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A defensive investment strategy for portfolio alpha return and market risk reduction

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Summary

Introduction and motivation

Financial markets have become increasingly complex and volatile during the past years. In particular with the recent and still on-going financial crisis, many investors, both professional and retail, have experienced great difficulties in facing such violent changes. Most asset classes have also been highly correlated one to another in periods of market downfalls. This situation has increased the request for risk reduction, particularly for a wide class of investors characterized by strong risk-aversion.

This thesis addresses such need and presents an investment strategy which focuses on the reduction of the market risk in a portfolio while pursuing a medium term positive return. In particular, we consider a portfolio comprised of a core-asset and a hedging tool, together with a certain quantity of available cash.

1. Mathematical framework

We set a mathematical framework valid throughout the thesis based on the distinction between past sample data available at a certain present time and probabilistic evaluation of future evolution, a distinction which is repeatedly stressed throughout the work. Starting from time series of prices or values, we consider the stochastic processes given the daily and compound returns. A link between the ex-ante and ex-post perspective is established by assuming that probabilistic means, volatilities and covariances are equal to the available sample means, standard deviations and covariances.

2. Core-asset daily returns

The core-asset adopted in the strategy is selected in function of the correlation to a certain benchmark index and in consideration of the track record pertaining the capacity of outperforming this same benchmark, both of which we require to be as high as possible. An in-depth analysis of

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such correlation is performed through a linear regression method which leads to the finding of the *alpha* and *beta* parameters and allows for a decomposition of the asset return into a component given the market return and another one which is considered independent to it and defined as *alpha dynamics*, the return process of which is presented and analyzed. Particular attention is put in the comparison between expected return at a certain time and the return actually delivered at the following time step, which leads to the definition of three types of errors. The first, defined as *hedgeable error*, does not affect the effectiveness of the hedge, the second is an *estimation error* which has conceptual significance and the third being the stochastic component of the alpha dynamics and as such defined as *alpha error*.

3. Core-asset compound returns

As an effect of the hedge, the alpha dynamics characteristics will be transferred to the portfolio and for this reason the compound return and growth over a long period of time is considered. In particular, we define the expected value of growth and its variance and volatility, with both exact and approximated formulae. When incurring into an approximation, numerical evidence is provided. Although at a daily time scale the mean alpha return is one order of magnitude inferior to its volatility, if we allow the growth process to evolve for a sufficient period defined as *waiting time*, then the same growth will achieve positive mean results with a given level of confidence, a fact which will be reflected into the portfolio growth. We then reconsider the linear regression scheme on a non-daily basis and outline a method to estimate future regression on such periods when we do not possess sufficient data to rely on sample past values. This method permits to find expected values of parameters A and B, being the equivalent on such time scale of the daily α and β and as a consequences expected compound returns of the core-asset and its alpha dynamics. Again, we then perform a comparison between this expectation and the one actually delivered at the end of the period, and highlight the emergence of the same three types of errors as those arising in the daily case.

4. Hedging tools

At this point we introduce two different types of securities liable to be adopted as hedging tools. The first is a subclass of the Exchange Traded Funds in their Short version, which are issued in order to perform an inverse daily return to the related index return. These products are however affected by tracking errors which reduce their effectiveness importantly and need to be used only with daily frequency, given that the replication feature is guaranteed only on such time frame.

On the other side we consider the possibility of entering into Futures contracts on the benchmark index with short positions, which allow for to realize the desired hedge with daily and also with a longer time frequency. The dynamics of returns for both the Short ETFs and the Futures is presented.

5. Portfolio return with daily hedging

We now implement the strategy and compare the portfolio return in the case of Short ETFs and Futures as hedging tools. The hedge is realized at a given time step by performing an evaluation of which will be the return of the portfolio at the following day and which would be the index return component in absence of the hedge. This determines the quantity of hedging tool to be traded. At the next time step the portfolio will achieve a different return from the expected one, and the difference is now composed of only two terms, the estimation error and the alpha error as previously introduced. These strategies require a certain cash allowance, and both the estimated and actual cash evolution is presented. In the end it results that from all points of view Futures perform better than ETFs and allow the hedged portfolio to experience a stable growth path and an increase on a number of risk-return performance indexes.

6. Portfolio return with non-daily hedging

As mentioned, Futures may be traded on a longer time scale than the daily one, as long as we are able to evaluate with sufficient accuracy the future correlation on such time frame between the asset and the benchmark index. This possibility simplifies remarkably the implementation of the strategy and its accessibility. The reason for which on the contrary Short ETFs may not be used in this way is presented. As in the daily hedging case, we consider the error components arising from the execution of the hedge based on expected compound returns and the actual ones delivered by the underlying securities. In this case the estimation error assumes major relevance and determines the reliability of such approach.

7. Portfolio return under real trading conditions

Finally, we introduce in the dynamics a number of realistic implementation conditions, such as transaction costs, bid-ask spread, actual interest rates, together with tracking errors for ETFs, dividends and non-arbitrage prices for Futures. Numerical simulation on real data is performed for three sample cases and it results that the Futures portfolio performs well also under such realistic trading conditions, whereas the ETF portfolio is heavily affected.

Annexes

Annexes are provided which summarize the most relevant mathematical formulae, the main implementation data and all the performance graphs for the three sample cases.

Conclusions

This defensive strategy, particularly when hedging with Futures, allows for a significant reduction of risk whilst pursuing an absolute return on the portfolio. It may be implemented in periods of uncertainty on market trends or under phases of high volatility without the necessity of winding out established investments. The fund-picking activity is crucial and an accurate analysis of the *alpha* dynamics is necessary. As a result, under certain conditions, the reduction in volatility and improvement in the risk-return performance can be significant, and thus encounter the necessity for risk reduction felt by many investors exposed to the volatility of markets.
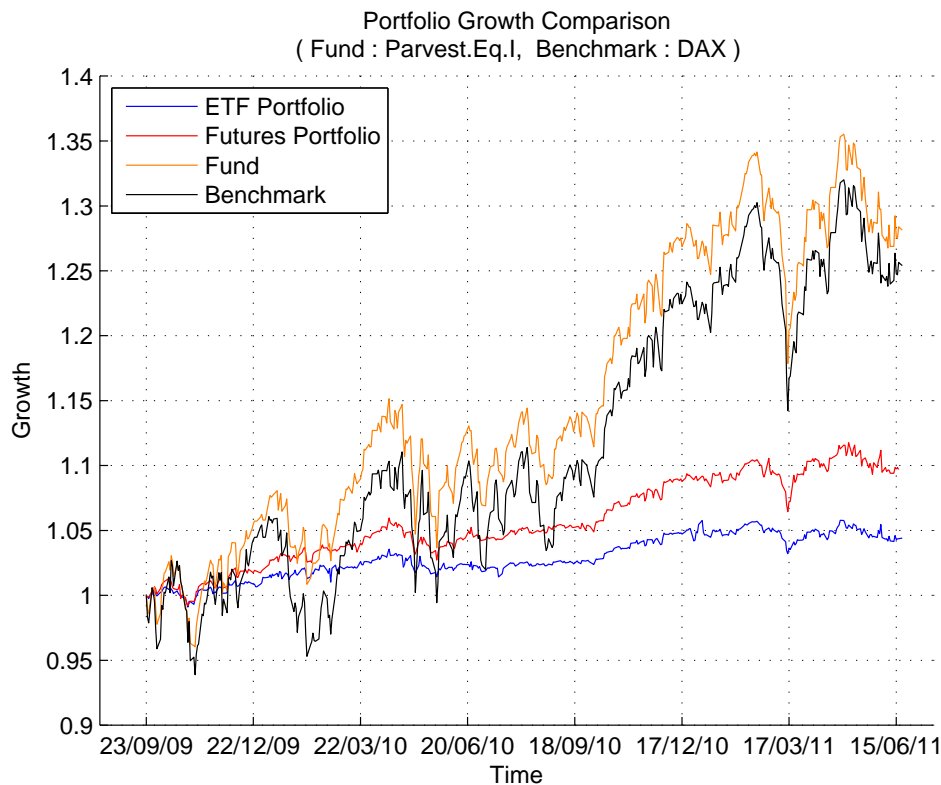

Figure 1: Portfolio growth with Futures and Short ETFs compared to the core-asset and the benchmark index growth for a sample case on the German equity market.