0,t}=1$; the yield-to-maturity is

$$y_{n,t} = -\frac{1}{n}\ln P_{n,t} \tag{1.16}$$

As described in Equation 1.15, the real bond price is obtained by iterating forward one-period expectations of the bond price for n periods. While future states of the economy are not known at time t, agents can only make expectations conditional on the available information at time t. In order to account for all possible future states for both ϵ and the posterior beliefs ξ for n periods, the bond price is solved numerically on a grid.

As explained in the previous Section, if we assume σ_h to be high enough to let the precautionary saving effect dominate, cases in which the posterior beliefs are biased towards σ_h are such that this scenario applies. In those cases, the precautionary saving motive implies agents' willingness to save long-term, because they know that high volatility states have a limited duration and eventually the volatility will go back to the low level: in this case, the "term structure of agents' beliefs" is downward sloping. In terms of bond pricing, a high perceived rollover risk makes investors incline to lock-in bond portfolios, allowing equilibrium prices of long-term bonds to be higher with respect to short-term ones, i.e. an inversion of the yield curve.

1.3.3 Nominal yield curve

Denote by $\pi_t = \ln\Pi_t$ the natural logarithm of the price level and introduce inflation $\Delta\pi_t$ as a first order autoregressive, exogenous state process (AR(1)) following Cox, Ingersoll and Ross (1985) and Bekaert, Engstrom and Grenadier (2004):

$$\Delta\pi_{t+1} = \eta_0 + \psi_0 \Delta\pi_t + \sigma_{\Delta\pi} v_{t+1} \tag{1.17}$$

Denote also by ρ the linear correlation between v_{t+1} and ϵ_{t+1} (i.e., the innovation in consumption growth). The nominal bond price is equal to the expected discounted nominal payoff.:

$$P^{\$}_{n,t} = E_t\left[M^{\$}_{t+1} P^{\$}_{n-1,t+1}\right] = F^{\$}_n(s_t) e^{A_n + B_n \Delta\pi_t} \tag{1.18}$$

with

$$\begin{aligned} F^{\$}_n(s_t) &= E_t[e^{\rho(B_{n-1}-1)\sigma_{\Delta\pi}\epsilon_{t+1}} M_{t+1} F^{\$}_{n-1}(s_{t+1})] \\ A_n &= A_{n-1} + (B_{n-1}-1)\eta_0 + \frac{1}{2}(B_{n-1}-1)^2 \sigma^2_{\Delta\pi}(1-\rho^2) \\ B_n &= (B_{n-1}-1)\psi_0 \end{aligned}$$

The SDF of the nominal security ($M^{\$}$) is the ratio between the SDF of the real bond and the one-period gross inflation:

$$M^{\$}_{t+1} = e^{-\Delta\pi_{t+1}} M_{t+1} \tag{1.19}$$

After some algebra, the nominal bond price becomes

$$P^{\$}_{n,t} = const * \sum_{j\in\{h,l\}} \xi_{t+1|t}(j) E^{(\epsilon)}_t \left[M_{t+1} e^{\rho(B_{n-1}-1)\sigma_{\Delta\pi}\epsilon_{t+1}} F^{\$}_{n-1,t+1} | \sigma_{\zeta_{t+1}} = \sigma_j, \xi_{t+1|t}\right] \tag{1.20}$$

with

$$const = e^{A_{n-1} + (B_{n-1}-1)(\eta_0 + \psi_0\Delta\pi_t) + 0.5(B_{n-1}-1)^2\sigma^2_{\Delta\pi}(1-\rho^2)}$$

and

$$M_{t+1} = e^{\ln\delta - \gamma g + \gamma(1-\phi)(s_t - \bar{s}) - \gamma[\lambda(s_t)+1]\sigma_{\zeta_{t+1}}\epsilon_{t+1}}$$

Appendix A.3 reports the proof of the nominal bond pricing formula; note that, assuming correlated innovations of the two state processes, the expected value in Equation 1.20 can be expressed as a function of ϵ only. The yield-to-maturity of the nominal bond is

$$y^{\$}_{n,t} = -\frac{1}{n}\ln P^{\$}_{n,t} \tag{1.21}$$

The nominal bond price has two additional components with respect to the real bond price: a scale factor that depends on inflation volatility (in $const$) and an extra term in the expectation part of Equation 1.20, i.e. $\exp\{\rho(B_{n-1}-1)\sigma_{\Delta\pi}\epsilon_{t+1}\}$. The extra term is key to get the intuition for the role of inflation. This term is a positive function of the product between ρ, ψ_0 (through B) and $\sigma_{\Delta\pi}$. If ρ is negative, as reflecting the existing negative correlation between consumption growth and inflation, the extra term adds to the precautionary saving effect in its impact on the level and the slope of the term structure. Indeed, the agents' willingness to make precautionary savings now depends not only on beliefs of the future consumption volatility states, but also on inflation volatility: the higher the volatility of inflation, the higher the need for precautionary savings.

With respect to the real term structure, we do not need posterior beliefs that are as biased towards σ_h to have precautionary savings prevail; if $\sigma_{\Delta\pi}$ is sufficiently high, the nominal yield curve can invert even though posterior beliefs are such that the real one is upward sloping.

We also compute the nominal risk premium up to a constant term, which once again depends on surplus consumption and agents' posterior probabilities:

$$\begin{aligned} E_t\left(r^{\$}_{n,t+1} - r^{\$}_{1,t+1}\right) &= const + E_t\Big(\ln F^{\$}_{n-1}(s_{t+1})\Big) - \ln F^{\$}_{n}(s_t) - \\ &- \gamma(1-\phi)(\bar{s}-s_t) + \ln \sum_{j\in\{h,l\}} \xi_{t+1|t}(j) e^{\frac{1}{2}(-\gamma[\lambda(s_t)+1]\sigma_j - \rho\sigma_{\Delta\pi})^2} \end{aligned} \tag{1.22}$$

Proof is in Appendix A.4.

1.4 Empirical analysis

This Section covers the application of the model described in Section 1.3 to US consumption and inflation data. The estimation of the parameters of the Markov switching process is carried out in Subsection 1.4.1. Then, we solve the model and discuss the behaviour of the slope of the term structure in Subsection 1.4.2. Finally we simulate from the model and report descriptive statistics in in Subsection 1.4.3.

1.4.1 Parameter estimation

We estimate the parameters of the Markov switching model by maximum likelihood. Real per capita consumption expenditures on nondurable goods and services are taken from the US Bureau of Economic Analysis. Following Yogo (2006), we restrict our sample to post 1952 data to avoid the exceptionally high consumption growth that followed World War II. Results are reported in Table 1.3; sample data are from 1952Q1 to 2016Q3.

Average consumption growth is estimated at 0.49 per cent per quarter, while volatility equals 0.22 per cent in the low state and 0.56 per cent in the high state (i.e., the latter is 2.5 times bigger than the previous). The low volatility state is slightly less persistent: the probability that high consumption growth volatility will persist next period is 0.93, while for the low volatility state such probability is 0.88. Consumption growth and posterior probabilities are depicted in Figure 1.4.

Data on the monthly CPI index are taken from the Bureau of Labor Statistics database; inflation is constructed as quarter-on-quarter log returns, where quarterly CPI are values of the last month of the quarter. Estimates of the three parameters of the AR(1) process for inflation are reported in the bottom panel of Table 1.3. The long-term mean of the autoregressive process is 0.85 per cent, and inflation volatility is 0.82 per cent, higher than the volatility of consumption growth in high state. The correlation with consumption growth is estimated to be equal to -0.11.

1.4.2 Model solution

The pricing of nominal and real bonds is obtained from the Euler equations; for numerical computations, the series method of Wachter (2005) is preferred to the fixed-point method of CC. Bond prices are computed numerically on a quadratic grid including combinations of a grid for s_t and a grid for $\xi_{t+1|t}$. Figure 1.5 plots the short term real rate and the implied real yield spread with respect to a grid of values for the surplus-consumption ratio given the posterior probabilities $\xi_{t+1|t} = (1, 0)$ (left panels) and $\xi_{t+1|t} = (0, 1)$ (right panels).

Figure 1.5 shows that a time-varying consumption growth volatility in habit models is sufficient to allow the real term structure to invert.

If agents expect a low volatility state with probability one, the short-term real yield is a decreasing function of the surplus-consumption ratio so the model can accommodate countercyclical real short-term rates (upper left panel); moreover, the equilibrium real term structure is inverted for values of S_t below a certain threshold (lower left panel). If the agent instead thinks that in the short-term the volatility of consumption growth is going to be high, precautionary saving is always prevailing on intertemporal substitution: the short-term real yield is procyclical (upper right panel) and the real term structure is inverted for all possible S_t.

Figure 1.6 shows how the short- and long-term real yields change as a function of the posterior probability to be in the low volatility state ($P(\sigma = \sigma_l)$ given a low S_t (left panel) or a high S_t (right panel). Both short- and long-term real yields are increasing with the probability of a low volatility state. The term structure is inverted when the agent is confident that next period's volatility will be either high or low ($P(\sigma = \sigma_l) = 0$ or $P(\sigma = \sigma_l) = 1$), or if he thinks that high volatility will be more likely (i.e., $P(\sigma = \sigma_l) < 0.5$)); with high S (right panel) the real yield curve is inverted only if the probability of high volatility in the short term is higher than a certain threshold.

Figure 1.7 depicts the short-term nominal and real yields as a function of S_t when the agent expects low volatility state (left panel) or high volatility state (right panel) with probability one. Note that nominal yields are always above real yields due to the effect of expected inflation.

We now focus on the nominal curve, studying its sensitivity to different calibrations of the long-term mean of the inflation process. Figure 1.8 displays 3-month and 5-year nominal yields for different levels of expected inflation when the agent expects a low volatility state (lower panels) or high volatility states (upper panels). We consider expected inflation equal to its long-run mean (0.85 per cent, middle panels), and to plus and minus two unconditional standard deviations (right and left panels, respectively).

The equilibrium nominal yield curve is very sensitive to changes in expected inflation. If the agent expects low volatility (lower panels), the higher the long-term inflation expectations, the larger the level of the surplus consumption that yields an inverted nominal curve: for the case of a long-run mean of 0.85 per cent, the yield curve inverts for values of the S_t grid below 0.3 (lower middle panel); for the extreme cases of negative or highly positive long-term inflation expectations, the term structure inverts for lower or higher values of the surplus consumption, respectively (lower left and lower right panel). In other words, it takes a higher surplus consumption for the agents to feel in a good state.

Provided that inflation expectations are mean reverting, variations in short-term yields are the main responsible for the inversion. This is coherent with the mechanics explained, in a different setup, by

Kurmann and Otrok (2013). If instead the agent expects high volatility states (top panels), the nominal yield curve is inverted for almost every values of the surplus consumption ratio; moreover, the higher the long-term inflation expectations, the larger the gap between long- and short-term yields (top panels, from left to right). This suggests expected inflation is an important driver of the inversion of the nominal term structure, which is allowed to invert even when the real term structure does not.

1.4.3 Simulation

In order to replicate the path of interest rates observed in the US market during the sample period, we simulate 100,000 observations of quarterly consumption growth and inflation. The model is calibrated using the parameters in Table 1.3 and Table 1.1. Mean and standard deviations of 3-month, 1-year, 3-year and 5-year zero yields are reported in Table 1.2.

Model-implied values are very close, on average, to the observed ones (the largest difference is around 30 basis points, in absolute value). The mean of 3-month estimated nominal yields is 5.10 per cent, while the observed ones are on average 4.80 per cent; 5-year implied and observed nominal yields are equal to 5.89 and 5.91 per cent, respectively. The average positive slope of the time series is therefore matched. Real yields are much smaller than nominal ones, meaning that the inflation component is, on average, quite sizable. Simulated yields, both real and nominal, are less volatile than the market rates.

1.5 Conclusion

In this chapter, we propose a consumption-based asset pricing model that allows not only the nominal, but also the real term structure of interest rates to invert. The main ingredients are time-varying volatility and the learning behaviour of agents, both implied in the Markov switching model of consumption growth. Agents form posterior beliefs over future states of the economy. The perceived short-term macroeconomic risk can be so high that, in the trade-off between making intertemporal consumption smoothing and precautionary saving, the latter prevails, with saving propensity shifting from the short to the long-run. In terms of bond pricing, a high perceived rollover risk makes investors incline to lock-in bond portfolios, allowing equilibrium prices of long-term bonds to be higher with respect to short-term ones

The estimated stochastic discount factor could, in principle, be used to price other type of assets. The impact of macroeconomic risk on equity pricing is investigated by Lettau, Ludvigson and Wachter (2008) among others. The application on corporate bond pricing or derivative pricing can be an avenue of future research. This model is designed for default-free economies: another interesting avenue of research could be that of investigating the evolution of a bond term structure containing a risk premium related to the default of the bond's issuer. Equilibrium yield curves of different countries with different default risks could in this way be compared.

Parameters	Value
Utility Curvature γ	2.00
Habit persistence ϕ	0.97
Derived Parameters	
Discount rate δ	0.98
Long-run mean of log surplus consumption $\bar{s}$	-3.25
Maximum value of log surplus consumption s_{max}	-2.75

Table 1.1: Assumptions on the parameters of the investor's utility function

Maturity	Mean			St. Dev.		
	Real	Nominal	Data	Real	Nominal	Data
1	1.60	5.10	4.80	1.43	2.20	3.15
4	1.71	5.23	5.21	1.46	1.68	3.28
8	1.86	5.39	5.44	1.51	1.61	3.20
12	2.02	5.56	5.62	1.56	1.64	3.11
20	2.35	5.89	5.91	1.67	1.73	2.94

Table 1.2: Means and standard deviations of continuously compounded zero-coupon bond yields in the model and in the data

3-month, 1-year, 3-year and 5-year implied yields are compared with data from 1952Q1 to 2016Q3.

Δc	μ	σ_l	σ_h	p_{ll}	p_{hh}
	0.491	0.223	0.556	0.884	0.930
	(0.029)	(0.014)	(0.045)	(0.280)	(0.284)

$\Delta\pi$	η_0	η_1	σ_π
	0.265	0.696	0.573
	(0.058)	(0.036)	(0.035)

Table 1.3: Parameter estimates of the consumption growth and inflation processes.

Values are in percentage points. Non-annualized quarterly growth rates of consumption are computed using data on real consumption expenditures on nondurable goods and services taken from the US Bureau of Economic Analysis; inflation is constructed as quarter-on-quarter log returns, where quarterly CPIs are values of the last month of the quarter. CPI data are from the Bureau of Labor Statistics.

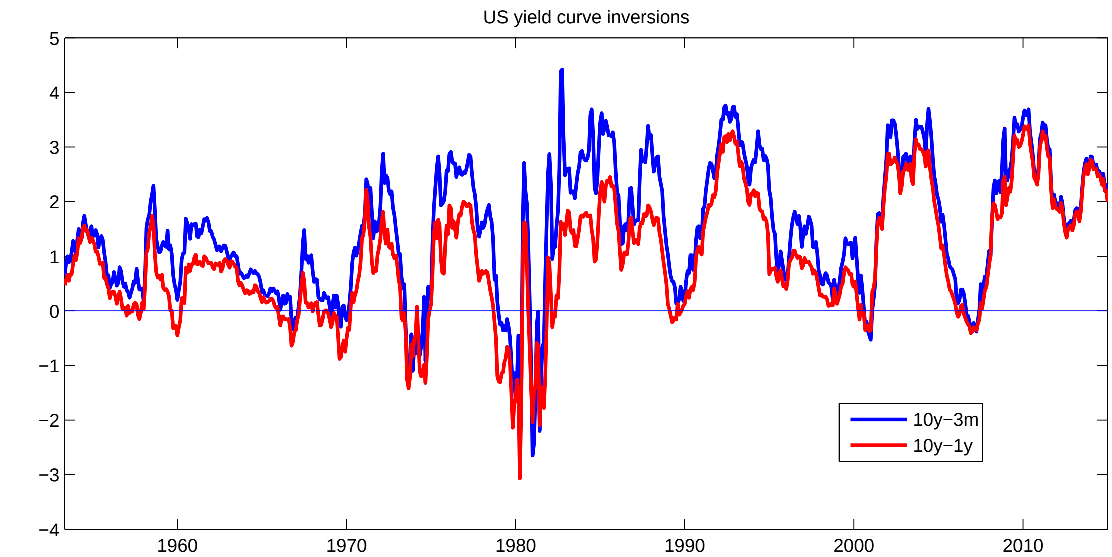

Figure 1.1: Two measures of the slope of the US Government yield curve.

10-year minus 3-month rates (blue line) and 10-year minus 1-year yields (red line).

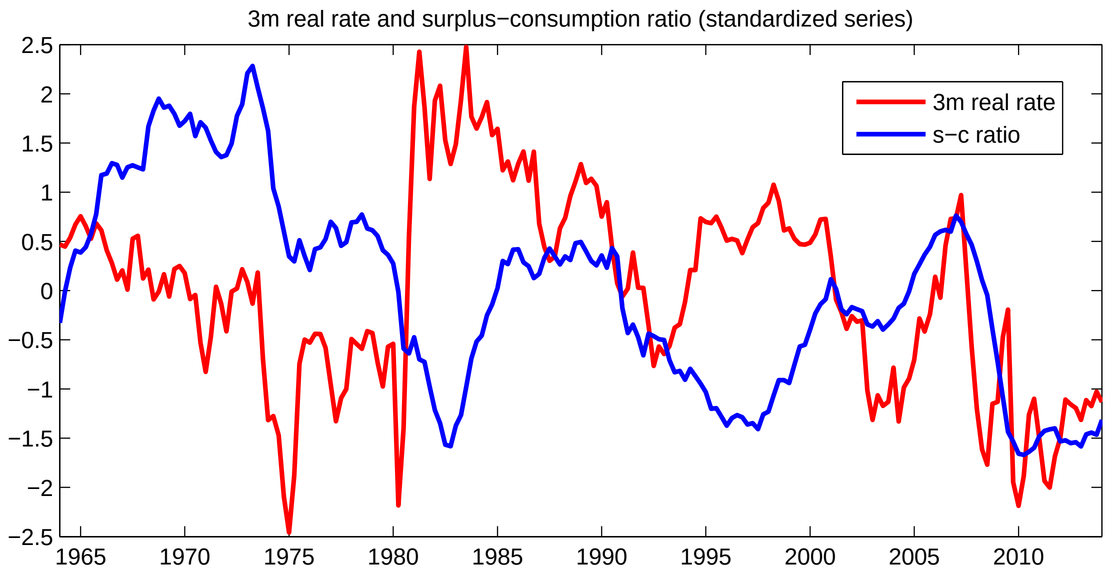

Figure 1.2: Real 3-month rate and surplus-consumption ratio.

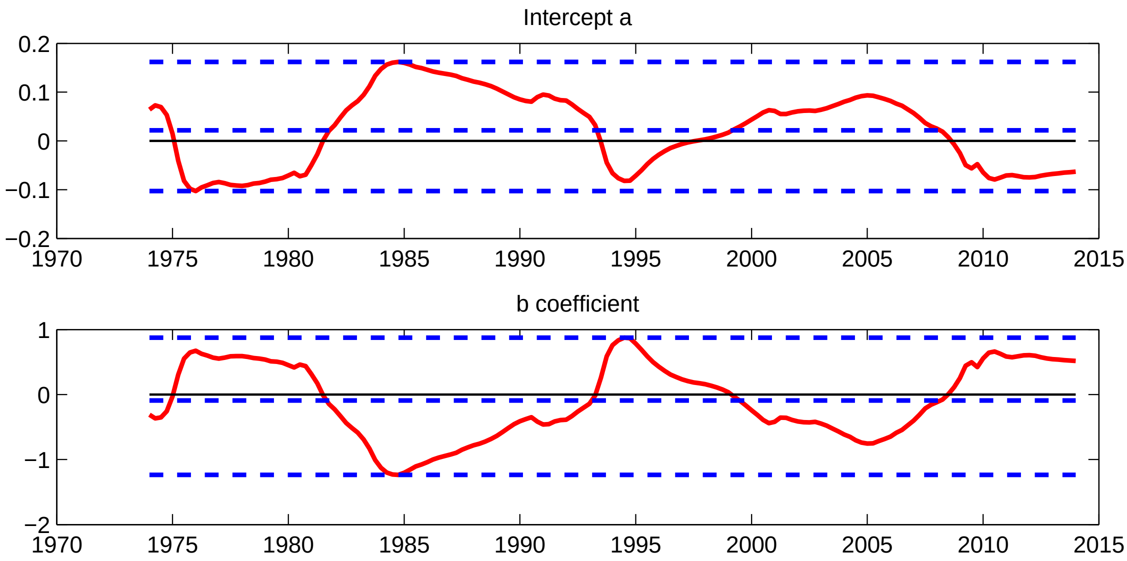

Figure 1.3: Rolling OLS estimate of a and b.

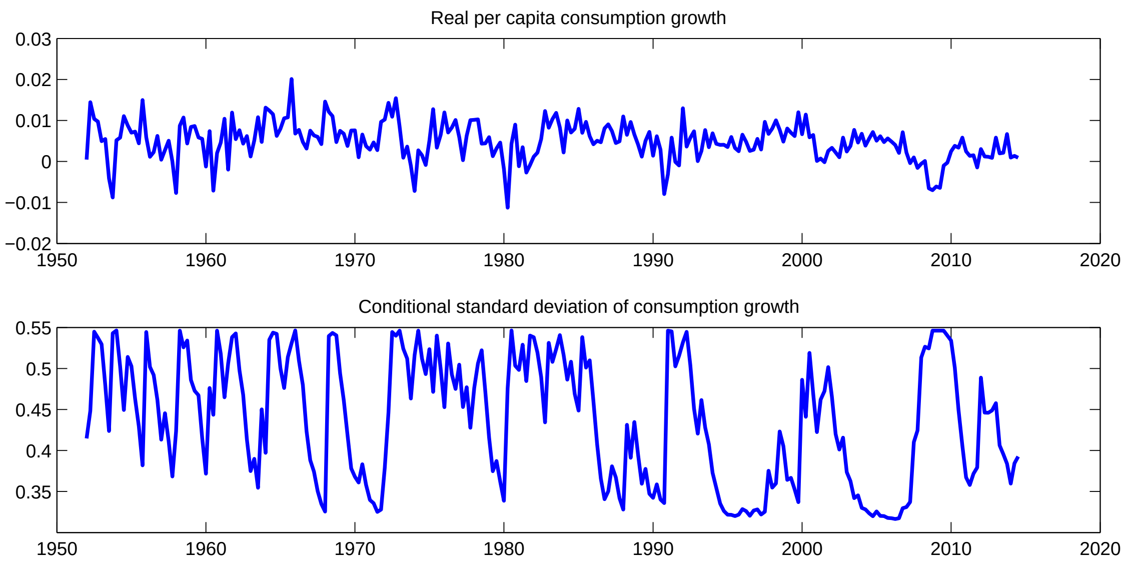

Figure 1.4: Output of the Markov switching estimate.

Top panel: real per capita consumption growth. Bottom panel: expected volatility of consumption growth.

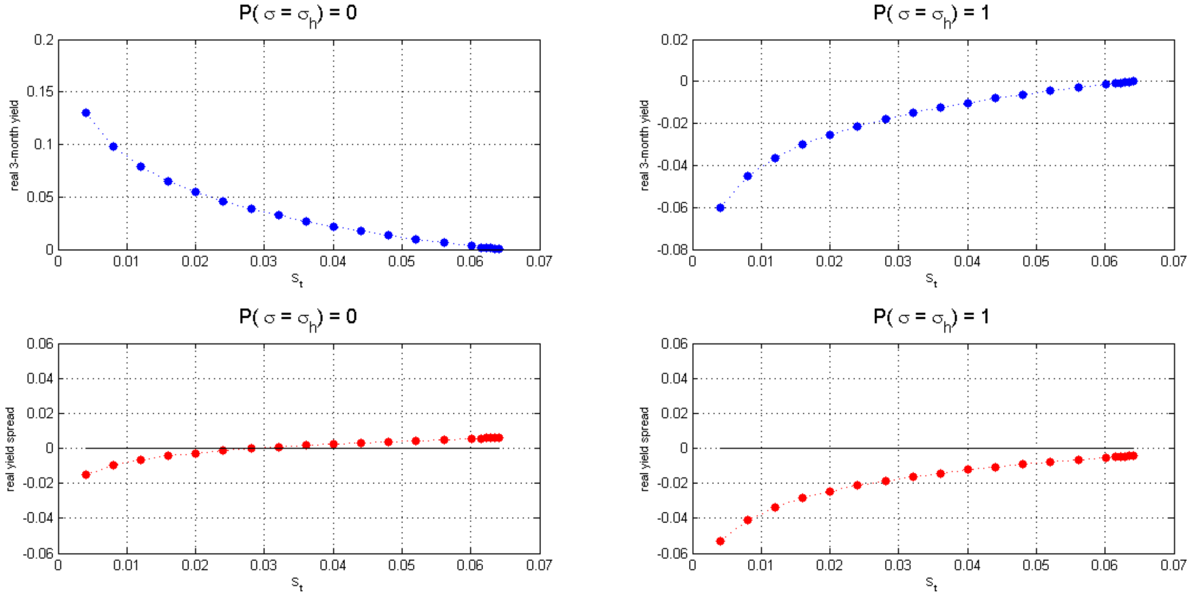

Figure 1.5: Real bond yields vs. surplus consumption when the expected state is low or high.

Continuously compounded yields on real bonds as a function of the surplus-consumption ratio implied by the posterior probabilities $P(\sigma = \sigma_h) = 0$ (left panels) and $P(\sigma = \sigma_h) = 1$ (right panels) and the parameters in Table 1.3 and Table 1.1.

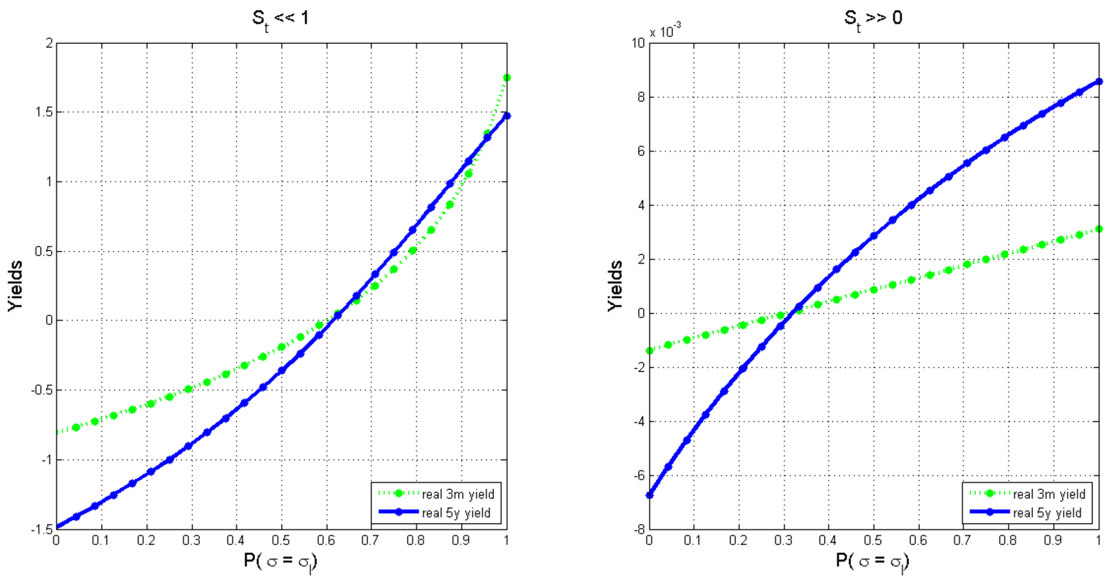

Figure 1.6: Real bond yields vs. posterior probabilities in case of low or high surplus.

Continuously compounded yields on real bonds as a function of the posterior probability to be in the low volatility state implied by a low S_t (left panel) and a high S_t (right panel) and the parameters in Table 1.3 and Table 1.1. The solide blue line represents the 5y yields; the dashed green line denotes 3m yields.

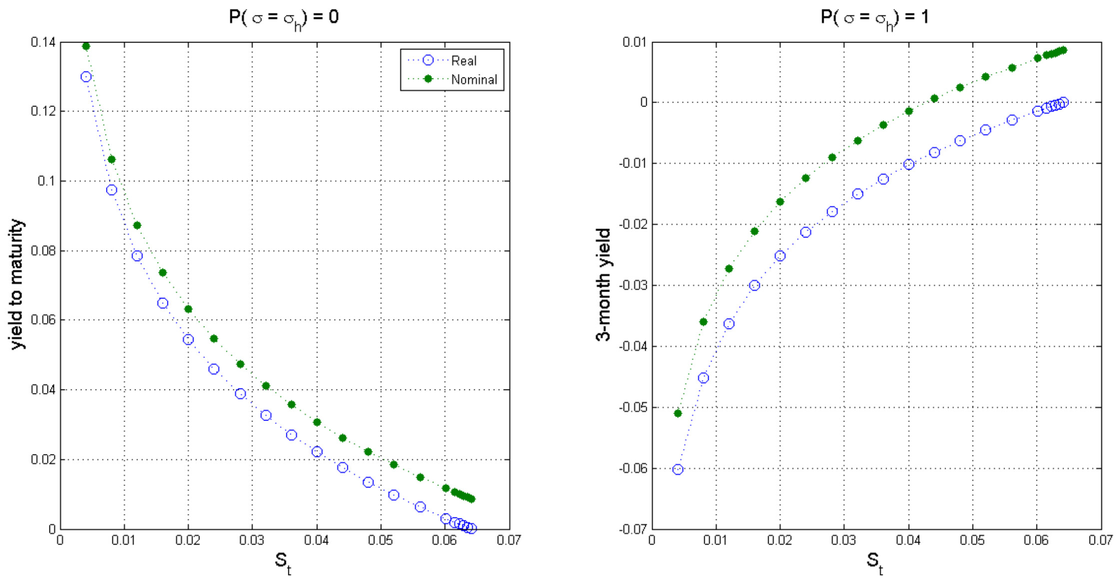

Figure 1.7: Real and nominal yields vs. surplus when the expected state is low or high.

Continuously compounded short-term yields on real and nominal bonds as a function of the surplus-consumption ratio implied by the posterior probabilities $P(\sigma = \sigma_h) = 0$ (left panel) and $P(\sigma = \sigma_h) = 1$ (right panel) and the parameters in Table 1.3 and Table 1.1. The blue line represents the real yield; the green line denotes the nominal yield.

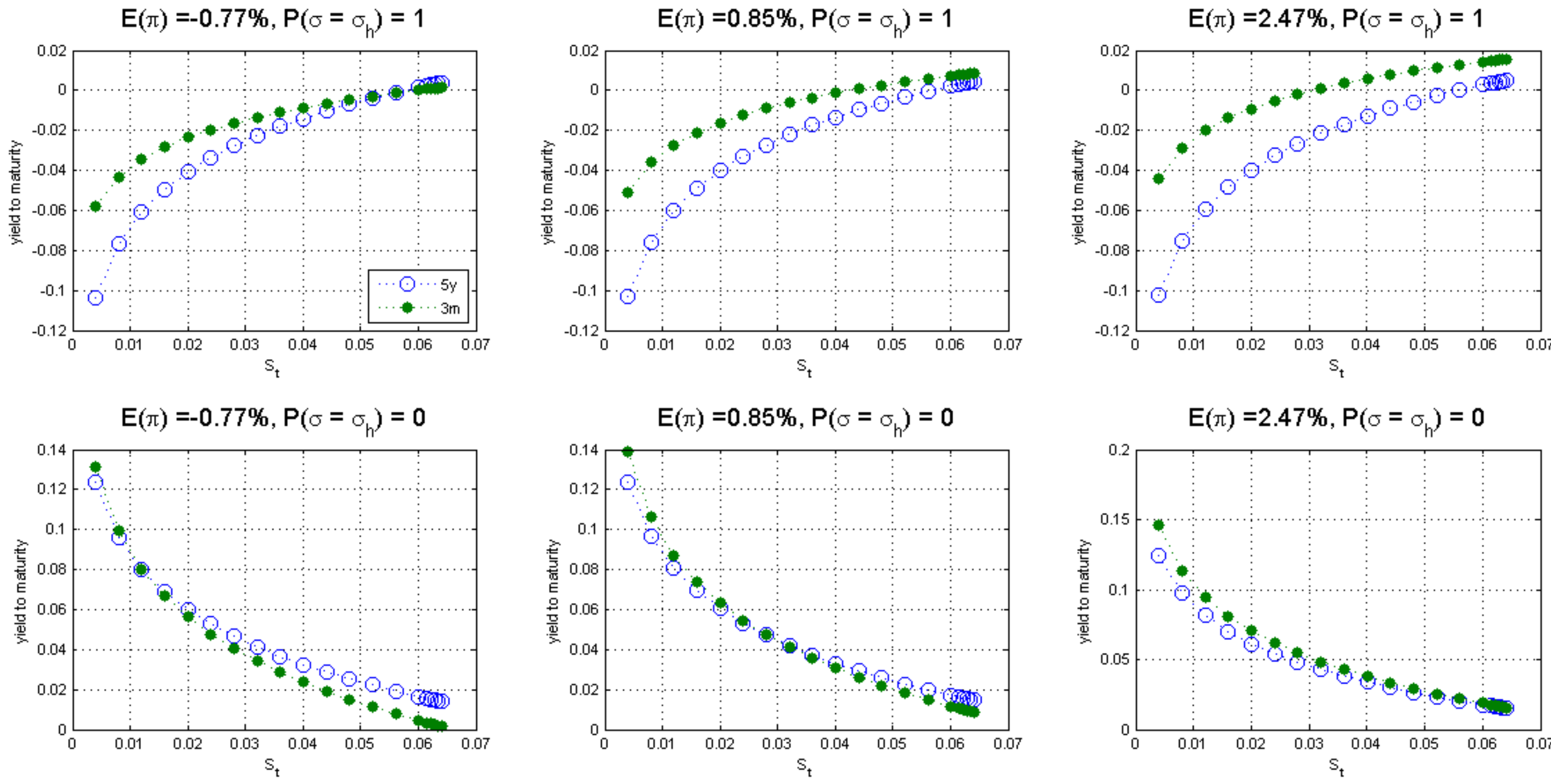

Figure 1.8: Nominal yields vs. surplus with different inflation expectations and high/low expected state.

Nominal continuously compounded bond yields as a function of the surplus-consumption ratio implied by the posterior probabilities $P(\sigma = \sigma_h) = 1$ (upper panels) and $P(\sigma = \sigma_h) = 0$ (lower panels) and the parameters in Table 1.3 and Table 1.1, for different values of expected inflation: long-term expectation (middle panels), long-term expectation minus and plus two standard deviations (left and right panels). Blue lines represent 5y yields; green lines denote 3m yields.

Chapter 2

Non-linear pass-through indicator of inflation expectations anchoring

2.1 Introduction

Since the end of 2011, the euro area, the United States and the United Kingdom have been experiencing a steady disinflation, which has become more pronounced since 2013 (see Figure 2.1). In addition, during 2014 long-term inflation expectations, both survey- and market-based, have also edged downwards (see Figure 2.2, 2.3, 2.4). A persistent departure from the central banks' targets could reveal a loss of trust in the ability of the monetary authority to bring inflation back; a de-anchoring of expectations can induce agents to postpone consumption and investment, eventually leading to a deflationary spiral.

In this chapter we propose a new indicator of inflation expectations anchoring based on a logistic model. By inspecting the comovement of daily changes in short and long-term inflation swap rates, it measures the odds that strong variations in short-term inflation compensations are channelled to large movements of the same sign in long-term ones.[1] We define an indicator of the pass-through of negative shocks to long-term inflation compensation (Left-Tail Pass-Through indicator, LTPT), to capture the risk of a downside de-anchoring of inflation expectations with respect to the policy target. In the same spirit , we also define an indicator of the pass through of positive shocks (Right-Tail Pass-Through indicator, RTPT) that matters in times of upside deanchoring concerns; in general, proxying downside (upside) de-anchoring concerns with long-term expectations falling below (above) the target, our *non-linear pass-through indicator* equals the LTPT (RTPT) in case of long-term expectations below (above) the target. In the current juncture, we focus on the behaviour of the LTPT (over time and with respect to the RTPT) to detect a possible downside de-anchoring. As far as we know, an indicator of anchoring based on the evidence of non-linear pass-through of inflation expectations has never been proposed in the literature.

Exploiting all the available inflation swap quotes, our results suggest that expectations in the US and

[1] Inflation swap rates are usually interpreted as a risk-neutral measure of markets' inflation expectations. While they do not represent pure expectations (they incorporate an inflation risk premium), the academia refers to them as inflation compensation (or breakeven inflation). As well as expectations, inflation risk premia are also informative in assessing the degree of anchoring, so we choose not to extract them from swap quotes; throughout the analysis, we also note that time series features of inflation compensation that are relevant in our model are not driven by risk premia (see Section 2.2).

UK have been mostly anchored between 2001 and 2016. Concerning the euro area, signs of de-anchoring have emerged in October 2014, with the LTPT reaching a peak at the end of the year; while an increase in the RTPT only followed some months later, then retrenching to pre-peak levels, the LTPT remained high and volatile also in 2015 and 2016. Since 2014, the asymmetric pass-through from short- to long-term inflation compensation suggests that the degree of anchoring of euro area inflation expectations may have diverged from the US's and UK's one.

The idea of measuring anchoring looking separately at coincident left- and right-tail daily variations in inflation swaps relies on the claim that the linear pass-through of shocks is not a sufficient statistic to identify anchoring.[2] Indeed, a transition from anchored to unanchored expectations implies a departure of beliefs on one side with respect to the target that is likely to modify the shape of the distribution of expectations: linear correlations help in identifying possible breaks in the average comovement, but are not enough to capture asymmetric changes in beliefs (skewness) or variations in uncertainty (variance).[3] By identifying tail variations in inflation compensation not in absolute terms, but relatively to the rolling estimation sample, we take into account changes in the shape of the distribution in assessing the tail pass-through.

In times of falling expectations and interest rates close to the zero lower bound, concomitant downswings in short- and long- term expectations could be more informative than concomitant upswings to identify a de-anchoring below the target, in light of the asymmetric ability of central banks in fighting disinflation with respect to inflation. A nonlinear approach to model anchoring is also motivated by two stylized facts documented in this chapter: (i) option-implied and econometric evidence on inflation compensation suggest that the inflation expectation process could be non-stationary and (ii) the recent increase in correlation between euro area short and long-term inflation swaps has been, in one case, non-linear.[4] Importantly, while we find heteroskedasticity, we note that skewness is not a feature of inflation compensation; therefore, the asymmetric pass-through that emerges in the case of the euro area cannot be ascribed to univariate time series characteristics (e.g., an increasing relevance of negative shocks) but is a genuine feature of the time-varying co-dependence.

Our work starts from the methods and evidence illustrated in Natoli and Sigalotti (2016). The authors proposed a set of copula-based and nonparametric comovement indicators and applied them to euro area inflation swaps and options to evaluate anchoring of inflation expectations; they find increasing average and tail correlations between inflation compensation and uncertainty at different horizons. With respect to those measures, the non-linear pass-through indicator has three main advantages: first of all, it only relies on quoted inflation swaps with no use of inflation options and of the burdensome calculations that this set of measures requires; secondly, being a single indicator, it allows a simpler identification of anchoring over time without the need of averaging out information coming from different measures;

[2]Ordinary pass-through models of inflation expectations assume that when expectations are firmly anchored to the central bank's target, long-term expectations should be quite stable, not reacting (as short-term expectations do) to inflation or macro data releases; according to this claim, linear correlations of long- with short-term expectations provide insights on the sensitivity of long-term beliefs and might reveal possible signs of de-anchoring.

[3]As pointed out by Gurkaynak, Levin and Swanson (2010), inflation targeting should help anchor market perceptions of the *entire* distribution long-term expectations.

[4]Concerning point (i), we observe that inflation compensations are autocorrelated and heteroskedastic, and in Section 2.2.2 we rule out the hypothesis that these features only reflect the dynamics of the inflation risk premium. The heteroskedasticity of inflation expectations is also considered in a recent econometric work using survey-based expectations, that points on time-varying uncertainty as the key feature of the inflation process (Grishchenko, Mouabbi and Renne, 2016).

lastly, compared to the DownTailCor and UpTailCor included in Natoli and Sigalotti (2016), it captures the *relative importance* of co-movement between variations in one tail with respect to any other co-variation, not only the *level* of that co-movement.

A growing literature investigating anchoring in the most recent period is reporting mixed results, especially for the euro area and the US. Strohsal and Winkelmann (2015) and Grishchenko, Mouabbi and Renne (2016) find more firmly anchored inflation expectations in the euro area than in the US, even though the latter note that the degree of anchoring in the euro area has decreased more than that in the US during the financial crisis. Concerning the euro area, some papers claim only mild reactions of inflation beliefs to macroeconomic announcements during the crisis (Autrup and Grothe, 2014) and post-crisis period (Scharnagl and Stapf (2015) and Speck (2016)), while others point to a higher sensitivity of longer-term inflation forecasts to shorter-term ones and to actual HICP inflation (Lyziak and Paloviita, 2016). Gurkaynak, Levin and Swanson (2010) compare the level of anchoring in the US and the UK from the 90's onwards, finding that, from initially high levels, the volatility and responsiveness to macro news of UK inflation compensation was substantially reduced after the adoption of the inflation target (similar results are reported in Mehrotra and Yetman (2014)). Our result is in line with Ehrmann (2014), who studies the stability of long-term beliefs in a panel of countries before and after inflation-targeting: under persistently low inflation, he finds that a sign of downside de-anchoring with respect to a target is that inflation expectations get revised down in response to lower-than-expected inflation but do not respond to higher-than-expected outturns.

This chapter is structured as follows: Section 2.2 describes the time series features of inflation compensation, discusses the implication of working with risk-neutral inflation expectations and comments on measures of average correlation; Section 2.3 explains in detail how the new estimator is constructed, and Section 2.4 discusses the results for the euro area, the United States and the United Kingdom. Section 2.5 concludes.

2.2 Inflation compensation and non-linear comovements

We first describe some time series features of inflation compensation, based on the analysis of inflation swaps. Then, we comment on two measures of average comovement between short- and long-term inflation compensation, documenting nonlinearities in the case of the euro area since 2014. Throughout the analysis, short and long-term inflation compensations are proxied by 1y1y and 5y5y forward inflation swap rates. In the following section, one- to ten-year inflation options are also used to estimate implied distributions of inflation compensation, giving further insights on the behavior of higher moments of inflation compensation since option data are available (i.e., since 2009).

2.2.1 Time series features of inflation compensation

An inflation swap is a derivative contract in which two parties agree to exchange a fixed amount of money with a floating amount linked to realized inflation on particular dates in the future. The underlying assets of swaps are the Harmonized Index of Consumer Prices excluding Tobaccos (HICPxT) for the euro area, the Consumer Price Index (CPI) for the United States and the Retail Price Index (RPI) for the United

Kingdom, all lagged by three months.

Figure 2.5 plots the first differences of 1y1y and 5y5y inflation swap rates for our three economies. As expected, time series on the right-hand side (5y5y) are much less volatile than those on the left-hand side (1y1y). We also observe that, in the post-crisis period, average daily variations are decreasing in magnitude over time, suggesting that, particularly for short-term inflation compensation, volatility might be time-varying. In the last part of the sample, big downswings or upswings in absolute terms are extremely rare, so we expect to find few large concomitant variations in short and long swaps. Risk-neutral probability distributions extracted from inflation options also suggest volatility of inflation compensation trending down since options are quoted (2009), see Figures 2.6 and 2.7.[5]

To formally investigate the time series features of inflation compensation, we fit three different models on the series of 1y1y inflation compensation: an AR(2), an AR(2)-GARCH(1,1) and an AR(2)-GJR(1,1), to test for the presence of autoregressive components, conditional heteroskedasticity and possible asymmetric effects of shocks on the volatility process. The reason why we focus on short-term inflation compensation is that, being at the origin of the pass-through, its non-stationarity features, if any, can mis-specify the nature of the co-movement unless taken into account into the pass-through specification.

Models are estimated on the all 2004–2016 sample and on the 2009–2016 subsample to focus on the post-crisis dynamics. Results for the euro area, United States and United Kingdom for the post-crisis period are shown in Table 2.1.[6] For each economy, at least one autoregressive coefficient and the ARCH and GARCH effects are significant in all models, confirming the presence of an autoregressive component and conditional heteroskedasticity. The asymmetry coefficient of the GJR specification is not significant for the euro area and the US, while it is so for the UK; however, in all the three cases, the AR(2)-GARCH(1,1) is the model that minimizes the Bayesian Information Criterion.

To summarize, we find significant heteroskedasticity in the inflation compensation process, that justifies a time-varying approach and the need to control for it in the anchoring specification; concerning asymmetry, we do not detect significant signs of asymmetries in short-term inflation compensation: evidence of asymmetric comovement with short- and long-term inflation compensation, if any, will not depend on the asymmetric relevance of positive and negative shocks on the short-term side.

2.2.2 Inflation risk premium

We investigate the comovement of inflation compensation to gain insights on the comovement of markets' inflation expectations. Inflation compensation incorporates an inflation risk premium in addition to the expectation of future inflation. For two main reasons, we choose not to extract risk premia and loosely refer to breakeven inflation as inflation expectations throughout the analysis.

First of all, the literature has shown that the identification of inflation risk premia is highly model-

[5]Inflation options are caps and floors. Caps (floors) are derivative contracts in which the holder has the right to receive compensation payments at the end of each period in which the inflation rate exceeds (falls below) an agreed-upon strike rate; for each economy, the underlying asset of options is he same as that of swaps. Risk-neutral probability distributions are estimated from year-on-year option quotes using the least absolute deviations (LAD) estimator, a semi-parametric technique developed in Taboga (2016) and first applied to euro area inflation swap by Natoli and Sigalotti (2016).

[6]Results for the all sample are available upon request.

dependent, provided that mixed results are given in terms of magnitude and sign.[7] Secondly, inflation risk premia are informative, as well as expectations, in assessing the degree of anchoring: indeed, the investors' willingness to pay large premia in order to protect themselves against a scenario of persistently low inflation would also signal high risks in terms of the central bank's credibility and ability to bring inflation back to target. Bauer and Christensen (2014) point out that risk-neutral probabilities are useful for policy analysis, as policymakers are worried about extreme outcomes just like investors: as stated in Kocherlakota (2013), policy decision making should take into account the evolution of risk-neutral probabilities, since it reflects changes in market participants' views about future possible outcomes.

However, it is important to check whether the heteroskedastic feature of inflation compensation is just driven by the dynamics of the inflation risk premium (usually considered highly volatile) or it is also a genuine feature of market-based inflation expectations. We take up this point by re-estimating the three univariate models of Section 2.2.2 (i.e., the AR(2), AR(2)-GARCH(1,1) and AR(2)-GJR(1,1)) on weekly estimates of market-based inflation expectations taken from Pericoli (2012).[8] Models are re-estimated only for the euro area and the US, for which data are available. Results (that are available upon request) show that not only inflation risk premia, but also short-term expectations have significant conditional heteroskedasticity, confirming the evidence reported in the previous Section.

2.2.3 Average correlation

According to standard univariate normality tests such as the Jarque-Bera and Kolmogorov-Smirnov tests, inflation compensation is non-Gaussian.[9] While normality is required to compute Pearson's rho linear correlation coefficient, the comovement of short- and long-term inflation compensation evaluated in this way can theoretically be spurious. We propose a comparison between Pearson and Spearman's rank correlation, the latter capturing monotonic correlation between series.

Figure 2.8 reports the two indices computed on 250-day rolling samples for the three economies (upper panel), as well as bootstrapped confidence bands for Spearman's correlations (lower panel). Looking at the left graph in the upper panel, in the case of the euro area Pearson's and Spearman's correlations diverged since 2014: Pearson's rho increases at the begining of 2014, then decreased in 2015 while Spearman's rank started to increase later in 2014 remaining high in 2015 and 2016, in line with US and UK cases. Assessing the monotonic relationship between two variable, the Spearman's index is high when data in the two series have the same rank. In this way, cases in which extreme ranks correspond to tail instead of core variations are not identified.

Looking at lower panels of Figure 2.8, Spearman's rank correlations between 1y1y and 5y5y forward inflation swaps appear to be significantly positive since October 2014 (euro area), March 2015 (United States), July 2015 (United Kingdom). If correlation was a sufficient statistics, we would have judged

[7] The heterogeneity of the available estimates of inflation risk premia is highlighted also by Pericoli (2012), who provides a comparison of some estimates found in the literature and shows that there indeed are stark differences among them.

[8] In this paper, the author estimates a no-arbitrage affine Gaussian term structure model for the nominal and real zero-coupon interest rates implied in standard and index-linked government bonds, obtaining a decomposition of inflation compensation into an expectation and a risk premium component; as far as we know, this is the only paper providing estimates for the euro area on a weekly (instead of a monthly) basis.

[9] The null hypothesis of the Jarque-Bera test is that data come from a normal distribution with an unknown mean and variance, while the null hypothesis of the Kolmogorov-Smirnov test is that data come from a standard normal distribution. We run those two tests on inflation swaps at short and long maturities, rejecting the null hypothesis at the 5% level in all cases.

high de-anchoring risk in all the three economies.

2.3 Indicator of non-linear pass-through

In this section we introduce the LTPT and RTPT indicators. We first define the LTPT, then we extend the definition to the RTPT. The indicators are first defined on a fixed sample; then, we discuss their rolling estimate based on the definition of tail variations.

2.3.1 Fixed sample

LTPT Considering short and long-term inflation swaps in first differences, we divide the observations for each variable into two groups, using dummy variables. While we focus on the reactions to extreme negative short-term variations, we only need to cluster our observations into left-tail and non-left-tail events. We construct two dummies in the following way: the first one takes the value 1 if daily revisions in short-term inflation compensation are below a specific quantile in the left tail area of the distribution and 0 otherwise; the second variable is equal to 1 in case of long-term revisions below the same quantile and 0 otherwise. As we saw in the previous Section, short-term inflation compensations have wider fluctuations than longer-term ones, so quantiles should be defined on each series separately to avoid mis-identifications of tail variations.

Namely, let $\{s_t\}$ and $\{l_t\}$ be the time series of daily short term and long term inflation compensation and $\{\Delta s_t\}$ and $\{\Delta l_t\}$ are the time series of daily changes. Let q^s and q^l be the chosen quantile of the empirical distributions of Δs_t and Δl_t, respectively. Then at a given date t, we define the binary variables x_t and y_t as

$$x_t = \begin{cases} 1 & \text{if } \Delta s_t \leq q^s \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

and

$$y_t = \begin{cases} 1 & \text{if } \Delta l_t \leq q^l \\ 0 & \text{otherwise} \end{cases} . \tag{2.2}$$

Our indicator of de-anchoring is based on the following logistic regression:

$$p(y_t = 1|x_t) = \frac{e^{\alpha+\beta x_t}}{1 + e^{\alpha+\beta x_t}} \tag{2.3}$$

where $p(y_t = 1|x_t)$ is the conditional probability of a left-tail revision of long-run expectations at time t.[10] The ratio of the latter probability to its complement ($p(y_t = 0|x_t)$) defines the odds of y being equal to 1 with respect to y being 0, i.e. the odds of a left-tail revision in long-term inflation compensation versus a non-extreme change, conditional on the value of x. Being x dichotomous, one can assess the relative probability of the events of interest by calculating the ratio of the odds when $x = 1$ against $x = 0$, i.e. the ratio of the odds of a large downward revision in long-term inflation compensation

[10] In order to obtain a meaningful interpretation of the estimated slope coefficient in terms of odds ratio, the logit model is preferred to a probit.

when the change in short-term one is extremely negative over the odds of the same revision being associated to a different short-term variation, either mildly negative or positive. This quantity, that we name *Left-Tail Pass-Through* (LTPT) indicator, is known as the odds ratio and is given by the exponential of β:

$$LTPT = \frac{p(y=1|x=1)}{p(y=0|x=1)} \bigg/ \frac{p(y=1|x=0)}{p(y=0|x=0)} = e^{\beta}. \tag{2.4}$$

Values of the LTPT above (below) one indicate that the odds that left tail long-term revisions are associated to variations in the left tail of short-term ones are greater (smaller) than the odds that variations in long-term inflation compensation are coupled with other changes in short-term views. Obviously, the statistical significance of the LTPT is linked to the statistical significance of the β coefficient.[11]

RTPT In the same spirit as Section 2.3.1, we construct our input dummies for the logistic regression in the following way: the first one takes the value 1 if daily revisions in short-term inflation swaps are *above* a specific quantile in the right tail area of the distribution and 0 otherwise; the second variable is equal to 1 in case of long-term revisions *above* the same quantile and 0 otherwise.

Defining Q^s and Q^l as the chosen quantiles of the empirical distributions of Δs_t and Δl_t, x_t and y_t are now defined as

$$X_t = \begin{cases} 1 & \text{if } \Delta s_t \geq Q^s \\ 0 & \text{otherwise} \end{cases} \tag{2.5}$$

and

$$Y_t = \begin{cases} 1 & \text{if } \Delta l_t \geq Q^l \\ 0 & \text{otherwise} \end{cases}. \tag{2.6}$$

Equation 2.3 and equation 2.9 will become

$$p(Y_t = 1|X_t) = \frac{e^{\gamma+\delta X_t}}{1+e^{\gamma+\delta X_t}} \tag{2.7}$$

$$p_t\left(y_t(\bar{Q}^l) = 1|x_t(\bar{Q}^s)\right) = \frac{e^{\gamma+\delta_t x_t(\bar{Q}^s)}}{1+e^{\gamma+\delta_t x_t(\bar{Q}^s)}} \tag{2.8}$$

2.3.2 Tracking the pass-through over time

To observe the evolution of the LTPT over time, we need to split the sample and estimate regression (2.3) on rolling windows. One of the main issues that arise is the choice of the quantiles q^s and q^l in each subsample.

The two most straightforward ways to set the quantiles are to compute them in the whole sample and refer to them in each rolling window estimate (*absolute thresholds*) or to re-calculate quantiles

[11] The significance of the coefficient β_t can be assessed by testing the hypothesis $H_0 : \beta_t = 0$. The z-statistic is computed as

$$z = \frac{\hat{\beta}_t}{\sqrt{\hat{var}(\hat{\beta}_t)}}$$

and is distributed as a standard normal distribution in large samples. Standard errors for e^{β_t} are the exponential of those of β_t.

at each iteration on the current subsample of observations (*time-varying thresholds*). If $\bar{q}^s$ and $\bar{q}^l$ are the quantiles estimated over the whole sample, and $\{q_t^s\}$ and $\{q_t^l\}$ are ones computed on each rolling window, equation (2.3) might be rewritten as:

$$p_t\left(y_t(\bar{q}^l) = 1|x_t(\bar{q}^s)\right) = \frac{e^{\alpha+\beta_t x_t(\bar{q}^s)}}{1+e^{\alpha+\beta_t x_t(\bar{q}^s)}} \qquad \textit{absolute thresholds} \tag{2.9}$$

and

$$p_t\left(y_t(q_t^l) = 1|x_t(q_t^s)\right) = \frac{e^{\alpha+\beta_t x_t(q_t^s)}}{1+e^{\alpha+\beta_t x_t(q_t^s)}} \qquad \textit{time-varying thresholds} \tag{2.10}$$

Defining tail variations with respect to the whole sample or relatively to each subsample entails two different views on the underlying inflation swap process. If the risk-neutral distribution of expected inflation is considered to be quite stable over time, then it could be preferable to identify tail variations as extreme variations with respect to the whole sample; on the contrary, if market-based expectations are significantly time-varying, then defining tail changes on the whole sample might lead to misleading results.

We have observed that the distribution of inflation compensation is substantially time varying and that average daily variations are decreasing in magnitude over time, such that big downswings or upswings in absolute terms are, in the last part of the sample, extremely rare. Because a fixed-threshold LTPT indicator could have, for this reason, very low power in testing anchoring, we choose to set time-varying thresholds. Nonetheless, an estimate of the fixed-LTPT is provided as a robustness check (Section 2.4.1).

2.4 Empirical estimates

In this section we discuss the evolution of the nonlinear pass-through indicator for the three economies, focusing on the last part of the sample where it coincides with the LTPT. Estimates are conducted on 250-day rolling windows, and bootstrapped confidence bands are obtained with 1000 replications. Our (relative) thresholds are set to be 10th percentile (LTPT) and 90th percentile (RTPT). The analysis is conducted on the January 2004 – May 2016 sample, so results of 250-day rolling estimates start from January 2005.[12]

Figure 2.9 shows the evolution of the LTPT indicator in the euro area. No relevant episodes of downside de-anchoring are detected before the global financial crisis. Between mid-2013 and mid-2014 the indicator started to grow, reaching a peak in late 2014. It's interesting to compare its evolution with the one of the RTPT (lower panel): while the LTPT started to increase in October 2014, the RTPT only followed in December, reaching a lower peak than the LTPT and then retrenching to 2013 levels. On the contrary, the LTPT remained high and volatile also in 2015 and 2016.

The dynamics of the indicator could interestingly be linked to some market event happening in 2015: a sharp decrease of the indicator at the beginning of 2015 happens on the eve of the extension of the Asset Purchase Programme of the European Central Bank (ECB) to include public sector securities; the new increase of the indicator in the summer of 2015 coincides with turbulences in the Chinese stock

[12] Inflation swaps quotes before January 2004 are not publicly available.

markets, and the fall observed in the following December precedes the announcement of further stimulus by the ECB by a couple of days.

Concerning the US case, Figure 2.10 shows a modest, non-significant increase of the LTPT at end-2014, then retrenching back at end-2015 (upper panel). No significant differences between left and right tail comovements are also reported (lower panel). For the UK, Figure 2.11 displays a moderate increase in the non-linear pass-through in the early months of 2015, comparable to the US case; while showing still increasing figures, the current level of the indicator is not comparable to the contemporaneous level observed for the euro area. As in the case of the US, no significant differences between left and right tail comovements are reported (lower panel).

Figure 2.12 shows the LTPT together for the three economies including the significance threshold (i.e., $exp(\beta) = 1$, lower black line) as well as the threshold of one-to-one pass-through (i.e., $exp(\beta) = exp(1) = 2.7183$, upper black line). The US and UK indicators never fall above the lower back line; in the case of the euro area, the lower confidence band is instead never below 1 (i.e., the indicator is always significant) since end 2014, with periods of over-reactions (i.e., indicator above the upper black line) in 2015 ad 2016.

2.4.1 Robustness checks

The empirical analysis shows a sharp increase in the de-anchoring risk for the euro area in late 2014, partly reversed during 2015, and no relevant episodes of de-anchoring for the United States and United Kingdom. In this section we propose alternative estimates of the LTPT and RTPT indicators as a robustness check.

Fixed-threshold LTPT First of all, referring to the issue raised in Section 2.3.2, we compute alternative estimates of the LTPT using fixed thresholds. Results for the euro area with bootstrapped confidence bands are shown in Figure 2.13.[13] As the non-linear pass-through indicator, the fixed-threshold LTPT peaked around end-2014; however, confirming our intuition, the very low number of identified tail variations makes confidence bands extremely large, suggesting very low power as an anchoring test.

Window length and tail level Secondly, we re-estimate the indicators using different specifications of the parameters: (i) different absolute tails; (ii) different sizes of the rolling window. Results for the euro area for the LTPT (left panels) and RTPT (right panels) are shown in Figure 2.14.[14]

Upper panels compare the baseline specification of the left and right tail cutoffs (10th and 90th percentiles) and two alternative ones (5th-95th and 15th-85th percentiles). Overall, the narrative remains the same for the three specifications. The higher volatility in the LTPT indicator based on a left-tail threshold at 5% can be explained by the smaller number of tail-labeled observations. Lower panels compare estimates carried out using 180- and 360-day rolling windows in addition to the 250-day baseline specification: results remain broadly unchanged under the alternative specifications of the window size.

[13] LTPT with fixed thresholds for the US and UK are available upon request.

[14] As before, results for the US and UK are available upon request.

Controlling for daily oil returns Third, as in other papers dealing with the pass-through of inflation expectations, we control for a possible direct effect of oil returns on long-term expectations (see Miccoli and Neri (2015) and Buono and Formai (2016)). This is motivated by the somewhat puzzling increased correlation between oil returns and 5y5y forward inflation swaps observed as of recently.[15] We expand our logit model by including daily oil returns, proxied by daily returns on front month WTI futures contracts. Equation 2.10 becomes

$$p_t\left(y_t(q_t^l) = 1 | x_t(q_t^s)\right) = \frac{e^{\alpha+\beta_t x_t(q_t^s)+\gamma_t oilret_t}}{1+e^{\alpha+\beta_t x_t(q_t^s)+\gamma_t oilret_t}} \tag{2.11}$$

Results reported in Figure 2.15 show that the LTPT does not change if daily oil returns are included in the model.

2.5 Conclusions

In this chapter we propose a new indicator of the degree of anchoring of inflation expectations based on a logistic model of inflation compensation. We apply it to euro area, United States and United kingdom inflation swaps. While Pearson's and Spearman's correlations signal increasing comovements in the three economies, the inspection of tail comovements using the non-linear pass-through indicator provides a different picture. No de-anchoring risk in the United States and the United Kingdom is observed; on the contrary, since 2014 the degree of de-anchoring in the euro area has diverged from the US's and UK's one.

A preliminary analysis and some robustness checks rule out the possibility that results are driven by features of the time series of inflation compensation – such as the fact that inflation compensations contain an inflation risk premium or the possibility that the series are skewed towards negative shocks –, or by model characteristics like parameter specifications or specific omitted variables.

[15] Oil shocks should affect long-term expectations only through their effect on short-run inflation expectations.

Euro area

	AR(2)			AR(2)-GARCH(1,1)			AR(2)-GJR(1,1)		
param	coeff	std error	tstat	coeff	std error	tstat	coeff	std error	tstat
Constant	-0.0688	0.1269	-0.5422	-0.1379	0.0780	-1.7678	-0.1479	0.0888	-1.6657
AR(1)	-0.3522	0.0119	-29.5905	-0.2640	0.0286	-9.2314	-0.2652	0.0288	-9.2231
AR(2)	-0.1188	0.0126	-9.4115	-0.0709	0.0262	-2.7089	-0.0704	0.0261	-2.6968
Const Var	30.0159	0.4466	67.2173	3.2323	0.2464	13.1162	3.0513	0.2432	12.5479
GARCH(1)	–	–	–	0.7270	0.0197	36.8839	0.7402	0.0195	37.8810
ARCH(1)	–	–	–	0.1704	0.0169	10.0684	0.1500	0.0230	6.5078
Asymmetry	–	–	–	–	–	–	0.0253	0.0300	0.8427
BIC		11691.9545			**11288.5844**			11295.6980	

United States

	AR(2)			AR(2)-GARCH(1,1)			AR(2)-GJR(1,1)		
param	coeff	std error	tstat	coeff	std error	tstat	coeff	std error	tstat
Constant	0.0564	0.1823	0.3095	0.0097	0.1307	0.0740	0.0278	0.1439	0.1930
AR(1)	-0.3772	0.0156	-24.1776	-0.3467	0.0206	-16.8126	-0.3455	0.0207	-16.6750
AR(2)	-0.1601	0.0173	-9.2719	-0.1267	0.0221	-5.7210	-0.1254	0.0223	-5.6322
Const Var	59.9203	1.1035	54.2989	0.8926	0.1896	4.7077	0.9127	0.1957	4.6628
GARCH(1)	–	–	–	0.8837	0.0083	106.9763	0.8830	0.0084	104.4942
ARCH(1)	–	–	–	0.1078	0.0089	12.0818	0.1150	0.0141	8.1653
Asymmetry	–	–	–	–	–	–	-0.0132	0.0178	-0.7413
BIC		12983.9645			**12579.9319**			12587.1799	

United Kingdom

	AR(2)			AR(2)-GARCH(1,1)			AR(2)-GJR(1,1)		
param	coeff	std error	tstat	coeff	std error	tstat	coeff	std error	tstat
Constant	0.0824	0.2071	0.3976	0.0215	0.1552	0.1385	-0.0554	0.1694	-0.3269
AR(1)	-0.4200	0.0132	-31.8982	-0.4014	0.0248	-16.1749	-0.4048	0.0248	-16.3114
AR(2)	-0.1638	0.0172	-9.5426	-0.1127	0.0277	-4.0621	-0.1126	0.0284	-3.9719
Const Var	79.9590	1.3351	59.8882	6.6857	0.3649	18.3203	6.3224	0.3663	17.2589
GARCH(1)	–	–	–	0.7209	0.0125	57.7780	0.7352	0.0126	58.5553
ARCH(1)	–	–	–	0.2133	0.0158	13.4782	0.1622	0.0186	8.7225
Asymmetry	–	–	–	–	–	–	0.0787	0.0299	2.6346
BIC		13523.1762			**13071.9794**			13075.7547	

Table 2.1: Model estimate on 1y1y forward inflation swap rates.

Sample: 1 October2009 - 31 May 2016. Sample: 1 October2009 - 31 May 2016.

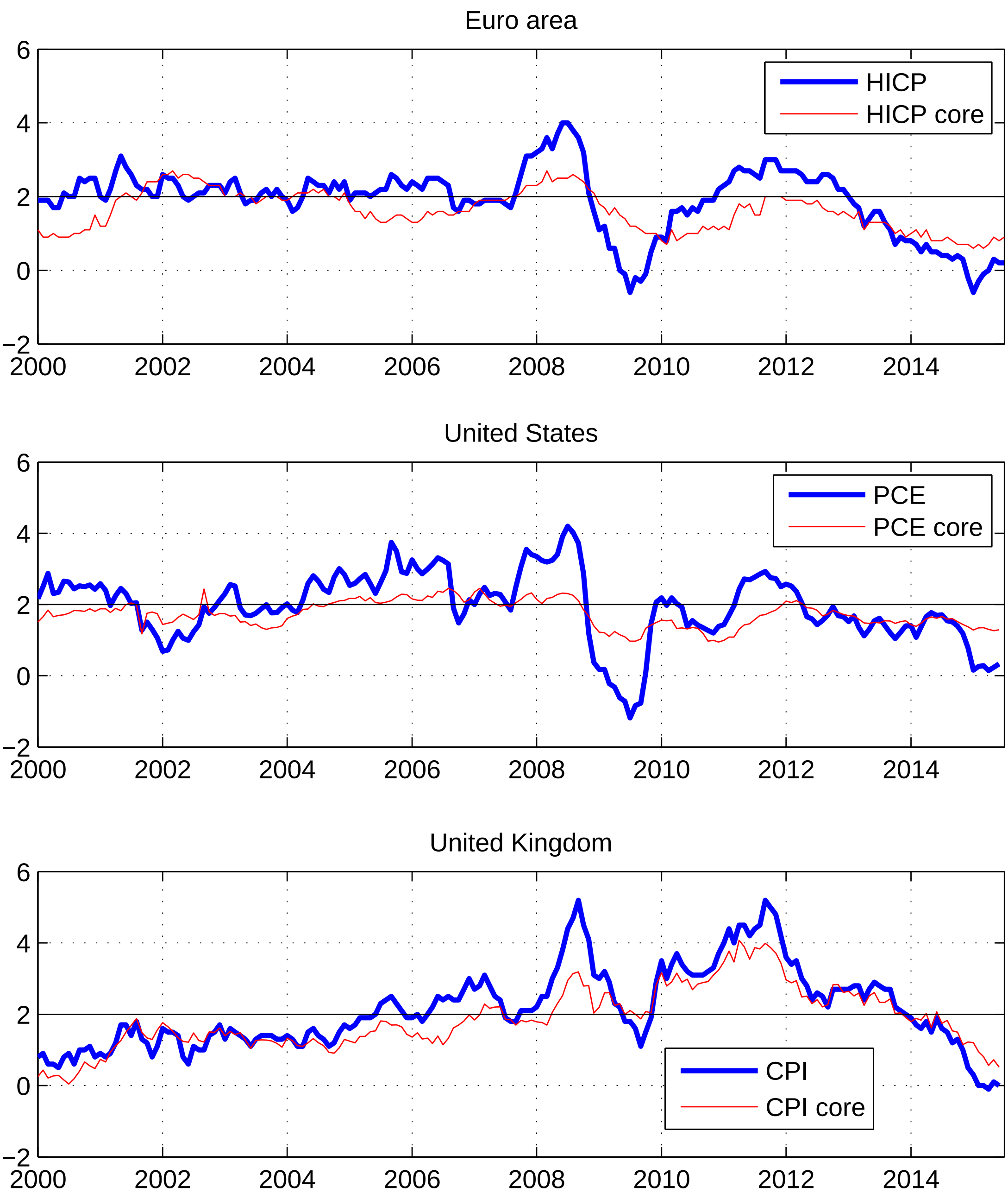

Figure 2.1: Inflation targets in the EA, US and UK.

Core indices are all items less food and energy

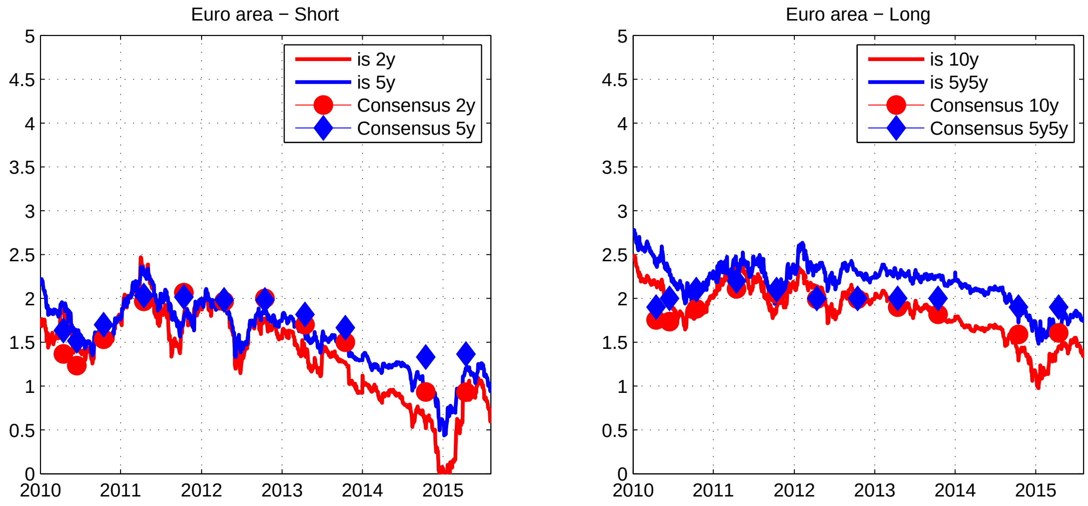

Figure 2.2: Market-based and survey based inflation expectations for the euro area.

The underlying measures of inflation are HICPxT (inflation swaps) and CPI (survey-based).

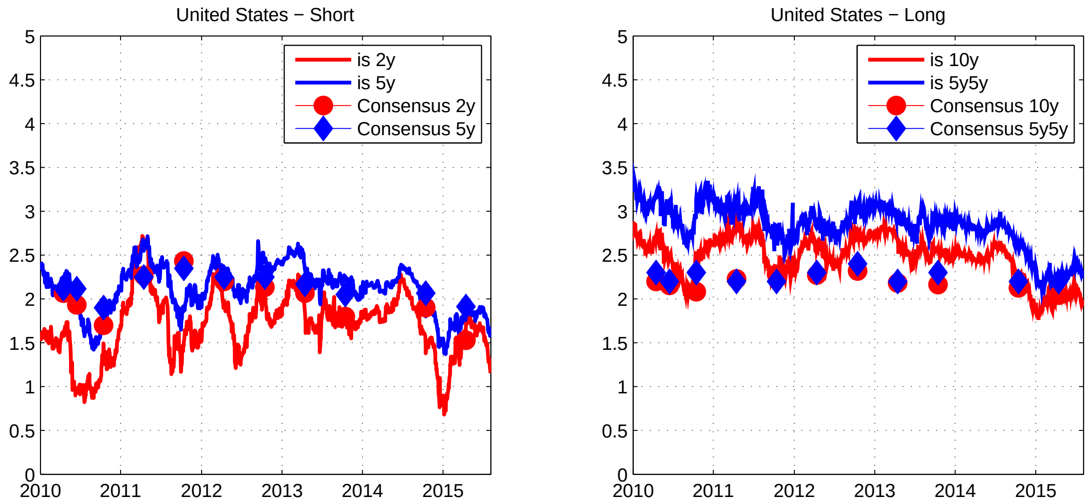

Figure 2.3: Market-based and survey based inflation expectations for the United States

The underlying measure of inflation is the CPI index.

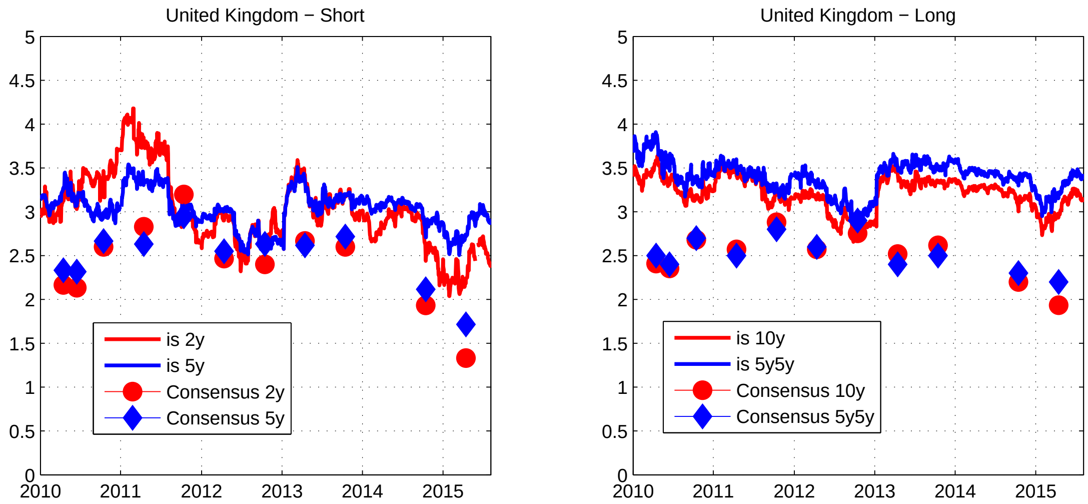

Figure 2.4: Market-based and survey based inflation expectations for the United Kingdom.

The underlying measures of inflation are RPI (inflation swaps) and CPI (survey-based).

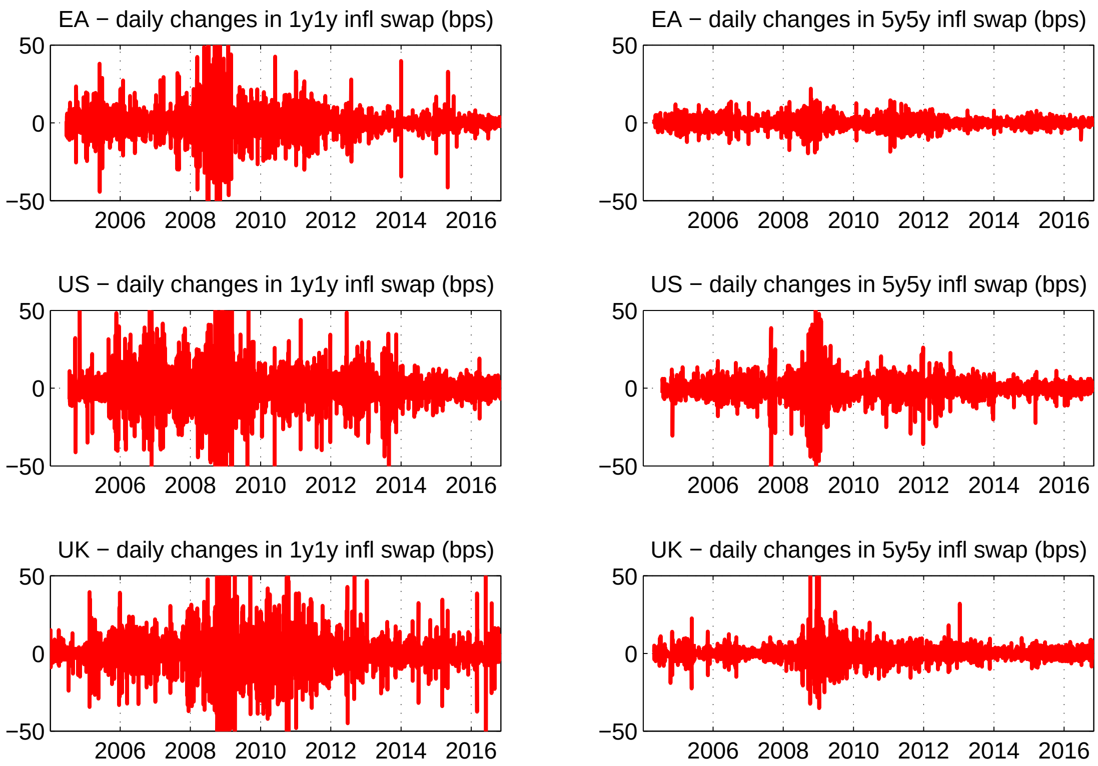

Figure 2.5: Daily changes in 1y1y and 5y5y forward inflation swap rates.

The sample is January 2004 to May 2016

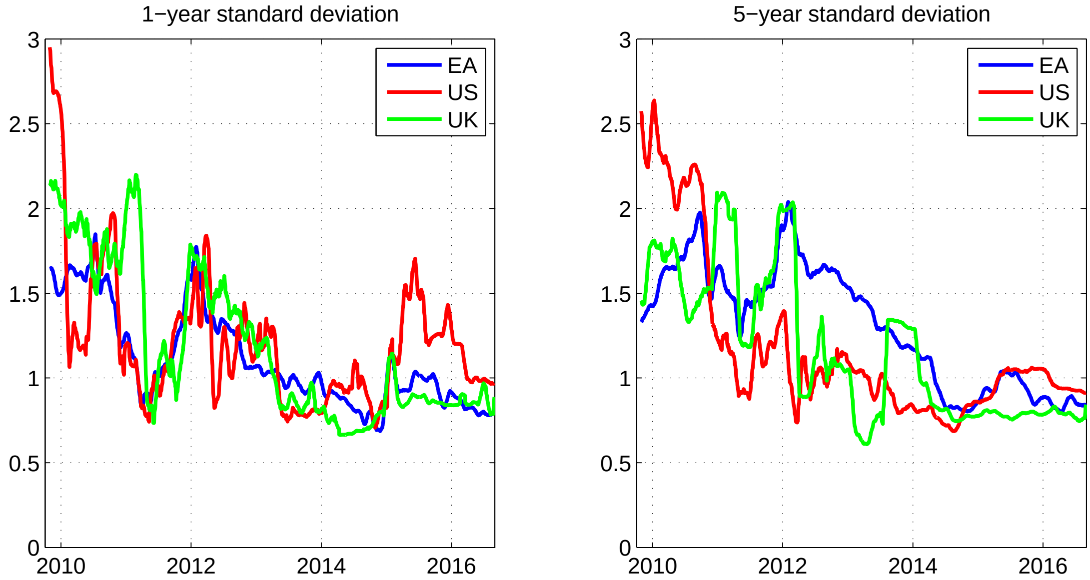

Figure 2.6: Short- and long-term inflation compensation uncertainty.

Uncertainty is proxied by standard deviations of option-implied distributions. 20-day moving averages are reported. The sample is October 2009 to May 2016.

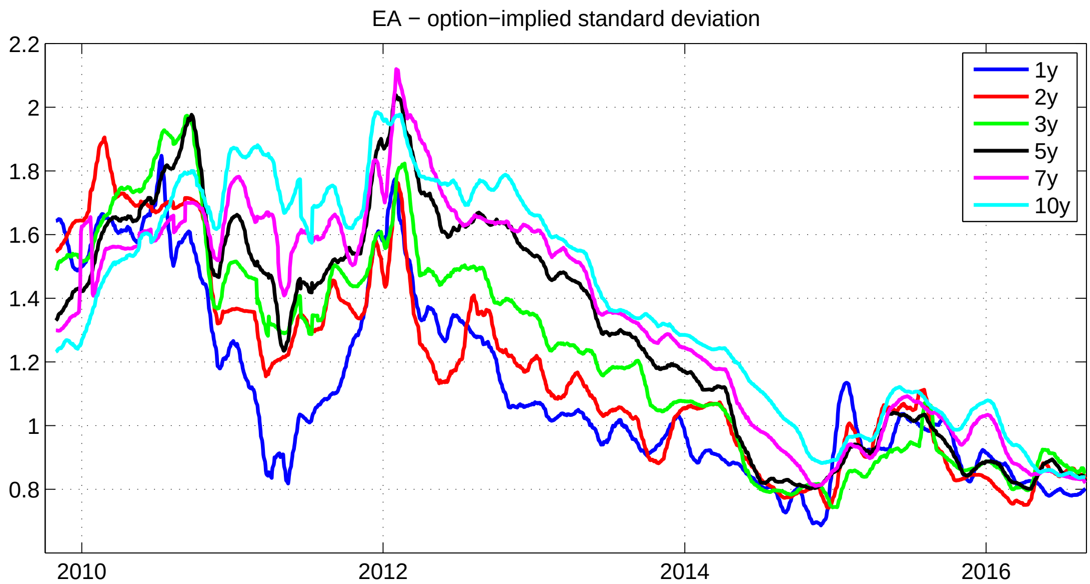

Figure 2.7: One- to ten-year inflation compensation uncertainty in the euro area.

Uncertainty is proxied by standard deviations of option-implied distributions. 20-day moving averages are reported. The sample is October 2009 to May 2016

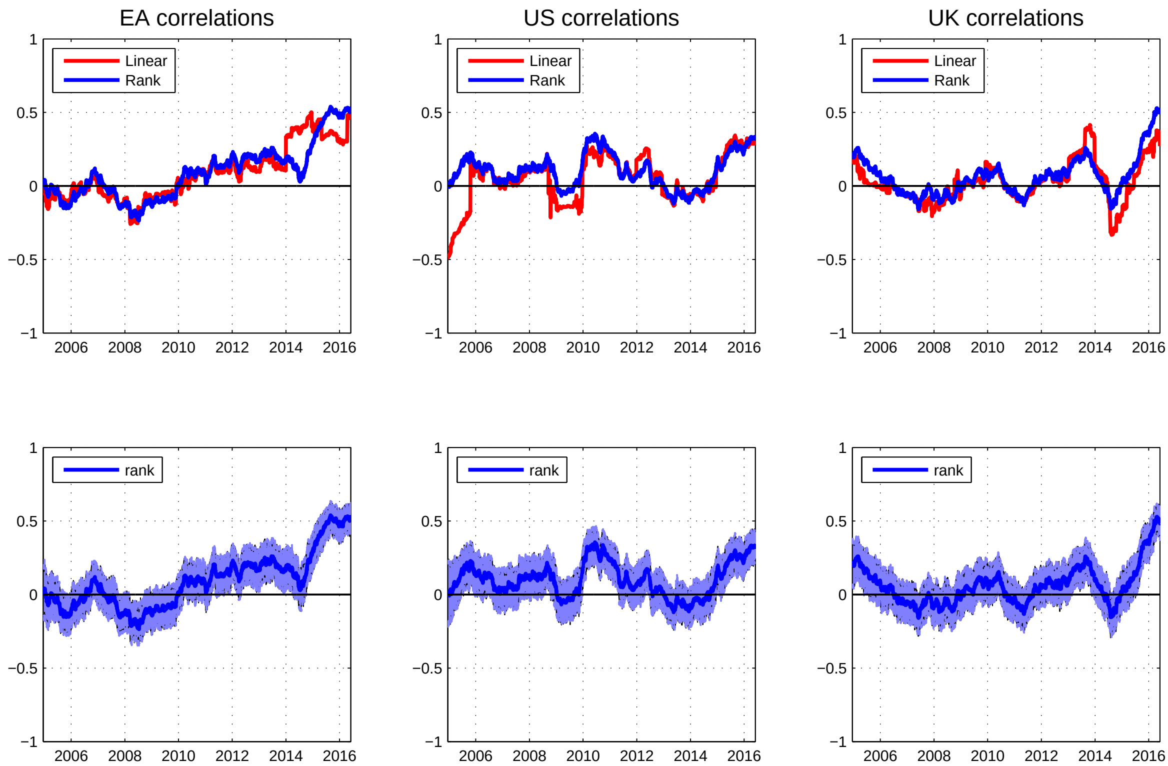

Figure 2.8: Correlations between 1y1y and 5y5y inflation swaps.

Upper panels: linear (Pearson's) vs. rank (Spearman's) correlations; lower panel: rank correlations with bootstrapped confidence bands computed with 1000 replications. The sample is January 2005 to May 2016.

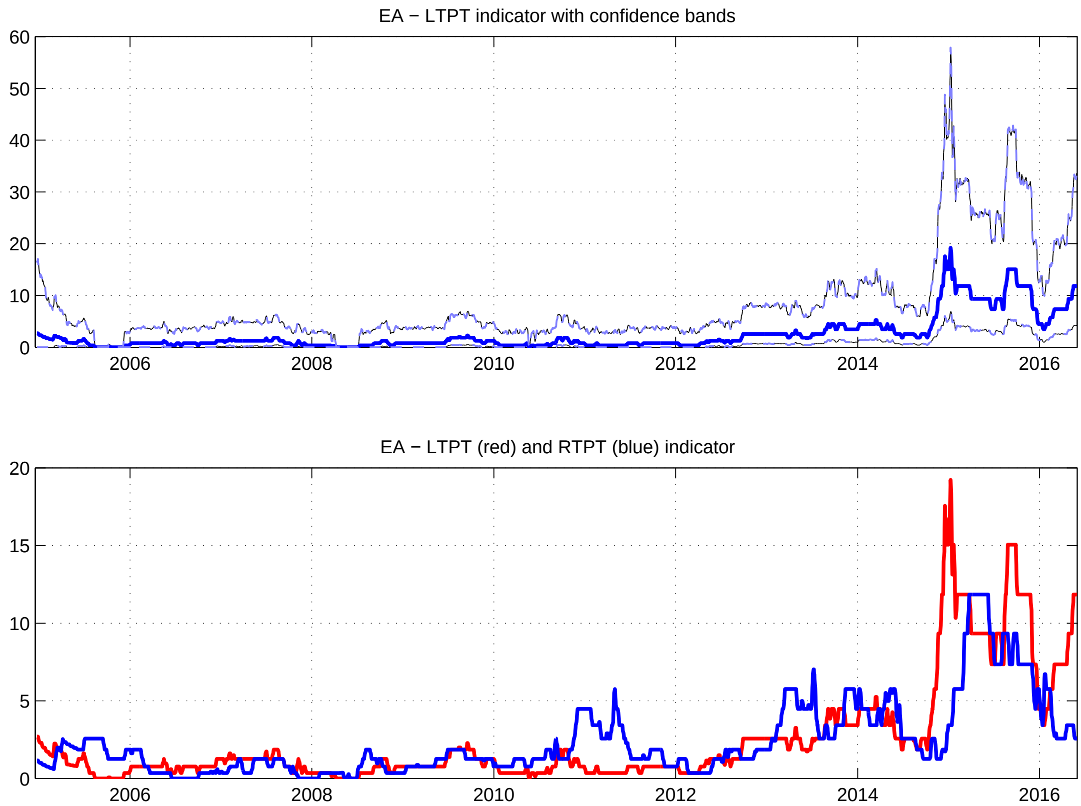

Figure 2.9: EA - LTPT indicator with confidence bands and LTPT vs. RTPT.

Bootstrapped confidence bands are computed with 1000 replications. Five-day moving averages are reported. The sample is January 2005 to May 2016.

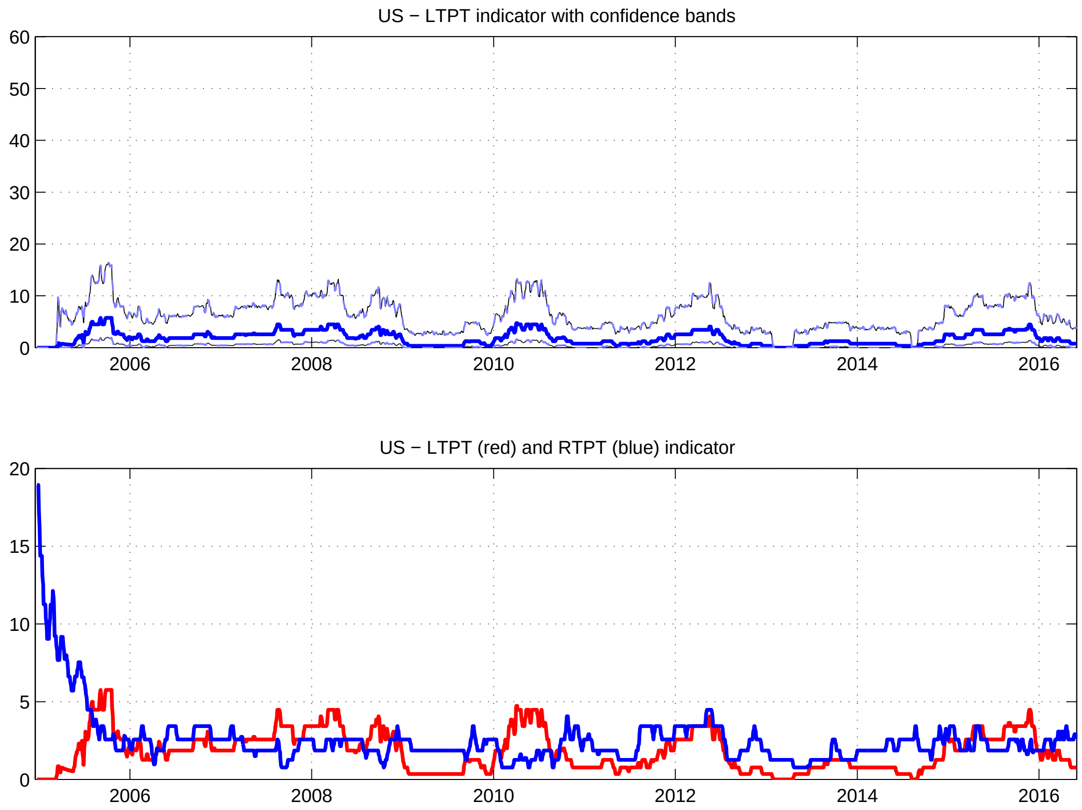

Figure 2.10: US - LTPT indicator with confidence bands and LTPT vs. RTPT.

Bootstrapped confidence bands are computed with 1000 replications. Five-day moving averages are reported. The sample is January 2005 to May 2016.

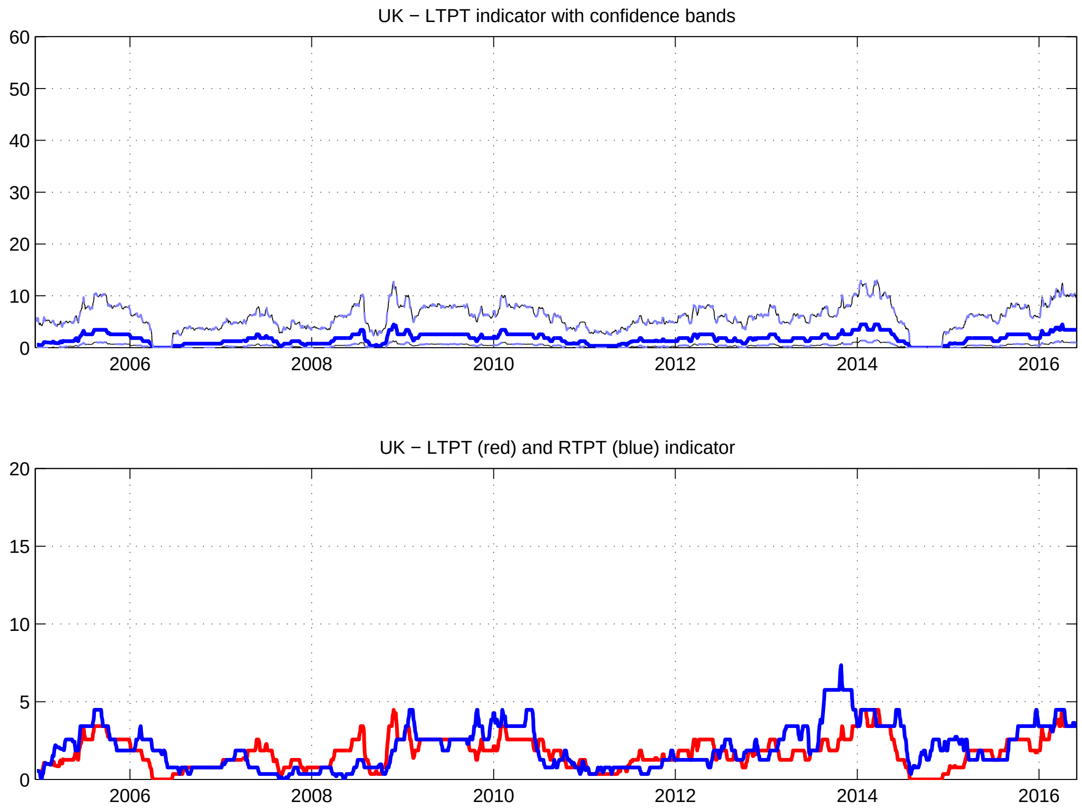

Figure 2.11: US - LTPT indicator with confidence bands and LTPT vs. RTPT.

Bootstrapped confidence bands are computed with 1000 replications. Five-day moving averages are reported. The sample is January 2005 to May 2016.

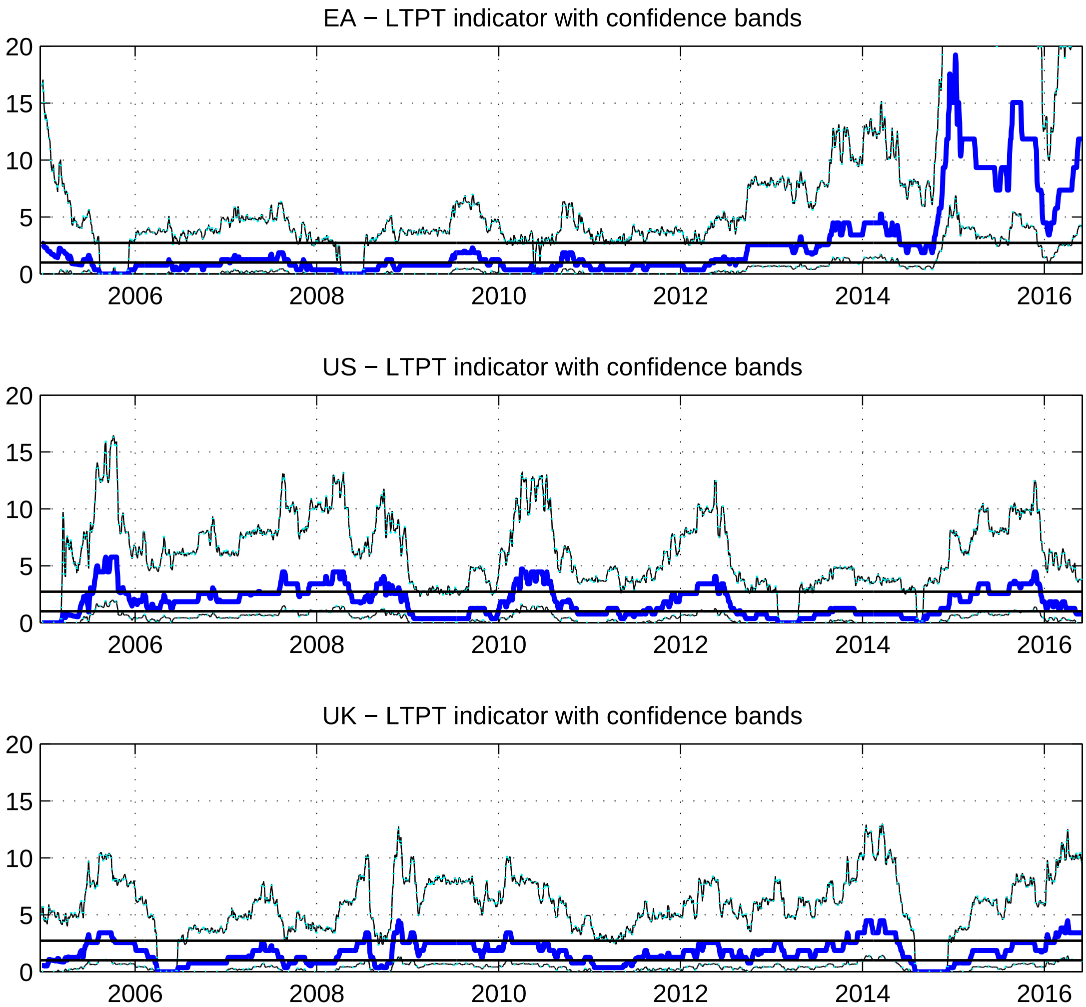

Figure 2.12: LTPT indicator (with confidence bands) for EA, US and UK.

In each plot, the lower black line indicates the significance threshold (LTPT=1) and the upper one indicates one-to-one pass-through (LTPT=exp(1)=2.7183). Bootstrapped confidence bands are computed with 1000 replications. Five-day moving averages are reported. The sample is January 2005 to May 2016

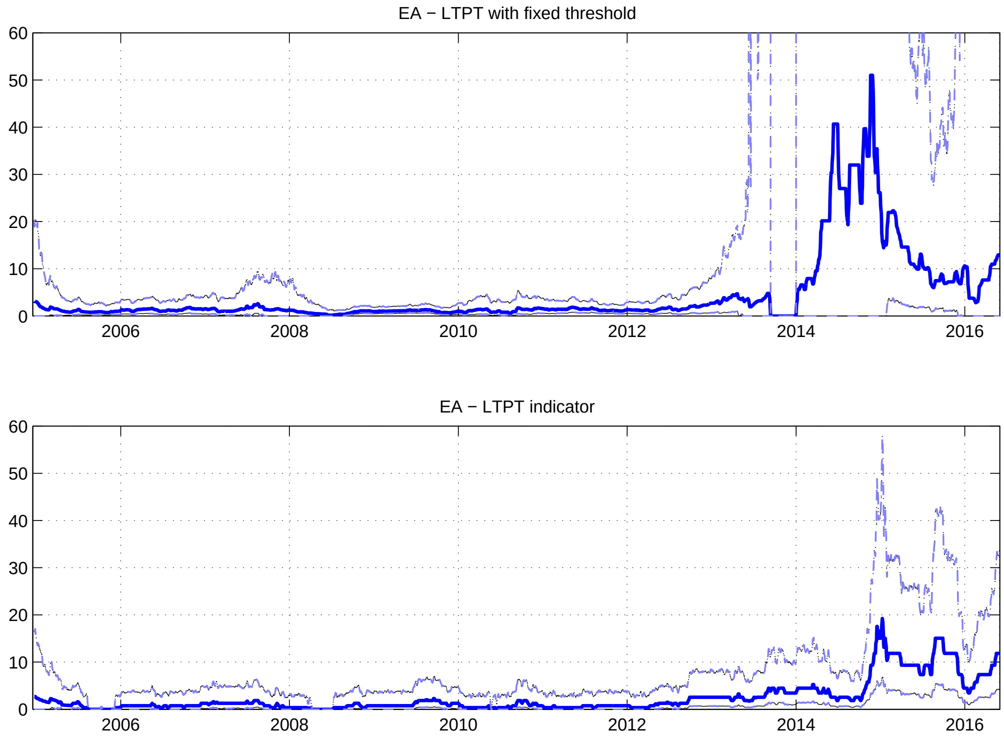

Figure 2.13: LTPT computed with fixed thresholds vs LTPT.

Bootstrapped confidence bands are computed with 1000 replications. Five-day moving averages are reported. The sample is January 2005 to May 2016.

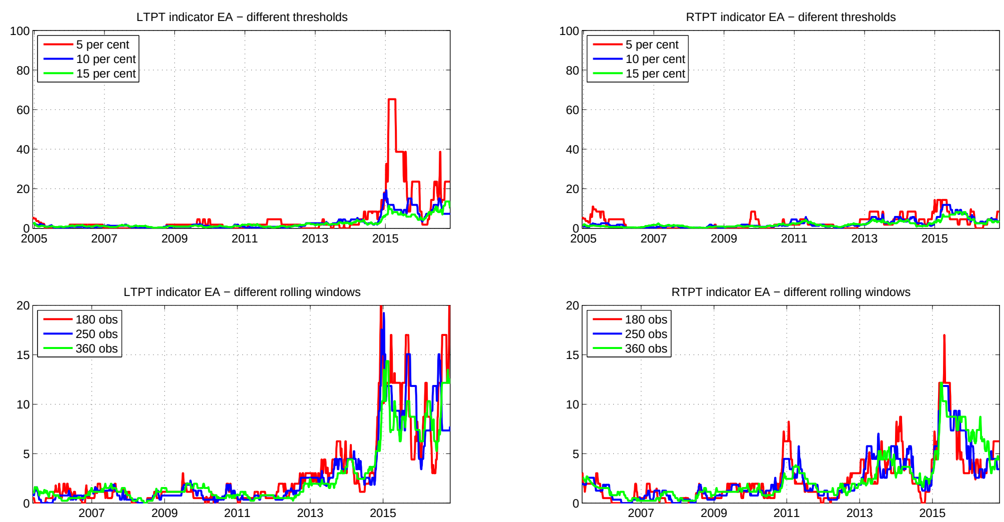

Figure 2.14: Robustness checks for the LTPT and RTPT.

Five-day moving averages are reported. The sample is January 2005 to May 2016.

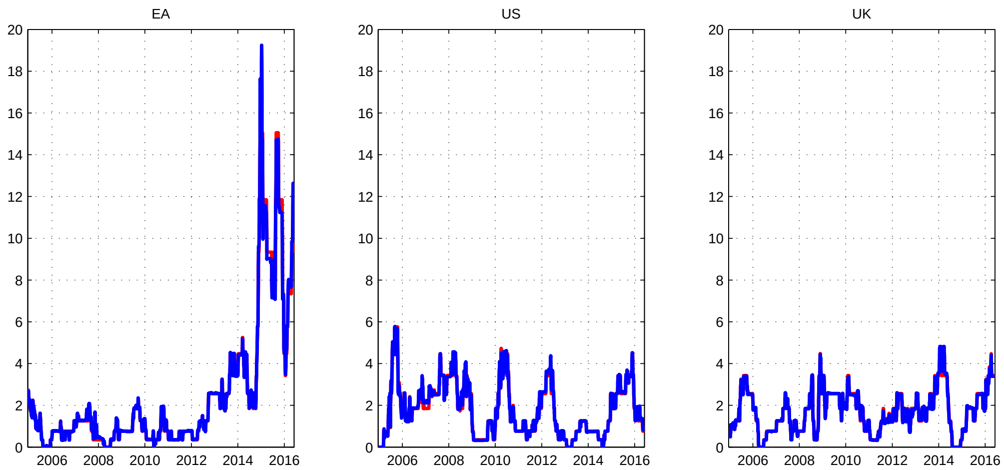

Figure 2.15: Robustness checks for the LTPT to a logit model that controls for daily oil returns. Five-day moving averages are reported. The sample is January 2005 to May 2016.

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Appendix A

Appendix to Chapter 1

A.1 Reference model

The standard CC framework and its extension in Wachter (2006) are described below. Representative investors have preferences over consumption with respect to a slow-moving reference level X_t, that is an exogenous habit level (the "keeping up with the Joneses" features motivated in Abel (1990)). The surplus-consumption ratio is the only state variable; a lognormal stochastic discount factor is defined and the one-period risk-free rate is derived in closed form from the Euler equation.[1]

The agent maximises

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma} \tag{A.1}$$

where C is consumption and X is an exogenous consumption habit level. The key variable on which consumer's choices are based is the surplus-consumption ratio, defined as

$$S_t \equiv \frac{C_t - X_t}{C_t} \tag{A.2}$$

Consumption growth is assumed to be a random walk

$$\Delta c_{t+1} = g + v_{t+1}, \qquad v_{t+1} \sim N(0, \sigma_v), \tag{A.3}$$

and the log of the surplus-consumption ratio is calibrated in a way that ensures procyclicality: it is the weighted sum of a constant term, an autoregressive component and the consumption shock v_{t+1} with a positive time-varying coefficient $\lambda(s_t)$. This term $\lambda(s_t)$ is a sensitivity parameter defined as a square root function of past values of the process; g being the average growth rate of consumption, γ the exponential

[1] With respect to the standard Lucas (1978) framework with power utility, habit preferences introduce some conceptual differences. The closed-form risk-free rate in the standard model under uncertainty is

$$r_{t,t+1} = -\ln\delta + \gamma \mathrm{E}_t(\Delta c_{t+1}) - \frac{\gamma^2}{2}\mathrm{VAR}_t(\Delta c_{t+1})$$

While interest rates depend only on contemporaneous consumption shocks in the original framework, here the state variable is backward looking and mean-reverting, depending on past shocks other than the contemporaneous one. Secondly, risk aversion is now time varying (γ/S_t): for a constant γ, it falls during booms and increases during recessions (it is countercyclical).

parameter of the power utility and ϕ the habit persistence parameter (assuming values between 0 and 1), s_{t+1} follows

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(\Delta c_{t+1} - g) \tag{A.4}$$

with

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1 & \text{if } s \leq s_{max} \\ 0 & \text{otherwise} \end{cases} \tag{A.5}$$

and

$$s_{max} = \bar{s} + \frac{1}{2}(1-\bar{S}^2), \qquad \bar{S} = \sigma_v \sqrt{\frac{\gamma}{1-\phi}} \tag{A.6}$$

As CC shows, the functional forms of $\lambda(s_t)$ and $\bar{s} = \ln \bar{S}$ are such that: (i) the risk-free rate is constant; (ii) habit is predetermined at the steady state $s_t = \bar{s}$; (iii) habit is predetermined near the steady state and moves nonnegatively with consumption everywhere.

Wachter (2006) applies an alternative specification suggested by CC, that verifies requirements (ii) and (iii) but allows the short-term rate to be a linear function of the state. The functional form of λ is left unchanged, but $\bar{S}$ is now calibrated in the following way:

$$\bar{S} = \sigma_v \sqrt{\frac{\gamma}{1-\phi-b/\gamma}} \tag{A.7}$$

Given this specification, the stochastic discount factor is

$$\begin{aligned} M_{t+1} &= \frac{\beta U_c(C_{t+1} - X_{t+1})}{U_c(C_t - X_t)} = \\ &= \beta \exp(-\gamma(g + (\phi - 1)(s_t - \bar{s}) + (1 + \lambda(s_t))(\Delta c_{t+1} - g))) \end{aligned} \tag{A.8}$$

From the closed-form specification of M_{t+1} it is straightforward to derive the formula of the risk-free rate, using the log normality assumption:

$$r_t^f = \ln \frac{1}{E_t M_{t+1}} = -\ln\delta + \gamma g + \gamma(\phi-1)(s_t-\bar{s}) - \frac{\gamma^2\sigma_{t+1}^2}{2}(1+\lambda(s_t))^2 \tag{A.9}$$

A.2 Market-implied real interest rates

Professional forecasters started to produce estimates of CPI inflation expectations at the beginning of the 80's, so those can not be used to retrieve real rates (by subtracting inflation expectations from nominal rates) before that date. We instead follow the procedure proposed in Chapter 3 of the April 2014's *World Economic Outlook* of the IMF: inflation expectations are computed as out-of-sample forecasts from a simulated autoregressive process of inflation. In this way we can estimate real rates for the whole sample (up to the 1960's).

Denoting P_t the monthly consumer price index at time t, an autoregressive model with 12 lags ($AR(12)$) is fitted on the variable $\gamma_t = lnP_t - lnP_{t-12}$; the estimation is carried out on a rolling window of 60 months in order to mitigate the effect of parameter instability. Model-based inflation expectations for horizon j are computed using out-of-sample forecasts of γ_t. Real rates are then recovered as

$$r_{n,t} = r^{\$}_{n,t} - \frac{(1-g)}{(1-g^n)} \sum_{i=1}^{n} g^i E_t \pi_{t,t+1}$$

where $r_{n,t}$ and $r^{\$}_{n,t}$ are the real and nominal rates at time t on a bond of maturity n, $E_t\pi_{t,t+i}$ is the inflation expectation at time t for period $t+i$ and $g = (1+\bar{r^{\$}})^{-i}$, with $\bar{r^{\$}}$ being the average nominal rate. The real rate is therefore equal to the nominal rate minus a weighted average of the inflation expectation over the entire life of the bond.

A.3 Pricing of real and nominal bonds

Let $P_{n,t}$ denote the price of a real bond maturing in n periods, and $P^{\$}_{n,t}$ the price of a nominal bond. Prices are computed as expectations of the future compounded SDFs until maturity.

The real price is determined recursively from the Euler equation (1.15) with boundary condition $P_{0,t} = 1$. Note that $P_{n,t}$ is a function of the posterior probability $\xi_{t+1|t}$. We solve for these functional equations numerically on a grid of values for the state variable $\xi_{t+1|t}$. Conditional on $\xi_{t+1|t}$, the price of the bond is a function of s_t alone, so equation (1.15) can be rewritten as

$$\begin{aligned}
P_{n,t} &= E_t\left[\delta\left(\frac{C_{t+1}}{C_t}\frac{S_{t+1}}{S_t}\right)^{-\gamma} P_{n-1,t+1}\right] \\
&= E_t\left[M_{t+1}P_{n-1,t+1}\right] \\
&= \sum_{j\in\{h,l\}} \xi_{t+1|t}(j) E_t[M_{t+1}P_{n-1,t+1}|\sigma_{\zeta_{t+1}} = \sigma_j, \xi_{t+1|t}] \\
&= \sum_{j\in\{h,l\}} \xi_{t+1|t}(j) E_t[e^{\ln\delta - \gamma[g+(1-\phi)(\bar{s}-s_t)+(\lambda(s_t)+1)\sigma_j\epsilon_{t+1}]} P_{n-1,t+1}|\sigma_{\zeta_{t+1}} = \sigma_j, \xi_{t+1|t}]
\end{aligned}$$

The last expectation can be solved using numerical integration on a grid of values for s_t, conditional on being in state j.

Analogously, the nominal bond price is equal to the expected discounted nominal payoff:

$$P^{\$}_{n,t} = E_t[M_{t+1}\frac{\Pi_t}{\Pi_{t+1}}P^{\$}_{n-1,t+1}] \tag{A.10}$$

In order to compute the nominal bond prices we introduce inflation as an additional state variable. Using the law of iterated expectations and conditioning on realizations of the shock to the level of the consumption growth, we can prove that

$$P^{\$}_{n,t} = F^{\$}_{n,t}\exp\{A_n + B_n\Delta\pi_t\} \tag{A.11}$$

with

$$\begin{aligned} F^{\$}_{n,t} &= E_t[M_{t+1}\exp\{\rho(B_{n-1}-1)\sigma_{\Delta\pi}\epsilon_{t+1}\}F^{\$}_{n-1,t+1}] \\ A_n &= A_{n-1}+(B_{n-1}-1)\eta_0+0.5(B_{n-1}-1)^2\sigma^2_{\Delta\pi}(1-\rho^2) \\ B_n &= (B_{n-1}-1)\psi_0 \end{aligned}$$

The boundary conditions are $F^{\$}_{0,t}=1$, $A_0=0$, and $B_0=0$.

The proof is by induction. Suppose equation (A.11) is true for $P^{\$}_{n-1,t+1}$. Then, from the Euler equation it must be that

$$\begin{aligned} P^{\$}_{n,t} &= E_t[M_{t+1}\frac{\Pi_t}{\Pi_{t+1}}\exp\{A_{n-1}+B_{n-1}\Delta\pi_{n+1}\}F^{\$}_{n-1,t+1}] \\ &= E_t[M_{t+1}\exp\{-\eta_0-\psi_0\Delta\pi_t-\sigma_{\Delta\pi}v_{t+1}+A_{n-1}+B_{n-1}(\eta_0+\psi_0\Delta\pi_t+\sigma_{\Delta\pi}v_{t+1})\}F^{\$}_{n-1,t+1}] \\ &= \exp\{A_{n-1}+(B_{n-1}-1)(\eta_0+\psi_0\Delta\pi_t)\}E_t[M_{t+1}F^{\$}_{n-1,t+1}\exp\{(B_{n-1}-1)\sigma_{\Delta\pi}v_{t+1}\}] \end{aligned}$$

If we use the law of iterated expectations twice and condition on $\xi_{t+1|t}$, that is the posterior probability at time $t+1$, and then on ϵ_{t+1}, that is the error on the level of consumption growth we have

$$\begin{aligned} P^{\$}_{n,t} =& \exp\{A_{n-1}+(B_{n-1}-1)(\eta_0+\psi_0\Delta\pi_t) \\ & \sum_{j\in\{h,l\}}\xi_{t+1|t}(j)E_t[M_{t+1}F^{\$}_{n-1,t+1}\exp\{(B_{n-1}-1)\sigma_{\Delta\pi}v_{t+1}\}|\sigma_{\zeta_{t+1}}\epsilon_{t+1},\sigma_{\zeta_{t+1}}=\sigma_j,\xi_{t+1|t}] \end{aligned}$$

given that

$$(B_{n-1}-1)\sigma_{\Delta\pi}v_{t+1}|\sigma_j\epsilon_{t+1}\sim N(\rho(B_{n-1}-1)\sigma_{\Delta\pi}\epsilon_{t+1},(B_{n-1}-1)^2\sigma^2_{\Delta\pi}(1-\rho^2))$$

we have

$$\begin{aligned} P^{\$}_{n,t} =& \exp\{A_{n-1}+(B_{n-1}-1)(\eta_0+\psi_0\Delta\pi_t)+0.5(B_{n-1}-1)^2\sigma^2_{\Delta\pi}(1-\rho^2) \\ & \sum_{j\in\{h,l\}}\xi_{t+1|t}(j)E_t[M_{t+1}F^{\$}_{n-1,t+1}\exp\{\rho(B_{n-1}-1)\sigma_{\Delta\pi}\epsilon_{t+1}\}|\sigma_{\zeta_{t+1}}=\sigma_j,\xi_{t+1|t}] \end{aligned}$$

Therefore, equation (A.11) is satisfied with

$$\begin{aligned} F^{\$}_n(s_t) &= E_t[M_{t+1}\exp\{\rho(B_{n-1}-1)\sigma_{\Delta\pi}\epsilon_{t+1}\}F^{\$}_{n-1,t+1}] \\ A_n &= A_{n-1}+(B_{n-1}-1)\eta_0+0.5(B_{n-1}-1)^2\sigma^2_{\Delta\pi}(1-\rho^2) \\ B_n &= (B_{n-1}-1)\psi_0 \end{aligned}$$

A.4 Nominal risk premium

Let's compute the nominal risk premium

$$E_t\left(r^{\$}_{n,t+1}-r^{\$}_{1,t+1}\right) \tag{A.12}$$

Using formula (1.18) we have that

$$E_t\Big(r^{\$}_{n,t+1}\Big) = E_t\Big(\ln F^{\$}_{n-1}(s_{t+1}) + A_{n-1} + B_{n-1}\Delta\pi_{t+1} - \ln F^{\$}_{n}(s_t) + A_n + B_n\Delta\pi_t\Big) =$$
$$= cost + E_t\Big(\ln F^{\$}_{n-1}(s_{t+1})\Big) - \ln F^{\$}_{n}(s_t) + B_{n-1}\underbrace{(\eta_0 + \psi_0\Delta\pi_t)}_{E_t(\Delta\pi_{t+1})} - B_n\Delta\pi_t =$$
$$= cost + E_t\Big(\ln F^{\$}_{n-1}(s_{t+1})\Big) - \ln F^{\$}_{n}(s_t) + \psi_0\Delta\pi_t$$

where the last equality comes from $B_n = (B_{n-1} - 1)\psi_0$.
For the second term, we know that $r^{\$}_{1,t+1} = 1/\ln(M^{\$}_{t+1})$ and

$$E_t\Big(M^{\$}_{t+1}\Big) = E_t\Big(e^{-\Delta\pi_{t+1}}M_{t+1}\Big) =$$
$$= E_t[e^{-(\eta_0+\psi_0\Delta\pi_t+\sigma_{\Delta\pi}v_{t+1})}e^{\ln\delta-\gamma[g+(1-\phi)(\bar{s}-s_t)+(\lambda(s_t)+1)\sigma_{\zeta_{t+1}}\epsilon_{t+1}]}]$$

By using the same methodology that we applied for the formula of the nominal bonds, we have

$$E_t\Big(M^{\$}_{t+1}\Big) = \exp(\ln\delta - \gamma(g + (1-\phi)(\bar{s} - s_t)) - \eta_0 - \psi_0\Delta\pi_t + 0.5\sigma^2_{\Delta\pi}(1-\rho^2))$$
$$\sum_{j\in\{h,l\}} \xi_{t+1|t}(j)\exp(0.5(-\gamma(\lambda(s_t)+1)\sigma_j - \rho\sigma_{\Delta\pi})^2)$$

so

$$r^{\$}_{1,t+1} = 1/\ln(M^{\$}_{t+1}) =$$
$$= -\ln\delta + \gamma(g + (1-\phi)(\bar{s} - s_t)) + \eta_0 + \psi_0\Delta\pi_t - 0.5\sigma^2_{\Delta\pi}(1-\rho^2)) -$$
$$- \ln\Big(\sum_{j\in\{h,l\}} \xi_{t+1|t}(j)\exp(0.5(-\gamma(\lambda(s_t)+1)\sigma_j - \rho\sigma_{\Delta\pi})^2)\Big)$$

Therefore the nominal risk premium is

$$E_t\Big(r^{\$}_{n,t+1} - r^{\$}_{1,t+1}\Big) = cost + E_t\Big(\ln F^{\$}_{n-1}(s_{t+1})\Big) - \ln F^{\$}_{n}(s_t) -$$
$$- \gamma(1-\phi)(\bar{s} - s_t) + \ln\Big(\sum_{j\in\{h,l\}} \xi_{t+1|t}(j)\exp(0.5(-\gamma(\lambda(s_t)+1)\sigma_j - \rho\sigma_{\Delta\pi})^2)\Big) \tag{A.13}$$

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