Entrepreneurs Meet Financiers*

Evidence from the Business Angel Market

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Abstract

This paper formalizes and estimates the process of *search and matching* between entrepreneurs and financiers in the business angel (BA) market. Our theoretical model describes the market for entrepreneurial finance as a fair in which the two sides of the market can meet bilaterally and transform a rough entrepreneurial idea into a real start-up firm. We then collect a new dataset from the BA markets of 17 developed countries for the period 1996-2014, and we estimate the aggregate matching function expressing the number of deals as a function of the number of submitted entrepreneurial projects and of business angels. Empirical findings confirm the technological features assumed in the theoretical literature: positive and decreasing marginal returns to both inputs (*stepping on toes* effect), technological complementarity across the two inputs (*thick market* effect) and constant returns to scale. We discuss the theoretical and policy implications of these findings.

Keywords: Entrepreneurial finance, angel investors, matching function estimation, constant returns to scale.

JEL Classification: 031, C78, L26.

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1 Introduction

Over the past few decades, the market for entrepreneurial finance -that is, finance provided to risky, innovative ventures- has received increasing attention in the economic and financial literature (Chemmanur and Fulghieri, 2014). Although still representing a small fraction of the overall investments in innovation, funds provided by such institutions as venture capitalists (VCs) and business angels (BAs) have been rapidly growing in size and in prominence.[1] For instance, a recent study by the OECD (2011) shows that the number of angel networks operating in the US and in Europe has roughly tripled in the ten years from 1999 to 2009. Moreover, the amount of investments provided by business angels and the number of deals have been increasing during the 2000s despite the financial crisis. Finally, these investments tend to be concentrated in the most innovative sectors. For instance, in Europe and in the US, biotechnology, ICT and environmental technologies absorb around 60% of the overall angel investments (OECD, 2011).

In this paper, we begin to investigate the *aggregate* behavior of the angel market. With the help of a new dataset, we study the interaction between the demand and the supply of funds and identify empirically the main aggregate characteristics of the "production process" of entrepreneurial ventures financed by BAs.

Recent theoretical literature describes this production process as one in which potential entrepreneurs search for funds to finance their ideas, and financiers (or capitalists) search for good ideas to finance. In this perspective, a start-up enterprise is the result of a successful matching between the demand side (entrepreneurs) and the supply side (financiers) of this market (Section 2 provides the main references on this literature).

The complex process of search and matching is usually and conveniently represented at aggregate level via the use of a *matching function* (Petrongolo and Pissarides, 2001), representing the number of deals (matches) as a function of the number of would-be

[1] Differently from VCs, which usually gather funds from institutional investors, BAs (also called angel investors) traditionally refer to wealthy individuals that invest their own funds in entrepreneurial ventures. The amount invested in each project by a BA is, on average, lower -and more concentrated on early stage ventures- than that invested by a VC. On the other hand, BAs and VCs share the following feature: they are expected to contribute to the project not only with financial investments but also with managerial and technical expertise (Shane, 2009; Maxwell et al., 2011). To see how the angel finance behavior has been changing in the last few years, a useful reference is Mason et al. (2016).

entrepreneurs and of financiers. The rationale behind the matching function is the possibility of capturing succinctly all market imperfections and all dimensions of heterogeneity without the need to specify them in detail.[2] Indeed, whether the matching function is a valid representation of the market for entrepreneurial finance ultimately rests on its empirical adequacy. In the words of Petrongolo and Pisarrides (2001, p. 392), "like the other aggregate functions, [the] usefulness [of the matching function] depends on its empirical viability and on how successful it is in capturing the key implications of the heterogeneities and frictions in macro models". This consideration provides the main motivation to the present paper.

The purpose of this paper is to estimate the aggregate matching function between entrepreneurs and financiers in the business angel market, and to verify the technological features commonly assumed in the theoretical literature. To reach this goal, we proceed as follows.

We first introduce formally the matching function and its technological characteristics. Most of the theoretical literature looking at entrepreneurial finance as a search and matching process assumes a well-behaved matching function (often in a Cobb-Douglas form) with the following technological characteristics: (i) positive and decreasing marginal returns to both inputs (entrepreneurs and financiers); (ii) a positive complementarity across the two inputs; (iii) constant returns to scale (CRS). The first and the second features are intuitive: the marginal effect of an increase in the number of entrepreneurs on the number of successful matches (i) is positive but decreasing in the number of entrepreneurs -and, of course, the same holds for financiers (*stepping-on-toes* effect), (ii) is increasing in the number of financiers, and *viceversa* (*thick-market* effect). The third feature, instead, captures the relation between the size of the market and its efficiency. CRS imply a sort of neutrality of size: specifically, if all inputs (entrepreneurs and financiers) increase by a given factor (say, they all double), the output (the number of deals) should increase by exactly the same factor (that is, it should also double).

In Appendix A, we embed this matching technology into a parsimonious model of search and matching between entrepreneurs and financiers. The model describes the market for entrepreneurial finance as a fair in which the two sides of the market can meet bilaterally and transform a rough entrepreneurial idea into a real start-up firm. We formally prove that the scale elasticity of the matching function disciplines the

[2] Few papers delve into the microeconomic foundations of the search and bargaining frictions in the entrepreneurial market. See, for instance, Silveira and Wright (2010, 2016), Chiu et al. (2011).

number of equilibria admitted by the model.[3] In particular, if returns to scale are increasing, the model may admit more than one equilibrium (Proposition 1). We also offer an intuitive economic interpretation to equilibrium multiplicity. While the model is useful to gain some perspective on the (theoretical and policy) implications of the technological hypotheses usually made, it is relatively self-contained, which is why we have decided to relegate it to a technical appendix: readers not interested in the details of the analytical framework may skip it and still be able to appreciate our empirical findings.

Finally, we estimate the matching process between entrepreneurs and financiers using a unique dataset on business projects financed by angels gathered in Business Angel Networks (BANs). We collect yearly data for the period 1996-2014 across 17 developed countries on (i) the number of business angels (financiers), (ii) the number of projects submitted to them (potential entrepreneurs), (iii) the number of deals (successful matches). With these data, we estimate three specifications of the matching function, taking the number of projects and angels as inputs and the number of deals as output. We start from a non-linear estimation of a log-constant-elasticity-of-substitution matching function. We then estimate a log-Cobb-Douglas matching function. Finally, and in line with the empirical literature on matching function estimation, we consider the more general form of transcendental logarithmic (or simply "translog") matching function.

Across all the three functional forms estimated, the matching process systematically holds the technological characteristics assumed in the theoretical literature: positive and decreasing marginal returns to both inputs and positive technological complementarity across the two inputs. According to the best performing model specification (the translog matching function), a 1% increase in the number of submitted projects leads to a 0.70% increase in the number of matches, while a 1% increase in the number of BAs leads to a 0.23% increase in the number of matches. We then test the returns to scale of the estimated matching function, and thus indirectly verify the empirical plausibility of multiple equilibria. Our evidence is against the presence of strong economies of scale in the BA market. Finally, we find that, in countries characterized by a higher GDP per capita and in countries with a less developed venture capital market, the matching process is more efficient -in the sense of delivering a higher number of deals given the same amount of inputs. The last finding suggests some degree of substitution between

[3] Classical references are Diamond (1982, 1984).

the venture capital market and the business angel market (Hellman and Thiele (2015); Hellman et al., 2017).

Our empirical findings have a number of implications for the aggregate behavior of the BA market. First, the thick-market effect -whereby the presence of entrepreneurs in the market stimulates the entry of financiers, and *viceversa*- might contribute to explain an empirical regularity about the entrepreneurial finance activity, namely, its pronounced geographic concentration (Mason, 2007; Chen et al., 2009). Secondly, the presence of strong economies of scale and, possibly, of equilibrium multiplicity is frequently used to justify policy intervention (Parker, 2009). In providing the first, admittedly preliminary, evidence of constant returns to scale in the market for angel investment, this paper adds a note of prudence in justifying policy intervention via this line of argument. Third, given that our empirical analysis is exclusively focused on deals finalized within BANs, our evidence on constant returns to scale suggests that these BANs are effective in reducing the search frictions between entrepreneurs and financiers, and thus it provides a policy rationale for promoting the development of such networks. We further discuss these issues in the last section.

The rest of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we introduce the matching function and state our testable hypotheses. In Section 4, we carry out the empirical analysis. Section 5 discusses the results and concludes the paper. The formal model and other technicalities are relegated to two appendices at the end of the manuscript.

2 Related Literature

This paper is broadly related to the literature on the financing of innovative entrepreneurial ventures (see, among others, Kortum and Lerner, 2000; Da Rin et al., 2011; Chemmanur and Fulghieri, 2014; and references therein). While the traditional focus of the equity-based entrepreneurial finance literature has been the venture capital market (Gompers and Lerner, 2004), research efforts have, more recently, also been devoted to business angels (Amatucci and Sohl, 2007; Mason, 2006). We now discuss the literature on angel finance most closely related to our work.

A number of papers have investigated the characteristics of angel investors, both of individual angels and of groups. Findings suggest that individual angels are usually either former entrepreneurs or ex-managers (Aernoudt, 1999; De Clercq et al., 2006;

Ibrahim, 2008; Politis and Landström, 2002; Politis, 2008; Prowse, 1998; Wetzel, 1983). The literature has also tried to identify the determinants of the angels' decision to join a group. Kerr et al. (2014) identify five benefits from group membership: i) higher investment size; ii) greater risk diversification over multiple investments; iii) economies of scale for legal and due diligence costs; iv) possibility of receiving and screening more valuable projects; v) opportunity to work with more experienced angel investors who can provide better monitoring strategies.

A second line of research has focused on the angels' investment decisions. Carpentier and Suret (2015), Mitteness et al. (2012) and Maxwell et al. (2011) find that the entrepreneurs' knowledge and/or experience of the industry significantly matter in the initial stages of deal evaluation. Related to this, Wetzel (1983), Croce et al. (2017) and Warnick et al. (2018) find that the characteristics of the management team are carefully evaluated by angels. In particular, a lack of motivation in the management team, the naivety or a weak entrepreneurial spirit of the entrepreneur are among the main reasons for rejection during the investor screening stage. Similarly, Murnieks et al. (2016) and Balachandra et al. (2014) add that entrepreneurs' passion, tenacity and a positive attitude towards coaching and mentoring are crucial for the likelihood of receiving angel finance.

A third stream of literature has concentrated on the contractual terms of the angel investments and on the ownership structure of the ventures financed. A few papers find that angel-backed deals are generally more entrepreneur-friendly and the governance mechanisms less stringent than those of more formal investors, the reason being that angels often retain the majority of voting rights and are more actively involved with the venture. Ibrahim (2008), Goldfarb (2014) and Wiltbank and Boeker (2007) argue that, when BAs co-invest as a group, contracts are more sophisticated and more similar to those provided by venture capitalists, in terms of amounts, stage of investment and incentives.

The post-investment stage has been comparatively less explored by the scientific literature. Some papers have focused on the exit strategy and on the rates of return from angel investments. Capizzi (2015) and Mason and Harrison (2002) find that high rates of return are observed for investments held for longer than three years and for divestments in the form of trade sale or IPO. Co-investments between business angels and venture capitalists are more likely to receive further venture capital funding and to result in an IPO or acquisition (Croce et al., 2018; Mason and Harrison, 2002), although only few angel-backed ventures reach the IPO stage (Shane, 2008). In general, ventures receiving capital from BAs are more likely to receive subsequent funding from formal investors compared to non-angel backed ventures (Kerr et al., 2014).

A significant research effort has been devoted to the relationship between the VC and the BA markets. Theoretical contributions include Hellman and Thiele (2015) and Kim and Wagman (2016). The former investigate the interdependencies between the two types of investors and how these affect market size and competition, as well as company evaluations and success rates. The latter analyze an early-stage entrepreneurs' choice between angel and venture capital and predict that more successful entrepreneurs are more likely to choose VC financing to angel financing in the first investment round. Among the empirical analyses, Dutta and Folta (2016) compare the relative contributions to innovation and successful exits by angel groups and venture capitalists. They find that the latter help ventures generate significantly higher innovation impact compared to the former. Hellman et al., (2017) empirically investigate the question as to whether the venture capital and angel investors are substitutes or complements. They find that they are dynamic substitutes, in that companies that obtain angel funding are less likely to obtain subsequent VC funding, and *viceversa*.

While this growing body of literature relies primarily on microeconomic data and focuses on the characteristics of the investment process, the aggregate functioning of the BA market and its industrial organization remain largely unexplored, mainly because data suitable to pursue this line of research are hardly available (Cumming and Zhang, 2016). The value added of this paper is to start filling this gap: with the help of a new dataset, we identify empirically the technological features of the entrepreneurial process financed by angels, using variation across countries and time. This exercise helps shed some light on the relative importance of the demand and the supply in the matching process, as well as on the market frictions inherent in this process and how these frictions affect the aggregate behavior of the BA market.

Finally, our formal approach is inspired by the recent search and matching literature on entrepreneurial finance. From a theoretical perspective, depicting entrepreneurial finance as a *search and matching* process between entrepreneurs and financiers has become a consolidated practice in the economic literature of entrepreneurship.[4] This paper is closely linked to that stream of literature, whose inspiring idea is clearly expressed in Phelps (2009). The most relevant theoretical contributions for our purposes include Inderst and Muller (2004), Michelacci and Suarez (2004), Wasmer and Weil

[4]This approach is rooted into solid theoretical foundations, as we explain in the next section.

(2004).[5] All of them assume a well-behaved matching technology with the characteristics described above. Sorensen (2007) and Cipollone and Giordani (2016) also provide empirical validations of the matching models by using appropriate data on venture capital market. In this perspective, our contribution can be seen as a first attempt to test empirically a set of common theoretical hypotheses contained in such class of models against business angels market data.

3 Entrepreneurial Finance as a Search and Matching Process

According to the search and matching literature cited in the previous section, the notion of a frictionless market characterized by well-behaved demand and supply can hardly capture the most salient features of the entrepreneurial finance market. In the words of Phelps (2009, p. 50), "the classical supply-and-demand apparatus does not apply to the core market of capitalist economies -the capital market, particularly the market for capital going to entrepreneurs' innovative projects". The main reason is that market participants -entrepreneurs and financiers- are highly *heterogeneous* along several dimensions (such as skills, location, beliefs, preferences) and are imperfectly informed about each other. The meeting of demand and supply of financial funds for entrepreneurship is then a costly and time-consuming process: it takes time and resources for entrepreneurs to find the right financier, and it takes time and resources for financiers to find a promising entrepreneurial project. Hence, and still in the words of Phelps (2009, p. 52), "the capital market is a sort of matching process that matches a financier to an entrepreneur who the former sees as having a model compatible with his own model". Search theory becomes useful to model all the frictions inherent in the meeting between demand and supply in the entrepreneurial finance market.[6]

[5] Prior to finance, search theory has been extensively used in several fields of economics, such as labor economics, monetary theory, and the theory of marriage. Rogerson et Al. (2005) contains a survey of applications for the labor market but also a list of references for applications in other fields.

[6] It is also reasonable to conjecture that such frictions are even more pronounced in the BA market than in the formal VC market, as the former is more focused on early-stage -and, thus, probably more uncertain- investments. Indeed, the fact that BA investments tend to be more local than other entrepreneurial investments suggests the existence of higher search costs. On the other hand, and as argued by Mason et al. (2016), the rise of angel networks has served the purpose of lowering such costs.

Inspired by this line of reasoning, we describe the production process of new entrepreneurial ventures via the following aggregate matching function:

$$M = M(L_E, L_K), \tag{1}$$

where M, L_E, L_K denote, respectively, the number of successful matches, would-be entrepreneurs and financiers (both here and in the next section, subscripts E,K will refer to variables associated to, respectively, entrepreneurs and financiers).

Theoretical models of search and matching typically assume positive and decreasing marginal returns to both inputs, that is to say

$$(i)\ \partial M/\partial L_j > 0 \text{ and } (ii)\ \partial^2 M/\partial L_j^2 < 0 \text{ for } j = E, K, \tag{Hp 1}$$

implying that the number of matches is increasing in both inputs at decreasing rate. Part (ii) of (Hp 1) implies a sort of stepping-on-toes (or congestion) effect in the entrepreneurial finance market, in that the individual probability of being funded for an entrepreneur (M/L_E) is decreasing in the number of competing entrepreneurs (and the same for financiers).

A second commonly assumed technological feature deserving empirical scrutiny is the existence of a positive complementarity across the two inputs. Two inputs are technological complements when the marginal productivity of one input is increasing in the use of the other input, that is, when the cross-partial derivative of the matching function is strictly positive:

$$\partial^2 M/(\partial L_j \partial L_{-j}) > 0 \text{ for } j = E, K. \tag{Hp 2}$$

In our context, this would imply that the impact of one additional potential entrepreneur on the output (of funded business ventures) is increasing in the number of financiers, and *viceversa*. This assumption captures the existence of a thick-market effect, whereby the individual probability of being funded for an entrepreneur (M/L_E) is increasing in the number of financiers (and *viceversa*). It also has a number of relevant implications in terms of the aggregate behavior of the sector that we will discuss in Section 5.

Finally, the third common assumption made in the literature is a unitary scale elasticity (or, simply, constant returns to scale, CRS) of the matching function, that is:[7]

$$M\left(\lambda L_E, \lambda L_K\right) = \lambda M\left(L_E, L_K\right) \text{ for } \lambda \in R_+. \tag{Hp 3}$$

Intuitively, this hypothesis implies scale neutrality: if there are twice as many entrepreneurs and twice as many financiers, a CRS matching function delivers twice as many entrepreneurial ventures.

(Hp 1), (Hp 2), and (Hp 3) are closely linked to each other. (Hp 1) captures the congestion effect, that is, the negative externality that each entrepreneur (financier) imposes on other entrepreneurs (financiers) when participating in the matching market. (Hp 2), instead, captures the thick-market effect, that is, the positive externality that each entrepreneur causes on financiers (and *viceversa*) when participating in the matching market. The scale elasticity of the matching function can be interpreted as representing the "sum" of these two externalities: the higher the scale elasticity, the stronger the thick-market effect compared to the congestion effect. (Hp 3) on unitary scale elasticity implies that these two opposite externalities exactly cancel each other out. In this case, agents are compensated according to their contributions to the match formation, and the matching market is said to be *efficient*, in the specific sense that the social value of the marginal product from entry is equal to the expected private return from entry (Hosios, 1990). This feature will turn out to be useful when, in the last section, we discuss the policy implications of our empirical findings.

In the next section, we verify (Hp 1), (Hp 2), and (Hp 3) against the available data from the business angel market.

4 Estimating the Search and Matching Process

This section is dedicated to estimating the function describing the production process of new entrepreneurial ventures, the so-called matching function, defined in expression (1). We use data on the BA market to verify three commonly assumed technological features of the matching function. (Hp 1) states that the marginal productivities of both inputs are positive and diminishing. This feature implies a stepping-on-toes effect, whereby the individual probability of being funded for an entrepreneur (or financier) (M/L_j, $J = E, K$) is decreasing in the number of competing entrepreneurs (financiers). (Hp 2) states that the matching function exhibits positive cross-partial derivative. This

[7] Formally, the scale elasticity is given by the sum of the elasticities of the output with respect to each input.

feature implies a thick-market effect, in that the individual probability of being funded for an entrepreneur is increasing in the number of financiers, and *viceversa*. (Hp 3) imposes constant returns to scale: if all inputs are scaled by a common factor $\lambda > 0$, then the number of matches is also scaled by the same factor λ. If this hypothesis is verified, then markets with a larger scale would not systematically offer a more (or less) efficient matching of entrepreneurs to financiers. As argued in the Introduction, our focus on the BA market is justified by its growing importance in the financing of innovative entrepreneurs. On the other hand, the scarcity of empirical work is due to lack of data on the angels' activity. This paper represents a first attempt to explore the "technological" characteristics of the matching process using aggregate data from the BA market.[8]

The plan of this section is as follows. The first subsection describes the dataset. The second subsection presents the matching function models. The third subsection presents the findings. The last subsection carries out a robustness analysis.

4.1 The Data on the Business Angel Market

A key challenge for the estimation of function (1) is the search of suitable data for our three variables of interest, L_E, L_K, M. Our dataset considers the activity undertaken by those angel investors which are gathered in networks, clubs, alliances and have a relation either with the Center for Venture Research (CVR) - for the United States - or with the European Association for Business Angels (EBAN) - for European countries. Let us consider them in order.

In the US, the Center for Venture Research (CVR) at the University of New Hampshire reports yearly and quarterly information about the angel investor market in the US as a whole, providing details on the number of active investors, the overall investment size by industry and by stage of investments, the contribution to job creation, the yield rates (defined as the percentage of deals on the total number of submitted projects) and the role of women and minority entrepreneurs. These data are collected through a survey that CVR submits to private investor clubs, angel alliances and matching networks in the US. The survey's responses are then published in quarterly analysis reports.

In Europe, the European Association for Business Angels (EBAN) keeps the most comprehensive record of visible business angel activity based on the information col-

[8]The implications of our findings will be thoroughly discussed in Section 5.

lected through a survey addressed to Business Angel Networks, Business Angel Federations, VC funds and Accelerators. The survey is conducted yearly and typically reports activities which have taken place in the previous year.[9] It contains aggregate country-specific information of the angel market size, namely, the number of projects submitted to the angels, the number of angels, the number of deals and, sometimes, the average amount of the deal.

As a result, our empirical analysis relies on (i) publicly available data on US business angels activity from the CVR, and on (ii) data on European business angels activity gathered from official EBAN's publications (note that, as part of geographical Europe, EBAN statistics also include Russia). We have created a unique dataset of yearly observations across 17 developed countries on 1) the total number of entrepreneurial projects submitted to each business angel as a proxy for L_E, 2) the number of business angels as a proxy for L_K, and 3) the number of deals as a proxy for M. We handle an unbalanced panel dataset of EU-15 countries (except Luxembourg and Ireland), plus Norway, Poland, Russia and the US over the period 1996-2014, for a total number of 105 observations. A summary description of these data is provided in Table 1.

INSERT TABLE 1 HERE

As shown in Table 1, the size of the angel market along our dimensions of interest is remarkably different across countries and, within each country, over time (as shown by the large value of the standard deviation). In particular, the Anglo-Saxon countries (UK and USA) display the largest number of business angels and of deals, followed by the most populous European countries (Germany and France).

A look at the evolution over time of our data reveals an increasing trend over the observed period in the size of the angel market for the whole sample of countries (particularly pronounced for the European continental countries). To give a rough estimate of this expansion along our time span, we compute the growth rate of each dimension between the first and the last period of observation for each country, and then we take the median values over the whole set of countries. We find that, over the

[9] In particular, the collection of data is pursued through a web form activated from the EBAN website or directly e-mailed to the Secretariat of EBAN through a pre-formatted survey. Results are published in EBAN's yearly documents (the "European directory of business angel networks in Europe" until 2009 and in the EBAN's Statistics Compendium from 2010).

period 1996-2014, the number of submitted projects, business angels and deals in the median country has respectively grown by 91%, 140% and 96%.

4.2 Matching Function Models

Using the data illustrated above, we carry out pooled and fixed-effect estimations of different specifications of the matching function. We start with the logarithmic transformation of the constant-elasticity-of-substitution (CES) function and then move to a Cobb-Douglas matching function. We finally consider a generalization of the Cobb-Douglas, the so called transcendental logarithmic (or simply, translog) matching function. We now describe them in order and specify the parameter conditions under which each of them satisfies (Hp 1), (Hp 2), and (Hp 3) (while relegating all technicalities to Appendix B).

CES Matching Function. The constant-elasticity-of-substitution (CES) function reads as follows:[10]

$$M_{i,t} = A\left[\delta_E\left(L_E\right)_{i,t}^{\Psi} + \delta_K\left(L_K\right)_{i,t}^{\Psi}\right]^{\frac{v}{\Psi}} \exp(\boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{i,t} + \varepsilon_{i,t}), \tag{2}$$

where $M_{i,t}$ is the number of deals in country i at time t; $\left(L_E\right)_{i,t}$ and $\left(L_K\right)_{i,t}$ are the number of projects submitted and of business angels in country i at time t, respectively; $\mathbf{c}_{i,t}$ is a vector of controls; v is the return-to-scale parameter; δ_E and δ_K are share parameters (with $\delta_E + \delta_K \equiv 1$), A is a scale technology parameter.

We now state the parameter restrictions on (2) implied by each of the three hypotheses above. It can be shown that: (a) part (i) of (Hp 1) (positive marginal returns to both inputs) requires v, A, δ_j (for $j = E, K$) being strictly positive; (b) part (ii) of (Hp 1) (decreasing marginal returns to both inputs) further requires $\Psi < 1$; (c) (Hp 2) (positive technological complementarity across the two inputs) is verified whenever $\delta_E, \delta_K, v, A, (v - \Psi) > 0$; (d) (Hp 3) (constant returns to scale) requires $v = 1$. Finally, it can be easily demonstrated that (Hp 2) and (Hp 3) together imply part (ii) of (Hp 1).

Cobb-Douglas Matching Function. The Cobb-Douglas matching function can be written as

[10] A CES-type function is a function that exhibit a constant elasticity of substitution across inputs. It is widely used in economics to represent either preferences or production technology.

$$M_{i,t} = A\,(L_E)_{i,t}^{\beta_E}\,(L_K)_{i,t}^{\beta_K}\exp(\boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{i,t}+\varepsilon_{i,t}), \tag{3}$$

where, again, $(L_E)_{i,t}$,$(L_K)_{i,t}$,$M_{i,t}$ denote, respectively, the number of projects submitted, business angels and deals in country i at time t; β_E and β_K are the output elasticities of submitted projects and business angels, respectively. Output elasticity measures the responsiveness of the number of matches to a change in input. For example, a $\beta_E = 0.45$ implies that a 1% increase in the number of submitted projects leads to a 0.45% increase in the number of matches. Finally, the expression $A\exp(\boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{i,t}+\varepsilon_{i,t})$ (where A is a positive constant and $\mathbf{c}_{i,t}$ is a vector of controls) can be interpreted as the "total factor productivity" of the aggregate matching function, proxying for the degree of *efficiency* of the matching technology (the higher this term, the higher the number of deals obtained with the same amounts of inputs).

In order for (Hp 1), (Hp 2) and (Hp 3) to be verified, function (3) must satisfy the following parameter restrictions. Taken together, part (i) of (Hp 1) and (Hp 2) require $\beta_E, \beta_K, A > 0$; part (ii) of (Hp 1) requires β_E, β_K being jointly lower than 1; (Hp 3) on constant returns to scale implies $\beta_E + \beta_K = 1$.

Without loss of generality, we estimate the following log-linear transformation of the Cobb-Douglas function:

$$\ln M_{i,t} = \ln A + \beta_E \ln (L_E)_{i,t} + \beta_K \ln (L_K)_{i,t} + \boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{i,t} + \varepsilon_{i,t}. \tag{4}$$

Translog Matching Function. The transcendental logarithmic (or simply, translog) matching function can be written as[11]

$$\begin{aligned}\ln M_{i,t} \;=\; & \ln A + \beta_E \ln (L_E)_{i,t} + \beta_K \ln (L_K)_{i,t} + \beta_{EK} \ln (L_E)_{i,t} \ln (L_K)_{i,t} + \\ & + \beta_{EE}\left[\ln (L_E)_{i,t}\right]^2 + \beta_{KK}\left[\ln (L_K)_{i,t}\right]^2 + \boldsymbol{\beta}_{\mathbf{c}}\mathbf{c}_{i,t} + \varepsilon_{i,t}.\end{aligned} \tag{5}$$

Compared to the Cobb-Douglas, the translog adds (i) an interaction term between the logs of the two inputs, in order to verify the existence of input complementarity not only in levels but also in *elasticities* (in which case β_{EK} would be significantly positive) and (ii) the squares of the two log-inputs to verify whether the elasticity of each input is decreasing with the log-value of that input (that is, whether $\beta_{EE}, \beta_{KK} < 0$).

[11] Useful references for the translog include Warren (1996), Yashiv (2000) and Kangasharju et al. (2005).

The scale elasticity of a translog is defined by $\epsilon = \epsilon_E + \epsilon_K$, where $\epsilon_E = \beta_E + \beta_{EK} \ln (L_K)_{i,t} + 2\beta_{EE} \ln (L_E)_{i,t}$ is the elasticity of new deals with respect to the number of submitted projects, and $\epsilon_K = \beta_K + \beta_{EK} \ln (L_E)_{i,t} + 2\beta_{KK} \ln (L_K)_{i,t}$ is the elasticity of new deals with respect to the number of BAs. Hence, and differently from the Cobb–Douglas case, input elasticities are not constant but depend on the scales of both inputs. Theoretical hypotheses can be summarized as follows. Part (i) of (Hp 1) requires $\epsilon_E, \epsilon_K > 0$; part (ii) of (Hp 1) needs $\beta_{EE}, \beta_{KK} < 0$; (Hp 2) is verified when $\beta_{EK} > 0$; finally, (Hp 3) requires $\epsilon = 1$.

4.3 Results

The three hypotheses are tested against several specifications of the log-CES, the log-Cobb Douglas and translog matching function, which differ with respect to the set of controls included in vector $\mathbf{c}_{i,t}$. We now present the results from the estimation of the three matching models in order.

CES Matching Function. The results from the non-linear estimations of the log-CES matching function are shown in Table 2. In model 1, the $\mathbf{c}_{i,t}$ vector includes a time trend only; model 2 adds country group dummies;[12] model 3 adds interaction terms between the time trend variable and the country group dummies; model 4 and model 5 replicate models 2 and 3, respectively, but replace country group dummies with country dummies. Including interaction terms between country identifiers and the time trend serves to capture the changing role of country-specific characteristics on the observed evolution of the matching function in presence of an unbalanced panel dataset. Since the log-CES is a non-linear function, estimation is conducted via *non-linear* least squares method (see Davidson and MacKinnon, 2004).

Results are in line with the theoretical hypotheses. In all specifications (i) parameters δ_E, δ_K, v, A are strictly positive, and the test for the null hypothesis that δ_E, δ_K, v, A are jointly null is strongly rejected (as requested by part (i) of (Hp 1)); (ii) v is significantly positive and around 1, Ψ is always not significantly different from zero, and the null hypothesis of the joint test for the null hypothesis that $\delta_E, \delta_K, v, A, (v - \Psi)$ are

[12] Countries have been aggregated in the following five groups: 1) Continental European countries (Austria, Belgium, Germany and the Netherlands); 2) Mediterranean European countries (France, Greece, Italy, Portugal and Spain); 3) Scandinavian countries (Denmark, Finland, Norway and Sweden); 4) Eastern European countries (Poland and Russia); 5) Anglosaxon countries (United Kingdom and the United States).

jointly null is strongly rejected (in line with (Hp 2)); (iii) the null hypothesis on the F-test on $v = 1$ (that is, (Hp 3) on constant returns to scale) cannot be rejected. As claimed above, (Hp 2) and (Hp 3) imply part (ii) of (Hp 1).[13]

INSERT TABLE 2 HERE

Finally, with a Ψ not significantly different from zero, we do not reject the hypothesis of a unitary elasticity of substitution between the two inputs.[14] Given that the CES collapses to a Cobb-Douglas function when $\Psi \to 0$, we now turn to a Cobb-Douglas specification of the matching function.

Cobb-Douglas Matching Function. Function (4) is estimated via a robust regression approach to deal with the presence, in the dataset, of outliers that can distort the ordinary least squares (OLS) estimator. Adopting the graphical tool proposed by Rousseeuw and Van Zomeren (1990), Figure 1 shows that several outliers are present, suggesting that there is a serious risk that the OLS estimator be strongly attracted by outliers (Rousseeuw and Leroy, 1987). To tackle this issue, we adopt a Huber's monotonic M-estimator in order to avoid giving greater importance to observations with very large residuals and, consequently, distorting the parameters' estimation in the presence of outliers.[15]

INSERT FIGURE 1 HERE

[13] In addition, the joint test for the null hypothesis that δ_E, δ_K, v, A are null and $\psi = 1$ is strongly rejected in all specifications.

[14] In fact, the Hicks elasticity of substitution between the two input factors is given by $1/(1-\Psi)$.

[15] Huber's monotonic M-estimator combines the M-estimator with an initial step that removes high-leverage outliers, based on Cook's distance (Hamilton, 1991). In particular, the adopted estimator takes the Cook's D for each observation by first running an OLS regression (Cook's distance is a way to estimate the influence of a data point when performing a least-square regression analysis). Then the procedure drops observations with Cook's distance greater than 1 and proceeds with iteratively re-weighted least squares, with each observation being assigned higher weights the smaller the computed residuals. In order to improve the estimator efficiency, two types of weighting procedure are used: Huber weighting and biweighting. The iterating procedure stops until the estimated coefficients converge, that is, until the maximum change between the weights from one iteration to the next is below tolerance.

We estimate four specifications of (4) which differ according to the set of variables included in the vector of controls $\mathbf{c}_{i,t}$. In model 1, the $\mathbf{c}_{i,t}$ vector includes a time trend variable only; model 2 adds year and country dummies; model 3 adds to model 1 country-specific time trends; model 4 augments model 3 with relevant macroeconomic indicators to proxy for country-specific macroeconomic changes which might affect the matching process as well as the input variables over the years. In particular, we consider the following variables: the amount of domestic credit to private sector provided by banks and that provided by the financial sector, both as a percentage of GDP (The World Bank database), the rate of GDP growth (The World Bank database), the level of GDP per capita (The World Bank database) and the amount of funds venture capitalists invested during the year (sources NVCA for the US, EVCA for Europe, RVCA for Russian Federation).

Results are shown in Table 3 and are in line with the theoretical predictions. (Hp 1) and (Hp 2) are not rejected since the share parameters are jointly significantly positive and different from one: a 1% increase in the number of submitted projects (business angels) leads to positive increase in the number of deals by between 0.37% (0.38%) and 0.64% (0.60%). The F-test on the parameter restriction $\beta_E + \beta_K = 1$ does not reject the hypothesis of constant returns to scale (Hp 3).

INSERT TABLE 3 HERE

Translog Matching Function. Model (5) is also estimated via robust regression. Table 4 displays the results of two translog matching function models: model 1 includes country dummies and country-specific time trends among the $\mathbf{c}_{i,t}$ vector of controls; model 2 augments model 1 with the same set of relevant macroeconomic indicators used for the Cobb-Douglas specification.

INSERT TABLE 4 HERE

Also in this case, we find evidence in favor of the three hypotheses stated above. Coefficients all have the expected signs and are highly statistically significant. In particular, the log-interaction term (β_{EK}) is significantly positive, meaning that the impact of a 1% increase in one input (the number of business angels or the number of submitted projects) is increasing in the log-value of the other input. On the other hand, the coefficients on the squared log-inputs (β_{EE}, β_{KK}) are significantly negative and jointly different from zero, suggesting that the impact of a 1% increase in one input is decreasing in the log-value of that input. The higher R-squared and the significant impacts of the additional terms compared to the previous specifications suggest a superior performance of the translog function in fitting our data.

In model 1, the two input elasticities, ϵ_E and ϵ_K, calculated at the sample median of the explanatory variables $\ln(L_E)_{i,t}$, $\ln(L_K)_{i,t}$ are jointly significantly positive and equal to 0.70 and 0.23, respectively (that is, respectively higher and lower than those obtained under the Cobb-Douglas estimation). These two numbers can be interpreted as follows: a 1% increase in the number of submitted projects leads to a 0.70% increase in the number of matches; similarly, a 1% increase in the number of BAs leads to a 0.23% increase in the number of matches. Finally, the scale elasticity of the matching function, $\epsilon = \epsilon_E + \epsilon_K$, is equal to 0.93, and the null hypothesis of constant returns to scale ($\epsilon = 1$) cannot be rejected.

While the values of the estimated elasticities are similar in model 2, two macroeconomic indicators are found to be highly significant: the GDP per capita and the size of the VC market. We find that, in countries characterized by a higher GDP per capita and in countries with a less developed venture capital market, the matching process is more efficient -in the specific sense of delivering a higher number of deals given the same inputs. Column 2 of Table 4 shows a positive elasticity of the number of matches to the value of GDP per capita: *coeteris paribus*, a 1% increase in the value of GDP per capita is associated to a 2.1% increase in the number of matches. This is not surprising, given that countries with a higher GDP per capita are wealthier and more developed: it is then reasonable to expect that, in those countries, the matching process is more efficient. On the other hand, a 0.1% increase in the amount of venture capital investments over GDP is associated with a 7.3% decrease in the number of matches, all other things being equal. This negative correlation suggests some degree of substitution (or competition) between between business angels and venture capitalists.

The efficiency of the matching process is also related to country dummies and country-specific time trends, which capture country-specific time variant and invariant characteristics not explicitly accounted for by the control variables (such as the quality of institutions, the regulatory framework, etc.). To test their significance, we compute an F-test for the joint significance of the coefficients (the null hypothesis being that all coefficients are not significantly different from zero against the alternative hypothesis that at least one coefficient is different from zero). The p-value associated with the resulting F-statistic is significant at 1%, and the null hypothesis of irrelevant role of these variables is rejected.

Finally, in order to shed some light on the explanatory power of the country-specific variables, we also compare the adjusted R-squared of model 2 with that of the same model but in which these variables are turned off (nested model). Introducing country-dummies and country-specific time trends improves the adjusted R-squared from 93% to 99%, suggesting that institutional characteristics -not captured by the input and the control variables- are able to explain 6% of the total variance of the matching process.

4.4 Robustness Analysis

The presence of *unobservable shocks* affecting either the decision to finance a project (that is, the matching process within each BA) or the number of agents on either side of the market (namely, the number of BAs or the number of submitted projects) might represent a source of estimation bias. For example, the number of projects submitted to a BA is very likely to depend on the efficiency of the matching process (which, as we have seen above, depends on such fundamental characteristics as the quality of institutions regulating the market). Therefore, random shocks to matching efficiency affect the number of matches both directly through the matching technology and indirectly through entrepreneurs' behavior. In this case, the number of projects submitted could be endogenous, and our previous estimation strategy would fail to account for that endogeneity (Borowczyk-Martins et Al., 2013).

To tackle this possible source of estimation bias, we run a system-GMM estimator on both Cobb-Douglas and Translog matching function models. GMM provides a convenient framework to cope with bias in panel data models and, more generally, to deal with endogeneity in the input variables. In particular, we adopt the so-called one step system-GMM estimator on our two models, both with and without additional controls. Following Blundell and Bond (1998), the choice of employing a one step system-GMM estimator is motivated by the large finite sample biases due to the time series persistence properties of some of the variables (in particular, the number of BAs). This estimator considers a system of equations formed by the equation in first-differences (to eliminate unobserved firm-specific effects) and the equation in levels. The system panel results are reported in Table 5. In the first three columns, we treat the explanatory variables $\ln(L_E)$, $\ln(L_K)$ (and, where appropriate, their interactions) as predetermined;[16] in the last three columns, we treat the same variables as endogenous or correlated with past, current or future values of the error term.[17]

INSERT TABLE 5 HERE

The results from the System-GMM estimations confirm the hypotheses. In all specifications, the elasticities of deals to both input shares are significantly positive and decreasing with the log-value of each input. Moreover, inputs are found to be technological complements. Finally, although the magnitudes of estimated coefficients differ across specifications, the null hypothesis of constant returns to scale is always not rejected.

The consistency of the GMM estimator depends on the validity of the instruments, which we test using two specification tests. The first is the standard Sargan test of overidentifying restrictions, which tests the validity of the instruments by analyzing the sample analog of the moment conditions used in the estimation process. The second test examines the hypothesis that there is no second-order serial correlation in the first-differenced residuals. The panel system-GMM estimates pass the specification tests. According to the Sargan statistic test for overidentifying restrictions reported at the bottom of Table 5, the set of instruments are valid, and the Arellano-Bond test for AR(2) implies that the error terms in levels are not serially correlated.

Table 6 summarizes the estimated input elasticities of the Cobb-Douglas and translog models with the most complete set of explanatory variables, under the three different estimation methods utilized (robust regression, system-GMM with explanatory variables treated as predetermined and system-GMM with explanatory variables treated as endogenous). For the Cobb-Douglas, depending on the model specification, a one percentage point increase in the number of BAs implies an increase in the number of

[16] In econometric models, this implies that the current period error term is uncorrelated with current and lagged values of the predetermined variable but may be correlated with future values.

[17] To prevent that a large number of instruments overfits the instrumented variables, biasing the results towards those of robust regressions, instruments are computed using only a limited number of lags (see note of Table 5 for details). Since we are dealing with an unbalanced panel with irregular patterns of missing observations, we have also estimated the same model applying backward orthogonal deviations transformation to the instruments for the transformed equation. Results are presented in Table A1 and are qualitatively and quantitatively similar to those obtained under the traditional GMM estimators which use instruments in levels.

deals by between 0.23% and 0.46%, while a one percentage point increase in the number of submitted projects implies an increase in the number of deals by between 0.54% and 0.91%. In the translog model, the values of the elasticities to submitted projects, calculated at the sample median of the explanatory variable, are more stable across the different estimation methods (between 0.51% and 0.70%); the estimated elasticities to the number of BAs, instead, range from 0.23% to 0.61%. Finally, evidence in favor of constant returns to scale is robust across all model specifications.

INSERT TABLE 6 HERE

5 Discussion and Conclusions

This paper has described the market for entrepreneurial finance as a decentralized market in which entrepreneurs and financiers search and match with each other. After introducing a standard aggregate matching technology to capture such relationship, we have specified the most common hypotheses imposed in the literature, and we have explicitly demonstrated that such hypotheses have important theoretical implications. Using a new dataset on the visible angel market, we have then estimated and compared three different specifications of the matching function (CES, Cobb-Douglas and translog), and we have verified empirically the previously stated hypotheses.

Across all model specifications, the estimated matching function exhibits positive and decreasing marginal productivities of its two inputs and a positive degree of technological complementarity between them. Moreover, the robust empirical evidence in favor of constant returns to scale suggests that the stepping-on-toes externality and the thick-market externality tend to cancel each other out and, hence, that the BA market organized through angel networks is able to produce an optimal set of entry conditions for market participants.

Finally, we have also found that the efficiency in the production process of entrepreneurial ventures is significantly influenced by, among others, the level of GDP per capita, the degree of development of the VC market, and other country-specific characteristics (possibly proxying for the quality of institutions prevailing in each country). Needless to say, the picture of the angel market that we have gained from the empirical analysis is somewhat limited by the nature of the data at our disposal, and further research efforts should be devoted to analyzing the determinants of the efficiency of this firm creation process. In the rest of the section, we further discuss these limitations and suggest avenues for future research.

5.1 Limitations

Let us briefly discuss two additional sources of potential estimation bias not tackled in Subsection 4.4 devoted to robustness analysis. First, as in most surveys, participation is voluntary, and the number and the identity of respondents might differ yearly. Moreover, given that the surveys are primarily addressed to established angel networks, the investments that individuals make on their own or with *ad hoc* groups of friends -the so-called "invisible market" (OECD, 2011)- are not included in our dataset. Therefore, the figures presented in each report might be not representative of each country's angel market, and the empirical analysis may then be characterized by the so called "voluntary response bias".

Secondly, due to a limitation intrinsic in the aggregate nature of our dataset, we are not able to explicitly control for angel *syndication*, that is, for the possibility that more than one angel finance the same entrepreneurial venture. This might generate two sources of bias: one via the number of entrepreneurs, the other via the number of financiers. Let us analyze them in order. First consider an entrepreneur submitting her project to multiple angels participating in the survey ("visible" angel syndication). In this case, we would observe multiple applications containing the *same* project and, hence, an overestimation of the number of fund-seeking entrepreneurs. This would in turn produce an underestimation of the marginal effect of the number of fund-seeking entrepreneurs. This is a possible bias via the number of potential entrepreneurs. Next consider an entrepreneur submitting her project to an angel participating in the survey. If this angel decides to finance the project by collecting funds also from angels *not* participating in the survey ("invisible" angel syndication), we would observe an underestimation of the number of potential financiers and, hence, an overestimation of the marginal effect of the number of angels on the matching outcome. This is a possible bias via the number of potential financiers.

With these two *caveat* in mind, the fact remains that, if one is interested in analyzing the aggregate behavior of the angel market, data obtained from the "supply-side" of the market (that is, from BA associations and networks) currently represent the richest source of information on the angels' investment activity. Supranational institutions (such as the European Commission) and important database providers (such as the

Bureau Van Dijk) consider EBAN annual surveys as the most detailed source of data on the visible fraction of the BA market in Europe.[18] The most recent empirical literature on angel finance often relies on data collected from angel groups and networks organized in BANs or syndicates (Kerr et al., 2014; Lerner et al., 2015). Although the relatively small sample size may limit the generalizability of our findings to other countries or time spans, our exercise can then be considered as a first attempt to investigate the aggregate behavior of the visible angel market by exploiting state-of-the-art sources of information on this market.

It would be useful to replicate our empirical analysis using deeper or alternative sources of information on the BA market. For example, access to micro-level information from the BA associations on angel activity could help solve the angel syndication bias discussed above. On the other hand, "demand-side" data sources (such as business surveys or national business registers), could help capture, at least in part, the "invisible" BA activity through the ventures in which the financiers have invested. Finally, institutional data obtained from active entrepreneurship policies (such as tax incentives, subsidies or government-sponsored entrepreneurship funds and grants) could also provide a valuable source of information on the supply and the demand for angel finance.

5.2 Implications and Avenues for Future Research

Let us conclude this paper discussing two broad implications following from our empirical findings and, in particular, from verifying Hypothesis 2 and Hypothesis 3. The first is concerned with the aggregate behavior of the BA sector possibly implied by Hypothesis 2. The second is related to the role of the policy maker in fostering entrepreneurial activity as implied by hypothesis 3. Let us tackle them in order.

Markets for entrepreneurial finance are typically characterized by a high degree of geographic concentration (Mason, 2007; Chen et al., 2009). The existence of a thick-market effect verified in Hypothesis 2 might contribute to rationalize this phenomenon. The thick-market effect implies that entrepreneurs are more willing to enter into a market that is crowded by financiers, and *viceversa*. This feature stimulates the clusterization of entrepreneurial activity. Needless to say, we are only claiming that the complementarity across the two sides of the BA market is compatible with

[18] In Europe, the Bureau Van Dijk has chosen EBAN as a data partner for the inclusion of business angels and seed funds *deals* information in Zephyr.

the geographic pattern documented by the empirical literature; proving that it *does* generate such pattern is out of the scope of this paper and, in our view, is a promising and interesting avenue for future research in this field.[19]

As is well known to economists, a variant of the so called "infant industry argument" suggests that, whenever a sector is characterized by strong economies of scale, the role of the policy maker may be crucial, in the very early stages of development, in fostering its growth. This argument is popular in the entrepreneurial policy debate,[20] and it is inspired by the theoretical literature (Cooper and John, 1988; Cooper, 1999). Models admitting multiple equilibria provide a powerful narrative behind the need for policy intervention: as equilibrium multiplicity implies the existence of coordination failures -that is, of equilibria characterized by sub-optimally low paces of economic activity-, government intervention may serve to spur a *virtuous* cycle, that is, to favor the coordination of economic agents towards a superior equilibrium configuration.[21] In providing the first, admittedly preliminary, evidence of constant returns to scale in the visible market for angel investment, this paper adds a note of prudence in justifying direct policy intervention on the demand or on the supply of angel finance via this specific line of argument.

The evidence on constant returns to scale has, however, an additional and more specific policy implication. As discussed in Subsection 4.1, our empirical analysis is exclusively concentrated on angel deals conveyed via Business Angels Networks (BANs). In the words of Mason (2009, p. 542), the role of the networks is "to improve the efficiency of information flow in the market by providing a channel of communication which enables entrepreneurs seeking finance to come to the attention of business angels and at the same time enables business angels to receive information on investment opportunities without compromising their privacy". In other words, networks act as a market intermediaries that reduce the search costs between entrepreneurs and the -often invisible- angels, while they play no role in the investment decision process: "the

[19] Notice also that other explanations have been proposed which are based on the existence of alternative network effects, such as knowledge spillovers, information spillovers, agglomeration benefits etc. (see, among others, Jaffe et al., 1993; Audretsch and Feldman, 1996; Guiso and Schivardi, 2011 and Parker, 2009).

[20] In the words of Lerner (2010, p.42), "every hub of cutting-edge entrepreneurial activity in the world today had its origins in proactive government intervention". Lerner (2010) also provides abundant anecdotal evidence in favor of this view. See also Mazzucato (2013).

[21] Models of entrepreneurship admitting multiple equilibria include Parker (2005), Ghatak (2007) and Landier (2004).

business angels make their own investment decisions, undertake their own due diligence and negotiate their own term sheet directly with the entrepreneur" (Mason, 2009, pp. 542).[22] By using data on deals finalized within these networks, this paper provides a cross-country assessment of the role of BANs in improving the efficiency of information flow in the market and thus in favouring the match between demand and supply for early-stage financing. For the reasons discussed in Section 3, the evidence on constant returns to scale of the matching process suggests that these networks provide *efficient* matching mechanisms between demand and supply, in that they help reduce search frictions and thus facilitate the entry of entrepreneurs and financiers. This result encourages public support for promoting the emergence of Business Angels Networks as effective facilitators of angel-backed entrepreneurial ventures. Needless to say, the evidence provided in this paper is far from conclusive in any dimension, and further work remains to be done to evaluate the aggregate behavior of the angel market as well as the performance of angel networks in facilitating the meeting between entrepreneurs and financiers.

References

[1] Amatucci, F. M. and J. E. Sohl (2007). Business angels: investment processes, outcomes and current trends, in A. Zacharakis and S. Spinelli Jr. (eds.), Entrepreneurship: The Engine of Growth, Volume 2, Westport CT, Praager, pp. 87–107.

[2] Aernoudt, R. (1999). "Business angels: Should they fly on their own wings?". *Venture Capital: An International Journal of Entrepreneurial Finance*, 1(2),pp. 187-195.

[3] Audretsch, D.B. and M.P. Feldman (1996). "R&D Spillovers and the Geography of Innovation and Production", *American Economic Review*, 86(3), pp. 630-640.

[4] Balachandra, L., H. Sapienza and D. Kim (2014). "How critical cues influence angels' investment preferences". *Frontiers of Entrepreneurship Research*, 34(1), 1-16.

[22] The role of BANs as market facilitators has so far received a little and mixed assessment. Most evaluations have been conducted for the UK (see, among the others, Mason and Harrison, 1997, 1999; Mason, 2009). Collewaert et al. (2010) find a positive impact of BANs in Flanders.

[5] Blundell, R., S. Bond (1998). "Initial conditions and moment restrictions in dynamic panel data models". *Journal of Econometrics*, 87, pp. 115-143.

[6] Borowczyk-Martins, D., G. Jolivet and F. Postel-Vinay (2013). "Accounting for Endogeneity in Matching Function Estimation". *Review of Economic Dynamics*, 16, pp. 440-451.

[7] Capizzi, V. (2015). "The returns of business angel investments and their major determinants". *Venture Capital: An International Journal of Entrepreneurial Finance*, 17(4), pp. 271-298.

[8] Carpentier, C., and J.-M. Suret (2015). "Angel Group Members' Decision Process and Rejection Criteria: A Longitudinal Analysis". *Journal of Business Venturing*, 30, pp. 808–821.

[9] Chemmanur, T. and P. Fulghieri (2014). "Entrepreneurial Finance and Innovation: An Introduction and Agenda for Future Research". *The Review of Financial Studies*, 27 (Special Issue "Entrepreneurial Finance and Innovation"), pp. 1-19.

[10] Chen, H., P. Gompers, A. Kovner, and J. Lerner (2009). "Buy Local? The Geography of Successful and Unsuccessful Venture Capital Expansion". NBER Working Paper, No. 15102.

[11] Chiu, J., C. Meh, and R. Wright (2011). "Innovation and Growth with Financial, and other, Frictions", NBER Working Paper no. 17512.

[12] Cipollone, A. and P.E. Giordani (2016). "Market Frictions in Entrepreneurial Innovation: Theory and Evidence". LUISS, Mimeo.

[13] Collewaert, V., Manigart, S. and R. Aernoudt (2010). "Assessment of Government Funding of Business Angel Networks in Flanders". *Regional Studies*, 44(1), pp. 119-130.

[14] Cooper, R., and A. John (1988). "Coordinating Coordination Failures in Keynesian Models". *Quarterly Journal of Economics*, 103, 441-63.

[15] Cooper, R.W. (1999). "Coordination Games, Complementarities and Macroeconomics". Cambridge University Press, Cambridge.

[16] Croce, A., F. Tenca and E. Ughetto (2017). "How business angel groups work: Rejection criteria in investment evaluation". *International Small Business Journal*, 35(4), pp. 405–426.

[17] Croce, A., M. Guerini and E. Ughetto (2018). "Angel Financing and the Performance of High-Tech Start-Ups". *Journal of Small Business Management*, 56(2), pp.208-228.

[18] Cumming, D and M. Zhang (2016). "Angel Investors around the World". Mimeo, York University, CA.

[19] Da Rin, M., T. Hellmann, M. Puri. (2011). "A survey of venture capital research". In: Constantinides, G., et Al. (Eds.), *Handbook of the Economics of Finance*, Vol.2. North Holland.

[20] Davidson, R., and J. G. MacKinnon (2004). "Econometric Theory and Methods". New York: Oxford University Press.

[21] De Clercq, D., V. H. Fried, O. Lehtonen and H. J. Sapienza (2006). "An entrepreneur's guide to the venture capital galaxy". *Academy of Management Perspectives,* 20(3), pp. 90-112.

[22] Diamond, P. (1982). "Aggregate Demand Management in Search Equilibrium". *Journal of Political Economy*, 90, pp. 881-894.

[23] Diamond, P. (1984). "Money in Search Equilibrium". *Econometrica*, vol. 52(1), pp. 1-20.

[24] Dutta, S., & Folta, T. B. (2016). "A comparison of the effect of angels and venture capitalists on innovation and value creation". *Journal of Business Venturing*, 31(1), pp. 39-54.

[25] Ghatak, M. (2007). "Entrepreneurial income, occupational choice, and trickle-up policies". *Journal of Economic Theory*, 137, pp. 27–48.

[26] Goldfarb, B. D., G. Hoberg, D. Kirsch and A. Triantis (2014). "Are Angels Different? An Analysis of Early Venture Financing". Robert H. Smith School Research Paper No. RHS 06-072.

[27] Gompers, P.A. and J. Lerner (2004). "The Venture Capital Cycle". MIT Press, Cambridge, MA.

[28] Guiso, L. and F. Schivardi (2011). "What Determines Entrepreneurial Clusters?". *Journal of the European Economic Association*, 9(1), 61-86.

[29] Hamilton, L. C. (1991). "How robust is robust regression?". *Stata Technical Bulletin,* 2, pp. 21–26, College Station, TX: Stata Press.

[30] Hellman, T. and V. Thiele (2015). "Friends or foes? The interrelationship between angel and venture capital markets". *Journal of Financial Economics*, 639-653.

[31] Hellman, T.F, P. Shure and D. Vo (2017). "Angels and Venture Capitalists: Substitutes or Complements?". Saïd Business School WP 2015-2.

[32] Hosios, A.J. (1990). "On the Efficiency of Matching and Related Models of Search and Unemployment". *The Review of Economic Studies*, 57(2), pp. 279-298.

[33] Ibrahim, D. M. (2008). "The (not so) puzzling behavior of angel investors". *Vanderbilt Law Review,* 61, pp. 1405–1452.

[34] Inderst, R. and H. Muller (2004). "The Effect of Capital Market Characteristics on the Value of Start-up Firms". *Journal of Financial Economics*, 72, pp. 319-56.

[35] Jaffe, A., M. Trajtenberg and R. Henderson (1993). "Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations". *Quarterly Journal of Economics*, 63(3), 577-598.

[36] Kangasharju, A., J. Pehkonen and S. Pekkala (2005). "Returns to scale in a matching model: evidence from disaggregated panel data". *Applied Economics*, 37(1), pp. 115-118.

[37] Kerr, W.R., J. Lerner and A. Schoar (2014). "The Consequences of Entrepreneurial Finance: Evidence from Angel Financing". *Review of Financial Studies*, 27, 20–55.

[38] Kim, J.H. and L. Wagman (2016). "Early-stage entrepreneurial financing: A signaling perspective". *Journal of Banking and Finance*, 67, 12-22.

[39] Kmenta, J. (1967). "On Estimation of the CES Production Function". *International Economic Review*, 8, pp. 180-189.

[40] Kortum S. and J. Lerner (2000). "Assessing the contribution of venture capital to innovation". *Rand Journal of Economics*, 31, pp. 674-692.

[41] Landier, A. (2004). "Entrepreneurship and the Stigma of Failure". Mimeo, Stern School of Business, New York University.

[42] Lerner, J. (2010). "Boulevard of Broken Dreams: Why Public Efforts to Boost Entrepreneurship and Venture Capital Have Failed and What to Do About It". Princeton University Press, Princeton, NJ.

[43] Lerner, J., A. Schoar, S. Sokolinski, and K. Wilson (2015). "The Globalization of Angel Investments". Bruegel Working Paper Series, 2015/09.

[44] Mason, C. M. (2006). Informal sources of venture finance, in S. C. Parker (ed.), The Life Cycle of Entrepreneurial Ventures, New York, Springer, pp. 259–99.

[45] Mason, C. M. 2007. "Venture Capital: A Geographical Perspective." In *Handbook of Research on Venture Capital*, edited by H. Landström, 86–112. Cheltenham: Edward Elgar.

[46] Mason, C.M. (2009). "Public Policy Support for the Informal Venture Capital Market in Europe: A Critical Review". *International Small Business Journal*, 27(5), pp. 536–556.

[47] Mason, C.M. and R.T. Harrison (1997). "Business Angel Networks and the development of the informal venture capital market in the UK: is there still a role for the public sector?". *Small Business Economics*, 9(2), pp. 111-123.

[48] Mason, C.M. and R.T. Harrison (1999). "Public policy and the development of the informal venture capital market: UK experience and lessons for Europe". In: K. Cowling K. (Ed.), *Industrial Policy in Europe*. London: Routledge.

[49] Mason, C.M., R.T. Harrison (2002). "Is it worth it? The rates of return from informal venture capital investments". *Journal of Business Venturing* 17, 211–236.

[50] Mason, C.M., T. Botelho and R. Harrison (2016). "The Transformation of the Business Angel Market: Empirical Evidence and Research Implications". *Venture Capital: An International Journal of Entrepreneurial Finance*, 18(4), pp. 321-344.

[51] Maxwell, A.L., S.A. Jeffrey, and M. Lévesque (2011). "Business angel early stage decision making". *Journal of Business Venturing*, 26(2), pp.212-225.

[52] Mazzucato, M. (2013). "The Entrepreneurial State: debunking public vs. private sector myths". Anthem Press: London, UK.

[53] Michelacci, C. and J. Suarez (2004). "Business Creation and the Stock Market". *Review of Economic Studies*, 71, pp. 459-481.

[54] Mitteness, C. R., M. S. Baucus and R. Sudek (2012). "Horse vs. Jockey? How stage of funding process and industry experience affect the evaluations of angel investors". *Venture Capital: An International Journal of Entrepreneurial Finance*, 14(4), pp. 241-267.

[55] Murnieks, C.Y., M.S. Cardon, R. Sudek, T.D. White, W.T. Brooks (2016). "Drawn to the fire: the role of passion, tenacity and inspirational leadership in angel investing". *Journal of Business Venturing,* 31 (4), 468–484.

[56] OECD (2011). "Financing High-Growth Firms: The Role of Angel Investors". OECD Publishing.

[57] Parker, S. (2005). "Explaining regional variations in entrepreneurship as multiple occupational equilibria". Journal of Regional Science, 45, pp. 829–50.

[58] Parker, S. (2009). "The Economics of Entrepreneurship". Cambridge (UK): Cambridge University Press.

[59] Petrongolo, B., and C.A. Pissarides (2001). “Looking into the Black Box: A Survey of the Matching Function.” *Journal of Economic Literature*, 39(2): 390–431.

[60] Phelps, E. (2009). "Toward a Model of Innovation and Performance Along the Lines of Knight, Keynes, Hayek and M. Polanyi". In Z. Acs, D. Audretsch and R. Strom (Eds.), Entrepreneurship, Growth and Public Policy, Cambridge University Press, Cambridge.

[61] Politis, D. (2008). "Business angels and value added: what do we know and where do we go?". *Venture Capital: An International Journal of Entrepreneurial Finance*,10(2), pp. 127-147.

[62] Politis, D., and H. Landström (2002). "Informal investors as entrepreneurs: The development of an entrepreneurial career". *Venture Capital: An International Journal of Entrepreneurial Finance*, 4(2), pp. 78-101.

[63] Prowse, S. (1998). "Angel investors and the market for angel investments". *Journal of Banking and Finance*, 22(6-8), pp. 785-792.

[64] Rogerson, R., R. Shimer, and R. Wright (2005). "Search-Theoretic Models of the Labor Market: A Survey". Journal of Economic Literature, Vol. XLIII, pp. 959–988.

[65] Rousseeuw, P. J. and A. Leroy (1987). "Robust Regression and Outlier Detection". New York: John Wiley and Sons.

[66] Rousseeuw, P. J. and B. Van Zomeren (1990). "Unmasking Multivariate Outliers and Leverage Points", *Journal of the American Statistical Association*, 85, 633-639.

[67] Shane, S. (2008). "The importance of Angel investing in financing the growth of entrepreneurial ventures". Working Paper 331. Small Business Administration, Office of Advocacy, U.S.

[68] Shane, S. (2009). "Fool's Gold? The Truth Behind Angel Investing in America". Oxford University Press. New York.

[69] Silveira, R. and R. Wright (2010). "Search and the Market for Ideas". *Journal of Economic Theory*, 145(4), 1550-1573.

[70] Silveira, R. and R. Wright (2016). "Venture Capital: A Model of Search and Bargaining". *Review of Economic Dynamics*, 19, pp. 232–246.

[71] Sorensen, M. (2007). "How Smart is Smart Money? A Two-Sided Matching Model of Venture Capital". *Journal of Finance*, 62, pp. 2725–2762.

[72] Warnick, B.J., C.Y. Murnieks, J.S. McMullen, and W.T. Brooks (2018). "Passion for entrepreneurship or passion for the product? A conjoint analysis of angel and VC decision-making". *Journal of Business Venturing*, 33, pp.315-332.

[73] Warren, R. S. (1996). "Returns to Scale in a Matching Model of the Labour Market". *Economics Letters*, 50(1), 135-142.

[74] Wasmer, E. and P. Weil (2004). "The Macroeconomics of Labor and Credit Market Imperfections". *American Economic Review*, 94, pp. 944-963.

[75] Wetzel, W. E. (1983). "Angels and informal risk capital". *Sloan Management Review*, 24(4), pp. 23-34.

[76] Wiltbank, R. and W. Boeker (2007). "Returns to angel investors in groups". Available at: http://ssrn.com/paper=1028592.

[77] Wiltbank, R., S. Read, N. Dew, and S.D. Sarasvathy (2009). "Prediction and control under uncertainty: Outcomes in angel investing". *Journal of Business Venturing*, 24(2), pp.116-133.

[78] Yashiv, E. (2000). "The Determinants of Equilibrium Unemployment". *American Economic Review*, 90(5), 1297-1322.

A The Theoretical Framework

In this section, we introduce a simple, dynamic model of search and matching between entrepreneurs and financiers. The basic framework introduced in Subsection A.1 transposes the classical Diamond's (1982) *coconut* model to the entrepreneurial finance market and extends it to a two-sided search structure.[23] Subsection A.2 discusses the issue of equilibrium multiplicity inside this framework.

A.1 The Fair of New Entrepreneurial Ideas

The world is populated by a measure $\overline{L}_E$ of entrepreneurs and a measure $\overline{L}_K$ of financiers who must decide whether to participate or not in a fair of new ideas. Time is continuous, and new ideas arrive randomly to the entrepreneurs according to a Poisson process with parameter σ. In order for such ideas to become marketable innovations, however, entrepreneurs need to meet financiers and convince them about their profitability. This process of search and matching occurs inside the fair. The model revolves around the entry decisions of the two types of agents, which in turn depend on the costs and on the expected benefits of attending the fair. Let us analyze them in order.

[23] See also Cipollone and Giordani (2016) for a more general version of the framework developed here.

An entrepreneur with a new idea has to decide whether to pursue it or abandon it and wait for the next one. To pursue it, each entrepreneur has to pay an entry cost into the fair, c_E, representing the cost of developing and submitting the project to the financiers. This cost is idiosyncratically drawn from a cumulative distribution function $F(c_E)$ in the support $[0, \overline{c_E}]$. On the other hand, each financier sustains an entry cost, c_K, capturing the cost of screening, evaluating and selecting the entrepreneurial projects. This cost is distributed according to a cumulative distribution function $G(c_K)$ in the support $[0, \overline{c_K}]$.[24]

Let $L_E \leq \overline{L}_E$ and $L_K \leq \overline{L}_K$ denote, respectively, the endogenous number of entrepreneurs and financiers participating in the fair (that is, those that have paid their respective entry cost). An entrepreneurial venture is the result of a process of successful search and matching between an entrepreneur and a financier both attending the fair. The production process of new entrepreneurial ventures is described by the aggregate matching function (1). The instantaneous probabilities of matching are defined by $\alpha_E = M/L_E$ and $\alpha_K = M/L_K$.

Entrepreneurs and financiers are assumed risk neutral and to discount the future via the exogenous riskless interest rate r. The expected benefit for entrepreneurs (financiers) is obtained via a standard dynamic programming argument. In particular, we determine the values of being *inside* and *outside* the fair for each type of agent, so that the expected benefit from entering into the fair is given by the difference between these two values.

For an entrepreneur, the value of being outside the fair (and thus of waiting for a new idea) is denoted by V_E^0 and defined by the following asset equation:

$$rV_E^0 = \sigma \int_0^{c_E^*} \left(V_E^1 - V_E^0 - c_E\right) dF(c_E), \tag{6}$$

where c_E^* (to be determined at equilibrium) is the highest cost for which there is still entry, and V_E^1 denotes the value of being inside the fair (that is, the entrepreneur's expected payoff from the venture). This latter value is defined by[25]

$$rV_E^1 = \alpha_E \left(\theta\pi + V_E^0 - V_E^1\right), \tag{7}$$

[24] c_E and c_K may alternatively be interpreted as outside options, that is, as opportunity costs of devoting to entrepreneurship for, respectively, entrepreneurs and financiers.

[25] Given our exclusive focus on the steady state equilibrium of the model, both here and in the financiers' asset equations, we impose $\dot{V}_j^h = 0$ for $h = 0, 1$ and $j = E, K$.

where π represents total instantaneous profits originating from the venture, and $\theta \in (0,1)$ is the entrepreneurs' fraction of these profits.

These asset equations have the usual interpretations. Equation (6) captures the entrepreneurs' return from being outside the fair as the instantaneous probability of a new idea (σ) times the corresponding payoff, which is given by the capital gain associated with participating in the fair ($V_E^1 - V_E^0$) minus the entry cost (c_E).[26] Equation (7) represents the entrepreneurs' return from being inside the fair as the chance of matching the right financier (α_E) times the sum of the profit share ($\theta\pi$) plus the capital gain or loss associated with exiting from the fair ($V_E^0 - V_E^1$).

The entrepreneurs' expected benefit from attending the fair is then measured by the difference $V_E^1 - V_E^0$. Subtracting (6) from (7), and solving the resulting equation by $V_E^1 - V_E^0$, we obtain

$$V_E^1 - V_E^0 = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF\left(c_E\right)}{r + \alpha_E + \sigma F\left(c_E^*\right)}. \tag{8}$$

Let us now turn to financiers. Their expected payoff from being outside the fair is denoted by V_K^0 and defined by the following asset equation:

$$rV_K^0 = V_K^1 - V_K^0 - c_K, \tag{9}$$

where V_K^1 denotes the expected value from participating in the fair of ideas, which is defined by

$$rV_K^1 = \alpha_K \left[\left(1-\theta\right)\pi + V_K^0 - V_K^1\right], \tag{10}$$

where $\left(1-\theta\right)\pi$ is the financiers' profit share.

The economic interpretation of these two asset equations is analogous to that of (6) and (7). The difference between (6) and (9) is simply due to the fact that, when assessing the value of being outside the fair, each entrepreneur does not know his/her entry cost (as it is specific to the idea he/she has yet to come up with), while each financier knows his/her own entry cost (as it is time-invariant by hypothesis).

Solving the system made up of (9) and (10) by $V_K^1 - V_K^0$, we obtain

$$V_K^1 - V_K^0 = \frac{\alpha_K \left(1-\theta\right)\pi + c_K}{r + \alpha_K + 1}, \tag{11}$$

[26] The integral over c_E reflects the fact that the entrepreneur does not know his/her entry cost before being "hit" by the new idea.

as the financiers' expected benefit from fair attendance.

We are now ready to formalize the optimal entry decisions for both entrepreneurs and financiers. At each point in time, the choice of the $\overline{L}_E - L_E$ entrepreneurs who are outside the fair, as to whether to pursue their project or abandon it, depends on the relative costs and benefits of the project. The cost c_E is distributed according to $F(c_E)$, while the benefit is measured by the difference $V_E^1 - V_E^0$. There exists a marginal entrepreneur for whom $c_E^* = V_E^1 - V_E^0$. Substituting for the expression given in (8), we obtain

$$c_E^* = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{r + \alpha_E + \sigma F(c_E^*)}. \tag{12}$$

All entrepreneurs whose entry cost is lower than c_E^* find it profitable to participate in the fair. The expression above links the threshold cost c_E^* to the probability of successful matching for entrepreneurs α_E (and, hence, to the endogenous number of entrepreneurs and financiers attending the fair, L_E, L_K). It can be proven that this relation is positive: an increase in the probability of a successful matching (higher α_E) leads to an increase in the cutoff value of the entry cost (higher c_E^*).[27]

On the other hand, the chance of a successful matching with an entrepreneur is worth $V_K^1 - V_K^0$ to a financier. Given that the cost of this chance, c_K, is distributed according to $G(c_K)$, there exists a marginal financier for whom $c_K^* = V_K^1 - V_K^0$. Substituting for the expression given in (11), we obtain

$$c_K^* = \frac{\alpha_K (1-\theta) \pi}{r + \alpha_K}. \tag{13}$$

All financiers whose entry cost is lower than c_K^* find it profitable to participate in the fair. This expression captures the positive relationship between α_K and c_K^*.[28]

[27] Applying the implicit function theorem to expression (12), we obtain

$$\frac{dc_E^*}{d\alpha_E} = \frac{\theta\pi - c_E^*}{r + \alpha_E + \sigma F(c_E^*)}$$

which is always positive given that no entrepreneur would undertake a project whose payoff ($\theta\pi$) is lower than the entry cost (c_E^*).

[28] Differentiating expression (13), we obtain

$$\frac{dc_K^*}{d\alpha_K} = \frac{(1-\theta)\pi r}{(r+\alpha_K)^2} > 0.$$

A.2 Coordination Failures in Entrepreneurial Finance

We are now ready to characterize the stationary equilibrium/equilibria of the model. Along the steady state, the inflows and outflows from the fair of ideas must be equal for entrepreneurs:

$$\dot{L}_E = \sigma \left(\overline{L}_E - L_E\right) F\left(c_E^*\right) - L_E \cdot \alpha_E = 0, \tag{14}$$

and for financiers:

$$\dot{L}_K = \left(\overline{L}_K - L_K\right) G\left(c_K^*\right) - L_K \cdot \alpha_K = 0, \tag{15}$$

where $L_E \cdot \alpha_E = L_K \cdot \alpha_K = M$. Equation (14) captures the evolution of entrepreneurs over time. Along the steady state, the number of entrepreneurs deciding to enter into the fair $(\sigma \left(\overline{L}_E - L_E\right) F\left(c_E^*\right))$ must equalize the number of entrepreneurs who have successfully matched with financiers and have thus exited from the fair $(L_E \cdot \alpha_E)$. An analogous interpretation can be given to (15).

A stationary equilibrium for this economy is defined as any 4-tuple (L_E, L_K, c_E^*, c_K^*) that solves the four equations (12), (13), (14) and (15). The next proposition formally links the number of stationary equilibria to the returns to scale of the matching function.

Proposition 1 *If the matching function (1) is homogeneous of degree 1, the economy admits one and only one stationary equilibrium.*

Proof. First pose $\Omega \equiv L_K/L_E$. Given that (1) has CRS, we can write $\alpha_E \equiv M/L_E = m(\Omega)$, and $\alpha_K \equiv M/L_K = (1/\Omega)\, m(\Omega)$. The entry conditions, (12) and (13), are then both functions of Ω only, the former increasing, the latter decreasing, that is, $c_E^*\left(\overset{+}{\Omega}\right)$ and $c_K^*\left(\overset{-}{\Omega}\right)$. By substituting these functions, respectively, into (14) and (15), we obtain

$$L_E = \frac{\sigma \overline{L}_E \cdot F\left(c_E^*\left(\Omega\right)\right)}{1 + \sigma F\left(c_E^*\left(\Omega\right)\right)} \tag{16}$$

and

$$L_K = \frac{\overline{L}_K \cdot G\left(c_K^*\left(\Omega\right)\right)}{1 + G\left(c_K^*\left(\Omega\right)\right)}. \tag{17}$$

Standard differential calculus proves that $L_E(\Omega)$ defined in (16) is monotone increasing in Ω, while $L_K(\Omega)$ defined in (17) is monotone decreasing in Ω. Hence, the function defined as their ratio, $L_K/L_E(\Omega)$, is unambiguously decreasing in Ω. Given that it is

$\Omega \equiv L_K/L_E$, a stationary equilibrium is a fixed point of this function. We now prove that this function admits one and only one fixed point. Define $g(\Omega) \equiv L_K/L_E(\Omega) - \Omega$. There exist sufficiently low values of Ω such that $g(\Omega) > 0$, as well as sufficiently high values of Ω such that $g(\Omega) < 0$.[29] Given that $g(\Omega)$ is a continuous and monotone decreasing function in Ω, the intermediate value theorem guarantees the existence of one and only one Ω^* such that $g(\Omega^*) = 0$, that is, such that $L_K/L_E(\Omega^*) = \Omega^*$. Finally, it might still be the case that multiple equilibria exist, even though they are all characterized by a unique ratio Ω^*. This instance, however, can be excluded by the fact that $L_E(\Omega)$ and $L_K(\Omega)$, defined in (16) and (17), are monotone functions of Ω. ■

How do we interpret equilibrium multiplicity? For the sake of illustration, suppose that our economy admits two (non-degenerate) equilibria, respectively denoted by superscripts O,P, with $(L_j)^O > (L_j)^P$ for $j = E, K$ (an example of this kind is developed at the end of this section). These two equilibria can be interpreted as self-fulfilling equilibria triggered, respectively, by optimistic or pessimistic expectations. Whenever entrepreneurs expect a high number of financiers to be matched with ($L_K^e = (L_K)^O$ where the superscript e stands for "expected"), their number will be high as well, $(L_E)^O$. Similarly, whenever financiers expect a high number of entrepreneurs ($L_E^e = (L_E)^O$), their number will also be high, $(L_K)^O$. Equilibrium O can be labelled as the optimistic (or *thick*) equilibrium. Via a symmetric argument, expecting few entrepreneurs and financiers makes the agents converge towards the low-entry equilibrium P, which can be referred to as the pessimistic (or *thin*) equilibrium.

Given that in our model only profitable projects are pursued, whenever multiple equilibria exist, they can be Pareto-ordered from the lowest to the highest number of entrepreneurial ventures (matches) produced by the economy. Welfare is thus highest at the equilibrium characterized by the highest number of matches: all other equilibria are sub-optimal and are the result of a coordination failure between entrepreneurs and financiers. In this respect, an entrepreneurial finance model admitting multiple

[29] The standard assumptions on the matching function imply that

$$\lim_{\Omega \to 0} \frac{L_K}{L_E}(\Omega) = +\infty$$

and

$$\lim_{\Omega \to +\infty} \frac{L_K}{L_E}(\Omega) = 0.$$

Even though they are not necessary, these two results ensure the existence of the two regions where $g(\Omega) > 0$ and $g(\Omega) < 0$.

equilibria is a model in which *animal spirits* matter, in the sense that, whether a high or a low activity equilibrium is reached may depend on a self-fulfilling mechanism triggered by entrepreneurs' and financiers' expectations. Usually, this line of argument provides a theoretical justification of policy intervention. We further discuss the policy implications of our model in the concluding remarks.

Example. Consider the model developed in the previous sections and further suppose that (i) the matching function is Cobb-Douglas with the following technological features: $M = AL_E^{\beta_E} L_K^{\beta_K}$ with $A \in R_+$, $\beta_E, \beta_K < 1$ and $\beta_E + \beta_K > 1$ (increasing returns to scale), and that (ii) entry costs are the same for every entrepreneur and every financier, $\underline{c_E}$ and $\underline{c_K}$.[30] This economy admits three stationary equilibria. The first (thin) equilibrium is given by the pair $\left((L_E)^P, (L_K)^P\right)$ that solves the following system (made up of the two optimal entry conditions, (12) and (13)):[31]

$$\left\{ \begin{array}{l} \underline{c_E} = \frac{AL_E^{(\beta_E-1)} L_K^{\beta_K} \theta\pi + \sigma \underline{c_E}}{r + AL_E^{(\beta_E-1)} L_K^{\beta_K} + \sigma} \\ \underline{c_K} = \frac{AL_E^{\beta_E} L_K^{(\beta_K-1)} (1-\theta)\pi}{r + AL_E^{\beta_E} L_K^{(\beta_K-1)}}. \end{array} \right. \tag{18}$$

The second (thick) equilibrium is instead given by the pair $\left((L_E)^O, (L_K)^O\right)$ that solves the system given by the two steady-state conditions:

$$\left\{ \begin{array}{c} \sigma\left(\overline{L}_E - L_E\right) = AL_E^{\beta_E} L_K^{\beta_K} \\ \overline{L}_K - L_K = AL_E^{\beta_E} L_K^{\beta_K}. \end{array} \right.$$

Finally, the third (degenerate) equilibrium is given by $\left((L_E)^T, (L_K)^T\right) = (0, 0)$, representing the most extreme form of coordination failure across market participants. It is possible to prove that this and the thick equilibrium are stable, while the thin equilibrium is unstable.[32]

[30] In some respects, this example resembles the one developed by Diamond (1982) in Section IX.

[31] Note that, under constant returns to scale (that is, when $\beta_E = 1 - \beta_K$), system (18) would become

$$\left\{ \begin{array}{l} \underline{c_E} = \frac{A\left(\frac{L_K}{L_E}\right)^{\beta_K} \theta\pi + \sigma \underline{c_E}}{r + A\left(\frac{L_K}{L_E}\right)^{\beta_K} + \sigma} \\ \underline{c_K} = \frac{A\left(\frac{L_K}{L_E}\right)^{\beta_K - 1} (1-\theta)\pi}{r + A\left(\frac{L_K}{L_E}\right)^{\beta_K - 1}}. \end{array} \right.$$

which is impossible, given that it contains two equations in only one unknown (the ratio L_K/L_E). Hence, under CRS, this equilibrium would disappear.

[32] The two best response functions in (18) are in fact strictly convex whenever the matching function

B Matching Function Models

We here discuss a few technical details on the three matching function models estimated.

CES Matching Function. Let us explicitly derive the parameter restrictions on (2) implied by the three hypotheses. (a) Parameter restrictions $A, v, \delta_j > 0$ (for $j = E, K$) ensure that Part (i) of (Hp 1) holds because

$$\frac{\partial M_{i,t}}{\partial \left(L_j\right)_{i,t}} = Av\delta_j L_j^{\Psi-1}(\delta_j \left(L_j\right)_{i,t}^{\Psi} + \delta_{-j} \left(L_{-j}\right)_{i,t}^{\Psi})^{\frac{v}{\Psi}-1} \exp(\boldsymbol{\beta}_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t})$$

is higher than zero when A, v, δ_j (for $j = E, K$) are positive; (b) part (ii) of (Hp 1) further requires $\Psi < 1$: in fact, after a few algebraic steps we can write

$$\begin{aligned}\frac{\partial^2 M_{i,t}}{\partial \left(L_j\right)_{i,t}^2} &= Av\delta_j L_j^{\Psi-2} \cdot (\delta_j \left(L_j\right)_{i,t}^{\Psi} + \delta_{-j} \left(L_{-j}\right)_{i,t}^{\Psi})^{\frac{v}{\Psi}-2} \left[(\Psi - 1) \cdot \delta_{-j} \left(L_{-j}\right)_{i,t}^{\Psi} + (v-1) \cdot \delta_j \left(L_j\right)_{i,t}^{\Psi}\right] \cdot \\ &\quad \cdot \exp(\boldsymbol{\beta}_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t})\end{aligned}$$

which is strictly negative whenever $\Psi < 1$. (c) (Hp 2) is verified when the cross-partial derivative of (2), which is given by

$$\begin{aligned}\frac{\partial^2 M_{i,t}}{\partial (L_j)_{i,t} \partial (L_{-j})_{i,t}} &= Av\left(v - \Psi\right) \delta_j \delta_{-j} \left(\left(L_j\right)_{i,t} \left(L_{-j}\right)_{i,t}\right)^{\Psi-1} (\delta_i \left(L_j\right)_{i,t}^{\Psi} + \delta_{-j} \left(L_{-j}\right)_{i,t}^{\Psi})^{\frac{v}{\Psi}-2} \cdot \\ &\quad \cdot \exp(\boldsymbol{\beta}_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t}),\end{aligned}$$

is strictly positive, which occurs whenever $\delta_E, \delta_K, v, A, (v - \Psi) > 0$; (d) (Hp 3) (constant returns to scale) requires $v = 1$ by construction. Finally, given that $v = 1$ and $(v - \Psi) > 0$ imply $\Psi < 1$, (Hp 2) and (Hp 3) together imply part (ii) of (Hp 1).

Cobb-Douglas and Translog Matching Functions. By definition, a Cobb-Douglas matching function assumes that the elasticity of output with respect to each input -that is β_j,for $j = E, K$- is constant. The translog matching is a generalization of the Cobb-Douglas function in which the output elasticity is, instead, allowed to vary with the values of both inputs. Note also that the three estimated models, (2), (4) and (5), are all closely related to each other. The log-linear Cobb-Douglas matching function (4) is clearly nested into the translog specification (5) (and thus obtainable

is characterized by increasing returns to scale. The algebraic proof of this statement, as well as a graphical intuition of the three equilibria, are available upon request from the authors. We omit them for brevity.

from the latter imposing the following restriction: $\beta_{EK} = \beta_{EE} = \beta_{KK} = 0$). On the other hand, the translog specification can be obtained from a second-order Taylor approximation of the logarithmic transformation of the CES specification (2).[33]

INSERT TABLE A1 HERE

[33] More precisely, when the elasticity of substitution is *in the neighborhood* of unity, a two-input log-CES function may be approximated by a Taylor expansion which has the form of (5) under the following restrictions: $\beta_{EK} = -2\beta_{EE} = -2\beta_{KK}$ (Kmenta, 1967).

Table 1. Summary Statistics.

	No. Deals		No. Angels		No. Projects		Obs.	Period
	mean	sd	mean	sd	mean	sd		
Austria	5	3	71	40	66	21	11	1997-2007
Belgium	36	10	161	135	222	79	8	2000-2007
Denmark	17	29	76	86	38	30	4	2000-2003
Finland	8	6	210	152	36	13	9	1996-1999, 2003-2007
France	205	69	2,504	1,004	827	265	4	2005-2008
Germany	36	8	580	174	2,309	889	6	2000-2005
Greece	1	1	11	3	8	4	5	2003-2007
Italy	174	146	262	32	857	625	8	2004-2006, 2010-2014
Netherlands	50	24	196	189	178	75	8	1996-1999, 2002-2003, 2006-2007
Norway	3	1	133	38	37	12	3	2001-2003
Poland	4	2	56	28	100	83	4	2004-2007
Portugal	1	-	18	6	5	1	4	2000, 2002, 2004-2005
Russia	3	1	88	46	35	7	2	2005-2006
Spain	16	10	273	197	280	181	8	2000-2007
Sweden	43	28	284	157	358	306	4	2003-2006
UK	245	100	4,959	465	558	282	4	2005-2006, 2008-2009
USA	56,586	11,205	257,802	36,919	363,341	96,498	13	2002-2014
Total	7,050	19,094	32,352	86,079	45,318	124,513	105	1997-2014

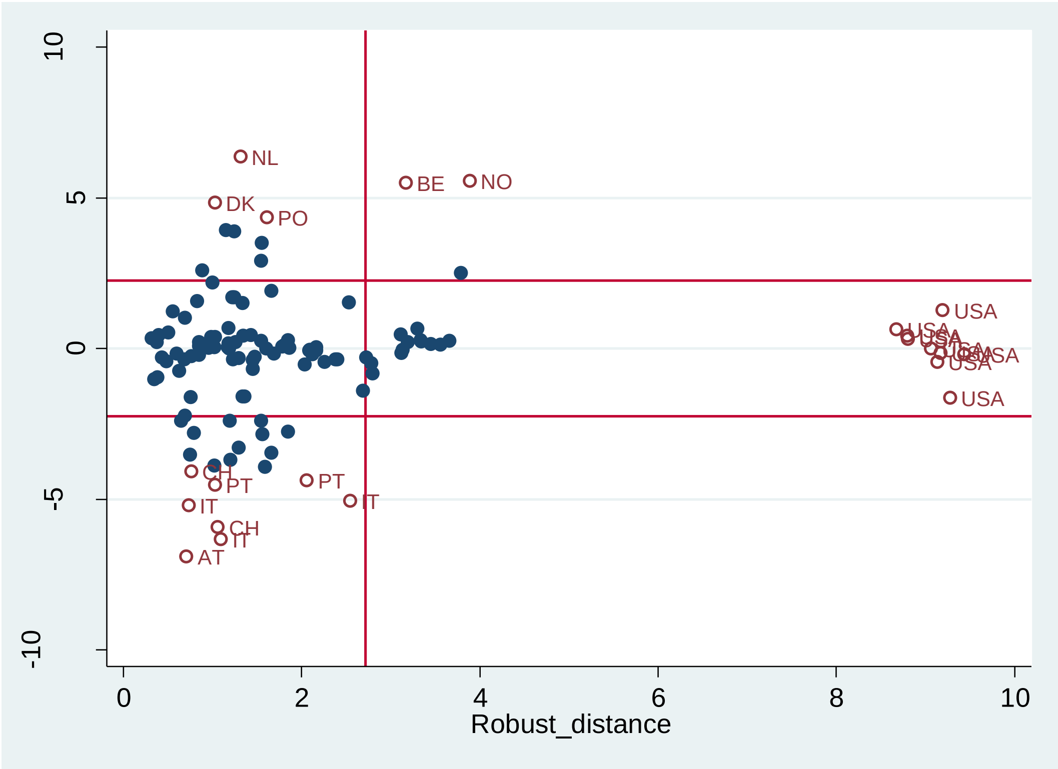
Table 2. Results from the nonlinear estimation of the log-CES matching function.

	(1)	(2)	(3)	(4)	(5)
logA	-3.5241***	-3.5375***	-2.2427***	-5.0414	-2.6440
	(0.4215)	(0.6246)	(0.8482)	(1.5066)	(1.5569)
δ_E	0.7296***	0.6205***	0.5615***	0.7738***	0.8879*
	(0.1527)	(0.1753)	(0.1820)	(0.1340)	(0.4170)
v	1.1263***	1.1723***	1.1407***	1.5249***	0.9450*
	(0.0612)	(0.1278)	(0.1294)	(0.4134)	(0.3590)
Ψ	0.3765	0.3332	-0.1658	-0.4123	-2.1804
	(0.8937)	(0.7561)	(0.7833)	(0.7686)	(5.8105)
Time trend	YES	YES	YES	YES	YES
Country group dummies	NO	YES	YES	NO	NO
Country group specific time trend	NO	NO	YES	NO	NO
Country dummies	NO	NO	NO	YES	YES
Country specific time trend	NO	NO	NO	NO	YES
N	105	105	105	105	105
R-squared	0.84	0.85	0.86	0.89	0.93
CRS p-value♦	0.1076	0.3925	0.4809	0.4062	0.3658
$\Psi \to 0$ p-value	0.6745	0.6604	0.8329	0.5931	0.7265

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

♦ The CRS p-value is an indicator of the strength of evidence against the null hypothesis of CRS. A p-value lower than the chosen significance level (α) means that the observed data is sufficiently inconsistent with the null hypothesis and that the null hypothesis may be rejected. Typically, using the standard $\alpha = 0.05$ cutoff, the null hypothesis is rejected when $p < .05$ and not rejected when $p > .05$.

Figure 1. Diagnostic plot of standardized robust residuals versus robust Mahalanobis distance of the vector of covariates from the vector of their means.



Note. The Mahalanobis distance of a multivariate vector $\mathbf{x}$ of $1 \times p$ dimension with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ is defined as: $D(x) = \sqrt{(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})}$, which follows a chi-squared distribution with p degree of freedom under normality. Observations lying at the right-hand side of the vertical limit (set at $\sqrt{X^2_{p,0.975}}$) are defined as *good leverage points* (for instance, the US). Their presence does not affect the OLS-estimation but it affects the statistical inference since they do deflate the estimated standard errors. Observations lying above or below the area delimited by the two horizontal limits (set at -2.25 and +2.25, respectively) are defined as *vertical outliers* and affect the estimated intercept of an OLS-estimation (Italy, Portugal, Denmark, Netherlands, Poland etc.). Observations lying both at the right-hand side of the vertical limit and outside the 95% confidence interval of the Standard Normal are considered *bad leverage points* (Belgium and Norway). Their presence significantly affects the OLS-estimates of both the intercept and the slope.

Table 3. Results from the robust estimations of Cobb-Douglas matching function specifications.

	(1)	(2)	(3)	(4)
β_K	0.3834***	0.3801**	0.5983***	0.4601***
	(0.0782)	(0.1511)	(0.1739)	(0.1585)
β_E	0.6414***	0.5950***	0.3726***	0.5408***
	(0.0695)	(0.0875)	(0.0886)	(0.0975)
Other controls	NO	NO	NO	YES
Time trend	YES	NO	YES	YES
Year dummies	NO	YES	NO	NO
Country dummies	NO	YES	YES	YES
Country specific time trend	NO	NO	YES	YES
N	105	105	105	105
R-squared	0.945	0.986	0.995	0.987
RTS	1.02	0.98	0.97	1.00
CRS p-value♦	0.4266	0.8426	0.8467	0.9942

*** p<0.01, ** p<0.05, * p<0.1

Table 4. Results from the robust estimations of Translog matching function specifications.

	(1)	(2)
β_K	0.5078***	0.5993***
	(0.1345)	(0.1355)
β_E	1.3356***	1.2052***
	(0.0553)	(0.0521)
β_{EK}	0.0612**	0.1699***
	(0.0266)	(0.0254)
β_{KK}	-0.0547***	-0.1143***
	(0.0184)	(0.0190)
β_{EE}	-0.0852***	-0.1296***
	(0.0118)	(0.0108)
$\beta_{\text{Credit by banks over GDP}}$		-0.0013
		(0.0020)
$\beta_{\text{Credit by financial sector over GDP}}$		0.0004
		(0.0021)
$\beta_{\text{Rate of GDP growth}}$		-0.0050
		(0.0071)
$\beta_{\text{VC investments over GDP}}$		-73.2509***
		(21.6392)
$\beta_{\text{Ln GDP per capita}}$		2.1061***
		(0.6738)
Time trend	YES	YES
Country dummies	YES	YES
Country specific time trend	YES	YES
N	105	105
R-squared	0.999	0.999
RTS	0.93	0.93
CRS p-value♦	0.1103	0.1208

*** p<0.01, ** p<0.05, * p<0.1

♦ The CRS p-value is an indicator of the strength of evidence against the null hypothesis of CRS. A p-value lower than the chosen significance level (α) means that the observed data is sufficiently inconsistent with the null hypothesis and that the null hypothesis may be rejected. Typically, using the standard $\alpha = 0.05$ cutoff, the null hypothesis is rejected when $p < .05$ and not rejected when $p > .05$.

Table 5. Results from the system-GMM estimations of the Cobb-Douglas and Translog matching function specifications.

	(1) (t-1)(t-2)	(2) (t-1)(t-2)	(3) (t-1)(t-2)	(4) (t-2)(t-3)	(5) (t-2)(t-3)	(6) (t-2)(t-3)
β_K	0.2452*	0.2310*	0.5331***	0.3269**	0.2926*	0.9695***
	(0.1387)	(0.1406)	(0.1915)	(0.1639)	(0.1540)	(0.0609)
β_E	0.9062***	0.9059***	1.9245***	0.8090***	0.8447***	1.7279***
	(0.0271)	(0.2574)	(0.1779)	(0.0554)	(0.2398)	(0.5042)
β_{EK}			0.1608***			0.0492**
			(0.0344)			(0.0215)
β_{KK}			-0.0818***			-0.0544***
			(0.0089)			(0.0070)
β_{EE}			-0.1953***			-0.1292*
			(0.0059)			(0.0709)
Other controls	NO	YES	YES	NO	YES	YES
N	105	105	105	105	105	105
RTS	1.15	1.14	1.10	1.14	1.14	1.12
CRS p-value	0.1751	0.5165	0.1383	0.2106	0.5005	0.1089
Sargan Test p-value	0.316	0.125	0.379	0.173	0.105	0.877
A-B test for AR(1) p-value	0.170	0.160	0.157	0.171	0.172	0.167
A-B test for AR(2) p-value	0.268	0.282	0.369	0.265	0.274	0.314

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Note: Clustered standard errors. The set of additional controls include: the rate of GDP growth (The World Bank database), the value of venture capital investments over GDP (EVCA, NVCA databases), domestic credit to private sector provided by banks over GDP (The World Bank database), domestic credit to private sector provided by financial sector over GDP (The World Bank database), the log value of GDP per capita (The World Bank database).
For estimations (1)-(2)-(3), the set of instruments includes the lagged levels of independent variables dated $(t-1)$ and $(t-2)$ and the lagged value of the dependent variable dated $(t-2)$ in the first-differenced equations and the corresponding lagged first-differences dated $(t-1)$ plus year dummies in the levels equations.
For estimations (4)-(5)-(6), the set of instruments includes the lagged levels of independent variables and the lagged value of the dependent variable dated $(t-2)$ and $(t-3)$ in the first-differenced equations and the corresponding lagged first-differences dated $(t-1)$ plus year dummies in the levels equations.
The CRS p-value is an indicator of the strength of evidence against the null hypothesis of CRS. A p-value lower than the chosen significance level (α) means that the observed data is sufficiently inconsistent with the null hypothesis and that the null hypothesis may be rejected. Typically, using the standard $\alpha = 0.05$ cutoff, the null hypothesis is rejected when $p < .05$ and not rejected when $p > .05$.

Table 6. Synthesis of input elasticities under different specifications.

		Robust Regressions	System-GMM	
			Predetermined explanatory variables	Endogenous explanatory variables
Cobb-Douglas	ε_K	0.46	0.23	0.29
	ε_E	0.54	0.91	0.84
Translog	ε_K	0.23	0.49	0.61
	ε_E	0.70	0.61	0.51

Note: In the case of the Cobb-Douglas matching function, $\varepsilon_K = \beta_K$ and $\varepsilon_E = \beta_E$.

Table A1. Results from the system-GMM estimations of the Cobb-Douglas and Translog matching function specifications: backward orthogonal deviations.

	(1) (t-1)(t-2)	(2) (t-1)(t-2)	(3) (t-1)(t-2)	(4) (t-2)(t-3)	(5) (t-2)(t-3)	(6) (t-2)(t-3)
β_K	0.3322*	0.2557*	1.1191***	0.3595*	0.2548*	1.1469***
	(0.1853)	(0.1371)	(0.0834)	(0.1876)	(0.1366)	(0.2571)
β_E	0.7928***	0.8849***	1.4605***	0.7631***	0.8854***	1.4545***
	(0.1651)	(0.2479)	(0.0803)	(0.1622)	(0.2476)	(0.1405)
β_{EK}			0.0273***			0.0431***
			(0.0037)			(0.0204)
β_{KK}			-0.0520***			-0.0613****
			(0.0101)			(0.0114)
β_{EE}			-0.1071***			-0.1153***
			(0.0170)			(0.0236)
Other controls	NO	YES	YES	NO	YES	YES
N	105	105	105	105	105	105
RTS	1.13	1.14	1.03	1.11	1.14	1.04
CRS p-value	0.1486	0.4840	0.6566	0.1472	0.4854	0.6380
Sargan Test p-value	0.113	0.121	0.383	0.112	0.107	0.178
A-B test for AR(2) p-value	0.266	0.280	0.377	0.267	0.280	0.376

*** p<0.01, ** p<0.05, * p<0.1

Note: Clustered standard errors. The set of additional controls include: the rate of GDP growth (The World Bank database), the value of venture capital investments over GDP (EVCA, NVCA databases), domestic credit to private sector provided by banks over GDP (The World Bank database), domestic credit to private sector provided by financial sector over GDP (The World Bank database), the log value of GDP per capita (The World Bank database).
For estimations (1)-(2)-(3), the set of instruments includes the lagged levels of independent variables dated (t − 1) and (t − 2) and the lagged value of the dependent variable dated (t − 2) in the first-differenced equations and the corresponding lagged first-differences dated (t − 1) plus year dummies in the levels equations.
For estimations (4)-(5)-(6), the set of instruments includes the lagged levels of independent variables and the lagged value of the dependent variable dated (t − 2) and (t − 3) in the first-differenced equations and the corresponding lagged first-differences dated (t − 1) plus year dummies in the levels equations.
The CRS p-value is an indicator of the strength of evidence against the null hypothesis of CRS. A p-value lower than the chosen significance level (α) means that the observed data is sufficiently inconsistent with the null hypothesis and that the null hypothesis may be rejected. Typically, using the standard $\alpha = 0.05$ cutoff, the null hypothesis is rejected when $p < .05$ and not rejected when $p > .05$.