An optimal Markovian consumption-investment problem in a market with longevity bonds

PhD Thesis by Vanessa Raso

Abstract

Lifetime has been constantly increasing (medical improvements, better life standards, etc) and so there is a demand for instruments hedging the longevity risk, i.e., the risk that members of some reference population might live longer, on average, than anticipated, for example, in the life companies' mortality tables. Longevity bonds are the first financial products to offer longevity protection.

We consider a market with (i) a riskless asset $G(t)$, (ii) a risk asset $S(t)$, (iii) a (zero coupon) bond $B(t, T)$ and (iv) a (zero coupon) longevity bond $L(t, T)$, furthermore we assume that the interest rate $r(t)$ and the mortality rate $\lambda(t)$ are observable stochastic processes, and that $(r(t), \lambda(t), S(t))$ is a Markovian diffusion. The bond is considered as a derivative on the stochastic interest rate $r(t)$, and the longevity bond as a derivative on $r(t)$ and the stochastic mortality intensity $\lambda(t)$. From a mathematical point of view the latter condition amounts to assume that bonds are deterministic functions of time and interest rate, while longevity bonds depend also on the interest rate.

The hedging problem in such a market is faced as an optimal intertemporal consumption-investment strategies for an agent maximizing the expected utility of her/his consumption rate until her/his death time. From a a mathematical point of view we consider it as a Markovian control problem (the controls being consumption-investment strategies) with random horizon τ, the investor's death time, and in order to take care of the budget constraints, besides $(r(t), \lambda(t), S(t))$ also the wealth process $V(t)$ is included in the state. Furthermore, including in the state also the process $z_0(t)$, defined by $dz_0(t) = -\lambda(t)z_0(t)\, dt$, one can reduce the Markovian control problem with random horizon time to a Markovian control problem with deterministic (infinite) horizon time. Morally the process $z_0(t)$ represents the conditional survival function of τ. we study the control problem also when the agent's portfolio, besides of the riskless and risk assets, consists of discrete time rolling both for ordinary bonds and longevity bonds. Rolling bonds are not anymore deterministic functions of time t and $r(t)$ (and of $\lambda(t)$ for longevity bonds), and depend also on the value of the interest rate (and of the mortality intensity for longevity bonds) at rolling times $t_k \leq t$. Nevertheless the consumption-investment optimization problem can still be put in a Markovian control problem setting.

We study this control problem in different settings, the most interesting being the case in which the agent's portfolio, besides of the riskless and risk assets, consists of discrete time rolling both for ordinary bonds and longevity bonds. Rolling bonds are not anymore deterministic functions of time t and $r(t)$ (and of $\lambda(t)$ for longevity bonds), and depend also on the value of the interest rate (and of the mortality intensity for longevity bonds) at rolling times $t_k \leq t$. Nevertheless the consumption-investment optimization problem can still be put in a Markovian control problem setting.

As a special case we consider the CRRA utility function. In this case, under regularity conditions, the value function can be characterized uniquely, and in order to compute it, one can suitably use Feynman-Kac representation.

The regularity conditions are satisfied when the when $S(t)$ is a Geometrical Brownian Motion, and $(r(t); \lambda(t))$ is a bidimensional CIR model: the drift coefficient of $\lambda(t)$ increases linearly with $r(t)$. The latter condition may be interpreted as follows: the interest rate growth may affect the active population mortality intensity, for instance, a large interest rate may diminish health care and prevention. The latter model has been simulated using Monte Carlo methods.