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## **Essays on Secular Stagnation**

Keywords: secular stagnation, bubbles, monetary policy, natural rate, market structure

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## Abstract

The dissertation develops, in two chapters, two themes related to Secular Stagnation.

In Chapter 1, I focus on Secular Stagnation and Bubbles. My starting point is an estimated vector-autoregression model and I provide empirical evidence on the existence of stock market bubbles and their response to a deleveraging shock. I show that a deleveraging shock triggers a persistent decline in loans and output, while stock prices fall on impact and only partly recover afterwards. By decomposing the stock price index within its fundamental and bubbly component, I show that its behaviour is almost entirely explained by the latter. I propose an OLG model and I show that bubbles exist if agents are financially constrained. The bubbly steady state is unstable and, after a deleveraging shock, the economy eventually reaches the undesirable bubbleless steady state, where Secular Stagnation may arise. I show that, in a sticky prices environment, by adopting an accommodative stance towards bubbles, monetary policy can ensure the stability of the bubbly steady state and the stationarity of the dynamics around it.

In Chapter 2, I focus on Secular Stagnation and Market Structure. I address the question on whether the market structure affects the equilibrium level of the real interest rate, defined as the rate consistent with full employment and stable inflation. I provide an empirical and a theoretical analysis on the link between the markup, as a proxy for the market structure, and the equilibrium interest rate. I uncover some evidence that higher markups are associated with lower real rates and that more market friendly economies display higher interest rates. I propose an OLG model with monopolistic competition to interpret these findings. I focus on the effects on the equilibrium of a change in market structure, both in an exogenous and endogenous markup framework. I show that an increase in the markup puts a downward pressure on the equilibrium interest rate and the economy enters Secular Stagnation. The key transmission channel works through the market for capital.

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# Introduction

The dissertation develops, in two chapters, two themes related to Secular Stagnation. Hansen (1939) defines Secular Stagnation as "sick recoveries which die in their infancy and depressions which feed on themselves and leave a hard and seemingly immovable core of unemployment." Later, Summers (2014) refers to Secular Stagnation as "the hypothesis that the IS curve has shifted back and down so that the real interest rate consistent with full employment has declined". Although there isn't a unique definition, key features of a Secular Stagnation equilibrium are: persistently low, and negative, real interest rates; depressed output and unemployment.

In Chapter 1, I focus on Secular Stagnation and Bubbles. There is widespread perception that deleveraging shocks may trigger asset price bubbles through ultra low interest rates. Krugman (2013) and Summers (2014) mention the risk that, in the aftermath of the recent deleveraging, excessively low interest rates may generate search for yield and asset price bubbles. The issue is part of the wide debate on "Secular Stagnation". Indeed, a corollary of the "Secular Stagnation" hypothesis is that the economy may need bubbles to recover. My starting point is an estimated vector-autoregression model and I provide empirical evidence on the existence of stock market bubbles and their response to a deleveraging shock. I show that a deleveraging shock triggers a persistent decline in loans and output, while stock prices fall on impact and only partly recover afterwards. By decomposing the stock price index within its fundamental and bubbly component, I show that its behaviour is almost entirely explained by the latter. I propose an OLG model and I show that bubbles exist if agents are financially constrained. The bubbly steady state is unstable and, after a deleveraging shock, the economy eventually reaches the undesirable bubbleless steady state, where Secular Stagnation may arise. I show that, in a sticky prices environment, by adopting an accommodative stance towards bubbles, monetary policy can ensure the stability of the bubbly steady state and the stationarity of the dynamics around it.

In Chapter 2, I focus on Secular Stagnation and Market Structure. In outlining the theory of Secular Stagnation, Hansen (1939) focuses on the role played by the process of capital formation in achieving full employment and on the key effect of population growth.

However, he notices that other factors may affect the achievement of full employment, slowing down the process of capital formation and thus progress. The development of monopolistic competition is among these factors and "the workability of free enterprise" is one of the main policy challenges. Despite this significant role, little attention has been paid on the relation between market structure and Secular Stagnation. I address the question on whether the market structure affects the equilibrium level of the real interest rate, defined as the rate consistent with full employment and stable inflation. I uncover some evidence that higher markups are associated with lower real rates and that more market friendly economies display higher interest rates. I propose an OLG model with monopolistic competition to interpret these findings. I focus on the effects on the equilibrium of a change in market structure, both in an exogenous and endogenous markup framework. I show that an increase in the markup puts a downward pressure on the equilibrium interest rate and the economy enters Secular Stagnation. The key transmission channel works through the market for capital. I assume that households can either save by making loans to young generations, whose borrowing is constrained by an exogenous limit, or by renting capital to firms. Capital is then used as a production input within the same period and pays a rental rate. I show that an increase in the markup, either exogenous or triggered by a decrease in the number of firms, reduces profits, production and return on capital. The equilibrium level of capital decreases leading to an excess of supply in the loans market which puts a downward pressure on the equilibrium interest rate. The markup thus magnifies the effects of a negative, supply-side, shock by transmitting it to aggregate demand.

# Chapter 1

## Deleveraging, Stock Market Bubbles and Monetary Policy

# Introduction

There is widespread perception that deleveraging shocks may trigger asset price bubbles through ultra low interest rates. Krugman (2013) and Summers (2014) mention the risk that, in the aftermath of the recent deleveraging, excessively low interest rates may generate search for yield and asset price bubbles. The issue is part of the wide debate on "Secular Stagnation", that is the concern expressed by Hansen (1939) and recalled by Summers (2014) that a negative equilibrium interest rate makes the economy stuck at suboptimal growth and deflation. A corollary of the "Secular Stagnation" hypothesis is that the economy may need bubbles to recover.

The objective of the project is to evaluate the role of stock market bubbles following a deleveraging shock. My starting point is a vector-autoregression (VAR) model on quarterly US data for GDP, the GDP deflator, a commodity price index, dividends, the federal funds rate, the S&P500 index and the series of loan volume. The focus is on the response of stock prices to an exogenous reduction in the loan volumes. The main results of the empirical analysis are the following. First, I show that a deleveraging shock is contractionary, and leads to a persistent decline in loan volumes, in GDP and in the GDP deflator. Stock prices fall on impact and only partly recover afterwards. Second, by decomposing the stock index within its fundamental and bubbly component, I show that its behaviour is almost entirely explained by the latter. Third, by allowing for time varying coefficients, the results confirm the relevance of the bubbly component in the recovery of the S&P500 and show the significant rise in its importance from the 90s, with a predominance in the very last part of the sample. Finally, I show that over time, the stochastic volatility of the stock price index has shown significant variation, and that there has been a convergence in the contribution of the deleveraging and monetary policy shocks.

I propose a 3-periods overlapping generations model with financial constraints and bubbles, and I focus on the existence condition of bubbles and on the effects on the equilibrium allocation of a deleveraging shock. I show that when agents are financially constrained, they positively value bubbles to smooth consumption and to invest the excess savings. The existence of bubbles raises the equilibrium interest rate and improves

the efficiency level at which the economy operates. However, the bubbly steady state is unstable, and any deleveraging shock eventually pushes the economy towards the bubbleless steady state, where Secular Stagnation may arise. I extend the economy to allow for output to be endogenously determined and for nominal rigidities. Monetary policy reacts to inflation and bubbles through an augmented Taylor rule. Formally, the existence of bubbles entails the AD curve containing an additional component, the bubble, which has implications for the transmission of monetary policy to the real economy. The effect of bubbles on aggregate demand strictly depends on whether the central bank reacts by "leaning against the wind" or through an accommodative stance. I show that, in the presence of binding nominal rigidities, if monetary policy adopts an accommodative stance towards bubbles, it can guarantee the existence of a stable bubbly steady state and stationarity of the dynamics around it, though it can not prevent a fall in output, inflation and bubble after a deleveraging shock.

The remainder of the paper is organized as follows. In section 1, I discuss the relation with the existing literature. Section 2 discusses the empirical motivation. In Section 3, I develop the baseline theoretical model with financial constraints and bubbles in a pure endowment economy. I focus on the role of bubbles following a deleveraging shock. I then extend the framework to allow for output to be endogenously determined and nominal rigidities. I discuss the steady states of the extended economy in Section 4 and the stochastic version in Section 5. Conclusions follow.

## **1.1 Relation with the existing literature**

The research contributes to the literature from an empirical and a theoretical perspective. For the empirical analysis, the paper relates to Furlanetto et al. (2014), Galì and Gambetti (2015) and Furlanetto et al. (2016) who focus on the reaction of stock prices to a monetary policy shock, and to Musso et al. (2011) who focus instead on the reaction of stock prices to loan supply shocks. Following Galì and Gambetti (2015), I decompose the asset price series within the fundamental and bubbly component and I study the reaction of stock prices and the main variables to an exogenous reduction of the loan volumes. The novelty of my contribution is to focus on the reaction to deleveraging shocks within

a framework which allows me to decompose the asset price series within the fundamental and bubbly component. I find that the reaction of the bubbly component differs from the fundamental one, and it helps explain the reaction of the stock prices series.

The theoretical model relates to three main strands in the literature. First of all, the paper relates to the literature on rational bubbles started by the seminal paper, Samuelson (1958). In line with Samuelson (1958) and Tirole (1985), I develop a theory of bubbles as remedy for dynamic inefficiency. In line with Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Martin and Ventura (2012, 2015) and, Dong et al. (2017), I introduce financial frictions and I show that bubbles exist when borrowing constraints bind. Moreover, the size of the bubble in the bubbly steady state increases as the borrowing constraint gets tighter. I model financial frictions in the baseline model following Jappelli and Pagano (1994).

The paper is also related to the wide literature on bubbles and monetary policy and to the "lean-vs-clean" debate. The paper is close to Galì (2014) and Dong et al. (2017). Galì (2014) examines the impact of monetary policy on asset bubbles through a simple OLG model with nominal rigidities. First of all, by the specific price in advance setting assumption, neither the existence nor the allocations associated with a deterministic bubbly equilibrium are influenced by monetary policy. In the stochastic scenario, "leaning against the wind" increases the volatility of the bubble growth and optimal monetary policy faces a trade-off between minimization of the volatility of the bubble and aggregate demand. I depart from Galì (2014) in three main ways. I allow for the existence of financial frictions and I show that the existence of bubbles depends on financial conditions. Secondly, I assume fixed supply for bubbles rather than creation of bubbles. In line with Tirole (1985) I consider scarcity one of the peculiarities of the bubbly asset. In Galì (2014), allowing for creation of bubbles guarantees the existence of a stable bubbly steady state, independently on monetary policy. Thirdly, I model price stickiness in the form of downward nominal wage rigidity. I show that, under binding nominal rigidity constraints, the stability of the deterministic bubbly steady state, and the stationarity of fluctuations around it, depend on the monetary policy's decision rule. A benign neglect approach as the strict inflation targeting regime proposed by Bernanke and Gertler (2000, 2001), has detrimental effects in my framework. In fact, like the "leaning

against the wind” approach, it leads to unstable patterns and the economy eventually reaches the bubbleless steady state equilibrium. Dong et al. (2017) adopt an infinite-horizon model with heterogeneous entrepreneurs and banking sector where bubbles play also a role as collateral. Through a richer environment than Galì (2014), they reach a similar conclusion, that ”leaning against the wind” may not be optimal, because, in their framework, it raises the volatility of inflation, though reducing the bubble’s one. Differently from Galì (2014), they show that monetary policy can affect the condition for the existence of the bubble and whether monetary policy should respond to asset bubbles depends on the particular interest rate rule and on the exogenous shock hitting the economy. The key reason for this result lies in the coexistence with money and a reserve requirement which generates a liquidity premium for the bubble. However, if the central bank follows a policy rule which targets the inflation and bubble, then the optimal coefficient on the asset bubble is negative but close to zero, and the gains from responding to the bubble are smaller the higher the coefficient associated with inflation. Differently from Dong et al. (2017), I show that the existence condition of bubbles depends on the response of monetary policy to bubbles. I uncover a specific relation between the coefficients of the interest rate rule, and I show that the higher the inflation coefficient, the more negative the coefficient associated with the bubble that ensures the existence and stability of the bubbly steady state. Moreover, while the main shocks in Dong et al. (2017) are a TFP and a non-fundamental sentiment shock, I focus on the effect of a deleveraging shock.

Finally, the paper builds on the literature on deleveraging and Secular Stagnation. The paper is close to Kocherlakota (2013) and Eggertsson et al. (2017). Kocherlakota (2013) shows that in an incomplete labor markets equilibrium, given a fixed interest rate, employment, and thus output, have to fall, after a contractionary shock, so as to satisfy the household’s intertemporal Euler equation. Eggertsson et al. (2017) first propose a theoretical framework to formalize the theory of Secular Stagnation. In particular, they propose an overlapping generations New Keynesian model in which one of the triggers for a permanent slump is a deleveraging shock, which creates an oversupply of savings and causes a permanent drop in the interest rate. I depart from Eggertsson et al. (2017) mainly in two ways. I allow agents to trade useless assets, the bubble, and I assume

that monetary policy adopts an augmented Taylor Rule and reacts to bubbles. I show that monetary policy should adopt an accommodative approach towards bubbles to avoid Secular Stagnation. Other papers discuss the role of bubbles within Secular Stagnation. Teulings (2016) shows that although bubbles are welfare enhancing, they are risky and do not implement the first best. A simple fiscal policy rule that commits to issue public debt, allowing for intragenerational transfers, turns out to be preferable under the financial stability point of view. Boullot (2016) shows that bubbles restore efficiency at the possible cost of a drop in potential output, and that their existence is independent of monetary policy. However, their macroeconomic effects depend on whether the economy is outside a liquidity trap. In line with Teulings (2016), he shows that fiscal policy is the right tool to prevent the formation of rational bubbles. Finally, Asriyan et al. (2016) discuss the role of conventional inflation targeting and unconventional monetary policy in a bubbly world where the pop up and burst of bubbles generate fluctuations in credit, investment and output. I focus on the stabilization role of monetary policy endowed with a Taylor-type reaction function.

## 1.2 Empirical Motivation

I estimate the response of stock prices to a deleveraging shock using a constant and a time-varying coefficients structural VAR. I follow Galì and Gambetti (2015) and I recover the fundamental component of the stock price series by computing the present value of the stream of dividends. Therefore, I am able to decompose the actual stock price series within the fundamental and bubbly component. I show that a deleveraging shock is contractionary and triggers a persistente decline in loans volume, output, inflation and, on impact, also in the stock price. The difference observed between the actual and theoretical S&P500 suggests that a significant bubbly component exists. The latter declines sharply on impact but recovers quickly afterwards in correspondence of a strong decrease in the federal funds rate. The bubbly component partly recovers jointly with GDP and the other variables after the contractionary shock.

### 1.2.1 The empirical model

**Data** I use quarterly US time series for the sample period 1960Q1-2011Q4. I augment the dataset in Galì and Gambetti (2015) with the series for loan volumes. The dataset contains series for GDP and its deflator, the World Bank commodity price index, the federal funds rate, the S&P500 stock price index and corresponding dividend series (both deflated by the GDP deflator series). As regards to loan volumes, I consider the series of outstanding amount of loans granted by financial intermediaries to private non-financial sector<sup>1</sup> (deflated by GDP deflator).

**Fundamental and bubbly component of stock prices** I follow Galì and Gambetti (2015) and I consider the price of an asset as  $Q_t = Q_t^F + Q_t^B$ . I define the fundamental component as the present discounted value of future dividends:

$$Q_t^F \equiv E_t \left\{ \sum_{k_1=0}^{\infty} (\Pi_{j=0}^{k_1-1} (1/R_{t+j})) D_{t+k} \right\}$$

or, rewriting it in log-linear form,

$$q_t^F = \text{const} + \sum_{k=0}^{\infty} \Lambda^k [(1 - \Lambda) E_t \{d_{t+k+1}\} - E_t \{r_{t+k}\}]$$

where lower case letters denote the logs of the original variables, and  $\Lambda \equiv \frac{\Gamma}{R} < 1$  is the ratio of the gross rate of dividend growth and interest rate along a balanced growth path.

Henceforth, it is possible to recover the bubbly component as the difference between the actual stock price series and the fundamental component.

**Identification** I choose a recursive (i.e. Cholesky) identification scheme as the baseline for short-run restrictions. Following Musso et al. (2011), I assume that the loan volumes series is the most endogenous variable. Moreover, I assume that neither GDP nor inflation and dividends are affected contemporaneously (within the same quarter) by a monetary

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<sup>1</sup>Source: BIS Total Credit Statistics. The series is Total Credit to Private Non-Financial Sector, Adjusted for Breaks, for United States, provided by domestic banks, all other sectors of the economy and non-residents. The "private non-financial sector" includes non-financial corporations (both private-owned and public-owned), households and non-profit institutions serving households as defined in the System of National Accounts 2008.

policy shock. Finally, I make two additional assumptions, that monetary policy does not react contemporaneously to innovations in the volume of loans and in the real stock prices, and that stock prices do not react contemporaneously to shocks in the volume of loans. The short-run restrictions that I use seem to do a good job in recovering shocks that are structurally interpretable not only based on the visual analysis of the impulse responses but also in line with the assumption for the sign restrictions in Gambetti and Musso (2016).

Let  $x_t \equiv [\Delta y_t, \Delta p_t^c, \Delta d_t, \Delta p_t, i_t, \Delta q_t, \Delta l_t^s]$  be the vector of variables where  $y_t$ ,  $p_t^c$ ,  $d_t$ ,  $p_t$ ,  $i_t$ ,  $q_t$  and  $l_t^s$  are the (log) output, (log) price level, (log) dividend series, (log) commodity price index, the short-term nominal interest rate, the (log) stock price index and (log) loans volume. The relationship between the variables and structural shocks can be represented by the following autoregressive model:

$$x_t = A_0 + A_1 x_{t-1} + \dots + A_p x_{t-p} + u_t$$

where  $A_0$  is the vector of intercepts and  $A_i$  for  $i = 1, \dots$  are matrices of coefficients. The lag-order is four. The vector of reduced form innovations,  $u_t$ , follows a white noise Gaussian process with mean zero and covariance matrix  $\Sigma$ . I assume that the reduced form innovations are a linear transformation of the underlying structural shocks  $\epsilon_t$ :

$$u_t \equiv S \epsilon_t$$

where  $E \epsilon_t \epsilon_t' = I$  and  $E \epsilon_t \epsilon_{t-k}' = 0$  for all  $t$  and  $k = 1, 2, 3, \dots$  and  $S$  is such that  $SS' = \Sigma$ .

Letting the fifth element in  $\epsilon_t$  be the monetary policy shock, then the adopted identification scheme implies that the fifth column of  $S$  has zeros as its first four elements, while the remaining two are unrestricted. The second assumption implies that the sixth and seventh elements in the fifth row of  $S$  are zero. The third and last assumption implies that the last element in the sixth row of  $S$  is zero as well. Let  $S$  be the Cholesky factor of  $\Sigma$ ,  $S = FD^{\frac{1}{2}}$ , where  $D$  is a diagonal matrix such that  $\Sigma = FDF'$  and  $F$  is lower triangular, with ones on the main diagonal.

The impulse response functions are then recovered as follows. Rewriting the model in companion form,

$$x_t = \mu + Ax_{t-1} + u_t$$

where  $x_t \equiv [x'_t, x'_{t-1}, \dots, x'_{t-p+1}]'$ ,  $u_t \equiv [u'_t, 0, \dots, 0]'$ ,  $\mu \equiv [A'_0, 0, \dots, 0]'$  and  $A$  is the corresponding companion matrix. By a local approximation,

$$\frac{\partial x_{t+k}}{\partial \epsilon_t^i} = \frac{\partial x_{t+k}}{\partial u'_t} \frac{\partial u_t}{\partial \epsilon_t^i} = B_k S^i \equiv C_k$$

where  $B_k \equiv [A^k]_{7,7}$ , given that  $[A^k]_{7,7}$  represents the first 7 columns and 7 rows of corresponding companion matrix.

## 1.2.2 Impulse responses to a deleveraging shock Constant Coefficients VAR

Figures 1.1 and 1.2 show the (cumulative) impulse responses of the main variables to a negative deleveraging shock, where the dashed lines delimit the space between the 16% and 84% percentiles. A tightening in the volume of loans leads to a persistent decline in loan volumes, in GDP and in the GDP deflator. The response of dividends is on impact similar to that of GDP but then they recover back to the initial level. As Fig. 1.2a shows, the stock price index also sharply declines in the short-run, but it recovers subsequently and ends up reaching a higher level than the initial one. Figure 1.2b shows the implied response of the fundamental component of the stock price. The latter increases on impact, given the relative stronger effect of lower interest rates, and then stabilizes at a slightly higher level than the initial one. The results show that there is a substantial difference, both in terms of the behaviour on impact and speed of recovery afterwards, between the response of the stock price index and its fundamental component which points to the existence of a non-neglegible bubbly component. Moreover, the behaviour of the latter goes in the opposite direction with respect to that of the fundamental component. The relative strenght of the bubbly component explains the actual behaviour of the stock price index. In fact, as Figure 1.2c shows, the bubble declines on impact but fastly recovers, in correspondence of a significant drop in the federal funds rate. The pattern of the bubble follows the economy's recovery afterwards. Finally, the gap between the observed and estimated fundamental price, as measured by the bubble, increases over time although the confidence bars are too large to reject the absence of different long run effects.

### 1.2.3 Evidence of time-variation

Following Galì and Gambetti (2015), I estimate a time-varying coefficients SVAR through Bayesian MCMC methods, and more specifically through the Gibbs sampling algorithm<sup>2</sup>. Figure 1.3 shows the evolution of the residual time-varying standard deviations. In particular, Figure 1.3 plots the posterior mean, 16th and 84th percentiles of the standard deviation of residuals of the GDP equation, of the interest rate equation (or monetary policy shocks), of the S&P500 Index equation, and of the loan volume equation (or loans shocks). For the GDP and the short-term interest rate, there are evident signs of a decrease in volatility. The pattern supports the "Great Moderation", in the late 80s. The asset prices and loan volume series do not share the same pattern. After an initial reduction and a low volatility period beginning in the early 1980s, the volatility of both the S&P500 and the loan volume series increases again at the beginning of the 2000s with a peak in 2009<sup>3</sup>. Overall, the evidence justifies the adoption of a stochastic volatility setup.

### 1.2.4 Impulse responses to a deleveraging shock

#### Time-varying Coefficients VAR

Figures 1.4 and 1.5 show the (cumulative) time-varying impulse response functions to a deleveraging shock. Loans, GDP, and its deflator display a persistent decline after the shock, particularly acute in the second part of the sample. The same holds true for dividends except for a brief period in the 1980s where the tightening in the loan volumes seems to have a slight positive impact on dividends. The reaction of the interest rate is particularly accommodative, though stronger in the first part of the sample. Concerning the reaction of stock prices, Figure 1.5a shows that the S&P500 declines on impact, and it recovers quickly afterwards. Observe how the rise in the S&P500 is particularly acute starting in the early 1990s. Notice that the pattern for the response of the fundamental component, Figure 1.5b, has changed little over time and it is pretty stable, in line with

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<sup>2</sup>See Primiceri (2005) and Galì and Gambetti (2015) for a detailed discussion of the procedure.

<sup>3</sup>As Musso et al. (2011) observe, phenomena related to credit conditions and asset prices may imply non-linear dynamics, especially in times of financial distress. Thus large residual standard deviations may suggest that the assumed linear structure of the model may not be satisfactory.

what obtained with the constant coefficients VAR. However, the relative importance of the fundamental component has decreased over time. Similarly to what shown in the previous section, as Figure 1.5c shows, the bubbly component drops on impact and fastly recovers in correspondence of the sharp decline in the nominal interest rate around the fourth/fifth quarter.

The rise in the bubbly component is particularly significant from the 1990s and predominant in the very last part of the sample. Moreover, the last part of the sample is also characterized by the lowest recovery in terms of output. The latter evidence could be related to distortions created by the zero lower bound constraint<sup>4</sup> and/or to the unconventional monetary policy measures.

### 1.2.5 Variance Decomposition

Figure 1.6 plots the estimates of the time varying volatility of asset prices conditional on deleveraging shocks, as implied by the estimated structural VAR. As a reference, I also plot the volatility conditional on monetary policy shocks, and the unconditional volatility. The pattern that emerges offers several insights. First of all, there is evidence of significant time-variation in the residual variances, with spikes in correspondence to the latest crises. The pattern of the S&P 500 shows similarity to that of real GDP (see also Gambetti and Musso (2014)). Secondly, both deleveraging and monetary policy shocks contribute to asset prices volatility. Starting from a dominant role of monetary policy shock in the 1970s, there has been a gradual convergence in the contribution of both shocks with their weights being extremely close since the late 1980s. The results extends the findings in Gambetti and Musso (2014) who show that the fraction of real GDP growth variance explained by loan volume shocks has increased in the recent decades.

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<sup>4</sup> The evidence may suggest that an insufficiently accommodative monetary policy, either because of a binding ZLB constraint or because of a "leaning against the wind" approach, is not beneficial. This latter phenomenon reminds of the Japanese experience. As Bernanke and Gertler (2000) show, the policy response of the BOJ to asset prices changed from being accommodative in the 79-89 to "leaning against the wind" in the 89-97, resulting in a "too tight" stance, especially in the face of the burst of the bubble occurred in 1991.

### 1.2.6 Sensitivity

I re-estimate the CC-SVAR by changing the ordering of the variables. The alternative specifications lead to results which are almost close to the baseline's ones. I consider the following alternatives:

$$x_t \equiv [\Delta y_t, \Delta p_t^c, \Delta d_t, \Delta p_t, \Delta q_t, \Delta l_t^s, i_t]$$

$$x_t \equiv [\Delta y_t, \Delta p_t^c, \Delta d_t, \Delta p_t, \Delta q_t, i_t, \Delta l_t^s]$$

$$x_t \equiv [\Delta y_t, \Delta p_t^c, \Delta d_t, \Delta p_t, \Delta l_t^s, \Delta q_t, i_t]$$

The first ordering implies that monetary policy responds to contemporaneous innovations both in the stock prices and in the volume of loans. By the second ordering, monetary policy responds only to contemporaneous innovations in the stock prices. Finally, by the third ordering, stock prices react contemporaneously to loans shock. Results are robust, in line with the baseline model.

## 1.3 A simple model

I develop a 3-periods deterministic, pure endowment overlapping generations model with population growth. Each individual receives income only in his middle age and no outside asset is available, but generations can borrow and lend to each other. In this case, young households borrow from the middle-aged ones, which in turn save for retirement when old. All cohorts are identical and population grows at a constant rate  $g = \frac{N_t}{N_{t-1}} - 1$ , where  $N_t$  is the size of population at time  $t$ . The objective of each individual is to maximize utility from consumption:

$$\log C_t^y + \log C_{t+1}^m + \log C_{t+2}^o \tag{1.1}$$

where I have assumed a unitary subjective discount factor.  $C^y$ ,  $C^m$  and  $C^o$  are consumptions of the household when young, middle-aged and old. Maximization of utility (1.1) is subject to the following constraints:

$$C_t^y = B_t^y \quad (1.2)$$

$$C_{t+1}^m = Y - (1 + r_t)B_t^y + B_{t+1}^m \quad (1.3)$$

$$C_{t+2}^o = -(1 + r_{t+1})B_{t+1}^m \quad (1.4)$$

$$(1 + r_t)B_t^y \leq \gamma Y \quad (1.5)$$

where equations (1.2), (1.3) and (1.4) are the budget constraints facing households of the generation born at time  $t$  in each period.  $B^y$  is the consumption loan from the middle-aged to the young, while  $B^m$  constitutes savings for the old age.  $Y$  is middle-aged income and  $r_t$  and  $r_{t+1}$  are the market real interest rate for  $t$ - $(t+1)$  and  $(t+1)$ - $(t+2)$ . Finally, Eq. (1.5) corresponds to an endogenous borrowing limit facing the household when young as in Jappelli and Pagano (1994). The constraint limits the amount of debt that the young can credibly commit to repay and, therefore, includes interest payments.

Denote by  $\phi$  the optimal fraction of lifetime income that the agent would borrow and consume when young in a frictionless economy<sup>5</sup>, then

**Assumption 1**  $\gamma < \phi$

If Assumption 1 holds, the borrowing constraint is always binding. Assumption 1 requires that the parameter  $\gamma$ , namely the loan-to-value ratio, is smaller than the optimal fraction, of lifetime income that the agent would borrow and consume when young in a frictionless economy.

By Assumption 1, then young agent's consumption equals the borrowing limit:

$$C_t^y = \frac{\gamma Y}{1 + r_t} \quad (1.6)$$

and the old at any time  $t$  consumes all his income so that:

$$C_t^o = Y - (1 + r_{t-1})B_{t-1}^m$$

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<sup>5</sup>In the appendix I provide the analysis of the benchmark, frictionless economy.

By optimality, the relevant, standard Euler Equation linking consumption growth to the real interest rate is the one between middle and old ages:

$$\frac{C_{t+1}^o}{C_t^m} = (1 + r_t) \quad (1.7)$$

Following Eggertsson et al. (2017), I analyze the equilibrium determination by focusing on equilibrium in the market for savings. By Walras' law goods market equilibrium holds. Equilibrium in the savings market requires demand for loans to equal supply:

$$N_t B_t^y = -N_{t-1} B_t^m$$

After normalizing by the middle-aged population size, the equilibrium condition in the savings market can be rewritten as

$$B_t^y(1 + g) = -B_t^m \quad (1.8)$$

Notice that the constraint on borrowing, Eq. (1.5), together with market clearing, Eq. (1.8), translates into a constraint on saving.

An *equilibrium* is a set of processes  $\{C_t^y, C_t^m, C_t^o, r_t, B_t^y, B_t^m\}$  that solve the households' problem and market clearing holds, given  $\gamma$  and  $g$ .

I rewrite Eq. (1.8) by defining the left-hand side as demand,  $L_t^d$ , and the right-hand side as supply of consumption loans,  $L_t^s$ . Demand for loans comes from young agents, using Eq. (1.6) to replace for  $B_t^y$ , while supply depends on net disposable income of middle generations. I recover the supply of loans by substituting out for  $C_t^m$  in the budget constraint for the middle-aged, Eq. (1.3), using the budget constraint for the old, the Euler Equation and the borrowing constraint for the young, so that:

$$L_t^d = \frac{(1 + g)}{(1 + r_t)} \gamma Y \quad (1.9)$$

$$L_t^s = \frac{1}{2} Y (1 - \gamma) \quad (1.10)$$

Notice that given the assumption on income, supply is perfectly unelastic and independent of the interest rate. By equating demand and supply, then I get the following equilibrium interest rate:

$$1 + r_t = (1 + g) \frac{2\gamma}{(1 - \gamma)} \quad \forall t \quad (1.11)$$

The equilibrium interest rate is a function of the growth rate and financial condition, and the tighter the financial constraints, the lower the equilibrium interest rate. Depending on the values of the parameters  $\gamma$  and  $g$ , the equilibrium interest rate may be negative and the economy may enter Secular Stagnation.

Moreover, the equilibrium allocation is dynamic inefficient. As shown in the appendix, for an allocation to be dynamically inefficient, the following condition must hold

$$r < g$$

Then, by Eq. (1.11), it follows that the allocation is dynamic inefficient whenever financial constraints are binding, or equivalently if

$$\gamma < \phi \quad (1.12)$$

In line with an established result in the literature, the existence of financial frictions generates low interest rates and dynamic inefficient equilibrium allocations<sup>6</sup> which makes room for a welfare improvement.

In the Appendix I solve the household's problem in a unique framework by providing the analysis of the Lagrangian multiplier. Coherently, the constraint is binding whenever  $r < g$ . Moreover, the lower the loan-to-value ratio,  $\gamma$ , the higher the shadow price of the constraint.

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<sup>6</sup>There has been an extensive debate on whether actual economies satisfy the dynamic efficiency condition. Firstly, Abel et al. (1989) translate the condition  $r > g$  into a relation between capital income and investment. They show that economies are indeed dynamic efficient since capital income is always higher than investment. Geerolf (2013) revises their calculation of capital income showing that they are in fact too sanguine. He shows that when one takes also into account decreasing return and monopoly power, then the condition for dynamic efficiency are verified for none of the advanced economies.

### 1.3.1 Rational Bubbles

I consider now the case that agents have the possibility to save through an outside asset, useless otherwise. In line with the literature on pure rational bubbles starting with the seminal paper by Samuelson (1958), I show that bubbles are welfare enhancing. Households maximize utility (1.1) subject to the budget constraints

$$C_t^y = B_t^y \tag{1.13}$$

$$C_{t+1}^m = Y - (1 + r_t)B_t^y + B_{t+1}^m - p_{t+1}^b a_{t+1} \tag{1.14}$$

$$C_{t+2}^o = -(1 + r_{t+1})B_{t+1}^m + p_{t+2}^b a_{t+1} \tag{1.15}$$

and the borrowing constraint

$$(1 + r_t)B_t^y \leq \gamma Y \tag{1.16}$$

In addition to the previous framework, agents have the possibility to save through an additional asset. The agent can purchase a quantity  $a_{t+1}$  paying a price  $p_{t+1}^b$  in his middle-age and he can sell the asset at a price  $p_{t+2}^b$  when old. Bubbles are in fixed supply<sup>7</sup>, equal to 1, and creation and trade come at zero cost. A bubble exists whenever the price of the asset is above its fundamental value, and since the newly introduced asset doesn't pay any dividend, then its fundamental value equals zero and a bubble exists whenever this asset is positively valued.

By optimality, Eqs. (1.6) and (1.7) hold together with the following no arbitrage condition:

$$p_t^b = \frac{p_{t+1}^b}{(1 + r_t)} \tag{1.17}$$

Eq. (1.17) states that the current price of the bubble must equal the price at which agents sell it in the future, discounted by the market interest rate.

In equilibrium markets for savings and for bubbles must clear. Hence, Eq. (1.8) holds together with market clearing for the bubbly asset:

$$N_{t-1}a_t = 1 \quad \forall t$$

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<sup>7</sup>The assumption of unit supply, as in Tirole (1985) and Farhi and Tirole (2012), can be relaxed. For example, Gali (2014) and Martin and Ventura (2012, 2015) assume creation of bubbles.

Letting  $b_t = p_t a_t$  denote the value of the bubble in per capita terms, at date  $t$ , I rewrite Eq. (1.17) as follows

$$b_{t+1} = \frac{1 + r_t}{1 + g} b_t \quad (1.18)$$

An *equilibrium* is a set of quantities  $(C_t^y, C_t^m, C_t^o, a_t, B_t^y, B_t^m)$  and prices  $(r_t, p_t^b)$ , given  $\{\gamma, g\}$ , which satisfy household's optimality, (1.6), (1.7), (1.18), together with the budget constraints, and market clearing conditions hold.

A bubbleless allocation, in which the price of the bubble is zero, is always an equilibrium for this economy. In fact,  $b_t = b_{t+1} = 0$  satisfies Eq. (1.18). I focus on the bubbly economy, such that  $b_t > 0$ . Proceeding as in the previous section, I analyze equilibrium determination by focusing on equilibrium in the market for savings. Therefore, I recover demand and supply of loans. While demand for loans is unchanged with respect to the bubbleless scenario, Eq. (1.9), supply is affected by the introduction of bubbles. The new supply schedule is:

$$L_t^s = \frac{1}{2} \left[ Y(1 - \gamma_{t-1}) - \left( b_t + b_{t+1} \frac{(1 + g)}{(1 + r_t)} \right) \right] \quad (1.19)$$

Supply of loans now depends on the interest rate and on the growth rate. Moreover, *ceteris paribus*, the supply of loans in the presence of bubbles is lower than the one in the absence of bubbles.

By equating demand and supply for loans, Eqs. (1.9) and (1.19), I get the equilibrium interest rate for loans:

$$1 + r_t = 2 \frac{(1 + g)\gamma_t Y}{Y(1 - \gamma_{t-1}) - b_t} + \frac{b_{t+1}(1 + g)}{Y(1 - \gamma_{t-1}) - b_t} \quad (1.20)$$

and after replacing (1.18) within (1.20), I get the equilibrium interest rate for the bubbly economy:

$$1 + r_t = 2 \frac{(1 + g)\gamma_t Y}{Y(1 - \gamma_{t-1}) - 2b_t} \quad (1.21)$$

The beneficial effects of the existence of bubbles are easily identifiable. Since  $b_t$  is strictly positive, the interest rate in the bubbly financially constrained economy is higher than

the interest rate in the corresponding economy in the absence of bubbles, see Eq. (1.11). Moreover, Figure 1.7 shows that bubbles are indeed welfare improving. I measure welfare as the individual's lifetime utility. For a given value of  $\gamma$ , welfare in the presence of bubbles is higher than welfare in the absence of bubbles. The reason is that bubbles raise welfare by helping agents to smooth consumption.

### 1.3.2 Equilibrium Dynamics

It is possible to recover the equilibrium dynamics by substituting out for  $r_t$  using Eq. (1.20) within the no arbitrage condition, Eq. (1.18).

A *deterministic bubbly equilibrium* is defined by a sequence  $\{b_t\}$  satisfying the following non linear difference equation:

$$b_{t+1} = f(b_t; Y, \gamma_t, \gamma_{t-1}) = \frac{2b_t\gamma_t Y}{Y(1 - \gamma_{t-1}) - 2b_t} \quad (1.22)$$

The function  $f()$  maps any  $b_t$  to a unique  $b_{t+1}$  and implies that, given  $\{b_0\}$ , there exists a unique path for  $\{b_t\}_{t=0}^{\infty}$ . Knowing the value of the bubble, it is possible to recover the equilibrium values for the remaining variables. The interest rate is uniquely determined by (1.21). Consumptions for the young, middle and old generations follow. Figure 1.8 shows the equilibrium dynamics and the two steady states<sup>8</sup>.

**Proposition 1** If  $r < g$ , and  $\gamma < \phi$ , given

$$\tilde{b}_0 = \frac{(1 - 3\gamma)Y}{2} \quad (1.23)$$

then, for any  $b_0 \in [0, \tilde{b}_0)$ , the equilibrium is asymptotically bubbleless and the interest rate converges to  $r = (1 + g)^{\frac{2\gamma}{(1-\gamma)}} - 1$ , as defined in Eq. (1.11). There exists a unique equilibrium with initial bubble  $b_0 = \tilde{b}_0$ . The equilibrium is stationary and the interest rate equals  $g$ . Proof in the Appendix.

Firstly, Proposition 1 states that the existence of bubbles is conditioned by the inefficiency of the bubbleless steady state and that, despite financial frictions, bubbles

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<sup>8</sup>I consider the following value for the parameters:  $Y = 10$ ,  $\gamma < \phi$ .

are welfare enhancing<sup>9</sup>. The existence of bubbles raises the level of efficiency at which the economy operates back to the benchmark, frictionless one: at the unique bubbly steady state, the interest rate equals the growth rate, and the economy is away from a potential Secular Stagnation. Notice that  $b_0$  is not predetermined but depends on household's choices made in  $t_0$ . Secondly, Proposition 1 clarifies the asymptotic behaviour of the economy and states that the bubbly steady state is unstable<sup>10</sup>.

### 1.3.3 Deleveraging

In this section, I focus on the effect on the equilibrium and on the bubble of a deleveraging shock. The motivation for addressing this question is twofold. On the one hand, from an empirical perspective, loan shocks are a key driver in the asset price, and asset price bubbles, volatility. On the other hand, from a theoretical point of view, financial conditions are shown to affect the existence condition of the bubble.

Thus, I consider a sudden and permanent reduction in the loan-to-value ratio,  $\gamma$ . Following Eggertsson and Krugman (2013), a deleveraging shock has been modeled as a downward revision in the borrowing limit. Figure 1.9 shows the equilibrium dynamics for a high  $\gamma$ , the light blue line, and a low  $\gamma$ , the dark blue line. Suppose that the economy starts at the bubbly steady state A, and a deleveraging shock occurs. The curve moves rightwards and, by definition (1.23), as  $\gamma$  drops, the new bubbly steady state level, C, consistent with tighter financial conditions is higher than the original one<sup>11</sup>. However, as Proposition 1 states, the bubbly steady state is unstable. Therefore, after the shock, the economy eventually crashes towards the bubbleless steady state, B, where Secular Stagnation may arise.

The result justifies the extension to an endogenous output framework with nominal

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<sup>9</sup>Bubbles emerge as a market based mechanism that allows for redistribution. There exist other interpretations for these inter- or intra- generational transfer schemes besides stock market bubbles as pure money, safe assets, government transfers, public debt as in Teulings (2016), or debt relief as in Fornaro (2013). Depending on the interpretations, different policy implications arise.

<sup>10</sup>A stable bubble steady state exists in a flexible prices economy if the economy features transaction costs or creation of bubbles.

<sup>11</sup>In the appendix I provide an economic intuition for why the new steady state level of the bubble is higher.

rigidities which allows me to evaluate the stabilizing role of monetary policy.

## 1.4 A model with endogenous output and nominal rigidities

I extend the model following Eggertsson et al. (2017) to allow output to be endogenously determined. There are downward nominal wage rigidities and monetary policy reacts through an augmented Taylor Rule. The goal of this section is to verify whether monetary policy plays any stabilizing role in managing asset bubbles.

**Households** The problem of the household is to maximize utility (1.1) subject to

$$C_t^y = B_t^y \quad (1.24)$$

$$C_{t+1}^m = \frac{W_{t+1}}{P_{t+1}}L_{t+1} + \frac{Z_{t+1}}{P_{t+1}} - (1 + i_t)\frac{P_t}{P_{t+1}}B_t^y + B_{t+1}^m - p_{t+1}^b a_{t+1} \quad (1.25)$$

$$C_{t+2}^o = -(1 + i_{t+1})\frac{P_{t+1}}{P_{t+2}}B_{t+1}^m + p_{t+2}^b a_{t+1} \quad (1.26)$$

$$(1 + i_t)\frac{P_t}{P_{t+1}}B_t^y \leq D_t \quad (1.27)$$

where  $W_{t+1}$  is the nominal wage rate,  $P_{t+1}$  is the aggregate price level,  $L_{t+1}$  is the labor supply and  $Z_{t+1}$  are the profits of the firm. Middle-aged agents supply inelastically labor  $\bar{L}$ . If the firms do not hire all available labor, then  $L_t$  may be lower than  $\bar{L}$ . For simplification I consider an exogenous borrowing constraint for the young which I assume to be binding, Eq. (1.27). Finally, I have considered the case that agent has access also to riskless, one period nominal debt denominated in money. The asset is in zero net supply, and the government controls its return, the nominal interest rate. Implicitly, the existence of money<sup>12</sup> implies that there is a lower bound on the nominal rate, i.e.  $i_t \geq 0$ .

The Euler Equation holds as follows:

$$\frac{1}{C_t^m} = \frac{1}{C_{t+1}^o}(1 + i_t)\frac{P_t}{P_{t+1}} \quad (1.28)$$

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<sup>12</sup>For a discussion about the introduction of money within an OLG framework, see Galí (2014.)

and combined with (1.7) imply the standard Fisher equation:

$$1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}} \quad (1.29)$$

The no arbitrage condition for the bubble holds as in (1.18).

**Firms** Firms are perfectly competitive and price-takers and they maximize period by period profits with respect to labor,  $L_t$ :

$$\max Z_t = P_t Y_t - W_t L_t \quad (1.30)$$

subject to the following Cobb-Douglas production function

$$Y_t = L_t^\alpha \quad (1.31)$$

where  $\alpha$  is the output elasticity of labor and, in equilibrium, the labor share of income. The firms' labor demand is given by the first order condition:

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \quad (1.32)$$

Following Eggertsson et al. (2017), I allow for downward nominal wage rigidities and I assume that households will never accept working for wages that fall below a wage norm,  $\tilde{W}_t$ , so that

$$W_t = \max \left\{ \tilde{W}_t, P_t \alpha \bar{L}^{\alpha-1} \right\} \quad \text{where} \quad \tilde{W}_t = \delta W_{t-1} + (1 - \delta) P_t \alpha \bar{L}^{\alpha-1} \quad (1.33)$$

where  $\delta$  represents the degree of nominal rigidity, while  $\bar{L}$  is the full employment labor level.

**Monetary Policy** Monetary policy sets the nominal rate according to the following augmented Taylor rule:

$$1 + i_t = \max \left\{ 1, (1 + i^*) \Pi_{t+1} \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left( \frac{b_t}{b^*} \right)^{\phi_b} \right\} \quad (1.34)$$

where  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ . The real interest rate responds systematically to fluctuations in inflation and in the size of the bubble with strength  $\phi_\pi$  and  $\phi_b$ , around their target values, when not

constrained. In line with Clain-Chamosset-Yvrard and Seegmuller (2015), I assume that monetary policy targets a steady state with bubble. As I have shown, the bubbly steady state is indeed always preferable to the bubbleless one. Coherently with this assumption, then I set  $1+i^* = (1+g)\Pi^*$ . I consider  $\phi_\pi > 0$  in line with the Taylor principle, while the sign of  $\phi_b$  is the focus of the incoming discussion. "Leaning against the wind" requires monetary policy to raise the policy rate in the face of a growing bubble, that is  $\phi_b > 0$ , while the opposite occurs under an accommodative stance.

An *equilibrium* is a collection of quantities  $\{C_t^y, C_t^m, C_t^o, B_t^y, B_t^m, a_t, L_t, Y_t, Z_t\}$  and prices  $\{P_t, W_t, r_t, i_t, p_t^b\}$  that solve the household's problem, the firm's problem together with the clearing market conditions and the policy rule, given an exogenous process for  $\{D_t\}$ .

I first provide an analysis of the steady states, and then I go through the stochastic framework. In order to analyze the steady state of the model, I first recover the aggregate supply and aggregate demand of the economy.

### 1.4.1 Aggregate supply

The aggregate supply specification follows Eggertsson et al. (2017). There are two regimes, depending on whether the lower bound on wages is binding or not.

**Flexible wages** Absent any friction in the labor market, the AS schedule is perfectly unelastic and labor demand equals the exogenous level of labor supply,  $\bar{L}$ ,

$$Y^{AS,f} = \bar{L}^\alpha \tag{1.35}$$

This condition holds whenever  $\Pi \geq 1$ <sup>13</sup>.

**Binding downward nominal wage rigidity** If the downward nominal wage rigidity constraint is binding, or  $\Pi < 1$ , then the real wage,  $w = \frac{W}{P}$ , is given by:

$$w = \frac{(1-\delta)\alpha\bar{L}^{\alpha-1}}{1-\delta\Pi^{-1}} \tag{1.36}$$

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<sup>13</sup>Define  $w_t = \frac{W_t}{P_t}$ , then by Eq. (1.33),  $w \geq \delta w\Pi^{-1} + (1-\delta)\alpha\bar{L}^{\alpha-1}$ . By labor demand schedule, the latter is equivalent to  $w \geq \delta w\Pi^{-1} + (1-\delta)w$  which holds whenever  $\Pi \geq 1$ .

Replacing the above equation within the labor demand schedule, Eq. (1.32), and using the production function, Eq. (1.31), I get the following AS schedule for  $\Pi < 1$ :

$$Y^{AS} = \left[ \left(1 - \frac{\delta}{\Pi}\right) \frac{1}{1 - \delta} \right]^{\frac{\alpha}{1-\alpha}} \bar{L}^\alpha \quad (1.37)$$

The above equation is a non linear Phillips Curve and implies a positive relation between output and inflation for the long run. As inflation increases, since wages are rigid, real wages decrease and firms hire more labor. Thus output increases. Notice that  $Y^{AS}=0$  for  $\Pi = \delta$ .

## 1.4.2 Aggregate demand

To derive the aggregate demand schedule for the bubbly steady state, I first recover the steady state size of the bubble, Eq. (1.23), in the endogenous output framework, with exogenous borrowing limit:

$$b = \frac{Y - 3D}{2} \quad (1.38)$$

which can be rewritten as:

$$Y = 3D + 2b \quad (1.39)$$

Given that the interest rate equals the growth rate,  $g$ , at the bubbly steady state, and given the policy rule, Eq. (1.34), I get an equation linking the inflation rate and bubble:

$$b = \left[ \frac{1 + g}{1 + i^*} \left( \frac{\Pi}{\Pi^*} \right)^{-\phi_\pi} \right]^{\frac{1}{\phi_b}} b^* \quad (1.40)$$

This equation links the bubble and inflation rate in a way that the equality between the steady state interest rate and growth rate is satisfied. The sign of the relation depends on monetary policy. Under the "leaning against the wind" approach, the bubble is a negative function of the steady state inflation gross rate and a positive function of the inflation target; viceversa under the accommodative stance. Finally, I replace Eq. (1.40) within Eq. (1.39) to get the AD curve, a relation between output and inflation:

$$Y^{AD} = 3D + 2b^* \left[ \frac{1 + g}{1 + i^*} \left( \frac{\Pi}{\Pi^*} \right)^{-\phi_\pi} \right]^{\frac{1}{\phi_b}} \quad (1.41)$$

As Eq. (1.41) shows, the sign of the relation between output and inflation depends on the policy parameter  $\phi_b$ . Whether monetary policy reacts to bubbles by "leaning against the

wind” or through an accommodative stance has implications for the steady state relation between aggregate demand and inflation. In the former case, Eq. (1.41) delivers the standard negative relation between output and inflation. In the latter case, aggregate demand is positively related to inflation. Graphically, the AD curve becomes an upward sloping curve. The rationale for this result is straightforward. The qualitative nature of Eq. (1.41) mirrors the underlying relation with the bubble. Under the accommodative stance, inflation and bubbles are positively correlated as can be seen through Eq. (1.40), and thus a positive correlation between inflation and output follows.

### 1.4.3 Equilibrium

The equilibrium is given by solving the system between the aggregate demand and supply schedules.

**Assumption 2** Assume that

$$\Pi > \frac{1}{1+g} \tag{1.42}$$

and

$$1+g > \frac{1}{\delta} \tag{1.43}$$

I follow Clain-Chamosset-Yvrard and Seegmuller (2015) and Teulings (2016) and I assume that the zero lower bound does not impose a binding constraint on monetary policy<sup>14</sup>. If Conditions (1.42) and (1.43) hold, the inflation rate is sufficiently high and the bubbly steady state features a positive nominal interest rate<sup>15</sup>. If Conditions (1.42) and (1.43) hold, then the zero lower bound does not impose a binding constraint on monetary policy as it states that the inflation rate is sufficiently high.

### 1.4.4 Deleveraging

Following the motivation discussed at the beginning of Section 1.3.3., I focus on the implications of a deleveraging shock in this extended setup. Consider the parameters

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<sup>14</sup>Bouillot (2016) provides an analysis of an environment with binding ZLB.

<sup>15</sup>Combine the bubbly steady state interest rate with the Fisher relation, Eq. (1.29), to verify that  $i_t \geq 0$  as long as  $(1+g)\Pi \geq 1$ .

in Table 1.1. The economy starts from the steady state A, and a deleveraging shock occurs so that the borrowing limit,  $D$ , drops to 0.25. Figures 1.10a and 1.10b show the scenarios under two alternative monetary policy approaches: the "leaning against the wind" approach,  $\phi_b = +0.2$ , and the accommodative approach,  $\phi_b = -0.2$ .

As Figure 1.10a shows, under the "leaning against the wind", there exists a new unique, bubbly equilibrium, B. A deleveraging shock has a negative effect both on output and inflation. By Eq. (1.38), it is possible to recover the value of the bubble.

Figure 1.10b shows the scenario under the accommodative approach. Two bubbly steady states exist, which I denote as the "high-bubble", B, and "low-bubble", C, steady states. The "high-bubble" steady state inherits the properties described for the bubbly steady state in the endowment economy. The bubble size increases, by Eq. (1.38), and, since prices are fully flexible, then the increase in the bubble translates into higher inflation, via Eq. (1.40). The "low-bubble" equilibrium is instead characterized by low output and inflation.

**Intuition for the existence of two bubbly steady states** Analytically, the existence of two steady states is due to  $\phi_b$  being negative. In fact the latter implies that, combining the AS and AD, Eqs. (1.37) and (1.41), results in a quadratic equation in either  $Y$  or  $\Pi$ , which has two, possibly distinct, roots. There exist two solutions, with high and low level of output. To get an intuition, consider the Euler Equation for the middle-aged agent:

$$U' \left( Y - D_{-1} - \frac{D}{1+g} - b \right) = (1+g)U' \left( D + (1+g)b \right) \quad (1.44)$$

If Condition (1.44) holds, the steady state allocation is dynamically efficient. The adjustment mechanism following a deleveraging shock depends on whether nominal rigidities bind or not. If nominal rigidities do not bind, the drop in the borrowing limit can be fully offset by an increase in the size of the bubble and inflation. If nominal rigidities bind, then a lower debt limit implies that output, and inflation, have to fall so as to satisfy the household's intertemporal Euler Equation. The bubble size is then endogenously determined through Eq. (1.38), and it may slightly decrease.

**Proposition 2** The "low-bubble" steady state exists if

$$\phi_b < -\epsilon\phi_\pi \frac{(1-\alpha)}{\alpha} \quad (1.45)$$

where  $\epsilon = \frac{1-\delta_w}{\delta_w}(1-3d)$ , given  $\delta_w = \frac{\delta}{\Pi} < 1$  and  $d = \frac{D}{Y}$ , is a small, positive number<sup>16</sup>. The "low-bubble" steady state is stable. Proof in the Appendix.

By Proposition 2, the existence of a stable bubbly steady state requires coordination between the two policy goals, and inflation targeting requires monetary policy to adopt an accommodative stance towards bubbles, that is to set  $\phi_b$  to be negative, for Condition (1.45) to be satisfied. Proposition 2 also states that the "low-bubble" steady state is stable. Unless monetary policy successfully coordinates expectations on the "high-bubble" steady state equilibrium, then the economy eventually reaches the "low-bubble" stable one.

#### 1.4.5 The case for a higher inflation target

Since the Great Recession, it has been argued that central banks should raise the inflation target. This policy would give monetary policy more room for further action, making the zero lower bound less likely to bind, see Blanchard (2010) and Ball (2014). Opponents argue that a higher inflation target would eventually have destabilizing effects and would lead to more bubbles, see Bernanke (2010). I take the opportunity for a brief excursus to discuss the implications of a raise of the inflation target in this specific framework.

Two lessons have emerged from the previous discussion. First, in a world with rational bubbles in fixed supply, the real interest rate equals the growth rate in the steady state equilibrium featuring bubbles. Thus, no policies other than the ones which target the growth rate can effectively raise the steady state equilibrium interest rate in this context. Second, bubbly steady states are in general unstable. The only exception is in a sticky prices scenario in which monetary policy adopts an accommodative stance towards bubbles.

To assess the effect of an increase of the inflation target, consider the aggregate demand schedule, Eq. (1.41). Independently on the chosen stance towards bubbles,

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<sup>16</sup>For parameters as in Table 1.1,  $\epsilon = 0.02$ .

an increase in  $\Pi^*$  is associated with an upward shift of the AD curve. Under "leaning against the wind", then this implies that the economy moves to an equilibrium with higher output and inflation, making condition (1.42) less likely to bind, while it broadens the gap between "low-" and "high-bubble" equilibria under the accommodative stance. Thus, raising the inflation target may have undesirable effects.

The rationale for this result follows from Eq. (1.40). While under flexible prices, a raise in the inflation target only translates into an increase in the steady state equilibrium inflation rate, the consequences under binding nominal rigidities depend on monetary policy: an increase in the inflation target raises the inflation rate, bubble and output under "leaning against the wind", but it has contractionary effects under accommodative stance. In fact, the increase in the inflation target lowers the rate at which the bubble increases over the target,  $\frac{b}{b^*}$ , thus requiring the steady state bubble to decrease. The drop in aggregate demand triggers a reduction in the equilibrium level of output and inflation. Thus, a better outcome could be reached by combining an accommodative approach with a fiscal, or financial, intervention which would move the "low bubble" steady state to the right.

## 1.5 Stochastic Economy

In this section, I consider a log linearized version of the equilibrium equations around the stable bubbly steady state. The  $\hat{\cdot}$  symbol on top of a variable indicates the deviation from its steady state value. As in Galì (2014), I define the martingale-difference process  $\xi_t$  to be the innovation in the bubble,  $b_t - E_{t-1}b_t = \xi_t$ .

The AS is either

$$\hat{y}_t = 0 \tag{1.46}$$

if  $\Pi > 1$ , or

$$\hat{\pi}_t = \frac{1}{\delta_w} \frac{1 - \alpha}{\alpha} (\hat{y}_t - \delta_w \hat{y}_{t-1}) \tag{1.47}$$

if  $\Pi < 1$  where  $\delta_w = \frac{\delta}{\Pi}$ . The AD results from the combination of the log linearized version of the bubble's dynamics, Eq. (1.18), the equilibrium rate for loans, Eq. (1.20),

the Fisher relation, Eq. (1.29), and the policy rule, Eq. (1.34):

$$\hat{b}_t = \hat{r}_{t-1} - \hat{g}_{t-1} + \hat{b}_{t-1} + \xi_t \quad (1.48)$$

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_b \hat{b}_t \quad (1.49)$$

$$\hat{r}_t + y_1 \hat{y}_t - d_1 \hat{d}_{t-1} - b_1 \hat{b}_t = \hat{g}_t + d_2 \hat{d}_t + b_2 E_t \hat{b}_{t+1} \quad (1.50)$$

where

$$y_1 = \frac{1}{1-d-b^y}, \quad d_1 = \frac{d}{1-d-b^y}, \quad b_1 = \frac{b^y}{1-d-b^y}$$

$$d_2 = \frac{2d}{2d+b^y}, \quad b_2 = \frac{b^y}{2d+b^y} = b_1$$

where  $d$  and  $b^y$  are the steady state ratios of the bubble and debt over output. I do not consider shocks to the growth rate, therefore  $\hat{g} = 0$ .

**Proposition 3** If prices are fully flexible, the equilibrium is described by the following difference equation for the bubble

$$\hat{b}_t = \frac{1+b_1}{1-b_2} \hat{b}_{t-1} \quad \text{where} \quad \frac{1+b_1}{1-b_2} > 1 \quad (1.51)$$

Bubbles only influence inflation with the effect being strictly positive if  $\phi_b < \frac{b_1+b_2}{1-b_2} = \bar{\phi}_b$ . Fluctuations around the bubbly steady state are not stationary.

If the downward nominal wage constraint is binding, the economy is described by a system of three equations in three variables,  $\hat{\pi}_t$ ,  $\hat{b}_t$ , and  $\hat{y}_t$  as functions of their past values and shocks. Bubbles affect also output with the effect being positive if  $\phi_b < \bar{\phi}_b$ . Fluctuations around the bubbly steady state are stationary if

$$\phi_b < -\epsilon \phi_\pi \frac{(1-\alpha)}{\alpha} \quad (1.52)$$

where  $\epsilon$  is a small, positive number<sup>17</sup>. Proof in the appendix.

Proposition 3 states that under flexible prices, monetary policy has no influence on the evolution of the bubble, and in line with the results from the pure endowment economy, fluctuations around the bubbly steady state are not stationary. In the sticky prices

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<sup>17</sup>Given the assumptions made on the parameters,  $\epsilon = 0.02$ .

environment, stationarity of equilibrium dynamics depends on monetary policy and the condition for (local) stationarity of the bubble around the steady state in the stochastic equilibrium, Eq. (1.52), corresponds to the condition of stability of that steady state under the deterministic equilibrium dynamics in Proposition 2. Moreover, the effects of bubbles on aggregate demand are positive under the accommodative stance such that (1.52) holds.

### 1.5.1 Deleveraging

Figures 1.11 and 1.12 show the behaviour of the main variables and the transition of the economy towards the new steady state following a permanent deleveraging shock. As anticipated, the new steady state features a lower level of income, inflation and bubble but displays an interest rate equal to the growth rate of the economy, i.e. dynamic efficiency. As the simulation makes clear, fluctuations, that is booms and busts, both in the bubble and in the main macroeconomic variables, are totally endogenous and explained by shocks in the credit market<sup>18</sup>.

Figure 1.11 shows the pattern of the variables for three different levels of  $\phi_b$ . The solid line corresponds to a mute accommodative policy,  $\phi_b = -0.2$ ; the dotted line to a mild accommodative policy,  $\phi_b = -0.6$ , and finally; the dashed line to a strong accommodative policy,  $\phi_b = -1.2$ . The key difference among the three scenarios is in the path followed by the bubble. The more accommodative the monetary policy approach, the larger the recovery of the bubble following the deleveraging shock. A larger recovery is associated with a slightly better performance for output. The effects are marginal though, and this is mostly motivated by the choice of an exogenous borrowing constraint. Allowing for an endogenous borrowing constraint formulation and bubbly collateral would lead to stronger effects on output.

Figure 1.12 shows the pattern of the variables for three different levels of  $\phi_\pi$ , given a mild response to the bubble. The solid line corresponds to a weak inflation targeting,  $\phi_\pi = 0.1$ ; the dotted line to a mild inflation targeting,  $\phi_\pi = 0.5$ ; the dashed line to an aggressive inflation targeting,  $\phi_\pi = 1.2$ . The weak inflation targeting scenario features the

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<sup>18</sup>Alternatively, Martin and Ventura (2012) introduce an exogenous random creation of bubbles which can be used as collateral. In their framework, fluctuations are triggered by shocks to bubble creation.

lowest volatility and the least drop of the interest rate and bubble. The intuition for this result follows. Since the bubble and inflation comove, inflation targeting and adopting an accommodative stance towards bubbles require the interest rate to adjust in the opposite directions, thus eventually neutralizing each other. Therefore, the milder the inflation targeting, the lower the neutralization effect to the detriment of the accommodative stance towards bubbles.

### 1.5.2 Sensitivity Analysis

I now examine the sensitivity of these results to changes in key parameters. Figure 1.13 reports the results from reducing the labor rigidity from 0.7 to 0.5 for the "High Flexibility" scenario and from raising it to 0.9 for the "Low Flexibility" one. Compared to the baseline case, the higher the level of flexibility, the deeper the drop in the bubble and inflation. Figure 1.14 reports the results from reducing the labor intensity from 0.68 to 0.5 for the "Low Intensity" scenario and from raising it to 0.9 for the "High Intensity" one. Compared to the baseline case, the lower the labor intensity, the deeper the drop in the bubble and inflation. The intuition behind the previous results is based on the high correlation existing between the bubble and inflation under an accommodative monetary policy stance. Since both the "low flexibility" and "high intensity" scenarios are associated with a flat AS schedule, then shocks on the AD side have zero or small effect on inflation, and thus on the bubble.

## 1.6 Conclusions

The objective of the project is to evaluate the role of stock market bubbles following a deleveraging shock. My starting point is an estimated vector-autoregression (VAR) on quarterly US data. I show that, following a deleveraging shock, the behaviour of the bubbly component of stock prices explains the path of stock prices. It falls on impact and partly recovers along with the main macroeconomic variables afterwards.

I adopt a general equilibrium theory and I show that when agents are financially constrained, they positively value bubbles to smooth consumption investing the excess savings. However, the bubbly steady state is unstable, and after any further deleveraging

shock, the economy eventually converges towards the bubbleless steady state, where Secular Stagnation may arise. I extend the economy to allow for output to be endogenously determined and for nominal rigidities, and I show that monetary policy should adopt an accommodative stance towards bubbles to guarantee the existence of a stable bubbly steady state and stationarity of dynamics around it. Therefore, following a deleveraging shock, monetary policy should, on the one hand, address the excess savings and, on the other hand, not fight the bubble by adopting an accommodative stance.

The results are based on a stylized model. I view as a necessary extension allowing for endogenous borrowing constraints and for the bubble to play a role also as collateral as in Miao and Wang (2015), Martin and Ventura (2015) and Dong et al. (2017).

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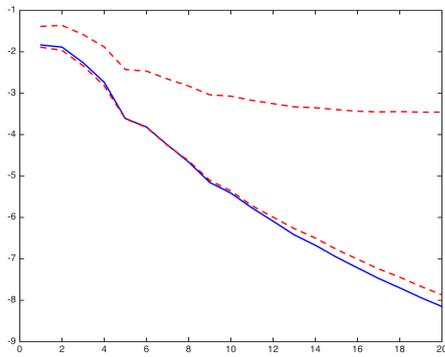
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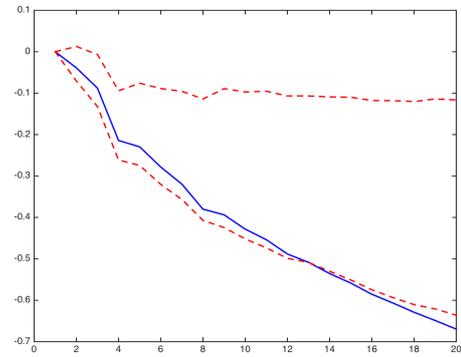
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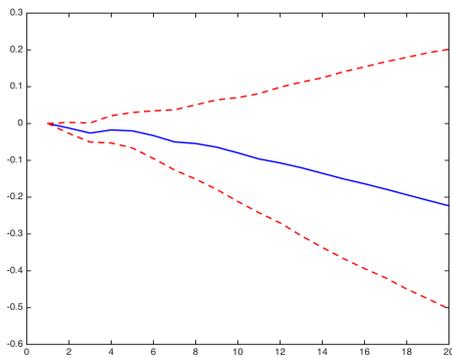
## 1.7 Figures



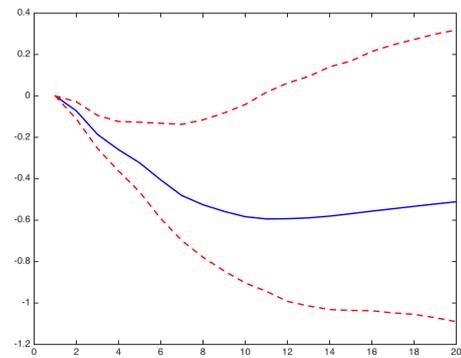
(a) Deleveraging shock



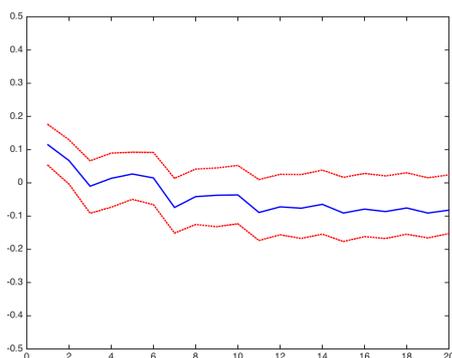
(b) GDP



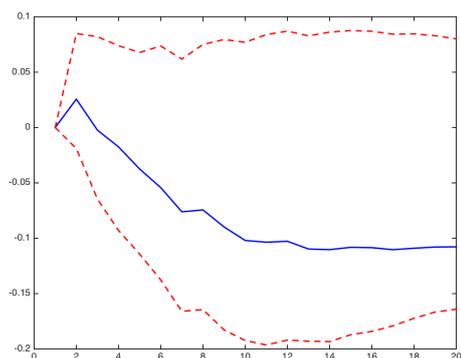
(c) GDP Deflator



(d) Dividends

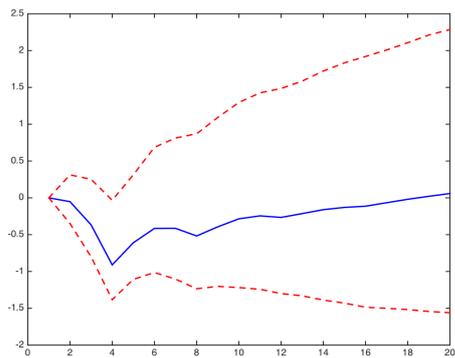


(e) Real Interest Rate

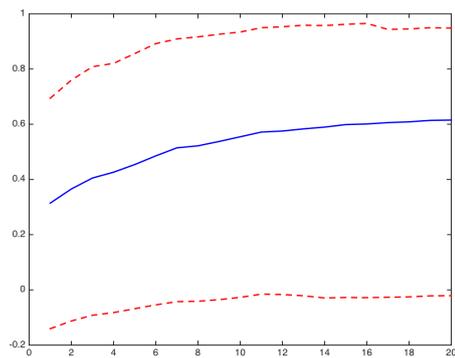


(f) Federal Funds Rate

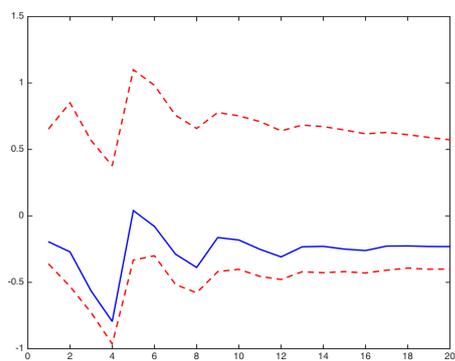
Figure 1.1: Estimated Responses to a deleveraging shock, Constant Coefficients VAR Part I.



(a) S&P 500

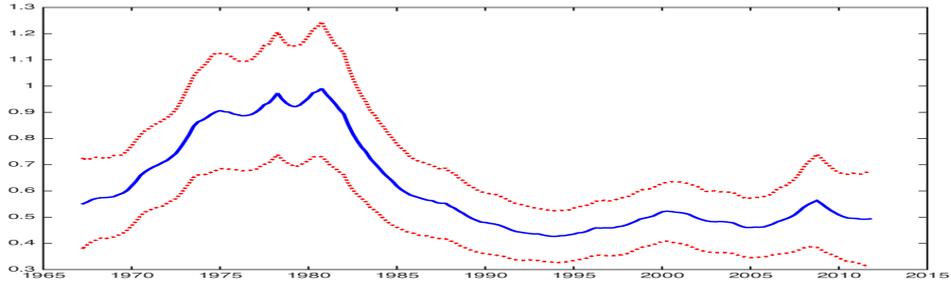


(b) Fundamental Component

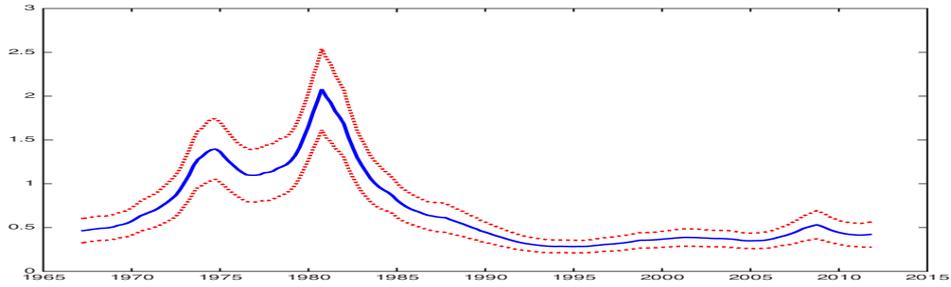


(c) Bubbly Component

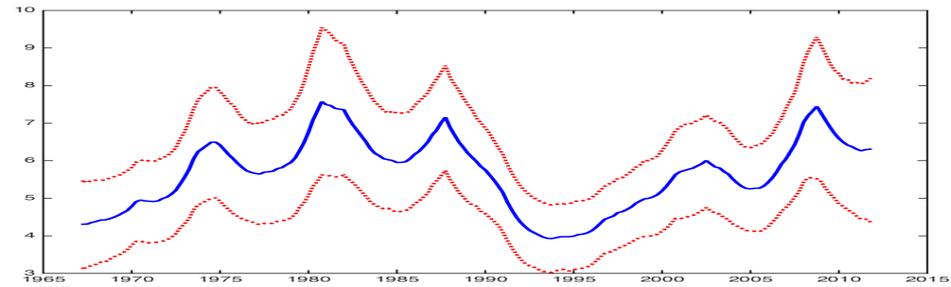
Figure 1.2: Estimated Responses to a deleveraging shock, Constant Coefficients VAR Part II.



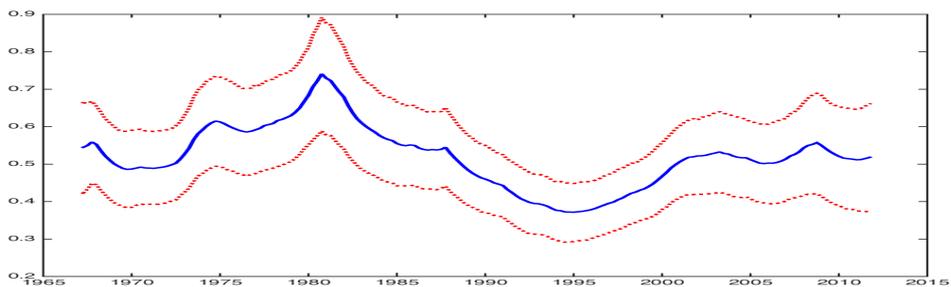
(a)



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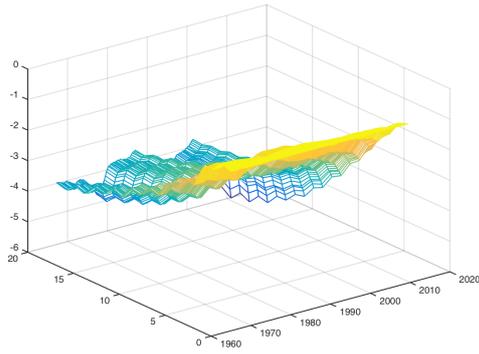


(c)

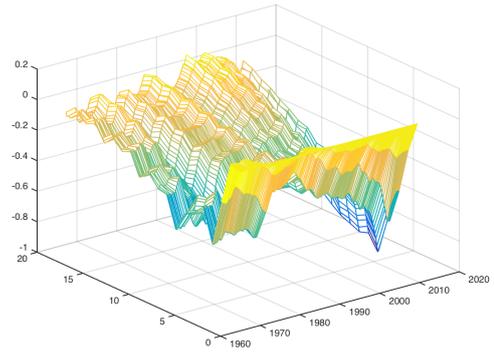


(d)

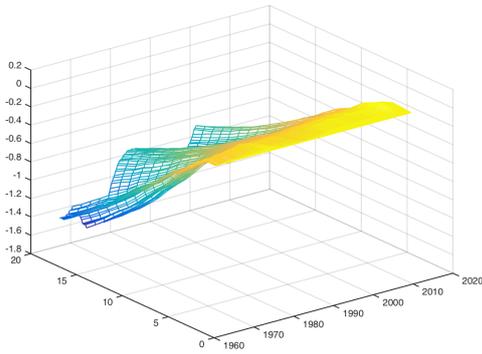
Figure 1.3: Posterior mean, 16th and 84th percentiles of the standard deviation of (a) residuals of the GDP equation, (b) residuals of the interest rate equation (or monetary policy shocks), (c) residuals of the S&P500 Index equation, and (d) residuals of the loan volume equation (or loans shocks).



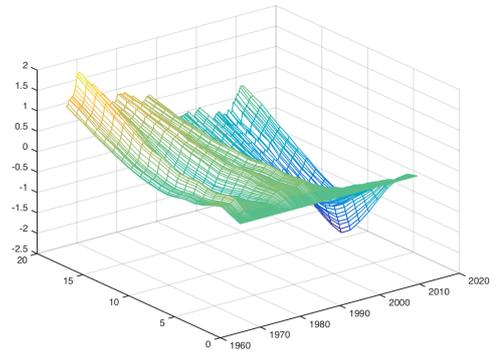
(a) Deleveraging Shock



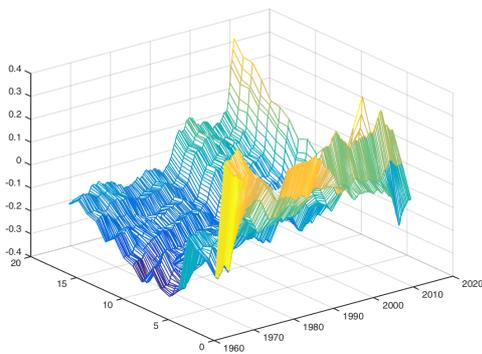
(b) GDP



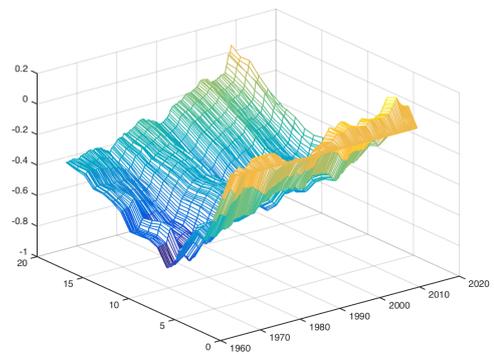
(c) GDP Deflator



(d) Dividends

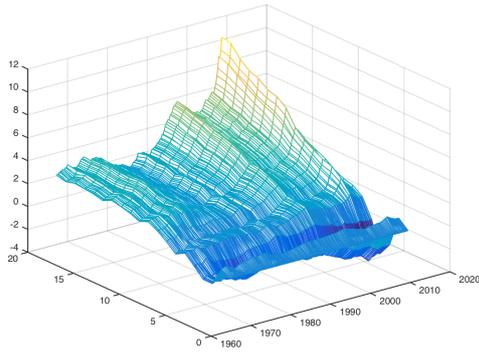


(e) Real Interest Rate

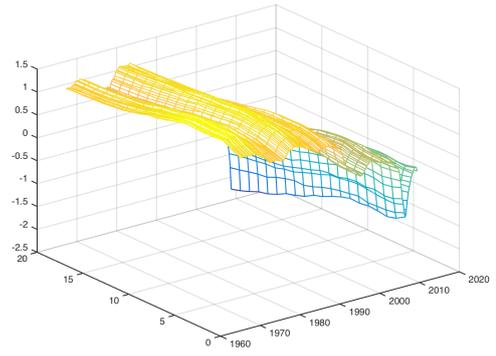


(f) Federal Funds Rate

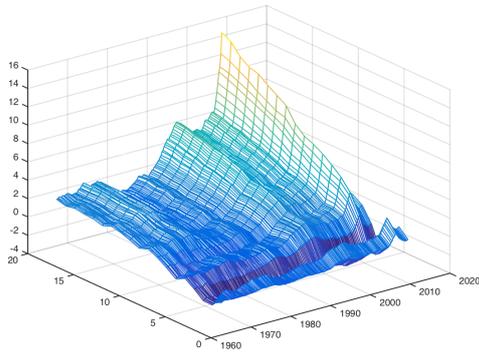
Figure 1.4: Estimated Responses to a deleveraging shock, Time-Varying Coefficients VAR Part I.



(a) S&P 500



(b) Fundamental Component



(c) Bubbly Component

Figure 1.5: Estimated Responses to a deleveraging shock, Time-Varying Coefficients VAR Part II.

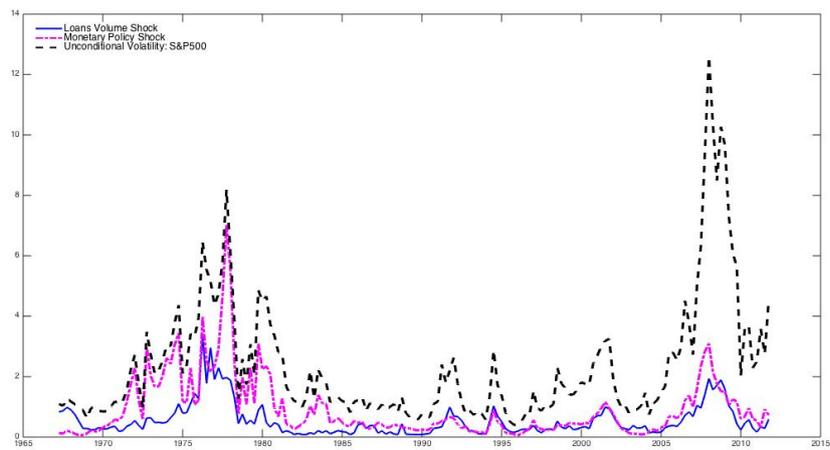


Figure 1.6: Conditional Variance: S&P 500

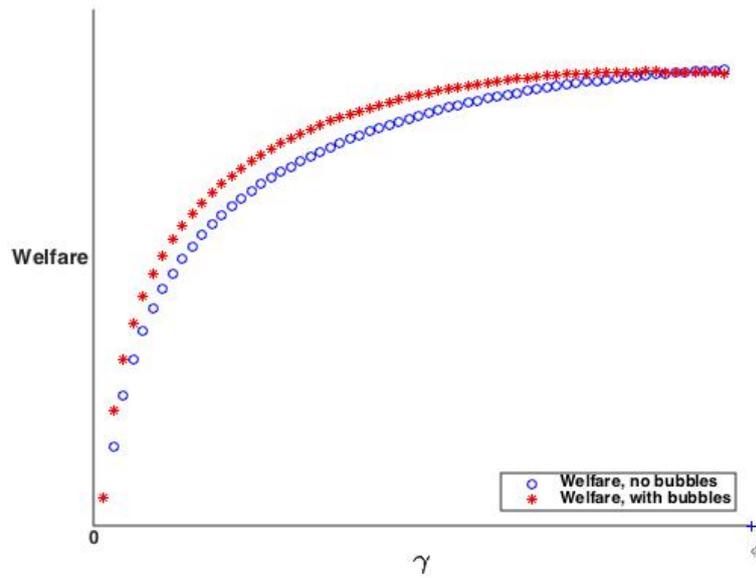


Figure 1.7: Measure of welfare, for given financial constraint. The red starred line represents welfare in the presence of bubbles, while the blue dotted line represents welfare in the absence of bubbles. I define welfare as lifetime utility for the generation born in  $t$ .

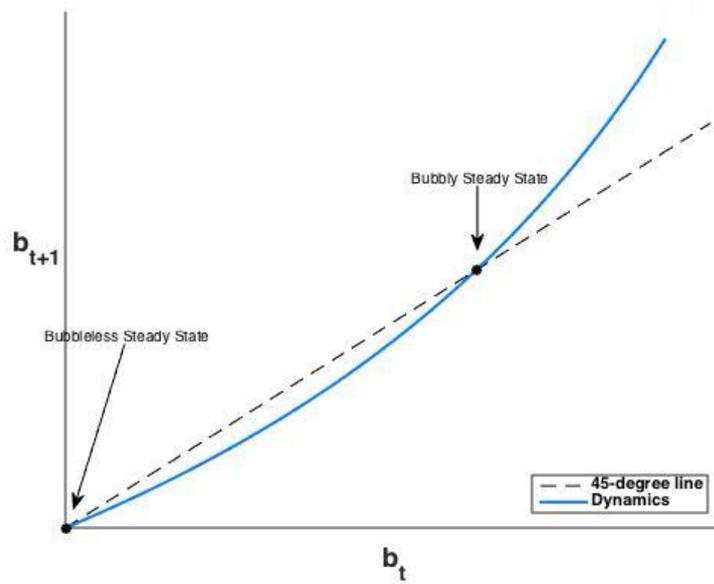


Figure 1.8: Equilibrium Dynamics.

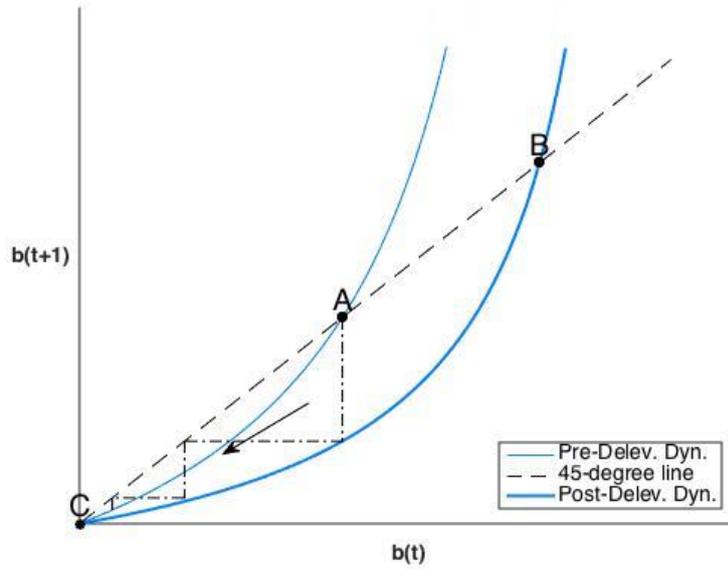
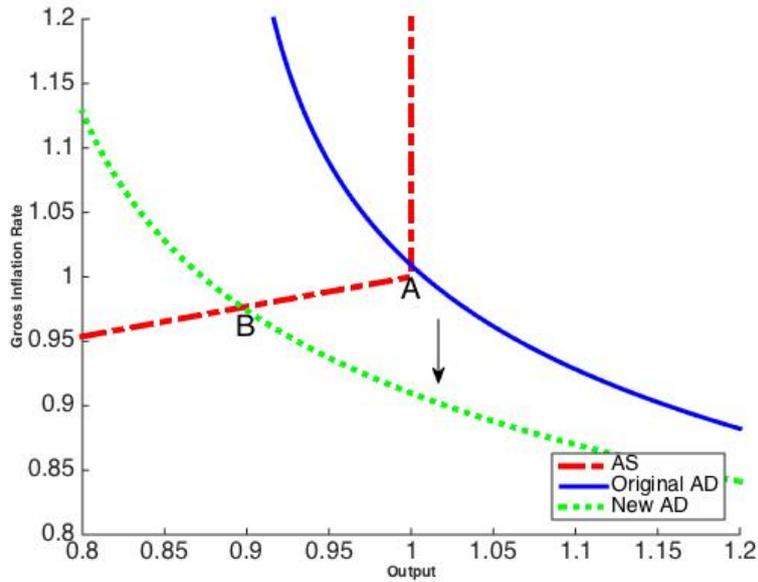


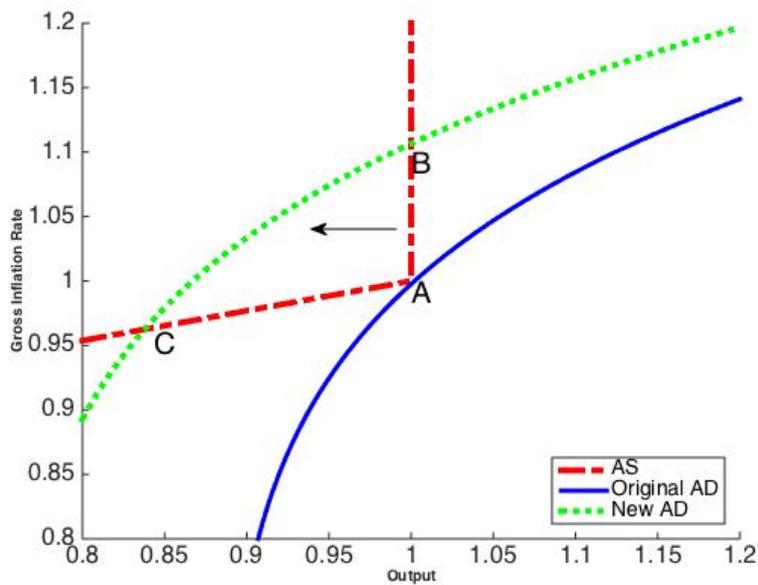
Figure 1.9: Transition after a deleveraging shock. The light blue line represents equilibrium dynamics before the shock, while the dark blue one the equilibrium dynamics after the shock. The initial steady state is A, when  $\gamma$  is high and bubbles are positively valued. As the shock occurs, and  $\gamma$  decreases, the equilibrium moves to C, the final bubbleless steady state. Point B represents the new bubbly steady state. Due to the instability of the latter, the economy eventually reaches the bubbleless steady state.

Description	Parameter	Value
Labor Supply	$L$	1
Discount factor	$\beta$	1
Taylor coefficient for $\Pi$	$\phi_\pi$	1.5
Labor share	$\alpha$	0.68
Bubble target	$b^*$	0.06
Wage adjustment	$\delta$	0.7
Inflation Target	$\Pi^*$	1.01
Borrowing Limit	$D$	0.295

Table 1.1: Parameters



(a) "Leaning against the wind"



(b) Accommodative Monetary Policy

Figure 1.10: Deleveraging Shock. The red dashed line represents the AS schedule; the blue solid line the original AD schedule before the shock, and; the green dotted line the AD schedule after the shock. The original steady state is in A. As the deleveraging shock occurs, the economy moves to B. Under the accommodative stance, two steady states exist, B or C.

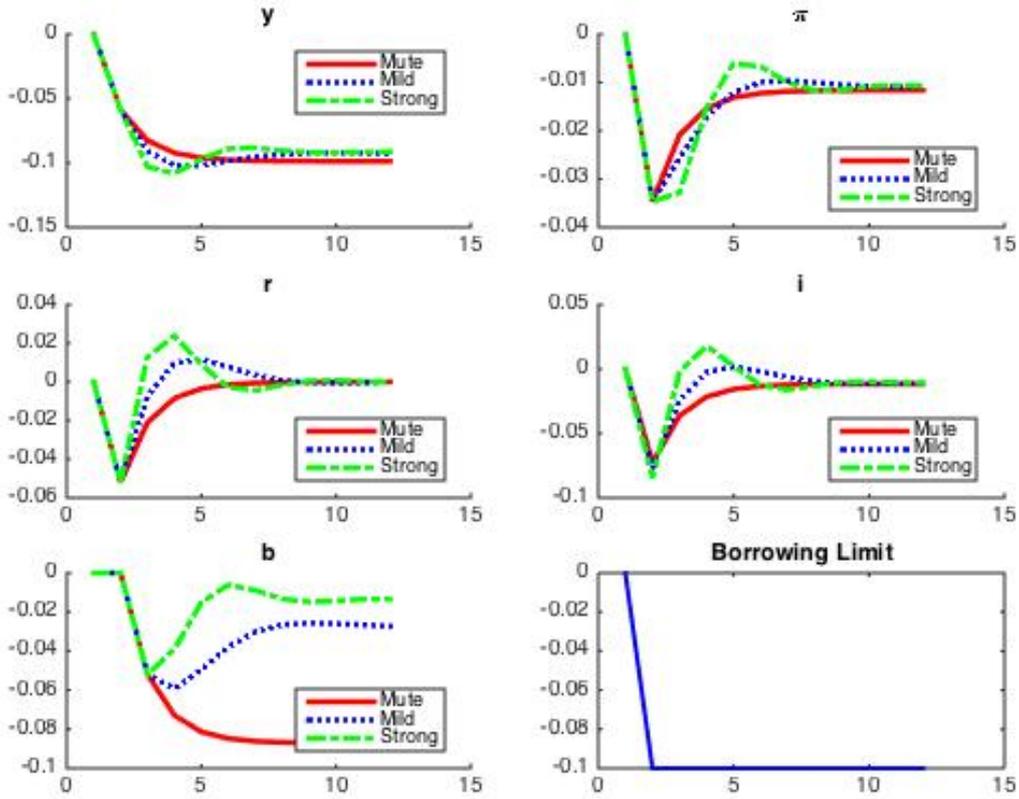


Figure 1.11: Responses after a permanent deleveraging shock. Variables are: output  $y$  and the bubble  $b$ , expressed as percentage deviation from steady state; inflation  $\pi$ , real and nominal interest rates,  $r, i$ , in terms of deviation from steady state. Parameters are as follows:  $\phi_\pi = 1.5$  and Mute:  $\phi_b = -0.2$ ; Mild:  $\phi_b = -0.6$ ; Strong:  $\phi_b = -1.2$ .

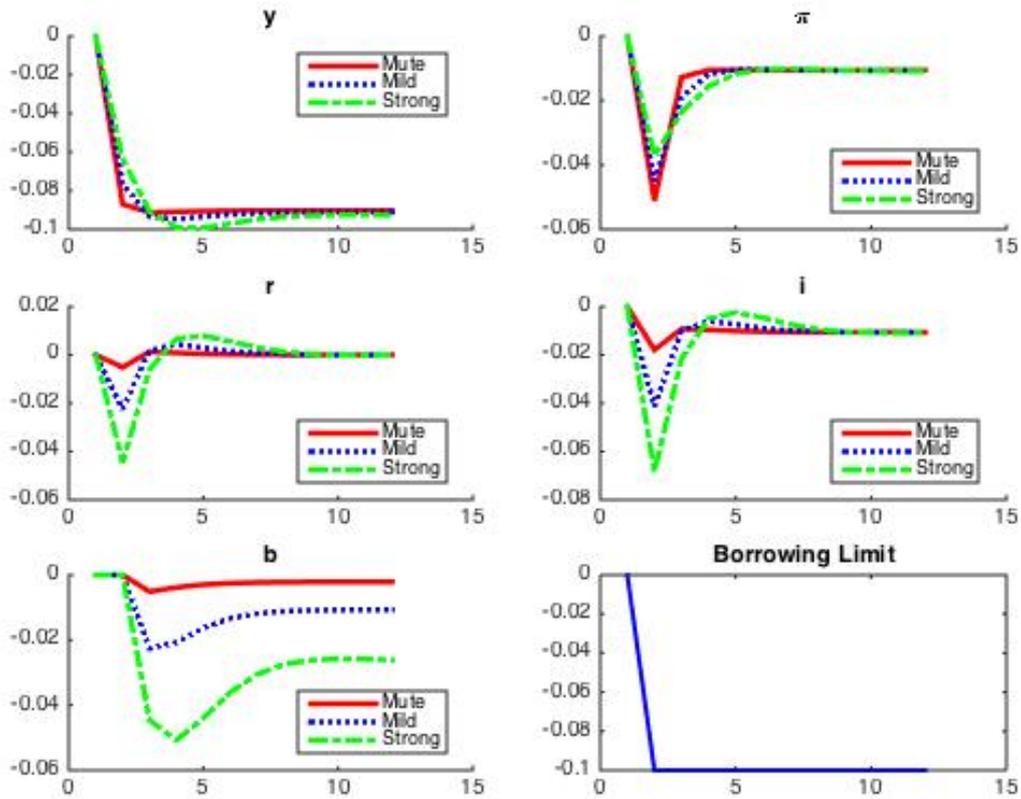


Figure 1.12: Responses after a permanent deleveraging shock. Variables are: output  $y$  and the bubble  $b$ , expressed as percentage deviation from steady state; inflation  $\pi$ , real and nominal interest rates,  $r, i$ , in terms of deviation from steady state. Parameters are as follows:  $\phi_b = -0.5$  and Weak:  $\phi_\pi = 0.1$ ; Mild:  $\phi_\pi = 0.5$ ; Aggressive:  $\phi_\pi = 1.2$ .

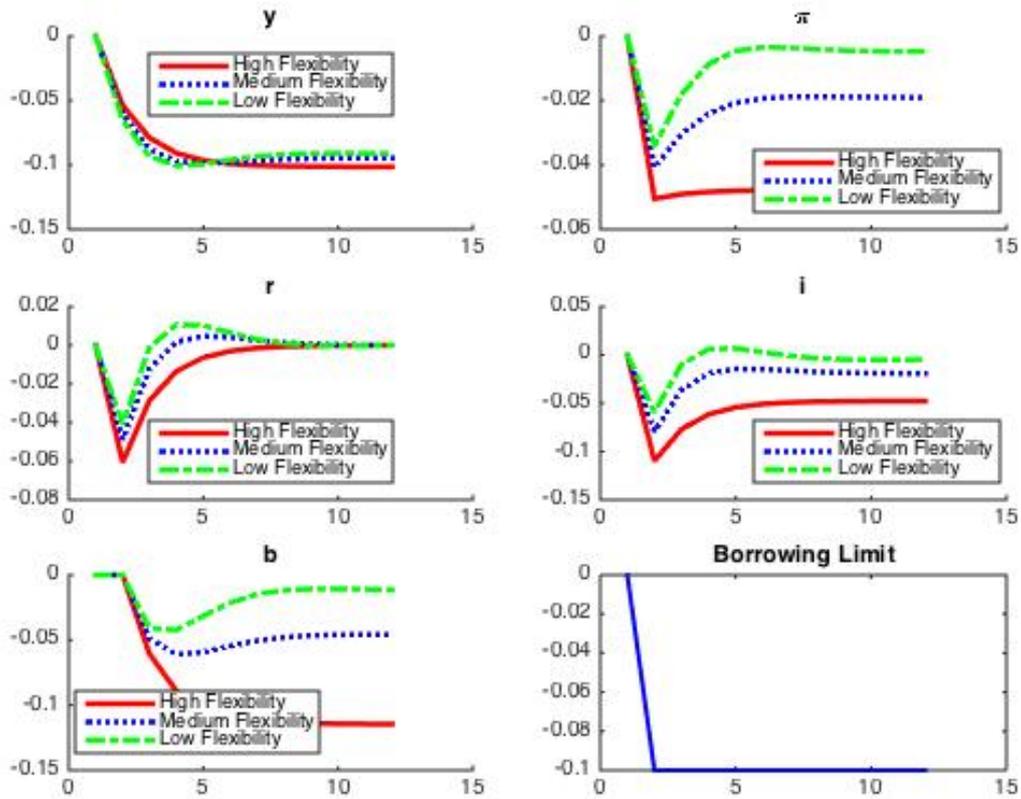


Figure 1.13: Responses after a permanent deleveraging shock. Variables are: output  $y$  and the bubble  $b$ , expressed as percentage deviation from steady state; inflation  $\pi$ , real and nominal interest rates,  $r, i$ , in terms of deviation from steady state. Scenarios are as follows: High Flexibility,  $\delta = 0.5$ ; Baseline,  $\delta = 0.7$ ; Low Flexibility,  $\delta = 0.9$ .

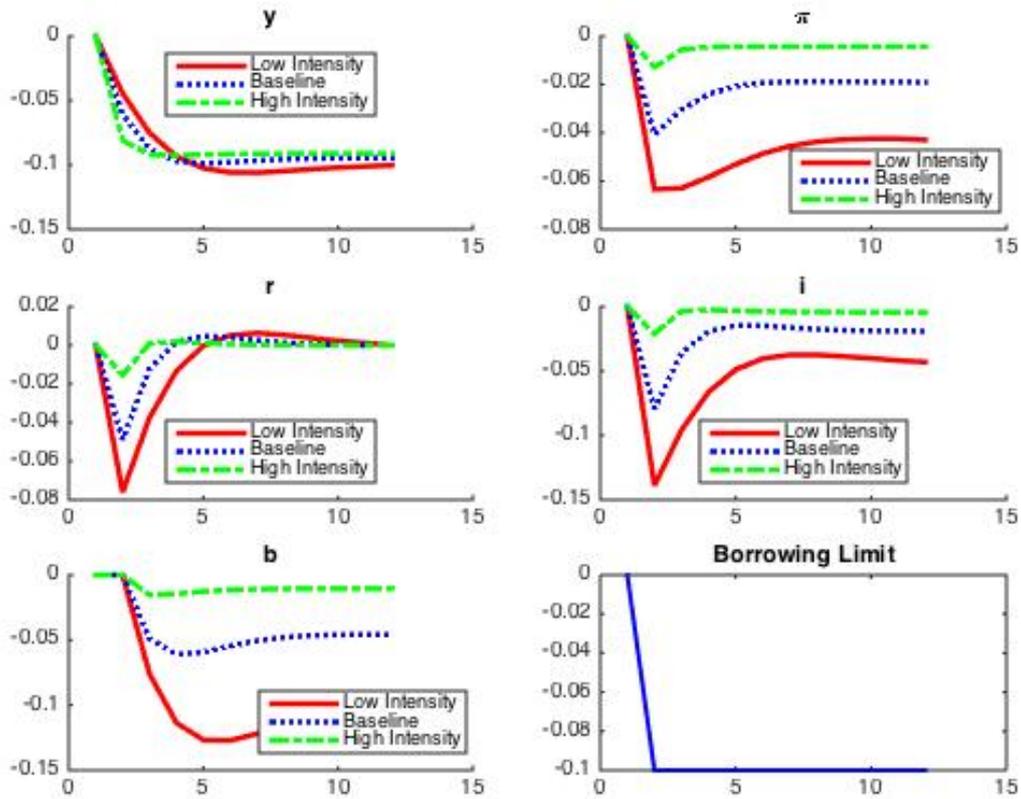


Figure 1.14: Responses after a permanent deleveraging shock. Variables are: output  $y$  and the bubble  $b$ , expressed as percentage deviation from steady state; inflation  $\pi$ , real and nominal interest rates,  $r$ ,  $i$ , in terms of deviation from steady state. Scenarios are as follows: Low Labor Intensity,  $\alpha = 0.5$ ; Baseline,  $\alpha = 0.68$ ; High Labor Intensity,  $\alpha = 0.9$ .

## 1.8 Appendix

### Frictionless Economy

The objective of each individual is to maximize utility from consumption over his life,

$$\log C_t^y + \beta \log C_{t+1}^m + \beta^2 \log C_{t+2}^o \quad (1.53)$$

where  $\beta$  is the subjective discount factor,  $\beta = 1$ , and;  $C^y$ ,  $C^m$  and  $C^o$  are the consumptions in young, middle and old ages.

Maximization of (1) is subject to the following constraints:

$$C_t^y = B_t^y \quad (1.54)$$

$$C_{t+1}^m = Y - (1 + r_t)B_t^y + B_{t+1}^m \quad (1.55)$$

$$C_{t+2}^o = -(1 + r_{t+1})B_{t+1}^m \quad (1.56)$$

where equations (2), (3) and (4) are the budget constraints for the young, the middle aged and the old.  $B^y$  is the consumption loan from the middle aged to the young, while  $B^m$  constitutes savings for the old age.  $Y$  is middle aged income and  $r_t$  and  $r_{t+1}$  are the market real interest rate for  $t$ - $(t+1)$  and  $(t+1)$ - $(t+2)$ . Optimality requires the consumption for the young to equal an optimal fraction,  $\phi$ , of the present value of lifetime income:

$$C_t^y = \phi \frac{Y}{1 + r_t} \quad (1.57)$$

where

$$\phi = \frac{1}{1 + \beta + \beta^2} = \frac{1}{3} \quad (1.58)$$

Notice that  $\phi$ , the optimal fraction of middle age income devoted to consumption in the youth, is exclusively a function of the subjective discount factor. Changes in the subjective discount factor affect the optimal level.

The savings market clears according to condition (1.8). The equilibrium allocation in the frictionless economy features the following interest rate

$$1 + r_t = 1 + g \quad (1.59)$$

The frictionless economy is therefore pareto efficient (Samuelson, 1958). The allocation corresponds to the central planner's one. In fact, given equal weights, the solution for the planning problem would be

$$\frac{U'(C^y)}{U'(C^m)} = \frac{U'(C^m)}{U'(C^o)} = 1 + g$$

## The household's problem

The household problem in the bubbleless economy can be synthetized by the following conditions:

$$U'(C_t^y) = (1 + r_t)U'(C_{t+1}^m) + \mu \quad (1.60)$$

$$\mu(B^y + \frac{\gamma Y}{1 + r_t}) = 0 \quad \text{with } \mu \geq 0 \quad (1.61)$$

$$U'(C_{t+1}^m) = (1 + r_{t+1})U'(C_{t+2}^o) \quad (1.62)$$

If  $\mu = 0$ , the constraint is not binding and both Euler Equations hold.

Assume that  $\mu \neq 0$ . Then the only relevant Euler Equation is the one between middle and old ages. By replacing (1.62) within (1.60), and substituting out for the old age consumption using the savings market clearing condition, (1.8), I get the following definition for the Lagrangian multiplier,  $\mu$ :

$$\mu = U'(C^y)[1 - \frac{1 + r}{1 + g}]$$

Notice that the Lagrangian multiplier represents the shadow price of the borrowing constraint and it is a decreasing function of  $\gamma$ . The sign of the multiplier is positive if  $r < g$ . The lower the interest rate, the higher the value of  $\mu$ . If the interest rate equals the growth rate, then  $\mu = 0$ .

## Proposition 1

First of all, by evaluating the two equilibrium conditions, (1.18) and (1.20), at the steady state, such that  $b_{t+1} = b_t = b$ , I recover the values for the interest rate and the bubble for the bubbly steady state:

$$\tilde{b}_0 = \frac{(1 - 3\gamma)}{2}Y$$

$$r = g$$

The steady state size of the bubble is decreasing in  $\gamma$  and increasing in income. Moreover, the steady state features a positively valued bubble,  $\tilde{b}_0 > 0$ , if and only if  $\gamma < \phi = \frac{1}{3}$ . Thus, the existence condition for the bubble corresponds to the condition for a dynamic inefficient bubbleless allocation, (1.12).

Consider Case 1, in which the initial bubble in time 0 is equal to the upper bound, that is  $b_{0,1} = \tilde{b}_0 = \frac{(1-3\gamma)}{2}Y$ . Then, by (1.22)

$$b_{1,1} = \frac{2^{\frac{(1-3\gamma)}{2}}\gamma Y^2}{Y(1-\gamma) - 2^{\frac{(1-3\gamma)}{2}}Y}$$

After simplifying, we verify that  $b_{1,1} = b_{0,1} = \tilde{b}_0 \forall t$ .

By the same token, by plugging  $b_{0,1}$  within (1.21), we see that  $r_{t,1} = g \forall t$ .

Consider now Case 2, in which the initial bubble is lower than the upper bound, that is  $b_{0,2} = \frac{(1-3\gamma)}{4}Y < \tilde{b}_0$ . By (1.22)

$$b_{1,2} = \frac{2^{\frac{(1-3\gamma)}{4}}\gamma Y^2}{Y(1-\gamma) - 2^{\frac{(1-3\gamma)}{4}}Y} = \frac{4\gamma}{1+\gamma} \frac{(1-3\gamma)}{4}Y$$

It's easy to see that  $b_{1,2} < b_{0,2}$  since  $\frac{4\gamma}{1+\gamma} < 1$ . Thus, if the economy starts with  $b_0 < \tilde{b}_0$ , it eventually reaches the bubbleless allocation. The interest rate in the limit is as defined in (1.11).

To assess the stability of the bubbly steady state, I loglinearize Eq. (1.22) around it and I obtain the following log-linear difference equation:

$$\hat{b}_t = \frac{1-\gamma}{2\gamma}\hat{b}_{t-1} \quad (1.63)$$

By Eq. (1.63) follows that the bubbly steady state is not stable since  $\frac{1-\gamma}{2\gamma} > 1$ . For  $\frac{1-\gamma}{2\gamma}$  to be less than 1,  $\gamma$  should be higher than  $\frac{1}{3}$ , a contradiction with the existence condition. Alternatively, consider the non-linear autonomous difference equation, (1.22). The bubbly steady state is an asymptotically stable equilibrium point if  $|f'(b)| < 1$ , where  $f'(b)$  is the first derivative of function  $f()$  evaluated at the bubbly steady state. The latter condition is verified for  $\gamma > \phi$ .

## Bubbles and Deleveraging

In this section I discuss why following a deleveraging shock, the equilibrium steady state size of the bubble increases. Let me focus first on equilibrium in the market for the bubble. I derive the steady state demand for the bubble from Eq. (1.18), so that

$$b_{t+1} - b_t = \frac{1+r_t}{1+g}b_t - b_t = 0 \quad \rightarrow \quad r = g$$

Demand for bubbles is perfectly unelastic at  $r = g$ . To derive the supply schedule, I proceed in a similar fashion considering Eq. (1.20) so that I get

$$b = \frac{1}{1 + \frac{1+r}{1+g}} \left[ \frac{1+r}{1+g} (1 - \gamma_{-1}) - 2\gamma \right] Y$$

I plot steady state demand and supply for bubbles in the plane  $(r, b)$ . As Figure 1.15 shows, along the optimal path, if  $\gamma$  drops, the steady state size of the bubble has to rise so as to satisfy the intertemporal Euler Equation:

$$U'(Y - \gamma_{-1}Y - \frac{\gamma_t Y}{1+g} - b^*) = (1+g)U'(+\gamma_t Y + (1+g)b^*) \quad (1.64)$$

### Proof

Consider the problem of the current middle generation at the bubbly steady state equilibrium as the deleveraging takes place. They choose consumption for middle and old ages and the bubble in order to maximize

$$\log C_t^m + \log C_{t+1}^o \quad (1.65)$$

$$C_t^m = Y - \gamma_{-1}Y + \frac{\gamma_t Y}{1+g} - b_t \quad (1.66)$$

$$C_{t+1}^o = -\gamma_t Y + (1+g)b_{t+1} \quad (1.67)$$

where I have already replaced the market clearing conditions. By optimality:

$$\frac{1}{C_t^m} = \lambda_1 \quad (1.68)$$

$$\frac{1}{C_{t+1}^o} = \lambda_2 \quad (1.69)$$

$$\lambda_1 = (1+g)\lambda_2 \quad (1.70)$$

where  $\lambda_1$  and  $\lambda_2$  are the Lagrangian multipliers associated with the budget constraints (1.66) and (1.67). The Euler Equation therefore implies:

$$U'(Y - \gamma_{-1}Y - \frac{\gamma_t Y}{1+g} - b^*) = (1+g)U'(+\gamma_t Y + (1+g)b^*) \quad (1.71)$$

Along the optimal path, if  $\gamma_t$  drops, the steady state size of the bubble has to rise so as to satisfy the Household's intertemporal Euler Equation.

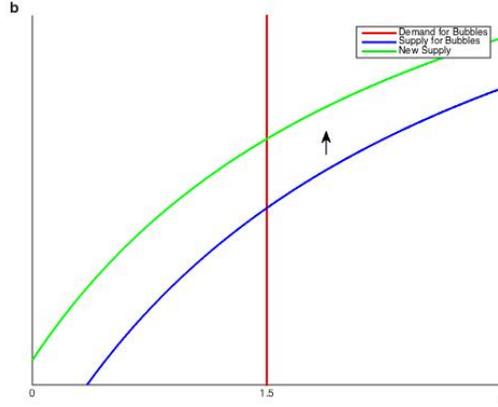


Figure 1.15: Steady state demand and supply for bubbles. The red, blue and green lines represent supply, demand before the shock and demand after the shock.

**Economic Intuition** A drop in the current loan-to-value ratio has two direct effects, both for current young and middle-aged generations. First of all, young agents demand less funds, by (1.9). Secondly, current supply of loans declines. In fact, the current price for the bubble,  $b_t$ , rises reflecting the increase in future demand<sup>19</sup>. Both markets clear with no need for an adjustment of the interest rate. Analytically, consider Eqs. (1.14), (1.18), (1.9), and (1.19). Following  $\Delta\gamma_t < 0$ :

1.  $\Delta L_t^d = \frac{1+g}{1+r_t} \Delta\gamma_t Y_{t+1}$
2.  $\Delta b_{t+1} = -\Delta\gamma_t Y_{t+1} \rightarrow \Delta b_t = \frac{1+g}{1+r_t} \Delta b_{t+1} \rightarrow \Delta L_t^s = -\frac{1+g_t}{1+r_t} \Delta b_{t+1}$

Eventually  $\Delta L_t^s = \Delta L_t^d$ . The existence of bubbles may rule out the possibility of a Secular Stagnation. A deleveraging shock may not have a permanent effect on the interest rate

<sup>19</sup>By the middle age budget constraint (1.14), the expected price for the bubble in (t+1) increases, being negatively related to financial conditions in (t). The reason is that current young generations have less debt to pay back when middle aged and they demand more bubbles.

as in the bubbleless economy in Eggertsson et al. (2017), but a temporary one as in the representative agent economy in Eggertsson and Krugman (2012).

Unfortunately, the bubbly steady state is unstable. Thus, the economy eventually crashes towards the bubbleless steady state following a deleveraging shock.

## Proposition 2

The existence of the "low-bubble" steady state equilibrium requires the slope of the AD to be higher than the slope of the AS. Analytically,

$$\left. \frac{\partial \Pi}{\partial Y} \right|_{AD} = -\frac{\phi_b}{\phi_\pi} \Pi (Y - 3D)^{-1} > \left. \frac{\partial \Pi}{\partial Y} \right|_{AS} = \frac{1 - \alpha}{\alpha} \frac{\Pi}{Y} \left(1 - \frac{\delta}{\Pi}\right)$$

After simplifying and arranging terms, the latter condition can be rewritten as follows:

$$\phi_b < -\epsilon \phi_\pi \frac{(1 - \alpha)}{\alpha}$$

Zooming on the "low-bubble" steady state, Figure 1.16 shows that it is stable. In order to study the dynamics, I label the AD curve as the  $\Delta Y = 0$ , and the AS as the  $\Delta \Pi = 0$ . They divide the plane  $(Y, \Pi)$  in four regions. Consider the  $\Delta Y = 0$  curve. If  $\Pi$  increases, i.e. we move into Regions I or IV, since under Accommodative Monetary Policy  $Y$  is positively related to  $\Pi$ , then  $Y$  increases; if  $\Pi$  decreases, i.e. we move into Regions II or III, instead,  $Y$  decreases. By the same token, consider the  $\Delta \Pi = 0$  curve, which is always upward sloping. If  $Y$  increases, i.e. we move into Regions III or IV,  $\Pi$  increases, while the opposite occurs in the area above the curve, Regions I and II. By combining the above dynamics, it is easy to verify that the "low-bubble" steady state is asymptotically stable.

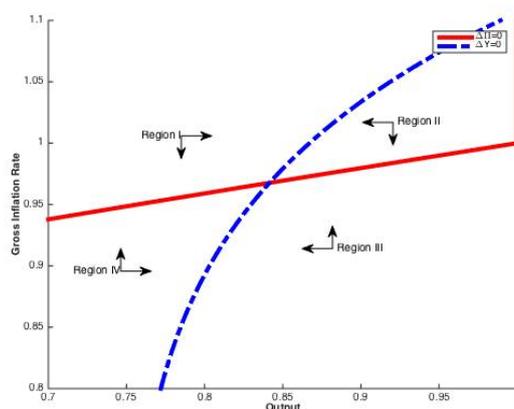


Figure 1.16: Low Bubble Steady State

## Proposition 3

### Full Employment

Under the flexible wages scenario, the equilibrium is given by (1.46) combined with aggregate demand side of the economy. In line with the steady state analysis performed in the previous section, in the flexible price equilibrium bubbles only influence inflation. The result is in line with Galì (2014). Inflation is uniquely pinned down by the following equation:

$$\hat{\pi}_t = \frac{1}{(1-b_2)\phi_\pi} \left\{ d_2 \hat{d}_t + d_1 \hat{d}_{t-1} + \left[ b_2 + b_1 - \phi_b(1-b_2) \right] \hat{b}_t \right\}$$

where

$$\hat{b}_t = \phi_\pi \hat{\pi}_{t-1} + (1 + \phi_b) \hat{b}_{t-1} + \xi_t$$

Notice that the impact of  $\hat{b}_t$  on inflation depends on the approach adopted by monetary policy. It's strictly positive for any

$$\phi_b < \frac{b_1 + b_2}{1 - b_2} = \bar{\phi}_b$$

Concerning dynamics and stationarity, if wages are fully flexible, monetary policy plays no role in affecting the dynamics of the bubble which are not stationary. In fact, I combine the above dynamics of the bubble with one period lagged dynamics of inflation, abstracting from shocks since are not relevant to establish stability, and I obtain that the equilibrium is synthesized by the following difference equation for the bubble:

$$\hat{b}_t = \frac{1 + b_1}{1 - b_2} \hat{b}_{t-1} \quad \text{where} \quad \frac{1 + b_1}{1 - b_2} > 1$$

As in the endowment economy described in Section 1, in the absence of nominal rigidities and independently of monetary policy, the bubbly steady state is unstable<sup>20</sup>.

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<sup>20</sup>It can be shown that  $\frac{1+b_1}{1-b_2} > 1$  is in fact  $\gamma < \frac{1}{3}$ .

## Binding Downward Nominal Wage Rigidity

If the constraint on wages is binding, after solving the system between the AD and the AS, (1.47), we get the following equation describing output gap,  $\hat{y}_t$ :

$$\hat{y}_t = \frac{1}{\left[ y_1 + \frac{1}{\delta_w} \phi_\pi \frac{(1-b_2)(1-\alpha)}{\alpha} \right]} \left\{ d_1 \hat{d}_{t-1} + d_2 \hat{d}_t + (1-b_2) \hat{g}_t + \phi_\pi (1-b_2) \frac{(1-\alpha)}{\alpha} \hat{y}_{t-1} + \hat{b}_t \left[ b_2 + b_1 - \phi_b (1-b_2) \right] \right\}$$

The impact of  $\hat{b}_t$  depends on the value of  $\phi_b$  and it's strictly positive if  $\phi_b < \bar{\phi}_b$ . Concerning dynamics and stationarity, the economy is described by a system of three equations in three variables,  $\hat{\pi}_t$ ,  $\hat{b}_t$ , and  $\hat{y}_t$  as functions of their past values and shocks. For simplification, I focus only on the autonomous component of the equilibrium conditions abstracting from shocks since they are not relevant for stability. The equilibrium is

described by the system  $X_t = \Phi X_{t-1}$  where  $X_t = \begin{bmatrix} \hat{\pi}_t \\ \hat{y}_t \\ \hat{b}_t \end{bmatrix}$  and the matrix  $\Phi$  is defined

as follows:

$$\Phi = \begin{bmatrix} \frac{-1}{\delta_w} \frac{1-\alpha}{\alpha} \chi_2 \phi_\pi & \frac{1}{\delta_w} \frac{1-\alpha}{\alpha} \frac{(1-b_2) \phi_\pi \frac{1-\alpha}{\alpha}}{y_1 + \chi_1} - \frac{1-\alpha}{\alpha} & -\frac{1}{\delta_w} \frac{1-\alpha}{\alpha} \frac{\chi_2 (1+\phi_b)}{y_1 + \chi_1} \\ \frac{-\chi_2 \phi_\pi}{y_1 + \chi_1} & \frac{(1-b_2) \phi_\pi \frac{1-\alpha}{\alpha}}{y_1 + \chi_1} & \frac{-\chi_2 (1+\phi_b)}{y_1 + \chi_1} \\ \phi_\pi & 0 & 1 + \phi_b \end{bmatrix}$$

where  $\chi_1 = (1-b_2) \phi_\pi \frac{1-\alpha}{\alpha} \frac{1}{\delta_w}$  and  $\chi_2 = [\phi_b (1-b_2) - b_1 - b_2]$ .

I am concerned with the values of the policy parameters,  $\phi_b$  and  $\phi_\pi$ , that make sure that the eigenvalues of matrix  $\Phi$  are inside the unit circle. I take as given the deep parameters linked to established economy's features<sup>21</sup>. The matrix  $\Phi$  displays eigenvalues inside the unit circle if and only if

$$\phi_b < -\epsilon \phi_\pi \frac{(1-\alpha)}{\alpha}$$

where  $\epsilon$  is a small, positive number<sup>22</sup>.

<sup>21</sup>I assume  $\delta_w = 0.8 < 1$  and  $d = 0.3 < \phi$ . Henceforth, I recover  $b^y = 0.05$ ,  $b_1 = b_2 = 0.08$ ,  $y_1 = 1.54$ .

<sup>22</sup>Given the assumptions made on the parameters,  $\epsilon = 0.02$ .

## Chapter 2

# Market Structure and Secular Stagnation

# Introduction

In outlining the theory of Secular Stagnation, Hansen (1939) focuses on the role played by the process of capital formation in achieving full employment and on the key effect of population growth. However, he notices that other factors may affect the achievement of full employment, slowing down the process of capital formation and thus progress. The development of monopolistic competition is among these factors and "the workability of free enterprise" is one of the main policy challenges. Despite this significant role, little attention has been paid on the relation between market structure and Secular Stagnation. In this research, I provide an empirical and a theoretical analysis on the link between the markup, as proxy for the market structure, and the equilibrium real interest rate.

Looking at the United States, two key features of the last three decades have been the decrease in the equilibrium interest rate and deep changes in the market structure. Figures 2.1a and 2.1b show that the interest rate and the weighted average cost of capital have experienced a downtrending path since the 90s, though the latter has been masked by the bubbly episodes, the IT and the housing bubbles. The US are not an isolated case and, as most recently discussed by Fisher (2016), interest rates have dropped, even below zero, in an increasing number of countries. Moreover, significant changes in the market structure have occurred: decrease in the number of firms; increase in M&A activity; increase in markups, and; increase in concentration. More specifically, Figure 2.1c shows the decrease in the number of listed companies<sup>1</sup>, while Figures 2.1d and 2.2a show, respectively, the increase in M&A activity and in the markup<sup>2</sup>. Concerning the measurement of the markup, I follow the baseline methodology adopted in Galì (1995) and in Nekarda and Ramey (2013), and I compute the markup as the inverse of the labor income share, resting on the assumption of a Cobb-Douglas production function<sup>3</sup>. Table 2.1 shows that an increase in concentration has occurred in most of the industries. I consider the concentration index released by the Census Bureau, based on the share of

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<sup>1</sup>The US "listing gap" has been analyzed recently by Dojige et al. (2015). They show that the high delist rate is explained by an unusually high rate of acquisitions of publicly-listed firms.

<sup>2</sup>Blonigen and Pierce (2016) provide evidence of increased markups after M&As.

<sup>3</sup>Rotemberg and Woodford (1991, 1999) provide a discussion of models of markup determination and alternative estimation approaches.

sales of the top 50 companies. Data are available until 2012, though a report by CEA (2016) confirms the trend for the most recent years. Looking at aggregate firms data, two key patterns emerge. On the one hand, the burden of R&D expenditures has increased over time, accounting for an increasing share of M&As explained by "economies of scale" reasons, Figure 2.2b. On the other hand, there has been a decrease in capital expenditures and gross investment. Figures 2.2c and 2.2d show the series for capital expenditures and gross investment scaled by total assets. The two variables are highly persistent, and trend downward during the sample though investment in particular shows a clear business-cycle pattern, growing in expansion and falling during recession<sup>4</sup>.

Motivated by these stylized facts, I investigate the relation between the equilibrium interest rate and markup as a proxy for the market structure. I collect data for 14 advanced economies and I recover the equilibrium interest rate and the markup series which are not directly observable. The main results of the empirical analysis are the following. I uncover some evidence that higher markups are associated with lower equilibrium real rates. I show that the main qualitative findings are robust to the use of alternative measures of the markup series. The results are also robust when controlling for additional determinants for the equilibrium interest rate. Finally, I consider the correlation between three alternative indices, released by the OECD and World Bank, which indirectly measure the degree of competitiveness, and an index for long term interest rates released by the OECD, for the year 2013, for the 14 advanced economies. The results show that the more competitive the economies, the higher the interest rate.

I propose a 3-periods OLG model with monopolistic competition to interpret the empirical evidence. In the baseline model, I assume that the markup is an exogenous function of the elasticity of substitution and I show that an increase in the markup puts a downward pressure on the equilibrium interest rate. The zero lower bound on the interest rate however breaks the adjustment mechanism and Secular Stagnation arises, that is "an equilibrium where the equilibrium real interest rate consistent with full employment is permanently negative", Eggertsson et al. (2017). In the extended model, I allow for the markup to be endogenously determined. The markup negatively depends on the number

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<sup>4</sup>Gruber and Kamin (2015) show that the fall of corporate investment goes beyond their model's predictions and it is inconsistent with the view of a cautious investment strategy by firms.

of active firms in the market which in turn is determined in equilibrium by the zero profit condition. I show that an increase in sunk costs reduces profits and the number of firms and triggers the increase in the markup.

The key economic channel for the markup to affect the equilibrium interest rate is the market for capital. I assume that households can either save by making loans to young generations, whose borrowing is constrained by an exogenous limit, or by renting capital to firms. Capital is then used as a production input within the same period and pays a rental rate. I show that an increase in the markup, either exogenous or triggered by a decrease in the number of firms, reduces profits, production and return on capital. The equilibrium level of capital decreases leading to an excess of supply in the loans market which puts a downward pressure on the equilibrium interest rate. The markup thus magnifies the effects of a negative, supply-side, shock by transmitting it to aggregate demand.

The remainder of the paper is organized as follows. In Section 1, I discuss the relation with the existing literature. In Section 2, I discuss the empirical motivation. In Section 3, I develop the baseline model with exogenous markup and I investigate the effects of an increase in the latter. In Section 4, I extend the baseline model to allow for the markup to be endogenously determined. Conclusions follow.

## 2.1 Relation with the existing literature

The paper is related to different strands of literature. From an empirical point of view, the paper is related on one hand to the literature which focuses on the ongoing changes in the structure of the market and their implications, as in Autor et al. (2016, 2017), Barkai (2016), Kahle and Stulz (2016), to name a few. On the other hand, the paper relates to the literature which focuses on the equilibrium, neutral, natural interest rate and its determinants. Though there are slight conceptual differences among the three definitions<sup>5</sup>, I refer to the most generic "equilibrium" to mean "the equilibrium rate consistent with output at potential and price stability", Laubach and Williams (2003). As far as I know, there has been no attempt to investigate the relation between the market

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<sup>5</sup>Williamson (2017) provides a brief discussion on this point.

structure and the equilibrium interest rate. Laubach and Williams (2003, 2015) and Hamilton et al. (2015) consider the relation with the growth rate. Pescatori and Turunen (2015) builds on Laubach and Williams (2003) to estimate the natural rate and to assess the role of the global savings glut and policy uncertainty as its determinants. Rachel et al. (2015) consider also other determinants such as public investment, demographics, inequality and, relative price of capital. Holston et al. (2016) and Gourinchas et al. (2016) focus on the international dimension. Hall (2016) consider the role of heterogeneity across investors. Del Negro et al. (2017) focus on the effect of the rise in the premium for safety and liquidity<sup>6</sup>.

From a theoretical perspective, the paper is related to the literature on Secular Stagnation and it is highly close to Eggertsson et al. (2017). They build a 3 periods OLG model with nominal rigidities and they focus on the role of deleveraging, drop in population growth, rise in inequality, reduction in the price of capital and in the share of labor as triggers for Secular Stagnation. They introduce an exogenous markup in the final goods sector in the quantitative exercise to match the reduction in the labor share as one of the key moments in the data and they find that the reduction in labor share accounts only for a small share of the decline in the equilibrium interest rate. My analysis differs from the one in Eggertsson et al. (2017) for two key reasons. On the one hand, I consider the increase in the markup to be the key shock behind the decline observed in both labor and capital shares, as shown in Barkai (2016). On the other hand, Eggertsson et al. (2017) do not explore the channel through which changes in market structure generate negative interest rates, and Secular Stagnation, which is instead the primary focus of this project. I allow for monopolistic competition in the intermediate goods sector, and I explore the implications of a change in the market structure both in an exogenous and endogenous markup framework.

The paper is further related to the literature on the effects of monopolistic competition starting with the seminal papers by Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987). Blanchard and Kiyotaki (1987) first highlight the existence of an "aggregate

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<sup>6</sup>Blanchard et al. (2017) add a twist on the Secular Stagnation hypothesis suggesting that pessimism about the future affects output and inflation through subdued demand and supply so that the interest rate undershoots the long run value.

demand externality” such that underproduction due to monopolistic competition is magnified through macroeconomic interactions. The latter is traditionally interpreted through the ”price channel”: if starting from a monopolistically competitive equilibrium, a shock occurs and affects prices, then aggregate demand reacts and output must adjust. I focus on a novel channel that works through the asset market and the return on capital. Barkai (2016) show that the macroeconomic implications of declining competition and increasing markups are the decline in labor and capital shares and the increase of the output and investment gap. I focus on a long-term perspective and I show that declining competition and increasing markups trigger the decline in the natural interest rate.

The paper is also related to Galí (1995), Peretto (1996), Bilbiee et al. (2012) and Cacciatore et al. (2015) who study the implications for macroeconomic fluctuations with an endogenous determination of the number of firms. Sunk costs are needed to bound and determine the equilibrium number of firms. The way sunk costs are introduced, whether they are entry or production sunk costs, is crucial. If sunk costs only burden production as in Peretto (1996), then entry is frictionless and the number of firms is a jumping variable which is free to adjust at its equilibrium value at any point in time. If sunk costs burden entry as in Bilbiee et al. (2012) and Cacciatore et al. (2015), then profits are allowed to vary and the number of firm is a state variable. In the latter scenario it makes sense to characterize the short-term as described by a fixed number of firms, while in the former there is no such difference.

Concerning the natural interpretation for sunk costs, they are generally referred to as R&D expenditures<sup>7</sup>. Autor et al. (2016) and Kahle and Stilz (2016) provide evidence that there has been an increase in R&D expenditures in the recent decades. Moreover, Autor et al. (2016) provide the empirical evidence that technological change has made markets increasingly concentrated so that the firms with higher productivity increasingly capture a larger share of the market. They find that the industries that became more concentrated were also the industries in which productivity increased the most. In the extended theoretical model, I show that the increase in sunk costs has contractionary effects by triggering a decrease in the number of active firms and thus an increase in the

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<sup>7</sup>One of the reasons for this interpretation lies on the accounting rules. Both under the IAS 38 and the US-GAAP, R&D expenses, unlike capital expenditures, are not capitalized.

markup.

Finally, the paper relates to the literature on the effects of structural reforms that aim to reduce markups by promoting competition. Blanchard and Giavazzi (20013), Bayoumi et al. (2004), Eggertsson et al. (2013), Fernández-Villaverde et al. (2014), and Gerali et a. (2015) show the beneficial effects of structural reforms in different contexts. In particular, Gerali et a. (2015) discuss the key role played by physical capital in magnifying the wealth effect. Eggertsson et al. (2013) show that although structural reforms have a long-term positive effect on output, they do have contractionary effects in the short-run. However, there are two major differences with Eggertsson et al. (2013) which potentially may lead to different results: the presence of capital and the endogeneity of the number of firms. First of all, accounting for firms' incentives to operate, though in a frictionless entry economy, eliminates the short-term distortions. Moreover, as postulated in Fernández-Villaverde (2014), the introduction of capital turns out to be the essential channel for the transmission of the effects of structural changes.

## 2.2 Empirical Motivation

The empirical evidence aims to uncover the qualitative relation between the equilibrium interest rate and the market structure. First, following a dynamic approach, I study the correlation between the equilibrium interest rate and the series of the markup, both at the US and cross-country level. Secondly, through a static perspective, I study the correlation between an index for long-term interest rate and three alternative indices released by the OECD and the World Bank that qualify the degree of competitiveness of countries.

I define the ex-ante real interest rate as the equilibrium interest rate, that is a rate consistent, on average, with output at potential and stable inflation. I follow the approach adopted by Hamilton et al. (2015) and I build the series for the equilibrium interest rate as the nominal short-term policy rate minus expected inflation<sup>8</sup>. I follow the

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<sup>8</sup>The assumption behind the methodology in Hamilton et al. (2015) is that "if policymakers are setting the nominal interest rate, so that on average, the output gap is zero, inflation is equal to target, and expected inflation is equal to target, then the ex ante real interest rate will equal the equilibrium interest rate as defined by the authors", Mester (2014).

common approach in the literature of inferring expected inflation from the forecast of an autoregressive model based on the previous year inflation. The autoregressive model is estimated through 10 years rolling windows to allow for time varying coefficients. With quarterly data, the forecasting equation is a fourth-order autoregression:

$$\pi_t = c + \phi_{1,t}\pi_{t-1} + \phi_{2,t}\pi_{t-2} + \phi_{3,t}\pi_{t-3} + \phi_{4,t}\pi_{t-4} + \epsilon_t$$

The ex-ante real interest rate is thus

$$r_t = i_t - [\hat{c} + \phi_{1,t}\pi_t + \phi_{2,t}\pi_{t-1} + \phi_{3,t}\pi_{t-2} + \phi_{4,t}\pi_{t-3}]$$

Figure 2.3 shows that the equilibrium real interest rate has varied considerably over time. Moreover, it has experienced a downward trend since the 80s.

I investigate the sign of the correlation between the equilibrium interest rate and a variable related to the market structure for the United States. I choose the markup as a proxy for market structure and I build the series as the inverse of the labor income share. The approach is justified by the assumption of a Cobb-Douglas production function.

The correlation coefficient for the full sample is high and equal to -0.6466. Figure 2.4a shows the scatter plot. Figure 2.4b instead is related to the peak-to-peak analysis. Peaks considered are the ones defined for the US business cycles according to NBER. Data for interest rate and markup are the averages in the period between peaks. The analysis of correlation delivers a value equal to -0.5677. Finally, Figure 2.4c shows the scatter plot for the 40 quarters moving averages. The correlation in this case is equal to -0.6412. Finally, by dividing the sample in three main subperiods, I observe that the correlation has increased over time in absolute terms. For the first subperiod, 1964q1 to 1990q1, the correlation equals -0.1943; for the second, until 2000q1, the correlation increases to -0.2631; in the last part of the sample the correlation reaches the value -0.7567. Though the correlation between the two variables has varied over time, the key qualitative finding is thus robust across different samples.

I extend the analysis to 14 advanced economies. I consider annual data for per capita GDP, population, nominal interest rate, inflation and labor share for the United States, Japan, Spain, Germany, France, Italy, Norway, Sweden, Denmark, UK, Canada,

Australia, Switzerland and, The Netherlands for the period 1980-2008<sup>9</sup>. Figure 2.5 shows that all the countries have experienced a long-term decline in the equilibrium interest rate, ending in some cases in negative territory in the last part of the sample. Figure 2.6 shows that the 14 economies also share a similar upward trend in the markup. The correlation analysis at the cross-country level confirms the previous findings. The correlation coefficient is negative and significantly different from zero, though smaller than the one for the United States. For the full sample, the correlation is -0.2354, while the one for the 10 years moving average further drops to -0.1591. To evaluate the robustness of the results concerning the role of the markup as one of the determinants of the equilibrium interest rate, I perform a panel regression analysis controlling for additional determinants. The coefficient associated with the markup is negative and statistically significant across almost all model specifications. Detailed results are shown in the Appendix.

Finally, through a static analysis, I provide evidence that more market friendly countries do have higher interest rates. I consider three different measures of competitiveness and I study the correlation with a measure for the interest rate. I consider the following indices released for 2013 for the above-mentioned 14 advanced economies: the Product Market Regulation Index released by the OECD, the Easy of Doing Business Indicator released by the World Bank and the Strength of Legal Rights Index released by the OECD. The indices qualify the degree of competitiveness of an economy through different perspectives. The PMR Index goes from 0 to 6 reflecting increasing restrictiveness of regulatory provisions for competition, covering issues related to state-control, barriers to entrepreneurship and barriers to trade and investment. For the EDB Index, I consider the Distance to Frontier score that helps to assess to absolute level of regulatory performance over time and it measures the distance of each country to the frontier, which is the best performance observed on each of the indicators across all economies in the Doing Business sample since 2005. The DTF goes from 0 to 100, reflecting the increasing closeness to the frontier. The Strength of Legal Rights Index

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<sup>9</sup>I use data for the nominal interest rate and inflation from Schularick and Taylor (2012) dataset. Per capita GDP is from Barro-Ursua Macroeconomic Data (2010). The markup is the inverse of the Share of Labour Compensation in GDP at Current National Prices, Source: University of Groningen, University of California, Davis.

goes from 0 to 12 with higher scores meaning better protection of legal rights. As for the interest rate, I consider the government bonds indicator collected by the OECD. Figures 2.7a, 2.7b and 2.7c show the scatter plots for the three indices. Leaving out Italy and Spain, which turn out to be the worst performers and apparently outliers among the selected countries<sup>10</sup>, then the scatter plot shows that the more market friendly the economies are, the higher the interest rates. Those countries with the lowest PMR score, or the highest DTF score and the highest Strength of legal rights index have the highest interest rates.

### 2.2.1 Robustness Check

I replicate the analysis of the correlation between the equilibrium interest rate and the markup for the United States by employing different measurement approaches for the markup series. Following Nekarda and Ramey (2013), I test the following three alternative measures:

- Markup with overhead costs. I only consider the series of production workers in the private sector, excluding overhead costs. I compute the markup as minus the log of the share of production workers, constructed by multiplying BLS quarterly data on employment, average weekly hours, and average hourly wages of production workers in the private sectors and then dividing by current dollar output in private business.
- Markup with CES production function computed as

$$\mu^{CES} = -\ln s + \left(\frac{1}{\sigma} - 1\right)[\ln Y - \ln(ZhN)]$$

where  $s$ ,  $h$  and  $N$  are the labor share in the private business sector, the average weekly hours and the employment data provided by BLS;  $\sigma$  is set equal to 0.5 following Chirinko (2008), and;  $Z$  is the quarterly data on TFP provided by Fernald (2012). I employ the unadjusted series, that is the standard Solow residual.

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<sup>10</sup>The evidence is not surprising since the two countries have been involved in the European debt crisis, a multiyear debt crisis which took place in Europe since 2009.

- Markup with CES production function where Z is the utilization adjusted TFP growth series, Fernald (2012).

Table 2.2 shows the results for the correlation analysis between the equilibrium ex ante real interest rate and the markup series under the baseline, the "overhead costs" and the two "CES production function" specifications. The correlation coefficient is negative and significantly different from zero, though it is much lower than the one under the baseline specification. Whether I employ the unadjusted or the adjusted TFP measures does not deliver different results.

## 2.3 A simple model with exogenous markup

In this section, I develop a simple model to interpret the empirical evidence. I adopt a 3 periods OLG model and I allow for output to be endogenously determined, for monopolistic competition and for nominal rigidities. I show that an exogenous increase in the markup puts a downward pressure on the equilibrium interest rate and triggers Secular Stagnation.

### Households

Households live for 3 periods and they obtain income only in their middle age. In this case, they borrow when young from the middle-aged households and they save for retirement when middle-aged. Population grows at a rate  $g_t = \frac{N_t}{N_{t-1}} - 1$ , where  $N_t$  is the size of population at time t. Each individual seeks to maximize the following utility function:

$$\max U = \log C_t^y + \beta \log C_{t+1}^m + \beta^2 \log C_{t+2}^o \quad (2.1)$$

where  $\beta$  is the subjective discount factor, and  $C_t^y$ ,  $C_{t+1}^m$  and  $C_{t+2}^o$  are the consumptions of the household when young, middle-aged and old. The household faces the following

budget constraints

$$C_t^y = B_t^y \quad (2.2)$$

$$C_{t+1}^m + K_{t+1} + (1 + r_t)B_t^y = w_{t+1}L_{t+1} + r_{t+1}^k K_{t+1} + Z_{t+1} + B_{t+1}^m \quad (2.3)$$

$$C_{t+2}^o + (1 + r_{t+1})B_{t+1}^m = (1 - \delta)K_{t+1} \quad (2.4)$$

$$(1 + r_t)B_t^y \leq D_t \quad (2.5)$$

where Eq. (2.2), (2.3) and (2.4) are the budget constraints for the young, the middle and the old ages, and (2.5) is the borrowing constraint for the young. Consumption of the young equals  $B^y$ , which denotes borrowing. When middle-aged, the agent has two alternative assets for savings: he can either make loans to young households, or invest in capital that is rented to firms. Income for the middle-aged is composed of labor income,  $w_{t+1}$  is real wage, that is  $w = \frac{W}{P}$  being  $W$  the nominal wage; profits,  $Z_{t+1}$ , and; return on capital. Capital is rented out in the same period as when investment takes place, enters as an input factor in the production function and has a real rental rate equal to  $r_t^k$ . When old, the agent consumes all his savings and the return from the sale of depreciated capital, where depreciation occurs at the rate  $\delta$ . Concerning Eq. (2.5), I assume that the exogenous borrowing constraint for the young is always binding. The latter assumption implies that consumption for the young equals the limit:

$$C_t^y = \frac{D_t}{1 + r_t} \quad (2.6)$$

Households' optimization implies:

$$\frac{1}{C_t^m} = \beta \frac{1 + r_t}{C_{t+1}^o} \quad (2.7)$$

$$(1 - r_t^k)C_{t+1}^o = \beta(1 - \delta)C_t^m \quad (2.8)$$

Eq. (2.7) is the consumption Euler Equation, while Eq. (2.8) defines the optimal choice for capital. By no arbitrage, combining the two equations gives the relation between the rental rate and the real interest rate as follows:

$$1 + r_t = \frac{(1 - \delta)}{1 - r_t^k} \quad (2.9)$$

I consider the case that agent has access also to riskless, one period nominal debt denominated in money. The asset is in zero net supply, and the government controls

its return, the nominal interest rate. Implicitly, the existence of money<sup>11</sup> implies that there is a lower bound on the nominal rate, i.e.  $i_t \geq 0$ . The Euler Equation reads as follows

$$\frac{1}{C_t^m} = \frac{1}{C_{t+1}^o} (1 + i_t) \frac{P_t}{P_{t+1}} \quad (2.10)$$

### Final good producers

The final good producers buy intermediate goods, package  $Y_t$ , and resell it to consumers in a perfectly competitive market. In particular, they seek to maximize profits

$$\max P_t Y_t - \int_0^1 p_t(i) y_t(i) di \quad (2.11)$$

where  $Y_t = (\int_0^1 y_t(i)^{\frac{\sigma-1}{\sigma}} di)^{\frac{\sigma}{\sigma-1}}$ .  $P_t$  and  $p_t(i)$  are the price of the final and intermediate goods respectively, where  $P_t \equiv (\int_0^1 p_t(i)^{\sigma-1} di)^{\frac{1}{1-\sigma}}$ . Maximization implies the following demand schedule for variety  $i$ :

$$p_t(i) = \left( \frac{y_t(i)}{Y_t} \right)^{-\frac{1}{\sigma}} P_t \quad (2.12)$$

### Intermediate good producers

Intermediate good producer of variety  $i$  seeks to maximize nominal profits,

$$\max Z_t(i) = p_t(i) y_t(i) - W_t L_t(i) - P_t r_t^k K_t(i) \quad (2.13)$$

subject to a Cobb-Douglas production function in capital and labor,

$$y_t(i) = A K_t(i)^{1-\alpha} L_t(i)^\alpha$$

and the demand schedule, (2.12). The first order conditions for the problem above are

$$\begin{aligned} \frac{W_t}{p_t(i)} &= \left( 1 - \frac{1}{\sigma} \right) \alpha \frac{y_t(i)}{L_t(i)} \\ \frac{P_t r_t^k}{p_t(i)} &= \left( 1 - \frac{1}{\sigma} \right) (1 - \alpha) \frac{y_t(i)}{K_t(i)} \end{aligned}$$

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<sup>11</sup>For a discussion about the introduction of money within an OLG framework, see Galí (2014.)

Given the symmetry of the model, all firms charge the same price,  $p(i) = p(j) = P$  and produce the same amounts,  $y(i) = y(j) = Y$ . Letting  $K_t$  and  $L_t$  denote the aggregate capital stock and labor supply, it follows from the first order conditions above, that

$$w_t = \frac{W_t}{P_t} = \left(1 - \frac{1}{\sigma}\right) \alpha \frac{Y_t}{L_t} \quad (2.14)$$

$$r_t^k = \left(1 - \frac{1}{\sigma}\right) (1 - \alpha) \frac{Y_t}{K_t} \quad (2.15)$$

By the firms' demand for labor and capital, marginal products of production inputs equal real marginal costs plus a markup. The presence of market power generates a wedge between the marginal product and the cost, and both the real wage and the return on capital are lower than their counterparts under perfect competition<sup>12</sup>. I define

$$\mu \equiv \frac{\sigma}{\sigma - 1}$$

as the measure of monopoly/market power. Equilibrium prices thus are not exogenous but are determined endogenously and depending on market power. The latter is a decreasing function of the elasticity of substitutability across varieties,  $\sigma$ . As  $\sigma \rightarrow \infty$ , the economy tends to the perfect competition benchmark.

## Monetary Policy

Monetary policy sets the nominal rate according to a standard Taylor rule:

$$1 + i_t = \max \left( 1, (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right) \quad (2.16)$$

I set  $\Pi^* = 1$  and  $i^* = 0$ , so that the zero lower bound for the steady state nominal rate,  $i \geq 0$ , binds at  $\Pi = 1$ .

### 2.3.1 Equilibrium

#### Labor Market

When wages are fully flexible, equilibrium labor equals fixed supply,  $\bar{L}$ . However, I assume the presence of a downward nominal wage rigidity constraint, such that nominal wages

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<sup>12</sup>Under perfect competition, the solution is  $w_t^* = \alpha \frac{Y_t}{L_t}$  and  $r_t^{k*} = (1 - \alpha) \frac{Y_t}{K_t}$ .

can never fall below the wage norm<sup>13</sup>,  $\tilde{W}_t$ :

$$W_t = \max \left\{ \tilde{W}_t, P_t w_t^f \right\}$$

where  $w_t^f$  is the real wage that would prevail in the flexible wages economy, defined as follows

$$w_t^f = \left( 1 - \frac{1}{\sigma} \right) \alpha A K_t^{1-\alpha} \bar{L}^{\alpha-1}$$

and  $\tilde{w}_t = \frac{\tilde{W}_t}{P_t}$  is such that

$$\tilde{w}_t = \gamma \frac{w_{t-1}}{\Pi_t} + (1 - \gamma) w_t^f$$

### Asset Market

Equilibrium in the market for loans requires that borrowing of the young equals the savings of the middle-aged so that

$$N_t B_t^y = -N_{t-1} B_t^m$$

or, equivalently,

$$(1 + g_t) B_t^y = -B_t^m \quad (2.17)$$

where I have normalized by middle-aged population size. I label the left-hand side as demand of loans,  $L_t^d$ , and the right hand side as supply of loans,  $L_t^s$ . Demand comes from the young, constrained agents, while supply of loans depends on total lending from middle-aged agents. Thus, equilibrium can be rewritten as

$$L_t^d = L_t^s$$

where

$$L_t^d = (1 + g_t) B_t^y = \frac{1 + g_t}{1 + r_t} D_t \quad (2.18)$$

$$L_t^s = -B_t^m = \frac{\beta}{1 + \beta} (Y_t - D_{t-1}) - \frac{\beta}{1 + \beta} \left( 1 + \frac{1}{\beta(1 + r_t)} \right) K_t \quad (2.19)$$

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<sup>13</sup>The existence of wage rigidities and the distinction between "insider" and "outsider" workers is one of the explanations in the "hysteresis" literature, that is one of the reasons why shocks that generate unemployment may have long-term effects. Blanchard and Summers (1986) show that wage bargaining, a key feature of labor markets, can help explain the substantial unemployment persistence.

where I have used Eq. (2.5) to derive the demand schedule and the Euler Equation, Eq. (2.7), together with the budget constraints, Eqs. (2.3) and (2.4), to derive the supply schedule. Demand of loans is a negative function of the interest rate, while it is a positive function of the exogenous borrowing limit,  $D_t$ , and growth rate. Supply of loans is an increasing function of the discount factor,  $\beta$ , net income, and interest rate, while it is negatively affected by the available stock of capital. Ceteris paribus, an increase in the available stock of capital reduces supply in the market for loans.

Equilibrium in the market for capital is such that households are willing to purchase any amount of capital for a given real rental rate which satisfies the no-arbitrage condition, Eq. (2.9). The equilibrium amount of capital is pinned down by firms' demand which can be recovered through Eq. (2.15). For future reference, demand for capital is negatively related to the markup.

By combining (2.18) and (2.19), I get the equilibrium interest rate for the loans market:

$$1 + r_t = \frac{1 + \beta}{\beta} \frac{(1 + g_t)D_t + \frac{(1-\delta)}{1+\beta}K_t}{Y_t - D_{t-1} - K_t} \quad (2.20)$$

For later use, the equilibrium interest rate is a positive function of capital. This follows from the fact that, ceteris paribus, an increase in capital reduces supply in the market for loans, putting an upward pressure on the interest rate. At this point it is useful to clarify that Eq. (2.20), evaluated at output equal to its potential, defines the natural interest rate,  $r^f$ , that is the interest rate consistent with full employment in equilibrium. By substituting out for capital, it follows that the natural interest rate is a negative function of the markup. Finally, equilibrium in the goods market follows by Walras' law.

The *equilibrium* is a set of quantities  $\{Y_t, L_t, C_t^m, C_t^o, B_t^m, K_t\}_{t=0}^{\infty}$  and prices  $\{i_t, \Pi_t, r_t^k, w_t\}_{t=0}^{\infty}$ , given exogenous processes for  $\{g_t, D_t\}_{t=0}^{\infty}$  that satisfy households' optimization, Eqs (2.7) and (2.8), together with budget constraints; firms' optimization, Eqs. (2.14) and (2.15) given the production function, and; monetary policy rule. Market clearing conditions hold.

### 2.3.2 Steady State

I consider the steady states of the economy. There are two scenarios, depending on whether the gross inflation rate is greater or lower than 1. On the AS side, the downward nominal wage rigidity constraint binds when the steady state features a negative inflation rate, that is  $\Pi < 1$ . Thus, labor is rationed. In fact, if the downward nominal wage rigidity is binding, the following equation holds:

$$w_t = \gamma \frac{w_{t-1}}{\Pi_t} + (1 - \gamma)w_t^f$$

By combining the latter with Eq. (2.14) and the definition for the flexible wage,  $w_t^f$ , and evaluating the resulting expression at the steady state such that  $K_t = K_{t-1} = K$  and  $L_{t-1} = L_t = L$ , I get the following relation between the steady state equilibrium labor and inflation:

$$L^{\alpha-1} \left(1 - \frac{\gamma}{\Pi}\right) = (1 - \gamma)\bar{L}^{\alpha-1}$$

When labor is rationed, the steady state equilibrium level of employment is below full employment, that is  $L < \bar{L}$ , and it must be that  $\left(\frac{1-\gamma}{1-\Pi}\right)^{\frac{1}{1-\alpha}} < 1$  which implies  $\Pi < 1$ . By construction whether the constraint on wages is binding or not depends only on the degree of flexibility in the labor market, that is the parameter  $\gamma$ , and on the gross inflation rate.

On the AD side, by assumption, the zero lower bound constraint for monetary policy initially binds as  $\Pi < 1$ . In general the kink of the aggregate demand curve occurs at the inflation rate at which the zero lower bound constraints monetary policy, that is  $\Pi_{kink} = \left(\frac{1}{1+i^*}\right)^{\frac{1}{\phi_\pi}} \Pi^*$ .

Therefore if  $\Pi > 1$ , the economy is described by the following set of conditions:

$$\begin{aligned} r^k &= \left(1 - \frac{(1-\delta)}{1+r}\right) \\ r^k &= (1-\alpha)\left(1 - \frac{1}{\sigma}\right)\frac{Y}{K} \\ Y &= AK^{1-\alpha}\bar{L}^\alpha \\ Y &= D + \frac{1+\beta}{\beta}\frac{1+g}{1+r}D + K\left(1 + \frac{(1-\delta)}{\beta(1+r)}\right) \end{aligned}$$

where, by the policy rule, inflation is such that  $1 + r = \Pi^{\phi_{\pi}-1}$ . The fact that the steady state marginal product of capital cannot be negative,  $r^k \geq 0$ , implies that

$$r \geq -\delta$$

Observe that the assumption of a positive rate of depreciation is key to avoid a zero lower bound on the real interest rate. In the absence of depreciation, capital would act as a perfect storage of wealth thus imposing a zero lower bound on the real interest rate as well.

If  $\Pi < 1$ , instead, the economy is described by the following set of conditions:

$$\begin{aligned} L &= \left( \frac{1 - \frac{\gamma}{\Pi}}{1 - \gamma} \right)^{\frac{1}{1-\alpha}} \bar{L} \\ r^k &= \left( 1 - \Pi(1 - \delta) \right) \\ r^k &= (1 - \alpha) \left( 1 - \frac{1}{\sigma} \right) \frac{Y}{K} \\ Y &= AK^{1-\alpha}L^\alpha \\ Y &= D + \frac{1 + \beta}{\beta}(1 + g)\Pi D + K \left( 1 + \frac{\Pi}{\beta}(1 - \delta) \right) \end{aligned}$$

I solve the model in both cases by substituting out for  $K$ ,  $L$ ,  $r^k$  and  $r$  to obtain the two equations in the two endogenous,  $Y$  and  $\Pi$ . For future reference, I compute the steady state level of capital by combining the first two equations for the scenario  $\Pi > 1$ , or the second and third ones for the scenario  $\Pi < 1$ , and I get:

$$K = \left( 1 - \frac{1}{\sigma} \right) (1 - \alpha) \left( 1 - \frac{1 - \delta}{1 + r} \right)^{-1} Y \quad (2.21)$$

By Eq. (2.21), the steady state level of capital is inversely related to the markup.

### 2.3.3 A change in market structure

Consider the parameters as listed in Table 2.3, Column 1. I assume that the nominal interest rate is consistent with the Taylor rule,  $1 + i^* = (1 + r^f)\Pi^*$ , where  $r^f$  is the natural interest rate as defined in Eq. (2.20) under full employment. By this assumption, it follows that any shock affecting the natural interest rate will alter the kink of the aggregate demand curve.

The goal of the project is to explore the implications for the equilibrium allocation and for the natural interest rate of a change in the market structure. Therefore, I assume that there is a decrease in the elasticity of substitution across goods, and  $\sigma$  drops from 10 to 2, which triggers an increase in the markup and a drop in the natural interest rate to -6%. Figure 2.8 shows the AD and AS schedules, before and after the shock. As the markup increases, and the natural interest rate drops, the kink of the aggregate demand curve moves inwards, and both the AD and AS curves shift leftwards. The economy goes from Full Employment, point A in Figure 2.8, to Secular Stagnation, point B.

From the supply side, there are two forces at work. Consider the upper portion of the AS schedule, and potential output as defined by the following equation:

$$Y = A^{\frac{1}{\alpha}} \left[ \left(1 - \frac{1}{\sigma}\right) (1 - \alpha) \left(1 - \frac{1 - \delta}{1 + r}\right)^{-1} \right]^{\frac{1-\alpha}{\alpha}} \bar{L}$$

where I have substituted out for capital using Equation (2.21) within the production function. As the above equation makes clear, an increase in the markup reduces capital and thus potential output, for a given level of inflation. Therefore, the AS curve moves leftwards. Moreover, aggregate supply is further reduced due to an aggregate demand externality. Market power in fact indirectly affects also aggregate demand and the decrease in potential output induces a leftward shift in the AD schedule. Moreover, recall the equilibrium in the market for loans and, specifically, the loans supply schedule, Eq. (2.19). Supply of loans negatively depends on the level of capital. The decrease in the steady state level of capital increases the supply of loans, putting a downward pressure on the equilibrium interest rate for loans. The existence of the zero lower bound prevents the rate from adjusting and the economy experiences Secular Stagnation. Deflation raises the real wage and rental rate of capital which further depresses firms' demand for inputs and equilibrium output. Hence, a sufficiently deep change in the market structure will move the economy away from Full Equilibrium, triggering Secular Stagnation.

## 2.4 A model with endogenous markup

In this section I assume that the markup depends on the number of firms which is endogenously determined in equilibrium and depends on the level of sunk costs, proxy for

$R\&D$  expenditures, that each firm has to bear. The task of this section is to construct an equilibrium with free entry and free exit where all firms face the same production technology and to characterize the effects of an increase in sunk costs.

Following Galì (1995), I assume that the elasticity of substitution and markup are endogenous, and depend on the size of the market for intermediate goods. Specifically,  $\sigma$  is a function of  $n$ , the number of intermediate goods, and has the following properties:

$$\frac{\partial \sigma(n)}{\partial n} = \sigma'(n) > 0 \quad \lim_{n \rightarrow 0} \sigma(n) = \bar{\sigma} > 1 \quad \lim_{n \rightarrow \infty} \sigma(n) \rightarrow \infty$$

### Final good producers

Final good producers seek to maximize profits as in Eq. (2.11) and they adopt the following technology

$$Y = [n^{-(1-\frac{1}{\mu(n)})} \int_0^n y(i)^{\frac{1}{\mu(n)}} di]^{\mu(n)}$$

where  $y(i)$  is the quantity of intermediate goods  $i \in [0, n]$  and  $\mu : R^+ \rightarrow R^+$  is a continuously differentiable function with the following properties:

$$\frac{\partial \mu(n)}{\partial n} = \mu'(n) < 0 \quad \lim_{n \rightarrow 0} \mu(n) = \bar{\mu} > 1 \quad \lim_{n \rightarrow \infty} \mu(n) = 1$$

The above properties follow from the assumption about the elasticity of substitution among goods,  $\sigma(n) = \frac{\mu(n)}{\mu(n)-1}$ .

The solution to the final good producers' problem gives the following demand schedule for the variety  $i$ :

$$y_t(i) = \frac{Y_t}{n_t} \left( \frac{P_t}{p_t(i)} \right)^{\sigma(n_t)} \quad (2.22)$$

where the aggregate price index,  $P$ , is  $P \equiv \left[ \frac{1}{n} \int_0^n p(i)^{1-\sigma(n)} di \right]^{\frac{1}{1-\sigma(n)}}$ .

### Intermediate good producers

Each firm  $i$  seeks to maximize profits, Eq. (2.13), subject to the demand schedule, Eq. (2.22), and a production function which accounts for sunk costs:

$$y_t(i) = AK_t(i)^{1-\alpha} L_t(i)^\alpha - \Omega$$

where  $\Omega$  represents exogenous sunk costs, meant as the minimum level of R&D expenses required for production. I follow Peretto (1996) and I assume frictionless entry so that the number of firms is a jumping variable which is free to adjust at its equilibrium value at any point in time such that profits are zero. By optimality follows:

$$\begin{aligned} K_t(i) : \quad & \left(1 - \frac{1}{\sigma(n_t)}\right) \left(\frac{n_t}{Y}\right)^{-\frac{1}{\sigma(n_t)}} P_t y_t(i)^{-\frac{1}{\sigma(n_t)}} (1 - \alpha) A K_t^{-\alpha}(i) L_t(i)^\alpha = P_t r_t^k \\ L_t(i) : \quad & \left(1 - \frac{1}{\sigma(n_t)}\right) \left(\frac{n_t}{Y}\right)^{-\frac{1}{\sigma(n_t)}} P_t y_t(i)^{-\frac{1}{\sigma(n_t)}} \alpha A K_t^{1-\alpha}(i) L_t(i)^{\alpha-1} = W_t \end{aligned}$$

Given the symmetry of the model, all firms face the same marginal costs, hence equilibrium prices and quantities are identical across firms. Equalities of prices, by Eq. (2.22), implies that  $y(i) = \frac{Y}{n}$ , and so,  $K(i) = \frac{K}{n}$ , and  $L(i) = \frac{L}{n}$ . Hence, the aggregate production function in equilibrium can be written as follows:

$$Y_t = A K_t^{1-\alpha} L_t^\alpha - n_t \Omega \quad (2.23)$$

and first order conditions as

$$w_t = \left(1 - \frac{1}{\sigma(n_t)}\right) \alpha \frac{(Y_t + n_t \Omega)}{L_t} \quad (2.24)$$

$$r_t^k = \left(1 - \frac{1}{\sigma(n_t)}\right) (1 - \alpha) \frac{(Y_t + n_t \Omega)}{K_t} \quad (2.25)$$

where I have used the production function and the fact that  $W_t = w_t P_t$ .

Moreover, in equilibrium the number of firms is such that the zero-profit conditions is always satisfied. Analytically, real profits are defined as follows:

$$\begin{aligned} Z_t = Y_t - w_t L_t - r_t^k K_t = \\ Y_t - \left(1 - \frac{1}{\sigma(n_t)}\right) \alpha \frac{Y_t + n_t \Omega}{L_t} L_t - \left(1 - \frac{1}{\sigma(n_t)}\right) (1 - \alpha) \frac{Y_t + n_t \Omega}{K_t} K_t = \\ \frac{(\mu(n_t) - 1) Y_t - n_t \Omega}{\mu(n_t)} \end{aligned} \quad (2.26)$$

where I have used Eqs. (2.24), (2.25) and the definition for the markup,  $\mu$ . By Eq. (2.26) profits depend on the number of firms, level of sunk costs and aggregate demand.

### 2.4.1 Steady State

I focus on the steady state equilibria. As in the baseline model, there are two scenarios, depending on whether the gross rate of inflation is above or below 1.

Case  $\Pi > 1$

$$\begin{aligned}
r^k &= \left(1 - \frac{(1-\delta)}{1+r}\right) \\
r^k &= \left(1 - \frac{1}{\sigma(n)}\right)(1-\alpha)\frac{AK^{1-\alpha}\bar{L}^\alpha}{K} \\
Y &= AK^{1-\alpha}\bar{L}^\alpha - n\Omega \\
Y &= D + \frac{1+\beta}{\beta}\frac{1+g}{1+r}D + K\left(1 + \frac{(1-\delta)}{\beta(1+r)}\right)
\end{aligned}$$

with  $1+r = \Pi^{\phi_\pi - 1}$  by the Taylor rule.

Case  $\Pi < 1$

$$\begin{aligned}
L &= \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{1}{1-\alpha}}\bar{L} \\
r^k &= \left(1 - \Pi(1-\delta)\right) \\
r^k &= \left(1 - \frac{1}{\sigma(n)}\right)(1-\alpha)\frac{AK^{1-\alpha}L^\alpha}{K} \\
Y &= AK^{1-\alpha}L^\alpha - n\Omega \\
Y &= D + \frac{1+\beta}{\beta}(1+g)\Pi D + K\left(1 + \frac{\Pi}{\beta}(1-\delta)\right)
\end{aligned}$$

Finally, the zero profits condition then implies that the equilibrium number of firms,  $n^*$ , satisfies the following condition:

$$n^*:\frac{Y}{n} = \frac{\Omega}{\mu(n) - 1} \quad (2.27)$$

**Assumption 1** I assume

$$\sigma(n) = \sigma n$$

where  $\sigma > 0$ . The latter implies  $\mu(n) = \frac{\sigma n}{\sigma n - 1}$ .

By the zero profits condition, then

$$n^* = \frac{1 + \sqrt{1 + \frac{4\sigma Y}{\Omega}}}{2\sigma} \quad (2.28)$$

The equilibrium number of firms,  $n^*$ , is a positive function of output and a negative function of sunk costs,  $\Omega$ . Moreover, by combining the zero profit condition with the aggregate production function<sup>14</sup> and Eq. (2.25), I obtain the equilibrium rental rate of capital,  $r^k$ :

$$r^k = (1 - \alpha) \frac{AK^{1-\alpha}L^\alpha}{K} - (1 - \alpha) \frac{\sqrt{AK^{1-\alpha}L^\alpha}}{K} \sqrt{\frac{\Omega}{\sigma}} \quad (2.29)$$

As the above equation makes clear, the final effect of sunk costs is negative. Indeed, by Eq. (2.25), sunk costs have a direct and an indirect effect on the rental rate of capital. The direct effect is positive, since sunk costs raise the marginal productivity of capital. The indirect one works through the reduction in the number of active firms. Sunk costs in fact reduce the number of firms through Eq. (2.28), which in turn increases the markup and lowers the return. By Eq. (2.29), it is clear that the indirect, negative effect prevails over the direct, positive one.

The system of equilibrium conditions, as it is, is not easily manageable. In order to obtain a graphical representation of the equilibrium and to perform the comparative statics that I am interested in, I define  $\hat{x}_t = \frac{x_t}{n_t}$  as the generic variable  $x_t$  expressed in terms of number of firms in the market,  $n_t$ , and I rewrite the equilibrium conditions as follows:

$$r^k = \left(1 - \frac{(1 - \delta)}{1 + r}\right) \quad (2.30)$$

$$r^k = \left(1 - \frac{1}{\sigma n}\right) (1 - \alpha) \frac{\hat{Y} + \Omega}{\hat{K}} \quad (2.31)$$

$$\hat{Y} = A\hat{K}^{1-\alpha}\hat{L}^\alpha - \Omega \quad (2.32)$$

$$\hat{Y} = \frac{n_{-1}}{n} \hat{D} + \frac{1 + \beta}{\beta} \frac{1 + g}{1 + r} \hat{D} + \hat{K} \left(1 + \frac{(1 - \delta)}{\beta(1 + r)}\right) \quad (2.33)$$

$$n = \frac{\hat{Y} + \Omega}{\sigma \Omega} \quad (2.34)$$

where  $1 + r = \Pi^{\phi_\pi - 1}$  and  $\hat{L} = \bar{L}$  if  $\Pi > 1$ , while  $1 + r = \Pi^{-1}$  and  $\hat{L} = \left(\frac{1 - \gamma}{1 - \gamma}\right)^{\frac{1}{1-\alpha}} \bar{L}$  if  $\Pi < 1$ .

The equilibrium conditions are in fact now easily comparable to the ones obtained for

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<sup>14</sup>By combining the zero profit condition with the production function, it follows that  $n^*$  can be written as  $n^* = \sqrt{\frac{AK^{1-\alpha}L^\alpha}{\sigma\Omega}}$ .

the baseline model and the key channels through which the market structure affects the equilibrium allocation are easily identifiable. The number of firms enters in Eq. (2.31) through the term  $\left(1 - \frac{1}{\sigma n}\right)$  which measures the inverse of the effective markup which is decreasing in the number of firms. Moreover, when shocks have occurred, such that the equilibrium number of firms has changed and  $n_{-1} \neq n$ , a change in the number of firms has a further effect on aggregate demand, Eq. (2.33), by relaxing or tightening the borrowing constraint through its effect on the equilibrium interest rate.

As for the solution of the baseline model, I substitute out for  $K$ ,  $L$ ,  $r^K$ ,  $r$  and  $n$  within (2.32) and (2.33) to get the AS and AD schedules as functions of the two endogenous variables,  $\hat{Y}$  and  $\Pi$ . In particular, by combining (2.30) and (2.31), I get the equilibrium level of capital,  $\hat{K}$ :

$$\hat{K} = \left(1 - \frac{1}{\sigma n}\right)(1 - \alpha)\left(1 - \frac{(1 - \delta)}{1 + r}\right)^{-1}(\hat{Y} + \Omega) = \frac{(1 - \alpha)(1 + r)}{\delta + r}\hat{Y}$$

where I have used (2.34) for the second equality. Compare the first equality with the counterpart for the baseline model, (2.21). Two novel elements emerge, that is the equilibrium level of capital depends on sunk costs and on the number of firms. By the second equality, after substituting out for the number of firms, Eq. (2.34), I get the equilibrium level of capital as a function solely of the two endogenous variables.

## 2.4.2 A change in market structure

Following the analysis in the baseline model, I focus on the effect of a change in the market structure. Employing an endogenous markup model helps me track the behaviour of firms and the number of active firms. The latter is determined in equilibrium and depends on the cost structure. As discussed in the Introduction, the empirical evidence shows indeed that the recent decades have been characterized by deep changes in the cost structure, and by a substantial increase in sunk costs such as R&D expenses, compliance costs. Therefore, I consider the case of an increase in sunk costs, from  $\Omega = 1.35$  to  $\Omega = 1.74$ , which delivers a neutral interest rate equal to -0.0036. Figure 2.9 shows the effects on the equilibrium allocation, given the parametrization described in Table 2.3, Column 2. The economy goes from Full Employment, point A, to Secular Stagnation, point B and the equilibrium number of firms, as defined by (2.28), declines by 0.2%.

The rationale builds on the one discussed for the baseline model. On the aggregate supply side, two forces are at work. First of all, the increase in sunk costs depresses profits and production leading to a decrease in the number of firms, which in turn widens the markup, and depresses potential output. The AS schedule, and consequently the AD schedule, both shift leftwards. Moreover, as in the baseline model, an aggregate demand externality worsens the consequences of the negative shock as the zero lower bound binds and the economy enters Secular Stagnation. Differently from the baseline model, the extended model shows that the decline in the number of firms has also a positive effect on aggregate demand, since  $\frac{n-1}{n} > 1$ . The rationale is that since the reduction in the number of firms triggers a reduction in the equilibrium interest rate, the latter leads to a slight relaxation of the borrowing limit which stimulates spending, though not enough to compensate the negative effects.

## 2.5 Conclusions

In this research, I focus on market structure and Secular Stagnation and I provide an empirical and a theoretical analysis on the link between the markup and the equilibrium interest rate. I uncover some evidence that higher markups are associated with lower real rates. Moreover I show that countries with better product market regulation have higher interest rates. I propose an OLG model with monopolistic competition to interpret the empirical evidence. I show that an increase in the markup puts a downward pressure on the equilibrium interest rate, and Secular Stagnation occurs. The key transmission channel is the market for capital. Households are given two alternatives for savings, either by making loans to the young generations whose borrowing is constrained by an exogenous amount, or by renting capital to firms. The increase in the markup reduces the equilibrium level of capital leading to an excess of supply in the market for loans which puts a downward pressure on the equilibrium interest rate. The zero lower bound breaks the adjustment mechanism and the economy enters Secular Stagnation.

Further research is needed, both for the empirical and theoretical analysis. On the empirical side, an additional implication of the analysis is that the cost of capital should fall differentially in industries that are experiencing larger increase in the markup. From

a theoretical perspective, I am working to extend the model for a quantitative evaluation which eventually may add important insights on the transition dynamics and on the policy implications. I leave these extensions for future research.

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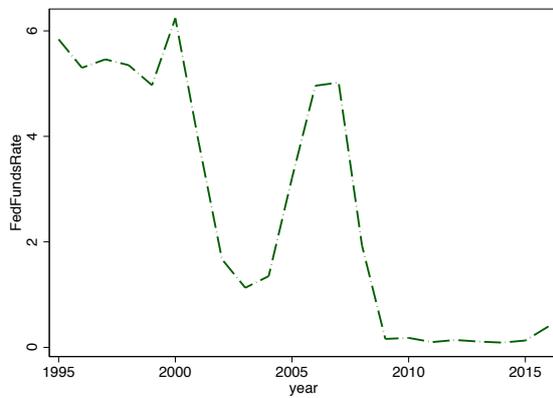
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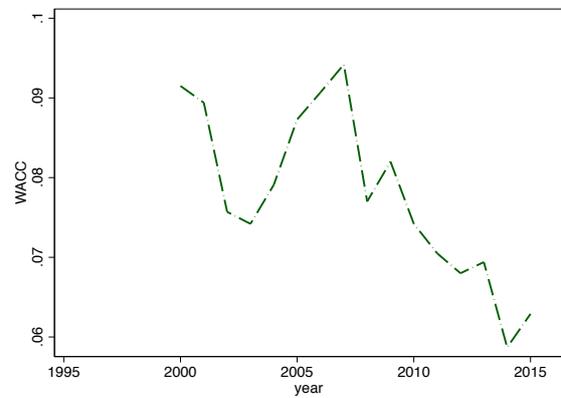
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## 2.6 Figures



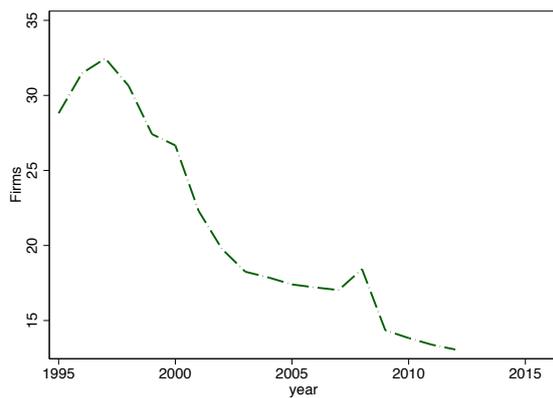
(a) 3-Month Treasury Bill.

Source: Board of Governors, Federal Reserve



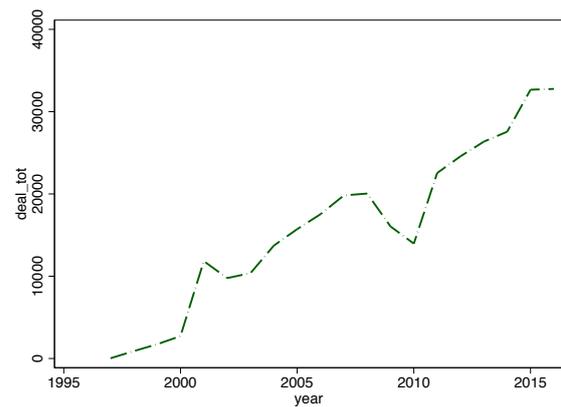
(b) Weighted Average Cost of Capital.

Source: Damodaran Online.



(c) Number of listed companies.

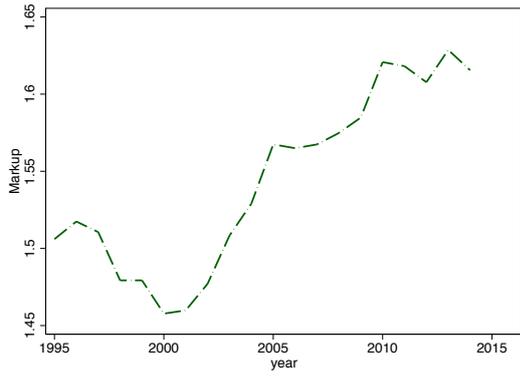
Source: World Bank



(d) Number of M&As, total deals.

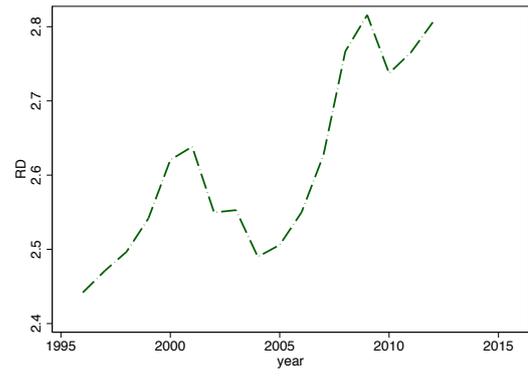
Source: Zephyr.

Figure 2.1



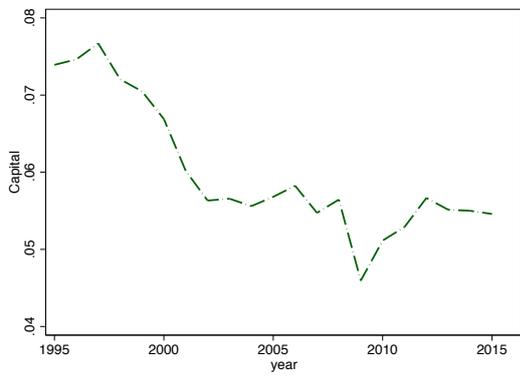
(a) Markup, author's calculations.

Source: BLS



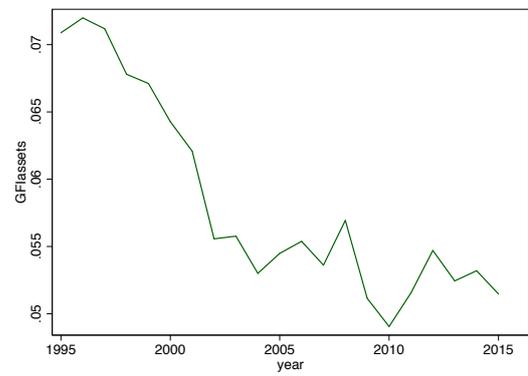
(b) Expenditures for R&D.

Source: World Bank



(c) Total capital expenditure, share of Total Assets for Nonfinancial Business.

Source: Financial Accounts of the US



(d) Gross Fixed Investment, share of Total Assets for Nonfinancial Business.

Source: Financial Accounts of the US

Figure 2.2

Industry	1997	2007	2012	Change 1997-2012
Transportation and Warehousing	30.7	42.7	42.1	+37 %
Professional, Scientific, and Technical Services	16.2	18.3	18.8	+16 %
Retail Trade	25.7	33.3	36.9	+35.9%
Arts, Entertainment and Recreation	21.8	19.5	19.6	-10%
Finance and Insurance	38.6	46.0	48.5	+25.6%
Administrative/Support	22.0	23.0	23.7	+7.7%
Wholesale Trade	20.3	24.9	27.6	+36%
Healthcare and Assistance	18.8	15.1	17.2	+8.5%
Real Estate Rental and Leasing	19.5	26.1	24.9	+27.7%
Accommodation and Food Services	21.1	23.7	21.2	+0.05%
Utilities	64.5	70.1	69.1	+7%
Other Services, Non Public Admin	12.8	11.3	10.9	-14.8%
Educational Services	19.6	22.3	22.7	+15.8%

Table 2.1: Change in Market Concentration by Sector, 1997, 2007 and 2012. Market concentration is measured as the share of sales for the 50 largest firms. Source: Economic Census, Census Bureau.

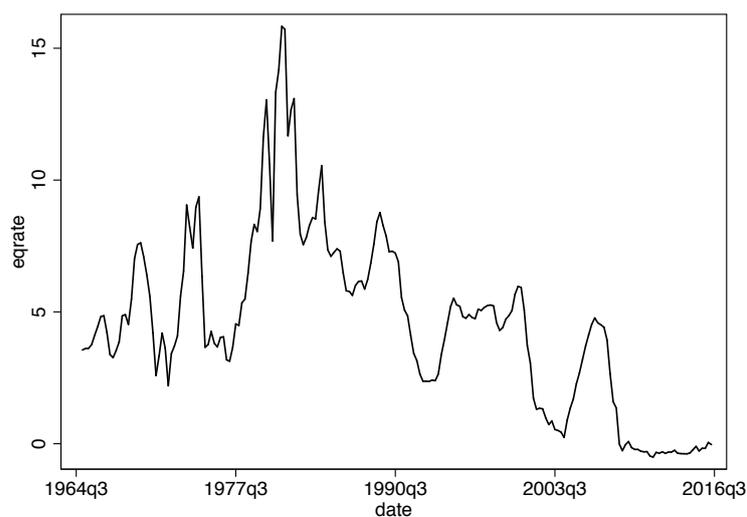
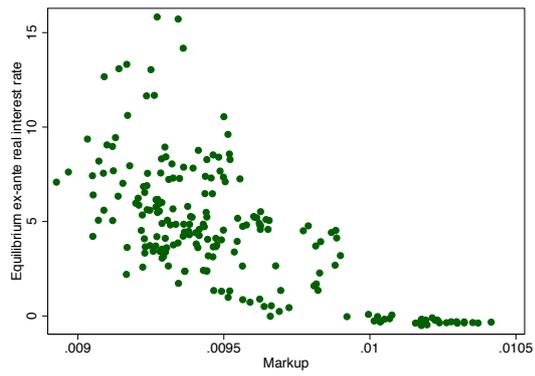
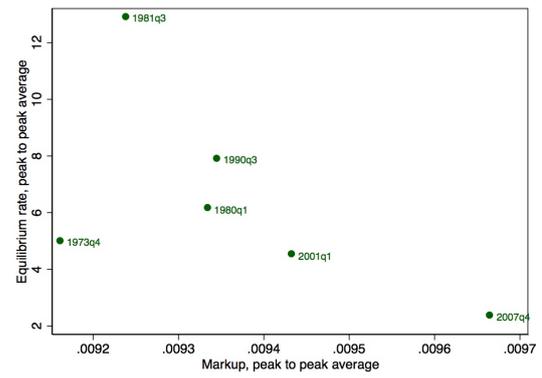


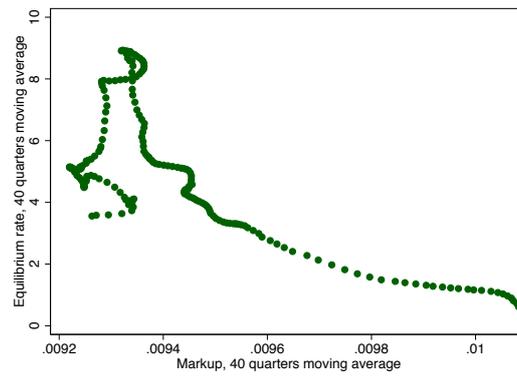
Figure 2.3: Equilibrium Real Interest Rate for the US



(a) Full Sample



(b) Peak-to-Peak



(c) 40-quarters Moving Averages

Figure 2.4: Analysis of Correlation between the Equilibrium Real Interest Rate and the Markup.

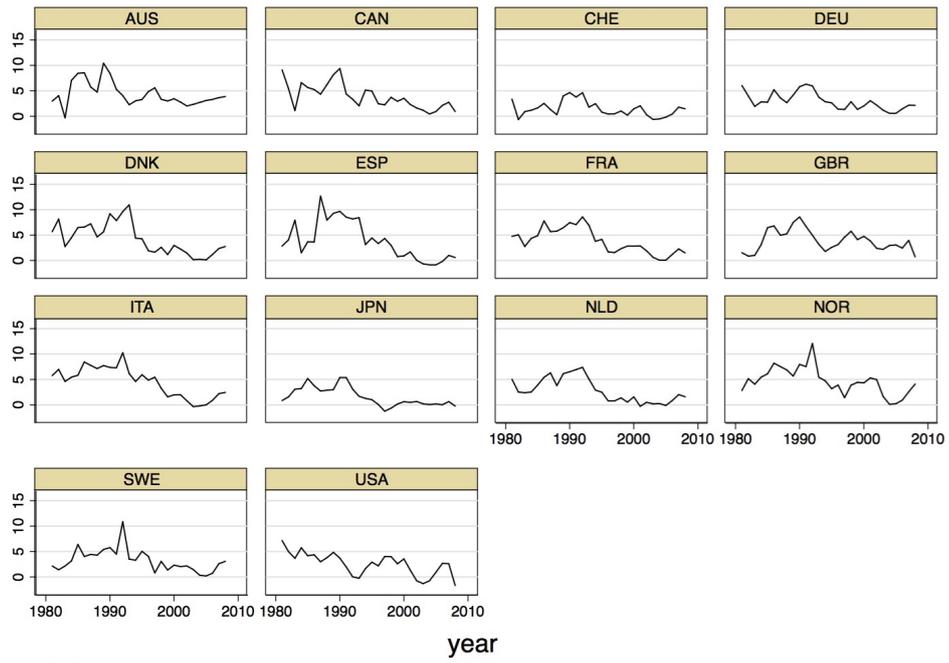


Figure 2.5: Equilibrium Real Interest Rate

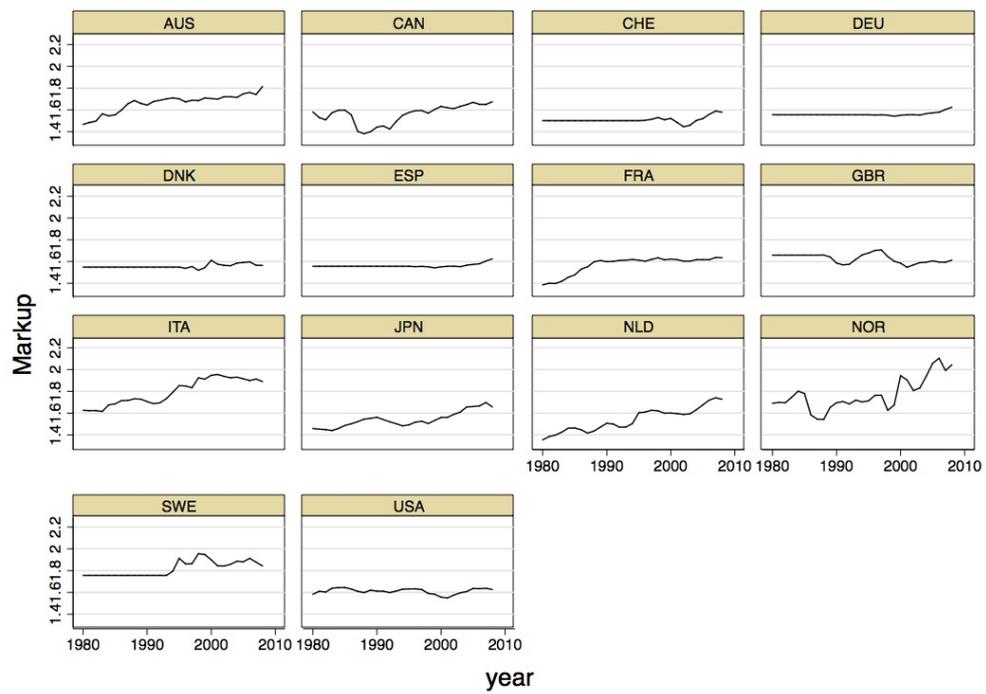
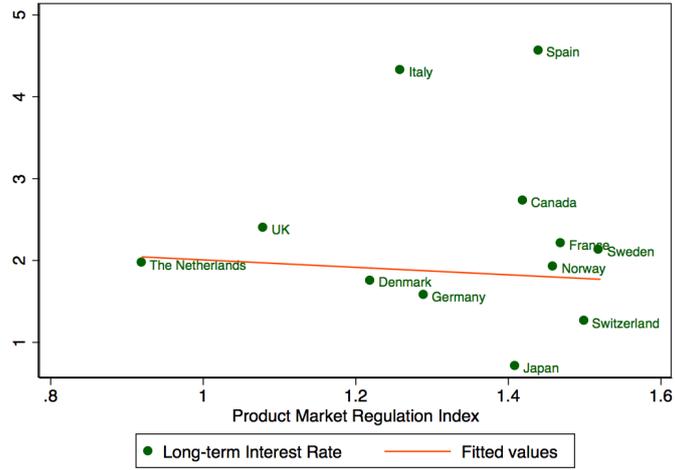
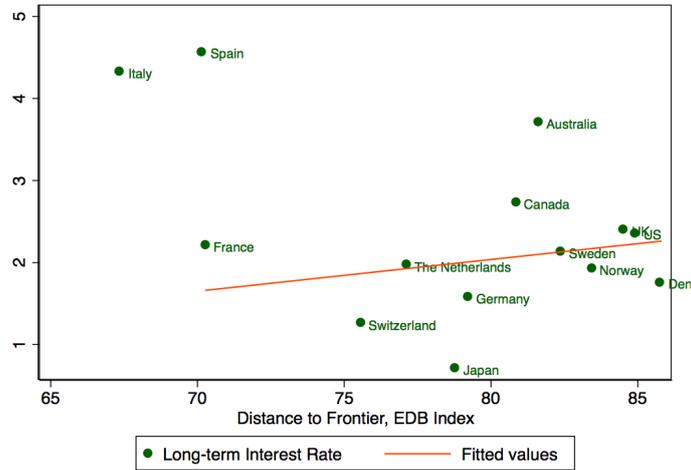


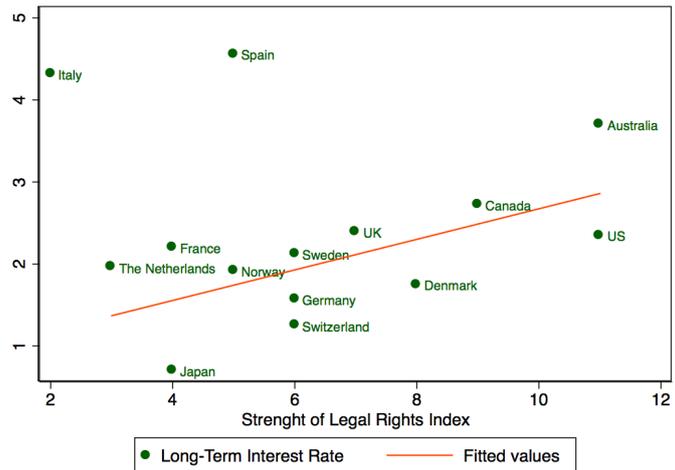
Figure 2.6: Markups



(a) PMR Index vs Interest Rate.



(b) DTF Score vs Interest Rate



(c) Strength of Legal Rights Index vs Interest Rate

Figure 2.7: Alternative measures for competitiveness vs  $1/t$  interest rate indicator.

	Baseline	Overhead Costs	CES Prod.Funct. unadj-Z	CES Prod.Funct. adj-Z
Full Sample	-0.6466	-0.4710	-0.4680	-0.4680
Moving Average	-0.6412	-0.3821	-0.3557	-0.3557
Peal-to-Peak	-0.5677	-0.3690	-0.3250	-0.3250

Table 2.2: Analysis of Correlation between the equilibrium ex ante real interest rate and the markup series under different measurement approaches, for the US.

Description	Parameter	(1) Value	(2) Value
Labor Supply	$\bar{L}$	1	1
TFP	A	1.2	3.4
Discount factor	$\beta$	0.99	1
Taylor coefficient for $\pi$	$\phi_\pi$	1.5	1.5
Labor share	$\alpha$	0.9	0.5
Elasticity	$\sigma$	10	0.001
Wage adjustment	$\gamma$	0.4	0.6
Inflation Target	$\Pi^*$	1	1
Borrowing Limit	D	0.285	0.2
Depreciation	$\delta$	0.6	0.6
Growth rate	g	0.001	0.05
Sunk Costs	$\Omega$	N.E.	1.35

Table 2.3: Columns (1) and (2) refer to the baseline and extended model.

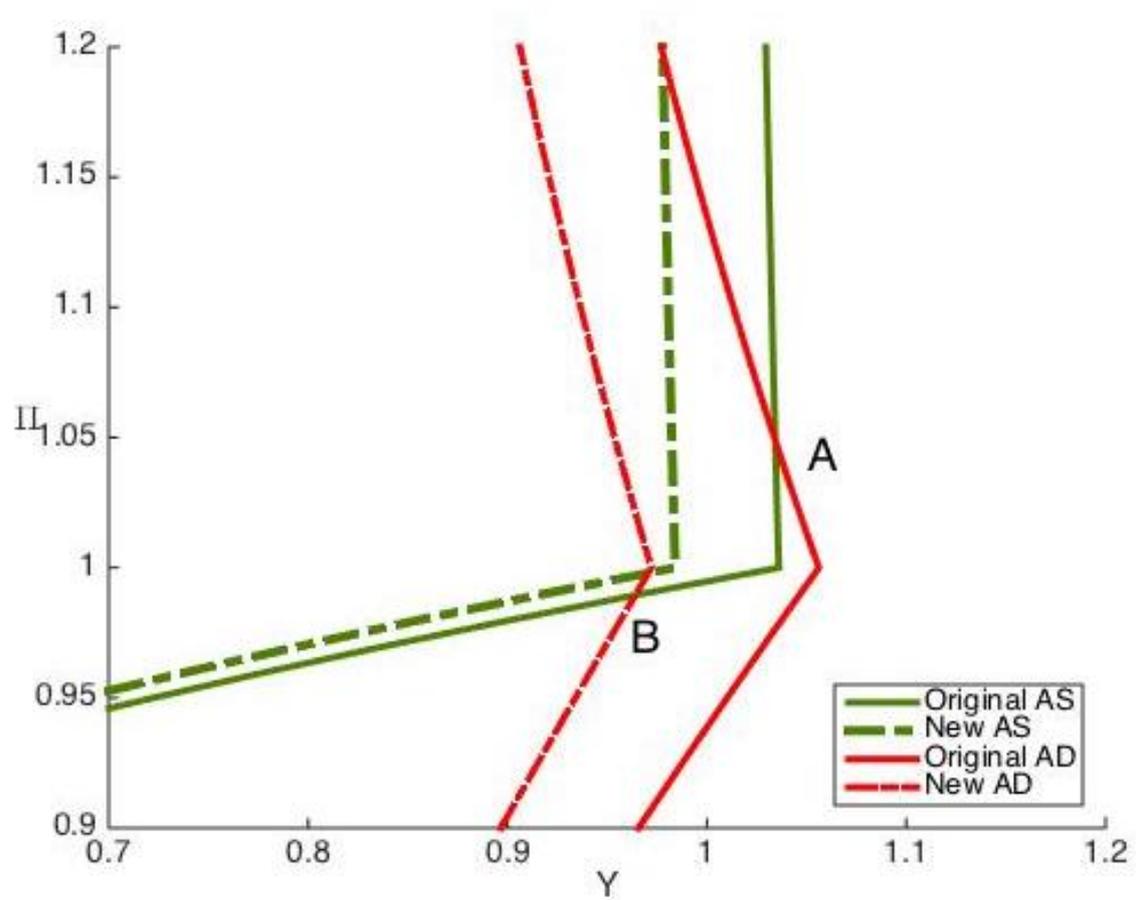


Figure 2.8: Effects of a change in market structure in the baseline Model. The solid lines represent the AS and the AD schedules before the shock, and the original steady state is in A. As the shock to the elasticity of substitution occurs, the markup increases and both the AS and the AD schedules move leftwards, the dashed lines. The economy moves to the new steady state, B.

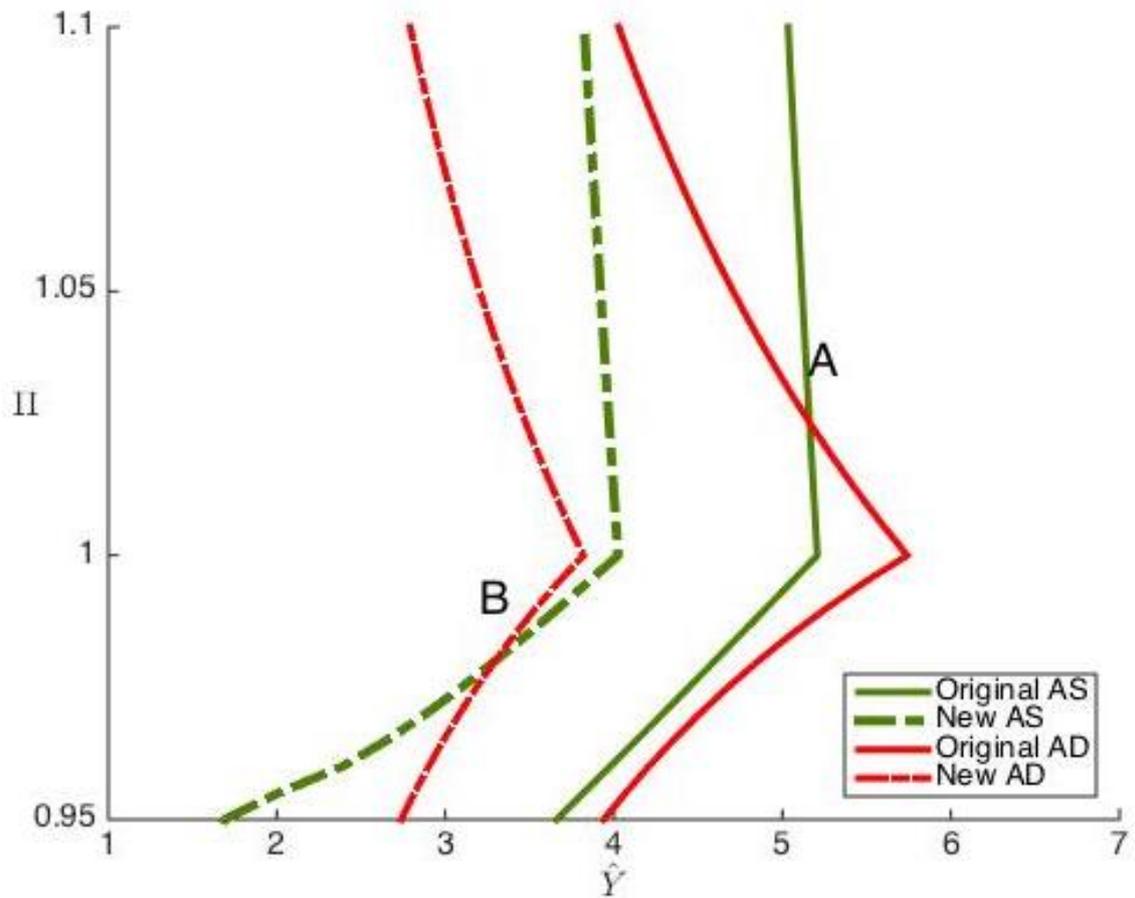


Figure 2.9: Effects of a change in market structure in the extended Model. The solid lines represent the AS and the AD schedules before the shock, and the original steady state is in A. As the shock to the level of sunk costs occurs, both the AS and the AD schedules move leftwards, the dashed lines. The economy moves to the new steady state, B.

## 2.7 Appendix

### Empirical evidence: Robustness

Tables 2.4 and 2.5. In Table 2.4 I report the results from the regression of the equilibrium interest rate over the change in the markup, Model 1. In Model specification 2, I have added country fixed effects to the OLS model, and, keeping country fixed effects, model specification 3 then adds time fixed effects. In the first two specifications the markup coefficient is significantly different from zero, negative and statistically significant. In model 3, instead, the time-component is highly significant recalling the result in Schularick and Taylor (2012) about a "common global time component" driving financial cycles. I consider model specification 2 as my preferred baseline specification henceforth. I then extend the model to allow for the additional explanatory variables. Table 2.5, Column 1 shows the results from the model where the independent variables are the change in the markup, GDP growth rate and population growth rate. In addition, I also consider the role of policy variables which the literature refers to as determinants of the long-run dynamics of the interest rate, Orr (1995). I consider growth in government assets, broad money (M2 or M3), and narrow money (M0 or M1). The markup coefficient is statistically significant and negative across all model specifications. The performance of the model slightly improves as additional determinants are considered. The growth and population rate coefficients are positive and statistically significant across all model specifications. Among the policy variables, the coefficient associated with government assets is positive, though not statistically significant, while the ones associated with broad and narrow money are both negative and highly statistically significant.

	1	2	3
	Baseline	Country Fixed Effect	Time Fixed Effect
Markup	-0.08*** (0.037)	-0.09*** (0.034)	-0.03 (0.003)
F-Test	4.42***	7.14***	19.78***
adj-R2	0.01	0.12	0.57

Table 2.4: Results from the panel regression where the dependent variable is the equilibrium ex-ante interest rate, the independent variable is the change in the markup. Model 2 and 3 add country and time fixed effects.

	Full	+Gov. Assets	+BroadMoney	+NarrowMoney
Markup	-0.10*** (0.034)	-0.10 *** (0.035)	-0.10*** (0.034)	-0.11*** (0.037)
Growth	0.16** (0.078)	0.16*** (0.084)	0.17** (0.081)	0.18*** (0.076)
Population	2.52** (0.646)	2.43*** (0.646)	2.49*** (0.629)	2.44*** (0.620)
Policy Variable	-	0.003 (0.008)	-0.07** (0.035)	-0.07*** (0.012)
F-test	9.31***	8.59***	9.31***	10.25***
adj-R2	0.1786	0.18	0.19	0.23

Table 2.5: Results from the panel regression. The dependent variable is the equilibrium ex-ante interest rate. Model specification "Full" considers the change in markup, growth and population rates as independent variables. The other specifications add growth in government assets, broad and narrow money.

