PhD Thesis:

Rescuing private DSGE

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In 2008, financial crisis hit US economy, provoking a strong and persistent slowdown in economic activity. Monetary authorities reacted bringing the interest rate at their minimum (Zero Lower Bound). After more than 5 years, the official interest rates are still there. The Great Recession lasted more than expected and the growth trend of the last five years have been substantially lower than in the previous twenty years. The employment rate and the total number of hours worked in the US economy are still below their pre-crisis level. The long period of low interest rates and disappointing recovery pushed many economists to mention the possibility that major developed economies had slipped into a secular stagnation trap. Though last quarters observations show sign of a stronger recovery on both side of the Atlantic, economic growth trends and macroeconomic performance looks still very far in terms of stability form the Great Moderation era.
DSGE models have been developed during the Great Moderation years. The main use has been devoted to study the fluctuations happening around an average exogenous growth trend.
The policy makers goal has been to minimize the fluctuations. The recent crisis showed the inadequacy of the pre-crisis state-of-the-art DSGE models in several dimensions:

- Concerning the fluctuations, DSGE models were not able to explain the Great Recession without recurring to the occurrence of a very unlikely shock. To overcome this inadequacy, DSGE models have been complemented with financial sectors, financial frictions or brand-new type of shocks (i.e. Marginal Efficiency of Investment shock (Justiniano et al. (2011)).

- Concerning the solution methods, the Great Recession stressed the role of non linearities inside the economy (stochastic volatility, occasionally binding constraint, the zero lower bound on the interest rate). This issue has been addressed by the development of a series of non-linear solution techniques. On the same front, non-linear estimation techniques became more and more important. Still, the curse of dimensionality seems to limit the diffusion of the non-linear solution and estimation techniques, especially to handle medium-scale DSGE models, for which fully non-linear methods are impractical.

- Concerning the mean: as growth trend seems affected by the Great Recession, DSGE models started to incorporate knowledge sector, $R\&D$ sectors and so forth to model endogenous growth in the DSGE models (Comin and Gertler, Bianchi (2014), Guerron Quintana (2014)).

The goal of these thesis is to fix some open issues related to the use of empirical DSGE models after the end of the Great Moderation.

In the first chapter, a simple set of techniques going under the name of Approximate Bayesian Computation (ABC) is proposed to perform non-linear DSGE estimation. In fact, non-linear model estimation is generally perceived as impractical and computationally burdensome. This perception limited the diffusion on non-linear models estimation.

ABC is a set of Bayesian techniques based on moments matching: moments are obtained simulating the model conditional on draws from the prior distribution . An accept-reject criterion is applied on the simulations and an approximate posterior distribution is obtained by the accepted draws.

A series of techniques are presented (ABC-regression, ABC-MCMC, ABC-SMC). To assess their small sample performance, Montecarlo experiments are run on AR(1) processes and on a RBC model showing that ABC estimators outperform the Limited Information Method (Kim, 2002), a GMM-style estimator.

In the remainder, the estimation of a new-Keynesian model with a zero lower bound on the interest rate is performed. Non-gaussian moments are exploited in the estimation procedure.

In the second chapter of the thesis, I try to explore the relations between growth trend of the economies and business cycles fluctuations, focusing on the role played by housing medium term fluctuations.

In many economies' recent experiences, housing market volatile fluctuations have been blamed as responsible for driving or at least influencing the trend at which economies were growing (US, Japan and Spain to mention a few).

The second chapter inquires on the possibility that houses, playing the double role of durable consumption good and collateral, can affect the growth trend at which an economy grows.

This is done through the study of a medium scale DSGE model with heterogeneous agents and endogenous growth where housing prices fluctuations influence the households' investment in technology, with a final effect on the growth trend.

It turns out that against the general wisdom, an exogenous increase in the appetite for housing generates a temporary decline in the growth trend. Conversely, the temporary relaxation of the borrowing constraints for debtors is able to generate the positive comovement between housing prices and growth trend observed in the last twenty years across the developed economies. The more indebted the economy, the larger the degree of exposure to this type of fluctuations will be.

Estimating Non-Linear DSGEs with the Approximate Bayesian Computation: an application to the Zero Lower Bound

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Abstract

Non-linear model estimation is generally perceived as impractical and computationally burdensome. This perception limited the diffusion on non-linear models estimation. In this paper a simple set of techniques going under the name of Approximate Bayesian Computation (ABC) is proposed.

ABC is a set of Bayesian techniques based on moments matching: moments are obtained simulating the model conditional on draws from the prior distribution . An accept-reject criterion is applied on the simulations and an approximate posterior distribution is obtained by the accepted draws.

A series of techniques are presented (ABC-regression, ABC-MCMC, ABC-SMC). To assess their small sample performance, Montecarlo experiments are run on AR(1) processes and on a RBC model showing that ABC estimators outperform the Limited Information Method (Kim, 2002), a GMM-style estimator. In the remainder, the estimation of a new-keynesian model with a zero lower bound on the interest rate is performed. Non-gaussian moments are exploited in the estimation procedure.

1 Introduction

DSGE (Dynamic stochastic general equilibrium) models play an important role in Macroeconomic theory. In the last decade, they became the workhorse of many central banks. They are used to explain economic fluctuations from a general equilibrium perspective, to make forecasts on the path of macroeconomic variables, to advise policy makers in taking decisions.

Model estimation is a crucial step allowing economists to make quantitative statements in the framework of a probabilistic structure.

Great moderation years have seen the prevalence of linear methods: log-linearization to solve the model, Kalman filter to compute the likelihood and Bayesian techniques to estimate the model.

The incoming of the Great Recession, the presence of a lower bound reached by the policy interest rate, the general increase in volatility, the need to model a fraction of borrowing constrained households pushed researchers to inquire about the role of non-linearities in the economic models. Log-linearization and Kalman filter are not fit to represent some features of the data (presence of occasionally binding constraints, stochastic volatility, non-Gaussian shocks) and non-linear solution methods are being developed.

The Particle filter is the method usually applied in the estimation of non-linear models (Fernandez-Villaverde et al.). The Particle filter is computationally burdensome, especially to handle medium or large-scale DSGE models. Besides, the Particle filter necessitates measurement errors, to avoid the degeneracy of the particles and compute the likelihood. In many cases, given the size of the model, the standard deviation of the measurement errors is fixed in advance. All these issues limited the diffusion of non-linear estimation so far.

In this paper, Approximate Bayesian Computation (ABC),a set of techniques based on simulation and moments matching, is proposed as an alternative to estimate non-linear models. ABC techniques are presented. Two Montecarlo experiments on ABC methods and the Bayesian Limited Information Method (BLI) are assessed. The goal is comparing the small sample performance of the two estimators. Moreover, ABC is applied to the estimation of standard new-keynesian model with an occasional binding positivity constraint on the interest rate.

Approximate Bayesian Computation techniques are a set of techniques developed in natural sciences. The core mechanism in ABC (ABC-rejection) is the following:

- The model is simulated a large number of times, conditional on the vectors of parameters drawn from the prior distribution. Each simulation has the same sample size of the observed sample;
- Euclidean distance between the moments of each simulation and the observed ones is computed for each simulation;
- Each simulation is accepted or rejected if the Euclidean distance is below or above a tolerance level;
- The accepted draws are a sample of the approximate posterior distribution.

Drawing from the prior distribution can be very inefficient if the prior and the posterior distributions are very different. This causes very low acceptance ratios and may make simple ABC-rejection impractical.

To tackle this issue, a series of refinements have been developed:

- ABC-regression: the accepted draws are corrected with a post-sampling correction step;
- ABC-MCMC: the accept-reject is applied to explore the posterior distribution building a Markov Chain;
- ABC-SMC: the draws are iteratively sampled from the approximate posterior distribution.

In Economics, the estimator proposed by Creel and Kristensen (2013) in its Bayesian simulated version (Simulated Bayesian Indirect Likelihood estimator, SBIL) coincides with a variant of ABC (ABC-kernel). Creel and Kristensen provide asymptotic results for the estimator, compare the small sample performance of the estimator with the Simulated Method of Moments from a frequentist perspective: they compute the RMSEs with respect to the true values. They also apply the method in the estimation of a baseline DSGE model, solved with perturbation methods.

Instead, in this paper, the comparison is done between ABC methods and the Bayesian Limited Information Method (BLI). The BLI is a Bayesian method based on exploiting the likelihood of the moments. It can be intuitively thought as the Bayesian version of the Generalized Method of Moments and the Simulated Method of Moments. In DSGE estimation, it has been applied by Christiano, Trabandt and Walentin,(2010), and Christiano, Eichenbaum and Trabandt (2014) and it is getting more and more popular among researchers.

The comparison of small sample performance is done from a Bayesian perspective: the RMSEs are computed with respect to the Full likelihood posterior mean and the approximate posterior distribution are compared to the Full likelihood posterior distributions.

The Montecarlo experiments are run using an AR(1) model and a RBC model with three observables and identification issues. The persistence and the sample sizes of the models are diverse to check the different performances of the estimators.

ABC estimators outperform BLI estimator using small samples and high persistence processes. With large samples, they have the same performance, provided that the number of simulations is sufficiently large to get rid of the simulation effect. This hold both for the AR(1) and the RBC model.

BLI and GMM-style estimators exploit the information contained in the moments. GMM-style estimators build the likelihood function/objective function relying on the normality assumption of the moments distribution: moments and their variances are sufficient statistics of asymptotically normally distributed moments.

Instead, ABC estimators explore the whole distribution of the moments. This is a comparative advantage with respect to the BLI estimator, especially when the distribution of the moments is far from being normal and not centred around the population moment. This difference is more remarkable with small samples and high persistence. In that case, convergence of moments to the normal distribution is slower and the actual moments distribution substantially differs from the asymptotic distribution.

For this same type of reason, ABC can exploit non-Gaussian moments: binomially and multinomially distributed moment. As an example, in an estimation procedure of an economic models, ABC techniques can try to match the frequency of recessions (and expansion), of deflation (and inflation) and so forth.

These results paved the way for a real life application: the estimation of a newkeynesian model with occasionally binding positivity constraint (models with Zero Lower Bound, ZLB).

Models with occasionally binding constraints produce moments which do not respect the regularity assumption requested to apply GMM-style estimators. ABC is more fit to estimate such models, since the moments distribution is explored through the accept-reject method, taking into account the actual distribution of the moments.

Moreover, ABC permits to match non-gaussian moments: the probability of being at the ZLB, the number of episodes and so forth. Th non-linearity generated by the occasionally binding constraint and the gap between the notional interest rate and the zero lower bound is handled by fully non-linear methods or piecewise linear methods.

Perturbation methods (log-linearization, 2nd order approximation and so forth) cannot handle the solution of a model with occasionally binding constraint, since they approximate the solution around a steady state in which the zero lower bound is not binding.

In this paper, a piecewise linear approximation method is applied to the solution of a model with ZLB. The model is estimated according to an ABC-Sequential Montecarlo technique. ABC-SMC is helpful to tackle the curse of dimensionality increasing the acceptance ratio.

The estimates are exploited to produce some consideration abut the role of the ZLB in the economy.

Summing up, the contributions of this work are the following. ABC techniques are exposed and applied to the estimation of economic models. Moreover, a comparison with the Bayesian Limited Information is assessed from a Bayesian perspective: ABC estimators outperform GMM-style estimators in terms of RMSE (computed with respect to the Full likelihood estimator). This is particularly true dealing with small samples and highly persistent processes.

Besides, the estimation of a model with a Zero Lower Bound is performed, using gaussian and non-gaussian moments. The estimation is performed using six observable variables and a dataset including 2013Q3[1]. The reminder of the paper is the following. Section **??** presents the ABC techniques. In Section **??** a comparison between ABC estimator and the BLI estimator is assessed. Section **??** houses an estimation on a vanilla RBC. In Section **??** the model with ZLB is estimated. In section 6, the Conclusion is housed.

2 Approximate Bayesian Computation.

The Approximate Bayesian Computation (ABC) is a set of statistical techniques developed in population genetics at the end of the 90's (Pritchard, 2000). In the last decade, the methodology spread across all natural sciences, namely epidomiology, ecology and biology. ABC is based on moments matching: the moments of the model are matched with the ones observed from the data. Moments are simulated according to the observed sample size and inference is based on the Euclidean distance between the simulated moments and the observed ones.

The use of moments of ABC-techniques makes the methodology similar to the GMM-syile estimators: the Generalized Method of Moments (GMM, Hansen 1982) and its simulated version (the Simulated Method of Moments, SMM). In Section 3, a comparison between the ABC and a bayesian version of a GMM estimator (Kim,2002) is assessed. As it will become clear ABC estimators present a series of advantages with respect to the GMM-style estimators.

[1]Gust et al. estimate a similar model using only three observables

ABC are particularly fit in the estimation of models whose likelihood computation is troublesome or whose moments distribution prevents the use of GMM-style estimators (irregular moments distribution, non-gaussian moments, short samples).

ABC methods have a Bayesian structure: the moments matching procedure updates a prior distribution to deliver an approximate posterior distribution. Approximation is a result of using the moments rather than computing the likelihood function of the model.

The pseudo-algorithm by Pritchard (2000) clarifies the mechanism at the core of ABC methods and goes under the name of ABC-rejection:

- Draw θ_i from the prior distribution $p(\theta)$
- Simulate the model and get the variable $\mathbf{y}_i$
- Compute the summary statistics $\mathbf{s}_i$
- If the Euclidean distance $\rho||\mathbf{s}_i - \mathbf{s}|| < \epsilon$ accept θ_i otherwise reject it
- Repeat the procedure for N times

where $\mathbf{s}_i$ is the vector of moments from the simulated sample, $\mathbf{s}$ is the vector of moments of the observed data, ϵ is the tolerance level.

In other words, in ABC-rejection the model is simulated a number of times conditional on parameters drawn from the prior distribution. Moments from these simulations are computed and matched against the observed moments. For each simulation the Euclidean distance is computed. If the euclidean distance is smaller than a fixed threshold, the simulation is accepted. The parameters of the accepted simulations are a sample from the approximate posterior distribution.

The Bayes Rule of the Bayesian statistics is approximated:

$$P(\theta|\mathbf{y}) \propto L(\mathbf{y}|\theta)P(\theta) \rightarrow P(||\mathbf{s}_i - \mathbf{s}|| < \epsilon)P(\theta) \tag{1}$$

The likelihood function is approximated by the accept-reject step on the euclidean distances criterion.

If the moments used in the estimation are sufficient statistics of the model, for $\epsilon \to 0$ and $N \to \infty$ the sequence of θ's accepted converges to the posterior distribution.
A large number of simulations needs to be run to reduce the error introduced by the simulation step. When the number of parameters to infer increases, so does the number of moments to use. The probability that the Euclidean distance is below the threshold is smaller and and a larger number of simulations are run to obtain N accepted draws.
This brings to high inefficiency (the acceptance ratio gets small) and this may make simple ABC impractical (curse of dimensionality).
A series of more sophisticated method has been developed to tackle the curse of dimensionality.
Accepted simulations can be assigned a weight according to a kernel weighting function. The argument of the kernel is the euclidean distance: the smaller the distance, the larger the weight. In this paper, this method is called *ABC-kernel* and coincides with the simulated Bayesian version of the estimator proposed by Creel and Kristensen, 2012.
In order of time, the ABC-rejection is the first method developed and is at the core of the other more sophisticated methods. ABC methods are mainly divided in three big subsets:

- ABC-regression;
- ABC-MCMC;
- ABC-SMC.

The three groups adopt different strategies to tackle the curse of dimensionality and the low efficiency of Pritchard algorithm. In particular the first solution runs a post-sampling correction on the accepted parameters, the last two draw parameters more efficiently.

2.1 ABC with local linear regression

ABC-rejection is affected by the curse of dimensionality: to estimate a large set of parameters, we need to increase the number of summary statistics in the Euclidean distance computation. The probability of the simulated parameters to be accepted decreases and a higher number of simulations have to performed. This may have a huge impact on the feasibility of the estimation procedure. Besides, to increase the tolerance level can strongly compromise the approximation of the posterior distribution due to a larger simulation error. ABC-regression increases the efficiency of ABC through a post-sampling correction. Three main refinements are introduced after the accept-reject step:

- The moments are rescaled by their median absolute deviation: this transforms the previous rectangular acceptance region in a sphere.

- Each accepted simulation is assigned a weight according to its euclidean distance: the smaller the distance ρ_i, the larger the weight W_i. An Epanechnicov weighting function is generally used, but the algorithm is compatible with other kinds of kernel (normal, triangular and so forth). [2].

- The accepted parameters are corrected exploiting the result of a regression run after the accept-reject (hence the name ABC-regression). Each parameter is updated according to the result of a local linear regression of the accepted parameters on the discrepancies between simulated moments and observed ones (Beaumont et al. (2002)).

In ABC regression (Beumont,2002), ABC is equivalent to a problem of conditional density estimation, where a joint density distribution $P(\mathbf{s}_i, \theta_i)$ is updated through an accept-reject algorithm:

$$P(\theta|\mathbf{s}) = \frac{p(\mathbf{s}_i, \theta)}{I\left\{\rho|\mathbf{s}_i - \mathbf{s}| < \epsilon\right\}} \tag{2}$$

For this reason, conditional density estimation techniques (Fan an Gijbels, 1992) estimation are borrowed and incorporated in the ABC algorithms.
The ABC-regression pseudo-algorithm is:

- Draw θ_i from the prior $P(\theta)$;

[2] This correction coincides with the Indirect Likelihood Inference by Creel and Kristensen (2013)

- Simulate the model and obtain the observable variables $\mathbf{y}_i$;
- Compute the simulated moments $\mathbf{s}_i$ and the absolute standard deviation; for each moment k_j;
- Compute the Euclidean distance for each simulation;

$$\rho|s_i, s| = \sqrt{\sum_{j=1}^{s}(s_i/k_j - s/k_j)^2} \tag{3}$$

- Select the tolerance level such that a fraction of the simulated parameters is accepted $P_\epsilon = N/M$.
- Each accepted draw is assigned a weight according to the Epanechnikov kernel:

$$K_\epsilon(\rho_i) = \begin{cases} \epsilon^{-1}(1 - (\frac{\rho_i}{\epsilon})^2) & \rho_i \leq \epsilon \\ 0, & \rho_i > \epsilon \end{cases}$$

  ;

- Apply a local linear regression to the linear model:

$$\theta_i = \alpha + (\mathbf{s}_i - \mathbf{s})'\beta + \epsilon_i, \tag{4}$$

  for $i = 1, ..., N$.

- Adjust the parameter given the results of the local linear regression:

$$\theta^* = \theta - (\mathbf{s}_i - \mathbf{s})'\hat{\beta}, \tag{5}$$

  which is equivalent to compute: $\theta_i^* = \hat{\alpha} + \hat{\epsilon}_i$

The adjusted parameters associated to their kernel weights are random draws of the approximate posterior distribution.

The initial part of the ABC-regression is the simple ABC rejection. The accepted parameters are corrected given two assumptions on the relation between the parameters drawn and the summary statistics simulated:

- Local linearity: a local linear relationship between the discrepancies of the moments and the parameters holds in the vicinity of the observed moment s such that the parameters can be expressed by the following equation:

$$\theta_i = \alpha + (\mathbf{s}_i - \mathbf{s})'\beta + \epsilon_i; \tag{6}$$

- Errors ϵ_i's have zero mean, are uncorrelated and homoskedastic.

In general, linearity only in the vicinity of $\mathbf{s}$ is a more palatable assumption than *global* linearity. In the local linear regression to estimate the coefficients for α and β, the minimized object is:

$$\sum_{i=1}^{m} \left\{\theta_i - \alpha - (\mathbf{s}_i - \mathbf{s})^T \beta\right\}^2 K_\delta(||s_i - s||) \tag{7}$$

In ABC literature, Epanechnikov kernel function is the one more common but others are feasible. In Eq.(**??**), the only difference with respect to the standard OLS is that the squared errors are weighted according to the distance ρ_i associated to the parameter θ_i. The solution is represented by:

$$(\hat{\alpha}, \hat{\beta}) = (XWX)'(XW\theta) \tag{8}$$

Where $X = (\mathbf{s}_i - \mathbf{s})$ for $i = 1, ..., N$ and W is a diagonal matrix, where each non zero element is $K_\delta(||s_i - s||)$.

The estimates for α and β are used in the adjustment step, through the adjustment equation **??**.

In conditional density estimation terms: $E[\theta|\mathbf{s}_i = \mathbf{s}] = \alpha$.

The posterior mean coincides with the Nadaraya-Watson estimator (Nadaraya, 1964, Watson, 1964), as suggested by Blum and Francois (2010) :

$$\alpha = \frac{\sum_i \theta_i^* K_\delta(||(\mathbf{s}_i - \mathbf{s}||)}{\sum_i K_\delta||(\mathbf{s}_i - \mathbf{s})||)} \tag{9}$$

Blum and Francois (2010) add further step: a *correction for heteroskedasticity* in the adjustment step with non-linear regression in lieu of the local linear regression.

For the sake of simplicity, here the local linearity assumption is maintained allowing the variance of the errors to change with the moments (Beaumont, 2010). The heteroskedastic is:

$$\theta_i = \alpha + (\mathbf{s}_i - \mathbf{s})'\beta + \epsilon_i = \alpha + (\mathbf{s}_i - \mathbf{s})'\beta + \sigma_i * \xi_i, \tag{10}$$

where σ_i^2 is the variance of the error conditional on observing the simulated moments $Var[\theta|\mathbf{s}_i]$ and $\xi_i \sim N(0,1)$.

In this new procedure (ABC-regression with correction for heteroskedasticity) estimates α and β remain the same while in a further step the conditional variance for each draw is estimated. Finally, the correction mechanism is applied.

Blum and Francois model the conditional variance on the moments discrepancy by a second local linear model, borrowing from Fan and Yao (1998). A second local linear regression is run and the conditional variance for each draw σ_i is estimated :

$$log(\epsilon_i)^2 = \tau + (\mathbf{s}_i - \mathbf{s})'\pi + \upsilon_i, \tag{11}$$

where υ_i is *iid* with mean zero and common variance.

In this second local linear regression, the following object is minimized:

$$\min \left\{ log(\hat{\epsilon}_i)^2 - (\mathbf{s}_i - \mathbf{s})'\pi \right\} K_\delta(||\mathbf{s}_i - \mathbf{s}||) \tag{12}$$

where $\hat{\epsilon}_i$'s are the heteroskedastic errors estimated in the first regression.

The variance conditional on the observed moments is $\sigma^2 = Var[\theta|s]$ is obtained according to

$$\hat{\sigma} = \hat{\tau} \tag{13}$$

while the the variance conditional on each *simulated* moments is

$$\hat{\sigma}_i = \hat{\tau} + (s_i - s)'\hat{\pi} \tag{14}$$

Values obtained in **??** are used in the new post-sampling correction equation **??** where the magnitude of each *heteroskedastic* error ϵ_i is corrected by the estimated standard deviation $\hat{\sigma}_i$:

$$\theta^* = \hat{\alpha} + \frac{\hat{\sigma}}{\hat{\sigma}_i}\hat{\epsilon}_i \tag{15}$$

When the associated variance is higher (lower) than the variance conditional on the observed moments, the ratio $\frac{\hat{\sigma}}{\hat{\sigma}_i}$ is lower (higher) than 1 and the magnitude of the correction will be decreased (increased) with respect to the estimated $\hat{\epsilon}_i$.
ABC-regression allows to increase the tolerance level (i.e. increase the fraction of accepted simulations), making the algorithm computationally more efficient. Nonetheless, when the dimensionality of the parameters increases, the algorithm can deliver unstable results.
Besides, some problems in the adjustment step can arise when the local linearity assumption does not hold: when the observed moments lie at the boundary of the simulated moments, adjusted values can be updated outside the support of the prior distribution (extrapolating rather than interpolating). Some refinements have been found by the literature to fix this problem, but a general consensus has not been reached.
Before adopting ABC-regression, drawing scatter plots can be useful to assess the informativeness of the moments regard the parameters to infer. In particular, (local) linear relations between moments and parameters can be found. When the dimensionality of the problem makes both ABC-rejection and ABC-regression impractical, the ABC-SMC is the technique more fit to tackle the curse of dimensionality, as it will be shown in the final Section.

2.2 ABC-MCMC

ABC-rejection can have very low acceptance rate and sampling form the prior can be very inefficient.
ABC-MCMC methods draw parameters from a distribution closer to the posterior. This increases the acceptance rate of the algorithm.
The algorithm developed by Marjoram et al. (2003) is the following:

- For $t = 0$, Draw $\theta \sim \pi(\theta)$;
- For $t \geq 1$ draw from:
$$\theta^{'} \sim K(\theta|\theta^{t-1}); \tag{16}$$
- Simulate and produce the moments conditional on θ^t;
- If $\rho(S(x), S(y)) < \epsilon$
  - Draw $u \sim U(0, 1)$,
  - If
$$u \leq \frac{\pi(\theta')}{\pi(\theta)^{t-1}} \frac{K(\theta^{t-1}|\theta^{'})}{K(\theta^{'}|\theta^{t-1})} \tag{17}$$
    then, $\theta^t = \theta^{'}$; otherwise $\theta^t = \theta^{t-1}$

  otherwise $\theta^t = \theta^{t-1}$

The MCMC produced by the algorithm is an approximation of the posterior distribution. Problems associated with ABC-MCMC are mainly related to presence of multimodality and mixing problems.

2.3 ABC-Sequential Montecarlo

ABC methods can be highly inefficient and the need for too many simulations can make them impractical. The acceptance rates of ABC-rejection are very low. The ABC-regression cannot deal with a large number of parameters and multimodality. ABC-MCMC cannot deal with multimodality and can get stuck in low acceptance regions of the parameters support. ABC-SMC can overcome such inefficiency.

ABC-SMC nests ABC into the sructure of a SMC technique: the initial particles are drawn from a proposal distribution. Each particle is a vector of parameters. The distribution is iteratively updated to converge to the target distribution.

At each step particles are perturbed according to a Kernel function. Each particle is accepted or rejected according to the Euclidean distance, choosing a decreasing tolerance level such that $\epsilon_t \leq \epsilon_{t-1}$.

If accepted, the particle is assigned a weight taking into account the Kernel function. A resampling procedure is envisaged to avoid sample degeneracy (i.e. few particles ending up hoarding much of the weight).

The algorithm is the following:

1. Initialize the tolerance level sequence: $\epsilon_1, \epsilon_2, \epsilon_3 ... \epsilon_T$ and select a sampling distribution μ_i. Set the iteration indicator $t = 1$.

2. Set the particle indicator $i = 1$ and:

   - If $t = 1$, draw the swarm of particles $\{\theta_1 \theta_2 ... \theta_N\}$ from the importance distribution μ_1.
   - If $t > 1$, sample the new swarm $\left\{\theta^{**}_{i,t-1}\right\} i = 1^N$ with weights $\left\{W^{**}_{i,t-1}\right\}_{i=1}^{N}$ ad perturb each particle according to a transition kernel $\theta^{**} \sim K_t(\theta|\theta^*)$

3. Simulate the model to obtain x^{**} conditional on each particle : if $\rho(S(x^{**}, S(x_0)) < \epsilon_t$ accept the particle, otherwise reject.

4. If accepted, assign the particle a weight:

   - If $t = 1$, $W_{i,1} = \frac{\pi(\theta_{i,1})}{\mu_1(\theta_{i,1})}$.
   - If $t > 1$,

$$W_{i,t} = \frac{\pi(\theta_{i,t})}{\sum_{j=1}^{N} W_{t-1}(\theta_{t-1,j}) K_t(\theta_{t,i}|\theta_{t-1,j})} \tag{18}$$

where $\pi(\theta)$ is the prior distribution for θ.

5. Normalize the weights such that $\sum_{i=1}^{N} W_{t,i} = 1$.

6. Compute the Effective Sample Size (ESS):

$$ESS = \left[\sum_{i=1}^{N} (W_{t,i})^2)\right]^{-1} \quad (19)$$

If the ESS is below $N\frac{1}{2}$, resample with replacement the particles according to the weights $\{W_{i,t}\}_{i=1}^{N}$ and obtain the new population with new weights $W_{t,i} = \frac{1}{N}$.

7. If $t < T$, return to (2).

This method does not get stuck in low probability areas or is able t explore the whole support also in case of multimodality. It eases the inefficiency in case of significant mismatch between prior and posterior. All these reasons make it particularly fit for the estimation of non-linear DSGE models.

3 A comparison with the Bayesian Limited Information Method

In this section the performance of the ABC estimators is compared with an increasingly popular alternative: the Limited Information Method (Kim, 2002). Its Bayesian version (the Bayesian Limited Information Method, henceforth BLI) is often interpreted as the Bayesian counterpart of the GMM-style estimators.

The BLI is obtained by applying a Bayes Rule where the Prior distribution contains the extra data information and the likelihood is the joint probability of the *moments*, rather than of the data. Given that the Central Limit Theorem applies, the likelihood is obtained relying on the asymptotic normal distribution of the moments (i.e. on the Central Limit Theorem).

Given the vector of parameters θ, the sample moments $\hat{\gamma}$ and the estimated variance of the moments $\hat{V}$, The Approximate Posterior distribution $P(\theta|\hat{\gamma}, \hat{V})$ is obtained according to the Bayesian updating rule:

$$P(\theta|\hat{\gamma},\hat{V}) = \frac{P(\hat{\gamma}|\hat{\theta},V)P(\theta)}{P(\hat{\gamma}|V)} \tag{20}$$

where T is the number of moments,$\hat{\gamma}$ is the vector of sample moments, $\gamma(\theta)$ is the vector of analytical moments depending on the parameter θ, $P(\theta)$ is the prior distribution.
The likelihood $P(\gamma|\hat{\theta},\hat{V})$, conditional on $\hat{V}$, is computed according to:

$$P(\gamma|\hat{\theta},\hat{V}) = \frac{1}{(2\pi)^{(\frac{N}{2})}}|\hat{V}|^{-\frac{1}{2}} \exp\left\{-\frac{T}{2}\left(\hat{\gamma}-\gamma(\theta)\right)' V^{-1}\left(\hat{\gamma}-\gamma(\theta)\right).\right\}. \tag{21}$$

The role of moments and the Bayesian structure make the BLI the direct competitor of ABC estimators to check the small samples properties of the ABC estimator in a Bayesian framework. Interestingly, BLI can be interpreted as the Bayesian counterpart of the GMM and SMM estimators.
The relative performance of the ABC estimator with respect to the BLI method is measured in two Montecarlo exercises. The goal is to understand how much the presence of small samples and large persistence across the time series affect two estimators.
The criteria for the comparison are twofold:

- the Root Mean Square Error (RMSE) with respect to the Full likelihood Posterior Mean;
- The Overlapping Ratio between the 90% Credible Intervals of the Approximate Posterior distributions and the Full Likelihood Posterior Distribution (our target distribution).

These two criteria analyse the estimators from a Bayesian perspective. RMSE measures how close are the two estimators to the Full likelihood Bayesian estimator (the Posterior Mean).
The Overlapping Ratio captures which of the two methods deliver a better approximation

of the posterior distributions. The RMSE is obtained by:

$$RMSE = \frac{1}{N} \frac{\sum \left(\hat{\theta}_{app} - \hat{\theta}_{full} \right)^2}{\theta}, \tag{22}$$

where $\hat{theta}_{app}$ is the mean of the posterior of one of the two approximating methods, $\hat{theta}_{full}$ is the full likelihood posterior mean.

The Overlapping Ratio is obtained by:

$$OR = \frac{CI_{90\%,App} \cap CI_{90\%,Fl}}{CI_{90\%,App} \cup CI_{90\%,Fl}} \tag{23}$$

where $CI_{i-\%,App}$ is the $i-th$ Percentile of the Approximate Posterior distribution, $\cap$ stands for Intersection and $\cup$ for Union. The Overlapping Ratio is always included in the interval $[-1, 1]$. For example if the two intervals perfectly coincide the Overlapping Ratio equals 1, whereas if two degenerate posterior distributions do not overlap at all, the Overlapping Ratio equals -1.

The BLI estimator relies on the usual regularity assumptions of the GMM-style estimators. The normality assumption allows the GMM-style estimators to compute the likelihood of the moments focusing just on the first and the second moments of the moments distributions, and compute the quadratic objective function to updae the prior.

With ABC methodology, the moments distribution is studied by simulating the model, according to the observed sample size. The departure from the normality assumption and the kernel exploration of distribution delivers more reliable estimators than the GMM style estimators when dealing with small samples and highly persistent cases.

This result has been partially pointed out by Creel and Kristensen (2012) in their Indirect Likelihood Inference with which ABC shares the same intuition and similar asymptotic results.

In a first step, the experiment is run on simple AR(1) model. In the remainder of this section focus is on a RBC model subject to some weak identification issue.

3.1 Case 1: AR(1)

Despite its simple structure, the AR(1) process reproduce different estimation issues. Moreover, most exogenous processes generating stochasticity in DSGE models are AR(1) processes exhibiting different kind of persistence (from low persistence processes to Unit Roots).

The AR(1) model is estimated varying the sample size and the persistence of the process, in order to check if and when an estimator exploring the simulated distribution of the moments (ABC) outperforms one relying on the normality assumptions and focusing on the GMM-style quadratic objective function (BLI).

The estimation for each AR(1) process is run 1000 times. The sample sizes are 100, 300, 1000 observations. The autocorrelation factor tuning the persistence can assume the following values $\phi = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99]$.

Increasing the persistence of the process and decreasing the sample size should favour ABC estimators both in terms of RMSE and Overlapping Ratios.

Vice versa, the gap between the RMSEs and the ORs of the two estimators is expected to close increasing the sample size and lowering the persistence.

The moment in the matching procedure is the first order autocovariance. The Prior distribution is a Uniform prior $\sim U[0, 1]$. For the ABC AR(1) is simulated 10000 times, the Euclidean distances between the observed autocovariance and the simulated ones are computed and sorted out to select the first percentile of the distribution. The curse of dimensionality does not affect the estimation: 10000 simulations are enough to get rid of the error induced by the simulations since the moment is a scalar. For this reason, the correction step of the ABC-regression and the Kernel Weighting do not improve the estimation results upon the ABC-rejection procedure. Only results for ABC-regression are reported for sake of brevity.

For the Bayesian Limited Information method, the likelihood of the autocovariance is computed and the prior updated. The posterior distribution is studied with the Importance Sampling algorithm: as importance distribution the prior distribution is used and 10000 samples are drawn for each estimation.

The estimated variance $\hat{V}$ to condition the likelihood (and the posterior distribution)is computed with two alternatives: the HAC Variance Covariance Estimator or a bootstrapping procedure.

In the first case, the Newey-West estimator is computed, using a Bartlett Kernel and the bandwidth equal to $B(T) = floor(4 * (T/100)^{(2/9)})$, where T is the sample size.

An alternative method is inspired to the solution proposed in Christiano, Trebandt ad Walentin, where the covariance matrix is estimated through a bootstrap step. In the latter case, a first step estimator is computed to minimize a quadratic objective function using the identity matrix as variance covariance matrix. Afterwards, the AR(1) process is simulated for 1000 times (bootstrapping) to compute the autocorrelation for each bootstrap and the covariance of the moments $\hat{V}$ to compute the likelihood. Since working with one moment, the identity matrix at the initial step is simply the unity scalar and the covariance matrix is the covariance of the autocovariances computed in the bootstrap step.

Before exposing the results of the experiment, it can be interesting to give a quick look to the distribution of the autocovariances obtained by simulating the AR(1) process using different autocorrelations (from low persietency up to almost unit roots).

In Fig. **??**, the distribution of the sample autocovariances when $\phi = 0.5$ is reported. The sample size varies from 50 to 1000 observations. The distribution of the autocovariances converge quickly to a Normal distribution with the mean of the population autocovariance ($\gamma = \phi/(1-\phi^2)$, represented by the pink plane. In the the highly persistent case (Fig.**??**) when $\phi = 0.99$, the convergence to the normal distribution is much slower and even with a sample size of 5000 observations, the distribution of the autocovariance is skewed and not centred around the population autocovariance. These results render a simple intuition on the expected (and found) results in the Montecarlo experiments.

Fig. **??**, Fig. **??**,and Fig. **??** show the evolution of the RMSEs with respect to the Full likelihood posterior mean varying the persistence from $\phi = 0.1$ up to $\phi = 0.99$. The three figures are generated using different sample size: respectively 100, 300 and 1000 observations. Comparing the three figures, the gap between the two methods is larger in favour of ABC in small samples and reduces increasing the sample size. Moreover, for each sample size, increasing the persistence widens the gap in favour of ABC. Among the different approaches to estimate the variance covariance matrix of the moments $\hat{V}$, the HAC Newey-West estimator ensures smaller RMSEs especially in highly persistent cases and small samples, while the bootstrapping methods has smaller RMSE with low autocorrelations. In large samples, the RMSEs converge, at least up to $\phi = 0.95$. These results were widely expected in the light of the distributions juxtaposed in Figs. **??**,**??**.

Figs. **??** **??** and **??** show instead the evolution of the Overlapping Ratios passing from low autocorrelations to almost unit roots. Again, the samples are made of 100, 300 and 1000 observations. Our intuition is confirmed by the results: the OR gap between ABC methods and BLI is larger in general for highly persistent processes proving that ABC outperforms BLI in approximating the posterior distributions under certain conditions: the smaller the sample size, the larger the gap between the methods in favour of ABC. Also from this standpoint, results suggest that among the BLI estimators, the HAC estimation of the variance covariance matrix has a larger OR values than the Bootstrapping Procedure for persistent processes, while the opposite is true for the low persistent cases.

3.2 Case 2: A RBC with identification issues

In this second section, the performance of the two estimators is studied in a more complex and real world application. The experiment is run on a linear RBC model with three structural shocks and three observables. Again the comparison is run from a Bayesian perspective, taking the Full likelihood posterior distribution as reference and trying to capture to which extent the two approximate posterior distributions approximate the results of the true full likelihood posterior distributions.

The RBC estimated by Creel and Kristensen (2012) is a plain-vanilla RBC model with no problems of identification, given the simple structure of the model with just one structural shock on productivity. The RBC studied in this section encounters some identification issues concerning the preference parameters, due to the presence of three stochastic processes: a productivity shock on the production function, a shock on the preference affecting the labour supply and a shock on the interest rate requested by the household. The presence of these three shocks permit to estimate the full likelihood distribution using three observable variables without the need for measurement errors.

The households maximize the following expected sum of the utility functions:

$$maxE_t\left(\sum_{t=0}^{\infty}\beta^t\left(lnC_t-B_t\frac{H_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)\right) \tag{24}$$

subject to the budget constraint:

$$C_t+I_t=W_tH_T+D_tR_tK_t \tag{25}$$

. E_t stands for the expectation operator, C_t is the consumption, H_t are the hours offered by each household, B_t is the shock to the preference (namely the labour supply) (Rios-Rull et al., 2012) and D_t is the shock to the interest rate requested by the household like in Smets and Wouters (2007). β is the subjective discount factor and ν is the Frisch elasticity. Capital K_t is cumulated according to the following rule:

$$K_{t+1}=(1-\delta)K_t+I_t \tag{26}$$

where δ is the depreciation rate and I_t is the investment. Firms choose how much capital and hours to employ in the production function given the technology A_t:

$$Y_t=A_tK_t^{\alpha}H_t^{1-\alpha} \tag{27}$$

The market clearing is defined by:

$$Y_t = C_t + I_t \tag{28}$$

The economy is subject to the following three structural shocks:

$$log(A_{t+1}) = \rho_a log(A_t) + \sigma_a \epsilon_a \tag{29}$$

$$log(B_{t+1}) = \rho_b log(B_t) + \sigma_b \epsilon_b \tag{30}$$

$$log(D_{t+1}) = \rho_d log(D_t) + \sigma_d \epsilon_d \tag{31}$$

The technology shock A_t is the standard shock of the RBC literature (Kydland and Prescott,1983). The shock on the preferences B_t perturbs the labour supply hitting the marginal rate of substitution between consumption and leisure (see Rios-Rull, 2012). The shock on D_t is a shock on the interest rate requested by the households and can be interpreted as a shock to the risk premium.(Smets and Wouters, 2007). The model equilibrium is obtained by the following equations:

$$H_t = \left(\frac{1}{B_t} \frac{W_t}{C_t} \right)^{\gamma} \tag{32}$$

$$\frac{1}{C_t} = \beta \left(\frac{1}{C_{t+1}} \left((1-\delta) + D_{t+1} R_{t+1} \right) \right) \tag{33}$$

$$Y_t = C_t + I_t \tag{34}$$

$$K_{t+1} = K_t(1+\delta) + I_t \tag{35}$$

$$R_t = \alpha A_t K_t^{\alpha-1} H_t^{1-\alpha} \tag{36}$$

$$W = (1-\alpha) A_t K_t^{\alpha} H_t^{-\alpha} \tag{37}$$

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha} \tag{38}$$

Eq.**??** is the intratemporal choice between consumption and leisure, Eq.**??** is the Euler Equation. Equations **??** and **??** are the resource constraints and market clearing conditions completing the equilibrium of the model. Eq.**??** is the law of motion of capital and Eq. **??**, **??**, **??** are the exogenous processes.

The experiment adopts an informative prior distribution of the same fashion that Rios-Rull et al.(2012) use to estimate a state-of-the-art Real Business Cycle. Informativeness in the prior distribution eases the identification issues associated to the preferences parameters. Each Montecarlo experiment is made of 100 repetitions. The RMSE and the Overlapping Ratio are computed using different sample sizes: 100, 200, 500 observations. The data generating parameters are the following: $\beta = 0.95$, $\gamma = 2$, $\rho_a = 0.95$, $\rho_b = 0.95$, $\rho_d = 0.95$, $\sigma_a = 0.01$ $\sigma_b = 0.01$, $\sigma_d = 0.01$. The persistence of the processes plays in favour of of the ABC-estimators, especially in light of the results previously obtained in the AR(1) case. The moments are the covariances and the first order autocovariances of three observables: income Y_t, hours H_t and investments I_t.

The prior distribution is indicated in Table **??**. Concerning the ABC methods, RBC is simulated 5000 times, the tolerance level is such that the acceptance ratio of the simulations is equal to 5%. The results of ABC-rejection, ABc-kernel, ABC-regression and ABC-regression+HC, ABC-OLS (ABC-regression where the regression is simply linear) are reported.

The variance covariance matrix of the BLI estimator is obtained through the HAC Newey-West estimator.

For each Full likelihood and BLI estimation, a MCMC is drawn following the steps listed in An and Shorfheide (2007). Each chain contains 10000 draws with a burn-in period of 1000 draws.

Table **??** contains the results of the RMSE for the case of 100 observations, informative prior and high persistence of the process. ABC RMSEs are smaller than the BLI RMSEs. Tables **??** and **??** report the RMSEs respectively for 200 and 500 observations. The gap between the estimators is still in favour of the ABC.

Concerning the performance among the different ABC algorithms, ABC rejection and ABC-kernel provide the smaller RMSE. When the number of parameters increases, the large number of simulations needed may affect the results. ABC-regression reduce the number of simulations needed but at the cost of increasing the possible distortions in case of highly non-linear relations among parameters and moments. ABC-SMC can tackle the curse of dimensionality without the drawbacks associated to the ABC-regression.
Overlapping Ratios of the 90% credible intervals of the approximate posterior distributions and the Full likelihood posterior distribution are compared. ABC outperforms BLI method in approximating the full likelihood posterior distribution under the three different sample sizes: 100, 200 and 500 observations. The results are respectively reported in Tables **??**,**??**,**??**. The same reasoning expressed concerning the trade-off among the ABC estimators holds for the Overlapping Ratios.

4 An application to a plain-vanilla RBC

As a first example of ABC-estimation in a DSGE framework, we show how to apply ABC-rejection and ABC-regression in the estimation of the Real Business Cycle Model. The Real Business Cycle model (henceforth RBC) is the core of the DSGE model.
Because of its diffusion and centrality in economic theory, its working is well known among the economists and its simple structure make it a popular benchmark model to introduce new technical devices concerning the DSGE-literature.
The RBC is a Rational Expectation model in discrete time and it can be represented by the following set of equations:

$$E_t\{\beta[R_tK_{t+1}(\frac{C_{t+1}}{C_t})^{-\gamma}]\} = 1 \tag{39}$$

$$R_tK_t = A_t\alpha K_t^{\alpha-1} + 1 - \delta \tag{40}$$

$$Y_t = K_{t+1} - (1-\delta)K_t + C_t \tag{41}$$

$$Y_t = A_t K_t^{\alpha} \tag{42}$$

$$logA_{t+1} = \rho logA_t + \sigma\epsilon_{a,t+1} \tag{43}$$

$$\epsilon \sim N(0,\sigma^2) \tag{44}$$

Where K_t is the capital stock at time t, C_t is the consumption at time t, A_t is the productivity level, Y_t is the income and ϵ_t is the innovation to productivity .

Equation **??** is the Euler equation, Equation **??** ensures equilibrium on the capital market, Equation **??** is the feasibility constraint incorporating the law of motion for capital, and Equation**??** is the production function. Equation **??** is the exogenous process for productivity: it follows an autoregressive process of order 1 (AR(1)).

The parameters of the model are: β, α, δ, γ, ρ, σ. They respectively represent the subjective discount factor, the capital share in the production function, the capital depreciation rate, the persistence and the standard deviation of the productivity shock.

We proceed to calibrate the model to obtain a Data Generating Process so to run our estimation experiment: the generated time series will be treated as *observed* data. In particular, the subjective discount factor β is 0.95 and the intertemporal rate of substitution γ is 2. The capital share α is 1/3 while the depreciation rate δ is 0.025. The productivity persistence rate ρ is 0.90 while the standard deviation of the structural shock is 0.01.

The model is solved using standard log-linearization procedures.

4.1 The estimation

Concerning the parameter α and δ we fix them at their true value.

The prior distribution (Table **??**)gives the extra-data information. Concerning β and ρ having support between 0 and 1, we use *Beta* distributions. The elasticity γ has a normal prior around 2.10 while σ has a *Gamma Inverse* distribution.

We run the model with the DGP parameters and compute the *observed* moments. The observed sample is composed of 200 observations. For this experiment we select the covariances and the first order autocorrelations of consumption, interest rates and income (in total 9 moments are used, $\mathbf{s}$ has size 9 X 1). We produce 5000 simulations of the model, drawing from the prior distribution. Each simulation has the same sample size of 200 observations. The vector of moments are stuck in a matrix, where each row is a different simulation and each column reports a moments, (5000 x 9) (in our notation $\mathbf{S} = \{\mathbf{s}_i\}_1^{5000}$).

Moreover, the drawn parameters will be cast in a 5000 x 4 matrix (Θ).

The vector $\mathbf{s}$, the matrices $\mathbf{S}$ and Θ are the only ingredients to apply ABC techniques. To start, the basic ABC-rejection is applied with a tolerance level set such that the 5% of the simulations is accepted. In Table **??** the statistics of the ABC-posterior distribution are reported.

Given our diffuse priors, 5000 simulations are not sufficient to get rid of the simulation effect. Instead of increasing the number of simulations, we apply the ABC-regression method.

The accepted parameters are regressed on the discrepancies between observed and simulated moments. The results are exploited to correct the accepted parameters. Besides, the accepted parameters are given a Epanechinkov kernel weight.

The statistics of the corrected approximate posterior distribution are reported in Table **??**.

With respect to the ABC-rejection results, the posterior distributions are sharper and centered around the DGP values for all the parameters.

We finally apply the ABC -regression adding the correction for the heteroskedasticity (ABC-regression+HC). In Table **??**, we report the new corrected posterior distribution with ABC-regression +HC. The marginal posterior distributions are represented in Fig. **??**.

5 An application to the Zero Lower Bound

?? The financial crisis of 2008, the Great Recession and the following years of slow growth represent a big challenge to DSGE modelling and their estimation.

Since the beginning of the crisis and with different timings, many central banks lowered interest rates at their minimum and maintained them there for more than 5 years. In 2014Q4, interest rates in U.S, Euro Area, U.K., Sweden and other economic areas are still at the zero lower bound.

From a model perspective, the binding constraint on the policy rate and the gap between the Taylor rule implied interest rate and the actual one cause a non-linearity to take into account in the model solution.

In this section, ABC-SMC is applied on the estimation of a newkeynesian model with an occasional binding constraint on the zero lower bound.

The non-linear model is borrowed by Fernandez-Villaverde et al. (2012). The notation is the same for the sake of comparison between the calibrated values of the original papers and the estimates of this section.

As it appears clear from Section **??**, ABC techniques study the simulated distribution of the moments without relying on the normality and regularity assumptions made in the GMM-style estimators. Moreover, in ABC non gaussian moments can be exploited (i.e. binomial, multinomial etc.) Here, the following moments are used in the estimation together with the usual covariances:

- Frequency of the zero lower bound, number of episodes at the zero lower bound in the sample;
- Frequency of recession events, number of recession episodes;
- Frequency of deflation events, number of deflation episodes.

Still nowadays, just few papers tackle the estimation issues of the nekeynesian model including the period of the Zero Lower bound. Most of the estimated models use samples

which exclude the Zero Lower Bound /Christiano et al. (2014), Arouba and Shorfheide (2013).
Gust et al. estimate a newkeyesian model with a binding constraint on the interest rate using the particle filter. Their sample contains three observable variables to make inference on the structural parameters. They solve the model with a fully non-linear method. In this paper the model is solved according to the Piecewise linear solution, using the MATLAB routine provided by Iacoviello and Guerrieri (2014). The piecewise linear solution is quick to obtain with respect to the other non-linear methods and can handle medium-size DSGE models. This allows to obtain a large numbers of simulations in short range of time. Moreover, differently from Gust et al. the sample includes six observable variables and includes observations up to 2014Q3. The main exercise uses data starting for the beginning of the Great Moderation (1983Q1). Hence in the sample, the ZLB binds for more than one fifth of the sample. This, together with the use of non-conventional features of the data (frequency of the ZLB and so forth), tries to capture the effects of the exit from the Great Moderation in the estimate results.

5.1 The model

The model is a standard newkeynesian model with occasionally positivity constraint on the interest rate. A household maximizes her utility consuming and providing labour (the unique productive factor) to intermediate firms operating in monopolistic competition, readjusting prices according to Calvo type of contracts. The differentiated products are then assembled by retail firms operating in perfect competition.
Households maximise the following utility function separable in consumption c_t and labour l_t.

$$\sum_{i=0}^{\infty}\left(\prod_{i=0}^{t}\beta_i\right)\left\{logc_t - \psi\frac{l_t^{1+\phi}}{1+\phi}\right\} \tag{45}$$

where ϕ is the inverse of the Frisch labour supply elasticity and β_t is the subjective discount factor subject to stochastic fluctuations around the mean β:

$$\beta_{t+1} = \beta^{1-\rho_b}\beta_t^{\rho_b} exp(\sigma_b \epsilon_{b,t+1} \tag{46}$$

with $\epsilon_{b,t+1} \sim N(0,1)$. rho_b and σ_b are respectively the autocorrelation and the standard deviation of the AR(1) process.

The household maximizes her utility subject to the budget constraint:

$$c_t + \frac{b_{t+1}}{p_t} = w_t l_t + R_{t-1} b_t / p_t + T_t + F_t \tag{47}$$

where b_t is a nominal government bond that pays a nominal interest rate R_t. p_t is the price level, whereas T_t and F_t are respectively the lamp sum taxes and the profits of the firms.

Retail firms reassemble intermediate goods y_{it} and the technology:

$$y_t = \left(\int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{48}$$

with ϵ is the elasticity of substitution. Final producers maximize their profit taking into account intermediate goods prices p_{it}, final prices p_t. The demand for each good will follow:

$$y_{it} = \left(\frac{p_{it}}{pt} \right)^{-\epsilon} y_t, \tag{49}$$

and the price of the final good will be equal to:

$$p_t = \left(\int_0^1 p_{it}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{50}$$

The wholesale firms operate according to the production function:

$$y_{it} = A_t l_{it}, \tag{51}$$

where the productivity A_t evolves according to the law of motion:

$$A_t = A^{1-\rho_A} A_{t-1}^{\rho_A} exp(\sigma_A \varepsilon_{A,t}) \tag{52}$$

with $\varepsilon \sim N(0,1)$. The marginal costs are $mc_t = \frac{w_t}{A_t}$.

The firms choose their price according to a Calvo rule, where each period just a fraction $1-\theta$ firms can re-optimize their prices p_{it}. Firms will choose their price to maximize the profits:

$$\max_{p_{it}} E_t \sum_{\tau=0}^{\infty} \theta^{\tau} \left(\prod_{i=0}^{\tau} \beta_{t+1} \right) \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y_{it+\tau} \tag{53}$$

s.t.

$$y_{it} = \left(\frac{p_{it}}{p_t} \right)^{-\epsilon} y_t \tag{54}$$

where λ_{t+s} is the Lagrangian multiplier for the household in period $t+s$. Two auxiliary $x_{1,t}$ and $x_{2,t}$ are used to define the solution to the maximization problem:

$$\epsilon x_{1,t} = (1 - \epsilon * x_{2,t}) \tag{55}$$

$$x_{1,t} = \frac{1}{c_t} mc_t y_t + \theta E_t \beta_{t+1} \Pi_{t+1}^{\epsilon} x_{1,t+1} \tag{56}$$

$$x_{2,t} = \frac{1}{c_t} \Pi_t^* y_t + \theta E_t \beta_{t+1} \Pi_{t+1}^{\epsilon-1} \frac{\Pi_t^*}{\Pi_{t+1}^*} x_{2,t+1} = \Pi_t^* \left(\frac{1}{c_t} y_t + \theta E_t \beta_{t+1} \frac{\Pi_{t+1}^{\epsilon-1}}{\Pi_{t+1}^*} x_{2,t+1} \right) \tag{57}$$

where $\Pi_t^* = \frac{p_t^*}{p_t}$. Inflation dispersion will be equal to:

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta)(\Pi_t^*)^{1-\epsilon}. \tag{58}$$

The government sets the nominal interest rate:

$$R_t = max\left[R_t, 1 \right], \tag{59}$$

with the notional interest rate Z_t:

$$Z_t = R^{1-\rho_r} R_{t-1}^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}) \right)^{\phi_y} \right]^{1-\rho_r} m_t \tag{60}$$

with m_t being the monetary policy *iid* shock $m_t = exp(\varepsilon_{m,t}\sigma_m)$, $\epsilon_{m,t} \sim N(0,1)$. The gross interest rate is equal to the notional interest rate as long it is larger than 1, since it cannot be set below 1 (the zero lower bound, ZLB).

The government sets also the spending:

$$g_t = s_{g,t} y_t \tag{61}$$

$$s_{g,t} = s_g^{1-\rho_g} s_{g,t-1}^{\rho_g} exp(\sigma_g \varepsilon_{g,t}) \tag{62}$$

with $\varepsilon \sim N(0,1)$. Since the agents are ricardian, we can set $b_t = 0$.

After aggregation we obtain:

$$y_t = \frac{A_t}{v_t} l_t \tag{63}$$

with v_t is the loss of efficiency introduced by the price dispersion:

$$v_t = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\epsilon} di \tag{64}$$

Moreover, following the Calvo pricing properties we can write:

$$v_t = \theta \Pi_t^{\epsilon} v_{t-1} + (1-\theta)(\Pi_t^*)^{-\epsilon}. \tag{65}$$

5.2 The Equilibrium

The Equilibrium is given by the sequence

$$\{y_t, c_t, l_t, mc_t, x_{1,t}, x_{2,t}, w_t, \Pi_t, \Pi_t^*, v_t, R_t, Z_t, \beta_t, A_t, m_t, g_t, b_t, s_{g,t}\}_{t=0}^{\infty} \tag{66}$$

. The equilibrium is defined by the following equations.

The intertemporal and the intratemporal household F.O.Cs:

$$\frac{1}{c_t} = E_t \left\{ \frac{\beta_{t+1}}{c_{t+1}} \frac{R_t}{\Pi_{t+1}} \right\}, \tag{67}$$

$$\psi l_t^{\phi} c_t = w_t t \tag{68}$$

The solution of the maximization problem of the firms:

$$mc_t = \frac{w_t}{A_t}, \tag{69}$$

$$\epsilon x_{1,t} = (1 - \epsilon * x_{2,t}), \tag{70}$$

$$x_{1,t} = \frac{1}{c_t} mc_t y_t + \theta E_t \beta_{t+1} \Pi_{t+1}^{\epsilon} x_{1,t+1}, \tag{71}$$

$$x_{2,t} = \frac{1}{c_t} \Pi_t^* y_t + \theta E_t \beta_{t+1} \Pi_{t+1}^{\epsilon-1} \frac{\Pi_t^*}{\Pi_{t+1}^*} x_{2,t+1} = \Pi_t^* \left(\frac{1}{c_t} y_t + \theta E_t \beta_{t+1} \frac{\Pi_{t+1}^{\epsilon-1}}{\Pi_{t+1}^*} x_{2,t+1} \right). \tag{72}$$

The government equations are:

$$R_t = max\left[R_t, 1\right], \tag{73}$$

$$Z_t = R^{1-\rho_r} R_{t-1}^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}) \right)^{\phi_y} \right]^{1-\rho_r} m_t. \tag{74}$$

Inflation evolution and price dispersion:

$$1 = \theta \Pi_t^{\epsilon-1} + (1-\theta)(\Pi_t^*)^{1-\epsilon}, \tag{75}$$

$$v_t = \theta \Pi_t^{\epsilon} v_{t-1} + (1-\theta)(\Pi_t^*)^{-\epsilon}. \tag{76}$$

Market clearing conditions:

$$y_t = c_t + g_t, \tag{77}$$

$$y_t = \frac{A_t}{v_t} l_t. \tag{78}$$

The stochastic processes are:

$$\beta_{t+1} = \beta^{1-\rho_b} \beta_t^{\rho_b} exp(\sigma_b \epsilon_{b,t+1}, \tag{79}$$

$$A_t = A^{1-\rho_A} A_{t-1}^{\rho_A} exp(\sigma_A \varepsilon_{A,t}), \tag{80}$$

$$s_{g,t} = s_g^{1-\rho_g} s_{g,t-1}^{\rho_g} exp(\sigma_g \varepsilon_{g,t}), \tag{81}$$

$$m_t = exp(\varepsilon_{m,t} \sigma_m). \tag{82}$$

5.3 Solution method

Standard perturbation methods provide local solutions and cannot handle models with occasionally binding constraint without adaptation. For this reason, Fernandez-Villaverde et al. solve the model using a fully non-linear solution. In this paper, the model is solved by the piecewise linear solution method presented in Guerrieri and Iacoviello 2013. The routine codes are directly provided by the authors. Here the solution technique is shortly presented. For a more detailed exposition, see the original paper.

The piecewise solution method delivers a first order perturbation solution in a piecewise fashion. The presence of the occasionally binding constraint creates two regimes: in one the constraint is slack, in the other it is binding. In the current exercise, the unconstrained case is the reference regime, the constrained (ZLB binding) is the alternative. The solution that we obtain is not just the juxtaposition of two linear solutions: the policy coefficients depend on how long the regime is expected to last. How long the model lasts is influenced by the state vector. This feedback effect can produce an important non-linearity. The piecewise linear solution allows to obtain a large number of simulations and tackle the curse of dimensionality generated by dealing with solving non-linearly medium-scale economic models. [3]

To solve the model two conditions must hold:

[3] A drawback of this solution method is that it assumes that agents do not expect future shocks hitting the economy in the following periods. Hence precautionary savings are not considered.

- Blanchard-Khan conditions must hold in the reference regime;
- If the shocks hitting the economy take the model away from the reference regime to the alternative regime, in absence of future shocks the model must return to the reference regime.

For further details on the solution method see Guerrieri and Iacoviello (2014).

5.4 Estimation strategy

The model is estimated using six quarterly macroeconomic time series for the US economy, taken from FRED dataset and used as observable variables: the log difference of Real GDP per person, the log difference of Real Consumption, log hours worked, the log GDP deflator, the log difference of real wage, the log FED funds rate. The observable equations are the following:

$$log\Delta GDP_t = 100(y_t - y_{t-1}) + \gamma, \tag{83}$$

$$log\Delta CONS_t = 100(c_t - c_{t-1}) + \gamma, \tag{84}$$

$$log\Delta WAGES_t = 100(w_t - w_{t-1}) + \gamma, \tag{85}$$

$$logHOURS_t = 100 l_t + \bar{l}, \tag{86}$$

$$log\Delta DEFL_t = 100 * (p_t * \pi + \pi - 1), \tag{87}$$

$$logFEDDUNDS_t = 100(exp(r_t) * RSS - 1) - 1. \tag{88}$$

Where Δ is the difference operator, $RSS = \frac{\pi}{\beta}$. The type of dataset is very similar to the one used by Smets and Wouters, except for the investment that is excluded. As in Smets and Wouters, measurement errors are not necessary to estimate the model, differently from the particle filter case.

The ZLB period started in 2008Q3 up to the end of the sample. Different estimation exercises are performed. First, estimation is conducted according to four different time ranges:

- *Baseline*: the sample size spans from 1966Q1 to 2014Q3. It is the largest sample, it contains 185 observations and starts from the same quarter used in the main estimation exercise in Smets and Wouters (2003);
- *Great Moderation without the ZLB*: the dataset range goes from 1983Q1 until 2007Q4. The sample contains 96 observations and stops before the interest rate enters the Great Recession and hits the ZLB period;
- *Great Moderation and the Great Recession*: the sample spans from 1983 to 2014Q3 (125 observations). The economy is at the ZLB for approximately one fifth of the time. The final part of the sample contains thh Great Recession and the slow recovery.
- *The Great Volatility II*: the sample spans from 2001Q1 until 2014Q3 (57 observations). The economy is at the ZLB for approximately 40% of the sample.

In a first case, the estimation is performed using just the covariances and the variances of the observable variables. The moments are computed and matched conditional on the two different regimes. In a second exercise, the estimation is performed using covariances and non-guassian distributed moments:

- the frequency of being on the zero lower bound over the sample, the number of periods at the ZLB;
- the frequency of being in recession over the sample, the number of periods at of recession;
- the frequency of being in deflation over the sample, the number of periods at of deflation.

The priors used are common in literature and are listed in Table **??**. ABC-SMC procedure is applied, until convergence of the posterior distribution. The model is simulated for 30000 times at the first iteration. In the first iteration, the 5% of the simulations is accepted according to the Euclidean distance. After the first iteration, each swarm of particle contains 1500 simulations and the particles are perturbed according to the kernel $K(\theta_{i,t}^{**}|\theta_{i,t}^{*})$:

$$\theta_{i,t}^{**} \sim N(\theta_{i,t-1}^{*}, c * \Sigma), \tag{89}$$

where Σ is a diagonal matrix with the variances of the first iteration accepted parameters, scaled by a scalar $c = 0.02$, to tune the acceptance rate in the following iterations.
When $t = 1$, weights are assigned according to:

$$W_{i,1} = \frac{\pi(\theta_{i,1})}{\mu_1(\theta_{i,1})} = 1 \tag{90}$$

since the prior distribution $\pi(\theta)$ and the proposal $\mu_1(\theta_{i,1})$ coincide.
When $t \geq 2$ weights are assigned according to:

$$W_{i,t} = \frac{\pi(\theta_{i,t})}{\sum_{j=1}^{N} W_{t-1}(\theta_{t-1,j}) K_t(\theta_{t,i}|\theta_{t-1,j})}, \tag{91}$$

where $W_{i,t}$ is the weight of particle i at iteration t. $K_t(\theta_{t,i}|\theta_{t-1,j})$ is the kernel of the perturbation step. Particles of parameters are resampled when the effective sample size is smaller than 750 (half of the accepted sample obtained after first accept-reject).
The tolerance level is decreased of 0.1% at each iteration.
The convergence for the approximate posterior distribution can be intuitively checked confronting the different approximate posterior distribution obtained at each iteration. In this case, ten iterations are enough to insure convergence of the posterior distributions. (Fig. **??**)

5.5 Estimation results.

Results for the four different samples are reported in Tables **??**-**??**.
Estimate results are standard. Across the different sub-samples, standard deviations are larger for the GM+ZLB and the GV-II periods and smaller for the GM period. Moreover, autocorrelations for the preference AR(1) process and the TFP AR(1) process are larger for GM+ZLB and the GV-II periods. Autocorrelation for the government spending process is larger for the baseline and the GM period.
Concerning the monetary policy, no significant differences emerge, except for the autocorrelation of the interest rate, which is smaller in the GM period compared to the other sub-samples estimates.
To check how these estimates affect the dynamics of the model, for each subsample estimate the model is shocked for two consecutive periods by two preference shocks in a row. The magnitude of each shock is equal to two standard deviations of the shock. Such shocks send the model onto the Zero lower bound in all the examples, except for the Great Moderation period. As expected, the latter case is the one where variables move more moderately (Fig.**??**). The largest variations are found in the GV II case.
These results suggest that even if the sub-samples differ only for small fractions of data, the estimation results provided evidence for different behaviour of the main variables.
If the non-gaussian moments are included estimates do not vary much compared to the case with only gaussian moments. This is probably attributed to the fact that the amount of new information introduced with the non gaussian moments is relatively small compared to the one provided by the moments already in use. Moreover, these moments seem to concern more the parameters affecting the steady state value of inflation and interest rates (the inflation target Π and the subjective discount factor β). As a result, the impulse responses look very similar (Fig.**??**). The GM period is the one with the smallest reactions to the shock. With respect to the case with only gaussian moments, except for the steady state values for interest rate and inflation which are lower.
From this simple exercise, the non gaussian moments used (duration of ZLB, frequency of

the ZLB, duration of deflation and so forth) appear to be useful to improve the efficiency in the identification of steady state parameters. A further investigation is required on this topic.

6 Conclusion

In this paper, Approximate Bayesian Computation techniques have been applied to the estimation of economic models.

Two Montecarlo experiments have been assessed to analyse the small samples properties of ABC techniques. ABC performance is compared to the one of the Bayesian Limited Information Method (BLI), Kim (2002). BLI can be interpreted as a Bayesian version of GMM-style estimators.

ABC outperforms BLI both using an AR(1) at different persistence and an RBC model with large persistence. The performance is analysed through the lens of Bayesian criteria:

- The RMSE with respect to the Full likelihood posterior mean;
- The Overlapping ratio between the approximate posteriors and the full likelihood posterior distribution.

This result holds stronger when dealing with small sample and large persistence data generating processes: ABC does not automatically rely on the normality assumption made on the moments. ABC explores the whole moments distribution.

Other estimation exercises are provided.

ABC-rejection and ABC-regression are applied to a vanilla RBC model.

A newkeynesian model with occasionally binding constraint is applied. The model is solved and simulated using the piecewise linear approximation by Iacoviello and Guerrieri 2013. ABC-SMC is applied to tackle the curse of dimensionality. The model is estimated with different sub-samples. Results show different behaviours of the main variables accoridng to the sub-sample estimates used to get the dynamics of the model. Great Moderation impulse responses strongly differ from the ones obtained using estimates that took into account the Great Recession and the ZLB period.

Estimation is performed also using non-gaussian moments, like the frequency of hitting the zero lower bound or the number of ZLB episodes. In this case, inference is affected by using these unconventional features of the data, contributing to increase the efficiency in the identification of some parameters.

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Table 1: Prior distribution for the parameters to estimate

Parameter	Distribution	Mean	Std.Dev.
β	*Beta*	0.97	0.04
γ	*Normal*	2.10	0.05
ρ	*Beta*	0.92	0.15
σ	*Gamma Inverse*	0.012	0.010

Table 2: Posterior distribution (ABC-rejection)

Sum.Stat.	β	ρ	γ	σ
DGP Params.	0.95	2.00	0.90	0.0100
Min.:	0.8213	0.8065	0.7658	0.0038
2.5% Perc.:	0.8628	0.8293	1.2326	0.0049
Median:	0.9455	0.9095	2.0865	0.0090
Mean:	0.9372	0.9050	2.1065	0.0090
Mode:	0.9532	0.9194	1.9913	0.0077
97.5% Perc.:	0.9748	0.9603	3.0564	0.0135
Max.:	0.9810	0.9712	3.3160	0.0142

RBC estimated parameters through ABC-rejection, 200 oservations, 5000 simulations, 5% acceptance rate.

Tables

Table 3: Posterior distribution (ABC-regression)

Sum.Stat.	β	ρ	γ	σ
DGP Params.	0.95	2.00	0.90	0.0100
Min.:	0.8783	0.8534	1.6111	0.0083
Weighted 2.5 % Perc.:	0.9159	0.8716	1.6964	0.0089
Weighted Median:	0.9438	0.9094	1.8929	0.0097
Weighted Mean:	0.9417	0.9106	1.9183	0.0097
Weighted Mode:	0.9445	0.9084	1.8314	0.0098
Weighted 97.5 % Perc.:	0.9545	0.9477	2.2998	0.0104
Max.:	0.9623	0.9620	2.5228	0.0105

RBC estimated parameters through ABC-regression, 200 observations, with 5000 simulations, 5% of acceptance rate.

Table 4: Posterior distribution (ABC-regression + HC)

Sum.Stat.	β	ρ	γ	σ
DGP Params.	0.95	2.00	0.90	0.0100
Min.:	0.8510	0.7241	1.7208	0.0086
Weighted 2.5 % Perc.:	0.9009	0.8689	1.7795	0.0090
Weighted Median:	0.9481	0.9082	1.9074	0.0097
Weighted Mean:	0.9437	0.9096	1.9246	0.0097
Weighted Mode:	0.9528	0.9035	1.8780	0.0097
Weighted 97.5 % Perc.:	0.9628	0.9481	2.1512	0.0102
Max.:	0.9713	0.9709	2.4564	0.0104

RBC estimated parameters through ABC-regression + Correction for heteroskedasticity, 200 observations, with 5000 simulations, 5% of acceptance rate.

Table 5: Prior distribution for the RBC parameters

Parameter	**Distribution**	**1**	**2**
β	*Beta*	0.95	0.02
γ	*Normal*	2	0.50
ρ_a	*Beta*	0.95	0.04
ρ_b	*Beta*	0.95	0.04
ρ_d	*Beta*	0.95	0.04
σ_a	*Gamma Inverse*	0.01	4
σ_b	*Gamma Inverse*	0.01	4
σ_d	*Gamma Inverse*	0.01	4

Prior distribution: Informative Prior

Table 6: RMSE, sample size=100 obs.

Methods	β	γ	ρ_a	ρ_b	ρ_d	σ_a	σ_b	σ_d
ABC-rej	0.01395	0.04079	0.01812	0.01566	0.01609	0.27268	0.22648	0.12772
ABC-ker	0.01456	0.04394	0.01871	0.01596	0.01666	0.27522	0.22939	0.13000
ABC-OLS	0.01406	0.06961	0.02131	0.02157	0.02014	0.27040	0.26608	0.16532
ABC-regr	0.01415	0.07220	0.02195	0.02180	0.02079	0.27223	0.26811	0.16567
ABC-HC	0.01920	0.10839	0.02755	0.03006	0.02448	0.26240	0.28406	0.22597
BLI	0.03729	0.05116	0.04154	0.03172	0.02695	0.67502	0.87365	0.30317

RMSE obtained in a Montecarlo experiment, 100 repetitions. The sample contains 100 observations. Case: High peristency and Informative Priors. ABC-rej= ABC-rejection, ABC-ker=ABC-rejection + kernel weighting, ABC-OLS= ABC + OLS Regression Step; ABC-regr= ABC-regression with Local Linear Regression, ABC-HC=ABC-regression + Correction for Heteroskedasticity

Table 7: RMSE, sample size=200 obs.

Methods	β	γ	ρ_a	ρ_b	ρ_d	σ_a	σ_b	σ_d
ABC-rej	0.01231	0.05162	0.01738	0.01691	0.01543	0.25187	0.22934	0.10386
ABC-ker	0.01332	0.05151	0.01798	0.01763	0.01606	0.25104	0.23001	0.10843
ABC-OLS	0.01237	0.06588	0.02006	0.02174	0.02197	0.24180	0.26175	0.13565
ABC-regr	0.01269	0.06876	0.01998	0.02186	0.02150	0.24271	0.26266	0.14553
ABC-HC	0.01655	0.09258	0.02385	0.02764	0.02650	0.22664	0.26542	0.20675
BLI	0.03418	0.10956	0.04294	0.05040	0.02682	0.58462	0.72849	0.59571

RMSE obtained in a Montecarlo experiment, 100 repetitions. The sample contains 200 observations. Case: High peristency and Informative Priors. ABC-rej= ABC-rejection, ABC-ker=ABC-rejection + kernel weighting, ABC-OLS= ABC + OLS Regression Step; ABC-regr= ABC-regression with Local Linear Regression, ABC-HC=ABC-regression + Correction for Heteroskedasticity

Table 8: RMSE, sample size=500 obs.

Methods	β	γ	ρ_a	ρ_b	ρ_d	σ_a	σ_b	σ_d
ABC-rej	0.01093	0.05065	0.02034	0.01764	0.01665	0.22934	0.27019	0.13820
ABC-ker	0.01110	0.05388	0.02042	0.01761	0.01669	0.22629	0.26472	0.14590
ABC-OLS	0.01081	0.08508	0.01765	0.01776	0.02078	0.22260	0.28040	0.19021
ABC-regr	0.01068	0.08764	0.01759	0.01755	0.02098	0.22184	0.28060	0.19879
ABC+HC	0.01205	0.11679	0.01875	0.01930	0.02520	0.20846	0.27512	0.26526
BLI	0.03742	0.06969	0.03863	0.03228	0.04938	0.58462	0.93056	0.53128

RMSE obtained in a Montecarlo experiment, 100 repetitions. The sample contains 500 observations. Case: High peristency and Informative Priors. ABC-rej= ABC-rejection, ABC-ker=ABC-rejection + kernel weighting, ABC-OLS= ABC + OLS Regression Step; ABC-regr= ABC-regression with Local Linear Regression, ABC-HC=ABC-regression + Correction for Heteroskedasticity

Table 9: OR100, sample size=100 obs.

Methods	β	γ	ρ_a	ρ_b	ρ_d	σ_a	σ_b	σ_d
ABC-rej	0.60627	0.81372	0.75159	0.80626	0.70472	0.44003	0.50961	0.70015
ABC-ker	0.67059	0.87735	0.76656	0.80266	0.80791	0.49469	0.58173	0.76644
ABC-OLS	0.36064	0.81120	0.72085	0.72093	0.75838	0.28645	0.39473	0.75102
ABC-regr	0.35319	0.80143	0.72284	0.72073	0.75071	0.28332	0.38944	0.74532
ABC-HC	0.55281	0.73430	0.66426	0.63165	0.68748	0.40299	0.42275	0.71252
BLI	0.04772	0.88188	0.66390	0.33132	0.34849	0.05231	-0.03619	0.43781

Overlapping Ratio obtained in a Montecarlo experiment, 100 repetitions. The sample contains 100 observations. Case: High peristency and Informative Priors. ABC-rej= ABC-rejection, ABC-ker=ABC-rejection + kernel weighting, ABC-OLS= ABC + OLS Regression Step; ABC-regr= ABC-regression with Local Linear Regression, ABC-HC=ABC-regression + Correction for Heteroskedasticity

Table 10: OR200, sample size=200 obs.

Methods	β	γ	ρ_a	ρ_b	ρ_d	σ_a	σ_b	σ_d
ABC-rej	0.57967	0.79299	0.73966	0.79252	0.67809	0.42241	0.47965	0.69985
ABC-ker	0.65300	0.87342	0.77590	0.78717	0.79739	0.48200	0.54320	0.76554
ABC-OLS	0.29644	0.82433	0.69746	0.68340	0.72241	0.22776	0.34818	0.74757
ABC-regr	0.29253	0.81414	0.69455	0.67834	0.72318	0.22574	0.34598	0.74664
ABC-HC	0.51729	0.77607	0.68972	0.67147	0.69955	0.39251	0.40124	0.71773
BLI	0.31990	0.77961	0.66926	0.62000	0.19545	0.12139	0.26556	0.43072

Overlapping Ratio obtained in a Montecarlo experiment, 100 repetitions. The sample contains 200 observations. Case: High peristency and Informative Priors. ABC-rej= ABC-rejection, ABC-ker=ABC-rejection + kernel weighting, ABC-OLS= ABC + OLS Regression Step; ABC-regr= ABC-regression with Local Linear Regression, ABC-HC=ABC-regression + Correction for Heteroskedasticity

Table 11: OR500, sample size=500 obs.

Methods	β	γ	ρ_a	ρ_b	ρ_d	σ_a	σ_b	σ_d
ABC-rej	0.50557	0.79718	0.68900	0.76053	0.64924	0.36181	0.44401	0.64670
ABC-ker	0.55337	0.86392	0.73684	0.76653	0.75746	0.40674	0.49992	0.73275
ABC-OLS	0.20349	0.79516	0.56611	0.58313	0.66112	0.13250	0.31733	0.69377
ABC-regr	0.20260	0.78619	0.56501	0.58256	0.65981	0.13104	0.31516	0.68630
ABC-HC	0.47321	0.76688	0.70070	0.74279	0.63753	0.34438	0.44529	0.67628
BLI	0.05993	0.83720	0.70892	0.31765	0.68671	0.11578	-0.04109	0.59651

Overlapping Ratio obtained in a Montecarlo experiment, 100 repetitions. The sample contains 500 observations. Case: High peristency and Informative Priors. ABC-rej= ABC-rejection, ABC-ker=ABC-rejection + kernel weighting, ABC-OLS= ABC + OLS Regression Step; ABC-regr= ABC-regression with Local Linear Regression, ABC-HC=ABC-regression + Correction for Heteroskedasticity

Par	Prior Distr	Prior Mean	Prior St.Dev.
β	*Beta*	0.997	
θ	*Beta*	0.7	0.1
ϕ_y	*Gamma*	0.2	0.1
ϕ_π	*Gamma*	2.2	1
ρ_R	*Beta*	0.7	0.2
ϵ	*Gamma*	6	1
ϕ	*Gamma*	1	2
π	*Uniform*	1.002	1.007
$\bar{l}$	*Normal*	0	0.5
γ	*Normal*	0	0.5
ρ_A	*Beta*	0.70	0.20
ρ_G	*Beta*	0.70	0.20
ρ_U	*Beta*	0.80	0.10
σ_A	*InvGamma*	0.005	4
σ_G	*InvGamma*	0.005	4
σ_M	*InvGamma*	0.005	4
σ_U	*InvGamma*	0.005	4

Table 12: Prior distribution for the estimation of the newkeynesian model with the occasionally binding ZLB

Parameter	$5\%CI$	Mean	$95\%CI$
β	0.99591	0.996279	0.99671
θ	0.71878	0.764166	0.804385
ϕ_y	11 0.116811	0.158159	0.214062
ϕ_π	1.45885	1.85648	2.28268
ρ_R	0.643356	0.74771	0.836872
ϵ	6.74608	7.07615	7.45351
ϕ	0.15195	0.402306	0.759111
π	1.00207	1.00243	1.00295
ρ_A	0.602841	0.692637	0.775175
ρ_G	0.688226	0.735817	0.765022
ρ_U	0.825232	0.909938	0.978777
σ_A	0.00401074	0.0058359	0.00846274
σ_G	0.00394885	0.00479752	0.00609127
σ_M	0.00325742	0.00443026	0.00595078
σ_U	0.00401563	0.00548303	0.00745046
$\bar{l}$	0.0300085	0.144853	0.344083
γ	0.00592662	0.115779	0.312684

Table 13: Estimates for the Baseline sample (1966Q1-2014Q3), using only gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

Parameter	$5\%CI$	Mean	$95\%CI$
β	0.99483	0.99520	0.99583
θ	0.66356	0.70538	0.73836
ϕ_y	0.10252	0.15220	0.19152
ϕ_π	1.49748	1.82866	2.25048
ρ_r	0.45842	0.51579	0.61003
ϵ	5.11345	5.56714	6.00983
ϕ	0.19322	0.59177	1.33045
π	1.00595	1.00621	1.00667
ρ_A	0.72697	0.82690	0.91660
ρ_G	0.67377	0.71112	0.76632
ρ_U	0.73527	0.83825	0.94086
σ_A	0.00325	0.00453	0.00576
σ_G	0.00337	0.00379	0.00423
σ_M	0.00338	0.00425	0.00494
σ_U	0.00352	0.00483	0.00623
l	0.00330	0.12036	0.29326
γ	0.00866	0.24216	0.53122

Table 14: Estimates for the Great Moderation sub-sample (1983Q1-2008Q3), using only gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

Parameter	$5\% CI$	Mean	$95\% CI$
β	0.9967	0.9971	0.9974
θ	0.7403	0.7719	0.7973
ϕ_y	0.1385	0.1715	0.2073
ϕ_π	1.5818	1.817	2.2302
ρ_r	0.6532	0.7498	0.8106
ϵ	5.6922	5.9553	6.2826
ϕ	0.1547	0.5162	0.9885
π	1.0030	1.0035	1.0040
ρ_A	0.8421	0.9438	0.9923
ρ_G	0.7334	0.7806	0.8140
ρ_U	0.4984	0.5965	0.6743
σ_A	0.0030	0.0044	0.00634
σ_G	0.0038	0.0044	0.00508
σ_M	0.0039	0.00508	0.0067
σ_U	0.0031	0.0044	0.0055
l	0.0443	0.1680	0.3280
γ	0.1508	0.3175	0.5380

Table 15: Estimates for the Great Moderation +ZLB sub-sample (1983Q1-2014Q3), using only gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

Parameter	$5\%CI$	Mean	$95\%CI$
β	0.99512	0.9953	0.99583
θ	0.69172	0.73243	0.8051
ϕ_y	0.11763	0.14271	0.1891
ϕ_π	1.28688	1.9525	2.5373
ρ_r	0.64005	0.7029	0.8352
ϵ	5.43814	5.8696	6.0763
ϕ	0.10567	0.3181	0.8044
π	1.0039	1.0046	1.00536;
ρ_A	0.77037	0.8747	0.9265
ρ_G	;0.70265	0.7492	0.8218
ρ_U	0.41432	0.50462	0.5395
σ_A	0.0037	0.0053	0.0073
σ_G	0.0046	0.0062	0.0070
σ_M	0.0035	0.0046	0.0056
σ_U	0.0043	0.00504	0.0068
l	0.0740	0.3899	0.5464
γ	0.01824	0.10519	0.40064

Table 16: Estimates for the Great Volatility II sub-sample (2001Q1-2014Q3), using only gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

Parameter	$5\% CI$	Mean	$95\% CI$
β	0.99624	0.996852	0.997151
θ	0.706703	0.746625	0.791787
ϕ_y	0.114461	0.191332	0.264339
ϕ_π	1.49242	1.97145	2.36059
ρ_R	0.555455	0.6427	0.69883
ϵ	5.56305	5.88184	6.15413
ϕ	0.101903	0.353194	0.821647
π	1.00467	1.00515	1.00584
ρ_A	0.672021	0.748399	0.791278
ρ_G	0.719414	0.759741	0.789415
ρ_U	0.519773	0.59543	0.648594
σ_A	0.00337889	0.00437111	0.00567692
σ_G	0.00437548	0.00484554	0.00545708
σ_M	0.00336325	0.0043778	0.00548099
σ_U	0.0038067	0.0049121	0.00665101
l	0.0439899	0.202102	0.405959
γ	0.0383722	0.194152	0.348914

Table 17: Estimates for the Baseline sample (1966Q1-2014Q3), using gaussian and non-gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

Parameter	$5\%CI$	Mean	$95\%CI$
β	0.99569	0.9962	0.9967
θ	0.7462	0.7864	0.8190
ϕ_y	0.17864	0.2308	0.2727
ϕ_π	2.2062	2.5943	2.90280
ρ_r	0.3969	0.4664	0.5124
ϵ	6.2492	6.6182	7.1278
ϕ	0.5348	1.6724	3.07298
π	1.0062	1.0064	1.00679
ρ_A	0.7309	0.8182	0.9135
ρ_G	0.7257	0.7617	0.8020
ρ_U	0.5355	0.6251	0.71412
σ_A	;0.0039	0.0048	0.00587
σ_G	0.0034	0.0038	0.00441
σ_M	0.00314	0.0036	0.00428
σ_U	0.00404	0.0052	0.00680
l	0.5625	0.7337	0.9362
γ	0.0226	0.1072	0.2849

Table 18: Estimates for the Great Moderation sub-sample (1983Q1-2008Q3), using gaussian and non-gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

Parameter	$5\%CI$	Mean	$95\%CI$
β	0.995834	0.996229	0.996659
θ	0.763125	0.794579	0.84167
ϕ_y	0.123546	0.178209	0.232961
ϕ_π	1.79105	2.20896	2.62851
ρ_r	0.606357	0.694911	0.790621
ϵ	5.07277	5.3655	5.68538
ϕ	0.152892	0.355819	0.806334
π	1.00274	1.00314	1.00371
ρ_A	0.678949	0.796617	0.861448
ρ_G	0.738915	0.761976	0.803161
ρ_U	0.728303	0.84731	0.919905
σ_A	0.00349434	0.00444531	0.00644926
σ_G	0.00387025	0.00477542	0.00580725
σ_M	0.00317167	0.00445699	0.00583731
σ_U	0.00435183	0.00615152	0.00815232
l	0.0148726	0.204401	0.416093
γ	0.00826553	0.176484	0.317272

Table 19: Estimates for the Great Moderation +ZLB sub-sample (1983Q1-2014Q3), using gaussian and non gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

Parameter	$5\%CI$	Mean	$95\%CI$
β	0.9963	0.9968	0.9972
θ	0.6779	0.7133	0.7623
ϕ_y	0.1258	0.1706	0.2181
ϕ_π	1.3898	1.6264	2.0326
ρ_r	0.6158	0.7225	0.8099
ϵ	5.7602	6.1574	6.4772
ϕ	0.0867	0.2295	0.7533
π	1.0023	1.0028	1.0036
ρ_A	0.5577	0.6525	0.8684
ρ_G	0.7107	0.7650	0.7931
ρ_U	0.8613	0.9393	0.9914
σ_A	0.0042	0.0060	0.0076
σ_G	0.00280	0.0035	0.005
σ_M	0.0037	0.0046	0.0056
σ_U	0.0037	0.0048	0.0060
l	0.0139	0.1085	0.3500
γ	0.0292	0.1361	0.3746

Table 20: Estimates Great Volatility II sub-sample (2001Q1-2014Q3), using gaussian and non gaussian moments, ABC-SMC 10th iteration posterior mean and 5% and 95% credible interval values.

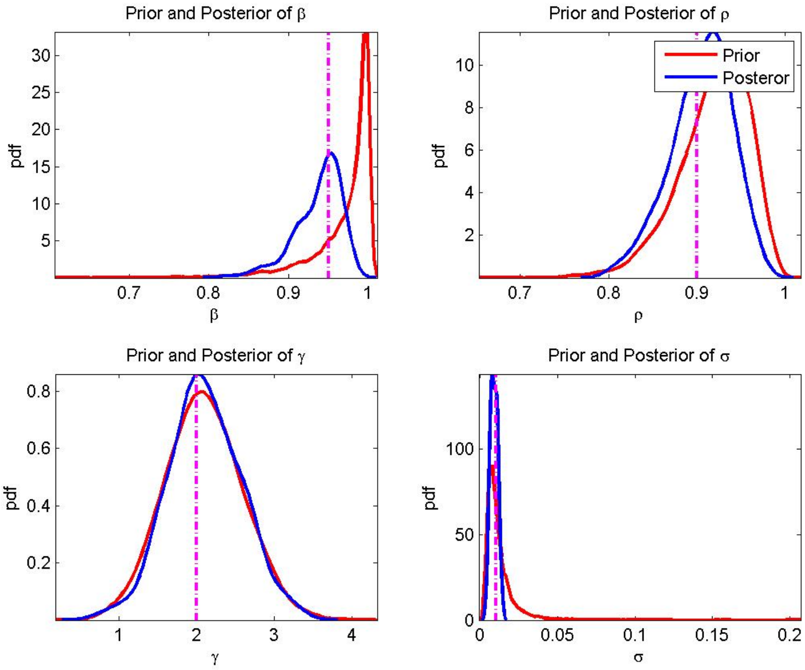


Figure 1: Marginal prior and posterior distributions for the estimated parameters with ABC-rejection, 200 observations, 5000 simulations, 5% of acceptance rate.

Figures

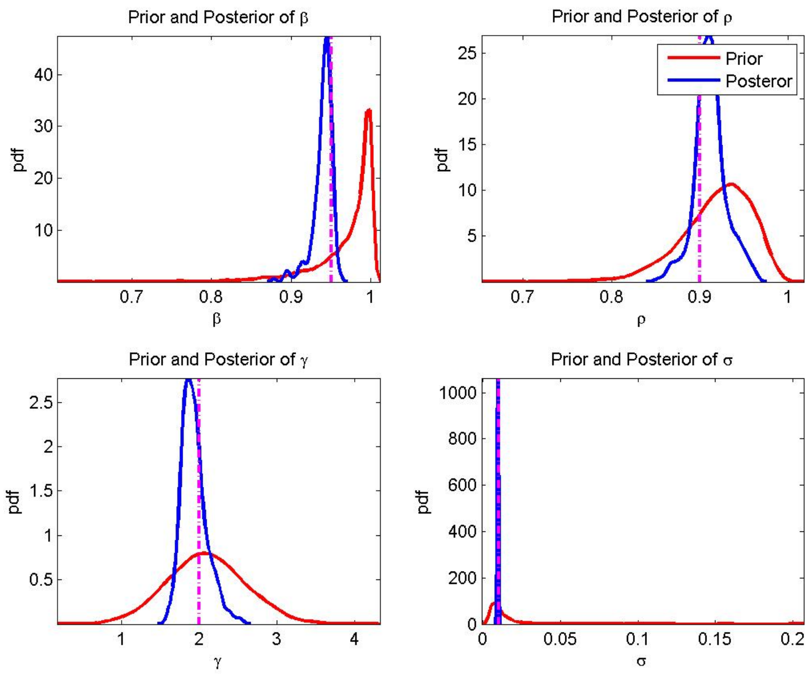


Figure 2: Marginal prior and posterior distributions for the estimated parameters with ABC-regression, 200 observations, 5000 simulations, 5% of acceptance rate.

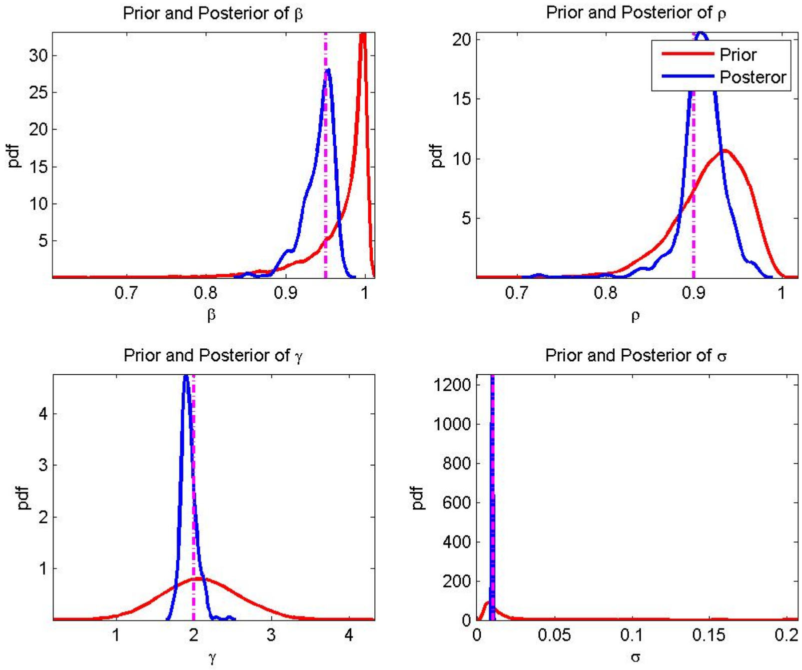


Figure 3: Marginal prior and posterior distributions for the estimated parameters with ABC-regression+HC, 200 observations, 5000 simulations, 5% of acceptance rate.

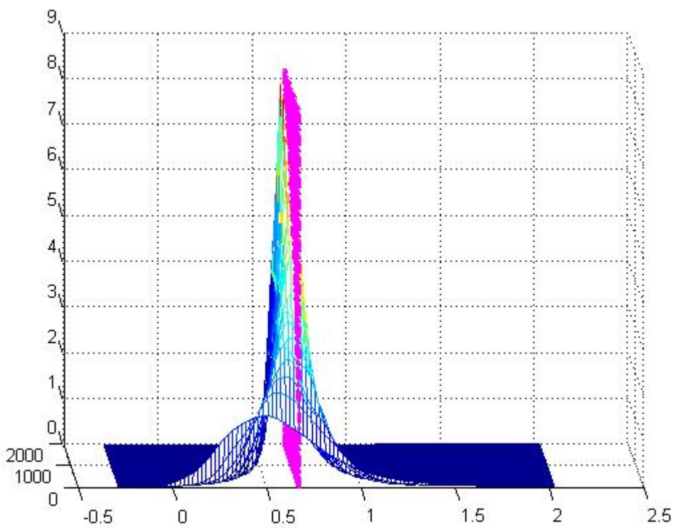


Figure 4: Distribution of the sample autocovariance for an AR(1) process with $\phi = 0.50$ for different sample sizes: from 50 to 2000 observations. The pink plane represents the population autocovariance.

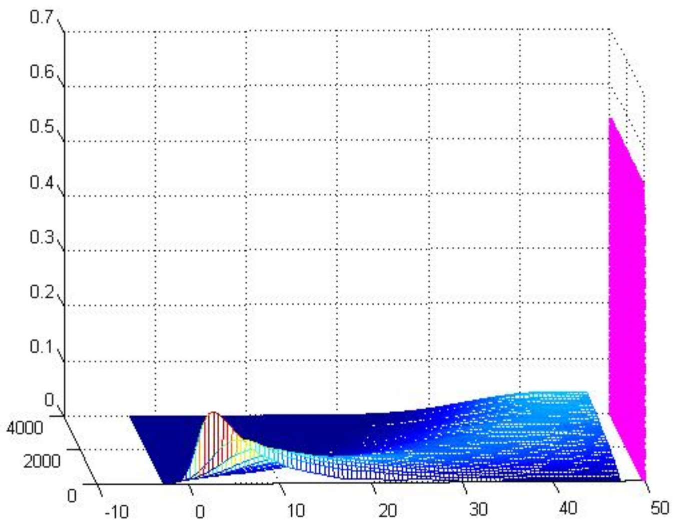


Figure 5: Distribution of the sample autocovariance for an AR(1) process with $\phi = 0.99$ for different sample sizes: from 50 to 4000 observations. The pink plane represents the population autocovariance.

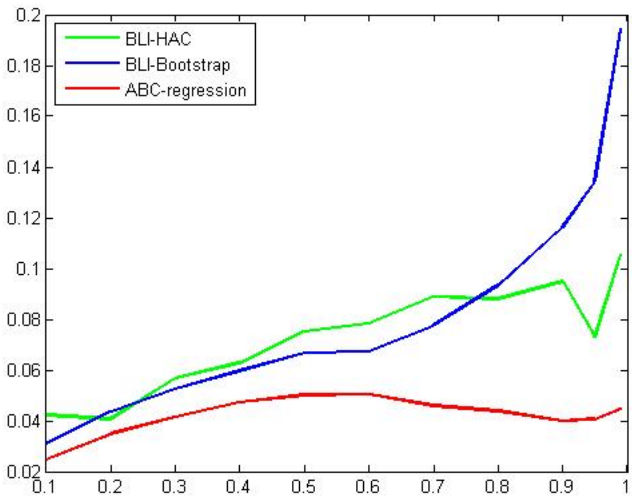


Figure 6: RMSE of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=100. Different autocorrelations on the horizontal axis.

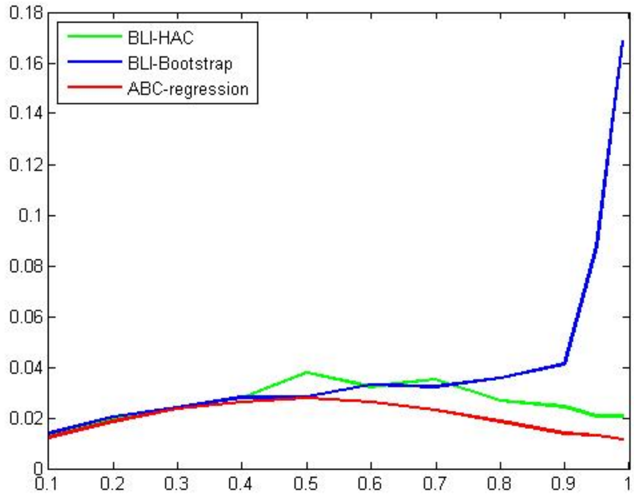


Figure 7: RMSE of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=300. Different autocorrelations on the horizontal axis.

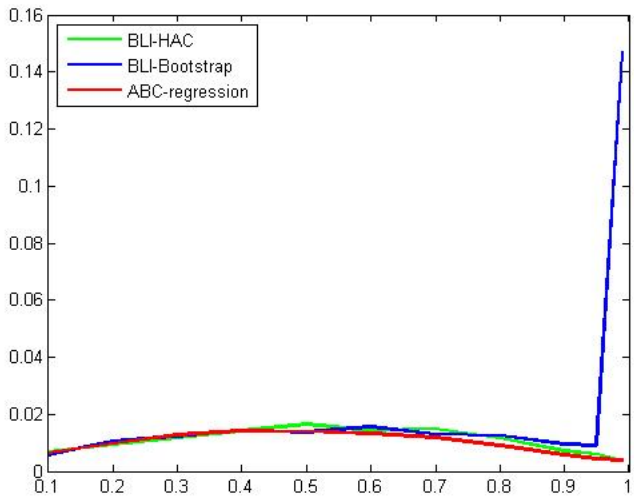


Figure 8: RMSE of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=1000. Different autocorrelations on the horizontal axis.

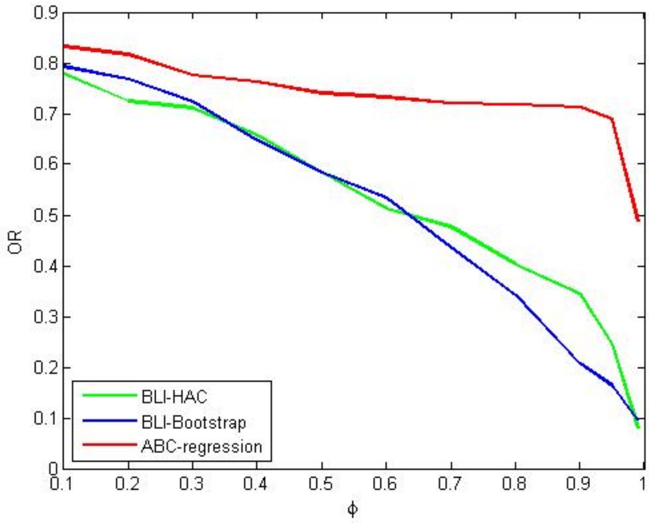


Figure 9: Overlapping Ratios of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=100. Different autocorrelations on the horizontal axis.

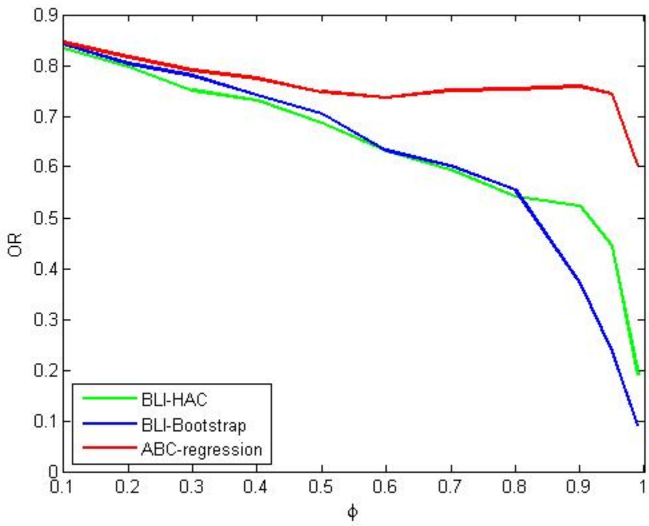


Figure 10: Overlapping Ratios of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=300. Different autocorrelations on the horizontal axis.

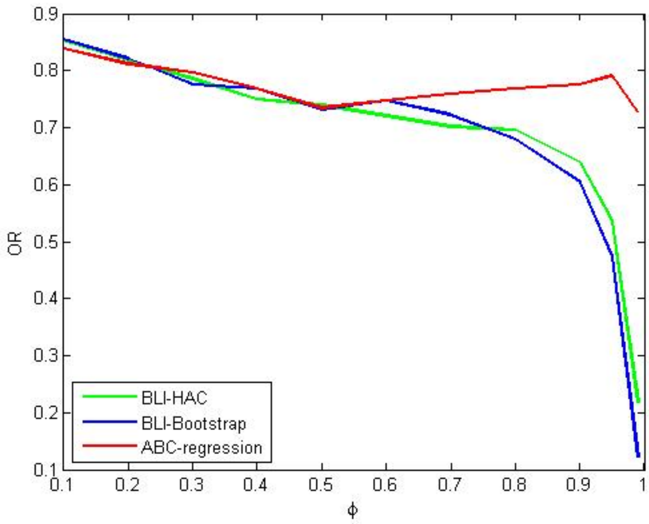


Figure 11: Overlapping Ratios of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=1000. Different autocorrelations on the horizontal axis.

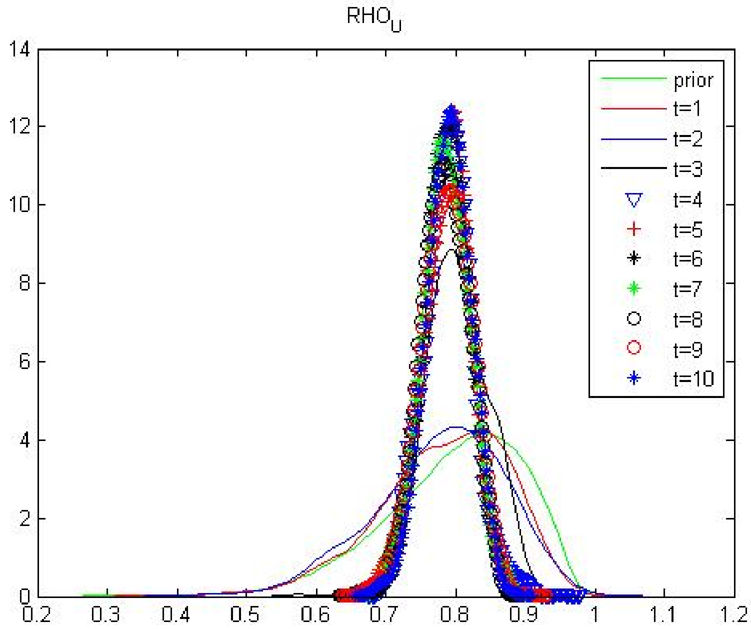


Figure 12: Approximate posterior distributions obtained for the first 10 iterations of the ABC-SMC for the parameter ρ_U in an estimation exercise (Estimation of the subsample Great Moderation + ZLB, using gaussian moments.

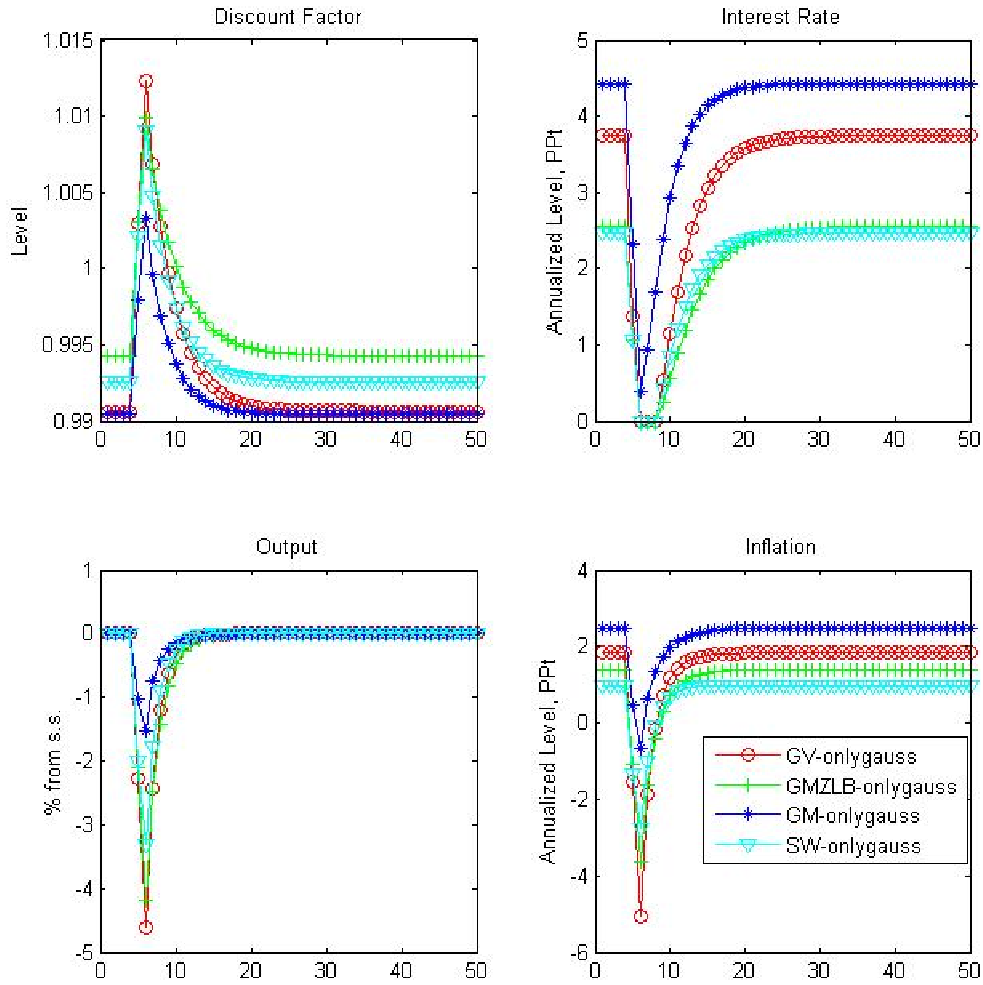


Figure 13: 2-standard deviation Impulse responses of preference shock for 4 different sub-samples: Baseline (SW), Great Moderation (GM), Great Moderation+Zero Lower Bound (GM+ZLB), Great Volatility-II (GV). ABC-SMC is performed using only gaussian moments. Priors and 10-th iteration approximate posteriors are reported.

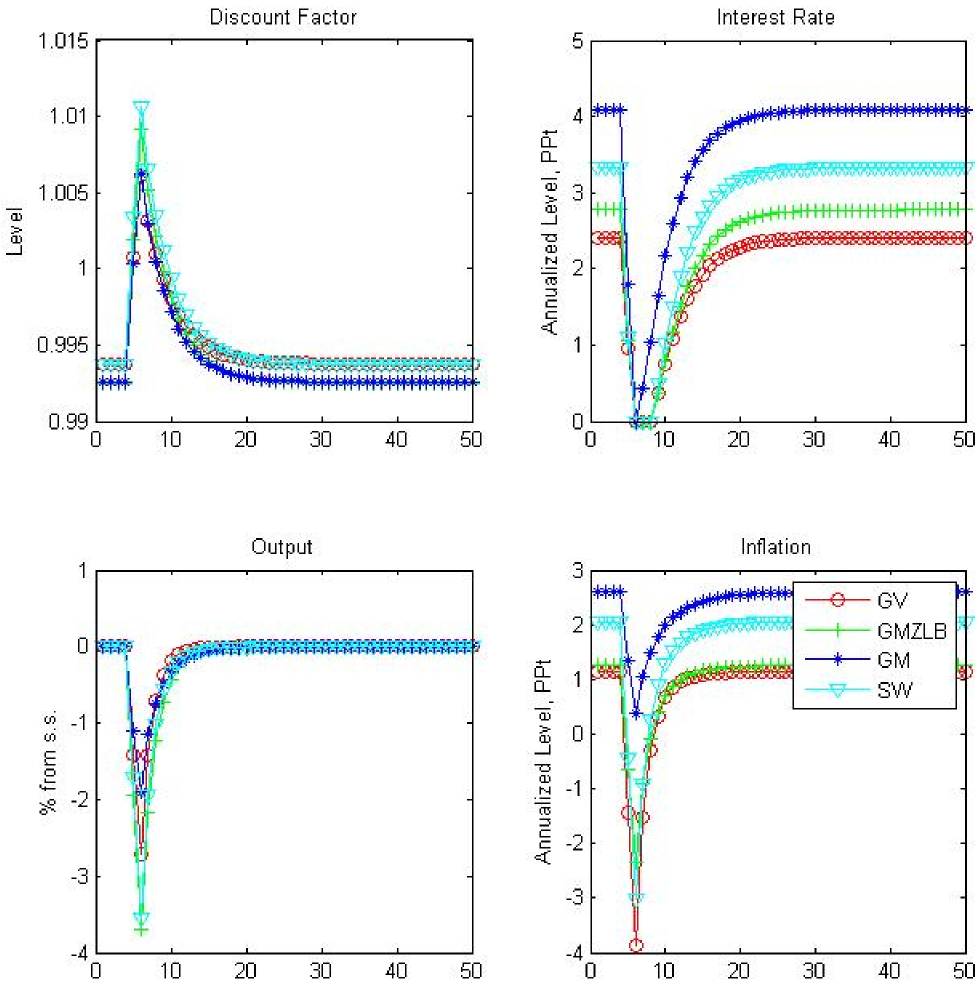


Figure 14: 2-standard deviation Impulse responses of preference shock for 4 different sub-samples: Baseline (SW), Great Moderation (GM), Great Moderation+Zero Lower Bound (GM+ZLB), Great Volatility-II (GV). ABC-SMC is performed using gaussian and non-gaussian moments. Priors and 10-th iteration approximate posteriors are reported.

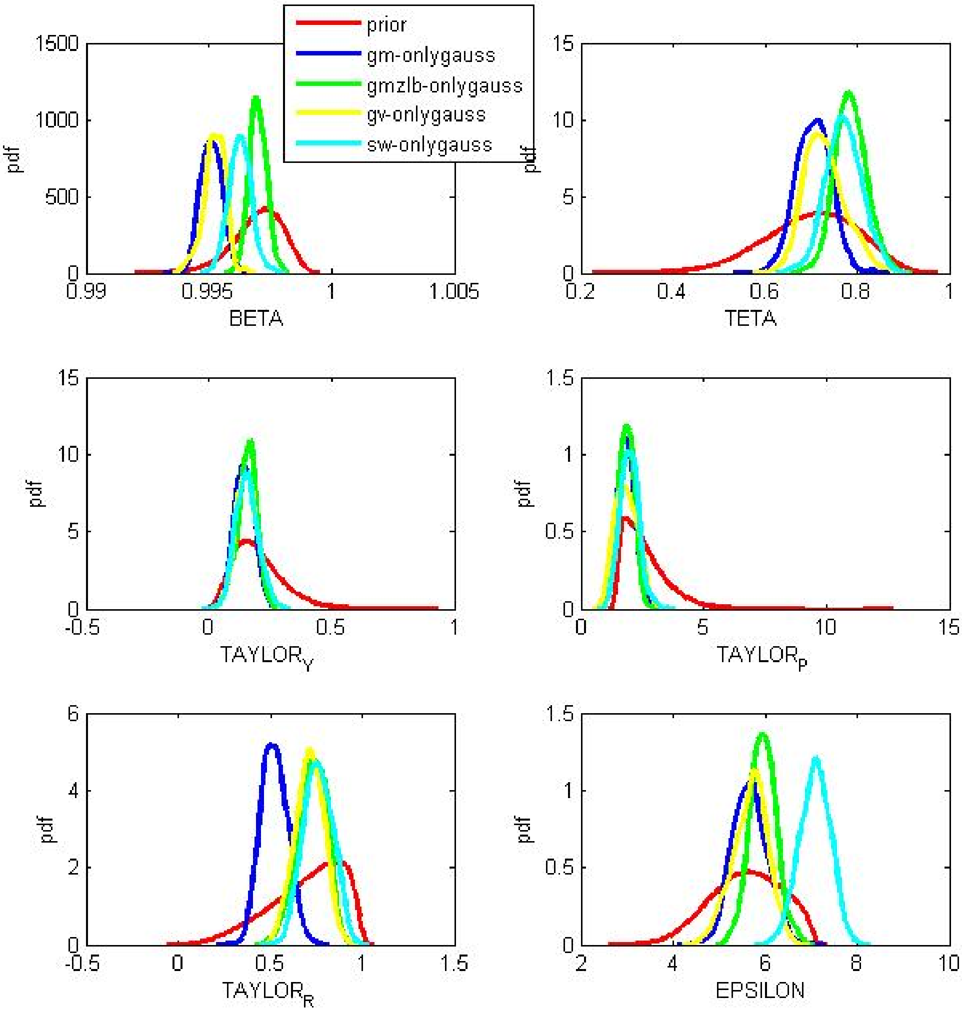


Figure 15: Prior and posterior distributions for the newkeynesian model for 4 different sub-samples: Baseline (SW), Great Moderation (GM), Great Moderation+Zero Lower Bound (GM+ZLB), Great Volatility-II (GV). ABC-SMC is performed using only gaussian moments. Priors and 10-th iteration approximate posteriors are reported

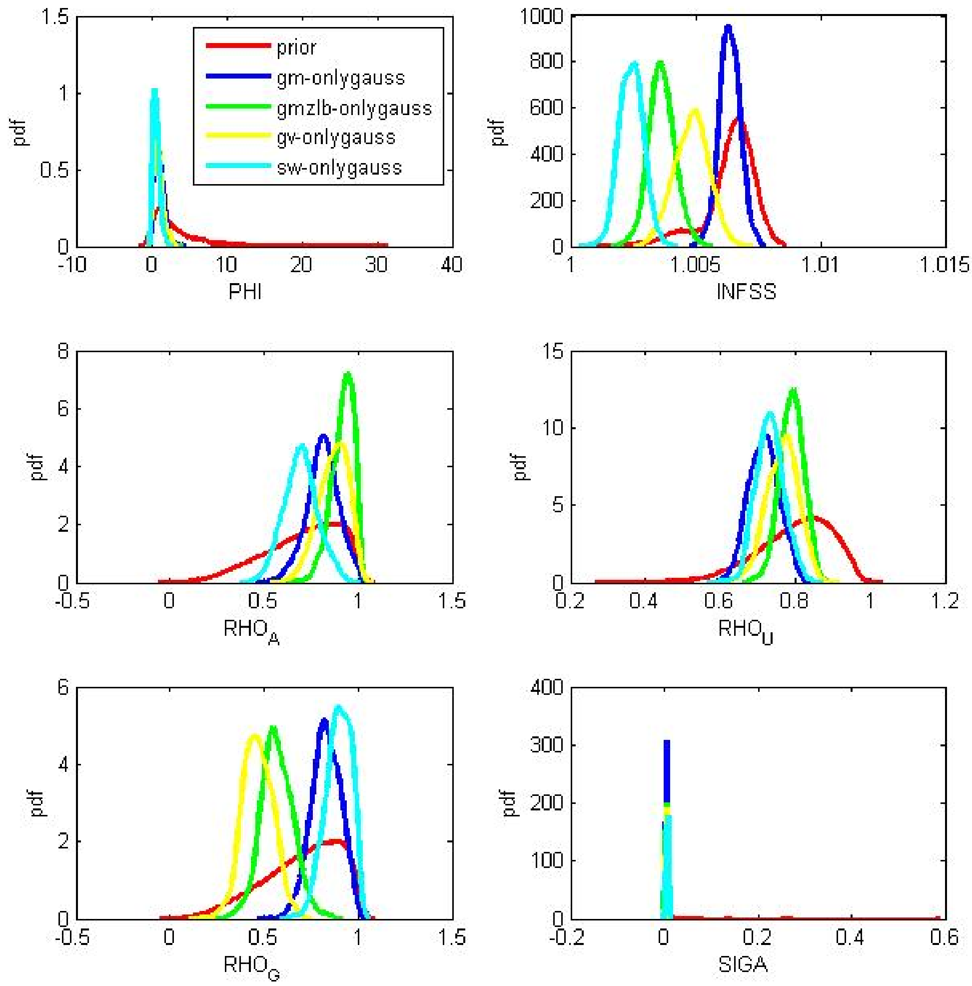
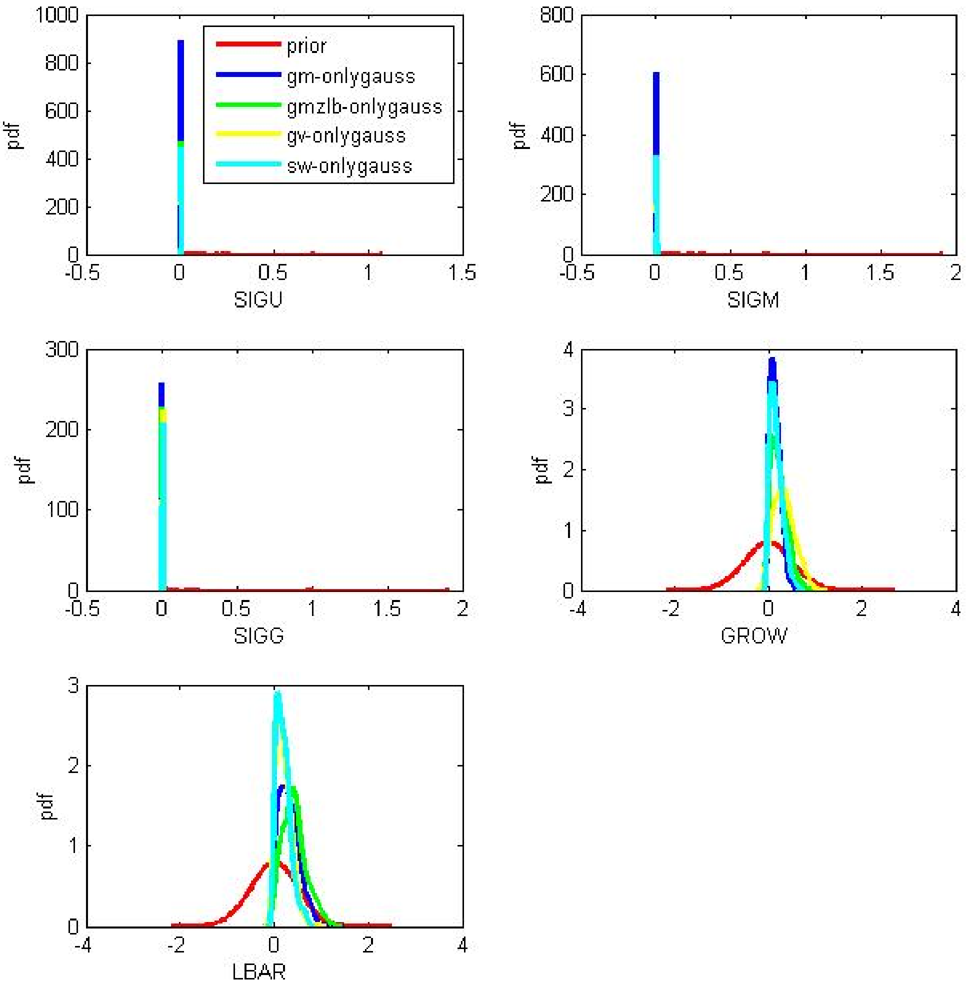


Figure 16: Prior and posterior distributions for the newkeynesian model for 4 different sub-samples: Baseline (SW), Great Moderation (GM), Great Moderation+Zero Lower Bound (GM+ZLB), Great Volatility-II (GV). ABC-SMC is performed using only gaussian moments. Priors and 10-th iteration approximate posteriors are reported



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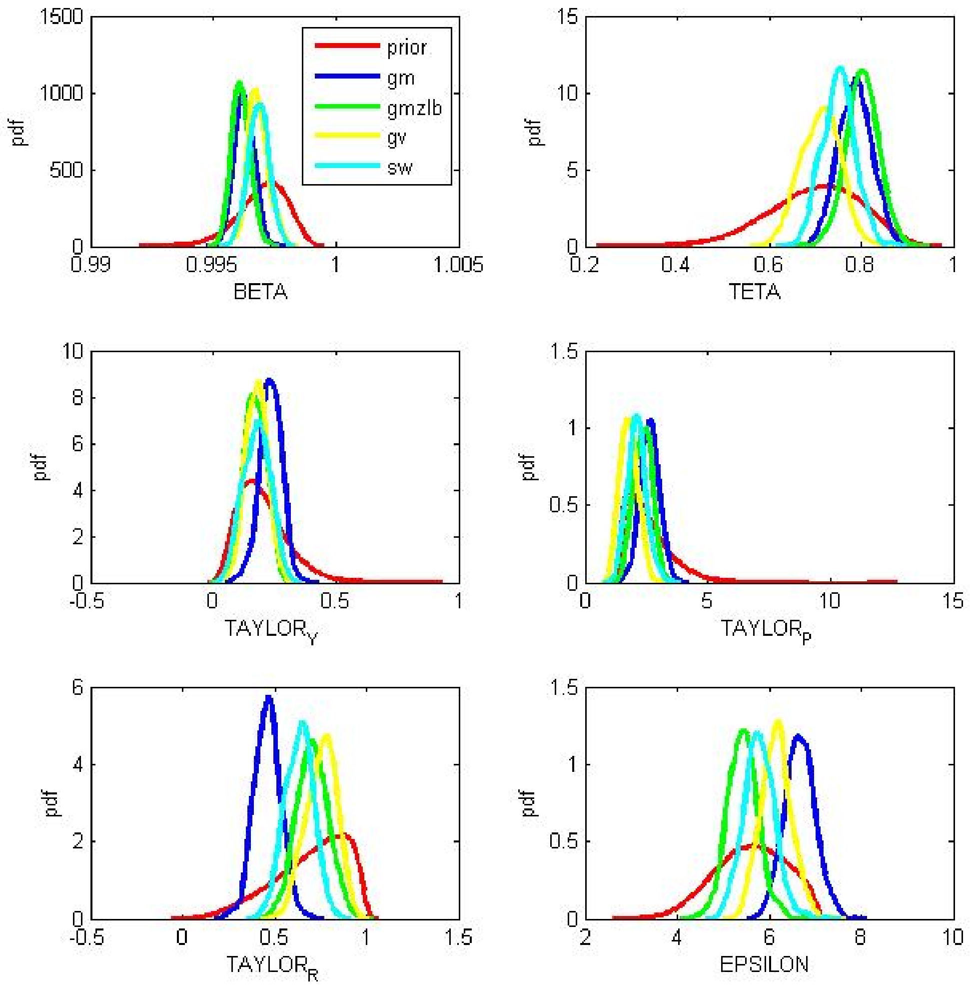

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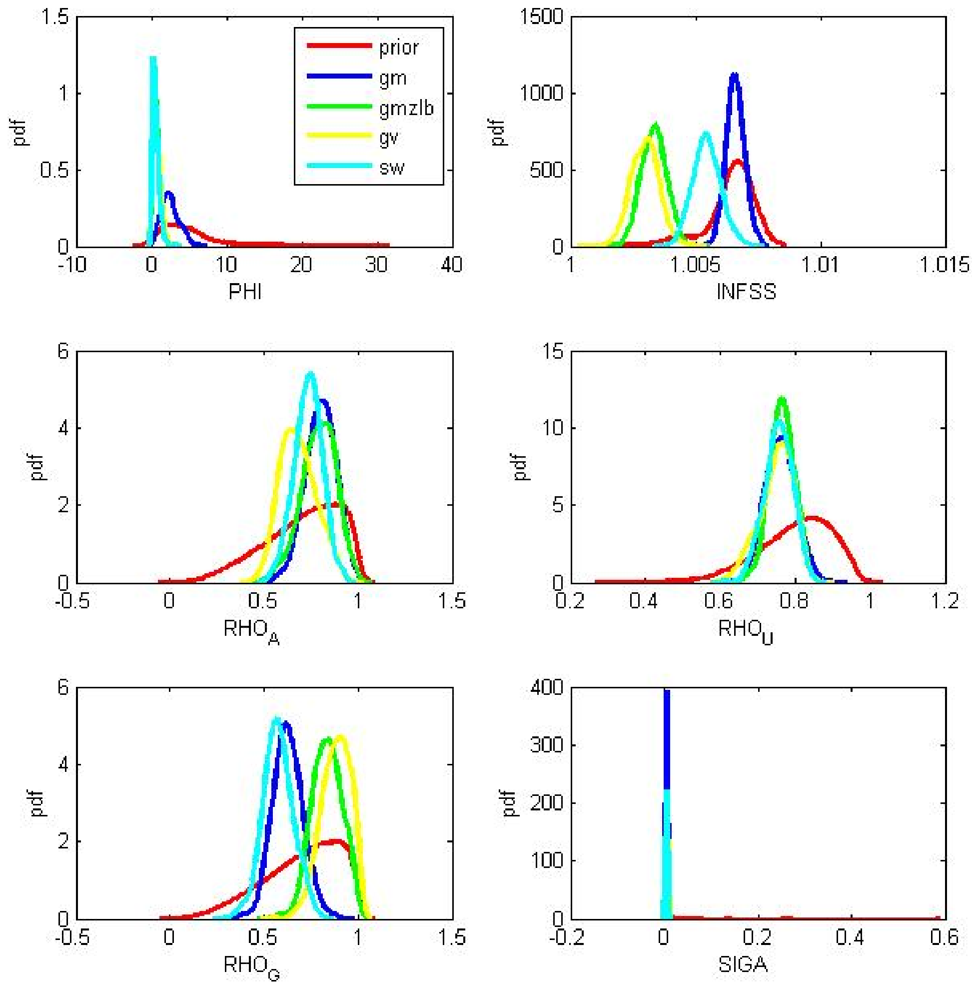

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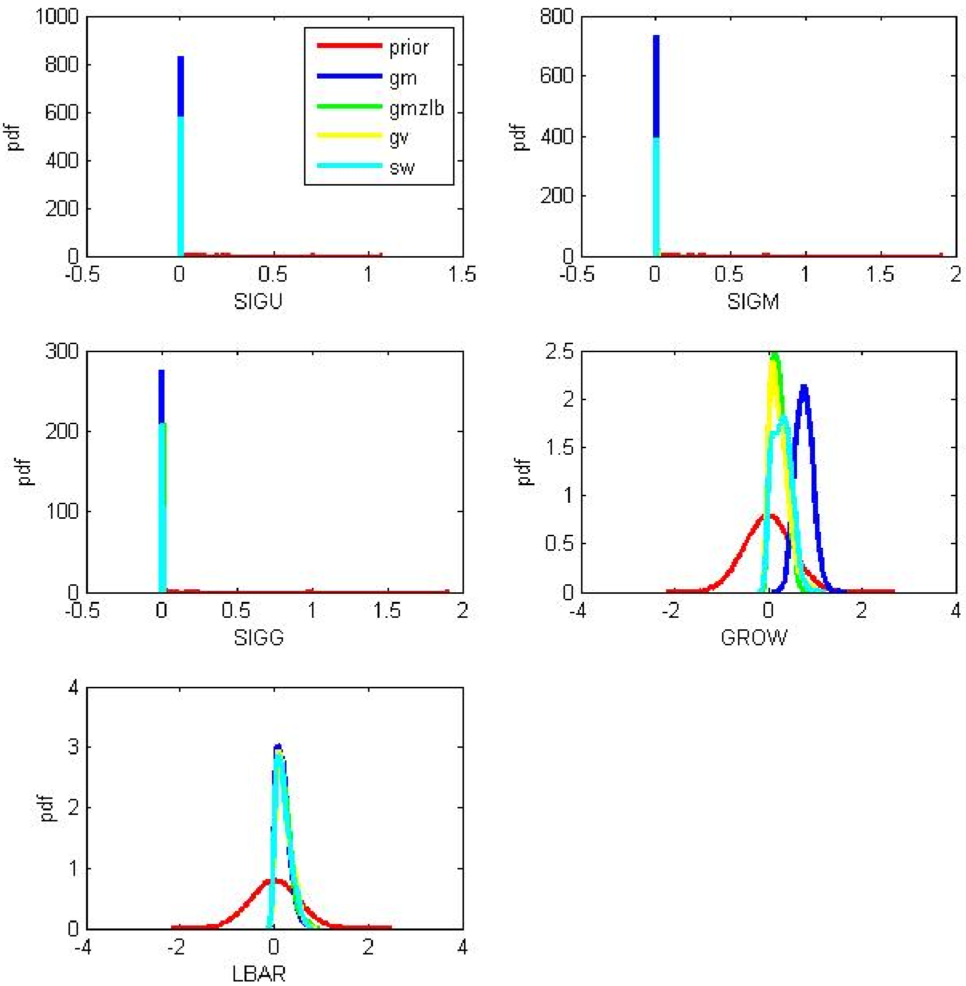

Figure 20: Prior and posterior distributions for the newkeynesian model with for 4 different sub-samples: Baseline (SW), Great Moderation (GM), Great Moderation+Zero Lower Bound (GM+ZLB), Great Volatility-II (GV). ABC-SMC is performed using gaussian and non-gaussian moments. Priors and 10-th iteration approximate posteriors are reported

Housing driven Growth: does it really exist?

Valerio Scalone

May 11, 2015

Abstract

In many economies' recent experiences, housing market volatile fluctuations have been blamed as responsible for driving or at least influencing the trend at which economies were growing (US, Japan and Spain to mention a few).
This paper inquires on the possibility that houses, playing the double role of durable consumption good and collateral, can affect the growth trend at which an economy grows. This is done through the study of a medium scale DSGE model with heterogeneous agents and endogenous growth where housing prices fluctuations influence the households' investment in technology, with a final effect on the growth trend. It turns out that against the general wisdom, an exogenous increase in the appetite for housing generates a temporary decline in the growth trend. Conversely, the temporary relaxation of the borrowing constraints for debtors is able to generate the positive comovement between housing prices and growth trend observed in the last twenty years across the developed economies. The more indebted the economy, the larger the degree of exposure to this type of fluctuations will be.

1 Introduction

2008-2009 Financial crisis marked the end of the Great Moderation era, a period of macroeconomic stability, sustained growth and stable inflation. The slow recovery that followed on both sides of the Atlantic pushed economists to inquire on the possibility that major developed economies had slipped in a Secular Stagnation trap, a period of depressed growth and low interest rates. These fears added to the acknowledgement that Japan's malaise was persisting after more than two decades from the housing bubble bust that precipitated it into the economic depression.

As a matter of fact, in a good fraction of economic chronicles, housing market boom-bustic cycles have been blamed as one of the possible initial triggers for financial crisis and economic slowdowns. Let aside the effects on short-term business cycles, a large part of economic chroniclers considers housing market fluctuations as responsible for driving or at least influencing medium-term macroeconomic fluctuations: periods of sustained growth would be accompanied by expansion in the nominal value of housing, alternating with periods of depressed growth marked by depressed nominal value of housing. This general wisdom lies on the foundations that houses are the bulk of collateral provided by borrowers to savers and that leveraging and deleveraging phases have an effect on the pace through which developed (and often highly indebted) economies grow (Japan, US and Spain's economies just to mention a few).

Until recently, DSGE models limited themselves to the analyse business cycles meant as fluctuations caused by economic shocks hitting the economy and temporarily moving the economy form the steady state growth path which was exogenously determined. Only recently pioneering works by Comin and Gertler (2006), Bianchi and Kung (2014) and Guerron-Quintana and Jinnay (2014) tried to reconcile the business cycle fluctuations with an endogenously varying technology growth.

The goal of this paper in to inquire on the role of the collateral fluctuations in affecting technology growth (i.e. the whole economy growth trend).

I ask if nominal collateral fluctuations have a role in influencing households decisions and namely investments in technology, identifying what could be the possible shocks and the pass-through creating a "collateral effect" on growth. I also ask how do the quantity (how much debt) and the quality (in terms of average duration) of debt affect this result.

To answer the question, a standard newkeynesian DSGE model with monopolistic competition and nominal frictions on prices and wages is built on Guerrieri and Iacoviello

(2014). The model features housing with the double role of consumption good and collateral; two heterogeneous agents (savers and borrowers) and a borrowing constraint on debt with a borrowing limit depending on the nominal value of collateral. I complement this model with a technology sector on the line of Bianchi and Kung (2014) where investment decisions in technology affect the technology accumulation (i.e. the growth trend of the economy).

This framework allows to identify the effects on technology growth coming from different types of shocks related to the housing market fluctuations.

In particular I assess the macroeconomic effects of three types of shocks:

- An housing demand shock increasing the marginal utility of housing consumption for both types of agents;
- An housing borrowers' demand shock increasing the marginal utility of housing consumption only for borrowers;
- A Loan to Value shock, temporarily expanding the borrowing constraints limit.

From the linear solution and the calibration some preliminary conclusions can be drawn.[1]

An expansionary housing shock (mimicking an housing market euphoria where housing demand exogenously increases) has a negative effect on technology investments: an appetite for houses makes the savers decreasing investments in technology and capital in favour of new housing spending.

An expansionary housing borrowers' demand shock has also negative effects on the growth trend due to the fact that an increase in housing from borrowers allows them to ask for more loans: the savers will lend theirs savings to borrowers and decrease investments in technology (lowering the economic growth trend).

Instead, an expansionary temporary increase in the LTV ratio increases the borrowers' housing consumption and housing prices, it further expands debt because of increase in

[1]In upcoming versions of the paper, non-linear solution will be performed using the Piecewise Linear solution method by Guerrieri and Iacoiello (2015) and estimation will be performed via Approximate Bayesian Computation methodology.

collateral nominal value. Being the housing prices higher than the steady state, it discourages savers from buying houses and makes them allocating their savings more on technology and physical capital investments, causing an increase in the growth trend of the economy.

In a sensitivity exercise, I inquire on the effects of these shocks when the share of borrowers is larger so to increase the debt of 70% with respect to the baseline scenario. It turns out that a larger debt increases the effect played by the LTV shock while reducing the effect of the housing shock: with a larger debt the increase in housing prices will be smaller if it comes from an increase in housing demand and larger if it is due to a relaxation of the borrowing constraints. In the fist case, crowding out of investments in technology in favour of housing spending will be smaller, while in the second case, savers will be pushed to increase their investments in technology.

As a final exercise, I study what happens if the average duration of debt is lower causing a larger sensitivity of debt to collateral values fluctuations, something that can be thought also as a more frequent rolling over of debt. This amplifies the effects of the LTV shock too, while keeping the effects of the housing shock substantially unchanged. Summing up, according to the model, positive comovements in housing and the economy medium term fluctuations can be explained more by a temporary relaxation of the borrowing constraints (a larger LTV ratio) rather than an exogenous housing market euphoria increasing the appetite of households for houses. The more indebted the economy and the more fragile its position in terms of average duration, the stronger the result.

To best of my knowledge, this is the first paper trying to assess a theoretical (and empirical after the estimation exercise will be run) link between medium-term housing market fluctuations and evolution of the growth trend of the economy, though the lens of a DSGE model. Non linear solution and non linear estimation will contribute to shed some light on the relative magnitude of these effects.

The paper is structured as follows. A literature review is housed in section 2 . In Section 3 the model is presented. The preliminary results are exposed in Section 4. Section 5 concludes. The stationary equations of the model are found in the Appendix.

2 Literature

This paper is related to three main strands of literature.

The recent financial crisis sparkled new interest for the role of financial frictions. Concerning this topic, Guerrieri and Iacoviello 2014 build a model with heterogeneous agents with occasionally binding borrowing constraints for the borrowers. An exogenous reduction in collateral demand lowers the borrowing limit for borrowers, forcing them to reduce consumption and borrowing. The increase of consumption of patients does not compensate for the reduction by the inpatients bringing a slowdown in overall consumption. Interestingly, the presence of the occasionally binding borrowing constraint ad a non-linear solution method deliver an asymmetric effect of collateral nominal value variation, where an increase does not have the same effects in absolute terms than does a decrease. I complement their model with a vertical innovation sector creating a link between collateral effects and growth trend. This allows to inquire about the role played by collateral demand in affecting medium term fluctuations and to check for any asymmetric relations among the collateral nominal price variations and growth trends.

Mendoza (2010) builds a SOE-DSGE model with occasionally binding constraint on borrowing to model the sudden stop and their asymmetric behaviour on the emerging markets' economies.

Guerron-Quintana and Jinnay (2014) implement a RBC with Kyotaki-Moore constraints on the entrepreneurs. Their model incorporates a technology sector with horizontal innovations. Liquidity shocks hitting entrepreneur's constraint limit her investments in innovation and lower the technology growth trend. Estimation is performed on the set of dynamic parameters. According to their analysis, liquidity shocks limiting the borrowing capacity of entrepreneurs had a crucial role in determining the 2008-2009 economic slowdown. With respect to their work, this paper presents the following contributions: it incorporates nominal frictions on prices and wages creating involuntary unemployment and incorporates a Taylor rule. Besides, this paper assigns a role to housing allowing to disentangle the source of the borrowing limit variations (collateral demand variations, LTV shocks).[2].

Christiano, Eichnebaum and Trabandt (2014) incorporate financial frictions into a standard DSGE model with nominal frictions and endogenous labour force supply. They estimate the model using pre-2008 data. According to their model a financial wedge and a preference shock represent the bulk of the causes determining the Great Recession.

More in general Del Negro et al. (2010), Jerman and Quadrini (2009), and Christiano Motto and Rostagno (2010) highlight the role of financial frictions as one of the main drivers of the Great Recession.

The paper adds to the stream of literature trying to reconcile growth and business cycle. Bianchi and Kung (2014) build a standard DSGE model with nominal frictions and monopolistic competition of the intermediate sector adding a technology sector with vertical innovation and utilization rate. In Bianchi and Kung's paper the model is solved linearly and estimated. The Marginal efficiency investment shock (MEI) is found to have the main role in leading both the business cycle and growth. The MEI shock already identified by Justiniano Primiceri and Tambalotti (2011) as one of the major sources of fluctuations is positively correlated with the Credit spread, measured as the difference between the high yield and the AAA corporate bond. Two main events are analysed through the lens of the model: the 00's Great Recession and the 70's Great Inflation. In the first, the technology investment, i.e the trend, is not as affected as in the IT bubble bust period. In the 70's, high inflation is explained also due to shocks lowering the technology investments. With respect to Bianchi and Kung, this paper complements the model with heterogeneous agents and a borrowing constraint depending on the nominal houses value. This allows to identify the effect the debt dynamics and houses fluctuations have on growth. [3]

[2]Moreover, the non-linear solution of the model will allow to determine the asymmetric effects of the financial frictions on growth and the interaction of the borrowing constraints limit dynamics with the ZLB

[3]The non-linear solution and the occasionally binding constraints on borrowing and the ZLB will also add as contribution to Bianchi and Kung (2014) allowing to quantify the role of the ZLB in lowering growth. This could make the model fit to explain Japan's housing bubble bust and weak growth history.

Other papers trying to reconcile business cycle and growth are Comin and Gertler 2006, where the authors detect medium-term fluctuations of output, productivity and productivity utilization rate featured by a pro-cyclicality relation. They integrate an RBC model with a technology sector with horizontal innovations. Their model is able to generate fluctuations similar to the ones detected in the empirical part of their paper.

In macrofinance, Kung and Schmidt (2013) adopt a DSGE model with long-term endogenous growth where recursive Epstein-Zin preferences make the agents caring for the long-term prospects of growth, explaining the high equity premium and the low and stable risk-free rate. Kung (2014) explain the term structure adopting a model in which endogenous growth wage mark up shocks generate a negative relation between inflation and output movement. An interaction with the monetary policy setting the interest rate via a Taylor rule creates the conditions to explain the term structure dynamics and its interaction in periods of weak versus sustained growth.

The model, particularly in its non-linear future version, is also related to the stream of literature that is trying to get some insight about the secular stagnation and the persistent period of weak growth featured in Japan in the Euro-area and until recently in the US. In particular, this paper will try to shed some light on the empirical relation existing between the zero lower bound and the prolonged weak growth periods.

Eggertson and Mehrotra (2014) build a three generations overlapping generations model where a debt-delevraging shock can lead to a permanent or very persistent period of stagnation, essentially due to an overbalance of saving over demand for loans pushing the necessary interest rate below the ZLB.

Benigno and Fornaro (2015) build a model capable of generating a permanent stagnation trap where low demand expectations push firms to decrease technology investment pushing the economy into a weak economy equilibrium. Liquidity trap and the ZLB exacerbate the reduction in demand and the consequent growth trend reduction. This paper, in the future non-linear version, will try to build an empirical DSGE mimicking a similar interaction between the inability of the monetary policy to offset consumption reduction and lower technology investments bringing to a reduction of the growth trend.

3 The model

3.1 General features of the model.

The model is a standard DSGE model, with nominal frictions, two types of agents (patients and inpatients), collateral in the utility function, investments in technology affecting the TFP (i.e the trend).

Guerrieri and Iacoviello (2014) is the starting point for demand side. There are two types of agents: patient and inpatients households. They both have collateral in their utility function. Inpatient households are (occasionally) borrowing constrained.

The supply side is is similar to Primiceri and Tambalotti 2009, complemented with endogenous TFP on the line of Bianchi and Kung 2014. Standard monopolistic competition for the wholesale sector and Calvo frictions to set prices and wages are also introduced.

Central bank sets the interest rate and is subject to the ZLB.

In this model, borrowers are borrowing constrained in the steady state, they supply labour hours, consume goods, trade housing with the savers and borrow from them.

Savers supply labour hours, consume goods and trade houses with borrowers, lend to borrowers. They accumulate capital and technology.

An increase in the housing detained by borrowers allows them to expand their debt and consume more. Conversely, when savers increase their share of houses they will also decrease investments in technology (and productivity).

The core mechanism of the model that links housing and technology growth concerns the way savers allocate their savings: a positive demand shock for housing pushes savers to buy more houses and invest less in technology. If the increase in the demand is exclusively due to borrowers, agents will increase their loans to the borrowers, decreasing investments in technology. Instead, if the borrowing conditions temporarily relax, the increase in housing prices due to a larger demand from borrowers, pushes savers to allocate their savings first in more loans. When the debt converges back to its steady value, being the housing prices larger, agents will invest the new available resources into technological investments.

3.2 The households

The patient and the inpatient households maximize the following respective utility functions:

$$E_0 \sum_{t=0}^{\infty} \beta^t z_t \left(\Gamma log\left(C_t - \epsilon C_{t-1}\right) + j_t log H_t - \frac{1}{1+\eta} n_t^{1+\eta} \right) \tag{1}$$

$$E_0 \sum_{t=0}^{\infty} \beta'^t z_t \left(\Gamma' log\left(C_t' - \epsilon C_{t-1}'\right) + j_t jb_t log H_t' - \frac{1}{1+\eta} n_t'^{1+\eta} \right) \tag{2}$$

where the prime symbol applies to inpatient households' variables. C_t is consumption, H_t is the collateral, and n_t are the hours worked. j_t is an AR(1) process propagating exogenous shocks to the demand for collateral, jb_t is an housing demand process for borrowers' housing demand only, z_t is the preference shock:

$$log(j_t) = (1-\rho_j)\bar{j} + \rho_j log(j_{t-1}) + u_{j,t} \tag{3}$$

$$log(jb_t) = (1-\rho_j)\bar{j} + \rho_{jb} log(jb_{t-1}) + u_{j,t} \tag{4}$$

$$log(z_t) = \rho_z log(z_{t-1}) + u_{z,t} \tag{5}$$

where $u_{j,t}$, $u_{jb,t}$ and z_t are nid shocks with variances σ_j^2, σ_{jb}^2 and σ_z^2.
The scaling factors $\Gamma = \left(\frac{\mu-\epsilon}{\mu-\beta\mu\epsilon}\right)$ and $\Gamma' = \left(\frac{\mu-\epsilon}{\mu-\beta'\mu\epsilon}\right)$; imply that in steady state the marginal utilities for consumption are equal to: $1/\bar{c}$ and $1/\bar{c}'$[4].

[4] μ is the steady state technology growth rate, i.e the steady state growth rate of the economy

The budget constraint for the patient agent is:

$$C_t+Q_tH_t+I_t+S_t+B_t = \frac{W_t n_t}{x_{w,t}}+Q_tH_{t-1}+[r_{K,t}u_{K,t}-a_K(u_{K,t})]\bar{K}_{t-1}+[r_{N,t}u_{N,t}-a_N(u_{N,t})]\bar{N}_{t-1}+\frac{R_{t-1}B_{t-1}}{\pi_t}+D \tag{6}$$

where $\bar{K}_t$ and $\bar{N}_t$ are the capital and the technology owned by the patient agent. $u_{K,t}$ and $u_{N,t}$ are the utilization rates for capital and technology. $K_t = u_{K,t}\bar{K}_{t-1}$ and $N_t = u_{N,t}\bar{N}_{t-1}$ are the capital and technology services rent from firms. $r_{K,t}$ and $r_{N,t}$ are the rental rates for capital and technology services. w_t is the wage per hour worked and $x_{w,t}$ is the wage mark up due to monopolistic competition in the labour market, Q_t is the price of collateral, I_t and S_t are the investments in capital and technology. B_t are the loans made to the inpatient households, R_t is the interest rate set by the central bank. $a_K(u_{K,t}) = \frac{1}{\sigma_k}\left(u_{k,t}^2 - 1\right)$ and $a_N(u_{N,t}) = \frac{1}{\sigma_n}\left(u_{n,t}^2 - 1\right)$ are the maintenance costs for capital and technology services. Dividends DIV_t derive from the mark up applied by firms. $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation.
The patient agent cumulates capital and technology according to the following laws of motion:

$$\bar{K}_t = (1-\delta_k)\bar{K}_{t-1} + \xi_{I,t}\left(I_t - \psi_k\frac{(I_t - I_{t-1})^2}{\bar{I}}\right) \tag{7}$$

$$\bar{N}_t = (1-\delta_n)\bar{N}_{t-1} + \xi_{S,t}\left(S_t - \psi_n\frac{(S_t - S_{t-1})^2}{\bar{S}}\right) \tag{8}$$

$\xi_{I,t}$ is an AR(1) process capturing marginal efficiency investment shocks (MEI), identified by Justiniano et al. as one of the main shocks driving the business cycle, highly correlated with the credit spreads for firms, mimicking tensions in the financial sector. In the model the shock affects the efficiency through which investments are converted into capital. $\xi_{S,t}$ is the equivalent for the technology accumulation process.

$$log(\xi_{I,t}) = \rho_I log(\xi_{I,t-1}) + u_{I,t} \tag{9}$$

$$log(\xi_{S,t}) = \rho_S log(\xi_{S,t-1}) + u_{S,t} \tag{10}$$

where $u_{I,t}$ and $u_{S,t}$ are nid processes with variances σ_I^2 and σ_S^2.

The inpatient household does not accumulate capital or technology, she buys and sells collateral, consumes, works and borrows from the patient household. Her budget constraint is:

$$C_t^{'} + Q_t H_t^{'} + \frac{R_{t-1}B_{t-1}}{\pi_t} = w_t^{'} n_t^{'} + Q_t H_{t-1}^{'} + B_t \tag{11}$$

Inpatient households are subject to the occasionally binding borrowing constraint:

$$B_t \leq \gamma \frac{B_{t-1}}{\pi_t} + (1-\gamma) Ms_t Q_t H_t^{'} \tag{12}$$

where Ms_t is the process for the *loan to value* ratio with respect to the collateral owned by the borrower, γ is the inertia of the borrowing limit.

$$Ms_t = (1-\rho_{Ms})\, M + \rho_{Ms} Ms_{t-1} + u_{Ms,t} \tag{13}$$

with $u_{Ms,t} \sim N(0, \sigma_{Ms}^2$.

3.3 Firms

The final good is produced in a market with perfect competition. An homogeneous final good is assembled according to the CES technology function:

$$Y_t^d = \left(\int_0^1 Y_{j,t}^{\frac{\lambda_{p,t}-1}{\lambda_{p,t}}} \, dj \right)^{\frac{\lambda_{p,t}}{\lambda_{p,t}-1}}. \tag{14}$$

where Y_t^d is the final output demanded, $\lambda_{p,t}$ is the goods elasticity evolving according to:

$$\lambda_{p,t} = \lambda_p + u_{mup,t} \tag{15}$$

where λ_t is the steady state elasticity and $u_{mup,t} \sim N(0, \sigma^2_{mup}$ is the price mark up shock. Firms maximize their profits and obtain the following demand function:

$$Y_{j,t} = Y^d_t \left(\frac{P_t(j)}{P_t}\right)^{-\lambda_{p,t}}. \tag{16}$$

where P_t is the price of the final good and $P_t(j)$ is the price of the intermediate good. The price of the final good is obtained by:

$$P_t = \left[\int_0^1 P_t(j)^{1-\lambda_{p,t}} dj\right]^{\frac{1}{1-\lambda_{p,t}}}. \tag{17}$$

The intermediate firm j produces the good according to the following production function:

$$Y(j)_t = K(j)^\alpha_t \left(A_t N(j)^\eta_t N_t^{1-\eta} n_t^{(1-\sigma)} n_t'^{\sigma}\right)^{1-\alpha} - F\bar{N}_t \tag{18}$$

with $N_t = \int_0^1 N(j)dj$ is the aggregate stock of technology, $(1-\eta)$ is the degree of technological spillovers. The stationary technology evolves according to:

$$log(A_t) = (1-\rho_A)log(A^*) + \rho_A log(A_{t-1}) + u_{a,t} \tag{19}$$

with $u_{a,t} \sim N(0, \sigma_a^2)$. A^* is picked to matched the balanced growth evidence as in Bianchi and Kung (2014) and Kung (2014).

Intermediate firms can re-optimize the price according to a Calvo rule: each period $1-\theta_P$ firms can optimally reset their price. The remaining part of the firms can index their prices by past inflation. The degree of indexation is controlled by the parameter $\chi_p \in [0,1]$. The maximization problem faced by intermediate firms is the following:

$$maxE_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^\tau \frac{u_{c,t+\tau}}{u_{c,t}} \left\{ \left(\prod_{s=1}^{\tau} \Pi^\chi \frac{p_{i,t}}{p_{t+\tau}} - mc_{t+\tau} \right) Y_{i,t+\tau} \right\} \tag{20}$$

s.t.

$$Y_{i,t+\tau} = \left(\prod_{s=1}^{\tau} \Pi^{\chi}_{t+s-1} \frac{p_{i,t}}{p_{t+\tau}}\right)^{-\lambda_{p,t}} Y^d_{t+\tau} \tag{21}$$

To solve the infinite horizon problem of profits maximization above and make it recursive, two auxiliary variables are used: g^1_t g^2_t. In order to solve the problem the following law of motions are derived:

$$g^t_1 = u_{c,t} mc_t Y^d_t + \beta\theta_p \left(\frac{\Pi^{\chi}_t}{\Pi_{t+1}}\right)^{-\lambda_{p,t}} g^1_{t+1}, \tag{22}$$

$$g^2_t = u_{c,t}\Pi^* Y^d_t + \beta\theta_p E_t \left(\frac{\Pi^{\chi}_t}{\Pi_{t+1}}\right)^{1-\lambda_{p,t}} \left(\frac{\Pi^*_t}{\Pi^*_{t+1}}\right) g^2_{t+1} \tag{23}$$

where $\Pi^*_t = \frac{p^*_t}{p_t}$ and $\lambda_{p,t} g^1_t = (\lambda_{p,t} - 1) g^2_t$ The price index will be equal to:

$$p_t^{1-\lambda_{p,t}} = \theta_p \left(\Pi^{\chi}_{t-1}\right)^{1-\lambda_{p,t}} + (1-\theta_p) p_t^{*1-\lambda_{p,t}} \tag{24}$$

3.4 The labour market

Households supply homogeneous labour hours to an intermediate union sector. The union differentiates labour and resell it to the labour packers. The labour packers resell it to the firms.

Importantly, markets for patient and inpatients are segregated. The labour union have market power and can set the wages taking into account the labour demand function of the labour packers for both households independently.

The labour packers operate in perfect competition, reassemble labour to be used by the intermediate firms according to the following functions:

$$n^d_t = \left(\int_0^{\sigma} n(l)_t^{\frac{\lambda_{w,t}-1}{\lambda_{w,t}}} dl\right)^{\frac{\lambda_{w,t}}{\lambda_{w,t}-1}}, \tag{25}$$

$$n'^d_t = \left(\int_{\sigma}^{1} n(l)_t^{'\frac{\lambda_{w,t}-1}{\lambda_{w,t}}} dl\right)^{\frac{\lambda_{w,t}}{\lambda_{w,t}-1}}, \tag{26}$$

where n_t^d and $n_t^{'d}$ are the hours supplied by savers and borrowers, $\lambda_{w,t}$ is the labour elasticity evolving according to:

$$\lambda_{w,t} = \lambda_w + u_{mup,t} \tag{27}$$

where λ_w is the steady state elasticity and $u_{muw,t} \sim N(0, \sigma^2_{muw}$ is the wage mark up shock. The labour packers buy differentiated labour and resell it to the firms. Maximizing their profits, the following demand function for labour is obtained:

$$n(l)_t = \left(\frac{W(l)_t}{W_t}\right)^{-\lambda_{w,t}} n_t^d. \tag{28}$$

$$n^{'}(l)_t = \left(\frac{W^{'}(l)_t}{W_t^{'}}\right)^{-\lambda_{w,t}} n_t^{'d}. \tag{29}$$

Labour packers operate in perfect competition. Combining the zero profit condition with this demand functions, the following wages are obtained:

$$W_t = \left(\int_0^\sigma W_t(l)^{1-\lambda_{w,t}} dl\right)^{\frac{1}{1-\lambda_{w,t}}} \tag{30}$$

$$W_t^{'d} = \left(\int_\sigma^1 W_t(l)^{'1-\lambda_{w,t}} dl\right)^{\frac{1}{1-\lambda_{w,t}}}. \tag{31}$$

Labour unions differentiate the labour and set the wages taking into account the labour demand function of the labour packers. Their presence allows the households to obtain a mark up for both types of agents, over their desired wages W_t^h and $W_t^{'h}$, where:

$$W_t^h = \frac{u_{n,t}}{u_{c,t}}, \tag{32}$$

$$W_t^{'h} = \frac{u^{'}_{n,t}}{u^{'}_{c,t}}, \tag{33}$$

where $u_{c,t}$ and $u_{n,t}$ are the marginal utilities for consumption and labour.

Labour unions are subject to Calvo frictions when setting the wages: θ_w can optimally reset wages and $1-\theta_w$ can partially index their wage by past indexation. The parameter $\chi_w \in [0,1]$ controls the degree of indexation.

Labour unions will maximize the following objective function for the savers:

$$maxE_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} \left\{ \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} W_{j,t} - W_{j,t+\tau}^{H} \right) n_{j,t+\tau} \right\} \tag{34}$$

s.t.

$$n_{j,t+\tau} = \left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \frac{W_{j,t}}{W_{t+\tau}} \right)^{-\lambda_{f,t}} n_{t+\tau}^{d} \tag{35}$$

An isomorphic maximization holds for the borrowers.

To solve the infinite horizon maximization problem and transform it into a recursive relation, an auxiliary variable f_t^s is used (f_t^b for the borrower). Two recursive laws of motion per agent will have to be solved with respect to the endogenous variables of the model:

$$f_t^s = \frac{\lambda_{w,t}-1}{\lambda_{w,t}} (W_t^*)^{(1-\lambda_{w,t})} u_{c,t} W_t^{\lambda_{w,t}} n_t^d + \beta\theta_w E_t \left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{1-\lambda_{w,t}} \left(\frac{W_{t+1}^*}{W_t^*} \right)^{\lambda_{w,t}-1} f_{t+1}^s, \tag{36}$$

$$f_t^s = \left(\frac{W_t}{W_t^*} \right)^{(\varphi+1)\lambda_{w,t}} n_t^{d1+\varphi} + \beta\theta_w E_t \left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{-\lambda_{w,t}(1+\varphi)} \frac{W_{t+1}^*}{W_t^*} f_{t+1}^s, \tag{37}$$

where W_t^* is the optimal wage. Again, isomorphic equations hold for the borrowers wage setting maximization problem. In a symmetric equilibrium, wages evolve according to:

$$W_t^{1-\lambda_{w,t}} = \theta_w \left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} \right)^{1-\lambda_{w,t}} W_{t-1}^{1-\lambda_{w,t}} + (1-\theta_w) W_t^{*1-\lambda_{w,t}}. \tag{38}$$

3.5 Monetary policy

The central bank sets the policy rate according to the rule:

$$R_t = max \left[1, R^{1-\rho_R} R_{t-1}^{\rho_R} \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi(1-\rho_R)} \left(\frac{\Delta Y_t}{\Delta Y} \right)^{\phi_y(1-\rho_R)} u_{M,t} \right] \tag{39}$$

where $u_{M,t}$ is an nid monetary policy shock with variance σ_M^2. R is the steady state interest rate and ΔY_t is the percentage variation of output, π^* is the inflation target.

3.6 The market clearing

The market clearing condition for the product is the following:

$$Y_t^d = a_k(u_{k,t})\bar{K}_{t-1} + a_n(u_{n,t})\bar{N}_{t-1} + C_t + I_t + S_t, \tag{40}$$

$$Y_t^s = \left(A_t u_{n,t}\bar{N}_{t-1} n^{d1-\sigma} n'^{d\sigma}\right)^{1-\alpha} \left(u_{k,t}\bar{K_{t-1}}\right)^{\alpha}, \tag{41}$$

where v_t^p is defined as the price dispersion:

$$v_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t}\right)^{-\lambda_{p,t}} di. \tag{42}$$

For the collateral:

$$H_t + H_t^{'} = 1. \tag{43}$$

4 Solution and Results

Once the FOC's are derived, the model is detrended. The model has an endogenous trend determined by the variable $\bar{N}_t$.

The model is detrended by $\bar{N}_t$.

Lower cases variables represent stationary variables.

The non-linear solution and the estimation of the model are currently being assessed by the author.[5]

[5] The model will be solved numerically using the Occbin toolbox routines provided by Guerrieri and Iacoviello (2014) to handle models with occasionally binding constraint. These routines incorporate the use of Dynare and provide a piecewise linear approximation solution.
Overall the model alternates among four regimes:

- The borrowing constraint and the Zero Lower Bound are both slack;
- Only the borrowing constraint is binding;
- Only the Zero Lower Bound is binding;
- Both the borrowing constraint and the ZLB are binding.

The piecewise linear solution is likely to deliver asymmetric behaviour of the variables according to the different sign of the shock. As an example when a positive housing demand shock hits the economy, borrowers may become non constrained for some period. This eases the effect of positive collateral nominal value fluctuations. Conversely, after a negative housing demand shock borrowers will remain constrained. This is going to produce asymmetric effects of housing shocks according to the sign.

Moreover, estimation will shed light on the magnitude and the role played by the different shocks in the economy.

For the remainder of the paper, results from the linear solution of the model will be presented. Standard calibration of the linear DSGE model can be applied and some interesting preliminary results can be drawn.

4.1 Calibration

The model is calibrated according to the estimate results obtained in the closest strand of DSGE literature. In Tab. 1, parameters values are reported together with their source. Most of the parameters are calibrated according to the the estimation performed in Guerrieri and Iacoviello (2014). Some of the parameters were originally calibrated by the authors. Concerning the technology sector and the nominal frictions, parameters have been calibrated according to the models by Guerron-Quintana and Jinnay (2014) and Smets and Wouters (2007). [6]

The subjective discount factor for the patient is 0.995, for the inpatient is $\beta^{'} = 0.993$. This ensures that the borrowers will be constrained in the steady state. Consumption habits parameter ϵ is equal to 0.6399. Frisch elasticity is equal to 1 since $\varphi = 1$.

Depreciation rate for capital δ_K is fixed at 0.025 whereas the one for technology is fixed at 0.04 to get a larger technology return rate.

The piecewise linear solution method is described in more depth by Guerrieri and Iacoviello (2015).

[6] The choice of not using parameters of the technology sector inferred in Bianchi and Kung (2014) hinges on the fact that the definition of $R\&D$ adopted in the future estimation will be more closer to the one by Guerron-Quintana.

Elasticities among intermediate goods and among intermediate labour (λ_p and λ_w are equal to 6, to match a steady state mark up on prices and wages of 20% with respect to the perfect competition case.

The costs of adjusting physical capital investment and technology investments (ϕ_K and ϕ_N) are equal to 5.03. In Bianchi and Kung (2014), the latter value is estimated to be much larger, in order to explain the extremely smooth path of $R\&D$ investments. In my case, investments in technology definition is wider and in line with the one provided by Guerron-Quintana and Jannay (2014), based on Nakamura (2003) definition of data on non-tangible assets, including $R\&D$ spending, patents, software, business plans and advertising. For this reason, I remained agnostic about the relative adjustment costs among capital and technology so far.

Nominal frictions parameters (stickiness and partial indexation) for prices and wages are both taken from Smets and Wouters (2007) estimation for the US economy.

Taylor rule values are standard for the post war period :$\rho_R = 0.52$, $\phi_P = 1.7385$ and $\phi_Y = 0.0796$.

σ is equal to 0.4151, meaning that 41.51% of agents are borrowing constrained, as obtained in the estimation exercise by Guerrieri and Iacoviello (2014).

The steady state Loan to Value ratio (M) is calibrated to 0.9.

Annual inflation target is 2%.

The growth trend steady steady state of technology (and of the whole economy) is endogenously determined by the structural parameters of the model and is equal to 2.04% per year.

Autocorrelations are are taken from Guerrieri and Iacoviello, except for the MEI in Technology shock (MEITech). This one is fixed according to the finding by Guerron-Quintana and Jinnay(2014).

The standard deviations have been implicitly fixed in the following theoretical exercise: in the following impulse responses, each shock produces a maximum absolute variation of 1% in income. In this way, it is possible to inquire on the effects that these shocks produce on the economy and ultimately on the medium-term fluctuations. The estimation exercise will provide information about the relative importance of each shocks according to the observed sample.

4.2 Do collateral nominal values play a role in the economy medium term fluctuations?

To answer the question, impulse responses of the model are compared. First of all, I focus on three different shocks:

- An expansionary housing demand shock: where the j_t is shocked increasing the marginal utility of housing for both agents;
- An expansionary housing demand shock only for borrowers;
- An expansionary LTV shock: where the LTV AR(1) process is shocked Ms_t in order to temporarily increase the Loan to Value Ratio.

According to the model and under the current calibration, a positive housing demand shock for both agents have a negative impact on the growth trend of the economy (Fig. 7). This result seems to contradict the general wisdom for which housing demand increases drive sustained growth period.

In fact, an increase in the demand of houses pushes patient agents to increase housing consumption since the ratio between marginal utility for houses and the marginal utility for houses is smaller than for borrowers, meaning that they are willing to pay more for houses with respect to the inpatient. This will drive housing prices up. Borrowers will deleverage due to their lower housing consumption. On the goods side, borrowers will increase their consumption and savers will decrease it. Due to the large spending in housing, patient will work more and inpatients will work less.

At the same time, patients will decrease investments in capital and importantly in technology, bringing to a reduction in the growth trend with respect to the steady state value.

An increase in marginal costs due to reduction in investments and hours worked by the borrowers will bring a larger inflation and a positive reaction of the interest rate set by the monetary policy.

In Fig. 3 and 4, impulse responses for the housing borrowers' demand are reported. In this case, marginal utility of housing is increased just for borrowers. This will increase borrowers' housing consumption and decrease savers' one, pushing up house prices and the collateral offered by borrowers to savers. Therefore borrowers will be able to increase their debt on impact. Savers will consume more, lend more to borrowers and invest less in capital and technology. This will negatively affect the growth trend of the economy (Fig. 7).

Marginal costs will increase pushing inflation up, causing an increase of the interest rate set by the central bank.

In Fig. 5 and 6, impulse responses to an expansionary LTV shocks are reported. Following a temporary increase in the LTV, borrowers will increase their debt and their housing consumption, driving up housing prices. Conversely, savers will consume more goods and less houses, they will lend more to savers and on impact will invest less in technology and capital, temporarily lowering technology accumulation and the economy's growth trend. Since the LTV converges back to its steady state, debt cumulated by borrowers decreases. Savers will allocate their savings between investments and housing and goods consumption. But since house prices are still larger than before and returns from capital and technology will be larger than their steady state values, savers will increase investments in capital and technology (increasing the economy's growth trend) and will buy less houses. When setting prices, firms will anticipate the reduction of marginal costs in the future due to an overall increase in investments and hours worked. Therefore, on impact inflation falls down the target and interest rates will decrease.

Summing up, according to the model positive housing demand fluctuations do not temporarily increase the growth trend of the economy, somehow against the general wisdom that housing bubbles and busts positively co-move with the medium-term fluctuations. On the contrary, housing demand shocks have a depressing effect on growth, pushing savers to invest less in technology, increase housing consumption or lend more to borrowers. Instead, an expansionary LTV shock has a positive effect on growth: borrowers increase their housing consumption driving up housing prices and ultimately pushing borrowers to invest more in technology and less in houses. This shock causes a general increase in debt, housing consumption for borrowers, an increase in output and in house prices and a positive effect on growth. Interestingly, a negative LTV shock generates opposite sign fluctuations where a sustained decline in housing price, deleveraging are associated with weak growth period, something that has the flavour of the slow recovery period following the Great Recession.[7] Besides, LTV shock is the only one among all the shocks included in the model for which there is a positive comovement among debt expansion, housing share for borrowers, output and the technological trend.

4.3 Sensitivity analysis

The following exercises are run to inquire on the role of indebtedness in the propagation of the shocks and their ultimate effects on the growth trend.

More debt

In the first experiment, the goal is to assess if an increase in indebtedness makes the economy more sensitive to the housing and the LTV shocks. The share of borrowers in the economy is increased to $\sigma = 0.7$. This causes an increase of 70% in the total debt of the economy.[8] Concerning the housing shock, it turns out that with a larger fraction of borrowers the sensitiveness of the model decreases (Fig. 12). In Figs. 8 and 9, a comparison between the baseline scenario impulse responses (solid lines) and the larger

[7] The actual role of the shock will be inferred in the estimation process.

[8] In a similar exercise not reported here, debt is increased on the intensive margin, lowering the subjective discount factor of the inpatient. Effects of housing and LTV shocks are very similar to the ones obtained varying the extensive margin of indebtedness (σ) and therefore are not reported here.

debt scenario ones (dashed lines) is showed. When the housing shock hits the economy, all agents will be more willing to buy houses, causing an increase of the price. The savers will be able to afford more houses and increase their share of houses. But the presence of more indebted agents make the increase of housing prices smaller. The reduction of the investments in technology (and in the trend) will be now smaller, due to an housing spending which is cheaper to the baseline case. Therefore, in a more indebted economy, housing shocks affect less medium term fluctuations.
Looking at the effects of the LTV shock, conclusions are opposite (10 and 11): a more indebted economy is more prone to LTV shocks in shaping medium-term fluctuations. An increase in the LTV ratio brings borrowers to increase their share of housing. In the alternative scenario, the fraction of agents benefiting of the LTV shock is larger, pushing houses prices up at the same level as in the baseline scenario despite a smaller variation in housing shares. Since borrowers now detain a smaller fraction of collateral the increase in debt will be smaller. This allows the saver to lend less to the borrower and anticipates the time when the she will start to increase her investments in technology (increasing the positive variation of the trend).

Shorter debt duration

In this paragraph I ask whether the type of debt detained by borrowers (long versus short duration) can also imply differences in the propagation of the housing and LTV shocks and their way of affecting the medium-term fluctuations.
In the model, equation 12 regulates the evolution of debt as weighted average of the past debt and and the nominal collateral value so to take into account the presence of a part of debt whose duration is longer than a quarter. The parameter γ assigns the degree of inertia to debt evolution.
In the exercise, the parameter γ is lowered to 0.2, in order to show what are the effects of the shocks when the debt evolution is more dependent on the collateral fluctuations,

mimicking a scenario where the average debt duration is shorter than in the baseline case. It turns out that housing shock propagation is only slightly affected by the variation in the debt duration, letting the smaller smoothness of debt response and minor quantitative variations for the other variables aside (Fig. 13-14).

Conversely, a smaller inertia in debt amplifies the LTV shocks effects Fig. 15-16). After an positive LTV shock, borrowers will be more keen in buying houses than in the baseline scenario, bringing a larger increase in debt. At the same time, once the LTV ratio starts to converge back to its steady state value, the smaller inertia of debt causes a faster decline in debt (and in borrowers' consumption). The anticipated deleveraging and high housing prices will push the saver to start to invest in technology earlier than before with the final effect of increasing the trend (Fig. 17).

5 Conclusion

This paper claims that housing market evolutions can affect the growth trend of the economy. Contrarily to the general wisdom for which an increase in appetite for housing can be seen as a source of a temporary increase in trend, this model shows that increase in the housing demand causes a decline in investment in technology (lowering the economy growth trend). Conversely, a positive comovement between housing prices and growth trend can be generated by the temporary relaxation of the borrowing debt limit.

These preliminary result is reached studying the effects of different types of shocks on a DSGE model with heterogeneous agents incorporating a technology sector, where collateral fluctuations affect the debt limit of borrowers. Housing prices play a crucial role in influencing the way savers allocate their savings among different options, that can either boost productivity (technological investments) or not having any effect, in case they invest in their own housing or lend to the borrowers.

Next steps will be devoted to:

- the non.linear solution to check if the shocks above may have an asymmetric effect due to the presence of occasionally binding constraints (the borrowing constraint and the Zero Lower Bound);
- the estimation of the model to assess the relative magnitude of the different shocks.

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Appendix

First Order Conditions and Market Clearing Equations

5.1 Patient Households

- Patient household marginal utilities:

$$u_{c,t} = \Gamma\left(\frac{1}{C_t - \epsilon C_{t-1}} - \frac{\beta\mu\epsilon}{C_{t+1} - \epsilon C_t}\right); \tag{44}$$

with

$$\Gamma = \left(\frac{\mu - \epsilon}{\mu - \beta\mu\epsilon}\right); \tag{45}$$

$$u_{n,t} = n_t; \tag{46}$$

$$u_{h,t} = \frac{j_t z_t}{H_t}; \tag{47}$$

- Patient Household Euler Equation:

$$u_{c,t} = \beta E_t u_{c,t+1} \frac{R_t}{\Pi_{t+1}}; \tag{48}$$

- Patient Household Labour Supply excluding the mark up deriving by trade unions maximization:

$$u_{n,t} = \frac{W_t^H}{x_{w,t}} u_{c,t}; \tag{49}$$

- Patient Household Collateral Demand:

$$u_{h,t} + \beta E_t u_{c,t+1} q_{t+1} = u_{c,t} q_t; \tag{50}$$

- Patient Household Euler Equation for Capital:

$$q_{k,t} = \beta E_t u_{c,t+1} \left(r_{k,t+1} u_{k,t+1} - a_k(u_{k,t+1}) \right) + (1 - \delta_k) q_{k,t+1} \beta; \tag{51}$$

- Patient Household Euler Equation for Technology:

$$q_{n,t} = \beta E_t u_{c,t+1} \left(r_{n,t+1} u_{n,t+1} - a_n(u_{n,t+1}) \right) (1 - \delta_n) q_{n,t+1} \beta; \tag{52}$$

- Patient Household capital accumulation equation:

$$u_{c,t} - \beta E_t q_{k,t+1} \xi_{I,t+1} 2\phi_k \Delta I_{t+1} = E_t q_{k,t} \xi_{I,t} (1 - 2\phi_k \Delta I_t); \tag{53}$$

with $\Delta I_t = \frac{I_t - I_{t-1}}{I}$.

- Patient Household technology accumulation equation:

$$u_{c,t} - \beta E_t q_{s,t+1} \xi_{S,t+1} 2\phi_n \Delta S_{t+1} = E_t q_{n,t} \xi_{S,t} (1 - 2\phi_n \Delta S_t); \tag{54}$$

with $\Delta S_t = \frac{S_t - S_{t-1}}{S}$.

- Patient Household Capital utilization:

$$r_{k,t} = ak(u_{k,t})' = \frac{2}{\sigma_k}; \tag{55}$$

with σ_k calibrated such that in steady state $u_k = 1$ and $ak(u_k) = 0$. This means that $\sigma_k = 2/r_k$.

- Patient Household Technology utilization:

$$r_{n,t} = an(u_{n,t})' = \frac{2}{\sigma_n}; \tag{56}$$

with σ_n calibrated such that in steady state $u_n = 1$ and $ak(u_n) = 0$. This means that $\sigma_n = 2/r_n$.

- Law of Capital accumulation:

$$\bar{K}_t = (1-\delta_k)\bar{K}_{t-1} + \xi_{I,t}\left(I_t - \psi_k \frac{(I_t - I_{t-1})^2}{\bar{I}}\right) \tag{57}$$

- Law of Technology accumulation:

$$\bar{N}_t = (1-\delta_n)\bar{N}_{t-1} + \xi_{S,t}\left(S_t - \psi_n \frac{(S_t - S_{t-1})^2}{\bar{S}}\right) \tag{58}$$

- Budget constraint for the patient Household:

$$C_t+Q_tH_t+I_t+S_t+B_t = W_tn_t+Q_tH_{t-1}+[r_{K,t}u_{K,t}-a_K(u_{K,t})]\bar{K}_{t-1}+[r_{N,t}u_{N,t}-a_N(u_{N,t})]\bar{N}_{t-1}+\frac{R_{t-1}B_{t-}}{\pi_t} \tag{59}$$

- Setting wages for the saver, using the auxiliary variable f_t^s:

$$f_t^s = \frac{\lambda_{w,t}-1}{\lambda_{w,t}}(W_t^*)^{(1-\lambda_{w,t})}u_{c,t}W_t^{\lambda_{w,t}}n_t^d+\beta\theta_w E_t\left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}}\right)^{1-\lambda_{w,t}}\left(\frac{W_{t+1}^*}{W_t^*}\right)^{\lambda_{w,t}-1} f_{t+1}^s, \tag{60}$$

$$f_t^s = \left(\frac{W_t}{W_t^*}\right)^{(\varphi+1)\lambda_{w,t}} n_t^{d1+\varphi} + \beta\theta_w E_t\left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}}\right)^{-\lambda_{w,t}(1+\varphi)}\left(\frac{W_{t+1}^*}{W_t^*}\right)^{-\lambda_{w,t}(1+\varphi)} f_{t+1}^s, \tag{61}$$

where W_t^* is the optimal wage for the savers.

$$W_t^{1-\lambda_{w,t}} = \theta_w\left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}}\right)^{1-\lambda_{w,t}} W_{t-1}^{1-\lambda_{w,t}} + (1-\theta_w)W_t^{*1-\lambda_{w,t}}. \tag{62}$$

5.2 Inpatient Households

- Inpatient household marginal utilities:

$$u^{'}_{c,t} = \Gamma^{'}\left(\frac{1}{C^{'}_{t} - \epsilon C^{'}_{t-1}} - \frac{\beta^{'}\mu\epsilon}{C^{'}_{t+1} - \epsilon C^{'}_{t}}\right); \tag{63}$$

with

$$\Gamma^{'} = \left(\frac{\mu - \epsilon}{\mu - \beta^{'}\mu\epsilon}\right); \tag{64}$$

$$u^{'}_{n,t} = n^{'}_{t}; \tag{65}$$

$$u^{'}_{h,t} = \frac{j_t j b_t z_t}{H^{'}_{t}}; \tag{66}$$

- Inpatient Household Euler Equation:

$$u^{'}_{c,t}(1 - \lambda_t) = \beta^{'} E_t u^{'}_{c,t+1} \frac{R_t - \gamma\lambda_{t+1}}{\Pi_{t+1}}; \tag{67}$$

- Inpatient Household Labour Supply without considering the trade union maximization:

$$u^{'}_{n,t} = W^{H'}_{t} u^{'}_{c,t}; \tag{68}$$

- Inpatient Household Collateral Demand:

$$u^{'}_{h,t} + \beta^{'} E_t u^{'}_{c,t+1} Q_{t+1} + u^{'}_{c,t}\lambda_t(1-\gamma)mQ_t = u^{'}_{c,t} q_t; \tag{69}$$

- Inpatient Household occasionally binding borrowing constraint:

$$\lambda_t\left(B_t - \gamma\frac{B_{t-1}}{\Pi_t} + (1-\gamma)\, M s_t Q_t H_t\right) = 0; \tag{70}$$

with λ_t being equal to 0 when the borrowing constraint is slack, $\lambda_t > 0$ when the borrowing constraint is binding.

- Budget constraint for the Inpatient Household:

$$C_t^{'} + Q_t H_t^{'} + \frac{R_{t-1} B_{t-1}}{\pi_t} = w_t^{'} n_t^{'} + Q_t H_{t-1}^{'} + B_t \tag{71}$$

- Setting wages for the borrower, using the auxiliary variable f_t^s:

$$f_t^b = \frac{\lambda_{w,t} - 1}{\lambda_{w,t}} \left(W_t^{'*}\right)^{(1-\lambda_{w,t})} u_{c,t} W_t^{'\lambda_{w,t}} n_t^{'d} + \beta^{'} \theta_w E_t \left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}}\right)^{1-\lambda_{w,t}} \left(\frac{W_{t+1}^{'*}}{W_t^{'*}}\right)^{\lambda_{w,t}-1} f_{t+1}^b, \tag{72}$$

$$f_t^b = \left(\frac{W_t^{'}}{W_t^{'*}}\right)^{(\varphi+1)\lambda_{w,t}} n_t^{'d1+\varphi} + \beta^{'} \theta_w E_t \left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}}\right)^{-\lambda_{w,t}(1+\varphi)} \left(\frac{W_{t+1}^{'*}}{W_t^{'*}}\right)^{-\lambda_{w,t}(1+\varphi)} f_{t+1}^b, \tag{73}$$

where $W_t^{'*}$ is the optimal wage for the borrowers.

$$W_t^{'1-\lambda_{w,t}} = \theta_w \left(\frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}}\right)^{1-\lambda_{w,t}} W_{t-1}^{'1-\lambda_{w,t}} + (1-\theta_w) W_t^{'*1-\lambda_{w,t}}. \tag{74}$$

5.3 Firms

- Firms' cost minimization: capital;

$$r_{k,t} u_{k,t} \bar{K}_{t-1} = r_{k,t} K_t = \alpha m c_t \left(A_t u_{n,k} \bar{N}_{t-1} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha} (u_{t,k} \bar{K}_{t-1})^{\alpha} = \alpha \varrho_t \left(Y_t + F \bar{N}_t\right) \tag{75}$$

- Firm' cost minimization: labour supplied by patient households;

$$W_t n_t = \varrho_t (1-\sigma)(1-\alpha) \left(A_t u_{n,k} \bar{N}_{t-1} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha} (u_{t,k} \bar{K}_{t-1})^{\alpha} = (1-\sigma)(1-\alpha) \varrho_t \left(Y_t + F \bar{N}_t\right) \tag{76}$$

- Firms' cost minimization: labour supplied by inpatient agents;

$$W_t^{'} n_t^{'} = \varrho_t \sigma (1-\alpha) \left(A_t u_{n,k} \bar{N}_{t-1} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha} (u_{t,k} \bar{K}_{t-1})^{\alpha} = \sigma (1-\alpha) \varrho_t \left(Y_t + F \bar{N}_t\right) \tag{77}$$

- Firms' cost minimization: technology;

$$r_{n,t}u_{n,t}\bar{N}_{t-1} = r_{n,t}N_t = \eta(1-\alpha)\varrho_t\left(A_t u_{n,k}\bar{N}_{t-1}n_t^{\sigma}n_t^{\prime(1-\sigma)}\right)^{1-\alpha}(u_{t,k}\bar{K}_{t-1})^{\alpha} = \eta(1-\alpha)\varrho_t\left(Y_t + F\bar{N}_t\right) \tag{78}$$

- Marginal costs:

$$\varrho = \frac{mc_t}{1+\eta(1-\alpha)} \tag{79}$$

- Production function:

$$Y_t = K_t^{\alpha}\left(A_t N_t n_t^{(1-\sigma)}n_t^{\prime\sigma}\right)^{1-\alpha} - F\bar{N}_t \tag{80}$$

- Price setting equations:

$$g_1^t = u_{c,t}mc_t Y_t^d + \beta\theta_p\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\lambda_{p,t}} g_{t+1}^1, \tag{81}$$

$$g_t^2 = u_{c,t}\Pi^* Y_t^d + \beta\theta_p E_t\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\lambda_{p,t}}\left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2 \tag{82}$$

where $\Pi_t^* = \frac{p_t^*}{p_t}$.

$$\lambda_{p,t}g_t^1 = (\lambda_{p,t}-1)g_t^2. \tag{83}$$

The price index will be equal to:

$$p_t^{1-\lambda_{p,t}} = \theta_p\left(\Pi_{t-1}^{\chi}\right)^{1-\lambda_{p,t}} + (1-\theta_p)p_t^{*1-\lambda_{p,t}} \tag{84}$$

5.4 Monetary policy

$$R_t = max\left[1, R^{1-\rho_R}R_{t-1}^{\rho_R}\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\phi_\pi(1-\rho_R)}\left(\frac{\Delta Y_t}{\Delta Y}\right)^{\phi_y(1-\rho_R)}u_{M,t}\right]; \tag{85}$$

5.5 Aggregation and market clearing

After aggregation, wage dispersion for savers v_t^w is defined as:

$$v_t^w = \int_0^{1-\sigma} \left(\frac{W_{j,t}}{W_t} \right)^{-\lambda_{w,t}} dj. \tag{86}$$

Wage dispersion will create a wedge between labour supplied and demanded:

$$n_t^d = \frac{1}{v_t^w} n_t, \tag{87}$$

Wage dispersion will evolve according to:

$$v_t^w = \theta_w \left(\frac{W_{t-1}}{W_t} \frac{\Pi_{t-1}^{\chi_w}}{\Pi_t} \right)^{-\lambda_{w,t}} v_{t-1}^w + (1-\theta_w) \left(\Pi_t^w \right)^{-\lambda_{w,t}}, \tag{88}$$

with $\Pi_t^w = \frac{W_t^*}{W_t}$. Isomorphic equations hold for the borrowers.
On the price setting side, aggregation delivers the following relations:

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{-\lambda_{p,t}} v_{t-1}^p + (1-\theta_p) \Pi_t^{*-\lambda_{p,t}}, \tag{89}$$

where v_t^p is defined as the price dispersion:

$$v_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\lambda_{p,t}} di. \tag{90}$$

Price dispersion creates a wedge between output supplied and demanded in the market clearing equation:

$$Y_t^d = \frac{Y_t^s}{v_t^p}, \tag{91}$$

where:

$$Y_t^d = C_t + I_t + S_t + a_k(u_{k,t})\bar{K}_{t-1} + a_n(u_{n,t})\bar{N}_{t-1} \tag{92}$$

and

$$Y_t^s = \left(A_t u_{n,t} \bar{N}_{t-1} n^{d1-\sigma} n'^{d\sigma} \right)^{1-\alpha} \left(u_{k,t} \bar{K}_{t-1} \right)^{\alpha} \tag{93}$$

While the market clearing condition for the collateral is:

$$H_t + H_t^{'} = 1. \tag{94}$$

Stationary equations

Lower case variables are stationary variables, obtained by detrending by $\bar{N}_t$.

$$c_t = \frac{C_t}{\bar{N}_t} \tag{95}$$

To make dynamic equations stationary, we make use of the ratio between the stock of technology in two periods μ_t:

$$\mu_t = \frac{\bar{N}_t}{\bar{N}_{t-1}}. \tag{96}$$

This variable is used to take into account the role of the time varying trend. For example, when detrending C_{t-1} by N_t:

$$\frac{C_{t-1}}{\bar{N}_t} = \frac{C_{t-1}}{\bar{N}_t}\frac{\bar{N}_{t-1}}{\bar{N}_{t_1}} = \frac{c_{t-1}}{\mu_t}. \tag{97}$$

Marginal utilities are detrended multiplying by N_t:

$$\tilde{u}_{c,t} = u_{c,t}\bar{N}_t. \tag{98}$$

5.6 Patient Households

- Patient household marginal utilities:

$$\tilde{u}_{c,t} = \Gamma\left(\frac{1}{c_t - \epsilon\frac{c_{t-1}}{\mu_{t-1}}} - \frac{\beta\mu\epsilon}{c_{t+1}\mu_{t+1} - \epsilon c_t}\right); \tag{99}$$

  with

$$\Gamma = \left(\frac{\mu - \epsilon}{\mu - \beta\mu\epsilon}\right); \tag{100}$$

$$\tilde{u}_{n,t} = u_{n,t} = n_t; \tag{101}$$

$$\tilde{u}_{h,t} = \frac{j_t z_t}{H_t} N_t = \frac{j_t z_t}{h_t}; \tag{102}$$

- Patient Household Euler Equation:

$$\tilde{u}_{c,t} = \beta E_t \frac{\tilde{u}_{c,t+1}}{\mu_{t+1}} \frac{R_t}{\Pi_{t+1}}; \tag{103}$$

- Patient Household Labour Supply:

$$u_{n,t} = w_t^H \tilde{u}_{c,t}; \tag{104}$$

- Patient Household Collateral Demand:

$$\tilde{u}_{h,t} + \beta E_t \frac{\tilde{u}_{c,t+1}}{\mu_{t+1}} q_{t+1} = \tilde{u}_{c,t} q_t; \tag{105}$$

- Patient Household Euler Equation for Capital:

$$\tilde{q}_{k,t} = \beta E_t \frac{\tilde{u}_{c,t+1}}{\mu_{t+1}} \left(r_{k,t+1} u_{k,t+1} - a_k(u_{k,t+1}) \right) + (1 - \delta_k) \tilde{q}_{k,t+1} \frac{\beta}{\mu_{t+1}}; \tag{106}$$

- Patient Household Euler Equation for Technology:

$$\tilde{q}_{n,t} = \beta E_t \frac{\tilde{u}_{c,t+1}}{\mu_{t+1}} \left(r_{n,t+1} u_{n,t+1} - a_n(u_{n,t+1}) \right) + (1 - \delta_n) \tilde{q}_{n,t+1} \frac{\beta}{\mu_{t+1}}; \tag{107}$$

- Patient Household capital accumulation equation:

$$\tilde{u}_{c,t} - \beta E_t \frac{\tilde{q}_{k,t+1}}{\mu_{t+1}} \xi_{I,t+1} 2\phi_k \Delta i_{t+1} = E_t \tilde{q}_{k,t} \xi_{I,t} (1 - 2\phi_k \Delta i_t); \tag{108}$$

with $\Delta i_t = \frac{(i_t - \frac{i_{t-1}}{\mu_t})}{i}$.

- Patient Household technology accumulation equation:

$$\tilde{u}_{c,t} - \beta E_t \frac{\tilde{q}_{n,t+1}}{\mu_{t+1}} \xi_{S,t+1} 2\phi_n \Delta i_{t+1} = E_t \tilde{q}_{n,t} \xi_{S,t} (1 - 2\phi_n \Delta s_t); \tag{109}$$

with $\Delta s_t = \frac{(s_t - \frac{s_{t-1}}{\mu_t})}{s}$.

- Patient Household Capital utilization:

$$r_{k,t} = ak(u_{k,t})' = \frac{2}{\sigma_k}; \tag{110}$$

with σ_k calibrated such that in steady state $u_k = 1$ and $ak(u_k) = 0$. This means that $\sigma_k = 2/r_k$.

- Patient Household Technology utilization:

$$r_{n,t} = an(u_{n,t})' = \frac{2}{\sigma_n}; \tag{111}$$

with σ_n calibrated such that in steady state $u_n = 1$ and $ak(u_n) = 0$. This means that $\sigma_n = 2/r_n$.

- Law of Capital accumulation:

$$\bar{k}_t = (1 - \delta_k) \frac{\bar{k}_{t-1}}{\mu_t} + \xi_{I,t} \left(i_t - \psi_k \frac{\left(i_t - \frac{i_{t-1}}{\mu_t} \right)^2}{i} \right); \tag{112}$$

- Law of Technology accumulation:

$$1 = \frac{(1 - \delta_n)}{\mu_t} + \xi_{S,t} \left(s_t - \psi_n \frac{(s_t - \frac{s_{t-1}}{\mu_t})^2}{s} \right); \tag{113}$$

- Budget constraint for the patient Household:

$$c_t+q_th_t+i_t+s_t+b_t = w_tn_t+q_th_{t-1}+[r_{K,t}u_{K,t}-a_K(u_{K,t})]\frac{\bar{k}_{t-1}}{\mu_t}+\frac{[r_{N,t}u_{N,t}-a_N(u_{N,t})]}{\mu_t}+\frac{R_{t-1}b_{t-1}}{\pi_t\mu_t}+div_t; \tag{114}$$

with $div_t = (1-mc_t)y_t$.

- Setting wages for the saver, using the auxiliary variable f_t^s:

$$f_t^s = \frac{\lambda_{w,t}-1}{\lambda_{w,t}}\left(w_t^*\right)^{(1-\lambda_{w,t})}\tilde{u}_{c,t}w_t^{\lambda_{w,t}}n_t^d+\beta\theta_wE_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\lambda_{w,t}}\left(\frac{w_{t+1}^*\mu_{t+1}}{w_t^*}\right)^{\lambda_{w,t}-1}f_{t+1}^s, \tag{115}$$

$$f_t^s = \left(\frac{w_t}{w_t^*}\right)^{(\varphi+1)\lambda_{w,t}}n_t^{d1+\varphi}+\beta\theta_wE_t\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\lambda_{w,t}(1+\varphi)}\left(\frac{w_{t+1}^*\mu_{t+1}}{w_t^*}\right)^{-\lambda_{w,t}(1+\varphi)}f_{t+1}^s, \tag{116}$$

where w_t^* is the optimal wage for the savers.

$$w_t^{1-\lambda_{w,t}} = \theta_w\left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{1-\lambda_{w,t}}\left(\frac{w_{t-1}}{\mu_t}\right)^{1-\lambda_{w,t}}+(1-\theta_w)w_t^{*1-\lambda_{w,t}}. \tag{117}$$

5.7 Inpatient Households

- Inpatient household marginal utilities:

$$\tilde{u}_{c,t}^{'} = \Gamma^{'}\left(\frac{1}{c_t^{'}-\epsilon\frac{c_{t-1}^{'}}{\mu_t}}-\frac{\beta^{'}\mu\epsilon}{c_{t+1}^{'}\mu_{t+1}-\epsilon c_t^{'}}\right); \tag{118}$$

with

$$\Gamma^{'} = \left(\frac{\mu-\epsilon}{\mu-\beta^{'}\mu\epsilon}\right); \tag{119}$$

$$\tilde{u}_{n,t}^{'} = u_{n,t}^{'} = n_t^{'}; \tag{120}$$

$$\tilde{u}_{h,t}^{'} = \frac{j_tjb_tz_t}{h_t^{'}}; \tag{121}$$

- Inpatient Household Euler Equation:

$$\tilde{u}_{c,t}^{'}(1-\lambda_t) = \beta^{'}E_t\frac{\tilde{u}_{c,t+1}^{'}}{\mu_{t+1}}\frac{R_t-\gamma\lambda_{t+1}}{\Pi_{t+1}}; \tag{122}$$

- Inpatient Household Labour Supply:

$$\tilde{u}^{'}_{n,t} = w^{'H}_t \tilde{u}^{'}_{c,t}; \tag{123}$$

- Inpatient Household Collateral Demand:

$$\tilde{u}^{'}_{h,t} + \beta^{'} E_t \tilde{u}^{'}_{c,t+1} q_{t+1} + \tilde{u}^{'}_{c,t} \lambda_t (1-\gamma) m q_t = \tilde{u}^{'}_{c,t} q_t; \tag{124}$$

- Inpatient Household occasionally binding borrowing constraint:

$$\lambda_t \left(b_t - \gamma \frac{b_{t-1}}{\Pi_t \mu_t} + (1-\gamma) M s_t q_t h^{'}_t \right) = 0; \tag{125}$$

with λ_t being equal to 0 when the borrowing constraint is slack, $\lambda_t > 0$ when the borrowing constraint is binding.

- Budget constraint for the Inpatient Household: *

$$c^{'}_t + q_t h^{'}_t + \frac{R_{t-1} b_{t-1}}{\pi_t \mu_t} = w^{'}_t n^{'}_t + q_t h^{'}_{t-1} + b_t \tag{126}$$

- Setting wages for the borrower, using the auxiliary variable f^s_t:

$$f^b_t = \frac{\lambda_{w,t} - 1}{\lambda_{w,t}} \left(w^{'*}_t \right)^{(1-\lambda_{w,t})} \tilde{u}_{c,t} w^{'\lambda_{w,t}}_t n^{'d}_t + \beta^{'} \theta_w E_t \left(\frac{\Pi^{\chi_w}_{t+s-1}}{\Pi_{t+s}} \right)^{1-\lambda_{w,t}} \left(\frac{w^{'*}_{t+1} \mu_{t+1}}{w^{'*}_t} \right)^{\lambda_{w,t}-1} f^b_{t+1}, \tag{127}$$

$$f^b_t = \left(\frac{w^{'}_t}{w^{'*}_t} \right)^{(\varphi+1)\lambda_{w,t}} n^{'d1+\varphi}_t + \beta^{'} \theta_w E_t \left(\frac{\Pi^{\chi_w}_{t+s-1}}{\Pi_{t+s}} \right)^{-\lambda_{w,t}(1+\varphi)} \left(\frac{w^{'*}_{t+1} \mu_{t+1}}{w^{'*}_t} \right)^{-\lambda_{w,t}(1+\varphi)} f^b_{t+1}, \tag{128}$$

where $w^{'*}_t$ is the optimal wage for the borrowers.

$$w^{'1-\lambda_{w,t}}_t = \theta_w \left(\frac{\Pi^{\chi_w}_{t+s-1}}{\Pi_{t+s}} \right)^{1-\lambda_{w,t}} \left(\frac{w^{'}_{t-1}}{\mu_t} \right)^{1-\lambda_{w,t}} + (1-\theta_w) w^{'*1-\lambda_{w,t}}_t. \tag{129}$$

5.8 Firms

- Firms' cost minimization: capital;

$$r_{k,t}u_{k,t}\bar{k}_{t-1} = r_{k,t}k_t = \alpha\varrho_t\left(A_t u_{n,k} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha}(u_{t,k}\bar{k}_{t-1})^{\alpha} \tag{130}$$

- Firm' cost minimization: labour supplied by patient households;

$$w_t n_t = \varrho_t(1-\sigma)(1-\alpha)\left(A_t u_{n,k}\frac{1}{\mu_t} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha}(u_{t,k}\frac{\bar{k}_{t-1}}{\mu_t})^{\alpha} \tag{131}$$

- Firms' cost minimization: labour supplied by inpatient agents;

$$w_t^{'} n_t^{'} = \varrho_t\sigma(1-\alpha)\left(A_t u_{n,k}\frac{1}{\mu_t} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha}(u_{t,k}\frac{\bar{k}_{t-1}}{\mu_t})^{\alpha} \tag{132}$$

- Firms' cost minimization: technology;

$$r_{n,t}u_{n,t} = \eta(1-\alpha)\varrho_t\left(A_t u_{n,k} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha}(u_{t,k}\bar{k}_{t-1})^{\alpha} \tag{133}$$

- Marginal costs:

$$\varrho_t = \frac{mc_t}{1+\eta(1-\alpha)} \tag{134}$$

- Production function:

$$y_t = \left(A_t u_{n,k}\frac{1}{\mu_t} n_t^{\sigma} n_t^{'(1-\sigma)}\right)^{1-\alpha}(u_{t,k}\frac{\bar{k}_{t-1}}{\mu_t})^{\alpha} \tag{135}$$

- Price setting equations:

$$g_t^1 = \tilde{u}_{c,t}mc_t y_t^d + \beta\theta_p\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\lambda_{p,t}} g_{t+1}^1, \tag{136}$$

$$g_t^2 = \tilde{u}_{c,t}\Pi^* y_t^d + \beta\theta_p E_t\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\lambda_{p,t}}\left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2 \tag{137}$$

where $\Pi_t^* = \frac{p_t^*}{p_t}$.

$$\lambda_{p,t} g_t^1 = (\lambda_{p,t} - 1) g_t^2. \tag{138}$$

The price index will be equal to:

$$p_t^{1-\lambda_{p,t}} = \theta_p \left(\Pi_{t-1}^{\chi}\right)^{1-\lambda_{p,t}} + (1-\theta_p) p_t^{*1-\lambda_{p,t}} \tag{139}$$

5.9 Monetary policy

$$R_t = max \left[1, R^{1-\rho_R} R_{t-1}^{\rho_R} \left(\frac{\Pi_t}{\Pi_t^*}\right)^{\phi_\pi(1-\rho_R)} \left(\frac{\Delta y_t}{\Delta y}\right)^{\phi_y(1-\rho_R)} u_{M,t}\right]; \tag{140}$$

with

$$\Delta y_t = \frac{y_t - \frac{y_{t-1}}{\mu_t}}{\frac{y_{t-1}}{\mu_t}} \tag{141}$$

and

$$\Delta y = \frac{y - \frac{y}{\mu}}{\frac{y}{\mu}} = \mu \tag{142}$$

5.10 Aggregation and market clearing

After aggregation, wage dispersion for savers v_t^w is defined as:

$$v_t^w = \int_0^{1-\sigma} \left(\frac{w_{j,t}}{w_t}\right)^{-\lambda_{w,t}} dj. \tag{143}$$

$$n_t^d = \frac{1}{v_t^w} n_t, \tag{144}$$

$$v_t^w = \theta_w \left(\frac{w_{t-1}}{w_t \mu_t} \frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\lambda_{w,t}} v_{t-1}^w + (1-\theta_w) \left(\Pi_t^w\right)^{-\lambda_{w,t}} \tag{145}$$

with $\Pi_t^w = \frac{w_t^*}{w_t}$. Isomorphic equations hold for the borrowers. On the price setting side, the following stationary relations hold:

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\lambda_{p,t}} v_{t-1}^p + (1-\theta_{p,t}) \Pi_t^{*-\lambda_{p,t}}, \tag{146}$$

where v_t^p, the price dispersion, is:

$$v_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\lambda_{p,t}} di. \tag{147}$$

$$y_t^d = \frac{y_t^s}{v_t^p}, \tag{148}$$

where:

$$y_t^d = c_t + i_t + s_t + a_k(u_{k,t}) \frac{\bar{k}_{t-1}}{\mu_t} + \frac{a_n(u_{n,t})}{\mu_t} \tag{149}$$

and

$$y_t^s = \left(A_t u_{n,t} n^{d 1-\sigma} n'^{d\sigma} \right)^{1-\alpha} \left(u_{k,t} \bar{k}_{t-1} \right) \frac{1}{\mu_t}^{\alpha} \tag{150}$$

While the market clearing condition for the collateral is:

$$h_t + h_t^{'} = 1. \tag{151}$$

Tables

Par	Value	Source	Par	Value	Source
β	0.995	*Guerrieri and Iacoviello (2014)*	α	0.33	*Guerrieri and Iacoviello (2014)*
β'	0.993		ρ_r	0.52	*Guerrieri and Iacoviello (2014)*
ϵ	0.6399	*Guerrieri and Iacoviello (2014)*	ϕ_p	1.7385	*Guerrieri and Iacoviello (2014)*
δ_K	0.025	*Guerrieri and Iacoviello (2014)*	ϕ_y	0.0796	*Guerrieri and Iacoviello (2014)*
δ_N	0.04		M	0.90	*Guerrieri and Iacoviello (2014)*
ϕ_K	5.03	*Guerrieri and Iacoviello (2014)*	$\bar{j}$	0.04	*Guerrieri and Iacoviello (2014)*
ϕ_N	5.03	*Guerrieri and Iacoviello (2014)*	γ	0.4547	*Guerrieri and Iacoviello (2014)*
π	1.005	*Guerrieri and Iacoviello (2014)*	ρ_A	0.7793	*Guerrieri and Iacoviello (2014)*
φ	1	*Guerrieri and Iacoviello (2014)*	ρ_{akk}	0.7651	*Guerrieri and Iacoviello (2014)*
λ_p	6	*Guerrieri and Iacoviello (2014)*[9]	ρ_{ann}	0.22	*Guerron-Quintana and Jinnay (2014)*
λ_w	6	*Guerrieri and Iacoviello (2014)*	ρ_z	0.7793	*Guerrieri and Iacoviello (2014)*
θ_P	0.66	*Smets and Wouters (2007)*	ρ_j	0.9934	*Guerrieri and Iacoviello (2014)*
θ_w	0.70	*Smets and Wouters (2007)*	ρ_{jb}	0.9934	
χ_p	0.2467	*Smets and Wouters (2007)*	ρ_{Ms}	0.8	
χ_w	0.58	*Smets and Wouters (2007)*	σ	0.4151	*Guerrieri and Iacoviello (2014)*

Table 1: Calibrated parameters: values and their source.

Figures

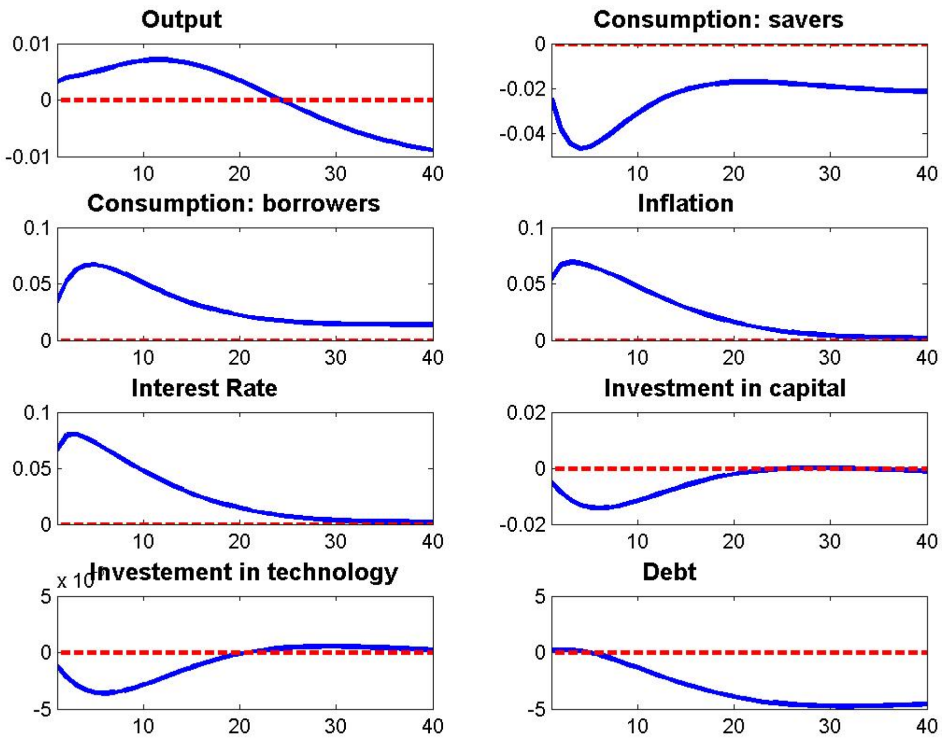

Figure 1: Impulse responses to a Housing Demand shock calibrated to obtain an maximum absolute variation of 1% in income.

Figure 2: Impulse responses to a Housing Demand shock calibrated to obtain an maximum absolute variation of 1% in income.

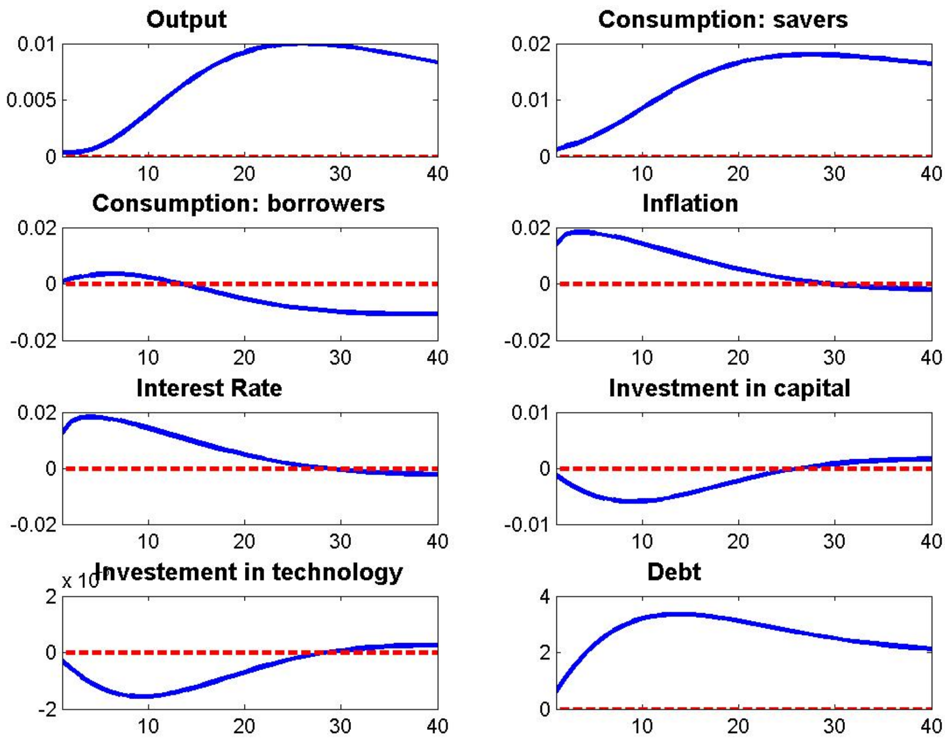

Figure 3: Impulse responses to a Housing Borrowers' Demand shock calibrated to obtain an maximum absolute variation of 1% in income.

Figure 4: Impulse responses to a Housing Borrowers' Demand shock calibrated to obtain an maximum absolute variation of 1% in income.

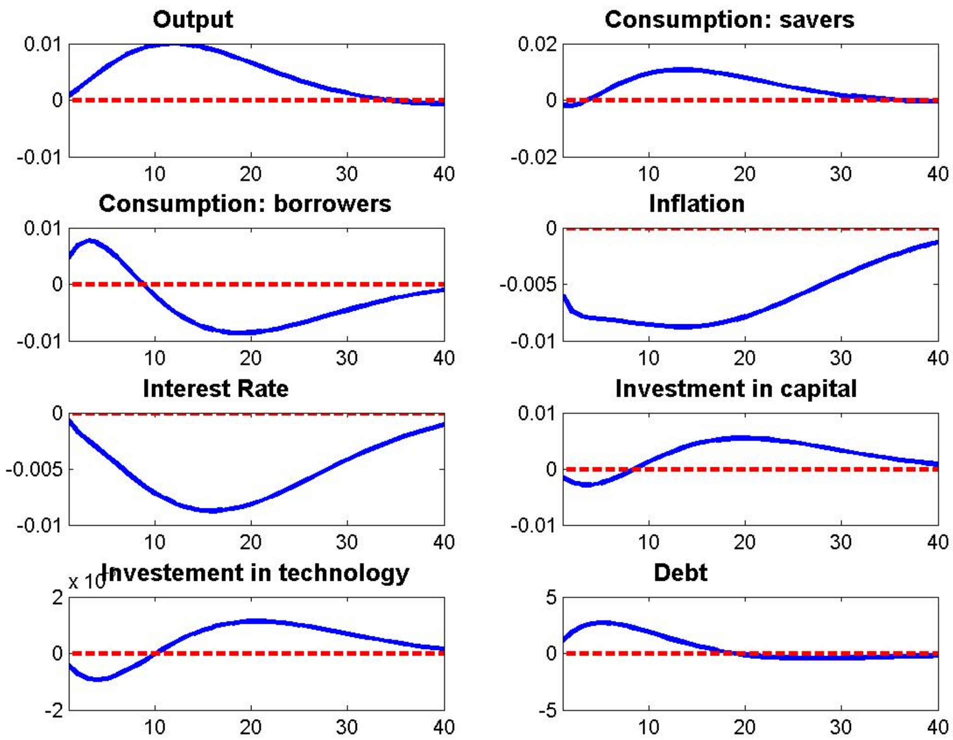

Figure 5: Impulse responses to a LTV shock calibrated to obtain an maximum absolute variation of 1% in income.

Figure 6: Impulse responses to a LTV shock calibrated to obtain an maximum absolute variation of 1% in income.

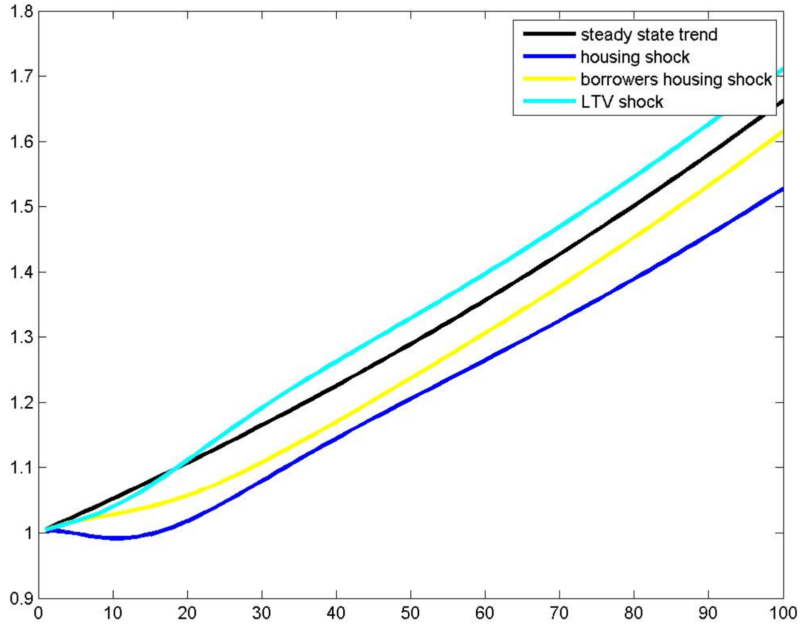

Figure 7: Medium term fluctuations obtained under standard calibration: variation of the growth trend in case of: Housing Demand shock (blue line), Housing Borrowers' Demand shock (yellow line), LTV shock (light blue line).

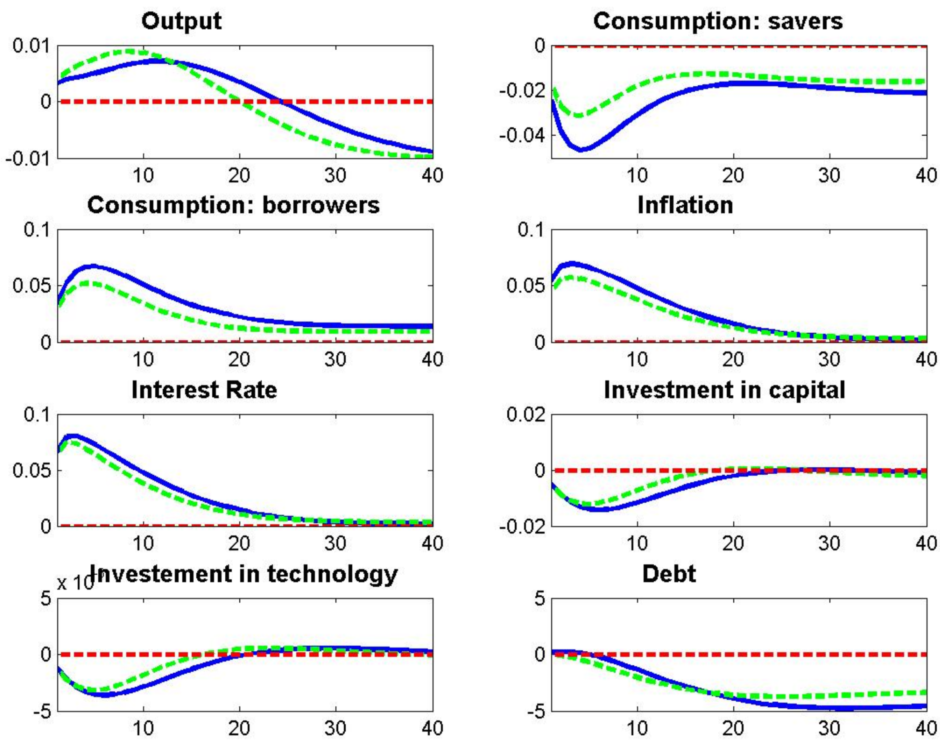

Figure 8: Impulse responses to a Housing Demand shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\sigma = 0.7$: larger fraction of borrowers (dashed line).

Figure 9: Impulse responses to a Housing Demand shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\sigma = 0.7$: larger fraction of borrowers (dashed line).

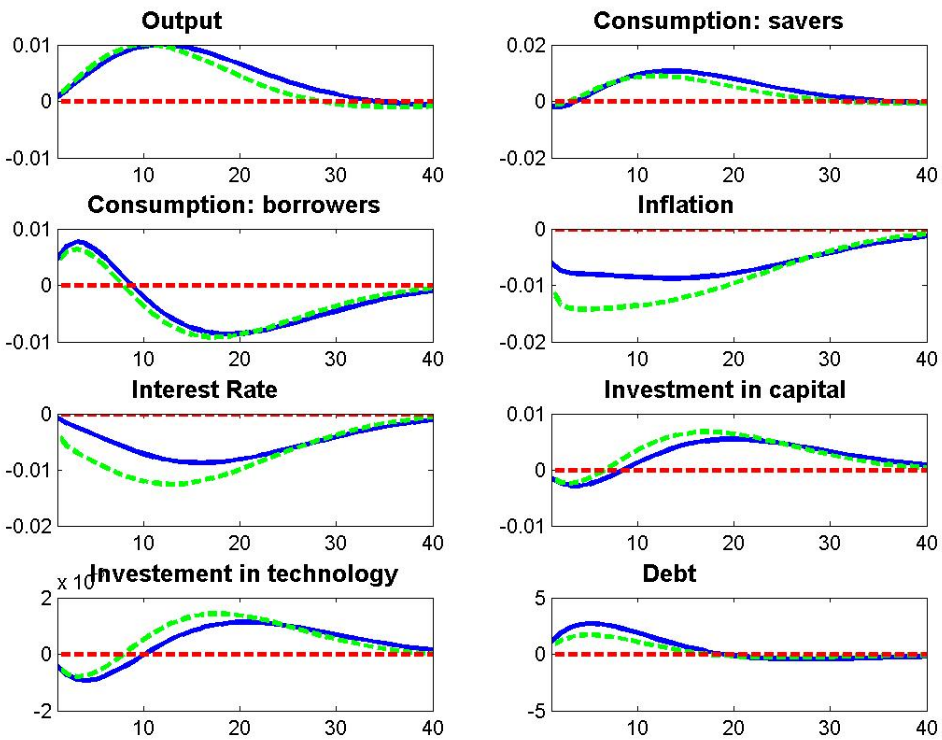

Figure 10: Impulse responses to a LTV shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\sigma = 0.7$: larger fraction of borrowers (dashed line).

Figure 11: Impulse responses to a LTV shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\sigma = 0.7$: larger fraction of borrowers (dashed line).

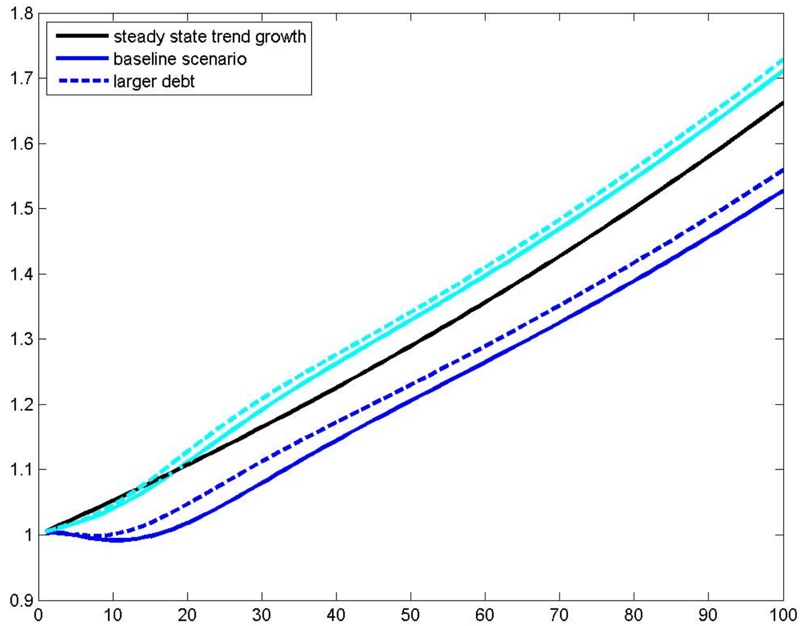

Figure 12: Medium term fluctuations obtained under standard calibration: variation of the growth trend in case of: Housing Demand shock (blue line), LTV shock (light blue line). Three different case per shock are analysed: baseline (solid line); $\sigma = 0.7$: larger fraction of borrowers (dashed line). Shocks are calibrated to obtain a maximum absolute variation of 2% in income.

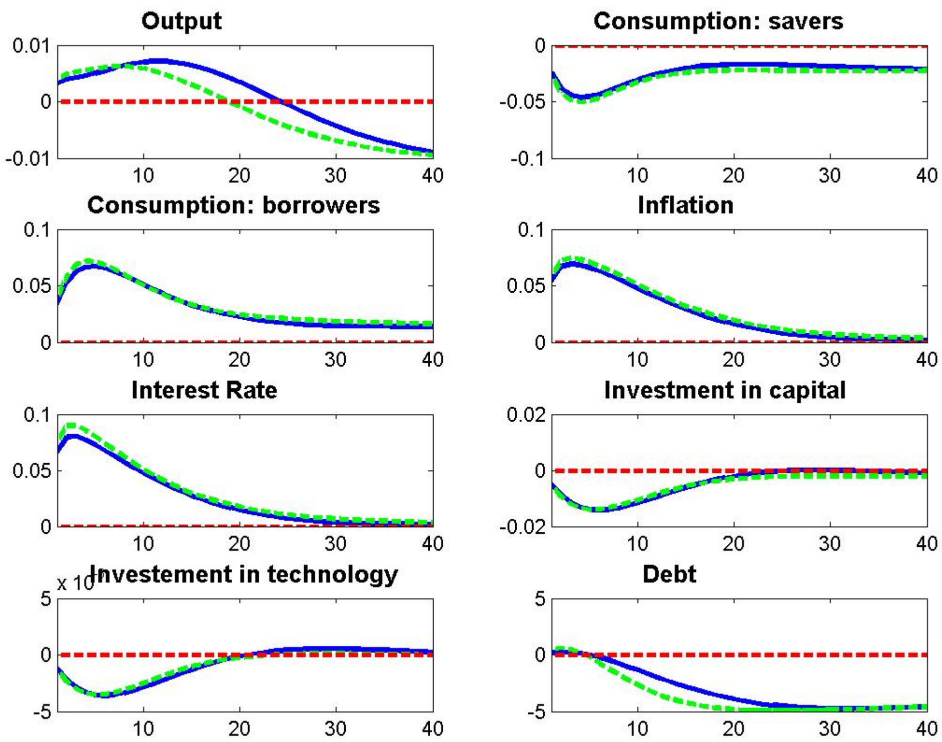

Figure 13: Impulse responses to a Housing Demand shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\gamma = 0.2$: smaller debt inertia (dashed line).

Figure 14: Impulse responses to a Housing Demand shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\gamma = 0.2$: smaller debt inertia (dashed line).

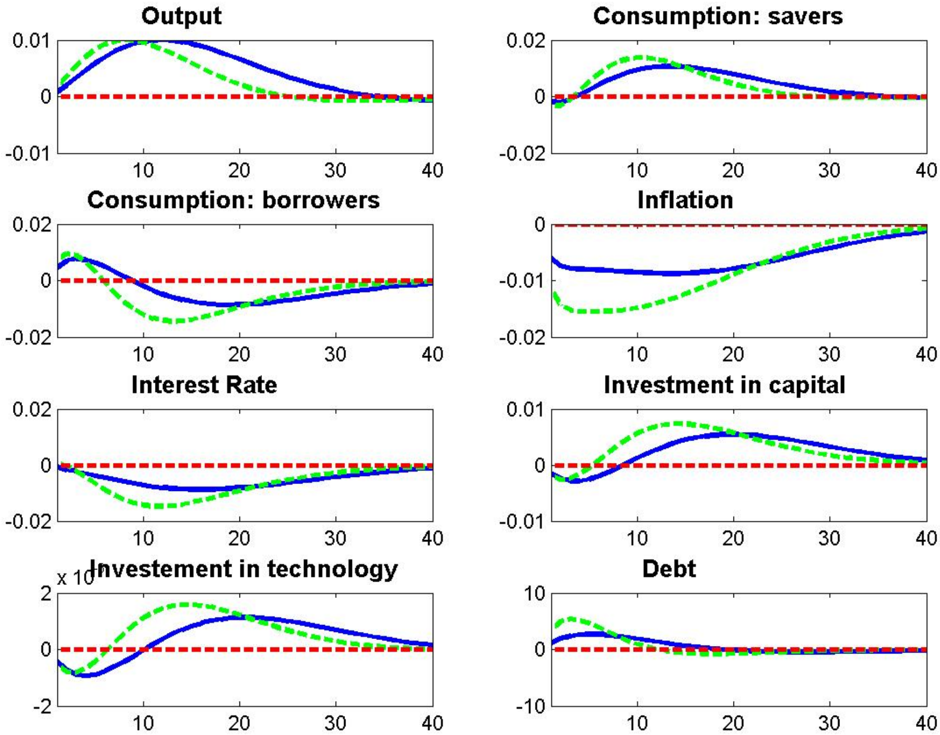

Figure 15: Impulse responses to a LTV shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\gamma = 0.2$: smaller debt inertia (dashed line).

Figure 16: Impulse responses to a LTV shock calibrated to obtain an maximum absolute variation of 1% in income: baseline (solid line); $\gamma = 0.2$: smaller debt inertia (dashed line).

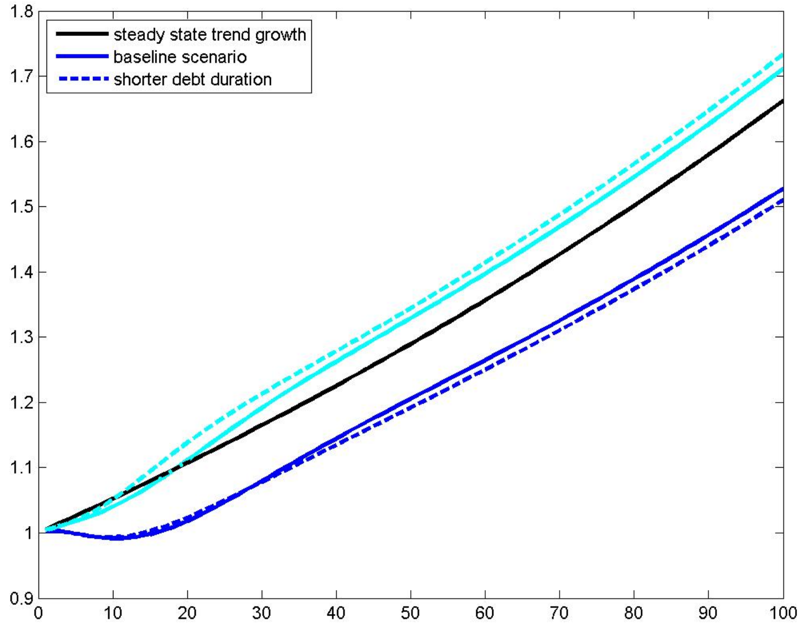

Figure 17: Medium term fluctuations obtained under standard calibration: variation of the growth trend in case of: Housing Demand shock (blue line), LTV shock (light blue line). Two different cases per shock are showed: baseline (solid line $\gamma = 0.4547$); $\gamma = 0.2$: smaller debt duration (dashed line). Shocks are calibrated to obtain a maximum absolute variation of 2% in income.