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Essays in Macroeconomics of Debt Deleveraging

Candidate: Federica Romei

Supervisor: Prof. Pierpaolo Benigno

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Dedicated to my extended family: my mom, my dad, Lorenzo, Enrico, Adriana and Sergio.

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This small success has many fathers.

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ABSTRACT

Essays in Macroeconomics of Debt Deleveraging

Federica Romei

This dissertation analyzes, in two chapters, how monetary and fiscal authorities can optimally manage debt reduction episodes.

In the first chapter I show how public debt deleveraging leads to a recession with different effects on real interest rates according to the fiscal instruments the government is using to reduce the debt. The fiscal authority should not depress much consumption of the agents who hold savings to improve the welfare of the ones who do not have access to financial markets. Moreover speed and timing of public deleveraging depend crucially on the type of instrument the fiscal authority uses to enforce it. Nominal rigidities, in this context, seem to be beneficial for the agents who cannot insure themselves through financial markets.

In the second chapter, written together with Prof. Pierpaolo Benigno, we show how deleveraging from high debt can provoke deep recession with significant international side effects. Due the debt reduction process, real and nominal variables can be subject to high fluctuations. All these movements are inefficient and interesting trade-offs emerge from the perspective of global welfare. Counterintuitively, we show that the optimal adjustment to global imbalances should not necessarily require large movements in the nominal exchange rate. Moreover we show that, whenever countries have an high degree of openness to trade, Central Banks needs to create a global liquidity trap to face the deleveraging shock.

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Introduction

This dissertation analyzes, in two chapters, how monetary and fiscal authorities can optimally manage debt reduction episodes. The first chapter studies what is the optimal public deleveraging speed for a fiscal authority in a closed economy context. The second chapter, written with Prof. Pierpaolo Benigno, instead, considers how a Central Bank should optimally react to an international private deleveraging episode.

More in detail, in the first chapter I study, in a context of heterogenous agents, incomplete markets and closed economy, what is the optimal deleveraging path that a fiscal authority needs to undertake when forced to reduce public debt. I consider a public deleveraging that may occur either through a public expenditure reduction or through an income taxation increase. I analyze, then, what are the consequences, on agents' welfare, of different speeds of deleveraging and different fiscal instruments. I focus, moreover, part of my analysis on how nominal rigidities interact with public deleveraging.

I find that under taxation experiment real interest rates tend to be very high and this is helpful for the class of agents who holds savings. When, instead, government uses public expenditure to reduce debt, real interest rates are below the steady state: This situation may be beneficial, if economy do not enter a liquidity trap, for the agents who do not participate in financial markets. I also find that, in most cases, agents who do not have access to the financial markets benefit from the presence of downward wages rigidities.

In the second chapter, we study a Central Bank who faces an international private debt deleveraging episode in a context of a two-country economy. We model a country,

H, as a net borrower and the other country, F, as a net saver and we shock the economy raising the cost of borrowing. Finally we assume the presence of a unique Central Bank (or equivalently two cooperative Central Banks) that maximizes the welfare of economy as a whole. The question we address is how the Central Bank can manage optimally this deleveraging episode.

We find that there are three channels through which the global economy can absorb the private deleveraging costs. The first is the reduction of the real interest rate of country H, the one who reduces his debt. The second is the expenditure-switching channel, namely a depreciation of the currency of country H in order to steal part of global demand. Unfortunately, movements in the nominal exchange rate lead to inefficient movements in the terms of trade, increasing the economy's welfare costs. The third mechanism that the Central Bank can use is the reduction of the nominal interest rate of the country F who is a net saver. According to the degree of home bias and the elasticity of substitution between goods produced in country H and F, Central Bank mixes the three channels to react optimally to the deleveraging shock.

To summarize, I focus first on a domestic episode of public debt reduction and then on an international episode of private debt reduction. In both experiments I find that choices of Fiscal and Monetary authority have non negligible consequences on agents' welfare. Understanding how to deal with a debt deleveraging, then, carries important policy implications.

CHAPTER 1

Need for the Right Speed: Public Debt Deleveraging and the Timing of Austerity

1.1. Introduction

Many countries have experienced in recent years a significant increase in the size of their public debt. While in some countries public debt increased largely as a consequence of the financial crisis, notably in the United States, in others, debt was already significantly high at the onset of the crisis. As some Eurozone economies struggled to refinance their debt, those who received assistance packages from institutions such as the International Monetary Fund or the European Central Bank were asked to implement plans to reduce their stock of liabilities. Even economies that did not receive assistance are now facing the question of how to deleverage. Remarkably, this episode of public debt deleveraging is set to occur after an unprecedented recession, whose effects have been felt differently according to agents' position in the wealth distribution.

This chapter studies the optimal deleveraging path a fiscal authority should undertake when forced to reduce public debt in a context of heterogenous agents and incomplete markets.

In my model, the government can use public expenditure or distortionary income taxation to deleverage. Agents differ with respect to their possibility of accessing financial markets: Only a subset of consumers can borrow or lend to smooth their consumption. In this context, public sector deleveraging has strong redistributional effects. Indeed, a large public debt implies high taxes (or low public expenditure) for all agents, while interest payments only accrue to those holding public bonds. High government debt is then a net transfer of wealth from the consumers who cannot trade financial assets to the ones who can. The timing of deleveraging and the fiscal instrument chosen to perform it are therefore not inconsequential in this context.

The choice of instrument through which the fiscal authority will reduce debt is important to determine dynamics of aggregate variables and agents' welfare. A reduction in public expenditure or an increase in distortionary taxation will lead to different types of recessions, with different effects on interest rates. In the first case, the real interest rate prevailing in the economy will fall. The government and agents who do not participate in financial markets will gain from low real interest rates. When the fiscal authority uses taxes to reduce the debt, instead, interest rates rise significantly. Consumers who hold public bonds will be the ones to benefit from this situation.

The presence of nominal rigidities and the Zero Lower Bound interact asymmetrically with the different types of recessions considered. Also, their effects on redistribution are non-trivial. When deleveraging occurs through a reduction in public expenditure a liquidity trap may occur, while downward wage rigidities are helpful for agents who have no access to financial markets. When, instead, deleveraging is brought about via taxation, presence of this kind of rigidities exerts an upward pressure on the real interest rate, worsening welfare of the economy as a whole.

1.2. Literature Reviews

My research is closely related to two different strands of the literature: one that has focused on private debt deleveraging and its interaction with monetary policy, and another one on optimal fiscal policy under commitment.

Papers in the literature on private debt deleveraging typically model such an event as an exogenous shock then analyzing the impact of different monetary policies in this context. Some recent papers, such as Guerrieri and Lorenzoni (2010), Krugman and Eggertsson (2012) or Philippon and Midrigan (2011) have studied debt deleveraging in a closed economy. Others, among which Fornaro (2012) Cook and Devereux (2012) and Benigno and Romei (2012), have focused on the consequences of private debt deleveraging in an international context. I depart from this literature in two way: I analyze a public debt deleveraging and I study how the deleveraging path impacts on agents welfare.

A second strand of literature analyzes optimal fiscal policy. In their seminal paper, Lucas and Stokey (1983) show how the public authority should react to a shock when it is possible to issue a state contingent debt, in a representative consumer framework. One of the main results is that the public instrument inherits the persistence of the shock. Aiyagari, Marcet, Sargent and Seppala (2002) analyze the same problem when the public authority cannot issue state contingent debt. They show that, under this circumstance, the public authority instrument is more persistent than the shock. Werning (2007) and Karantounias (2013) analyze the same problem, the former departing from the representative agent assumption, while the latter considering recursive preferences. Lastly Bandhari, Evans, Golosov and Sargent (2013) study optimal taxation with heterogenous agents and aggregate uncertainty. This literature aims to understand how a government can react optimally to a shock. This chapter, instead, aims to understand, what is the optimal government-induced shock on the economy.

My paper is close to the work of Röhrs and Winter (2014). They analyze what are the consequences of a public debt reduction under heterogenous agents, market incompleteness under flexible prices on aggregate welfare . My research differ in that I focus on the optimal deleveraging speed and on its interaction with nominal rigidities.

1.3. Model

1.3.1. Consumers

I consider a closed economy inhabited by two types of agents, that I call Savers and Hand-to-Mouth. There is a continuum of measure $1 - \chi$ of Savers and a continuum of measure χ of Hand-to-Mouth. There is no uncertainty, so all agents have perfect foresight. Following Weil (1992), Savers differ with respect to the type of financial markets they can have access to. Hand-to-Mouth are endowed at birth with one unity of equity in all the firms in the economy. They cannot hold any other type of financial assets. Savers, instead, are not only endowed with the same amount of equity in firms that borrowers have, but they are also able to trade in riskless, one-period, non-contingent bonds. Access to financial markets allows Savers to choose how to optimally allocate their consumption intertemporally. Hand-to-Mouth agents, on the other hand, are forced to solve a static problem in each period.

Guiso, Haliassos and Jappelli (2000), among others, show that the degree of participation in financial markets increases in wealth. Accordingly, I consider Hand-to-Mouth agents in this model to proxy for the lower quantiles of the wealth distribution in the economy. Savers, on the other hand, proxy for richer consumers who hold financial assets in their portfolio.

Savers and Hand-to-Mouth have identical preferences over streams of consumpion, C and government-provided services, G. Moreover, they enjoy leisure and they supply hours of labor l to firms in the economy. Agents' lifetime utility is:

$$\sum_{t=0}^{\infty} \beta^t U(C_t^j, G_t, l_t^j) \qquad for \quad j = \{H, S\} \tag{1.1}$$

where $U(.)$ is concave, twice differentiable and satisfies the Inada condition.

I assume that C and G are bundles of goods

$$C = \left[\int_0^1 c(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

and

$$G = \left[\int_0^1 g(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $c(i)$ and $g(i)$ are private and public consumption of a generic good i produced in the economy. I assume the presence of a continuum of measure one of goods, which are imperfect substitute with an elasticity of intratemporal substitution equal to ϵ in both aggregators.

Savers and Hand-to-Mouth to firms two different varieties of labor. Every agent supplies labor to all the firms. Total labor supply of each agent is then:

$$l_H = \int_0^1 l_H(i)di \qquad l_S = \int_0^1 l_S(i)di$$

where $l_H(i)$ ($l_S(i)$) is the amount of hours supplied to firm i by the agent of type H

(S). Since agents are indiffernt across firms to which they supply labor, an agent of type H (S) will receive the same wage W_H (W_S) from all firms.

Savers trade one-period riskless bond, b_t^s in unit of consumption good that pays a real interest rate, r_t. Hand-to-Mouth, instead, consume profits distributed by firms and their labor income in each period. Their respective budget constraints are as follows:

$$C_t^S = W_{S,t} l_{S,t}(1-\tau_t) - b_t^S + \frac{b_{t+1}^S}{1+r_t} + \varpi_t \tag{1.2}$$

and

$$C_t^H = W_{H,t} l_{H,t}(1-\tau_t) + \varpi_t \tag{1.3}$$

where ϖ are profits paid by each firm and τ_t is a proportional income tax charged by the fiscal authority.

Real interest rate is determined by the Fisher equation:

$$(1+r_t) = \frac{(1+i_t)}{\Pi_{t+1}}$$

where i_t is the nominal interest rate at time t and Π_{t+1} is the gross inflation at time $t+1$. Since the nominal interest rate cannot be negative, the real rate must be greater than the inverse of inflation, i.e.:

$$(1+r_t) \geq \frac{1}{\Pi_{t+1}} \tag{1.4}$$

Hand-to-Mouth do not face any intertemporal decision problem. In every period they decide consumption and labor supply, according to the first order condition:

$$\frac{U_l(c_t^H, G_t, l_{H,t})}{U_c(c_t^H, G_t, l_{H,t})} = -W_{H,t}(1-\tau_t) \tag{1.5}$$

Savers, on the other hand, decide how much to save as well as consumption and hours supplied, maximizing (1.1) subject to (1.2):

$$U_c(c_t^S, G_t, l_{S,t}) = \beta(1+r_t)U_c(c_{t+t}^S, G_{t+1}, l_{S,t+1}) \tag{1.6}$$

$$\frac{U_l(c_t^S, G_t, l_{S,t})}{U_c(c_t^S, G_t, l_{S,t})} = -W_{S,t}(1-\tau_t) \tag{1.7}$$

Savers smooth consumption intertemporally according to their Euler equation. Note that since Savers are the only type of agent that can optimally allocate their intertemporal pattern of consumption, their behavior will critically affect the price of the bond and the interest rate.

1.3.2. Firms

The economy is populated by a continuum of identical firms of measure one. Each firm has access to the following technology

$$y(i) = L(i)^{\omega} = (L_S(i)^{\alpha_S} L_H(i)^{\alpha_H})^{\omega} \tag{1.8}$$

allowing them to transform a mix of two different types of labor inputs, L_S and L_H into output of a differentiated good.

Total labor hired by each firm is a Cobb-Douglas aggregator of labor supplied by each type of agent, with $\alpha_S + \alpha_H = 1$ and $\alpha_S > \alpha_H$ to capture positive correlation between the distribution of wealth and the distribution of skills in the economy. Firms opereate under decreasing return to scale, so that $\omega < 1$.

Every firm

$$\min_{L_S(i),L_H(i)} W_S L_S(i) + W_H L_H(i) \tag{1.9}$$

subject to

$$y(i) \leq \bar{y}$$

From the optimization problem above, I derive

$$\frac{W_S L_S(i)}{\alpha_S} = \frac{W_H L_H(i)}{\alpha_H} = WL(i)$$

where the aggregate real wage W is defined as:

$$W = (kW_S^{\alpha_S} W_H^{\alpha_H})$$

where $k \equiv \left(\frac{1}{\alpha_S}\right)^{\alpha_S} \left(\frac{1}{\alpha_H}\right)^{\alpha_H}$.

Each firm i, competing under monopolistic competition, faces a demand schedule of the type:

$$y(i) = \left(\frac{p(i)}{P}\right)^{-\epsilon} [C + G] = \left(\frac{p(i)}{P}\right)^{-\epsilon} Y \tag{1.10}$$

where $p(i)$ is the price set by firm i and P is the price aggregator defined as:

$$P = \left(\int_0^1 p(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

Every firm faces a quadratic cost of adjusting nominal prices a-la Rotemberg which is measured in units of aggregate final output, i.e.:

$$\phi(p_t(i), p_{t-1}(i), Y_t) = \frac{\phi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)\bar{\Pi}} - 1\right)^2 Y_t.$$

where $\bar{\Pi}$ is steady state inflation and $\phi > 0$ determines the degree of nominal rigidity.

Firms maximize the present discounted sum of their profits:

$$\varpi(i)_t = \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{p_t(i)}{P_t} y_t(i) - W_t(i) L_t(i) - \phi(p_t(i), p_{t-1}(i), Y_t)\right] \tag{1.11}$$

subject to (1.8) and (1.10), where $\lambda_t \equiv (\chi U_c(c_t^H, G_t, l_t^H) + (1-\chi) U_c(c_t^S, G_t, l_t^S))$ is a weighted average of agents' specific stochastic discount factors. I can exploit the fact that every firm faces the same cost as well as the same demand schedule to drop the i index. First order condition for profits is as follows:

$$W_t = \frac{\omega}{\mu} Y_t^{1-\frac{1}{\omega}} + \frac{\phi\omega}{\epsilon}(\Pi_t - \bar{\Pi})\Pi_t Y_t^{1-\frac{1}{\omega}} - \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\phi\omega}{\epsilon}(\Pi_{t+1} - \bar{\Pi})\Pi_{t+1} Y_{t+1} Y_t^{-\frac{1}{\omega}} \tag{1.12}$$

where $\mu \equiv \frac{\epsilon}{\epsilon - 1}$ is the markup and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate. Output , Y_t, increases if either aggregate real costs decrease, W_t, if current inflation increases or if future inflation will decrease. From this equation it is possible to derive the standard positively sloped AS curve, since firms will supply more output whenever prices increase.

Additionally, firms are subject to a second type of rigidity: nominal wages are downwardly rigid for both types of labor. A generic firm i is a wage taker and does not consider this rigidity when maximizing their profits. Similarly, workers supply labor in a perfect competitive market and, as a consequence they also do not take into account the presence of this rigidity.

This friction is expressed as follows:

$$W_{S,t} \geq \psi W_{S,t-1}\Pi_t \qquad W_{H,t} \geq \psi W_{H,t-1}\Pi_t \tag{1.13}$$

where $\psi \in [0,1]$ is a measure of wage rigidity: $\psi = 0$ means that wages are free to move, while $\psi = 1$ leads to completely rigid nominal wage. I assume that the parameter ψ is identical for both labor markets.

Labor markets, whenever condition (1.13) is binding, will determine hours worked by the demand side, only. This will generate some involuntary unemployment due to high cost in that labor market. To summarize, when wages are free to move labor markets will clear, otherwise there will be an excessive supply of labor hours.

$$(L_{H,t} - \chi l_{H,t})\left(W_{H,t} \; - \psi W_{H,t-1}\Pi_t\right) = 0 \tag{1.14}$$

and

$$(L_{S,t} - (1-\chi) l_{S,t})\left(W_{S,t} \; - \psi W_{S,t-1}\Pi_t\right) = 0 \tag{1.15}$$

Notice that the economy can be in four different situations: either Savers' labor market or Hand-to-Mouth labor market or both or none can experience involuntary unemployment. The introduction of two different labor markets allows me to consider the redistributional effect of downward wage rigidities in this context.

1.3.3. Government and Central Bank

The government provides G_t units of non-rival consumption good in every period. Public expenditure is financed either by charging taxes on labor income or by issuing a bond in term of consumption good, B_t^G whose return, also denominated in units of consumption good, is the real interest rate, r_t. Government budget constraint is :

$$\tau_t(W_t^S l_t^S + W_t^H l_t^H) = G_t + \frac{B_{t+1}^G}{(1+r_t)} - B_t^G \tag{1.16}$$

Since a fraction of the population is not able to optimally smooth consumption, the fiscal authority, by borrowing and saving, can positively affect the welfare of such agents.

The objective of the Central Bank is to maintain inflation on target whenever possible. The Central Bank keeps the nominal interest rate at zero whenever the desired nominal interest is negative.

$$\begin{cases} \Pi_t = \bar{\Pi} \quad \text{if } i_t \geq 0 \\ i_t = 0 \quad \text{Otherwise} \end{cases} \tag{1.17}$$

1.3.4. Market Clearing Condition

Goods and financial markets clear, i.e.:

$$(1-\chi)C_t^S + \chi C_t^H + G_t = Y_t \tag{1.18}$$

$$B_t^G + (1-\chi)b_t^S = 0 \tag{1.19}$$

Labor markets clear only if the constraint (1.13) is not binding. Hence:

$$\left(L_{H,t} - \chi l_{H,t}\right)\left(W_{H,t} \ - \psi W_{H,t-1}\Pi_t\right) = 0$$

and

$$\left(L_{S,t} - (1-\chi) l_{S,t}\right)\left(W_{S,t} \ - \psi W_{S,t-1}\Pi_t\right) = 0$$

1.3.5. Equilibrium

Given a sequence of taxes, public expenditure and nominal interest rates $\{\tau_t, G_t\}_{t=0}^{\infty}$ *an equilibrium is a sequence of prices* $\{r_t, \Pi_t, W_t, W_t^S, W_t^H\}_{t=0}^{\infty}$ *and allocations,* $\{C_t^S, C_t^H, L_t^S, L_t^H, l_t^S, l_t^H, b_t^s, Y_t\}$ *such that:*

- *Given prices, Savers maximize* (1.1) *subject to* (1.2) *and Hand-to-Mouth maximize* (1.1) *subject to* (1.3) *;*
- *Every firm maximizes* (1.11) *subject to* (1.8) *and* (1.10) *;*
- *Goods, financial and labor markets clear,* (1.18), (1.18), (1.14) *and* (1.15)*;*
- *Government Budget Constraint is satisfied,* (1.16)*;*
- *Central Bank targets the inflation,* (1.17)*;*

1.4. Calibration

1.4.1. Quantitative Results

This section analyzes the effects of a public debt reduction on agents' welfare. The assumption made in this model that the economy is populated by two types of agents helps to capture unequal responses to fiscal shocks, shown to be important by Kanishka and Surico (2011), among others. These authors, highlight, in particular, that a variable determining different effects of fiscal changes is tightness of borrowing constraints, captured in this paper by the inability to borrow of Hand-to-Mouth agents.

I consider a scenario under which that the fiscal authority is forced to bring down its debt from an high level, B^H, to a low one, B^L in a determined time span. Such a debt reduction can be rationalized in several ways. For example, this path may be imposed by the existence of a supranational authority or by international investors willing to charge an infinite cost to the economy if at time T debt is not at the target, B^L.

Note that infinte paths are available to the fiscal authority to converge to the new steady state. I restrict my analysis to the class of monotonic decreasing deleveraging paths. This seems consistent with casual empirical evidence: as Southern European economies implemented austerity measures in response to the recent sovereign debt crisis, the proposed plans for public borrowing generally implied a mononotonically decreasing path for public debt.

I model the path of deleveraging as:

$$B_t^G = B_t^H + (B_t^L - B_t^H)\left(\frac{t}{T}\right)^{JB}$$

where I assume a strictly positive JB. Figure 1 show that JB is a measure of the speed of the public deleverage. A $JB < 1$ determines a convex deleveraging path (fast) while with a $JB > 1$ deleveraging is a concave process (slow).

Throughout the exercise, I remain agnostic about aggregate the welfare measure in this economy. I will proceed by analyzing separately the impact of different policies on the Hand-to-Mouth and Savers' welfare. This allows me to avoid imposition of

Pareto weights on the heterogeneous agents that populate this economy.

The following proposition shows how, under certain conditions on agents' value functions, studying individual consumers' welfare allows us to exclude a given set of Pareto dominated policies. This will allow us, on the other hand, to state that the optimal policy for the economy as a whole will belong to the complement of this set.

I define the welfare function as:

$$We(JB) = \sum_{t=0}^{\infty} \beta^t \left\{ aWe_H(JB) + (1-a)We_S(JB) \right\}$$

where $We_H(JB)$ and $We_S(JB)$ are the Hand-to-Mouth and Savers value function, respectively, as a function of JB and a is a generic weight that can take any value in the interval $[0,1]$. I define JB^H and JB^S, respectively as

$$JB^k = \arg\max_{JB} We_k(JB) \quad k = \{S, H\}.$$

Moreover, I define JB^W as:

$$JB^W = \arg\max_{JB} We(JB)$$

Proposition 1 *If agents' value functions are continuous, twice differentiable and concave in JB, with $JB^S < JB^H$ ($JB^H < JB^S$), then the JB^W that maximize W for a generic $a \in [0,1]$ will lie in the interval $[JB^S, JB^H]$ ($[JB^H, JB^S]$). All the JB who do not belong to this interval are Pareto dominated.*

Proof is in appendix A.

I will restrict the set of possible actions undertaken by the government to reduce debt to include either reductions in government expenditure or increases in tax rates, but not both. Obviously, fiscal authorities in the real world may adopt a combination of these. However, this restriction allows to consider separately the effects of different types of fiscal contractions.

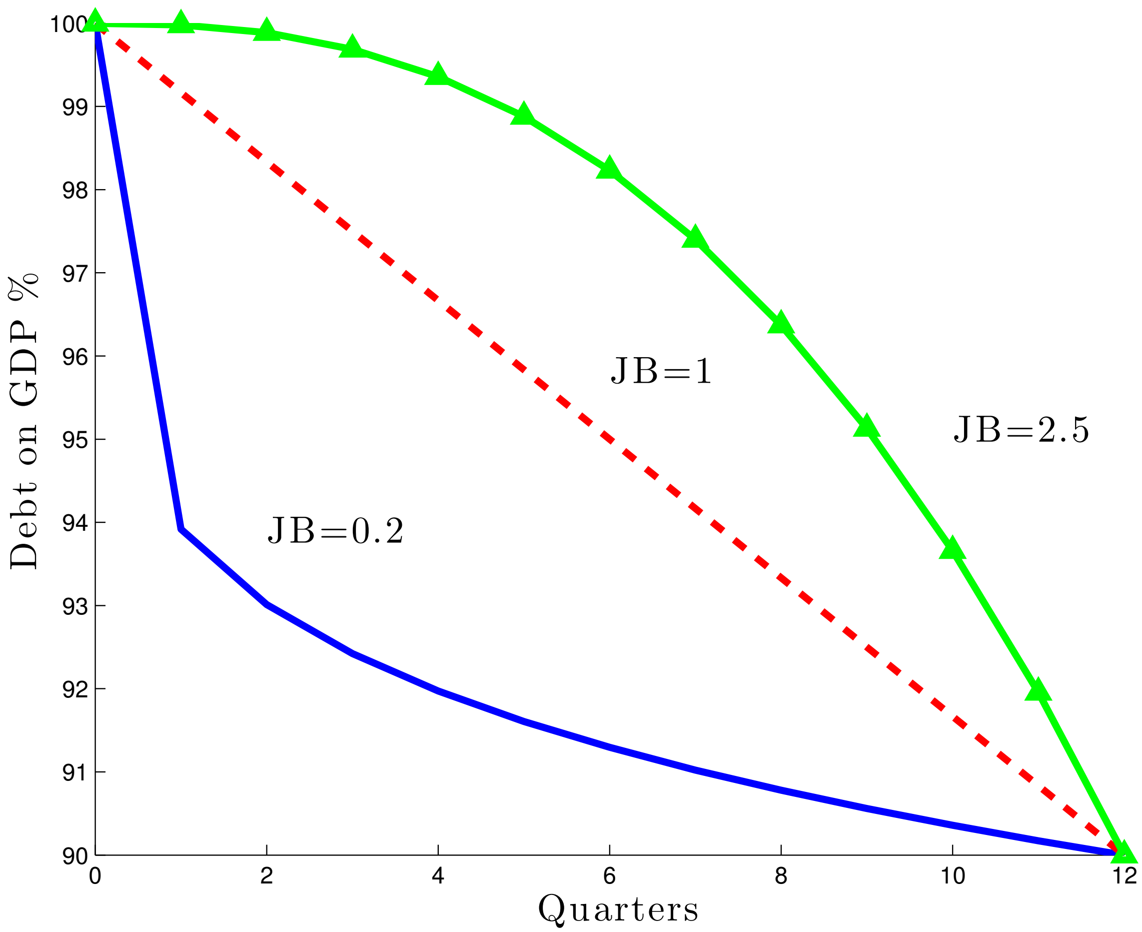

Figure 1: Figure shows different deleveraging path according to different JB. A $JB < 1$ (blue continuos line) represents a convex (fast) deleverage, a $JB = 1$ (red dotted line) stands for a smooth deleverage while a $JB > 1$ (green line) represents a concave (slow) deleverage.

1.4.2. Calibration

The model is calibrated quarterly. I use a global method to take into account non linearities that may arise from the Zero Lower Bound and downward nominal wage rigidities. Preferences take the following functional form:

$$U(c^j, G, l^j) = \frac{(c^{j\,\psi} G^{1-\psi})^{1-\rho}}{1-\rho} - \frac{l^{j\,(1+\eta)}}{(1+\eta)} \quad for\ j = \{S, H\}$$

where I set $\psi = .9$ and $\rho = \eta = 2$ in line with the literature[1]. I assume that 80% of consumers are Savers - $\chi = .2$. I set α_S equal to 0.8 such that individual labor income is identical for the Savers and for t Hand-to-Mouth[2]. I set ω, the parameter governing decreasing return to scale to equal 0.9.

I set real rate in line with literature at 2.5% and the steady state gross inflation equal to 1. I assume that the elasticity of substitution among the different varieties, ϵ, is equal to 8 such that the markup is 1.14.

I set tax in initial and final steady state to match the median payroll tax in US, $\tau = .127$. In 2013 US debt to GDP held by the public but not by the Federal Reserve System equalled 55%. More than half was held by foreign investors. Hence, I set the debt to GDP at yearly basis at 27.5% and I decrease this to 25% during the exercise.

In all the exercises, I set public expenditure as a share of GDP, s_g, to equal .931 in the initial steady state and equal to .938 in the final steady state. When debt reduction occurs through income tax I move in the first quarter s_g from .931 to .938. The time span of deleveraging, T, is equal to 4 quarters.

[1]De Walque, Smets and Wouters (2005)

[2]From firms' cost minimization $\frac{(1-\chi)}{\alpha_S} W^S l^S = \frac{\chi}{\alpha_H} W^H l^H$.

Parameters	Flexible Prices	Rotemberg Costs (RC)	RC and Wage Rigidities
η	2	2	2
ρ	2	2	2
χ	.2	.2	.2
α_S	.8	.8	.8
ω	.9	.9	.9
ϵ	8	8	8
τ	.127	.127	.127
$D_{GDP}^{initial}$	.275	.275	.275
D_{GDP}^{final}	.25	.25	.25
$s_g^{initial}$	.931	.931	.931
s_g^{final}	.938	.938	.938
ϕ	0	77	77
ψ	0	0	.97

Table 1: This table shows parameters values used for calibration under different the exercises of the chapter/paper.

1.5. Benchmark

In this section I assume that $\phi = \psi = 0$, so that prices are perfectly flexible, and that the nominal interest rate can go below zero, so that the constraint (1.17) does not hold. This formulation is helpful to understand the main mechanism behind the model and it will be the benchmark for all other exercises.

1.5.1. Public Expenditure

In this section I will consider the effects of of a reduction in public debt that is achieved by reducing public expenditure. The fiscal authority will create a shock on the demand side that will lead, independently on the speed of the deleverage, to an output recession. Savers, during the transition, are forced to consume more, as government is cutting back debt. Consequently, the real interest rate falls when deleveraging occurs. This is a key result since the government as well as Hand-to-Mouth consumers will benefit from a low rate.

It is important to note that, the less gradual the process of debt reduction, the more sizable will be the drop in the real rate. As a result, Hand-to-Mouth prefer a debt reduction that occurs in as few quarters as possible. Moreover, they would like deleverage to take place as soon as possible. The explanation is straightforward: assume that most of the debt reduction occurs in the last quarter. Knowing this, Savers will be willing to move resources from the future to the present. This will put an upward pressure on the real rate before deleveraging has been undertaken. The government would then pay a high interest rate on a large stock of debt. As a consequence, Hand-to-Mouth consumers dislike such a deleveraging path and they would prefer the debt reduction to occur as soon as possible.

As shown in Figure 2 , under this public expenditure experiment, Hand-to-Mouth prefer an extremely fast deleverage. The fiscal authority severely reduces public expenditure in the first quarter. Output collapses since firms face a drop in demand. Savers, being forced to save less, experience a boom consumption. The real rate decreases, reducing financing costs for the government. Note that, despite the aggressive policy stance, debt to GDP do not fall immediately due to the recession.

Movements in the real rate will also drive Savers' choice of optimal deleveraging path. Since they want to avoid an extreme drop in the real interst rate, they prefer, as Figure 2 shows, a relatively slow and smooth reduction of debt. Savers, knowing that they will experience a consumption boom in the last quarters, are willing to dissave in the first quarters. The real rate falls in the last quarter but, in the preceding periods, it will be relatively high. The economy will experience a prolonged output recession, mostly due to the fall in Hand-to-Mouth consumption and government expenditure. Debt to GDP will converge slowly to the new steady state.

Note that in both cases, in general equilibrium, a big fall in the real interest rate prevents large fluctuations in real variables.

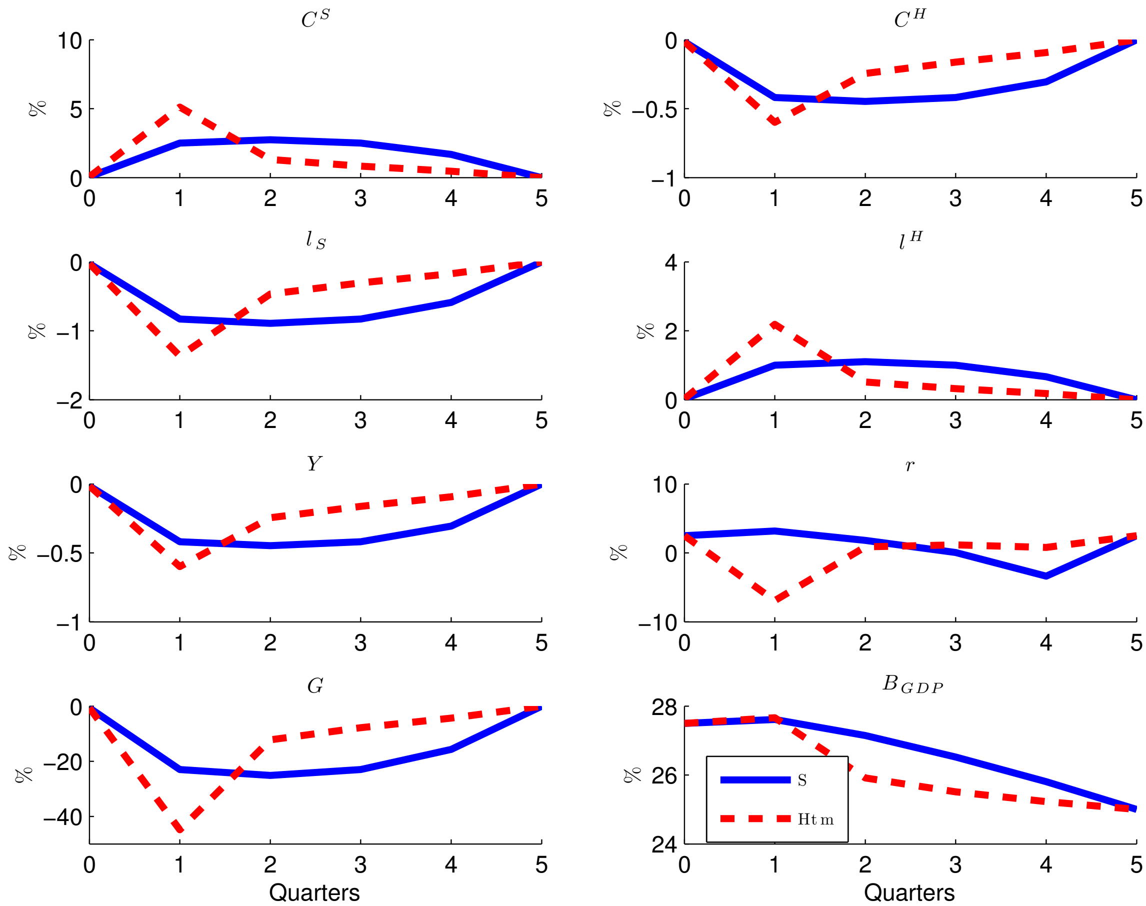

Figure 2: Figure show the optimal i.r.f. for the Hand-to-Mouth consumers (red dotted line) and for the Savers (blue continuos line) when fiscal authority reduces his debt using public expenditure. Real rate, r, and debt on GDP, B_{gdp} are in percentage levels, while the other variables are in percentage deviations from the final steady state.

1.5.2. Taxation

I consider now an experiment in which the fiscal authority raises income tax to finance reduction in debt.

As before, independently on the speed of deleverage, the economy will experience an output recession. Differently from before, however, the negative demand shock, will be only indirectly created by the government. Agents, facing high distortionary taxes, are poorer. As they consume less, output is depressed.

On the other hand, during the debt reduction, the real interest rate will now increase. Savers, indeed, face a reduction in their after-tax labor income. Having access to the financial market, they are then willing to borrow, putting upward pressure on the real rate. The government then has to deleverage when financing costs are high. Tax pressure on the agents needs to increase further to finance the debt reduction. Consumption falls by more, further increasing the pressure on real rate. The economy enters a vicious circle of deep output recession and high interest rate.

Note that, despite the fall in own private consumption and public expenditure, Savers gain during the transition, independently on the deleveraging speed. Their financial income increases, allowing them to enjoy more leisure when their wage is lowered by distortionary taxation.

Differently from the previous experiment, it is Savers who want to affect the interest rate the most, as this increases during the transition. Hence, they will prefer a very fast deleveraging. As Figure 3 shows, in their preferred path, the fiscal authority performs the whole debt reduction in the first quarter causing a deep, but short, recession. Debt to GDP will initially increase.

Hand-to-Mouth, under this experiment would prefer instead a slow debt reduction, as they want to reduce upward pressure on the real rate. As shown by Figure 3, the economy enters a long output recession. The real rate is close to the steady state, spiking in the last quarter. Compared to the previous case, Hand-to-Mouth consumption decreases by more, while labor supply increases due to the high real interest rate.

Under this experiment, again, debt to GDP initially increases due to the endogenous recession.

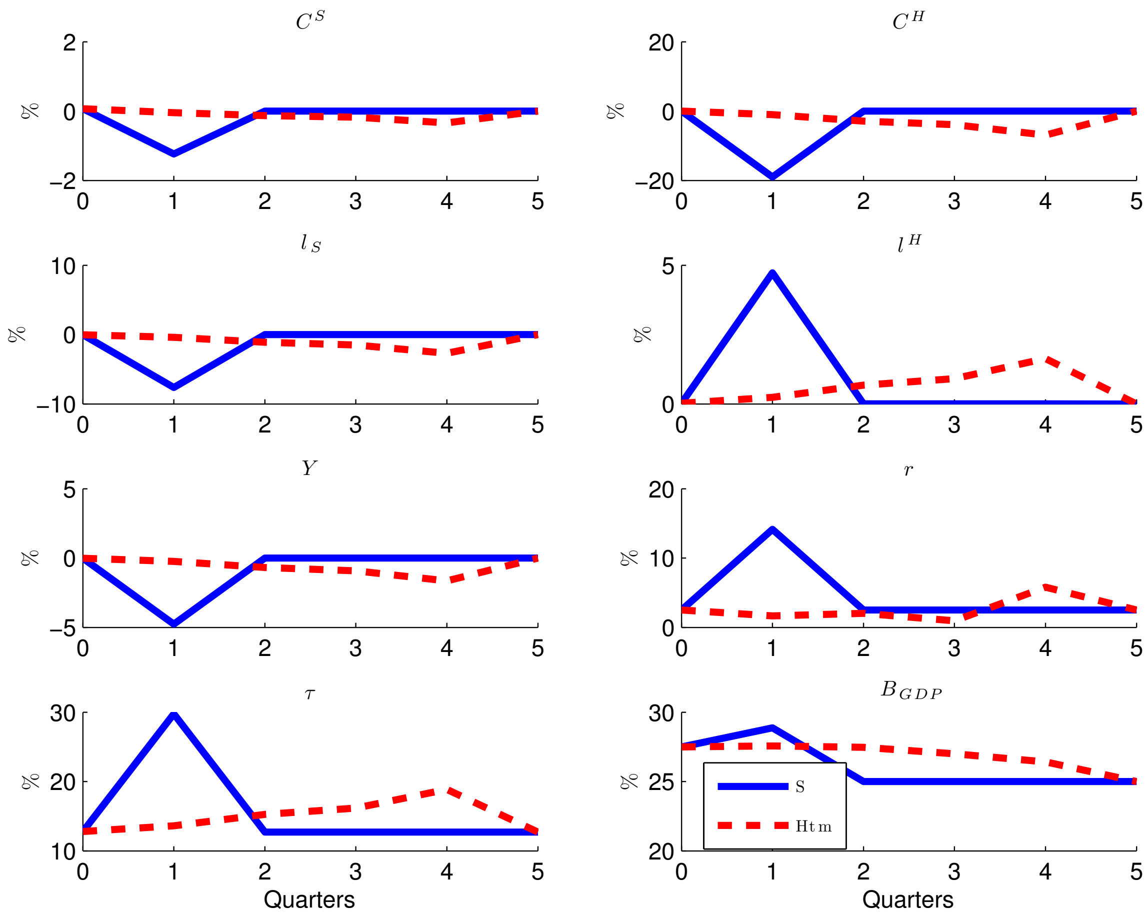

Figure 3: Figure show the optimal i.r.f. for the Hand-to-Mouth consumers (red dotted line) and for the Savers (blue continuos line) when fiscal authority reduces his debt using tax on labor. Real rate, r, debt on GDP, B_{GDP} and tax share, τ are in percentage levels, while the other variables are in percentage deviations from the final steady state.

1.5.3. Consumption Equivalent

In order to compare welfare under the two fiscal experiments I compute the consumption equivalent, c_e as:

$$\sum_{t=0}^{\infty} \beta^t U(C_t^j, G_t, l_t^j) = \frac{U(C^j - c_e^j, G^j, l^j)}{1-\beta} \quad j = \{S, H\}$$

where the right hand side is utility for consumer j computed along the simulations and C^j, G and l_j are consumption, public expenditure and labor, respectively, computed at the final steady state for consumer of type j. Following Lucas (1987), I define consumption equivalent as the decrease in steady-state consumption that makes agents indifferent between a constant consumption path and the time-varying one achieved in the simulation. Under this definition, a positive consumption equivalent amounts to a welfare cost while a negative consumption equivalent amounts to a welfare gain.

In Figure 4 I plot consumption equivalent as a percentage share of final steady state consumption for Hand-to-Mouth (first panel) and Savers (second panel) as a function of the speed of deleveraging, JB. Solid line refers to deleveraging achieved via a reduction in public expenditure, while the solid line refers to the increase in taxes experiment. [3]

In this flexible price setup, Hand-to-Mouth lose during the debt reduction, independently of the speed of deleveraging. On the other hand, Savers may gain or lose depending on the speed. The Figure shows how some deleveraging paths are Pareto dominated by others: for example, under tax experiment for which $JB > 1.75$, where 1.75 is the minimum consumption equivalent achievable by the Hand-to-Mouth, wel-

[3]Note that the consumption equivalent is a continuos and convex function in the debt reduction speed JB. It follows that agents' value functions are continuos and concave in JB. The assumptions of Proposition 1 are then satisfied.

fare of both agents would improve by choosing a lower JB .

Limiting the analysis to the interval of debt reduction speeds JB that are not Pareto dominated, Hand-to-Mouth lose less, in term of welfare, from a deleveraging achieved by a fall in public expenditure than from one achieved by an increase in taxes. The main difference between the two experiments is in the behavior of the real interest rate. Hand-to-Mouth, once they internalize the government budget constraint, are net borrowers. As a consequence, they dislike high real interest rates. Moreover, under the tax experiment, the economy enters a deeper recession than under the public expenditure case. Hand-to-Mouth agents, having no access to the financial market, will be more exposed to the cost of the recession.

On the other hand, Savers have strong preferences for deleveraging via taxation. Indeed, oppositely than Hand-to-Mouth, they hold public bonds, thus benefitting from high real interest rates.

Recently, in Southern European countries, there has been a large debate on how to implement austerity measures. One of the positions suggests to heavily tax savers, in order to achieve a more equal distribution. In light of this model, under flexible prices, this is detrimental for agents who have no access to financial markets. This result crucially depends on general equilibrium effects: Abstracting from debt reduction, let us consider the effects of policies that aim to subsidize Hand-to-Mouth agents by taxing the wealthy. Savers will work less in response, consuming their financial wealth instead. The real interest rate increases and output falls. A government who is a borrower will pay a high interest rate on his debt while tax revenues increase less due to the output recession. It is likely that, due to the recession and to the real interest rate rise, the government will have a hard time in achieving its objective.

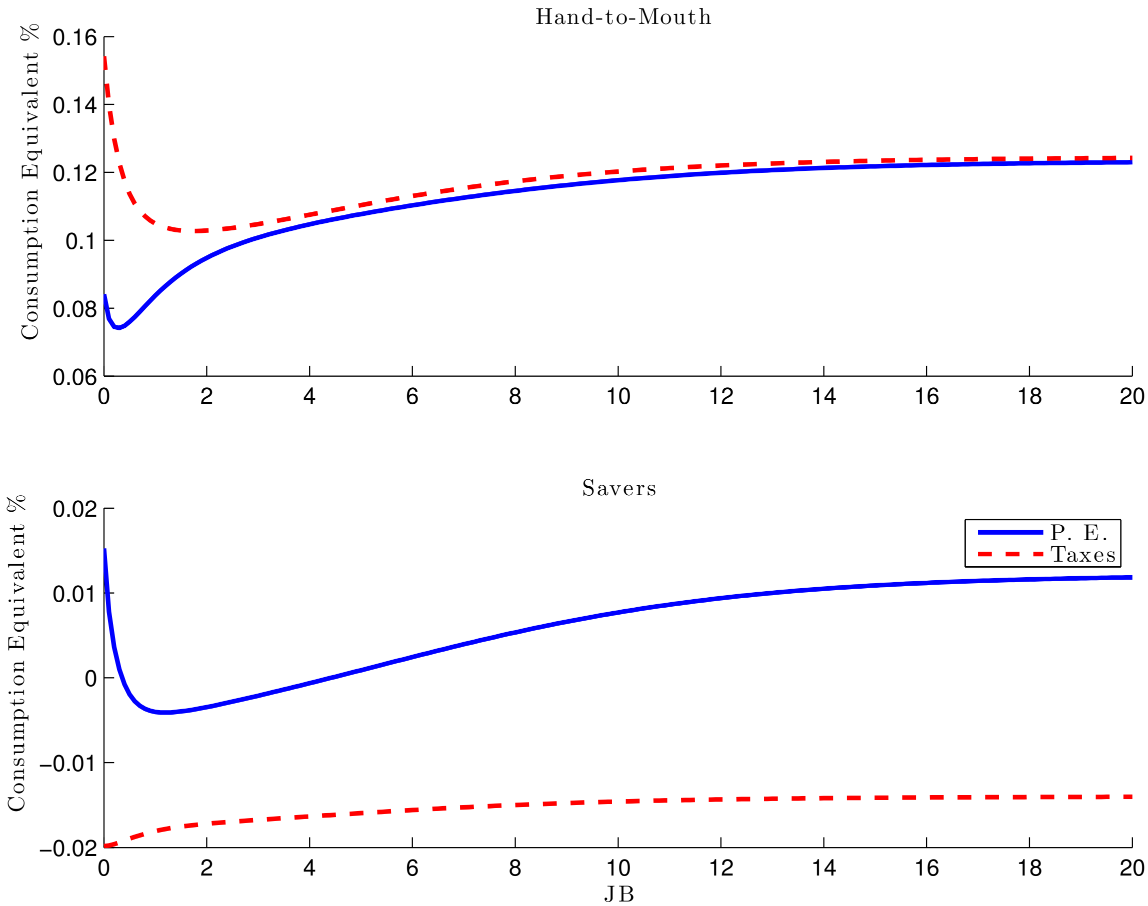

Figure 4: Figure shows consumption equivalent as a percentage of final steady state consumption for Hand-to-Mouth (first panel) and Savers (second panel) both under public expenditure (continuos blue line) and taxation (red dotted line). In both Figures consumption equivalent is a function of deleveraging speed, JB.

1.6. Nominal Rigidities

Some of the results in the previous section hinge fundamentally on the assumption of perfect price flexibility. Figure 2 shows that the real rate needs to go below zero whenever the government reduces his public expenditure in order to deleverage. Negative real interest rates prevent real variables from experiencing large fluctuations. While negative real interest rates have been observed in reality, this is generally not the case for nominal interest rates, placing a lower bound on real rates too. I assume then in this section that the Central Bank is constrained by the presence of the Zero Lower Bound on nominal interest rates (eq. (1.17)).

In the previous formulation, additionally, firms were able to absorb negative demand shocks by cutting nominal wages. Again, this might not be feasible since nominal wage are downwardly rigid in the data. To capture such features in the context of my experiment, I consider now positive Rotemberg adjustment costs and occasionally binding downward wage rigidities. In subsection 1.6.1 I assume the presence of Rotemberg costs without wage rigidities. Under this circumstance, I will analyze only how simulations under public expenditure change, since this is the only instance in which the real rate goes below zero. In subsection 1.6.2 I assume the presence of both Rotemberg costs and nominal wage rigidities.

1.6.1. Rotemberg costs and the Zero Lower Bound

In this section and the following, I introduce the presence of a sizeable price rigidity[4].

Under this formulation, the Central Bank sets a nominal interest rate that is equal to zero whenever it had to be negative in the flexible price case. This will affect government expenditure in two ways. On the one hand, the fiscal authority needs to

[4] Rotemberg cost $\phi = 77$

pay a higher real interest rate on public debt and, as a consequence, public expenditure will have to decrease more. Moreover, the fall in public expenditure will further depress output. Tax revenues fall and public expenditure decrease once more. Under the Zero Lower Bound, the economy enters a vicious cycle of low public expenditure and low output since, as shown by a strand of the literature[5], the public expenditure multiplier may be greater than one in this setup.

Note that, as Figure 5 shows, preferences of Savers and Hand-to-Mouth converge. As explained in the sections above, under flexible prices, extreme deleveraging paths - very slow or very fast - are the ones in which real interest rates fall more to clear the market. Consequently, if the Zero Lower Bound binds, these will be the costliest combinations for the economy. The Pareto optimal interval of values of JB shrinks substantially, as extreme choices have become very unfavourable. Due to the larger fall of productive government expenditure, Hand-to-Mouth consumers experience a welfare loss that is higher than the one under flexible prices. Savers switch from a welfare gain under flexible prices to a welfare loss.

It is now ambiguous whether Hand-to-Mouth prefers a debt reduction through taxation or public expenditure. For deleveraging paths of intermediate speed, as before, Hand-to-Mouth prefers a reduction in public expenditure. For extreme cases, however, as the Zero Lower Bound kicks in, he prefers the adjustment to take place through taxes. As deleveraging that takes place via expenditure becomes more costly, Savers, who preferred a taxation driven adjustment already, have their relative preferences unchanged.

Inroducing the Zero Lower Bound, then, has a significant effect on agents' welfare and their choices. A fiscal authority who wishes to contract public expenditure to reduce

[5]Eggertsson(2009).

debt needs to consider the possibility of entering a detrimental liquidity trap. Hence, depending on deleveraging speed, it may be the case that distortionary taxation is a more suitable policy instrument to achieve the desired result.

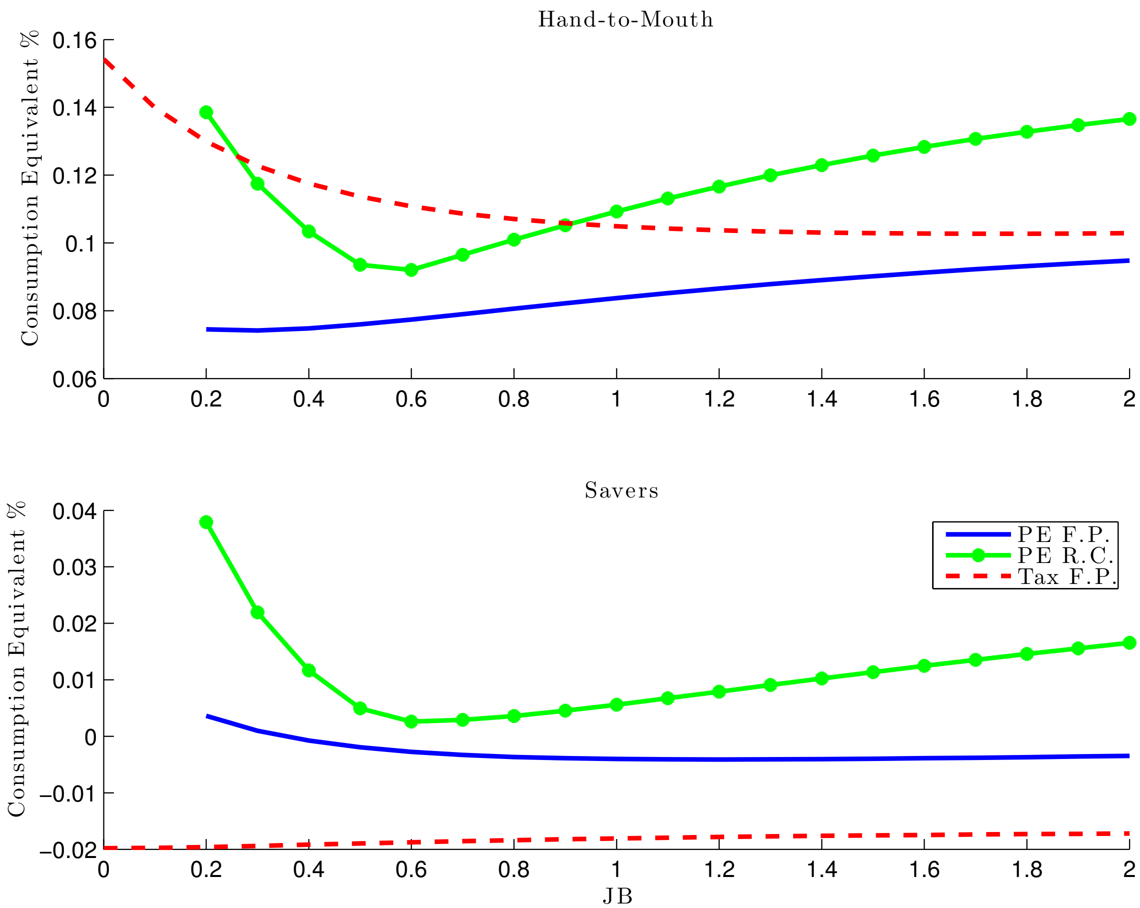

Figure 5: Figure shows consumption equivalent as percentage of final steady state consumption for Hand-to-Mouth consumers (first panel) and Savers (second panel) under public expenditure and flexible prices (blue line), under taxation and flexible prices (red dotted line) and under public expenditure and Rotemberg costs (green line). In both Figures consumption equivalent is a function of deleveraging speed, JB.

1.6.2. Wage Rigidities

Recently, a big strand of the literature, like Benigno and Ricci (2011), Schmitt-Grohé and Urîbe(2011) and Calvo et al. (2013) among others, has analyzed how nominal downward wage rigidities affect economies both in the short and in the long run. Downward wage rigidities, under certain condition, can magnify recessions : Since firms are not able to decrease their costs, they will decrease their production. In the next subsections I will study how the presence of downward wage rigidities together with Rotemberg costs, changes agents' welfare and preferences. In equation (1.13) I set ψ, the parameter governing the extent of wage rigidity, to equal 0.97. As I will show, downward nominal wage rigidities have a different impact according to the type of recession the economy is facing.

Public Expenditure

To understand results under downward wage rigidities, it is useful to consider how labor reacts during deleveraging under flexible prices. In this circumstance, when debt reduction occurs, output falls and firms demand less labor. Savers, on one hand, experiencing a boom in consumption, will enjoy more leisure. On the other hand, Hand-to-Mouth, experiencing a negative shock on their consumption and public expenditure, increase their labor supply. In equilibrium, then, Hand-to-Mouth wage falls more than Savers' one. Adding Rotemberg costs does not change labor markets dynamics but, as I explained above, the economy is lead into a liquidity trap.

The introduction of downward wage rigidities directly affects the Hand-to-Mouth labor market, since it is the one where wages fall the most. Firms, as a result of the increase in wages, will hire less Hand-to-Mouth and more Savers. As Figures 6 and 7 show, adding downward wage rigidities only affects significantly labor market variables while aggregate variables remain largely unchanged due to the liquidity trap. High labor market costs, indeed, exert an upward pressure on nominal prices, making deflation less severe. Nominal interest rate will be still at zero. Consequently, the real interest rate will either remain unchanged or slightly fall, either leaving unaffected or mildly dampening the extent of the recession.

Downward wage rigidities under a liquidity trap, then, will be a transfer of resources from Savers - who work more- to Hand-to-Mouth - who work less. This result critically depends on the presence of the liquidity trap. As detailed by Eggertsson (2010), when the Zero Lower Bound binds, the "paradox of flexibility" may hold. This is a situation in which flexible prices are more detrimental for the economy than rigid ones. Hence, in this case, nominal rigidities, especially regarding wages, will not exacerbate the cost of deleveraging on the economy.

As Figure 9 shows, downward wage rigidities reduce welfare costs for Hand-to-Mouth agents, while increasing costs for Savers independently on the speed of debt reduction. It is still ambiguous whether Hand-to-Mouth prefer public expenditure or taxation. Indeed, for a fast deleveraging, they bear a lower welfare cost under public expenditure, while, for a slow one, they are better off in the case of increase taxation. Agents' preferences regarding the speed of deleveraging under public expenditure remain, instead, unchanged.

A government willing to reduce debt through public expenditure needs to consider the possibility of entering a liquidity trap. If this is case, there are few deleveraging paths that are Pareto dominant, namely those that minimize the fall in public expenditure. Moreover, a fiscal authority who wants to redistribute resources from the rich to the poor may consider enforcing policies that increase the extent of wage rigidities. Note that, in this analysis, I do not consider idiosyncratic effects of unemployment. It could be the case, in a more accurate analysis, that, ex-post, unemployed Hand-to-mouth would be worse off under nominal wage rigidities. Indeed, the presence of these kind of rigidities, may decrease the probability to become employed once a worker is unemployed. This analysis is left for future research.

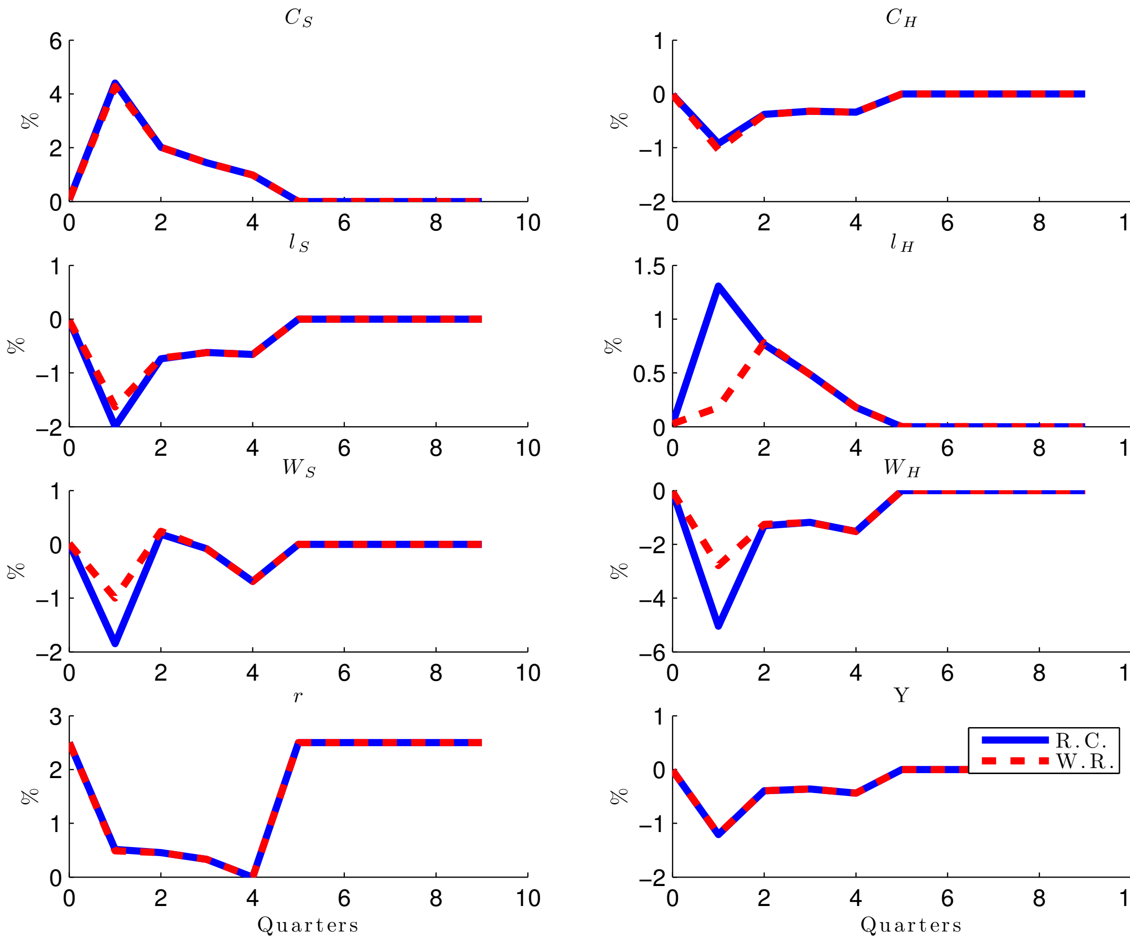

Figure 6: Figure shows i.r.f. for the Savers and Hand-to-Mouth optimal deleveraging path under Rotemberg costs and public expenditure experiment. Blue line are simulations under Rotemberg costs, while red dotted line under Rotemberg cost and wage rigidities. All the variable, except r, are in percentage deviation from the final steady state. Real rate, r, is in percentage level.

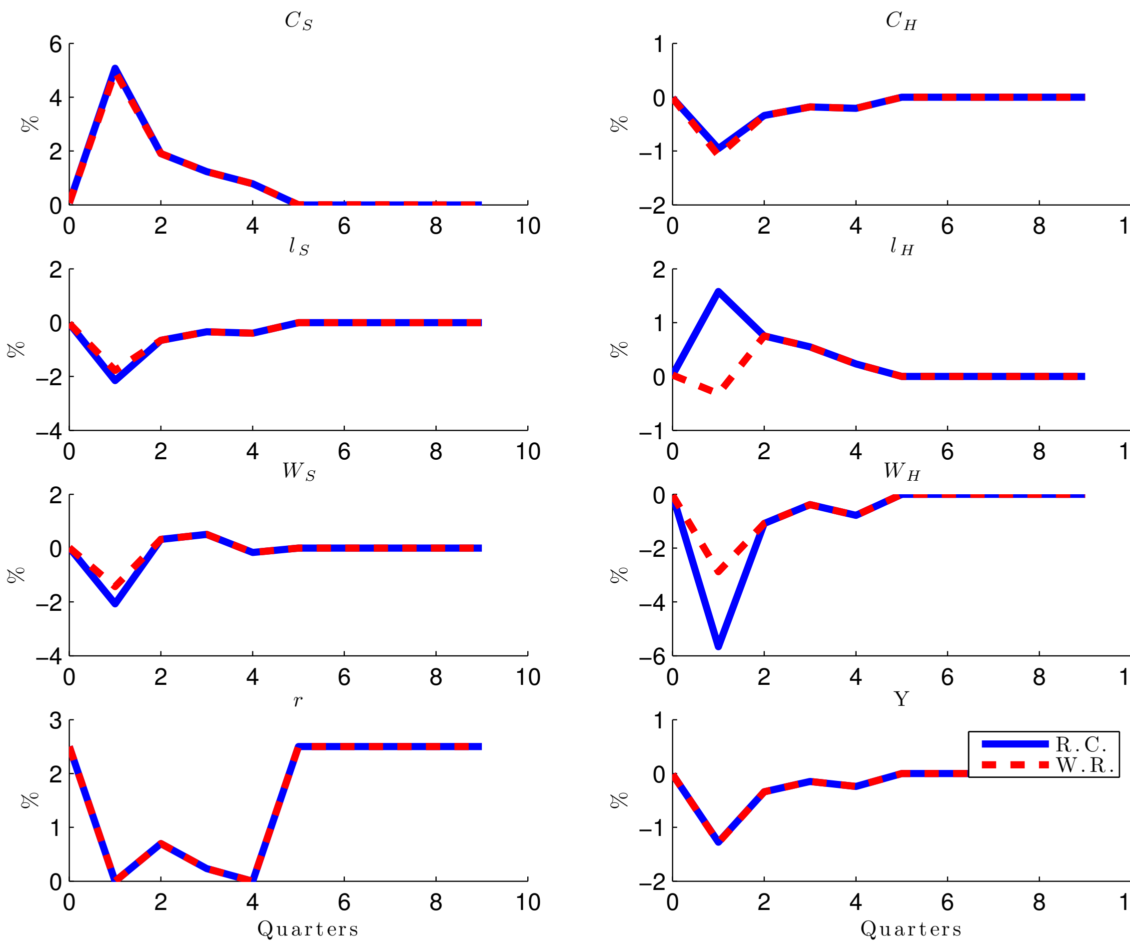

Figure 7: Figure shows the optimal deleveraging path for the Hand-to-Mouth under public expenditure experiment and wage rigidities. Blue line are simulations under Rotemberg costs, while red dotted line under Rotemberg cost and wage rigidities. All the variable, except r, are in percentage deviation from the final steady state. Real rate, r, is in percentage level.

Taxation

When a government is deleveraging through taxation, downward wage rigidities, whenever binding, are welfare decreasing. Since nominal wages increase, prices increases too, exerting upward pressure on the inflation rate. As a consequence, the Central Bank needs to raise the nominal interest rate and the government, paying an higher real interest rate,has to raise taxes. The recession is, then, deeper than under flexible prices. Differently than under the public expenditure experiment, aggregate variables dynamics change significantly with the introduction of nominal wage rigidities, as Figure 1.13 shows. There Savers optimal deleveraging path under flexible prices is compared with the same deleveraging path under wage rigidities.

Note that downward wage rigidities bind whenever the deleveraging speed is too slow or too fast, as shown in Figure 9. Under these combinations, indeed, real interest rates are very high in equilibrium, as explained in section 1.5.2. The higher interest rates, the higher will be the shock on Hand-to-Mouth consumption and, as a result, the higher will be their increase in labor supply. Consequently, under these debt reduction paths, nominal wages fall substantially. Under flexible prices Savers were the ones who preferred extreme debt deleveraging combinations, as they gained from high real interest rates. This beneficial effect is now dampened by the cost introduced by downward wage rigidities. They will prefer, then, the fastest debt reduction path under which the downward wage rigidity does not bind. Hand-to-Mouth, on the other hand, still find optimal the same debt reduction path they chose under flexible prices.

Despite the deep recession, Savers still gain in welfare term from a deleveraging that occurs via taxation. Hence they keep preferring this experiment to the public expenditure one . Hand-to-Mouth preferences change accordingly to the speed of debt reduction. If deleveraging is fast they prefer public expenditure, if instead it is slow they prefer taxation.

Finally, it is important to highlight that, after the introduction of downward wage rigidities, Savers' optimal path is closer to the Hand-to-Mouth one. If the government plays a Pareto dominant strategy, Hand-to-Mouth gain from the introduction of downward wage rigidities. Indeed, the Pareto dominant interval shrinks, excluding the combinations of deleveraging paths that decreased the most Hand-to-Mouth welfare under flexible prices. Then, even under the taxation experiment, they will gain from the introduction of downward wage rigidities.

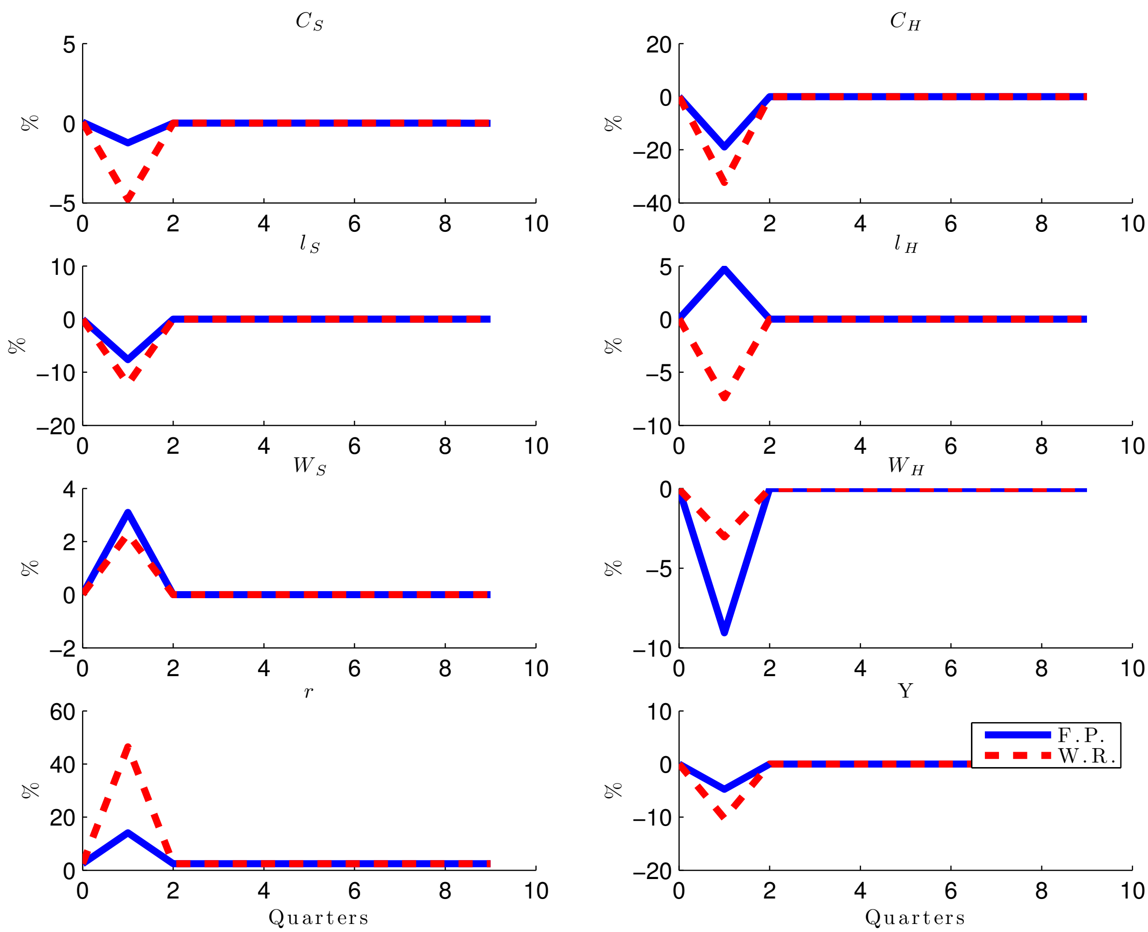

Figure 8: Figure shows i.r.f. under optimal Savers' deleveraging path in the flexible prices model (blue line). Red dotted line represents i.r.f under the same deleveraging when downward wages rigidities bind . All the variables, except r, are in percentage deviation from the final steady state. Real interest rate, r, is in percentage level.

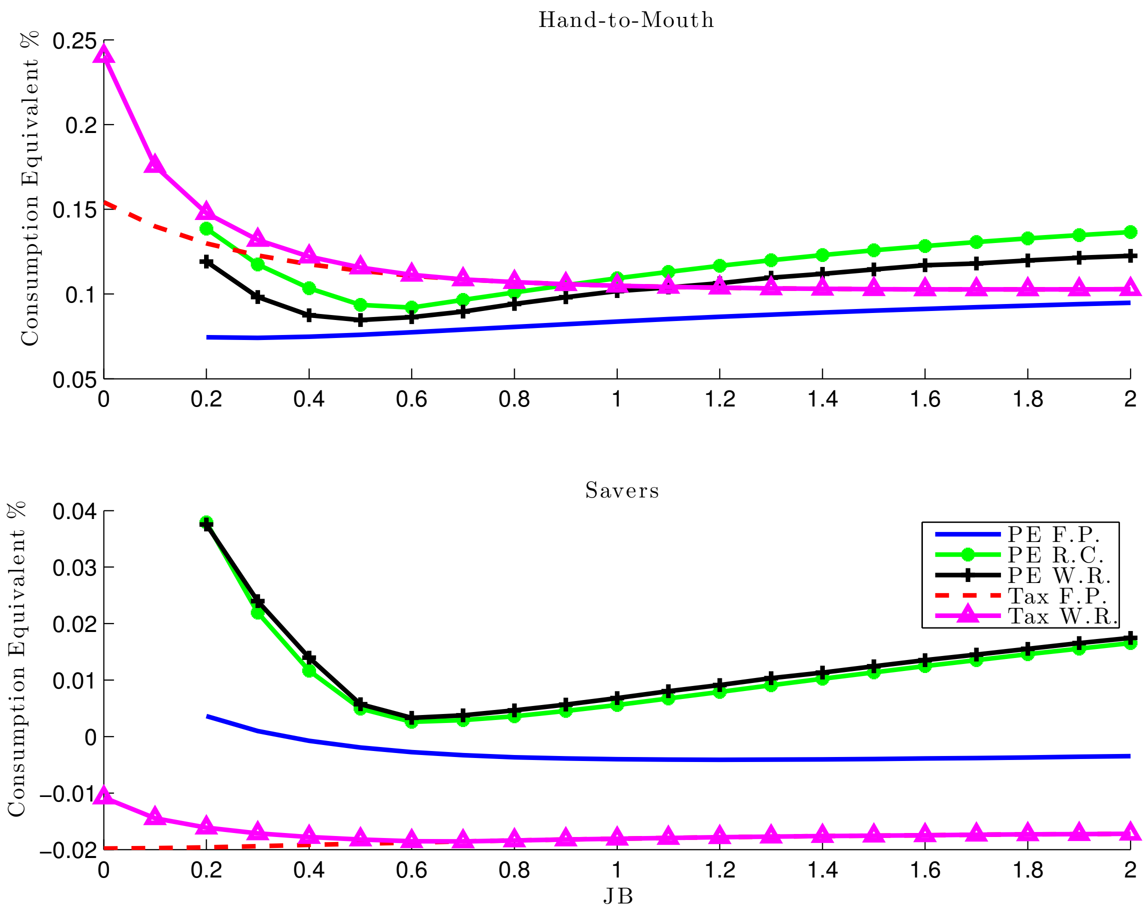

Figure 9: Figure shows consumption equivalent as percentage of final steady state consumption for Hand-to-Mouth (fist panel) and Savers (second panel) under different exercises: public expenditure (blue) and taxation (red) under flexible prices , public expenditure under Rotemberg cost (green), public expenditure (black) and taxation (magenta) under Rotemberg cost and Downward wage rigidities. Both Figures shows consumption equivalent as a function of deleveraging speed, JB.

1.7. Time Span

Lastly I analyze whether and how the debt reduction time span changes agents' welfare. On one hand a longer time span decreases the size of the per-quarter shock, on the other hand it takes more time for agents to achieve a constant level of consumption. To address this question I focus on the Pareto dominant interval of deleveraging paths. As all this combinations are Pareto optimal, the maximum achievable by one agent corresponds to the minimum for the other. Figures 10 and 11 show the minimum and maximum consumption equivalent[6] achievable by each type of agent as function of debt deleveraging time span, T. Figure 10 shows, for the public expenditure experiment, the relevant Pareto intervals under flexible prices, Rotemberg costs and downward wage rigidities (with Rotemberg costs). Figure 11 corresponds to the taxation experiment.

Under public expenditure Savers gain slightly from an increase in the time span. Nevertheless, the magnitude of the gain is negligible. The effect of increasing the time span on Hand-to-mouth welfare is instead significant and counterintuitive. As T increases, Hand-to-Mouth welfare worsens, in both best and worst case scenarios. Indeed, with a shorter time span, as the shock on output demand is larger, real interest rates tend to be lower. The higher will be the time span, the higher will be the real interest rate and the lower will be Hand-to-Mouth welfare. Moreover, Figure 10 also shows that the Pareto interval widens, meaning that the higher the time span, the more Savers and Hand-to-Mouth preferences will differ. Then, as T increases, the variability of outcomes increases for the Hand-to-Mouth.

Under the taxation experiment, Savers are, again, indifferent to the time span, both

[6] Remember that the consumption equivalent is decreasing in welfare.

under flexible prices and under downward wage rigidities. Hand-to-Mouth, on the other hand , experience an improvement in their welfare, as the time span increases . When a government reduces debt through taxation, interest rate are higher. As T increases, the per-quarter-shock is smaller and, as a consequence, interest rates increase less. Hand-to-Mouth, then, experience a welfare gain with an higher T.

Concluding, Savers are largely unaffected by the time span while this is not the case for Hand-to-Mouth. Under taxation they would prefer a longer time span of deleveraging while, under public expenditure, they want the fiscal authority to reduce debt in a short time span. Since this variable is unimportant from the Savers' point of view, once chosen an instrument, the government, can adjust T to maximize Hand-to-Mouth welfare.

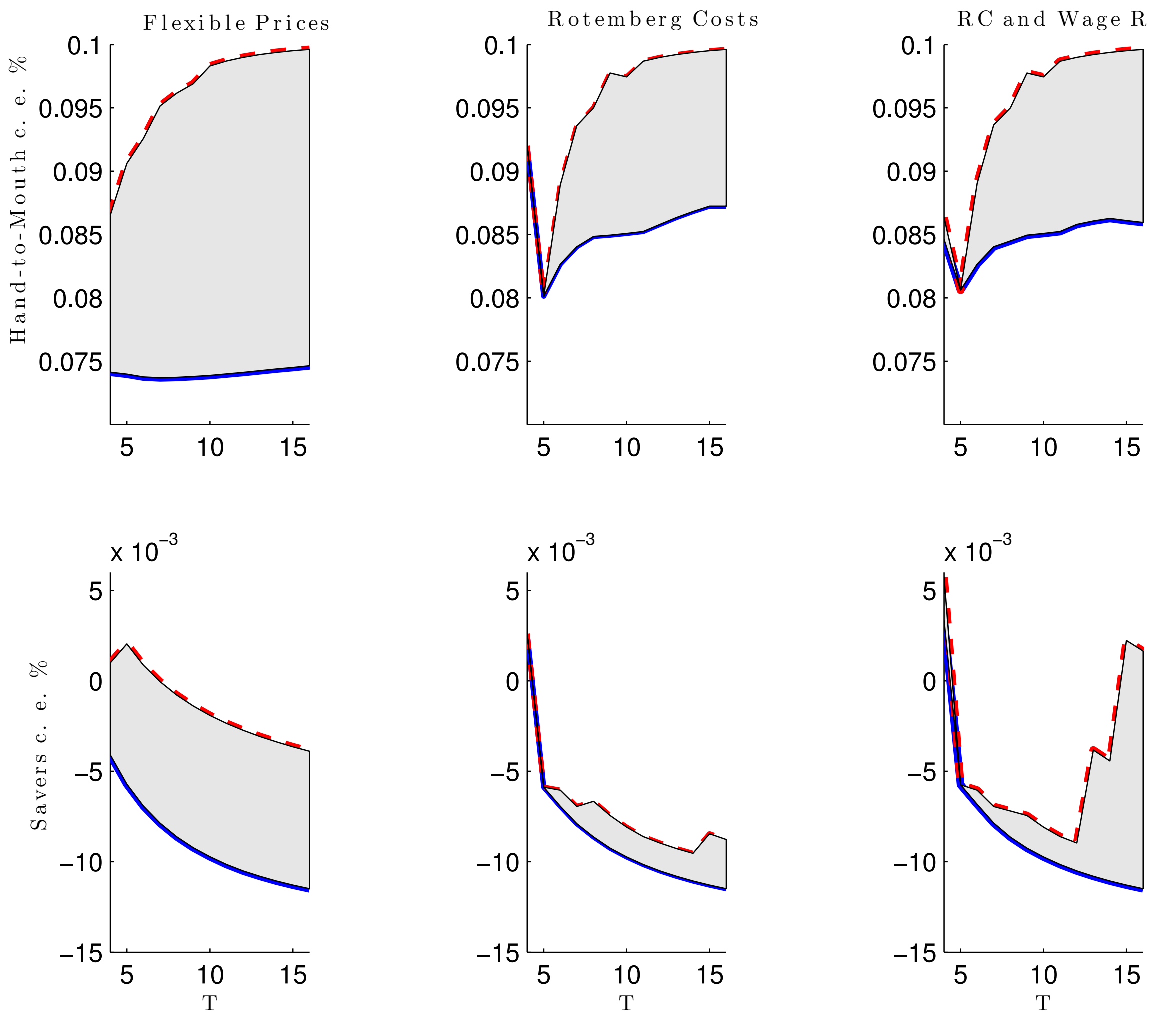

Figure 10: Figures show the minimum (blue continuos line) and maximum (red dotted line) consumption equivalent as percentage of final steady state consumption as a function of time span, T under the public expenditure experiment. The first row refers to Hand-to-Mouth while the second to the Savers. First column refer to Flexible Prices case, second to Rotemberg costs and the third to Rotemberg costs plus downward wage rigidities

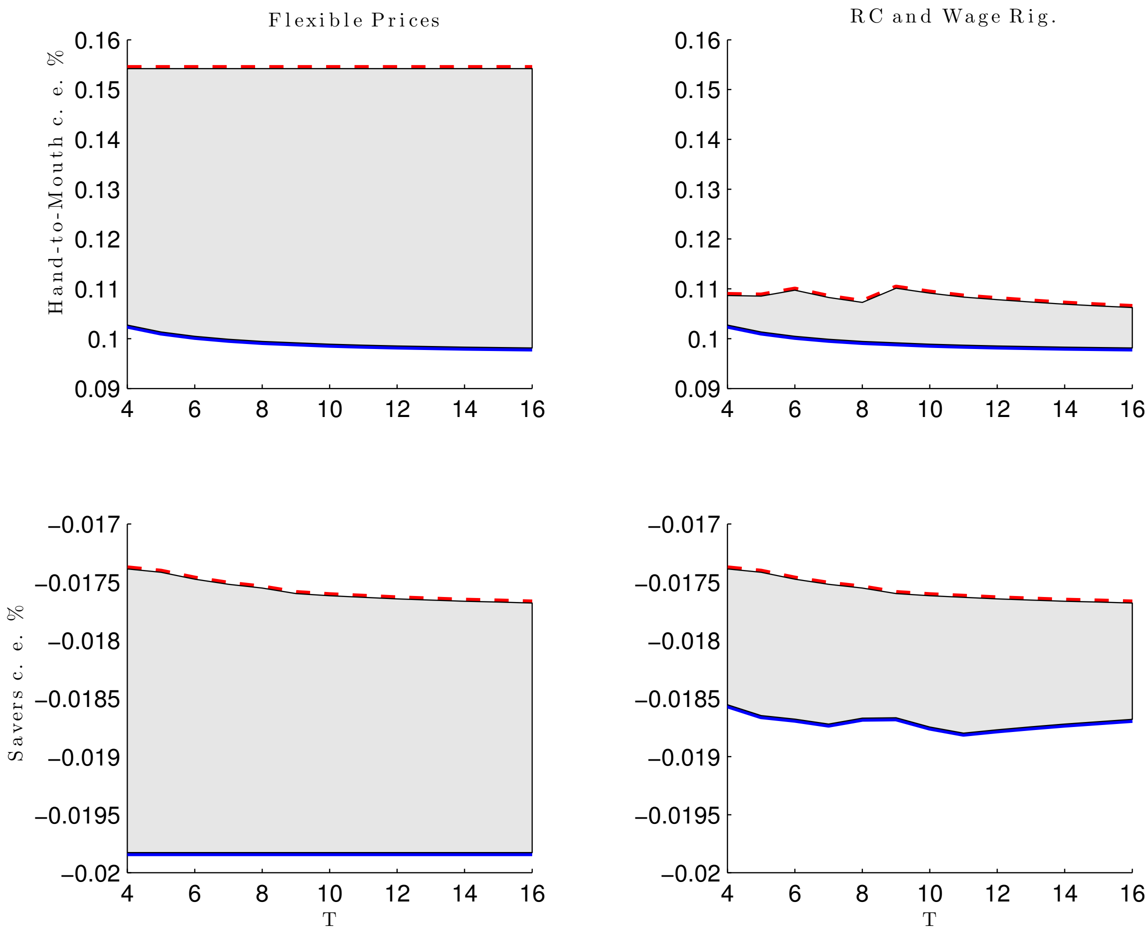

Figure 11: Figures show the minimum (blue continuos line) and maximum (red dotted line) consumption equivalent as percentage of final steady state consumption as a function of time span, T under the public expenditure experiment. The first row refers to Hand-to-Mouth while the second to the Savers. First column refer to Flexible Prices case, second to Rotemberg costs and the third to Rotemberg costs plus downward wage rigidities

1.8. Conclusions

I analyze welfare implications of different paths of public deleveraging on different types of agents. I find that, in a context of heterogenous agents and incomplete markets, the speed of deleveraging is not inconsequential for agents' welfare.

Deleveraging through income taxation leads to a recession with high real interest rates. In this situation, agents, who hold public debt prefer extreme deleveraging paths, while consumers who do not participate in the financial markets prefer slower one. An economy choosing, instead, to reduce public debt using public expenditure faces the risk to enter a liquidity trap. Under this circumstance, independently of their participation in financial markets, agents prefer the path that minimizes the fall in public expenditure.

Moreover, I find that downward wage rigidities are always detrimental for the agents who participate in financial market. These rigidities, acting as a sort of insurance in bad times, on the other hand, are helpful for the financial constrained.

The time span of deleveraging affects agents who do not participate in financial markets only, with little or no consequence for the others. Hence, the fiscal authority, should adjust this variable to maximize aggregate welfare, if unconstrained otherwise. Interesting results may emerge by relaxing some critical assumptions in this paper, namely the presence of exogenous wealth distribution and perfect foresight. Firstly, if poor agents were able to borrow, movements in the interest rate would directly affect them too. Moreover, the introduction of uncertainty can also affect the conclusions of this paper. For example, introducing uninsurable unemployed risk among the Hand-to-Mouth, I expect their preferences for nominal wage rigidities to change. Exploration of these avenues is left for future research.

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APPENDIX

Appendix A

Proposition 2 *If agents' value functions are continuos, twice differentiable and concave in JB with $JB^S < JB^H$ ($JB^H < JB^S$), then the JB^W that maximize W for a generic $a \in [0,1]$, will lie in the interval $[JB^S, JB^H]$ ($[JB^H, JB^S]$). All the JB those do not belong to this interval are Pareto dominated.*

Proof. Being an argmax JB^W satisfies the following condition:

$$We'(JB^W) = aWe'_H(JB^W) + (1-a)We'_S(JB^W) = 0 \tag{1.20}$$

where $We'(.)$,$We'_H(.)$ and $We'_S(.)$ are the first derivatives of $We(.)$, $We_H(.)$ and$We_S(.)$ with respect to JB.

By contradiction assume that $JB^W < JB^S$, then $We'_H(JB^W) > 0$ and $We'_S(JB^W) > 0$, then condition (1.20) will not be met. Again by contradiction assume that $JB^W > JB^H$, then $We'_H(JB^W) < 0$ and $We'_S(JB^W) < 0$, then condition (1.20) will not be met. If an argmax exists it should lie in the interval $[JB^S, JB^H]$. ■

Appendix B

Agents	Flexible Prices	Rotemberg Costs (RC)	RC and Wage Rigidities
		Taxation	
Hand-to-Mouth	1.75	1.75	1.75
Savers	0	0	0.7
		Public Expenditure	
Hand-to-Mouth	0.3	0.6	0.5
Savers	1.2	0.6	0.6

Table 2: This table shows optimal JB for Hand-to-Mouth and Savers both under public expenditure and taxation experiment.

CHAPTER 2

Debt Deleveraging and the Exchange Rate

[1]

2.1. Introduction

The decade leading up to the financial crisis was marked by divergences and disequilibria. Global imbalances have been at the center of the debate, with several economists warning against the unsustainability of the US external position. The euro area has experienced internal current account divergences, producing an enormous accumulation of debt. The crisis was most severe in the economies that had piled up too much private or public debt in one form or another. It is still being debated whether the divergences of the past actually caused the crisis or merely reflected other underlining problems.[2] In any case, the general tendency is for the crisis-ridden countries to reduce debt. In this deleveraging process, exchange-rate policies have been often placed under scrutiny, as in the case between US and China or in reference to the choice of irrevocably fixing exchange rates in the eurozone.

Debt deleveraging raises interesting questions on macroeconomic adjustment. A recent literature, spurred by the works of Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2010) and Philippon and Midrigan (2011), has studied the mechanism of adjustment to debt deleveraging but in closed economies. So far the international

[1] I wrote this chapter with Prof. Pierpaolo Benigno.
[2] An interesting discussion is in Obstfeld (2011), Obstfeld and Rogoff (2010).

consequences have been neglected. This is a gap that this work aims at filling given the importance of the aforementioned debate on global and European imbalances. There are two main contributions of this paper. First, to understand the international transmission mechanism of debt deleveraging. Second, to discuss its welfare consequences by asking how monetary and exchange-rate policies should be designed to better accommodate from a global perspective the ensuing macroeconomic adjustment.[3]

The transmission mechanism of a reduction in one country's external debt presents some familiar features with that of the old transfer problem, as discussed among others in Keynes (1929). Deleveraging forces debtor countries to cut spending sharply and depresses demand. Spending should increase in the rest of the world. But international relative prices might not be immune to the adjustment.[4] If the fall in demand is sharper for domestic goods, which is the case when there is home bias in consumption, the excess supply of these goods globally lowers their prices relative to foreign prices and expands overall demand for them, thus easing the depressive impact of deleveraging. These changes in relative prices are achieved by depreciation of the deleveraging country's currency. In the longer run, a country that has paid down part of its debt is richer than at first, since there is less debt to serve, so the demand for domestic goods is relatively higher. The exchange rate swings from short-term depreciation to appreciation in the long run. But, without home bias, deleveraging does not produce any movement in the exchange rate in both the short and long run.

Following the propagation mechanism described above, we could be tempted to con-

[3]It should be noted that none of the papers of Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2010) and Philippon and Midrigan (2011) deals with the welfare consequences of debt deleveraging.

[4]In the current debate on the unwinding of global imbalances, Feldstein (2011) and Krugman (2011a,b) have stressed the importance of exchange rate movements in correcting global imbalances.

clude that the exchange rate, and other international relative prices, should move substantially to mitigate the costs of deleveraging, but only when there is home-bias in goods consumption. Otherwise a fixed-exchange rate would be desirable. However, this conclusion is completely misleading if viewed from the perspective of a benevolent planner maximizing welfare in the global economy. Indeed, this planner dislikes any large variations of consumption, output and relative prices and would prefer, instead, to accommodate the adjustment in a smoother way. This is not feasible and interesting trade-offs emerge between output, consumption and terms-of-trade stabilization.

There are three available channels through which the global economy can absorb in a better way a deleveraging shock. The first channel is a pure domestic one, already emphasized by the closed-economy literature as in Eggertsson and Krugman (2012). The real interest rate in the deleveraging country should fall to reduce its borrowing costs while adjusting to a lower level of debt. To this end, a policy in which the interest rate of the deleverager stays at the zero-lower bound is desirable. The other two channels have instead an international dimension: the expenditure-switching channel and the fall in the real interest rate in the non-deleveraging countries.

The expenditure-switching channel driven by movements of the exchange rate is clearly desirable from a global perspective to the extent that can mitigate the output recession in the deleveraging country by shifting the burden of adjustment to other countries. However, it leads to costs in terms of movements in the terms of trade, which are unjustified by efficient shocks.[5] In general, the benevolent planner dislikes large variations of the exchange rate and other international relative prices. Indeed, when the expenditure-switching effect is stronger because domestic and foreign goods

[5] An efficient shock that could justify such movements is a productivity shock.

are more substitute, the optimal movements in the exchange rate are small. On the contrary, when the expenditure-switching effect is too weak, a depreciation of the currency can adversely reduce the real income of the country, making it even more poor. Also in this case, the exchange rate should not depreciate much.

A fall in the real interest rate in the non-deleveraging countries is also desirable to the extent that can raise foreign consumption to offset the recession in the deleverager.[6] However, the rise in consumption in the rest of the world is also unjustified by efficient shocks and therefore brings inefficiencies. When the expenditure-switching channel is more effective, the fall in the foreign real rate is less needed. On the contrary, when the expenditure-switching channel is weak, the real rate should fall substantially in the rest of the world. In this case a global liquidity trap can be desirable as when countries are more open to trade.

There are some earlier works related to our framework. Obstfeld and Rogoff (2001, 2005, 2007) also studied the exchange-rate implications of a sudden improvement in one country's current account balance, conducting some comparative-static experiments without analyzing the welfare consequences. Our focus here is on dynamic adjustment, on the role of monetary policy taking into account the zero lower bound and on optimal monetary policy from a global perspective. Policies at the zero lower bound, in an open economy, have been explored by Svensson (2001, 2003), Jeanne (2009) and Fujiwara et al. (2010, 2011), but in different models without debt deleveraging. There is also substantial literature on open economies analyzing credit-constrained economies and the implications of relaxing or restricting credit access for the equilibrium economy: see among others Aghion et al. (2001), Aoki et al. (2010) and Mendoza (2010) and more recently Devereux and Yetman (2010). In an

[6]This channel has a parallel in the closed-economy literature where a fall in the real rate raises consumption of savers.

open-economy model, Cook and Devereux (2011) have studied the optimal response to preferences' shocks which bring one country to the zero lower bound while appreciating its currency. In a recent work, Fornaro (2012) studies also international debt deleveraging emphasizing similar mechanisms of adjustment as in our framework. He is not concerned with welfare implications but analyzes mostly the occurrence of liquidity traps under a monetary union. Bhattarai et al. (2013) study instead optimal monetary policy in a currency-area model with financial frictions.

This paper is organized as follows. Section 2.2 describes a deleveraging shock in a simple two-country open-economy endowment model. Section 2.3 extends the basic model to include nominal rigidities and endogenous output. Section 2.4 discusses optimal policy from a global perspective. Section 2.5 performs some robustness analysis by varying the degrees of home bias and the elasticity of substitution between traded goods. Section 2.6 analyzes the case in which debt deleveraging concerns debt denominated in foreign currency. Section 2.7 concludes. An online appendix reports the main equations of the model and the solution method.[7]

2.2. A simple model

We adopt a simple two-country endowment economy to study how debt deleveraging in one country spreads to the rest of the world economy. The two countries are Home, denoted by H, and Foreign, denoted by F. Each country has an endowment of a good. The two goods, H and F respectively, are traded frictionlessly. The representative agent of country H maximizes utility from consumption

$$\sum_{t=0}^{\infty} \beta^t u(C_t),$$

[7] The online appendix is available via the journal's website.

where β is the discount factor with $0 < \beta < 1$. The consumption index C is a Cobb-Douglas aggregator of the consumption of the two goods, C_H (denoting Home goods) and C_F (denoting Foreign goods):

$$C = \left(\frac{C_H}{\alpha}\right)^{\alpha} \left(\frac{C_F}{1-\alpha}\right)^{1-\alpha}, \tag{2.1}$$

where $0 < \alpha < 1$ represents the share of consumption of goods H in the overall consumption basket, for a consumer of country H. Given the prices for the two goods, P_H and P_F, expressed in the currency of country H, the consumption-based price index of the Home country, P, is

$$P = P_H^{\alpha} P_F^{1-\alpha}.$$

Consumers in the Foreign country maximize their utility from consumption

$$\sum_{t=0}^{\infty} \beta^t u(C_t^*),$$

where the consumption basket C^* is:

$$C^* = \left(\frac{C_H^*}{1-\alpha^*}\right)^{1-\alpha^*} \left(\frac{C_F^*}{\alpha^*}\right)^{\alpha^*}, \tag{2.2}$$

and now α^*, with $0 < \alpha^* < 1$, is the weight given to goods F. The general price index in country F is:

$$P^* = P_H^{*(1-\alpha^*)} P_F^{*\alpha^*},$$

where P_H^* and P_F^* are the prices of goods H and F in the currency of country F.

The two goods are traded with no friction, and the law of one price holds

$$P_F = SP_F^*, \qquad P_H = SP_H^*,$$

where S is the nominal exchange rate, defined as units of Home currency per unit of Foreign currency. Preferences are biased towards domestic goods under the assumption that $\alpha = \alpha^* > 1/2$. In this case, our model generates deviations from purchasing power parity (PPP), in which the real exchange rate (Q) is proportional to the terms of trade $T = P_F/P_H$

$$Q = \frac{SP^*}{P} = \left(\frac{P_H}{P_F}\right)^{1-2\alpha} = T^{2\alpha-1}. \tag{2.3}$$

Given preferences and prices, demands for the goods are:

$$C_H = \alpha\left(\frac{P_H}{P}\right)^{-1} C, \qquad C_F = (1-\alpha)\left(\frac{P_F}{P}\right)^{-1} C,$$

$$C_H^* = (1-\alpha^*)\left(\frac{P_H^*}{P^*}\right)^{-1} C^*, \qquad C_F^* = \alpha^*\left(\frac{P_F^*}{P^*}\right)^{-1} C^*.$$

Consumers in the Home country receive in every period t an endowment $Y_{H,t}$ of good H, which they can sell at the price $P_{H,t}$; they consume a bundle C_t of goods H and F at price P_t; borrow or lend resources $D_{t+1}/(1+i_t)$, in units of currency of country H, and pay back or receive the face value of the funds lent in the previous period D_t. A positive value for D denotes nominal debt. D is the only asset traded internationally and $1+i$ is the one-period risk-free gross nominal interest rate on domestic currency.[8]

[8] Nominal bonds allow for meaningful asset trading even when consumption baskets are different across countries.

As a result, the flow budget constraint for consumers in the Home country is:

$$P_t C_t = P_{H,t} Y_{H,t} + \frac{D_{t+1}}{1+i_t} - D_t. \tag{2.4}$$

There is a limit on the amount of real debt that the agent can take in each period

$$\frac{D_t}{P_t} \leq k, \tag{2.5}$$

where $k > 0$. Similar constraints have been used in other open-economy models, such as Aoki et al. (2010), Devereux and Yetman (2010) and Mendoza (2010). They are justified in terms of the guarantees that international creditors require when borrowers have limited commitment. As in Eggertsson and Krugman (2012), we do not model the source of this constraint but interpret it as the maximum size of the debt that can be considered safe and that international investors are willing to lend to country H at each point in time. A change in this limit –in particular its reduction over time– constitutes the debt-deleveraging experiment analyzed here.[9] This drop can happen just for a change in confidence triggered by an internal banking or financial crisis –not modelled here– so that international investors are more reluctant to lend to country H. In the equilibrium that we are going to analyze, consumers in country H will be at the corner and (2.5) limits their borrowing capacity.

Looking now at country F, the flow budget constraint is:

$$P_t^* C_t^* = P_{F,t}^* Y_{F,t}^* + \frac{D_{t+1}^*}{S_t(1+i_t)} - \frac{D_t^*}{S_t}, \tag{2.6}$$

where $Y_{F,t}^*$ represents the endowment of good F and D_t^* the holding of nominal debt

[9] The parameter k should be not larger than the natural borrowing limit, defined as the present discounted value of all future income in units of the consumption good.

in units of currency H. Consumers in country F face a similar borrowing limit in units of their consumption basket:

$$\frac{D_t^*}{P_t^* S_t} \leq k^*, \tag{2.7}$$

for a positive k^*. In the equilibrium that we are going to analyze, consumers in country F will be creditors in international markets and the limit in (2.7) is not binding.

The optimal intertemporal allocation of consumption in country H is governed by the following Euler equation:

$$U_c(C_t) \geq \beta(1 + r_t)U_c(C_{t+1}), \tag{2.8}$$

where the home-country real interest rate is defined as

$$1 + r_t \equiv \frac{(1 + i_t)P_t}{P_{t+1}}.$$

Similarly, the Euler equation for consumers in country F is:

$$U_c(C_t^*) \geq \beta(1 + r_t^*)U_c(C_{t+1}^*), \tag{2.9}$$

where the foreign-country real interest rate is connected to the home-country real rate through

$$(1 + r_t) = (1 + r_t^*)\frac{Q_{t+1}}{Q_t}.$$

Both Euler equations hold with equality when the borrowing limit is not binding.

Equilibrium in goods and asset markets implies

$$Y_{H,t} = T_t^{1-\alpha}[\alpha C_t + (1-\alpha)Q_t C_t^*], \tag{2.10}$$

$$Y_{F,t}^* = T_t^{-\alpha}[(1-\alpha)C_t + \alpha Q_t C_t^*], \tag{2.11}$$

$$D_t + D_t^* = 0. \tag{2.12}$$

Combining the equilibrium in the goods market, the terms of trade can be written as

$$T_t = \frac{Y_{H,t}}{Y_{F,t}^*}\left(\frac{(1-\alpha)C_t + \alpha Q_t C_t^*}{\alpha C_t + (1-\alpha)Q_t C_t^*}\right), \tag{2.13}$$

while the real exchange rate follows from $Q_t = T_t^{2\alpha-1}$.

Two results can be read directly from equation (2.13). First, a relative abundance of Home over Foreign goods lowers Home prices relative to the Foreign (expressed in the same currency), worsening the Home terms of trade and depreciating its real exchange rate. If prices of goods are rigid in the endowment currency or if the monetary authority strictly targets the domestic price level, this corresponds to a nominal depreciation. Under these assumptions, in what follows, we use terms of trade, real and nominal exchange rates interchangeably.[10]

Second, and more important, home bias in consumption is crucial in order for deleveraging to influence the exchange rate. In fact, if preferences are identical across countries ($\alpha = 1/2$), the terms of trade are independent of the distribution of wealth and

[10]In the model with nominal rigidities the decomposition of the terms of trade into prices and exchange rate movements will follow naturally from the interaction between price rigidities and monetary policy.

just proportional to the ratio of the endowments of the two goods.[11] Instead, when there is home bias, the distribution of wealth and debt across countries can also affect relative prices through the demand channel. Imagine that deleveraging in the Home country reduces Home consumption. Since Home consumers demand more of their own goods, the fall in Home consumption depresses the demand for Home goods more than that for Foreign goods. The price of the Home goods relative to Foreign falls, worsening the Home terms of trade and depreciating the Home currency. In these cases, exchange rate management is a factor in the debt-deleveraging transmission mechanism.

2.2.1. Steady state

A deleveraging shock produced by a lowering of the debt limit k in the Home country requires some time to be absorbed. In this section we abstract from the adjustment process and compare the initial and final steady-state equilibria. We start from an initial steady state in which the distribution of wealth is such that consumers in the home country come up against their borrowing limit. This is a feasible choice because the initial distribution of wealth is indeterminate given that agents in the two countries share the same discount factor. Steady-state Home and Foreign real interest rates ($\bar{r}$ and $\bar{r}^*$) are tied to the subjective discount factor β

$$(1+\bar{r}^*) = (1+\bar{r}) = \frac{1}{\beta}, \tag{2.14}$$

where an upper bar denotes variables at their steady-state levels. Debt in the Home country is at the borrowing limit (2.5), and the steady-state level of consumption

[11] This is a standard result that depends on the assumption of Cobb-Douglas preferences, as in Cole and Obstfled (1991).

follows from the budget constraint (2.4)

$$\bar{C} = \bar{T}^{\alpha-1}\bar{Y}_H - (1-\beta)k. \tag{2.15}$$

Combining (2.3), (2.6) and (2.12) consumption in the Foreign country is given by

$$\bar{Q}\bar{C}^* = \bar{T}^{\alpha}\bar{Y}_F^* + (1-\beta)k. \tag{2.16}$$

The steady-state terms of trade can be simply obtained by appropriately incorporating (2.15) and (2.16) into (2.13)

$$\bar{T} = \frac{\bar{Y}_H}{\bar{Y}_F^*}\left[1 + (1-\beta)\left(\frac{2\alpha-1}{1-\alpha}\right)\frac{k}{\bar{T}^{\alpha-1}\bar{Y}_H}\right]. \tag{2.17}$$

Interestingly, the terms of trade and the real exchange rate depend on the level of debt and the distribution of wealth, but only when there is home bias in consumption, i.e. when $\alpha > 1/2$. When we move from a high- to a low-debt equilibrium (k falls), equation (2.17) shows that the terms of trade improve in the long run. Indeed, consumption for the constrained borrowers is higher in the final than in the initial steady state, since they have less debt and can service it at less real cost. On the contrary, Foreign consumers have to lower consumption. Since there is home bias, the demand for Home goods increases relative to that of Foreign goods in the long run, the terms of trade of country H improve and the real exchange rate rises. The interesting part of the exercise, however, is the short-run adjustment, which is completely different in form, actually swinging from a short-run currency depreciation to a long-run appreciation.

2.2.2. Adjustment to a deleveraging shock in country H

We now study the dynamic adjustment to a deleveraging shock that hits country H. Let us say that for exogenous reasons, there is a fall in the maximum amount of external debt that can be considered risk-free: the debt ceiling k drops from k_{high} to k_{low}. The adjustment takes place in two periods, the short run and the long run.

In the long run, denoted by a bar, the results of section (2.2.1) apply. The real interest rate follows from (2.14) while $\bar{T}$, $\bar{C}$, $\bar{C}^*$ and $\bar{Q}$ solve equations (2.3), (2.13), (2.15) and (2.16). With respect to the initial steady state, consumption in the Home country will be higher in the long run, since there is less debt to serve. Specularly, it will be lower in the Foreign country. The terms of trade of the Home country improves if there is home bias. Otherwise, it will be unaffected.

In the short run, the flow budget constraint of the Home country implies:

$$C = T^{\alpha-1}Y_H + \frac{k_{low}}{1+r} - k_{high}, \tag{2.18}$$

and Foreign consumption follows specularly

$$QC^* = T^{\alpha}Y_F^* - \frac{k_{low}}{1+r} + k_{high}. \tag{2.19}$$

Euler equations of the consumers in the Foreign country link short and long-run consumption through the real interest rate

$$\frac{1}{C^*} = \frac{1}{\bar{C}^*}\beta(1+r^*), \tag{2.20}$$

where we have assumed log utility, while the Euler equation of the Home consumer holds with an inequality because of the borrowing limit. In the short run, the Home and Foreign rates are related to the changes in the real exchange rate between the short and the long run

$$1 + r = (1 + r^*)\frac{\bar{Q}}{Q}. \tag{2.21}$$

Using short- and long-run consumption in the Euler equation (2.20) of country F, we obtain an expression for the short-run real interest rate

$$(1 + r) = \frac{1}{\beta}\left[\frac{\bar{T}^{\alpha}\bar{Y}_F^* + k_{low}}{T^{\alpha}Y_F^* + k_{high}}\right]. \tag{2.22}$$

The short-run real rate depends on movements in the terms of trade and debt positions between the short and the long run for a given path of output, which is exogenous and can be considered constant through the exercise. Equation (2.22) determine r, T given that $\overline{T}$ is also a function of k_{low} as discussed in the previous section. The additional equilibrium condition comes from combining (2.13), (2.18) and (2.19) into

$$T = \frac{Y_H}{Y_F^*}\left[1 + \frac{2\alpha - 1}{1 - \alpha}\frac{1}{T^{\alpha-1}Y_H}\left(k_{high} - \frac{k_{low}}{1 + r}\right)\right],$$

Some qualitative implications for the short-run terms of trade can be inferred already from this equation, again assuming home bias in consumption, which is necessary in order for the dynamic and the distribution of debt to affect the terms of trade. When country H is deleveraging with respect to the world, then it is easy to see that the terms of trade in the short run, T, will move to a higher level. Therefore, the exchange rate of the Home country will depreciate in the short run but appreciate in the long run. In a world without home bias, the terms of trade will be completely insulated from the deleveraging shock. As in the closed-economy model of Eggertsson and Krugman (2012), a deleveraging shock produces a fall in the Home real interest rate, as shown in (2.22), which is enhanced by the short and long-run movements of the terms of trade under the assumption of home bias. In this case, the real rate in the Home country falls more than the real rate of the Foreign country, as shown in (2.21).

For a first assessment of the magnitude of the impact of deleveraging on the world economy, we calibrate the model assuming that each period corresponds to one year. In the next section, we consider a more general environment in which deleveraging is spread endogenously over several periods, but in a quarterly model. Here, in a yearly model, considering a steady-state real rate of 2.5% per year we can calibrate $\beta^* = 0.9756$. We set $\alpha = 0.76$, which is consistent with the share of US non-durable consumption spending that goes to US-made products, as shown in Hale and Hobijn (2011). We set $k_{high} = 0.4$ to match the 40% of the US net external position in debt securities over GDP that Gourinchas, Govillot and Rey (2010) report for the year 2008. We imagine alternative scenarios in which external domestic debt over GDP, defined as d_{gdp}, is reduced from 40% to 30%, to 25% and 20%, respectively.[12] According to Gourinchas, Govillot and Rey (2010), the net external debt position of US before the crisis was around 20% in 2002 and around 30% in 2006.

Figure 1 shows the adjustment of Home and Foreign consumption, Home and Foreign real interest rates, the terms of trade and Home external debt as a fraction of GDP after a deleveraging shock. As discussed in Section (2.2.1), the terms of trade improve in the long run because the Home country reduces its debt exposure and so has more resources available to buy goods. Since there is home bias in preferences, the demand for domestic goods rises together with their relative price. In quantitative terms, the figure shows that all the variables display a negligible difference between the initial

[12] We normalize $\bar{Y}_H = 1$ so that $\bar{d} = k_{high}/\bar{Y}_H = 0.4$.

and final steady states. In the short run, the adjustment takes different direction and

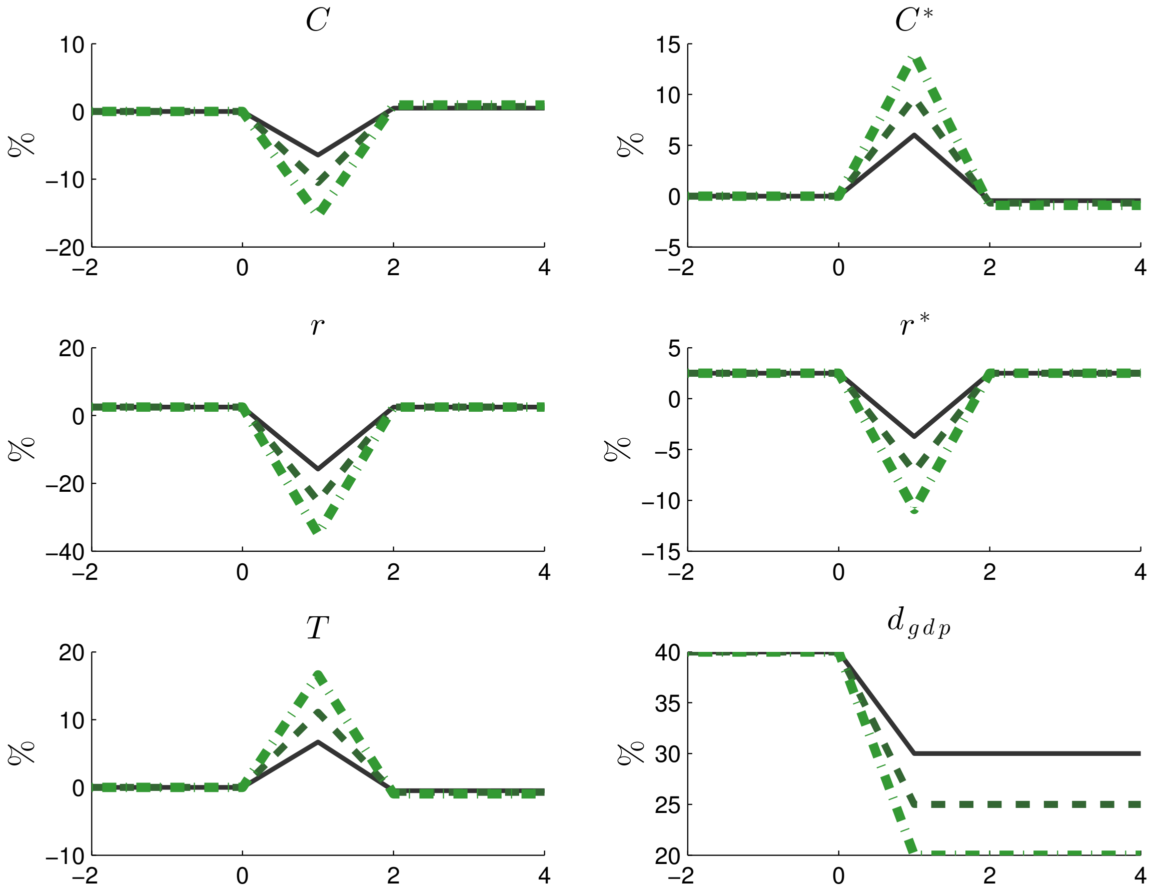

Figure 12: Responses of Home and Foreign consumption (C, C^*), Home and Foreign real interest rates (r, r^*), the terms of trade (T) and the Home external debt position over GDP (d_{gdp}), to a deleveraging shock that brings the external debt-to-GDP ratio down from 40% to 30%, 25%, 20%. The variables r, r^* and d_{gdp} are in percents the others are in percentage deviation with respect to the initial steady state.

magnitude. Home consumers must reduce their consumption drastically in order to repay debt. Because of home bias, aggregate demand for goods H drops more sharply, so the terms of trade worsen, implying a sharp depreciation of the Home currency close to 15% for severe shocks. Since in the short run deleveraging borrowers reduce their demand for goods more than in the long run, the real interest rate falls, an offsetting factor that generates more consumption by consumers in country F. The real interest rate falls more in H than in F, as is shown in equation (2.21), since the terms of trade (and the real exchange rate) rise in the short run before falling in the long run. Notice that starting from a real interest rate of 2.5% the deleveraging shock drives both Home and Foreign rates below zero; and when deleveraging is severe far below zero, to -20% or more. Consumption in country H can fall even up to 15% while that of country F specularly rises with the same magnitude.

2.2.3. Efficiency

The simple model shows that consumption, real interest rates and the terms of trade move significantly following a deleveraging shock. But, are these movements efficient? To address this question, we should define the efficient allocation in our model which critically depends on the efficient distribution of wealth. Since the latter changes during the deleveraging experiment, there is not an obvious choice. To sharpen our analysis, we can think at our deleveraging experiment as one that brings the world economy from an inefficient distribution of wealth to an efficient one. The Home country, for un-modelled reasons, has accumulated too much external debt and suddenly is forced to repay part of it to reach the efficient level.

To define the efficient allocation, we solve the maximization of the aggregate welfare

$$\sum_{t=0}^{\infty} \beta^t \left\{ \xi \ln(C_t) + (1-\xi) \ln(C_t^*) \right\}$$

for some Pareto weight ξ given the two resource constraints (2.10) and (2.11). In particular the Pareto weight ξ is chosen in such a way that in the final steady state the ratio of the marginal utilities of consumption is proportional to the real exchange rate

$$\frac{\xi}{1-\xi} \frac{\bar{C}^*}{\bar{C}} = \frac{1}{\bar{Q}}.$$

As shown in the Appendix, by taking a second-order approximation of the above objective function with respect to the final steady state and combining it with a second-order approximation of the resource constraints (2.10) and (2.11), a quadratic loss function follows

$$L_t = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\xi\tilde{C}_t^2 + (1-\xi)(\tilde{C}_t^*)^2 + \alpha(1-\alpha)\tilde{T}_t^2\right\} \tag{2.23}$$

which appropriately penalizes deviations of the target variables with respect to the final steady state, and through which it is possible to evaluate the costs of deleveraging. In the loss function (2.23), a variable with a tilde denotes a deviation of that variable with respect to the final efficient steady state. Any departure of consumption, Home and Foreign, and the terms of trade from their final steady-state values is costly. In particular, the terms of trade are a distinct objective since in a model with multiple goods misalignments of relative prices with respect to their efficient levels are costly because they produce a misallocation of real resources across different uses.

The world economy would be better off by forgiving the inefficient part of the Home-country external debt in a way to immediately achieve the efficient allocation. Obviously, this could entail non-negligible costs –not considered in our framework– which make this option not viable. However, even an orderly deleveraging process, with the large swings shown in Figure 1, can be costly. In particular, the costs can be as high as a 0.036% permanent reduction in the steady-state consumption of both countries when considering the worst scenario in which external debt drops from 40% to 20%. These costs are not huge but it should be noted that we are evaluating them in terms of a *permanent* reduction in steady-state consumption, while the adjustment process lasts only two periods. Instead, if we evaluate the costs in terms of a temporary fall in steady-state consumption for an already long ten-year horizon, the drop is around

0.16% in the worst scenario. For a five-year horizon it is around 0.31%.[13]

It is important to add that even in the case in which there is no home bias, and therefore the terms of trade do not move, there are costs of deleveraging according to (2.23). Whether the terms of trade (or the exchange rate) move or do not move should not be interpreted as a symptom of a correct or wrong adjustment to global imbalances.

Some lessons can be drawn from this simple open-economy model. In addition to the channel identified in the closed-economy literature –i.e. that deleveraging produces a fall in the real interest rate– there is an additional mechanism acting through the terms of trade but only if there is home bias in preferences. In this case, the exchange rate of the deleveraging country depreciates on impact and then appreciates in the long run. Consumption of the deleveraging country falls while that of the foreign economy increases given the low real rates. Moreover the real interest rate of the deleverager falls more than that of the other country, again under the assumption of home bias.

We have also shown that the adjustment process is in general inefficient, if we take the perspective that the final distribution of wealth reached after deleveraging is instead efficient. This observation opens room for policy options to mitigate the adjustment. However, the simple model presented in this section is of a rather limited use. Three options would be available: 1) debt forgiveness; 2) default; 3) transfers across countries. While the model is not suitable to evaluate the costs of the two former options, the latter one is also hard to enforce for completely disjoint political entities.[14] In what follows, we analyze the role of monetary policy in a framework in

[13] These costs might be also considered as an upper bound below which the costs of debt forgiveness or default could become a better option.

[14] A currency union can be an exception. Fornaro (2013) studies debt-relief policy, in the form of

which the above three options are not available or used.

There is a further limitation of the model presented above, namely the assumption of endowment economies which limits the extent to which the exchange rate can be an important element in the adjustment process. Indeed, relative-price movements driven by variations in the exchange rate can produce expenditure-switching effects across countries which can mitigate the costs for the deleverager. To this end, in the next section, we extend the model to allow for endogenous production. We also assume nominal rigidities, consistent with the empirical evidence on the real effects of monetary policy shocks, and study the relevance of different exchange-rate regimes and monetary policies in the adjustment to international deleveraging.

2.3. A model with endogenous production and nominal rigidities

The model used in this section closely follows those of the open-economy macro literature, such as Obstfeld and Rogoff (2001, 2005) and Benigno (2009). The new elements here with respect to the simple model of the previous section are nominal rigidities, endogenous output, debt deleveraging on a longer horizon and more general preference specifications.

Three factors can delay the adjustment to a deleveraging shock and create interesting dynamics. First, nominal rigidity slows the response of relative prices and can lead to a contraction in real output. Second, the zero lower bound on the nominal interest rate can prevent real rates from falling, depressing aggregate demand and output. Finally, the exchange-rate regime may either attenuate or amplify the response of real and nominal exchange rates.

Households in country H, a continuum of measure one, have preferences over con-

a transfer of wealth from creditors to debtors, in a closed economy.

sumption and work hours as follows:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}}{1-\rho} - \int_0^1 \frac{[L_t(j)]^{1+\eta}}{1+\eta} dj \right] \right\},$$

where $L_t(j)$ is hours worked of variety j; $\eta \geq 0$ the inverse of the labor-supply elasticity and $\rho > 0$ the inverse of the intertemporal elasticity of substitution in consumption. Every household can supply all varieties of labor; C is a consumption bundle of goods H and F given by

$$C = \left(\alpha^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-\alpha)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$

where α, with $\alpha \geq 1/2$, represents still the weight given to home-produced goods in the consumption bundle while θ, with $\theta > 0$, is the intratemporal elasticity of substitution between Home and Foreign-produced goods. The Cobb-Douglas case (2.1) of the previous section is nested when $\theta = 1$. Given this preference, the consumption-based price index is given by

$$P = \left(\alpha P_H^{1-\theta} + (1-\alpha) P_F^{1-\theta} \right)^{\frac{1}{1-\theta}}.$$

Differently from the previous section, we now assume that C_H is composed of a continuum of goods $c(h)$ of measure one all produced in country H, while C_F is a continuum of goods, $c(f)$, produced in country F:

$$C_H = \left[\int_0^1 c(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \qquad C_F = \left[\int_0^1 c(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 0$ is the elasticity of substitution across goods produced within a country.

The price indices P_H and P_F are:

$$P_H = \left[\int_0^1 P(h)^{1-\sigma} dh\right]^{\frac{1}{1-\sigma}} \qquad\qquad P_F = \left[\int_0^1 P(f)^{1-\sigma} df\right]^{\frac{1}{1-\sigma}},$$

where $P(h)$ and $P(f)$ are the prices of the goods h and f denominated in the currency of country H. Households in country H face the following flow budget constraint:

$$P_t C_t = \int_0^1 W_t(j) L_t(j) dj + \Psi_t + \frac{D_t}{1+i_t} - D_{t-1} - k_t P_t \cdot \tilde{\chi}\left(\frac{d_t}{k_t}, \frac{\bar{d}_t}{k_t}\right) \qquad (2.24)$$

where $W_t(j)$ is the nominal wage for the variety of work j and Ψ_t are firms' profits, which are distributed to the households in equal proportion. In the flow budget constraint (2.24) we have added a function $\tilde{\chi}(\cdot,\cdot)$, appropriately normalized, capturing costs of adjusting the debt position. The function depends on the real debt of the individual households, $d_t = D_t/P_t$, with respect to a threshold k_t and of the country's aggregate real debt, given by $\bar{d}_t$, again with respect to the same threshold. Excess borrowing, above a certain limit, is costly and may reflect intermediation frictions related to the monitoring that lenders perform when exerting too much credit. We assume that the function $\tilde{\chi}(\cdot,\cdot)$ is always non-negative, $\tilde{\chi}(\cdot,\cdot) \geq 0$ since it reflects only costs and not benefits. Moreover, the derivatives of the function with respect to individual debt, $\tilde{\chi}_d(\cdot,\cdot)$, and to aggregate debt, $\tilde{\chi}_{\bar{d}}(\cdot,\cdot)$, are non-negative $\tilde{\chi}_d(\cdot,\cdot) \geq 0$ and $\tilde{\chi}_{\bar{d}}(\cdot,\cdot) \geq 0$. In particular, we assume that $\tilde{\chi}_d(1,1) = 0$ which is a sort of optimality condition for individual borrowing saying that at the risk-free level of debt the marginal cost of increasing the borrowing capacity is zero. However, increases in aggregate debt above the risk-free threshold are costly at the margin, $\tilde{\chi}_{\bar{d}}(1,1) > 0$.[15] The assumption that the individual cost of borrowing depends also on aggregate debt is not only convenient for technical reasons, as it will be explained later, but also

[15]We further assume that the second derivatives are such that $\tilde{\chi}_{d,d}(1,1) + \tilde{\chi}_{d,\bar{d}}(1,1) > 0$.

because it might capture several features of the recent financial crisis. For the same characteristics of the individual borrowers, financial intermediaries might charge a different premium on lending depending on the aggregate conditions if these reflect different degrees of vulnerability of the financial system to systemic risk. The aggregate level of debt might be an important signal of this vulnerability since it might imply or predict a worsening of the balance sheets of intermediaries in the case in which more non-performing loans materialize when macroeconomic conditions worsen. Moreover, given the interdependence of the financial system, an overall higher level of aggregate debt might facilitate the contagion of a poor creditworthiness of some sectors of the economy to others, and therefore exacerbate adverse-selection problems in exerting credit to individual borrowers. In a nutshell, during the financial crisis, even reliable borrowers faced a worsening in their credit conditions because of the weakening of the overall financial system due to the high level of debt and leverage accumulated in the past.

The deleveraging experiment that we consider in this section involves a one-time reduction in the threshold k_t which, given the structure of the economy, produces a dynamic adjustment of debt and other aggregate variables. In particular, the zero-lower bound constraint is mainly responsible of the dynamic adjustment. We assume that k_t changes from $k_{\max}$ to $k_{\min}$.[16] This might capture a banking or financial crisis, or just a change in confidence, such that excess borrowing above the lower threshold is now costly. Therefore, borrowers need to deleverage. It should be noted that through the cost function the steady-state debt position of households is determined in our

[16]The one-period deleveraging experiment of the previous section can be also seen as a limiting case of the modelling device used in this section, when the cost of adjustment with respect to the threshold is infinite. Notice also that, as in the model of the previous section, any level of initial debt d such that $d \leq k_{\max}$ is consistent with the steady-state equilibrium, since $\tilde{\chi}(\cdot,\cdot) \geq 0$, $\tilde{\chi}_d(1,1) \geq 0$ and partial derivatives are non-negative. We set $d = k_{\max}$. The analysis further shows that when k falls to $k_{\min}$, the level of debt d is adjusted gradually to $k_{\min}$ from above.

model, a device often used in the literature. Moreover, as it will be shown later, the fact that the cost function depends on individual debt is going to be reflected, through optimality conditions of households, into a borrowing premium.[17]

Similarly, preferences of households in country F are:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{*1-\rho}}{1-\rho} - \int_0^1 \frac{[L_t^*(i)]^{1+\eta}}{1+\eta} di \right] \right\},$$

where C_t^* is now given by

$$C^* = \left((1-\alpha)^{\frac{1}{\theta}} (C_H^*)^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} (C_F^*)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

and the related consumption-based price index is

$$P^* = \left((1-\alpha)(P_H^*)^{1-\theta} + \alpha (P_F^*)^{1-\theta} \right)^{\frac{1}{1-\theta}}.$$

$L_t^*(i)$ represents hours worked of type i in foreign firms. The consumption bundles C_H^* and C_F^* and the appropriate consumption-based price indices P_H^* and P_F^* have the same structure as those of country H, whereas $P^*(h)$ and $P^*(f)$ are now the prices of the goods h and f expressed in the currency of country F. The law of one price holds for each traded good (i.e., $P(h) = SP^*(h)$ and $P(f) = SP^*(f)$) but, as explained in Section 2.2, there can be deviations from PPP because of Home bias. Households in country F face a flow budget constraint:

$$P_t^* C_t^* = \int_0^1 W_t^*(i) L_t^*(i) di + \Psi_t^* + TR_t^* + \frac{B_t^*}{1+i_t^*} + \frac{D_t^*}{(1+i_t)S_t} - B_{t-1}^* - \frac{D_{t-1}^*}{S_t},$$

[17] In a log-linear approximation, the model will be isomorphic to one in which the interest-rate is assumed to be elastic with repect to the individual and aggregate debt.

where $W_t^*(i)$ is nominal wage for the variety of work i, Ψ_t^* are Foreign profits. In writing the flow budget constraint of the foreign consumers, we are assuming that they can borrow and lend also in bonds denominated in their own currency, B_t^*, at the interest rate $1 + i_t^*$. However, we assume that these securities are in zero-net supply within the country. The only asset traded internationally is denominated in the currency of country H, and consumers of country F hold D_t^* units of it.[18] In writing the budget constraint for consumers in country F, we are neglecting the costs of adjusting their debt position because in the equilibrium that we are going to analyze these consumers will be creditors in the international financial markets, and therefore they are not subject to costs of excessive borrowing. Moreover, the borrowing costs of consumers in country H are transferred in terms of profits of intermediation to the consumers in country F. Indeed these profits are given by $TR_t^* = P_t k_t \tilde{\chi}\left(d_t/k_t, \bar{d}_t/k_t\right)/S_t$.[19]

Turning to the consumer's optimality conditions, the stochastic version of the Euler equation (2.9) still describes the intertemporal allocation of consumption in country F and holds with equality at an interior optimum

$$(C_t^*)^{-\rho} = \beta(1+i_t)E_t\left\{(C_{t+1}^*)^{-\rho}\frac{S_t}{S_{t+1}}\frac{P_t^*}{P_{t+1}^*}\right\},$$

while the Euler equation of the Home country changes to

$$(C_t)^{-\rho}\left\{1-(1+i_t)\psi\left(\frac{d_t}{k_t}\right)\right\} = \beta(1+i_t)E_t\left\{(C_{t+1})^{-\rho}\frac{P_t}{P_{t+1}}\right\},$$

where $\psi\left(d_t/k_t\right) = k_t\tilde{\chi}_d(d_t/k_t, \bar{d}_t/k_t)$ since in equilibrium $d_t = \bar{d}_t$. The cost of excessive

[18] In equilibrium $D_t + D_t^* = 0$.

[19] The welfare analysis of next section simplifies substantially under this assumption. Otherwise, goods market equilibria will be affected by the costs of intermediation and bring additional effects in the quadratic approximation of the objective function.

borrowing for households in country H endogenously implies a premium in addition to the interest rate paid. Note that when $d_t = k_t$, $\psi(\cdot) = 0$ and we retrieve the standard Euler equation.[20]

The Euler equation of households in country F with respect to holdings of securities denominated in their currency reads as

$$(C_t^*)^{-\rho} = \beta(1+i_t^*)E_t\left\{(C_{t+1}^*)^{-\rho}\frac{P_t^*}{P_{t+1}^*}\right\}.$$

By combining the two Euler equations for the households in country F, we get that the excess return of investing in foreign versus domestic currency is orthogonal to the stochastic discount factor of the foreign household

$$E_t\left\{\beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}\frac{P_t^*}{P_{t+1}^*}\left[(1+i_t^*)-(1+i_t)\frac{S_t}{S_{t+1}}\right]\right\}=0,$$

which in a log-linear approximation delivers the standard uncovered-interest-rate-parity condition.

In both countries there is a continuum of firms, each producing one of the goods. Firms use all the varieties of labor offered in the country, combining them through the following technologies

$$y(h) = \left[\int_0^1 L^h(j)^{\frac{\tau-1}{\tau}}dj\right]^{\frac{\tau}{\tau-1}} \qquad y^*(f) = \left[\int_0^1 L^f(i)^{\frac{\tau-1}{\tau}}di\right]^{\frac{\tau}{\tau-1}},$$

where τ is the elasticity of substitution across varieties of labor, with $\tau > 1$. We assume that firms operate under monopolistic competition, setting their prices in a

[20]The assumptions on the cost function $\tilde{\chi}(\cdot,\cdot)$ imply that $\psi_d(1) > 0$. Instead, if the cost function depends only on the first argument, it would be the case that $\psi_d(1) = 0$. In this case it follows that, in a first-order approximation of the equilibrium conditions, there will be no effect, both in the Euler equation of borrowers and in their budget constraint, of assuming costs of portfolio adjustment.

flexible way. It follows that $p_t(h) = P_{H,t} = \mu W_t$ for each h and $p_t^*(f) = P_{F,t}^* = \mu W_t^*$ for each f, where W_t and W_t^* are aggregate nominal wages in the respective currencies and the price markup is $\mu \equiv \sigma/(\sigma - 1)$. While prices are flexible, wages adjust in a staggered way following Calvo's model in which unions, grouping work of the same variety, have monopolistic power in setting wages. In each period, in country H (F), only a fraction $1 - \lambda$ ($1 - \lambda^*$) of the varieties of labor, with $0 < \lambda, \lambda^* < 1$, can have their wages reset according to the macroeconomic conditions and independently of the last adjustment. The remaining fraction of varieties of labor, of measure λ (λ^*), can only index their wages to the current inflation target, $\bar{\Pi}$ ($\bar{\Pi}^*$), which does not necessarily coincide with actual inflation. It is clear that wage rigidity translates directly into price rigidity, since we do not have productivity shock. The resulting aggregate-supply equations are standard for this kind of model. The set of equilibrium conditions is presented in detail in the online appendix.

2.4. Optimal adjustment to international deleveraging

In this section we ask how a benevolent central planner, maximizing the utility of the world economy, would optimally react to a deleveraging shock hitting country H. A natural objective of policy is the maximization of the weighted average of the utility of the consumers in the world economy, which is given by

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\xi \left(\frac{C_t^{1-\rho}}{1-\rho} - \int_0^1 \frac{[L_t(j)]^{1+\eta}}{1+\eta} dj \right) + (1-\xi) \left(\frac{C_t^{*1-\rho}}{1-\rho} - \int_0^1 \frac{[L_t^*(i)]^{1+\eta}}{1+\eta} di \right) \right] \right\}. \tag{2.25}$$

In particular, consistent with the previous analysis, we choose appropriately the weight ξ so that the steady state reached after deleveraging is efficient. As before, our experiment entails a reduction of debt that brings the economy from an inefficient allocation to an efficient one. To this end, we assume that there are appropriate subsidies which eliminate the monopolistic distortions in the labour markets. The final steady state is described by the following set of equilibrium conditions

$$\bar{C} = \bar{p}_H \bar{Y}_H - (1-\beta)\bar{\Pi}^{-1} k_{\min},$$

$$\bar{Q}\bar{C}^* = \bar{p}_F \bar{Y}_F^* + (1-\beta)\bar{\Pi}^{-1} k_{\min},$$

$$\bar{Y}_H = \bar{p}_H^{-\theta}[\alpha \bar{C} + (1-\alpha)\bar{Q}^{\theta}\bar{C}^*]$$

$$\bar{Y}_F^* = \bar{p}_F^{-\theta}[(1-\alpha)\bar{C} + \alpha \bar{Q}^{\theta}\bar{C}^*]$$

$$\bar{Y}_H^{\eta} = \bar{p}_H \bar{C}^{-\rho}$$

$$\bar{Y}_F^{*\eta} = \bar{p}_F \bar{Q}^{-1}(\bar{C}^*)^{-\rho}$$

$$1 = \alpha \bar{p}_H^{1-\theta} + (1-\alpha)\bar{p}_F^{1-\theta}$$

$$\bar{Q} = \left((1-\alpha)\bar{p}_H^{1-\theta} + \alpha \bar{p}_F^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

which clearly determine the allocation of $\bar{C}, \bar{C}^*$, $\bar{Y}_H$, $\bar{Y}_F^*$, $\bar{Q}$, $\bar{p}_H, \bar{p}_F$ given the level of debt $k_{\min}$ reached after deleveraging and the steady-state inflation rate in country H, $\bar{\Pi}$.[21] It is also easy to show that an efficient allocation should satisfy the condition

$$\frac{\xi}{1-\xi}\left(\frac{\bar{C}}{\bar{C}^*}\right)^{-\rho} = \frac{1}{\bar{Q}}$$

which indeed is the one determining the weight ξ given the above derived $\bar{C}, \bar{C}^*$ and $\bar{Q}$.

The fact that the new steady state is efficient simplifies a lot the analysis. Indeed, by taking a second-order approximation of (2.25) around the efficient steady state and

[21] Notice that one equation is redundant. We have also defined $p_H = P_H/P$ and $p_F = P_F/P$.

combining it with the resource constraints, we obtain an expression containing only quadratic terms which can be correctly evaluated through a first-order approximation of the equilibrium conditions. Details are left to the online appendix. Maximization of the above utility function corresponds to minimization of the following loss function

$$L_t = \frac{1}{2} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\varphi_1 \tilde{C}_t^2 + \varphi_2 (\tilde{C}_t^*)^2 + \varphi_3 \tilde{T}_t^2 + \varphi_4 \tilde{Y}_{H,t}^2 + \varphi_5 \tilde{Y}_{F,t}^{*2} + \varphi_6 (\pi_{H,t} - \bar{\pi})^2 + \varphi_7 (\pi_{F,t}^* - \bar{\pi}^*)^2 \right] \right\} \tag{2.26}$$

for some non-negative parameters φ_1, φ_2, φ_3, φ_4, $\varphi_5, \varphi_6, \varphi_7$ discussed in the online appendix; the variables $\tilde{C}_t$, $\tilde{C}_t^*$, $\tilde{Y}_{H,t}, \tilde{Y}_{F,t}^*, \tilde{T}$ represent log deviations with respect to the final steady state of the respective variables, while $\pi_{H,t}$ and $\pi_{F,t}^*$ are the Home and Foreign producer inflation rates and $\bar{\pi}$ and $\bar{\pi}^*$ are their respective targets such that $\bar{\pi} = \ln \bar{\Pi}$ and $\bar{\pi}^* = \ln \bar{\Pi}^*$.

According to the loss function (2.26), the benevolent central planner dislikes deviations of the producer inflation rates in each country from their respective targets. This captures the costs of wage dispersion due to misallocation of labor demand across varieties which have the same level of efficiency. Moreover deviations of output, consumption in each country and the terms of trade from their efficient levels are also penalized. It is optimal to keep the GDP inflation rates at their targets and at the same time to achieve immediate stabilization of output, consumption and the terms of trade at the efficient levels.

However, the efficient allocation can only be reached in the long run when deleveraging ends, while it is not feasible in the short run mainly for two reasons: first, as shown in the simple model of Section 2, an adjustment to a deleveraging shock brings about movements in output, consumption and relative prices whose direction contrasts with the efficient movements built into the objective (2.26), creating therefore important trade-offs; second, a policy of keeping GDP inflation rates at their targets $\bar{\pi}$ and $\bar{\pi}^*$ at all times may not be feasible because, as shown again in Section 2, a deleveraging shock produces negative real interest rates which, with constant inflation rates, require the nominal interest rates to go below zero and violate the zero-lower bound constraint.[22]

Given the loss function (2.26) we solve for the linear-quadratic optimal-policy problem taking into account the zero-lower bound constraints.[23] We analyze the effects of an unanticipated deleveraging shock that moves k immediately from $k_{\max}$ to $k_{\min}$. The shock produces a dynamic path of deleveraging which depends endogenously on policy. In particular we calibrate $k_{\max}$ and $k_{\min}$ such that the Home country external debt with respect to GDP moves from an initial level of 40% to reach a final steady state of 30% at the end of deleveraging. In particular $k_{\max}$ is set at 1.6095 while $k_{\min}$ at 1.2054. The model is calibrated quarterly. We set $\beta = 0.9938$ to imply a 2.5% real annual return on a yearly basis. The steady-state inflation rates are set to $\bar{\Pi} = \bar{\Pi}^* = 1.005$ to imply a 2% inflation rates on a yearly basis in each country. These imply that $\bar{\pi} = \bar{\pi}^* = 2\%$ at annual rates. We set the parameter $\alpha = 0.76$ as in previous section and calibrate the parameters σ and τ to 7.66, implying steady-state mark-ups in goods and labor market equal to 15%. The inverse of the elasticity of substitution in consumption, ρ, is set to 2, consistent with a number of studies, and the inverse of the labor supply elasticity, η, is set to 1.5, which is in the range of the estimates of De Walque et al (2005) in a two-country model of the euro area and the US. The degree of wage rigidities is also taken from De Walque et al. (2005); λ and λ^* are set equal to 0.8, which is consistent with their estimates and implies a duration of wages of 5 quarters in both countries (this is also in line with other micro

[22]Even in this context debt forgiveness or appropriately-defined transfers can achieve the efficient allocation provided each monetary policy follows its inflation target.

[23]See the online appendix for the details.

studies). Finally the elasticity of substitution across Home and Foreign goods is set at a unitary value consistent with what often assumed in several studies, $\theta = 1$. In the next section, we are going to perform robustness analysis along different assumptions on θ and α. Finally, the borrowing cost creates, in a log-linear approximation, a spread between the interest rate faced by the borrowers in the Home country and the risk-free rate. This spread depends on the distance between the level of borrowing and what is considered the risk-free threshold of debt

$$\hat{\imath}_t^b - \hat{\imath}_t \equiv \varpi_1(\hat{d}_t - \hat{k}_t).$$

In particular, as shown in the online appendix, $\hat{\imath}_t^b$, the effective borrowing rate, is the relevant nominal interest rate entering the log-linear approximation of the Euler equation and the external budget constraint of country H. The parameter ϖ_1 is set at 0.047 in such a way that on impact the drop in k, considering a constant debt d, produces a 4.5% spread at annual rates which is consistent with the peak of the TED spread observed during the US financial crisis.

Figures 13 and 14 show the optimal adjustment following the deleveraging shock under the benchmark calibration compared with a policy in which both countries aim at targeting GDP inflation at 2%, $\pi_{H,t} = \pi^*_{F,t} = 2\%$. These inflation-targeting policies are of particular interest since have been often found to be the welfare-maximizing policies under cooperation in open-economy models. Indeed, they are the optimal cooperative policies in our model in the absence of deleveraging shocks and under an efficient distribution of wealth across countries.[24] Moreover, the adoption of such inflation-targeting policies by many developed countries before the crisis makes them

[24]This is true in models in which there is producer-currency pricing as in the framework of this paper (see Benigno and Benigno, 2006). Engel (2011) shows that with local-currency pricing it is optimal to stabilize CPI inflation rates.

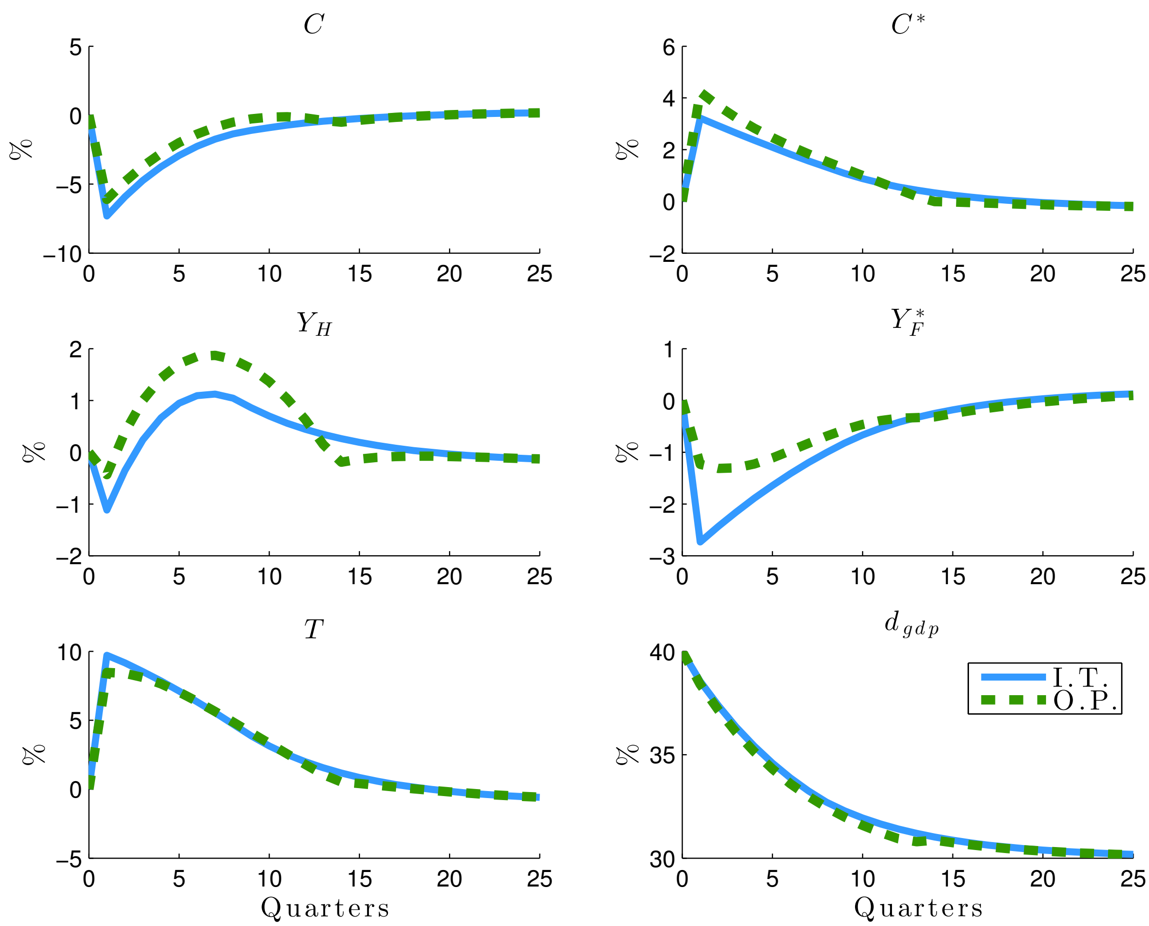

Figure 13: Impulse responses following the deleveraging shock under optimal policy in comparison with the policy in which both countries follow inflation-targeting policies $\pi_{H,t} = 2\%$ and $\pi^*_{F,t} = 2\%$, but can be constrained by the zero-lower bound. Variables are: Home and Foreign consumption $(C\,, C^*)$, Home and Foreign output (Y_H, Y_F^*), terms of trade (T), the level of external debt of country H as a percentage of its GDP (d_{gdp}). All variables, except for d_{gdp}, are in percentage deviations from the steady state.

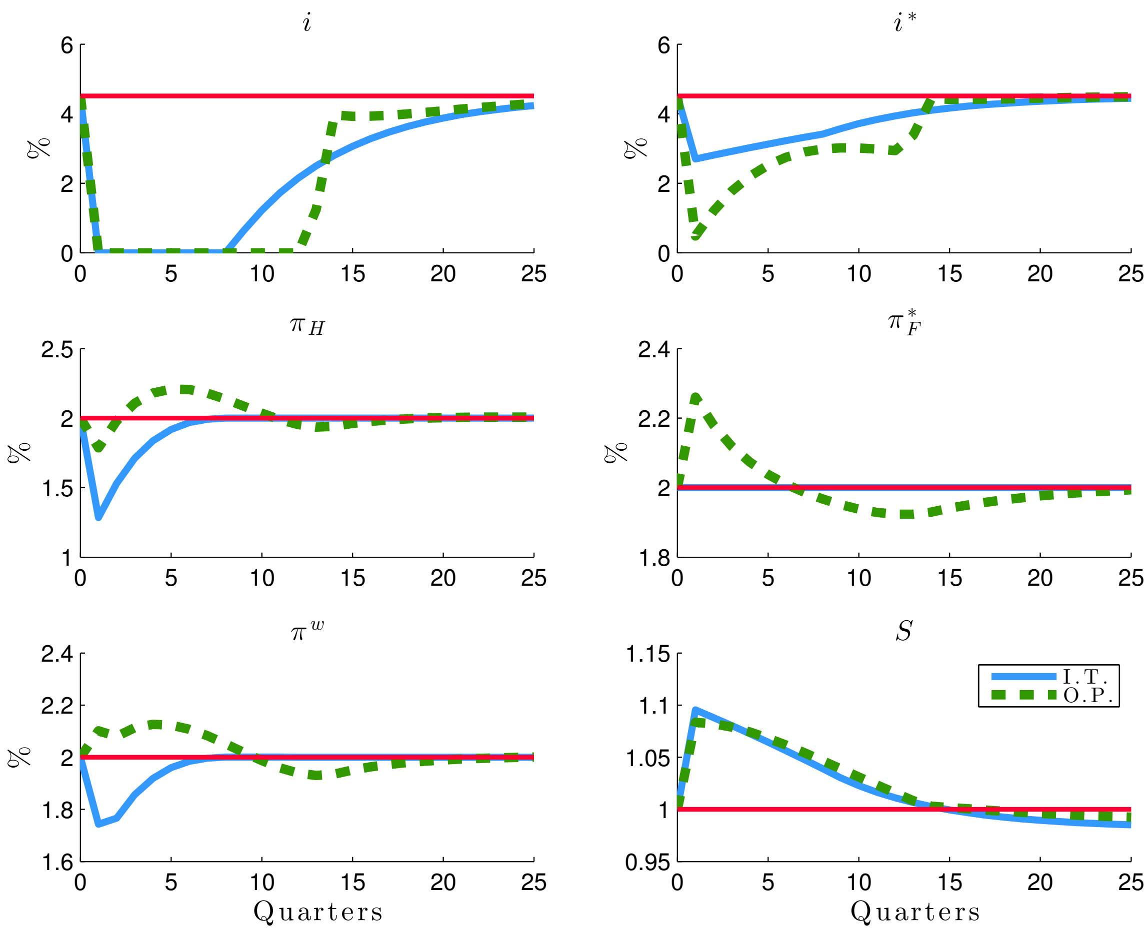

Figure 14: Impulse responses following the deleveraging shock under optimal policy in comparison with the policy in which both countries follow inflation-targeting policies $\pi_{H,t} = 2\%$ and $\pi^*_{F,t} = 2\%$, but can be constrained by the zero-lower bound. Variables are: Home and Foreign nominal interest rates (i,i^*), Home and Foreign producer inflation rates (π_H, π^*_F), world inflation (π^W), defined as $\pi^W = 1/2 \cdot \pi + 1/2 \cdot \pi^*$ where π and π^* are the Home and Foreign CPI inflation rates, the level of the nominal exchange rate (S); inflation and interest rates are in percents and annual rates.

an interesting benchmark of comparison to discuss how optimal policy changes when there is a deleveraging shock. As previously discussed, it should be noted that the latter couple of policies is not feasible since at the beginning of deleveraging they would imply a nominal interest rate for country H, below the zero-lower bound. Considering such a constraint, the GDP inflation rate in country H needs to fall under the target, as shown in Figure 14, while the economy stays in the "liquidity trap" for eight quarters. On the contrary, the zero-lower bound is not a constraint for country F.

As shown in Figure 13, inflation-targeting policies are quite costly for both economies in particular in terms of a contraction in output for several quarters. Moreover, the consumption recession in country H is particularly deep and counteracted only in part by an expansion in the consumption of Foreign households. The short-run depreciation of the deleverager's nominal exchange rate is sizeable and around 9%. External debt reaches slowly the efficient level after more than fifteen quarters.

As shown in the same figure, optimal policy improves substantially with respect to inflation-targeting policies. First, it should be noted that country H's external debt converges to the new steady state level of 30% of GDP earlier but still after four years. Even under optimal policy, the contraction of the deleverager's consumption is inevitable, although mitigated. There is now a larger increase in the other country's consumption. Most important, the output recession is milder in both countries and also shorter in country H. The better adjustment is achieved with less variations of the nominal exchange rate and the terms of trade. The improvements in the real economies are explained by the different monetary policies followed under optimal policy. Interest rate in the deleveraging country should be at the zero-lower bound for a longer horizon, up to three years.[25] In country F, the interest rate is also low but remains above the zero-lower bound. The fact that the real interest rates substantially fall in countries H and F mitigates the costs of deleveraging. The GDP inflation rates should fluctuate around their target: an increase in foreign inflation is needed at the beginning of deleveraging while inflation in country H initially falls below target, and then rises afterward. Interestingly, sub-optimal inflation-targeting policies even undershoot their inflation targets, in country H, because of the zero-lower bound constraint. This implies a global disinflation measured by world inflation, π^w. Instead, under optimal policy, world inflation stays above the 2% target, in particular at the beginning of the deleveraging episode.

The figures show three channels though which the benevolent planner can cope with a deleveraging shock in country H. First, it can lower the real interest rate in the deleverager to reduce its borrowing costs, mainly through policies in country H of low or zero nominal interest rate and inflation above target. Second, it can mitigate the consumption and output recession in country H by expanding consumption in country F through a lowering of the real rates in country F, again using policies of low or zero nominal interest rate and inflation above target in country F. Third, it can mitigate the output recession in country H through a worsening of the Home terms of trade and a depreciation of its nominal exchange rate to switch expenditure from foreign to domestic goods. However, all the identified channels imply costs in terms of the loss function (2.26) which should be appropriately weighted.

We now turn to investigate how alternative international transmission mechanisms change the way in which the benevolent planner uses the three channels identified

[25]The results that under optimal policy the stay at the zero-lower bound is longer than under inflation targeting is in line with the findings of Eggertsson and Woodford (2003) in a simple closed-economy model.

above.

2.5. Alternative international transmission mechanisms

How do the results of Figures 13 and 14 change under alternative international transmission mechanisms? We address this question through different assumptions on the elasticity of substitution between Home and Foreign goods, θ, and the degree of home bias in goods consumption, captured by α.

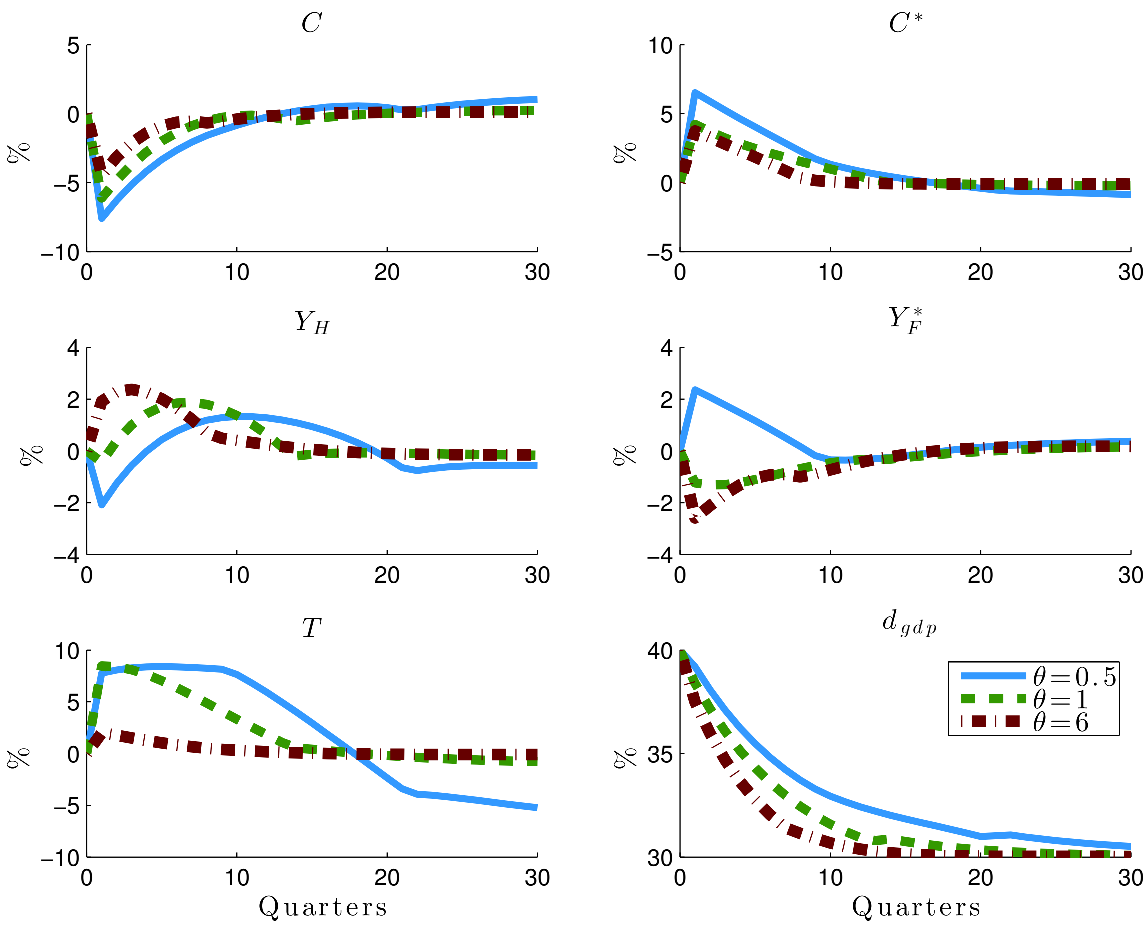

Figure 15: Impulse responses under optimal policy for different values of $\theta = 0.5, 1$ and 6. Variables are: Home and Foreign consumption (C, C^*), Home and Foreign output (Y_H, Y_F^*), terms of trade (T), the level of external debt of country H as a percentage of its GDP (d_{gdp}). All variables, except for d_{gdp}, are in percentage deviations from the steady state.

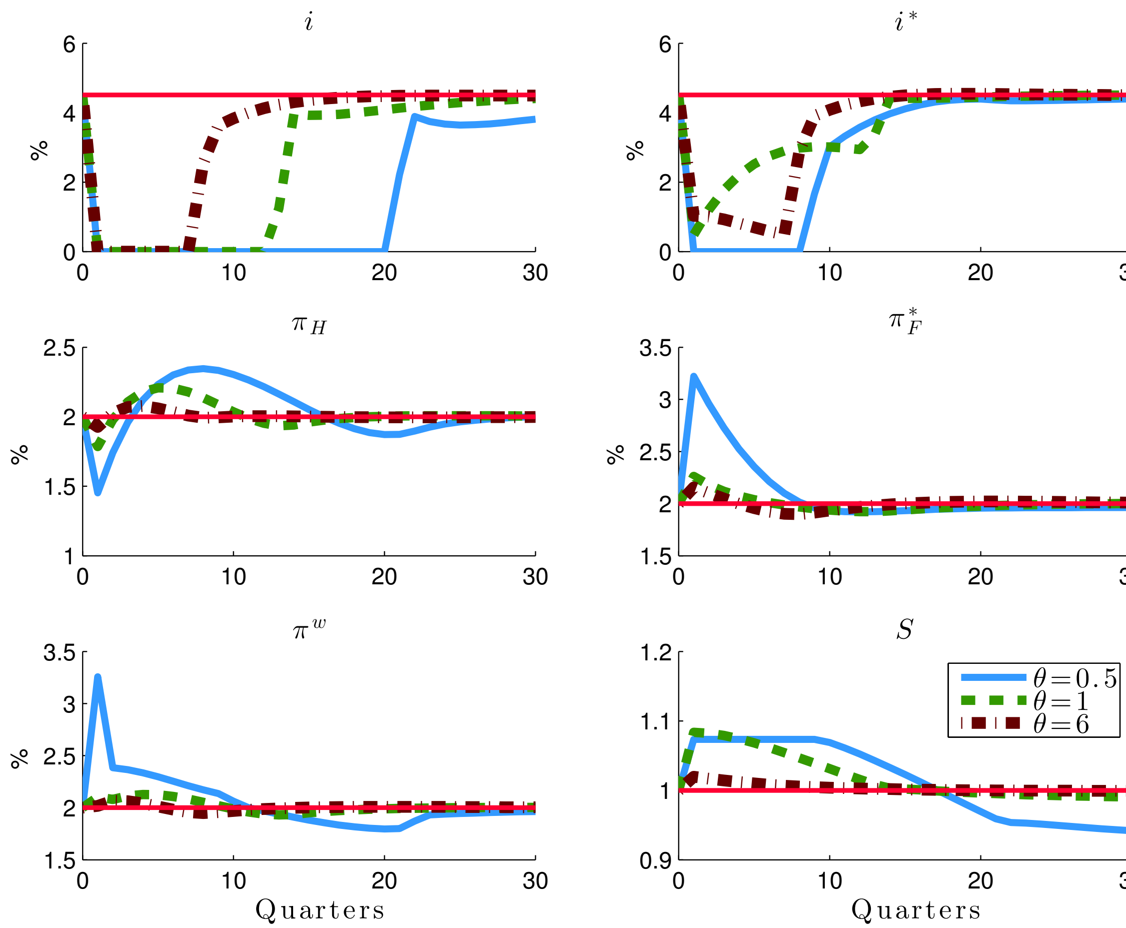

Figure 16: Impulse responses under optimal policy for different values of $\theta = 0.5, 1$ and 6. Variables are: Home and Foreign nominal interest rates (i,i^*), Home and Foreign producer inflation rates (π_H, π_F^*), world inflation (π^W), defined as $\pi^W = 1/2 \cdot \pi + 1/2 \cdot \pi^*$ where π and π^* are the Home and Foreign CPI inflation rates, the level of the nominal exchange rate (S); inflation and interest rates are in percents and annual rates.

The parameter θ measures the elasticity of substitution of consumption between Home and Foreign goods with respect to variations in their relative price. When θ is high, it suffices a small depreciation of the Home currency to create significant expenditure-switching effects from country F's goods to those of country H. The classical expenditure-switching channel is clearly stronger when the elasticity θ is larger. However, this is not all that matters for the international transmission mechanism. It should be noted, indeed, that real income depends critically on the position of θ with respect to the unitary value. The real income of country H in units of its own consumption can be written as

$$\frac{P_{H,t}Y_{H\,t}}{P_t} = \left(\frac{P_{H,t}}{P_t}\right)^{1-\theta} \left[\alpha C_t + (1-\alpha)Q_t^{\theta}C_t^*\right]$$

from which it follows that values of θ below the unitary value might imply that a worsening of the Home terms of trade or a depreciation of the Home currency, i.e. a fall in $P_{H,t}/P_t$, can have adverse effects on the Home country's real income making it harder to deleverage. This reminds phenomena of "immiserizing" growth in which a depreciation of the currency can increase production of a country but at the same time make it more poor with a reduction in its real income.

Figures 15 and 16 show how optimal policy changes for alternative values of θ around the benchmark value of one, namely we plot impulse responses under optimal policy for $\theta = 0.5, 1$ and 6. Consistently with the previous discussion a value of θ below one is harmful for the real income of country H. The consumption recession is much deeper, when $\theta = 0.5$, while the needed expansion in the foreign country is specularly stronger. Output recession is now deep in the Home country but not in the Foreign economy because the terms of trade does not vary much. Instead, for a higher $\theta = 6$, the expenditure-switching channel works to improve the deleverager's real income and can therefore mitigate its consumption recession. A smaller expansion in consumption is required in the foreign country, but an output recession is now unavoidable because of the effectiveness of the expenditure-switching effect.

The paths of the terms of trade and the nominal exchange rate deserve particular attention under the three scenarios. As already discussed, a deleveraging shock under home bias mechanically produces an initial worsening of the Home-country terms of trade and of its nominal exchange rate. However, as shown in the objective function (2.26), these movements are inefficient and the optimal policy should aim to reduce them by weighing them appropriately with the trade-offs implicit in the loss function. Indeed, when $\theta = 6$, the expenditure-switching channel is strong enough that a smaller depreciation of the currency is sufficient to substantially shift production from foreign to home goods. On the contrary, when $\theta = 0.5$, this channel is weaker and moreover real income of the deleverager's country is adversely hit. Even in this case, the optimal policy requires a smaller short-run depreciation of country $H's$ currency and now a substantial appreciation in the long run.

Interestingly, Figure 16 shows that the stay at the zero-lower bound depends on the alternative assumptions on θ. For low values of θ, the stay at the zero-lower bound for country H is exactly five years. Even country F is now forced to stay at the zero-lower bound and for a long period of two years. This is because the contraction in country H's consumption is larger and requires more expansion in country F's consumption, which can be stimulated by a larger fall in its real interest rate and therefore in the nominal interest rate. The stay at the zero-lower bound is completely avoided for higher values of θ where the nominal interest rate of country F stays above the constraint.

Speaking in terms of the three channels identified above, the expenditure-switching channel cannot be used when θ is low. Therefore, the benevolent planner has to rely more on the real interest-rate channels by lowering more the policy rates. A global liquidity trap may be optimal in this case. On the contrary when θ is high, the expenditure-switching channel is more effective, hence there is less need to lower the real interest rates, in particular in the foreign country. The liquidity trap is shorter for the deleverager and absent in country F.

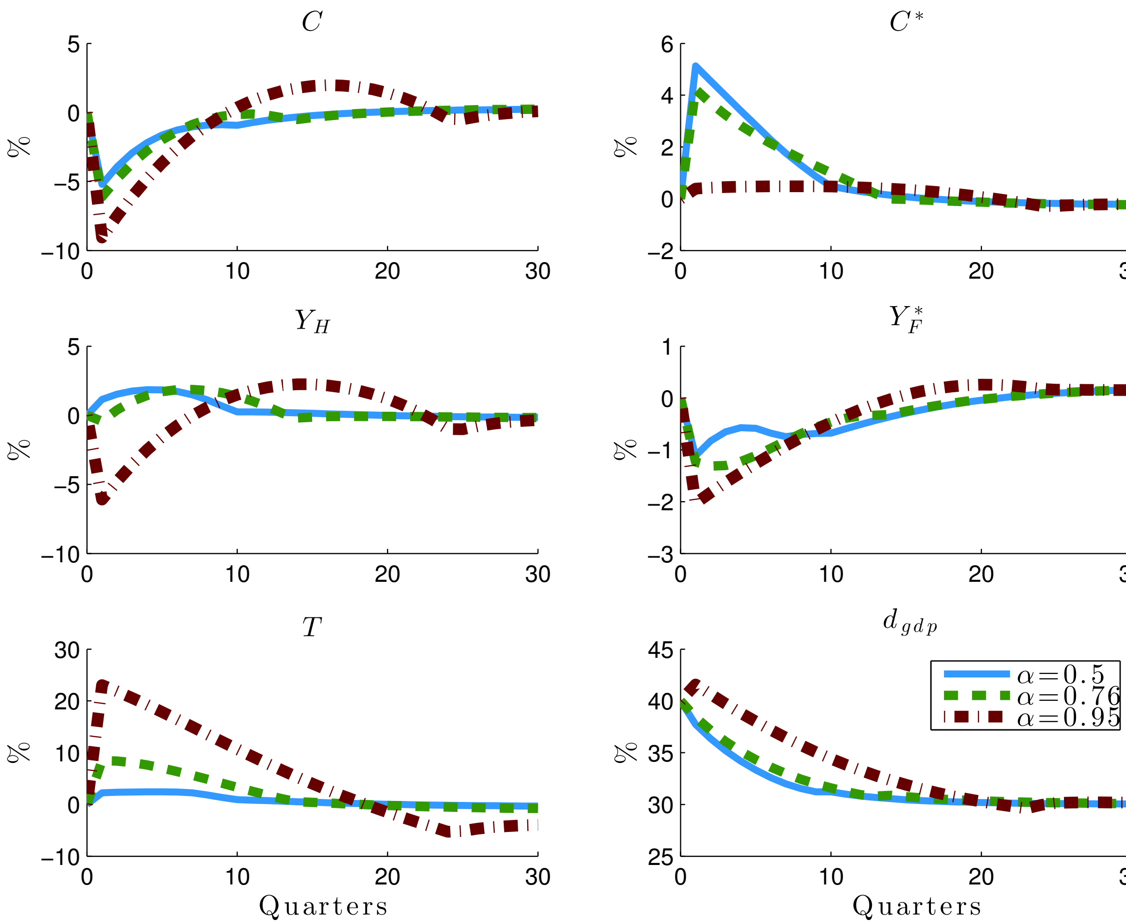

Figure 17: Impulse responses under optimal policy for different values of $\alpha = 0.5, 0.76$ and 0.95. Variables are: Home and Foreign consumption (C, C^*), Home and Foreign output (Y_H, Y_F^*), terms of trade (T), the level of external debt of country H as a percentage of its GDP (d_{gdp}). All variables, except for d_{gdp}, are in percentage deviations from the steady state.

The other dimension through which the international transmission mechanism of the

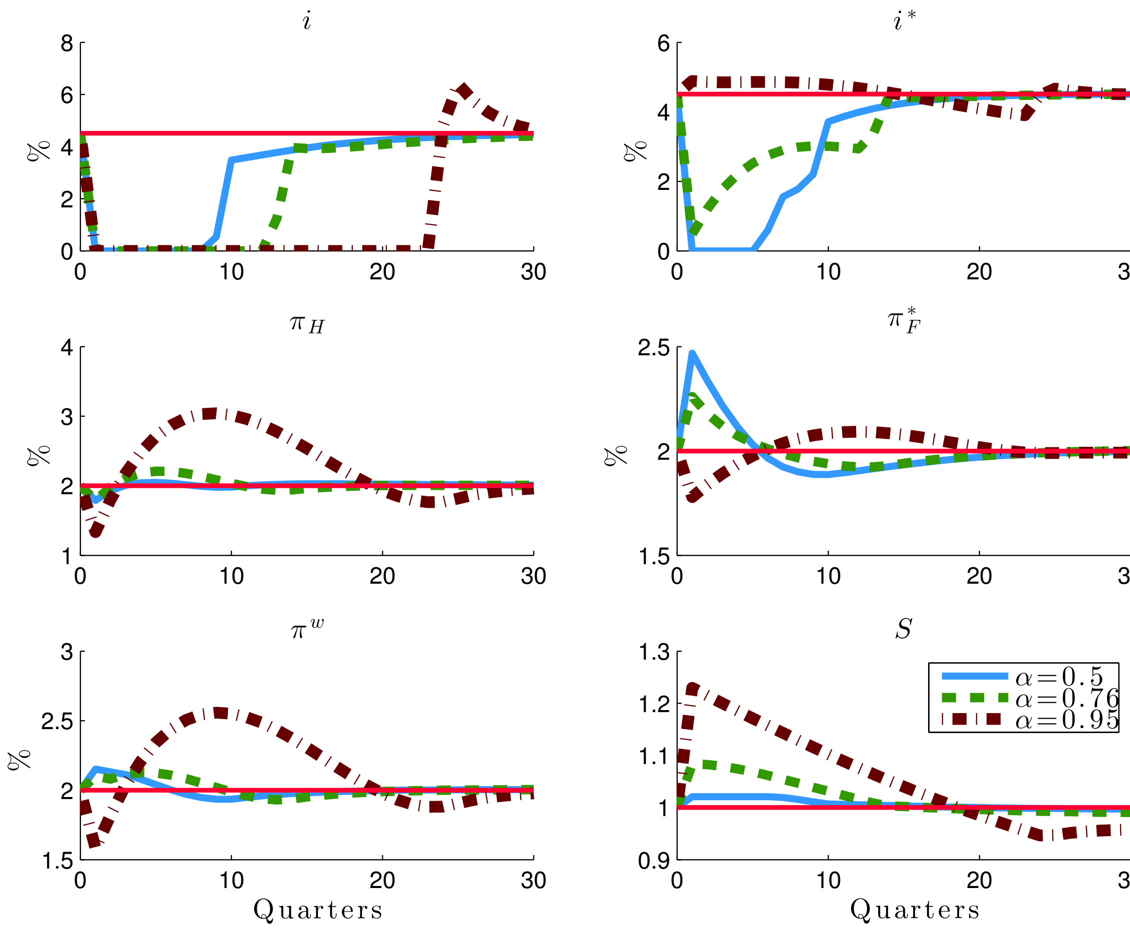

Figure 18: Impulse responses under optimal policy for different values of $\alpha = 0.5, 0.76$ and 0.95. Variables are: Home and Foreign nominal interest rates (i,i^*), Home and Foreign producer inflation rates (π_H, π_F^*), world inflation (π^W), defined as $\pi^W = 1/2 \cdot \pi + 1/2 \cdot \pi^*$ where π and π^* are the Home and Foreign CPI inflation rates, the level of the nominal exchange rate (S); inflation and interest rates are in percents and annual rates.

shock could be different is along alternative assumptions on the degree of home bias, captured by the parameter α. We have already discussed, in the simple model of Section 2, that the responses of terms of trade and nominal exchange rate depend on this assumption. The higher is the home bias, the more the terms of trade of the deleveraging country worsen implying a nominal exchange rate depreciation. However, according to (2.26), all these movements are costly and optimal monetary policy should be directed to mitigate them. Figures 17 and 18, fixing now θ at the benchmark unitary value, show how optimal policy changes for alternative values of α around the benchmark of 0.76, plotting also the impulse responses when there is no home bias, i.e. $\alpha = 0.5$, and for a high degree of home bias, $\alpha = 0.95$. As Figure 18 shows, the nominal exchange rate depreciates only slightly in absence of home bias. However, there is still an important international transmission of the shock since Home and Foreign real interest rates are more interconnected in this case.[26]

The fall in Foreign real rates stimulates consumption in country F to compensate for the fall in country H. Not surprisingly, given the fall in both real rates, foreign country is now forced to stay at the zero-lower bound and for a long time. When instead the home-bias parameter is large, the two economies behave like closed economies. Indeed, while the nominal exchange rate moves a lot without much ability to switch expenditure across goods, consumption and output in the Foreign economy are only marginally affected by the deleveraging of country H. In this case, the nominal interest rate in country F does not go to the zero-lower bound while the liquidity trap is longer in country H. With a high degree of home bias, the large depreciation of the exchange rate is helpless and the entire burden of adjustment is borne by the deleverager.

In terms of the three channel identified above, the only one available to cope with

[26] When $\alpha = 0.5$, the Home and Foreign real interest rates associated with the consumption baskets C and C^* are equal across countries, since $C = C^*$.

the shock, when α is high, is the fall in the real rate in the deleverager. Indeed, in this case, the economies are closed. For intermediate values of α, economies become more open and therefore the other two channels become more relevant. When α is close to 0.5, the exchange rate is marginally affected by the deleveraging shock and the benevolent planner lowers the real rates in both countries. A global liquidity trap emerges in this case.

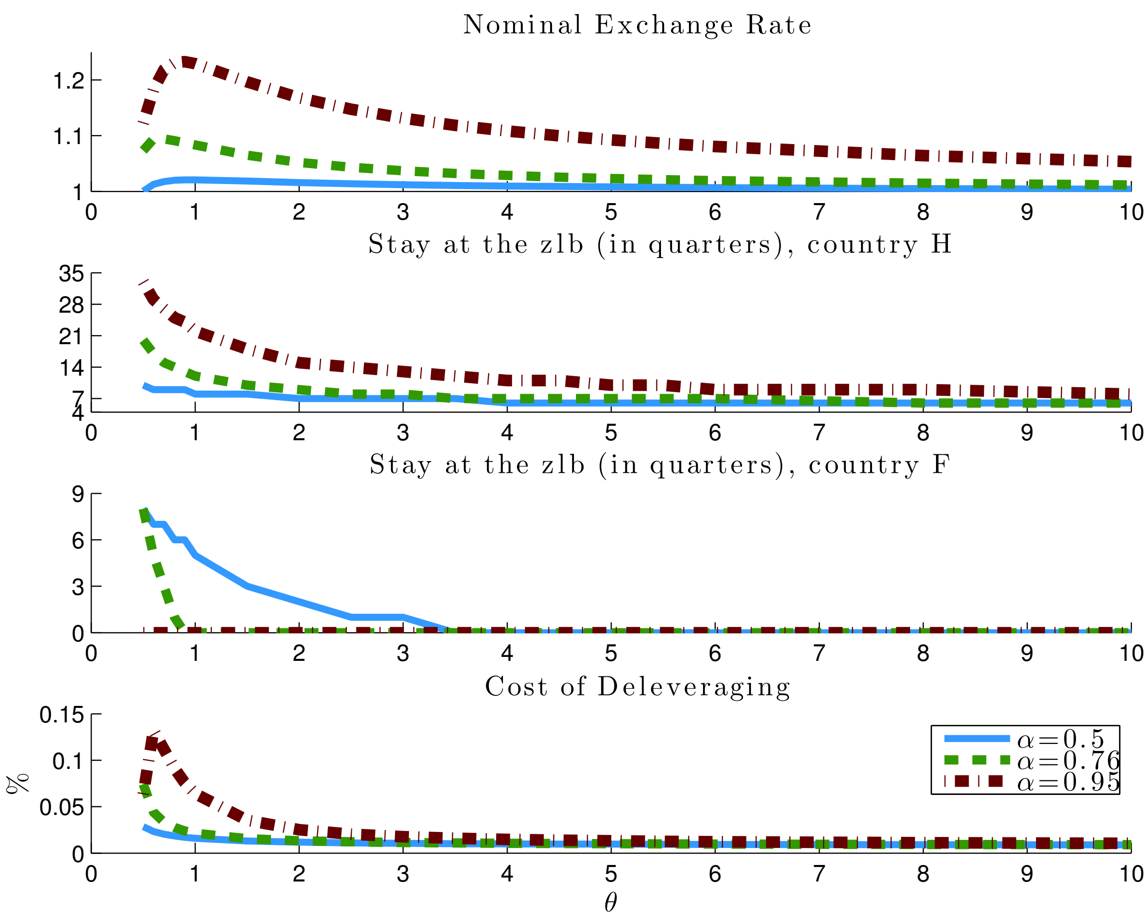

Figure 19: First chart: first-quarter level of the nominal exchange rate (S). Second chart: length of stay (in quarters) at the zero-lower bound for the deleveraging country H. Third chart: length of stay (in quarters) at the zero-lower bound for country F. Fourth chart: costs of deleveraging in units of a percentage change in steady-state consumption for both countries. All charts are done under optimal policy for different values of θ (x-axis) and for $\alpha = 0.5, 0.76$ and 0.95.

Figure 19 synthesizes some of the results of this Section by plotting in the following order, from top to the bottom, the first-quarter level of the nominal exchange rate, the length of stay (in quarters) at the zero-lower bound, for the Home and Foreign country, respectively, and the costs of the deleveraging shock under optimal policy. All these plots are done for a range of θ between 0.5 and 6 and for three values of $\alpha = 0.5, 0.76$ and 0.95.[27]

The Home-country exchange rate depreciates on impact the more, the higher the degree of home bias and the closer to the unitary value the elasticity of substitution is. Higher values of θ or values below one imply a smaller depreciation and even a short-run appreciation in absence of home bias. The length of the stay at the zero-lower bound becomes shorter as the elasticity of substitution θ increases. A low value of α reduces the stay for the deleveraging country at the expenses of a longer stay for the other economy. Finally, the bottom chart of Figure 19 shows the costs of deleveraging in terms of a permanent reduction, in percentage, in the steady-state consumption levels of both countries. The costs are particularly sizeable when the international transmission mechanism is weaker. In our model, this weakness depends on two factors: 1) a low elasticity of substitution, since it implies a weak expenditure-switching channel and may cause phenomena of "immiserizing" growth and 2) a high degree of home bias, since economies are closed and the shock can only be absorbed in the Home country.

2.6. Deleveraging and the original sin

In this section, we study how the transmission mechanism of international debt deleveraging and its efficient adjustment change when the external debt is denominated in foreign currency. Indeed, the analysis of previous sections might be appro-

[27] As discussed in Benigno (2009), in a similar class of models, there is no solution for low values of θ.

priate for countries like the US which has the exorbitant privilege of being able to borrow in its own currency, but not for emerging-market economies which are usually affected by the original sin of borrowing in foreign currency. We make few changes to our model to accommodate this case. In particular the flow budget constraint of households in country H is now written as:

$$P_t C_t = \int_0^1 W_t(j) L_t(j) dj + \Pi_t + \frac{S_t D_t}{1 + i_t^*} - S_t D_{t-1} - f_t P_t \cdot \tilde{\chi}\left(\frac{S_t D_t}{P_t}\frac{1}{f_t}, \frac{S_t \bar{D}_t}{P_t}\frac{1}{f_t}\right)$$

where indeed the currency denomination of debt is that of country F and the interest-rate paid on debt is the foreign interest rate $1 + i_t^*$. We have also changed the arguments of the cost function to reflect the new denomination of debt where now f_t represents the risk-free level of external debt that can be held without costs. Given this budget constraint, the Euler equations change appropriately. Details are left to the online appendix.

Figures 20 and 21 compare the results of optimal policy when debt of the deleveraging country is denominated in foreign currency with the benchmark case of domestically-denominated debt of Section 2.4.

Results are in some way surprising. The striking difference is in the response of the policy rates. Under the benchmark case of debt denominated in domestic currency, the liquidity trap is mainly affecting the deleverager's nominal interest rate. On the opposite, when debt is denominated in foreign currency, it is the foreign interest rate that should be forced to the zero-lower bound and for long time, almost three years as shown in Figure 21. This is intuitive since the borrowing cost for the deleverager depends now on foreign interest rates. To ease the costs of deleveraging, the benevolent planner tries to lower at most the foreign interest rate. Indeed, the domestic nominal

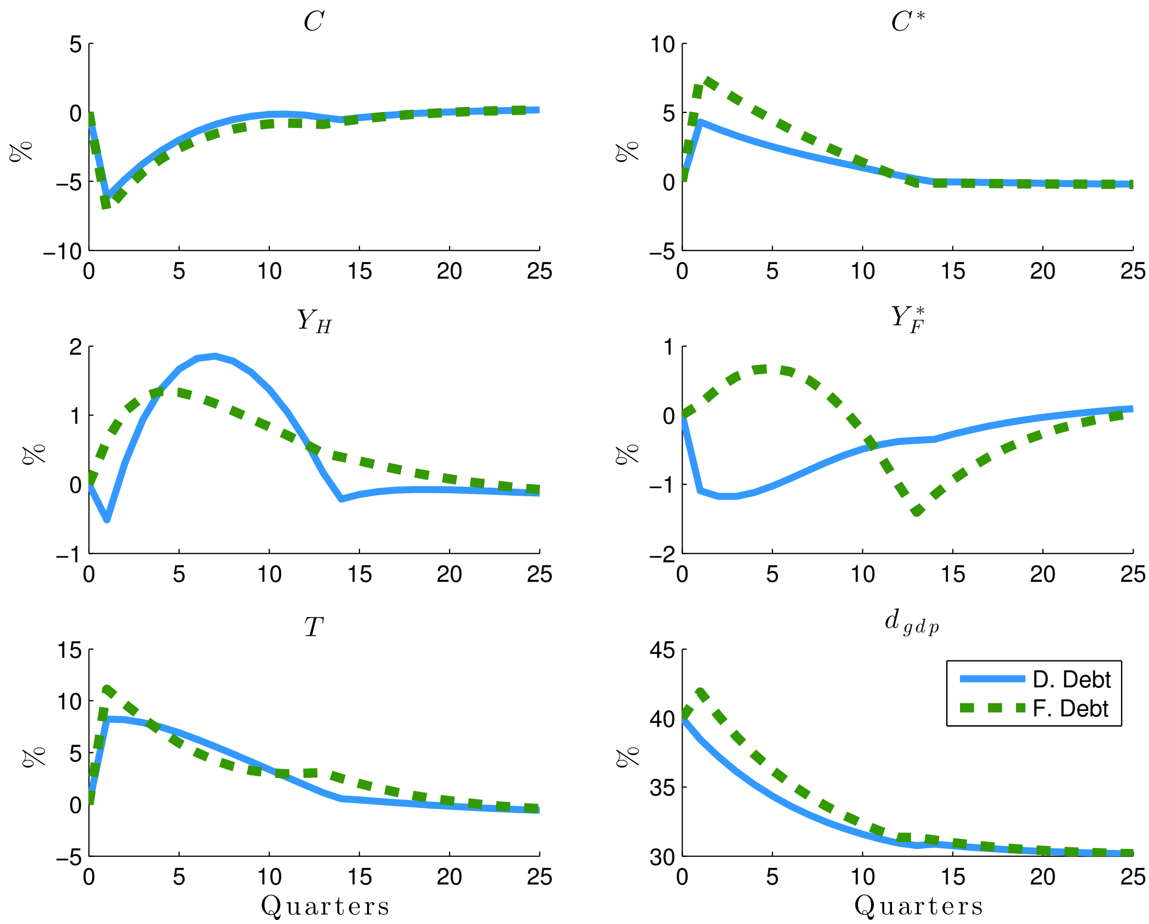

Figure 20: Impulse responses under optimal policy: comparison between the benchmark case of debt denominated in domestic currency versus the case of foreign-denominated debt. Variables are: Home and Foreign consumption (C, C^{*}), Home and Foreign output (Y_H, Y_F^{*}), terms of trade (T), the level of external debt of country H as a percentage of its GDP (d_{gdp}). All variables, except for d_{gdp}, are in percentage deviations from the steady state.

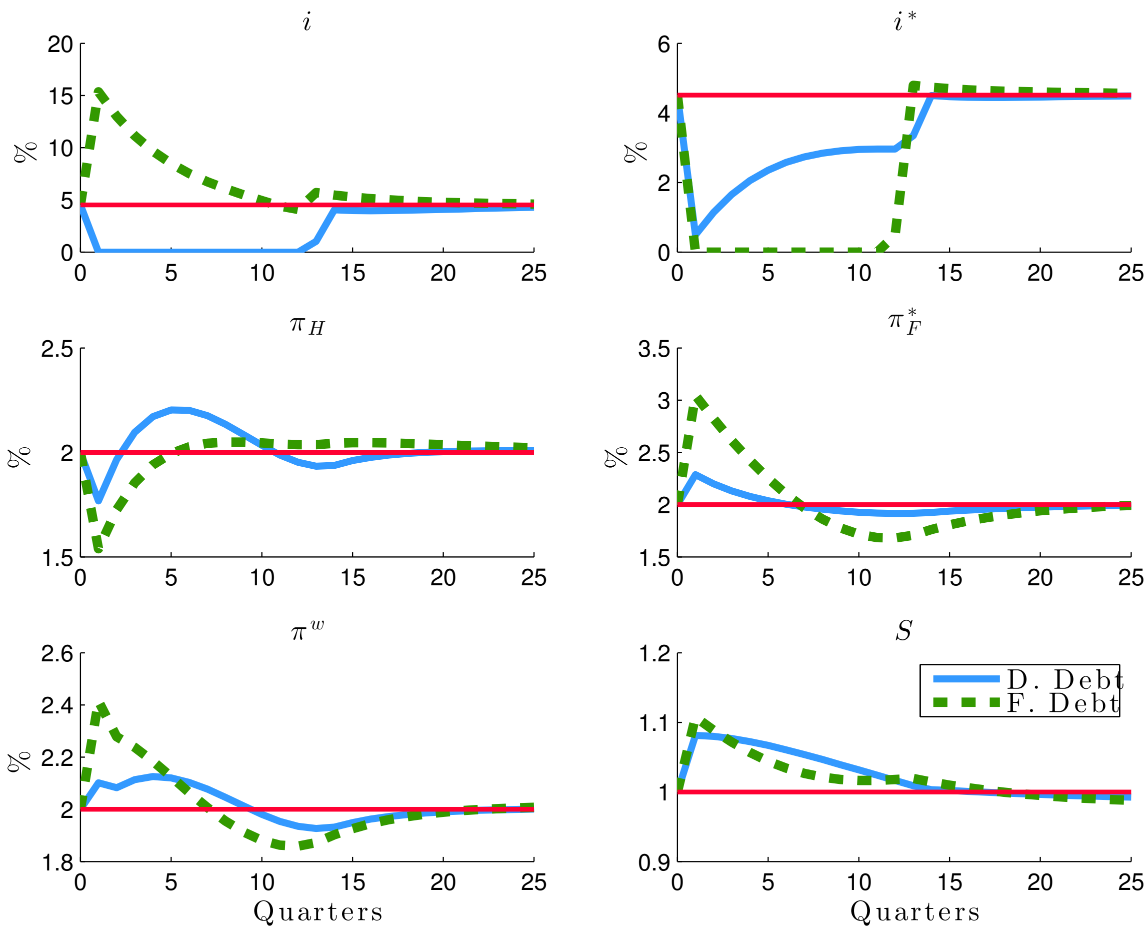

Figure 21: Impulse responses under optimal policy: comparison between the benchmark case of debt denominated in domestic currency versus the case of foreign-denominated debt. Variables are: Home and Foreign nominal interest rates (i,i^*), Home and Foreign producer inflation rates (π_H, π_F^*), world inflation (π^W), defined as $\pi^W = 1/2 \cdot \pi + 1/2 \cdot \pi^*$ where π and π^* are the Home and Foreign CPI inflation rates, the level of the nominal exchange rate (S); inflation and interest rates are in percents and annual rates.

interest rate is left to rise. The fact that now the foreign interest rate stays longer at the zero-lower bound implies also that the real rate in country F is lower for a long period which pushes up consumption in country F to a larger extent. On impact it rises by 7.5% as opposed to the 4% of the benchmark case. As a consequence, output expands more in country F.

It is also surprising to see that the exchange rate depreciates much more when debt is denominated in foreign currency rather than under the benchmark case. Indeed, a depreciation of the nominal exchange rate is even more costly in this case since it "inflates" the real resources needed by country H to pay back debt. The overall external debt to GDP initially rises and then falls at a slower pace toward the new steady-state value. However, these costs are outweighed by the benefits. The central planner, by worsening the Home-country terms of trade, can tilt production from the Foreign economy to the Home country. This is needed to mute the expansion in the Foreign economy, caused by a too low real rate. Otherwise, the over heating in country F would be even larger bringing more inefficiencies.

2.7. Conclusion

We have examined the international implications of debt deleveraging in one country within the world economy and studied how monetary policy should be set at the global level, focusing in particular on the reaction of the nominal exchange rate and policy rates. Deleveraging reduces aggregate demand and may lead to recession, as economic agents save to repay the debt. There are interesting international spillovers through trade and the exchange rate. The adjustment to a deleveraging shock naturally requires movements in two relative prices: namely the exchange rate and the real interest rate. The exchange rate, which is an international relative price, moves in such a way as to rebalance resources across countries. The deleveraging country's currency will depreciate in the short run and appreciate in the long-run. This depends critically on home bias in consumers' preferences. Since in the short run agents who are paying down their debt have less resources for consumption, the price of home goods should fall relative to the foreign, and a fall in the exchange rate will assist this adjustment. Once the debt has been repaid, however, agents have more resources to spend and in particular on domestic goods. This exchange rate movements produce expenditure-switching effects which favour production in the deleveraging country at the expenses of the rest of the world. The other important relative price in the adjustment, the real interest rate, will come down in both countries and fall more sharply in the deleveraging country. This fall in the real rates stimulates foreign consumption in order to mitigate the overall impact of the shock on the deleverager and the world economy.

The interesting and surprising result of this paper is that all these large variations in relative prices and quantities are inefficient, when seen from the perspective of a benevolent planner, who cares about world utility and sees the new level of external debt reached after deleveraging as the efficient one. This planner dislikes all the adjustment process described above and would like to immediately achieve the new equilibrium allocation characterized by lower debt. Therefore, important trade-offs emerge between stabilizing consumption, output and relative prices. The desirability of the expenditure-switching channel versus the Home and Foreign real-interest-rate channels depends on the elasticity of substitution in consumption between domestic and foreign goods and on the degree of home bias. Only for elasticities of substitution around the unitary value, the nominal exchange rate of the deleverager is left to depreciate in a sizeable way in the short run. Otherwise, it should move less or even appreciate when the elasticity of substitution is very low. For low degrees of home bias, the real interest rate should fall in a substantial way in the foreign economy and the burden of adjustment is more shared across countries. High degrees of home bias imply that all the burden is on the deleveraging country because economies are more closed. It is true that the nominal exchange rate and terms of trade vary substantially in these cases, but they are less effective in generating spillovers to the rest of the world.

In this study, we have concentrated on the role of monetary policy and alternative exchange-rate regimes in mitigating or amplifying the costs of debt deleveraging. The zero lower bound on nominal interest rates is a significant constraint in our analysis, because the natural rate of interest falls substantially. We have shown that whether zero-lower bound policies should be coordinated or not depends also on the international transmission mechanism. When the elasticity of substitution between foreign and domestic goods and/or the degree of home bias are low, a global liquidity trap should emerge as an optimal policy of a benevolent planner.

We have analyzed a very simple two-country open-economy model. The consequent limitations are essentially the price paid for the simplifications used. First, in the real world debt deleveraging affects a variety of agents in the economy: households, banks, firms and governments. Distinguishing them in the model would enhance realism and possibly enable us to differentiate the effects of deleveraging on the economy according on which agents are paying down their debt. It is likely that, however, the qualitative results implied by our simple framework would hold also in a more complex context. Second, the asset market structure has been kept very simple – only one asset traded internationally. This is a significant limitation, since the portfolio position of a country is much more complex and diversified involving assets and liabilities, in different currencies and instruments ranging from equity to debt. Finally, we have focused on the response of a benevolent policymaker maximizing welfare of the global economy and abstracted from a possible lack of international monetary policy coordination which might change in a substantial way the equilibrium allocation. In particular, among the three channels identified in this paper to cope with the deleveraging shock, the fall in the real rate of the non-deleveraging country and the depreciation of the nominal exchange rate requires some cooperation at the international level. Non-coordinated policies might result in sub-optimal equilibria with higher costs for the world economy. This is an interesting area of analysis which we leave to future research.[28]

[28] Fujiwara et al. (2011) discuss cooperative versus non-cooperative solutions in the emergence of global liquidity traps.

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APPENDIX

2.8. Derivation of loss function (2.23)

To derive (2.23), we take a second-order approximation around the final steady state of the following Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{t=0}^{\infty} \beta^t \left\{ \xi \ln(C_t) + (1-\xi)\ln(C_t^*) + \lambda_{1,t}(Y_{H,t} - (\alpha T_t^{1-\alpha} C_t + (1-\alpha) T_t^{\alpha} C_t^*) + \right. \\
& \left. + \lambda_{2,t}(Y_{F,t}^* - ((1-\alpha) T_t^{-\alpha} C_t + \alpha T_t^{\alpha-1} C_t^*)) \right\}
\end{aligned}
$$

First it should be noted that in the steady state the following conditions

$$\xi \bar{C}^{-1} = \bar{T}^{1-\alpha}\bar{\lambda}_1, \tag{2.27}$$

$$(1-\xi)(\bar{C}^*)^{-1} = \bar{T}^{\alpha}\bar{\lambda}_1, \tag{2.28}$$

$$\bar{T}\bar{\lambda}_1 = \bar{\lambda}_2, \tag{2.29}$$

hold together with the two resource constraints.

By taking a second-order approximation of the above Lagrangian around the above-defined steady state, we obtain

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{t=0}^{\infty}\beta^t\left\{\xi\bar{C}^{-1}(C_t-\bar{C})-\xi\bar{C}^{-2}(C_t-\bar{C})^2+(1-\xi)(\bar{C}^*)^{-1}(C_t^*-\bar{C}^*)+\right.\\
& -(1-\xi)(\bar{C}^*)^{-2}(C_t^*-\bar{C}^*)^2-\bar{\lambda}_1[\alpha(1-\alpha)\bar{C}\bar{T}^{-\alpha}(T_t-\bar{T})+\alpha\bar{T}^{1-\alpha}(C_t-\bar{C})+\\
& +\alpha(1-\alpha)\bar{T}^{-\alpha}(T_t-\bar{T})(C_t-\bar{C})-\alpha^2(1-\alpha)\bar{C}\bar{T}^{-\alpha-1}\frac{(T_t-\bar{T})^2}{2}+\\
& +\alpha(1-\alpha)\bar{C}^*\bar{T}^{\alpha-1}(T_t-\bar{T})+(1-\alpha)\bar{T}^{\alpha}(C_t^*-\bar{C}^*)+\\
& \alpha(1-\alpha)\bar{T}^{\alpha-1}(T_t-\bar{T})(C_t^*-\bar{C}^*)-\alpha(1-\alpha)^2\bar{C}^*\bar{T}^{\alpha-2}\frac{(T_t-\bar{T})^2}{2}]+\\
& -\bar{\lambda}_2[-\alpha(1-\alpha)\bar{C}\bar{T}^{-\alpha-1}(T_t-\bar{T})+(1-\alpha)\bar{T}^{-\alpha}(C_t-\bar{C})+\\
& -\alpha(1-\alpha)\bar{T}^{-\alpha-1}(T_t-\bar{T})(C_t-\bar{C})+\alpha(1+\alpha)(1-\alpha)\bar{C}\bar{T}^{-\alpha-2}\frac{(T_t-\bar{T})^2}{2}+\\
& +\alpha(\alpha-1)\bar{C}^*\bar{T}^{\alpha-2}(T_t-\bar{T})+\alpha\bar{T}^{\alpha-1}(C_t^*-\bar{C}^*)+\alpha(\alpha-1)\bar{T}^{\alpha-2}(T_t-\bar{T})(C_t^*-\bar{C}^*)+\\
& \left.+\alpha(\alpha-1)(\alpha-2)\bar{C}^*\bar{T}^{\alpha-3}\frac{(T_t-\bar{T})^2}{2}]\right\}
\end{aligned}
$$

in which it should be noted that all the linear terms cancel out using the steady-state relationship. The second-order terms can be simplified and the above expression collapses to

$$
\mathcal{L}=\sum_{t=0}^{\infty}\beta^t\left\{-\xi(\tilde{C}_t)^2-(1-\xi)(\tilde{C}_t^*)^{-2}-\bar{\lambda}_1\alpha(1-\alpha)\bar{C}\bar{T}^{1-\alpha}\frac{\tilde{T}_t^2}{2}-\bar{\lambda}_1\alpha(1-\alpha)\bar{C}^*\bar{T}^{\alpha}\frac{\tilde{T}_t^2}{2}\right\}
$$

which can be further written as

$$
\mathcal{L}=\sum_{t=0}^{\infty}\beta^t\left\{-\xi(\tilde{C}_t)^2-(1-\xi)(\tilde{C}_t^*)^{-2}-\alpha(1-\alpha)\xi\frac{\tilde{T}_t^2}{2}-\alpha(1-\alpha)(1-\xi)\frac{\tilde{T}_t^2}{2}\right\}
$$

from which the loss function in the text follows.

2.9. Model equilibrium conditions

The model of Section 3 is represented by the following 18 equilibrium conditions

$$(C_t^*)^{-\rho} = \beta E_t \left\{ (C_{t+1}^*)^{-\rho} \frac{(1+i_t)Q_t}{Q_{t+1}\Pi_{t+1}} \right\},$$

$$(C_t^*)^{-\rho} = \beta E_t \left\{ (C_{t+1}^*)^{-\rho} \frac{(1+i_t^*)}{\Pi_{t+1}^*} \right\},$$

$$(C_t)^{-\rho} \left\{ 1 - (1+i_t)\psi\left(\frac{d_t}{k_t}\right) \right\} = \beta E_t \left\{ (C_{t+1})^{-\rho} \frac{(1+i_t)}{\Pi_{t+1}} \right\},$$

$$C_t = p_{H,t}Y_{H,t} + \frac{d_t}{(1+i_t)} - \frac{d_{t-1}}{\Pi_t} - k_t\chi\left(\frac{d_t}{k_t}\right)$$

$$Y_{F,t}^* = p_{F,t}^{-\theta}\left[(1-\alpha)C_t + \alpha Q_t^\theta C_t^*\right]$$

$$Y_{H,t} = p_{H,t}^{-\theta}\left[\alpha C_t + (1-\alpha) Q_t^\theta C_t^*\right]$$

$$p_{F,t}^{\theta-1} = \alpha\left(T_t\right)^{\theta-1} + (1-\alpha)$$

$$p_{H,t}^{\theta-1} = \alpha + (1-\alpha)\left(T_t\right)^{1-\theta}$$

$$\left(\frac{1-\lambda^*\left(\frac{\Pi_{F,t}^*}{\bar{\Pi}^*}\right)^{\tau-1}}{1-\lambda^*}\right)^{\frac{1+\eta\tau}{\tau-1}} = \frac{F_t^*}{K_t^*}$$

$$F_t^* = (C_t^*)^{-\rho} p_{F,t}\frac{1}{Q_t}Y_{F,t}^* + \beta\lambda^* E_t\left[F_{t+1}^*\left(\frac{\Pi_{F,t+1}^*}{\bar{\Pi}^*}\right)^{\tau-1}\right]$$

$$K_t^* = \tilde{\mu}(Y_{F,t}^*)^{1+\eta} + \beta\lambda^* E_t\left[K_{t+1}^*\left(\frac{\Pi_{F,t+1}^*}{\bar{\Pi}^*}\right)^{\tau(1+\eta)}\right]$$

$$\left(\frac{1-\lambda\left(\frac{\Pi_{H,t}}{\bar{\Pi}}\right)^{\tau-1}}{1-\lambda}\right)^{\frac{1+\eta\tau}{\tau-1}} = \frac{F_t}{K_t}$$

$$F_t = (C_t)^{-\rho} p_{H,t} Y_{H,t} + \beta\lambda E_t \left[F_{t+1} \left(\frac{\Pi_{H,t+1}}{\bar{\Pi}} \right)^{\tau-1} \right]$$

$$K_t = \tilde{\mu} Y_{H,t}^{1+\eta} + \beta\lambda E_t \left[K_{t+1} \left(\frac{\Pi_{H,t+1}}{\bar{\Pi}} \right)^{\tau(1+\eta)} \right]$$

$$\frac{T_t}{T_{t-1}} = \frac{\Pi^*_{F,t}}{\Pi_{H,t}} \frac{S_t}{S_{t-1}}$$

$$Q_t = \left[(1-\alpha) p_{H,t}^{1-\theta} + \alpha p_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$\Pi_t = \frac{\left[\alpha \left(\Pi_{H,t} \right)^{1-\theta} + (1-\alpha) \left(T_t \Pi_{H,t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}}{\left[\alpha + (1-\alpha) \left(T_{t-1} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}}$$

$$\Pi^*_t = \Pi_t \left(\frac{Q_t}{Q_{t-1}} \right) \left(\frac{S_{t-1}}{S_t} \right)$$

which need to be solved for the following 20 unknowns C_t, C^*_t, i_t, Q_t, Π_t, i^*_t, Π^*_t, T_t, $Y_{H,t}$,.$Y_{F,t}$, d_t, $\Pi^*_{F,t}$, F^*_t, K^*_t, $\Pi_{H,t}$, F_t, K_t, $S_t/S_{t-1}, p_{H,t}, p_{F,t}$ given the inflation targets $\bar{\Pi}^*$ and $\bar{\Pi}$ where two further restrictions come from the policy rules, specified in the text. Notice that $\tilde{\mu}$ is composite mark-up including the mark-ups in the goods and labor markets, i.e. $\tilde{\mu} = \mu \cdot \mu_w$ where $\mu_w \equiv \tau/(\tau-1)$. We also have defined $p_{H,t} \equiv P_{H,t}/P_t$ and $p_{F,t} \equiv P_{F,t}/P_t$. Moreover, the zero-lower-bound constraint requires that $i_t \geq 0$ and $i^*_t \geq 0$. In the above equations, we have defined $\chi(d_t/k_t) = \tilde{\chi}(d_t/k, \bar{d}_t/k)$ since in equilibrium $\bar{d}_t = d_t$.

2.10. Model Solution

We define $y_t \equiv [z_t \; x_{t-1} \; w_t]$ as a vector of length n_y containing the control variables, z_t, of dimension n_z, the endogenous state variables, x_{t-1}, of dimension n_x and the exogenous state variables, w_t, of dimension n_w. In particular we may define vector of exogenous state variables more specifically, i.e.:

$$w_t \equiv \begin{bmatrix} k_t & i_t^z & i_t^{*z} & c_t & c_t^* & y_{H,t} & y_{F,t}^* & s_t \end{bmatrix}'$$

where k_t represents the safe level of debt, as defined in the main text; i_t^z and i_t^{*z} are two variables used to model the zero-lower bound on the nominal interest rates. Indeed in the log-linear approximation, the restriction that nominal interest rates should be above zero corresponds to have $\hat{\imath}_t \geq i_t^z$ and $\hat{\imath}_t^* \geq i_t^{*z}$. The variables c_t, c_t^*, $y_{H,t}$, $y_{F,t}^*$ and s_t are the defined in equation (2.40) and represent the log deviation between the final and initial steady state of C, C^*, Y_H, Y_F^* and T respectively.

Finally we define ϵ as a vector of length n_ϵ that collects innovations to the exogenous stochastic variables. Again we may define this vector more in detail:

$$\epsilon \equiv \begin{bmatrix} (\ln(k_{min}) - \ln(k_{max})) & -\ln(\frac{\Pi}{\beta} - 1) & -\ln(\frac{\Pi}{\beta} - 1) & \epsilon_c & \epsilon_{c^*} & \epsilon_{y_H} & \epsilon_{y_F^*} & \epsilon_s \end{bmatrix}'$$

where ϵ_x is defined as the log difference between the final and initial steady state for a generic variable X, i.e. $\epsilon_x \equiv \ln(\bar{X}) - \ln(X)$. The process for the exogenous state variables can be modeled as:

$$w_t = M_w w_{t-1} + \tilde{C}^t \; \epsilon$$

where M_w is an identity matrix of dimensions $n_w \times n_w$. $\tilde{C}^t$ is matrix of dimension $n_w \times n_\epsilon$ and it is an identity matrix when $t = 1$, otherwise it is a matrix of zeros.

We can write the model in a compact form as:

$$A \cdot y_{t+1} = B^t \cdot y_t + C^{t+1} \cdot \epsilon \tag{2.30}$$

where B^t and C^{t+1} are time-dependent matrices, A and B^t have dimension $n_y \times n_y$ and C^{t+1} has dimension $n_y \times n_\epsilon$. The matrix C^t is of the form

$$C^{t+1} = \begin{bmatrix} H \\ \tilde{C}^{t+1} \end{bmatrix},$$

where H is a matrix of zeros of dimension $(n_y - n_w) \times n_\epsilon$.

We consider a framework which is flexible enough to treat the possibility that either Home interest rate, i_t, is at zero-lower bound, or Foreign interest rate, i_t^*, is at zero-lower bound, or both, or none of them. B^t should be adjusted accordingly depending on the different cases.

We define B^a as the matrix characterizing the case in which both interest rates are at zero lower bound; $B^H(B^F)$ is the matrix characterizing the case where only Home (Foreign) interest rate is at zero lower bound while B^n refers to the case where both interest rates are not constrained by the zero-lower bound.

In the model of Section (2.4), we verify the following sequence of events: from 0 to T_1 both interest rates are at zero lower bound (T_1 can also be 0), from T_1 to T_2 only the interest rate in country H is at zero lower bound. From T_2 onwards both interest rates are above zero. This timing implies that:

$$B^t = \begin{cases} B^a & for \;\; t \in (0; T_1] \\ B^H & for \;\; t \in (T_1, T_2] \\ B^n & for \;\; t \in (T_2, \infty] \\ \end{cases}$$

where T_1 and T_2 are model specific and to be determined endogenously.

In the model of Section (2.6), we verify the following sequence of events: from 0 to $\tilde{T}_1$, the interest rate in country F is at the zero lower bound and, from $\tilde{T}_1$ onwards, both interest rates will be above zero.

This timing implies that:

$$B^t = \begin{cases} B^F & for \;\; t \in (0, \tilde{T}_1] \\ B^n & for \;\; t \in (\tilde{T}_1, \infty] \\ & \end{cases}$$

We can rewrite the system (2.30) by omitting the law of motion of the exogenous state variables:

$$\begin{bmatrix} \tilde{A}_1 & \tilde{A}_2 \end{bmatrix} \begin{bmatrix} z_{t+1} \\ x_t \end{bmatrix} = \begin{bmatrix} \tilde{B}_1^t & \tilde{B}_2^t & \tilde{B}_3^t \end{bmatrix} \begin{bmatrix} z_t \\ x_{t-1} \\ w_t \end{bmatrix} \tag{2.31}$$

where $\tilde{A}$ is a matrix of dimension $(n_y - n_w) \times (n_y - n_w)$ which is appropriately partitioned in the matrices $\tilde{A}_1$ and $\tilde{A}_2$, while $\tilde{B}$ is a matrix of dimension $(n_y - n_w) \times n_y$ which is appropriately partitioned in the matrices $\tilde{B}_1^t$, $\tilde{B}_2^t$, $\tilde{B}_3^t$.

We guess the following linear solution:

$$z_t = h_x^t x_{t-1} + h_w^t w_{t-1} + h_\epsilon^t \epsilon,$$

$$x_t = g_x^t x_{t-1} + g_w^t w_{t-1} + g_\epsilon^t \epsilon,$$

$$w_t = M_w w_{t-1} + \tilde{C}^t \epsilon,$$

We can plug the guessed solution into equation (2.31) and rearrange everything to get:

$$\begin{bmatrix} \tilde{A}_1 h_x^{t+1} + \tilde{A}_2 & -\tilde{B}_1^t \end{bmatrix} \begin{bmatrix} g_x^t \\ h_x^t \end{bmatrix} = \tilde{B}_2^t \tag{2.32}$$

$$\begin{bmatrix} \tilde{A}_1 h_x^{t+1} + \tilde{A}_2 & -\tilde{B}_1^t \end{bmatrix} \begin{bmatrix} g_w^t \\ h_w^t \end{bmatrix} = \tilde{B}_3^t M_w - \tilde{A}_1 h_w^{t+1} M_w \tag{2.33}$$

$$\begin{bmatrix} \tilde{A}_1 h_x^{t+1} + \tilde{A}_2 & -\tilde{B}_1^t \end{bmatrix} \begin{bmatrix} g_\epsilon^t \\ h_\epsilon^t \end{bmatrix} = \tilde{B}_3^t \tilde{C}^t - \tilde{A}_1 h_\epsilon^{t+1} - \tilde{A}_1 h_w^{t+1} \tilde{C}_t \tag{2.34}$$

Equations (2.32), (2.33) and (2.34) can be solved for the unknown matrices h_x^t, h_w^t, h_ϵ^t, g_x^t, g_w^t, g_ϵ^t working backward. Since we know that after T_2 (or $\tilde{T}_1$ in the model with foregn-denominated debt), there are no shocks and the interest rates are not constrained by the zero-lower bound, we can find the unknown time-invariant matrices h_x, h_w, h_ϵ, g_x, g_w, g_ϵ which applies for each $t \geq T_2$ (or $t \geq T_1$). Then starting from these matrices, we can get all the remaining matrices by using the above equations working backward. Given an initial guess on T_1, T_2 for one model and $\tilde{T}_1$ for the other model, we verify that the implied path of the nominal interest rates and the stay at the zero-lower bound are consistent with the guessed timing. Otherwise, we guess another T_1, T_2 or $\tilde{T}_1$, depending on the model.

2.11. Optimal policy

We take a second-order approximation of the welfare of world economy (2.25) around the final efficient steady state. First, notice that the objective can be written as

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\xi \left(\frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_{H,t}^{1+\eta}}{1+\eta} \Delta_t \right) + (1-\xi) \left(\frac{C_t^{*1-\rho}}{1-\rho} - \frac{Y_{F,t}^{*1+\eta}}{1+\eta} \Delta_t^* \right) \right] \right\}$$

where the indexes of price dispersion are defined as

$$\Delta_t \equiv \lambda \left(\frac{\Pi_{H,t}}{\bar{\Pi}_t}\right)^{(1+\eta)\tau} \Delta_{t-1} + (1-\lambda)\left(\frac{1-\lambda\left(\frac{\Pi_{H,t}}{\bar{\Pi}_t}\right)^{\tau-1}}{1-\lambda}\right)^{\frac{(1+\eta)\tau}{\tau-1}} \tag{2.35}$$

$$\Delta_t^* \equiv \lambda^* \left(\frac{\Pi_{F,t}}{\bar{\Pi}_t^*}\right)^{(1+\eta)\tau} \Delta_{t-1}^* + (1-\lambda^*)\left(\frac{1-\lambda^*\left(\frac{\Pi_{F,t}}{\bar{\Pi}_t^*}\right)^{\tau-1}}{1-\lambda^*}\right)^{\frac{(1+\eta)\tau}{\tau-1}} . \tag{2.36}$$

A second-order approximation of the objective function around the efficient steady state delivers

$$\begin{aligned} U_t \quad = \quad & \bar{U} + E_t\left\{\sum_{t=0}^{\infty}\beta^t[\xi[\bar{C}^{-\rho}(C_t-\bar{C}) - \bar{Y}_H^{\eta}(Y_{H,t}-\bar{Y}_H) - (1+\eta)^{-1}\bar{Y}_H^{1+\eta}(\Delta_t-1)+\right. \\ & \frac{1}{2}\bar{C}^{-\rho-1}(C_t-\bar{C})^2 - \frac{1}{2}\bar{Y}_H^{\eta-1}(Y_{H,t}-\bar{Y}_H)^2] + (1-\xi)[\bar{C}^{*-\rho}(C_t^*-\bar{C}^*)+ \\ & -\bar{Y}_F^{*\eta}(Y_{F,t}^*-\bar{Y}_F^*) - (1+\eta)^{-1}\bar{Y}_F^{*1+\eta}(\Delta_t^*-1) + \frac{1}{2}\bar{C}^{*-\rho-1}(C_t^*-\bar{C}^*)^2+ \\ & -\frac{1}{2}\bar{Y}_F^{*\eta-1}(Y_{F,t}^*-\bar{Y}_F^*)^2]] + \mathcal{O}(\|\cdot\|^3) \end{aligned}$$

where $\mathcal{O}(\|\cdot\|^3)$ contains terms of order higher than the second. We take a second-order approximation of the constraints

$$Y_{F,t}^* = p_{F,t}^{-\theta}\left[(1-\alpha)C_t + \alpha Q_t^{\theta} C_t^*\right],$$

$$Y_{H,t} = p_{H,t}^{-\theta}\left[\alpha C_t + (1-\alpha) Q_t^{\theta} C_t^*\right],$$

considering that

$$\alpha p_{H,t}^{1-\theta} + (1-\alpha)p_{F,t}^{1-\theta} = 1,$$

$$Q_t^{1-\theta} = (1-\alpha)p_{H,t}^{1-\theta} + \alpha p_{F,t}^{1-\theta}.$$

where, consistently with Appendix B, we define $p_{H,t} \equiv P_{H,t}/P_t$ and $p_{F,t} \equiv P_{F,t}/P_t$.

Combining the second-order approximation of the constraints with the second-order approximation of the utility function at the efficient steady state, we can obtain after some steps that

$$\begin{aligned} U_t \;=\; & \bar{U} + \xi \bar{C}^{1-\rho} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[-\rho \frac{\tilde{C}_t^2}{2} - \rho \frac{\bar{C}^* \bar{Q}}{\bar{C}} \frac{\tilde{C}_t^{*2}}{2} - \eta \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{\tilde{Y}_{H,t}^2}{2} - \eta \frac{\bar{p}_F \bar{Y}_F^*}{\bar{C}} \frac{\tilde{Y}_{F,t}^{*2}}{2} \right.\right. \\ & -\theta \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{\tilde{p}_{H,t}^2}{2} - \theta \frac{\bar{p}_F \bar{Y}_F^*}{\bar{C}} \frac{\tilde{p}_{F,t}^2}{2} + \theta \frac{\bar{C}^* \bar{Q}}{\bar{C}} \frac{\tilde{Q}_t^2}{2} \\ & \left. -(1+\eta)^{-1} \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} (\Delta_t - 1) - (1+\eta)^{-1} \frac{\bar{p}_F \bar{Y}_F^*}{\bar{C}} (\Delta_t^* - 1) \right] + \mathcal{O}(\| \cdot \|^3) \qquad (2.37) \end{aligned}$$

where we have transformed variables using the following relationship

$$X_t = \bar{X} \left(1 + \tilde{X}_t + \frac{1}{2} \tilde{X}_t^2 \right) + \mathcal{O}(\| \cdot \|^3)$$

for a generic variable X where $\tilde{X}$ denotes its log-deviation with respect to the final steady state. Notice that Δ_t and Δ_t^* in (2.37) are second-order terms which can be expressed in terms of the inflation rates by expanding through a second-order approximation (2.35) and (2.36). Using these approximations we can write (2.37) as

$$\begin{aligned} U_t \;=\; & \bar{U} + \xi \bar{C}^{1-\rho} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[-\rho \frac{\tilde{C}_t^2}{2} - \rho \frac{\bar{C}^* \bar{Q}}{\bar{C}} \frac{\tilde{C}_t^{*2}}{2} - \eta \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{\tilde{Y}_{H,t}^2}{2} - \eta \frac{\bar{p}_F \bar{Y}_F^*}{\bar{C}} \frac{\tilde{Y}_{F,t}^{*2}}{2} \right.\right. \\ & -\theta \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{\tilde{p}_{H,t}^2}{2} - \theta \frac{\bar{p}_F \bar{Y}_F^*}{\bar{C}} \frac{\tilde{p}_{F,t}^2}{2} + \theta \frac{\bar{C}^* \bar{Q}}{\bar{C}} \frac{\tilde{Q}_t^2}{2} \\ & \left. -\kappa \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{(\pi_{H,t} - \bar{\pi})^2}{2} - \kappa^* \frac{\bar{p}_F \bar{Y}_F^*}{\bar{C}} \frac{(\pi_{F,t}^* - \bar{\pi}^*)^2}{2} \right] + \mathcal{O}(\| \cdot \|^3) \qquad (2.38) \end{aligned}$$

where

$$\kappa \equiv \frac{\lambda \tau (1 + \eta \tau)}{(1-\lambda)(1-\lambda\beta)}, \qquad \kappa^* \equiv \frac{\lambda^* \tau (1 + \eta \tau)}{(1-\lambda^*)(1-\lambda^*\beta)},$$

and $\pi_{H,t} \equiv \ln \Pi_{H,t}$, $\pi^*_{F,t} \equiv \ln \Pi^*_{F,t}$, $\bar{\pi} \equiv \ln \bar{\Pi}$ and $\bar{\pi}^* \equiv \ln \bar{\Pi}^*$.

The objective (2.38) can be written also in the equivalent form

$$\begin{aligned} U_t \;\; = \;\; & \bar{U} + \xi \bar{C}^{1-\rho} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[-\rho \frac{(\hat{C}_t - c)^2}{2} - \rho \frac{\bar{C}^* \bar{Q}}{\bar{C}} \frac{(\hat{C}^*_t - c^*)^2}{2} - \eta \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{(\hat{Y}_{H,t} - y_H)^2}{2} \right. \right. \\ & -\eta \frac{\bar{p}_F \bar{Y}^*_F}{\bar{C}} \frac{(\hat{Y}^*_{F,t} - y^*_F)^2}{2} - \theta \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{(\hat{p}_{H,t} - p_H)^2}{2} - \theta \frac{\bar{p}_F \bar{Y}^*_F}{\bar{C}} \frac{(\hat{p}_{F,t} - p_F)^2}{2} + \theta \frac{\bar{C}^* \bar{Q}}{\bar{C}} \frac{(\hat{Q}_t - Q)^2}{2} \\ & \left. -\kappa \frac{\bar{p}_H \bar{Y}_H}{\bar{C}} \frac{(\pi_{H,t} - \bar{\pi})^2}{2} - \kappa^* \frac{\bar{p}_F \bar{Y}^*_F}{\bar{C}} \frac{(\pi^*_{F,t} - \bar{\pi}^*)^2}{2} \right] + \mathcal{O}(\| \cdot \|^3) \end{aligned} \tag{2.39}$$

where for a generic variable X, $\hat{X}$ denotes the log deviations with respect to the initial steady-state (before deleveraging) and x denotes the log difference between the final and initial steady state.

The objective function is now quadratic and can be appropriately evaluated by a log-linear approximation of the constraints around the initial steady state. By taking an approximation of the model equilibrium conditions presented in the above section of the Appendix, we respectively get

$$E_t \hat{C}^*_{t+1} = \hat{C}^*_t + \rho^{-1} [\hat{\imath}_t - E_t(\pi_{t+1} - \bar{\pi} + \hat{Q}_{t+1} - \hat{Q}_t)]$$

$$E_t \hat{C}^*_{t+1} = \hat{C}^*_t + \rho^{-1} [\hat{\imath}^*_t - E_t(\pi^*_{t+1} - \bar{\pi}^*)]$$

$$E_t \hat{C}_{t+1} = \hat{C}_t + \rho^{-1} [\hat{\imath}_t - E_t(\pi_{t+1} - \bar{\pi}) + \varpi_1(\hat{d}_t - \hat{k}_t)]$$

$$\hat{C}_t = \upsilon_1 [\hat{p}_{H,t} + \hat{Y}_{H,t}] - \upsilon_2 [\beta \hat{\imath}_t - (\pi_t - \bar{\pi})] + \upsilon_2 \beta \hat{d}_t - \upsilon_2 \hat{d}_{t-1} - \varpi_2 (\hat{d}_t - \hat{k}_t)$$

$$\hat{Y}^*_{F,t} = -\theta \hat{p}_{F,t} + \upsilon_3 \hat{C}_t + (1 - \upsilon_3)(\hat{C}^*_t + \theta \hat{Q}_t)$$

$$\hat{Y}_{H,t} = -\theta \hat{p}_{H,t} + \upsilon_4 \hat{C}_t + (1 - \upsilon_4)(\hat{C}^*_t + \theta \hat{Q}_t)$$

$$\hat{p}_{H,t} = -(1-\alpha)p_F^{1-\theta}\hat{T}_t$$

$$\hat{p}_{F,t} = \alpha p_H^{1-\theta}\hat{T}_t$$

$$\pi_{H,t} - \bar{\pi} = \phi[\eta\hat{Y}_{H,t} + \rho\hat{C}_t - \hat{p}_{H,t}] + \beta E_t(\pi_{H,t+1} - \bar{\pi})$$

$$\pi^*_{F,t} - \bar{\pi}^* = \phi^*[\eta\hat{Y}^*_{F,t} + \rho\hat{C}^*_t - \hat{p}_{F,t} + \hat{Q}_t] + \beta E_t\left(\pi^*_{F,t+1} - \bar{\pi}^*\right)$$

$$\hat{T}_t = \hat{T}_{t-1} + (\pi^*_{F,t} - \bar{\pi}^*) - (\pi_{H,t} - \bar{\pi}) + \Delta\hat{S}_t$$

$$\begin{aligned}\hat{Q}_t &= (1-\alpha)p_H^{1-\theta}Q^{\theta-1}\hat{p}_{H,t} + \alpha p_F^{1-\theta}Q^{\theta-1}\hat{p}_{F,t} \\ &= p_H^{1-\theta}p_F^{1-\theta}Q^{\theta-1}(2\alpha-1)\hat{T}_t\end{aligned}$$

$$\pi_t - \bar{\pi} = \alpha p_H^{1-\theta}(\pi_{H,t} - \bar{\pi}) + (1-\alpha)p_F^{1-\theta}[(\pi^*_{F,t} - \bar{\pi}^*) + \Delta\hat{S}_t]$$

$$\pi^*_t - \bar{\pi} = \pi_t - \bar{\pi} + \Delta\hat{Q}_t - \Delta\hat{S}_t$$

where $\phi \equiv \tau/\kappa$, $\phi^* \equiv \tau/\kappa^*$ while these parameters are evaluated at the initial steady-state

$$\upsilon_1 = \frac{p_H Y_H}{C}$$

$$\upsilon_2 = \frac{k}{\Pi C}$$

$$\upsilon_3 = \frac{(1-\alpha)C}{(1-\alpha)C + \alpha C^* Q^\theta}$$

$$\upsilon_4 = \frac{\alpha C}{\alpha C + (1-\alpha)C^* Q^\theta}$$

$$\varpi_1 \equiv (1+i)\psi_d(1)k$$

$$\varpi_2 \equiv \frac{\chi_d(1)}{C}.$$

where we define $\psi_d(1)$ and $\chi_d(.)$ as the partial derivatives of $\chi(d_t/k_t)$ and $\psi(d_t/k_t)$

with respect to d.[29]

Note that under the assumption $\varpi_1 = \varpi_2/\beta\upsilon_2$ we can re-write the Euler equation and the budget constraint of the Home country in the following ways

$$E_t\hat{C}_{t+1} = \hat{C}_t + \rho^{-1}[\hat{\imath}_t^b - E_t(\pi_{t+1} - \bar{\pi})]$$

$$\hat{C}_t = \upsilon_1[\hat{p}_{H,t} + \hat{Y}_{H,t}] - \upsilon_2[\beta\hat{\imath}_t^b - (\pi_t - \bar{\pi})] + \upsilon_2\beta\hat{d}_t - \upsilon_2\hat{d}_{t-1}$$

where the effective borrowing rate $\hat{\imath}_t^b$ is defined as

$$\hat{\imath}_t^b - \hat{\imath}_t = \frac{\varpi_2}{\beta\upsilon_2}(\hat{d}_t - \hat{k}_t) = \varpi_1(\hat{d}_t - \hat{k}_t).$$

We maintain this assumption when calibrating the model, as explained in the text.

Optimal policy solves the maximization of (2.39) under the above-defined constraints, taking into account the two zero-lower-bound constraints. The equilibrium conditions of the optimal policy problem can be written in the general form (2.30) and therefore similar steps to those described in that section are used to solve for the response of the endogenous variables to the deleveraging shocks.

Note that by using the above restrictions, we can further write the second-order approximation of the utility as

$$\begin{aligned} U_t \;\; = \;\; & \bar{U} + \xi\bar{C}^{1-\rho}E_t\left\{\sum_{t=0}^{\infty}\beta^t\left[-\rho\frac{(\hat{C}_t - c)^2}{2} - \rho\frac{\bar{C}^*\bar{Q}}{\bar{C}}\frac{(\hat{C}_t^* - c^*)^2}{2} - \eta\frac{\bar{p}_H\bar{Y}_H}{\bar{C}}\frac{(\hat{Y}_{H,t} - y_H)^2}{2}\right.\right. \\ & -\eta\frac{\bar{p}_F\bar{Y}_F^*}{\bar{C}}\frac{(\hat{Y}_{F,t}^* - y_F^*)^2}{2} - \theta\bar{p}_H^{1-\theta}\bar{p}_F^{1-\theta}\alpha(1-\alpha)\left(1 + \frac{\bar{C}^*\bar{Q}}{\bar{C}}\frac{1}{\bar{Q}^{2(1-\theta)}}\right)\frac{(\hat{T}_t - s)^2}{2} \\ & \left.-\kappa\frac{\bar{p}_H\bar{Y}_H}{\bar{C}}\frac{(\pi_{H,t} - \bar{\pi})^2}{2} - \kappa^*\frac{\bar{p}_F\bar{Y}_F^*}{\bar{C}}\frac{(\pi_{F,t}^* - \bar{\pi}^*)^2}{2}\right] + \mathcal{O}(\|\cdot\|^3) \end{aligned} \tag{2.40}$$

[29]The function $\chi(d_t/k_t)$ has been defined in Appendix B.

2.12. Model with deleveraging on foreign debt

In this section, we discuss the extension of the model to the case in which debt of the deleveraging country is denominated in foreign currency. In this case, the flow budget can be written as

$$P_t C_t = \int_0^1 W_t(j) L_t(j) dj + \Pi_t + \frac{S_t D_t}{1+i_t^*} - S_t D_{t-1} - f_t P_t \cdot \tilde{\chi}\left(\frac{S_t D_t}{P_t}\frac{1}{f_t}, \frac{S_t \bar{D}_t}{P_t}\frac{1}{f_t}\right) \quad (2.41)$$

where now the function capturing the adjustment costs of changing the debt position has arguments expressed in terms of individual and aggregate real debt, in units of the domestic price index, with respect to a threshold f_t.

The following equilibrium conditions characterize now the consumers' problems in the Home country:

$$(C_t)^{-\rho}\left\{1 - (1+i_t^*)\psi\left(\frac{d_t^*}{f_t}\right)\right\} = \beta(1+i_t^*)E_t\left\{(C_{t+1})^{-\rho}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\right\},$$

$$C_t = \frac{P_{H,t}Y_{H,t}}{P_t} + \frac{d_t^*}{(1+i_t^*)} - \frac{d_{t-1}^*}{\Pi_t}\frac{S_t}{S_{t-1}} - f_t\chi\left(\frac{d_t^*}{f_t}\right)$$

where we have defined $d_t^* = S_t D_t / P_t$ and

$$(C_t)^{-\rho} = \beta E_t\left\{(C_{t+1})^{-\rho}\frac{(1+i_t)}{\Pi_{t+1}}\right\},$$

since we are allowing for trading, within country H, of a risk-less bond denominated in domestic currency.

Note that in the final steady state now

$$\bar{C} = \bar{p}_H \bar{Y}_H - (1-\beta)\bar{\Pi}^{*-1}\bar{f},$$

$$\bar{Q}\bar{C}^{*} = \bar{p}_F \bar{Y}_F^{*} + (1-\beta)\bar{\Pi}^{*-1}\bar{f},$$

Finally the model equilibrium conditions in a first-order approximation are now

$$E_t \hat{C}_{t+1}^{*} = \hat{C}_t^{*} + \rho^{-1}[\hat{\imath}_t^{*} - E_t(\pi_{t+1}^{*} - \bar{\pi}^{*})]$$

$$E_t \hat{C}_{t+1} = \hat{C}_t + \rho^{-1}[\hat{\imath}_t - E_t(\pi_{t+1} - \bar{\pi}_t)]$$

$$E_t \hat{C}_{t+1} = \hat{C}_t + \rho^{-1}[\hat{\imath}_t^{*} - E_t(\pi_{t+1} - \bar{\pi}) + E_t \Delta \hat{S}_{t+1} + \tilde{\varpi}_1(\hat{d}_t^{*} - \hat{f}_t)]$$

$$\hat{C}_t = \upsilon_1[\hat{p}_{H,t} + \hat{Y}_{H,t}] - \tilde{\upsilon}_2[\beta \hat{\imath}_t^{*} - (\pi_t - \bar{\pi}) + \Delta \hat{S}_t] + \tilde{\upsilon}_2 \beta \hat{d}_t^{*} - \tilde{\upsilon}_2 \hat{d}_{t-1}^{*} - \tilde{\varpi}_2(\hat{d}_t^{*} - \hat{f}_t)$$

$$\hat{Y}_{F,t}^{*} = -\theta \hat{p}_{F,t} + \upsilon_3 \hat{C}_t + (1-\upsilon_3)(\hat{C}_t^{*} + \theta \hat{Q}_t)$$

$$\hat{Y}_{H,t} = -\theta \hat{p}_{H,t} + \upsilon_4 \hat{C}_t + (1-\upsilon_4)(\hat{C}_t^{*} + \theta \hat{Q}_t)$$

$$\hat{p}_{H,t} = -(1-\alpha) p_F^{1-\theta} \hat{T}_t$$

$$\hat{p}_{F,t} = \alpha p_H^{1-\theta} \hat{T}_t$$

$$\pi_{H,t} - \bar{\pi} = \phi[\eta \hat{Y}_{H,t} + \rho \hat{C}_t - \hat{p}_{H,t}] + \beta E_t(\pi_{H,t+1} - \bar{\pi})$$

$$\pi_{F,t}^{*} - \bar{\pi}^{*} = \phi^{*}[\eta \hat{Y}_{F,t}^{*} + \rho \hat{C}_t^{*} - \hat{p}_{F,t} + \hat{Q}_t] + \beta E_t \left(\pi_{F,t+1}^{*} - \bar{\pi}^{*}\right)$$

$$\hat{T}_t = \hat{T}_{t-1} + (\pi_{F,t}^{*} - \bar{\pi}^{*}) - (\pi_{H,t} - \bar{\pi}) + \Delta \hat{S}_t$$

$$\begin{aligned} \hat{Q}_t &= (1-\alpha) p_H^{1-\theta} Q^{\theta-1} \hat{p}_{H,t} + \alpha p_F^{1-\theta} Q^{\theta-1} \hat{p}_{F,t} \\ &= p_H^{1-\theta} p_F^{1-\theta} Q^{\theta-1} (2\alpha - 1) \hat{T}_t \end{aligned}$$

$$\pi_t - \bar{\pi} = \alpha p_H^{1-\theta}(\pi_{H,t} - \bar{\pi}) + (1-\alpha) p_F^{1-\theta}[(\pi_{F,t}^{*} - \bar{\pi}^{*}) + \Delta \hat{S}_t]$$

$$\pi_t^{*} - \bar{\pi} = \pi_t - \bar{\pi} + \Delta \hat{Q}_t - \Delta \hat{S}_t$$

where now

$$\tilde{\upsilon}_2 = \frac{f}{\Pi^* C}$$

$$\tilde{\varpi}_1 \equiv (1 + i^*)\psi_{d^*}(1) f$$

$$\tilde{\varpi}_2 \equiv \chi_{d^*}(1) f / C.$$