Abstract
We study economies where price stickiness arises due to the simultaneous presence of both menu and information costs. We identify the relative importance of these costs using firm’s survey data and analyze the response of prices and output following a permanent unexpected monetary shock. For a given frequency of price adjustment, we find that the information friction significantly amplifies the real effect of the shock when the shock is small, or when it is not known by firms. Instead, when the shock is large and known to firms the flexibility of prices increases and the real effects gradually vanish. (JEL: E23, E31, E52)

1. Introduction
Information frictions, such as costly gathering and processing of data, feature as a prominent explanation for the price stickiness that lies at the core of many macroeconomic models. Indeed, firms’ survey data in Fabiani et al. (2007) and lab experiments in Magnani, Gorry, and Oprea (2016) provide some direct evidence on the relevance of infrequent information processing. However, other sources of evidence...
suggest that such information frictions cannot fully account for price setting behavior. First, most sticky information models predict continuous price adjustments in the absence of other frictions, a prediction that contrasts with the evidence on infrequent price changes. Second, recent analyses of the cross-sectional size of price changes show that, once measurement error is taken into account, there are extremely few price changes of a small size, a prediction that stands in contrast with models featuring only cognitive frictions.¹

A possible resolution of the price setting patterns described above is to consider models that feature both an information cost as well as a physical menu cost for price changes. A few contributions have provided formal analyses of optimal decisions in settings where both frictions are present (for price setting see Bonomo, Carvalho, and Garcia 2010; Alvarez, Lippi, and Paciello 2011, in finance see Abel, Eberly, and Panageas 2007, 2013; Alvarez, Guiso, and Lippi 2012). These models are useful to analyze the steady state moments of an economy, such as the timing and size of price changes, but no characterization has been provided of how the simultaneous presence of the two frictions affects the propagation of an aggregate shock. This paper advances this line of research by solving for the general equilibrium and providing a full characterization of the propagation of an aggregate shock in an economy where firms face both an information friction, and are thus subject to spells of “inattentiveness”, as well as a physical menu cost.

Our contribution is methodological and has substantive economic implications. From a methodological point of view the key difficulty is that the problem has a two-dimensional state space: firms’ decisions concerning whether to observe (gather and process information) and/or whether to adjust prices, depend on the firm’s beliefs about their own profits, or markup, as well as on the uncertainty that surrounds such beliefs. We provide a rigorous characterization of this decision problem, characterize the steady state of an aggregate economy populated by such firms and analyze the propagation of a once and for all aggregate monetary shock.

Another contribution is to show that information frictions make monetary policy shocks more powerful. In particular, we show that for a given degree of aggregate price stickiness (i.e., a given frequency of price changes), an economy with a larger observation cost and smaller menu cost will feature a larger output effect in response to an unexpected monetary shock. Because of this differential effect of a monetary shock, we study the identification of the relative importance of menu and observation cost—while keeping constant the degree of price stickiness. To identify the relative importance of the two costs we use the implications of the steady state decision rules for the relative frequency of price reviews, an empirical proxy of the infrequent information gathering activity measured using firm’s survey data, and the frequency of price adjustments. Indeed in Alvarez et al. (2011) we establish the existence of a one-to-one mapping between the ratio of frequencies of reviews and adjustments and the ratio of the menu and observation costs for a stylized version of this model.

¹. On the lack of small price changes see the discussion in Section 3 of Cavallo (2016).
The main contribution of the paper is to illustrate the response of aggregate output to a monetary shock under alternative assumptions about two dimensions of the policy experiment: the (i) size of the monetary shock and (ii) whether the aggregate shock $\delta$ is known to the firms. We summarize the response of the economy by the cumulative impulse response of output to a once and for all monetary shock of size $\delta$, which we denote by $M(\delta)$. We show that if firms do not know the shock at the time it occurs, then the behavior of the model is the one of a pure time-dependent model with respect to the size of the shock. In particular, when the firms are unaware of the monetary shock (until their first observation occurs) then $M(\delta)$ is proportional to the size of the shock. The reason for this behavior is that firms will only consider adjusting their prices at the times when an observation was planned.$^2$ We contrast this result with the one that occurs if the monetary shock is known by firms as soon as it occurs, in which case $M(\delta)$ is hump shaped as a function of the size of the shock. This behavior is reminiscent of pure state-dependent models where, as the shock gets larger, more firms choose to pay the menu cost and adjust, and thus there is increasing flexibility. Our view, which we further elaborate in the conclusions, is that the most reasonable assumption is that monetary shocks are known immediately, especially when they are large. Thus, under our preferred assumption there is a cap to the power of monetary policy, as in traditional state-dependent models.

Our paper relates to a rich strand of literature that studies the role of information frictions in the propagation of aggregate shocks.$^3$ In our framework firms pay attention to the state only infrequently and, when they do, they receive a perfect signal on the relevant state of the price setting decision. Our approach differs from the rational inattention literature that followed Sims (2003), where agents can process a flow of new information every period, as we aim to obtain infrequent price reviews and relate them to infrequent price changes. Our technology of information acquisition does not allow firms to allocate their attention across different types of shocks that impact their price setting decision, as in Mackowiak and Wiederholt (2009). We also abstract from strategic interactions in the price setting and information acquisition decisions that, as in Hellwig and Veldkamp (2009), can create a wedge between the precision of available information about the nominal shock and the degree of price inertia.

Our paper also relates to a growing literature studying the propagation of aggregate shocks in economies that feature both sticky prices and information frictions. Several authors combine nominal rigidities with informational frictions (generally different from the specific one we consider here). Klenow and Willis (2007) allow for exogenously different frequencies of review of idiosyncratic and

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$^2$ Mackowiak and Wiederholt (2009) have argued that, when information about different types of shocks is equally available and costly, it is efficient for firms to pay less attention to aggregate shocks than to idiosyncratic shocks. This view suggests that it may be more appropriate to assume that shocks are not freely learned by agents when they occur. However, we find it interesting to also consider the case of a shock that is known to agents since, especially for large shocks, it seems reasonable that the news will be available to agents at a relatively lower cost. We return to this discussion in the conclusions.

$^3$ Early contributions include Phelps (1969), Lucas (1972), Barro (1976), and Townsend (1983).

The rest of the paper is organized as follows. The next section gathers evidence pointing to the presence of both menu and information cost. In Section 3 we describe the model, and characterize the firm’s optimal decision rules as well as the model in a general equilibrium. Section 4 discusses the choice of parameters and the calibration to the U.S. economy. Section 5 computes the output response to a monetary shock using the calibrated model economy. Section 6 briefly reviews the scope and robustness of the results and some avenues for future research.

2. Evidence on Menu and Information Costs

This section gathers three different kinds of evidence pointing to the presence of both menu and information cost. First we summarize evidence from survey data to document that firms review and adjust their price infrequently. Second, we gather evidence from scraped price data to argue that there are almost no small price changes. Third, we gather evidence from lab experiments on the presence of cognitive information costs.

2.1. Survey Evidence on Price Reviews and Price Adjustments

A price review is an activity related to the firm’s information gathering and processing that is necessary to evaluate the current price policy. The surveys have been conducted in several developed economies in an effort to gather new data on the firms’ pricing behavior.5 As observation and menu costs are among the most prominent microfoundations of price rigidity, these surveys explicitly aim to measure the relevance of these two frictions. In particular, the surveys elicit information on the frequency of price adjustments and the frequency of observation separately, as menu cost and observation cost imply both infrequent price adjustment and infrequent information acquisition. The typical survey question asks firms: “In general, how often do you review the price of your main product (without necessarily changing it)?”; with possible choices being yearly, semi-yearly, quarterly, monthly, weekly, and daily. The same surveys contain a separate question on the frequency of price changes.

4. A closely related paper in this literature is Demery (2012) who, building on the results of Alvarez et al. (2011), studies how the real effects of monetary shocks depend on the relative size of observation and menu costs, as we do here. We argue that the results of Demery differ from ours because of a flaw in his solution method. See Online Appendix D for a detailed explanation of our claim.

TABLE 1. Number of price-reviews and price-adjustments per year.

<table>
<thead>
<tr>
<th></th>
<th>AT</th>
<th>BE</th>
<th>FR</th>
<th>GE</th>
<th>IT</th>
<th>NL</th>
<th>PT</th>
<th>SP</th>
<th>EURO</th>
<th>CAN</th>
<th>U.K.</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Review</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2.7</td>
<td>12</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Adjust</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Medians

Percentage of firms with at least four price reviews or adjustments

<table>
<thead>
<tr>
<th></th>
<th>Review</th>
<th>Adjust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Review</td>
<td>54</td>
<td>11</td>
</tr>
<tr>
<td>Adjust</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

Note: Medians are computed over number of price adjustments and reviews per year. Sources: Fabiani et al. (2007) for the Euro area; Amirault et al. (2006) for Canada; Greenslade and Parker (2008) for the United Kingdom; Blinder et al. (1998) for the United States.

The robust finding of these surveys is that the frequency of price reviews is larger than the frequency of price adjustments. In Section 4 we will use these data to calibrate our model. Notice that to in order to map the frequency of observation and adjustment into their respective (observation and adjustment) costs requires a model: in a model where both frictions are present a low frequency of observation, which one would intuitively map into a high observation cost, might be caused by a high menu cost that reduces the expected benefit of observing.6

The upper panel of Table 1 reports the median yearly frequencies of price reviews and adjustments across all firms in surveys taken from various countries. The median firm in the Euro area reviews its price a bit less than three times a year, but changes its price only about once a year. The United Kingdom, the United States, and Canada have higher frequency of price changes than the Euro area, but also higher frequency of reviews, so that on average firms review more frequently than they adjust their price.

We notice that the comparison between the median frequencies of adjustments and reviews may be subject to measurement error because firms are often asked to choose the frequency of reviews among a discrete set of alternatives (e.g., daily, weekly, etc.), whereas they are asked to report a number with no restriction for the frequency of price adjustments.7 As a robustness check, the bottom panel of Table 1 reports the fraction of firms reviewing the price, and the fraction of firms changing the price, at least four times a year. It shows that the mass of firms reviewing prices at least four times a year is substantially larger than the corresponding one for price changes across all countries.

Next, we document that the frequency of price reviews is consistently higher than the frequency of price adjustments also at the firm level.8 Using firm level data, Table 2 classifies the answers of each firm in a sample of four countries in three mutually exclusive categories: (1) firms for which the frequency of price changes is

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6. See Alvarez et al. (2011) for a detailed discussion of this point.
7. See Online Appendix A for a discussion on the measurement error in the survey data.
8. Figure 6 in Alvarez et al. (2011) documents this fact across a number of industries (two digits NACE classification) in six OECD countries.
TABLE 2. Relative frequency of price changes and price reviews (firm level data).

<table>
<thead>
<tr>
<th>Percentage of Firms with:</th>
<th>Belgium</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Spaina</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Change &gt; Review</td>
<td>3</td>
<td>5</td>
<td>19</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>(2) Change = Review</td>
<td>80</td>
<td>38</td>
<td>11</td>
<td>38</td>
<td>89</td>
</tr>
<tr>
<td>(3) Change &lt; Review</td>
<td>17</td>
<td>57</td>
<td>70</td>
<td>46</td>
<td>11</td>
</tr>
<tr>
<td>No. firms</td>
<td>890</td>
<td>1,126</td>
<td>835</td>
<td>141</td>
<td>194</td>
</tr>
</tbody>
</table>

Note: Each column reports the percentage of firm-level records for which the frequency of price changes is greater, equal or smaller, than the frequency of price reviews. aFor Spain we only report statistics for firms that review four or more times a year. Sources: Table 17 in Aucremanne and Druant (2005) for Belgium, and our calculations based on the individual firm data described in Loupias and Ricart (2004), Stahl (2009), and Fabiani, Gattulli, and Sabbatini (2004) for France, Germany, and Italy, respectively; Section 4.4 of Alvarez and Hernando (2005) for Spain.

greater than the frequency of price reviews; (2) firms for which the two frequencies are equal, and (3) firms that change prices less frequently than they review them. The table shows that, for the large majority of firms in the sample, the frequency of price reviews is greater than the frequency of price adjustment.

This survey evidence indicates that firms review the level of their prices more often than they adjust them. This information is useful for modeling purposes: the data display very little presence of price changes in the absence of a review of information, a behavior the literature refers to as “price plans” or “indexation”.

2.2. Scraped Price Data

Next we report evidence showing that there is a very small fraction of small price changes. The source for this evidence is the Billion Price Project (BPP) dataset by Cavallo (2016). A unique feature of these data, which are made of actual retail price quotes scraped from online sellers (from several goods and countries), is that they are essentially free of the measurement error that is common in both CPI and scanner data, due to unit-value indices, composition effects, and time-aggregation biases.9 Section 3 in Cavallo (2016) documents that using scraped price data the size distribution of price changes features a very small mass of small price changes. This finding stands in stark contrast with the picture that emerges from traditional datasets where small price changes appear prominently. Figure 1 in Cavallo shows that this is largely the result of a time aggregation bias, by comparing scanner and scraped date for the same retailer, location, and time period. This finding is germane for our paper because the lack of small price change suggests the presence of a fixed cost of changing prices, that is,

9. See Eichenbaum et al. (2014b) for a discussion of the importance of measurement error in CPI data.
FIGURE 1. Response of log $c_t$ to a 1% monetary shock. All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, $n_a = 1.4$, and to the mean absolute size of price changes, $e_{\Delta p} = 0.085$.

It is evidence in favor of strictly positive menu costs. In other words, the “hole” in the middle of the distribution of the size of price changes that appears in Figure 1 by Cavallo is inconsistent with pure observation cost models that predict a normal-shaped distribution of the size of price changes, with a density that peaks at near-zero price changes. Instead, this figure is consistent with a model as Alvarez et al. (2011) in which firms face both an observation and a menu cost and optimally avoid implementing a tiny price change.

2.3. Experimental Evidence

Magnani et al. (2016) provide another piece of evidence that documents the importance of information frictions. They conduct a lab experiment and produce evidence on the presence of cognitive observation costs. The subjects participate in an experiment that mimics the one of a profit maximizing monopolist subject to a menu cost. The subjects
must decide, at any point in time, whether to adjust prices or not, and are paid in proportion to the expected discounted profit of a monopolistic competitive firm that needs to set prices subject to a fixed menu cost. The mathematical solution of the problem faced by the subjects is to follow a simple symmetric sS rule. Yet the subjects’ actual choices for adjustment in the lab follow a pattern that is different from an sS rule: they do not always adjust when the difference between the actual price and the ideal price hits a constant critical value. Rather, the patterns of price adjustment found in the lab are remarkably close to the ones produced by the optimal decision rule of a problem with a menu cost and a strictly positive observation cost. In particular, the distribution of the size of adjustments produced by the experiment is bimodal, with a hole in the middle and with long tails, consistent with the theoretical results in Alvarez et al. (2011). The authors interpret these results as evidence of cognitive observation costs.

3. The Model

This section describes the model, the general equilibrium and the monetary shock. We consider firms that set prices under two frictions: a standard fixed cost of adjusting the price, inducing infrequent price adjustments, and a fixed cost of observing the state, inducing infrequent information acquisition. In the model each firm plans two related choices: observing the state and adjusting the price. Our model is a general equilibrium version of the price setting problem studied in Alvarez et al. (2011), and embeds as special cases the “menu cost” model (e.g., Barro 1972; Dixit 1991) as well as the “observation cost” model (e.g., Caballero 1989; Bonomo and Carvalho 2004; Reis 2006). The menu cost model aggregates similarly to Golosov and Lucas (2007) and provides a useful benchmark of comparison since the predictions of this model have been extensively studied in the literature. The observation cost model is a general equilibrium version of Reis’s (2006) inattentive producers model that, with a constant fixed cost of observing the state, features reviews at approximately uniformly distributed times, and therefore behaves similarly to Taylor’s (1980) staggered price model. We consider an economy where money grows at the constant rate \( \mu \), and study the output effects of a one time unexpected permanent increase in money supply produced by models with different combinations of observation and menu costs, including the two special cases where one of the costs is zero.

There are two types of agents in this economy, a representative household and a unit mass of monopolistically competitive firms, each producing a different variety of consumption good. Firm \( i \)'s output at time \( t \) is given by \( Y_{i,t} = z_{i,t} l_{i,t} \), where \( l_{i,t} \) is the

10. In this experiment subjects are paid in proportion to a function that replicates the discounted value of profits for a monopolist competitive firm. Subjects observe the realization of a random walk with no drift, which represents the ideal profit maximizing price, as well as the current flow of the firm’s profits. At every moment subjects have to decide whether to move their current price toward the ideal one, which entails paying a fixed menu cost.
labor employed by firm \( i \) in production, and \( z_{i,t} \) is the firm’s idiosyncratic productivity that evolves according to

\[
d \log(z_{i,t}) = \gamma \, dt + \sigma \, dB_{i,t},
\]  

(1)

where \( B_{i,t} \) is a standard Brownian motion with zero drift and unit variance, the realizations of which are independent across firms.

3.1. The Household Problem

We assume that the (real) aggregate consumption \( c_t \) is given by the Spence–Dixit–Stiglitz consumption aggregate

\[
c_t = \left[ \int_0^1 \left( A_{i,t} \, C_{i,t} \right)^{(\eta-1)/\eta} \, \frac{d\delta}{\eta} \right]^{\eta/(\eta-1)} \quad \text{with} \quad \eta > 1,
\]  

(2)

where \( C_{i,t} \) denotes the consumption of variety \( i \) at time \( t \). There is a preference shock \( A_{i,t} \) associated to good \( i \) at time \( t \), which acts as a multiplicative shifter of the demand for good \( i \). We assume that \( A_{i,t} = 1/z_{i,t} \), so the (log) of the marginal cost and the demand shock are perfectly correlated.\(^{11}\)

Household’s preferences over time are given by

\[
\int_0^\infty e^{-\rho t} \left[ \frac{c_t^{1-\varepsilon}}{1-\varepsilon} - \xi \, L_t + \log \left( \frac{\hat{m}_t}{P_t} \right) \right] \, dt \quad \text{with} \quad \rho > 0,
\]  

(3)

where period \( t \) utility depends on consumption, \( c_t \), labor supply, \( L_t \), and cash holdings \( \hat{m}_t \) deflated by the price index \( P_t = \left[ \int_0^1 \left( A_{i,t}^{-1} \, p_{i,t} \right)^{(1-\eta)} \, d\delta \right]^{1/(1-\eta)} \). The household has perfect foresight on the path of money, nominal wages, nominal interest rates, nominal lump-sum subsidies, and aggregate nominal profits. Financial markets are complete, in the sense that all profits of firms are held in a diversified mutual fund. Since all aggregate quantities are deterministic, the budget constraint of the representative agent is

\[
\hat{m}_0 \geq \int_0^\infty Q_t \left[ \int_0^1 p_{i,t} \, C_{i,t} \, d\delta + R_t \hat{m}_t - \mu m_t - W_t L_t - D_t \right] \, dt,
\]  

(4)

\(^{11}\) We introduce this assumption for several reasons. First, the cross-section distribution of output will be stationary, and the maximum static profit of the firms is constant across the different productivity levels. Second, this version of the model with correlated demand and cost shocks has been analyzed in the literature by several authors (see Woodford 2009; Bonomo et al. 2010; Midrigan 2011; Alvarez and Lippi 2014), so it makes the results for our benchmark case comparable to the existing literature. Nevertheless in the Online Appendix G we solve the model without preference shocks, that is, \( A_{i,t} = 1 \) for all \( i \) and \( t \), and conclude that the assumption on preference shocks is irrelevant for the quantitative predictions of our benchmark economy.
where \( Q_t = \exp \left( - \int_0^t R_s \, ds \right) \) is the time zero price of a dollar delivered at time \( t \), \( R_t \) is the instantaneous risk-free net nominal interest rate (and hence the opportunity cost of holding money), \( m_t \) is the stock of money supply, \( W_t \) is the nominal wage, and \( D_t \) is aggregate nominal net profits rebated from all firms to households. The household chooses the buying strategy, \( C_{i,t} \), labor supply, \( L_t \), and money-holding, \( m_t \), so to maximize equation (3), subject to equation (4), and taking prices \( Q_t, P_t, R_t, W_t \), and initial money holdings, \( m_0 \), as given.

Using the equilibrium condition in the money market, \( \dot{m}_t = m_t D_t \), the first order condition for money holdings reads

\[
e^{-\rho t}/m_t = \zeta \, Q_t \, R_t,
\]

where \( \zeta \) is the Lagrange multiplier of equation (4). The first order conditions for consumption and labor supply are given by

\[
e^{-\rho t} \, c_{i,t}^{1/\eta - \epsilon} \, C_{i,t}^{-1/\eta} \, z_{i,t}^{-1+1/\eta} = \zeta \, Q_t \, P_{i,t},
\]

\[
e^{-\rho t} \xi = \zeta \, Q_t \, W_t.
\]

Taking logs and differentiating w.r.t. time equation (5), one obtains the following o.d.e., \( \dot{R}_t = R_t (R_t - \mu - \rho) \), which has two steady states, zero and \( \rho + \mu > 0 \). The steady state \( \rho + \mu \) is unstable: if \( 0 < R(0) < \rho + \mu \) then it converges to zero, and if \( R(0) > \rho + \mu \) it diverges to \(+\infty\). Thus, there exists an equilibrium where regardless of the sequence of firm prices, \( p_{i,t} \), we have

\[
R_t = R = \rho + \mu, \quad W_t = \xi \, R \, m_t, \quad \text{and} \quad \zeta = \frac{1}{m_0 R}.
\]

An important property of the equilibrium conditions in equation (8) is that the equilibrium wage is proportional to the money supply whose dynamics are exogenous. Notice that this result occurs since there are no strategic complementarities in price setting of the type emphasized by Woodford (2001). The profit-maximizing price of a firm will be independent of the other firms’ actions because the nominal marginal cost will be independent of the aggregate price level. Although allowing for these strategic complementarities may affect the propagation of monetary shocks, and likely amplify the size of their real effects, this property simplifies the solution of the model substantially.

3.2. The Firm Problem

In this section we analyze the price setting problem of the firm. We first define the firm profits, then we describe the information structure and the price adjustment technology, and finally present the dynamic programming problem of the firm.
The firm’s per period nominal profit, scaled by the economy money supply $m_t$, is

$$\Pi_{i,t} = c_t^{1-\eta} z_{i,t}^{1-\eta} R \left( \frac{p_{i,t}}{R_m} \right)^{-\eta} \left( \frac{p_{i,t}}{R_m} - \xi \right),$$  

(9)

where we used equations (6) and (7) to obtain an expression for the firms’ demand $C_{i,t}$, and the equilibrium condition in equation (8) to obtain an expression for the nominal wage. The price that maximizes the firm’s profit in equation (9) is given by

$$p_{i,t}^* \equiv \frac{\eta R_m}{\eta - 1} \frac{1}{z_{i,t}} \xi.$$  

(10)

Next we substitute $p_{i,t}^*$ into equation (9) to express the firm’s profit as

$$\Pi \left( \frac{p_{i,t}}{p_{i,t}^*}, c_t \right) = c_t^{1-\eta} F \left( \frac{p_{i,t}}{p_{i,t}^*} \right),$$  

(11)

where the period profit only depends on two state variables, and the function $F(\cdot)$ is

$$F(x) \equiv R \xi^{1-\eta} \left( x \frac{\eta}{\eta - 1} \right)^{-\eta} \left( x \frac{\eta}{\eta - 1} - 1 \right).$$

The firm profit only depends on the ratio $p_{i,t}/p_{i,t}^*$ and on the aggregate consumption level. We will refer to (the log of) the ratio $p_{i,t}/p_{i,t}^*$ as to the “price gap”, and denote it by $g_{i,t} \equiv \log(p_{i,t}/p_{i,t}^*)$, so that $g = 0$ is the gap that maximizes the period profits. It follows from the definition of $g$, and from the laws of motion of $W$ and $z$, that the dynamics of $g$ for any firm $i$, when firm $i$ is not adjusting the price, are given by

$$dg_{i,t} = (\gamma - \mu) dt + \sigma dB_{i,t}.$$  

(12)

We notice that the function $F(x)$ has a unique maximum at $x = 1$, so that $\Pi^*(c_t) \equiv \Pi(1, c_t)$ is the maximum profit per period. The maximum profit $\Pi^*(c_t)$ is independent of the firm’s idiosyncratic state $z_t$, but varies with the aggregate consumption. This is because, at the profit-maximizing price, the idiosyncratic demand shock faced by each firm exactly offsets the effect on profit of an idiosyncratic shock to productivity. Finally, we denote by $\bar{\Pi}$ the maximum profit evaluated at the steady state consumption $\bar{c}$, that is, $\bar{\Pi} \equiv \Pi^*(\bar{c})$.

3.2.1. The Costs of Price Adjustment and Price Reviews. Each firm faces two frictions. First, we assume that paying attention to economic variables that are relevant for the price setting decision is costly. Second, the firm has to pay a menu cost anytime it changes its price. We model this framework along the lines of Alvarez et al. (2011). In particular, we assume that firms do not observe their productivity $z_{i,t}$, or other variables informative about the firms’ relevant state, unless they decide to undertake a costly action, which we refer to as a review. After paying the observation cost the firm...
learns perfectly the current value of $z$. Firms have no information on the realizations of idiosyncratic productivity shocks until the next review. A price review requires a fixed amount of labor. Given that the cost of labor scaled by the money supply is a constant, we can express the value of the observation cost as a fraction of the steady state profit: $\theta \overline{\Pi}$, where $\theta > 0$ is a parameter. Similarly to the observation cost, each price change requires a fixed amount of labor. We express the value of this cost as a fraction of steady state frictionless profits: $\psi \overline{\Pi}$, where $\psi > 0$ is a parameter.

3.2.2. The Firm Recursive Problem. Under our assumptions no new information arrives between review dates. In principle, even absent new information, the firm could implement some price changes between two review dates, for example, to keep track of predictable changes in the price gaps, such as those due to its drift. We showed in Alvarez et al. (2011) that as long as the drift is “small” relative to its variance, the firms will find it optimal to adjust their price only upon observation of the state, so that “price plans” will not be implemented. Notice that this assumption is consistent with the empirical evidence on the average frequency of price reviews and adjustments, discussed in the previous section. Thus, to ease notation, we set up the firms’ problem so that no price adjustment occurs between review dates.

Let $\{\tau_{i,n}\}$ denote the dates where the subsequent reviews will take place. The subindex $n$ denotes the $n$th review date, whereas the subindex $i$ denotes a firm. These stopping times satisfy $\tau_{i,n-1} \leq \tau_{i,n} \leq \tau_{i,n+1}$. Thus, upon reviewing the state in period $t$, the value of a firm $i$ with price gap $g$ is given by $V_t(g) = \max \{\tilde{V}_t, \tilde{V}_t(g)\}$, where $\tilde{V}_t$ is the value of the firm conditional on adjusting the price,

$$
\tilde{V}_t = -(\theta + \psi) \overline{\Pi} + \max_{T,\hat{g}} \int_0^T e^{-r s} E \left[ \Pi(e^{g_{t+\hat{g}}}, c_{t+\hat{g}}) \mid g_{i,t} = \hat{g} \right] ds
$$

$$+ e^{-r T} E \left[ V_{t+T}(g_{i,t+T}) \mid g_{i,t} = \hat{g} \right],
$$

and $\tilde{V}_t(g)$ is the value conditional on $g$ and not adjusting the price,

$$
\tilde{V}_t(g) = -\theta \overline{\Pi} + \max_T \int_0^T e^{-r s} E \left[ \Pi(e^{g_{t+\hat{g}}}, c_{t+\hat{g}}) \mid g_{i,t} = g \right] ds
$$

$$+ e^{-r T} E \left[ V_{t+T}(g_{i,t+T}) \mid g_{i,t} = g \right],
$$

where $r$ is the equilibrium real discount rate that, from the household first order conditions, is equal to $\rho$. The firm value depends on the expected discounted sum of

12. See Section III of that paper. We also showed that for a wide range of parameters that are consistent with low inflation economies such as those discussed in Section 2, price plans or indexation would not be optimal. See Online Appendix C for a mapping from the model of this paper to the framework of Alvarez et al. (2011).
firm’s profits that, from equation (11), depend on the path of the price gap \( g_{t:t+s} \) and on the path of aggregate consumption \( \{c_{t+s}\} \). We notice that the value function \( V_t(g) \) depends on time \( t \) only because of the effect of \( \{c_{t+s}\} \): given perfect foresight, the current time \( t \) is enough to infer the future dynamics of the aggregate state. In steady state, that is, when \( c_t = \bar{c} \), the solution of the stationary value function characterizes completely the firm problem.\(^{13}\)

In the next proposition we use the process for the profit in equation (11) and the price gap \( g_{t:t} \) in equation (12) to express the firm’s problem as a function of the structural parameters. The homogeneity of the value functions with respect to the level of the steady state profits is used to normalize the value functions and simplify the state space.

**PROPOSITION 1.** Consider the problem of firm i evaluated at a review date \( t = \tau_{in} \), for a given path of aggregate consumption \( \{c_{t+s}\} \). Let \( v_t(g) \equiv V_t(g)/\Pi, \) \( \tilde{v}_t(g) \equiv \tilde{V}_t(g)/\Pi, \) and \( \tilde{V}_t(g) \equiv \tilde{V}_t(g)/\Pi. \) The firm maximizes the value function \( v_t(g) = \max \{\tilde{V}_t, \tilde{v}_t(g)\} \), where

\[
\tilde{v}_t = -\theta - \psi + \max_{T, \hat{g}} \int_0^T e^{-rs} \left( \frac{c_{t+s}}{\bar{c}} \right)^{1-\varepsilon \eta} f(\hat{g}, s) ds + e^{-rT} \int_{-\infty}^{\infty} v_{t+T}(\hat{g} + (\gamma - \mu)T + \sigma \sqrt{T}x) dN(x), \tag{13}
\]

\[
\tilde{v}_t(g) = -\theta + \max_{T} \int_0^T e^{-rs} \left( \frac{c_{t+s}}{\bar{c}} \right)^{1-\varepsilon \eta} f(g, s) ds + e^{-rT} \int_{-\infty}^{\infty} v_{t+T}(g + (\gamma - \mu)T + \sigma \sqrt{T}x) dN(x), \tag{14}
\]

and

\[
f(g, s) \equiv \eta e^{(\eta-1)} \left( (\mu - \gamma + \frac{s^2}{2\sigma^2})(\eta-1)s - g \right) - (\eta - 1)e^{\eta} \left( (\mu - \gamma + \frac{s^2}{2\sigma^2})\eta s - g \right),
\]

where \( N(\cdot) \) is the CDF of a standard normal distribution.

The proof of the proposition follows immediately from the recursive firm problem described above. The term involving \( \{c_{t+s}/\bar{c}\} \) reflects the impact of aggregate consumption on discounted profits. At standard parameter values such as \( \varepsilon \eta > 1 \) we have that higher expected growth in aggregate consumption is associated to lower expected discounted profits, since the firm discounts states of the world with higher aggregate consumption more, as can be seen from equation (11).

---

\(^{13}\) We study a version of the steady-state problem of equation (13) in Alvarez et al. (2011). In that paper the period profit was assumed to be a quadratic function of \( g \), which can be derived as a second order approximation to \( \Pi(c, \bar{c}) \) of equation (11).
The function \( f(g, s) = \mathbb{E} \left[ \prod (e^{R_{t+s}}, \bar{c}) / \Pi^*(\bar{c}) \big| g_t = g \right] \) is a measure of the expected growth of profits \( s \) periods ahead, conditional on inaction and constant aggregate consumption. The first term of \( f(g, s) \) depends on the expected growth in real revenues. The second term of \( f(g, s) \) depends on the expected growth in real marginal cost. We notice that \( f(g, 0) \) is maximized at \( g = 0 \) where it takes the value of 1. In choosing whether to adjust the price, the firm trades-off higher expected profits from changing the price against the menu cost \( \psi \), giving rise to an \( sS \) type of adjustment rule.

3.2.3. Optimal Decision Rules. The optimal decision rule for each review time is described by three values for the price gap, \( g_t < \hat{g}_t < \bar{g}_t \), and a function \( T_t(g) \), where the decision rules may vary over time because of variation in the aggregate state \( c_t \). After observing its price gap \( g \) at \( t \), the firm leaves its price unchanged if \( g \in (g_t, \bar{g}_t) \). Otherwise the firm changes its price gap to \( \hat{g}_t \). The function \( T_t(g) \) gives the (optimally chosen) time the firm will wait until the next review as a function of the price gap after the adjustment decision. In Alvarez et al. (2011) we provide an analytical characterization for these decision rules in the steady state where \( c_t = \bar{c} \).

3.2.4. The Firm’s Beliefs and the Decision Problem. In Proposition 1 we defined the firm’s state and value function only at times of observations. In this case the state is a scalar given by the price gap \( g \) right after the observation. Next we will use this value function and the corresponding decision rules to define and extend the firm’s decision problem for all times, that is, including times where the firm is not observing the state. This is useful to properly define an equilibrium with arbitrary initial conditions, as we explain below.

At any moment of time the firm’s state is given by its belief about its price gap, which we assume to be normally distributed with expected value and variance \((\bar{g}, \sigma^2)\). Beliefs are normally distributed because right after an observation the price gap evolves as an (unobserved) BM with drift \( \gamma - \mu \) and with innovation variance \( \sigma^2 \). Note that this means that beliefs evolve deterministically through time. For instance, right after an observation at time \( t \) we have \( \bar{g}_t = g_t \) and \( \sigma^2_t = 0 \). Letting a time interval \( s \) elapse after this observation, the firm’s beliefs are \( \bar{g}_{t+s} = \bar{g}_t + s(\gamma - \mu) \) and \( \sigma^2_{t+s} = \sigma^2_s \).

A firm with state \((\bar{g}, \sigma^2)\) decides a time horizon \( T \geq 0 \) until which it will keep its price constant and obtain no new information. At the end of this inaction period the firm will observe the price gap and decide whether to adjust its price. Recall that we have already derived the optimal value of observing its price gap as \( v_{t+T}(g) \) in Proposition 1. Thus a firm with a state \((\bar{g}, \sigma^2)\) solves,

\[
\omega_t(\bar{g}, \sigma^2) = \max_{T \geq 0} \int_{-\infty}^{\infty} \left[ \int_0^T e^{-rs} \left( \frac{R_{t+s}}{c} \right)^{1-\eta} f(\bar{g} + z\sigma, s) \, ds + e^{-rT} v_{t+T}(\bar{g} + T(\gamma - \mu) + z\sqrt{\sigma^2 + T\sigma^2}) \right] dN(z). \tag{15}
\]
Notice that if at time $t$ it is optimal to choose $T = 0$, then the firm will either observe immediately or observe and adjust. If instead $T > 0$ then the firm’s next observation will occur at time $t + T$. We can thus define the time until the next observation $T^*_t(\tilde{g}, \tilde{\sigma}^2)$ for a firm with beliefs $(\tilde{g}, \tilde{\sigma}^2)$. Given the optimal solution for $T$ implied by equation (15) it is convenient to define the indicator function $obs_t(\tilde{g}, \tilde{\sigma}^2)$, namely an indicator that the firm observes at time $t$ conditional on the state $(\tilde{g}, \tilde{\sigma}^2)$,

$$obs_t(\tilde{g}, \tilde{\sigma}^2) = \begin{cases} 1 & \text{if } T = 0 \\ 0 & \text{if } T > 0 \end{cases}.$$  

### 3.3. Equilibrium

The equilibrium is such that the household supplies labor $L_t$ to satisfy the demand from all the firms, and each firm $i$ supplies goods so that its output satisfies demand, that is, $C_{i,t} = Y_{i,t}$ for each $i,t$. As discussed above the equilibrium nominal wages and interest rates are given by equation (8), whereas the equilibrium prices $p_{i,t}$ of the different varieties are determined by the solution to the firm problem in Proposition 1 and depend on the path for the aggregate consumption $\{c_t\}$. In turn, using equation (2) and the household’s first order conditions in equations (5) and (6) for optimal demand $C_{i,t}$, the equilibrium aggregate consumption $c_t$ depends on the equilibrium prices $p_{i,t}$ of the different varieties. Thus we have a fixed point problem: finding the aggregate consumption sequence $\{c_t\}$ that generates optimal pricing decisions $p_{i,t}$ that are consistent with $\{c_t\}$.

We are now ready to describe the consistency conditions implied by the equilibrium, that is, a mapping from policies to a path of aggregate consumption $\{c_t\}$. As a consequence of the first order conditions in equations (5) and (6), and of the definition of $c_t$, the path of consumption has to satisfy

$$c_t = \left( \int \left( \frac{\xi}{\eta - 1} \right)^{1-\eta} \varphi_t(dg) \right)^{-1/(\eta - 1)},$$

where $\varphi_t(\cdot)$ is the cross-sectional distribution of price gaps, $g_{i,t}$, in period $t$. To explain how to obtain the distribution of price gaps $\varphi_t$ we turn to the discussion of an alternative state space for the firm’s problem representing the beliefs of the firm.

### 3.3.1. Firm’s Beliefs

As discussed above, for an encompassing characterization of the behavior of all firms (not just those that are observing, as done in Proposition 1) we use a two dimensional state space in terms of the firm’s expected price gap and the uncertainty surrounding it. This is essential to aggregate firms with different price gaps, including those that are not currently observing, and retrieve the distribution of price gaps needed to compute, for example, equilibrium output as in equation (17). We denote the time $t$ distribution of beliefs across firm as $\varphi_t(\tilde{g}, \tilde{\sigma}^2)$. Notice that given $\varphi_t$.
we can compute \( \varphi_t \) as follows:

\[
\varphi_t(g) = \int_{-\infty}^{\infty} \int_{0}^{\infty} \varphi_t(\tilde{g}, s \sigma^2) n \left( \frac{g - \tilde{g} - (\gamma - \mu)s}{\sqrt{s} \sigma} \right) \, ds \, d\tilde{g},
\]  

(18)

where \( n(\cdot) \) denotes the density of a standard normal distribution. This formula uses rational expectations, in that the actual distribution of price gaps coincides with the beliefs. In turn the distribution of beliefs \( \varphi_t \) is generated using an initial distribution \( \varphi_{t_0} \) and the optimal decisions rules for \( t \geq t_0 \) given by \( \{g_t, \hat{g}_t, \tilde{g}_t, T_t(\cdot)\}_{t \geq t_0} \). To streamline the presentation we illustrate the law of motion for this distribution only for the steady state.\(^{14}\)

3.3.2. Law of Motion of Distribution of Beliefs. Using the initial distribution of beliefs \( \varphi_{t_0}(\cdot, \cdot) \) and the optimal decision rules, summarized by \( \{obs_t(\cdot, \cdot), g_t, \hat{g}_t, \tilde{g}_t\}_{t \geq t_0} \), we describe the dynamics of the distribution of beliefs. The next expressions characterize such dynamics

\[
0 = (\gamma - \mu) \frac{\partial}{\partial \tilde{g}} \varphi_t(\tilde{g}, \tilde{\sigma}^2) + \sigma^2 \frac{\partial^2}{\partial \tilde{\sigma}^2} \varphi_t(\tilde{g}, \tilde{\sigma}^2) \quad \text{for} \quad obs_t(\tilde{g}, \tilde{\sigma}^2) = 0 \quad \text{and} \quad \tilde{\sigma}^2 > 0,
\]

(19)

\[
\varphi_t(\tilde{g}, \tilde{\sigma}^2) = 0 \quad \text{if} \quad obs_t(\tilde{g}, \til{\sigma}^2) = 1 \quad \text{and} \quad \til{\sigma}^2 > 0,
\]

(20)

\[
\varphi_t(\tilde{g}, 0) = \int_{-\infty}^{\infty} \int_{0}^{\infty} obs_t(\tilde{g}', \til{\sigma}^2) n \left( \frac{\tilde{g} - \tilde{g}'}{\til{\sigma}'} \right) \varphi_t(\tilde{g}', \til{\sigma}^2) \, d\til{g}' \, d\til{\sigma}^2
\]

\[
\text{for} \quad \tilde{g} \neq \tilde{g}_t \quad \text{and} \quad \tilde{g} \in [\tilde{g}_t, \tilde{g}_t],
\]

(21)

\[
\varphi_t(\tilde{g}_t, 0) = \int_{-\infty}^{\infty} \int_{0}^{\infty} obs_t(\tilde{g}', \til{\sigma}^2) n \left( \frac{\tilde{g}_t - \tilde{g}'}{\til{\sigma}'} \right) \varphi_t(\tilde{g}', \til{\sigma}^2) \, d\til{g}' \, d\til{\sigma}^2 \times \varphi_t(\tilde{g}', \til{\sigma}^2) \, d\til{g}' \, d\til{\sigma}^2.
\]

(22)

There are four equations corresponding to four different regions of the state space. The first two regions correspond to beliefs with some uncertainty about the price gaps, so that \( \til{\sigma}^2 > 0 \). There are two cases depending on whether it is optimal to observe the state or not. If observing is not optimal, then for any time interval of length \( \Delta > 0 \) we have:\n
\[
\varphi_{t+\Delta}(\tilde{g} + \Delta(\mu - \gamma), \til{\sigma}^2 + \Delta\til{\sigma}^2) = \varphi_t(\tilde{g}, \til{\sigma}^2),
\]

since the expected value of the beliefs

\( \text{\(^{14}\)} \) One could write an explicit expression for the law of motion of \( \varphi_t \) for any given path of decision rules.
have a drift $\gamma - \mu$ and its variance becomes more diffuse at rate $\sigma^2$. Differentiating
the equation with respect to $\Delta$ and taking the limit for $\Delta \to 0$ gives equation (19).
The second case, given by equation (20), corresponds to beliefs for which observation
is optimal, and hence there the mass disappears from that point. The third and fourth
regions correspond to beliefs where the firm observes the price gap so that $\tilde{\sigma}^2 = 0$. At
this point, firms split depending on whether they decide to adjust prices. If they do not
adjust, then firms keep their original price gap $\hat{\tilde{g}}$, as indicated by equation (21). If they
instead decide to adjust, the mass of adjusting firms is moved to the optimal return
point $\hat{\tilde{g}}$ as in equation (22).

3.3.3. Equilibrium as a Fixed Point. Summarizing, for a given initial condition $\varphi_{t_0}$
the equilibrium can be thought as a fixed point on the paths $\{c_t\}_{t \geq t_0}$ to itself. Given a
conjectured path $\{c_t\}_{t \geq t_0}$ we use Proposition 1 to solve for the path of decision rules.
Then, given the path of decisions rules $\{g_t, \hat{\tilde{g}}_t, \tilde{g}_t, T_t(\cdot)\}_{t \geq t_0}$, we generate the path of
$\{\varphi_t(\cdot)\}_{t \geq t_0}$ and the corresponding path of $\{\phi_t(\cdot)\}_{t \geq t_0}$ and construct the implied path
$\{c_t\}_{t \geq t_0}$ using equation (17), iterating until the conjectured path converges to the actual
one.

3.3.4. Steady State. In particular, a steady state equilibrium is characterized by an
invariant distribution $\varphi_t(\cdot) = \bar{\varphi}(\cdot)$ for each $t$, so that $c_t = \bar{c}$. As noticed by Golosov
and Lucas (2007) the cross-sectional distribution $\varphi(\cdot)$ enters the firm problem in
Proposition 1 only as a determinant of aggregate consumption $c_t$. This simplifies the
numerical solution of the problem as firms do not need to form expectations based
on the law of motion for the cross-sectional distribution, but only on the path of
the scalar: $\{c_t\}$. Moreover, although our definition of equilibrium and our numerical
solution take this general equilibrium feedback fully into consideration, its effect on the
decision rules is very small for realistic monetary shocks. The result that, in this model,
the general equilibrium effects are negligible for small monetary shocks is formally
established in closely related set-ups in Gertler and Leahy (2008) and Alvarez and
Lippi (2014).15

3.4. The Monetary Shock

Here we describe the monetary shock experiment that we perform in the paper. We
start the economy in steady state at some $t = t_0$ where economic agents expect $c_t = \bar{c}$
and $\varphi_t(g) = \bar{\varphi}(g)$ for all $g \in (-\infty, +\infty)$ and all $t \geq t_0$. The monetary shock takes the
form of an unforeseen, one time, permanent increase in the stock of money supply so

15. See Proposition 1 in Gertler and Leahy (2008) and Proposition 7 in Alvarez and Lippi (2014). This
result relies on the assumption of no strategic complementarity in price setting discussed above, so that
general equilibrium feedbacks only impact the firms’ discount factor but not their nominal marginal cost.
that

$$\log(m_t) = \begin{cases} \log(m_{t-T}) + \mu(t - T) + \delta & \text{for all } t \geq t_0 \text{ and } T > 0 \\ \log(m_{t-T}) + \mu(t - T) & \text{for all } t < t_0 \text{ and } T > 0 \end{cases},$$

where $\delta$ is the log-difference in money supply upon the realization of the shock. In our baseline analysis we assume that firms only learn about the realization of the shock after their first observation of the price gap. This is equivalent to assume that, in the spirit of the rational inattentiveness literature, firms do not pay attention to the changes in these variables, or in their own profits, unless they pay the observation cost.

Using equation (8), we notice that the shock to the level of money supply causes a proportional change in the nominal wage on impact, after which the nominal wage grows at the growth rate of the money supply, $\mu$. As the profit-maximizing price in equation (10) is a constant markup over nominal marginal cost, the monetary shock causes a parallel shift of size $\delta$ in the distribution of price gaps $\varphi_i(g)$. For instance, an increase in money supply of $\delta$ log-points causes a decrease on impact of size $\delta$ to the log-price gap for all $i$ at $t = t_0$. In solving for the response of the economy to the monetary shock we will compute the dynamics of the distribution of price gaps $\varphi_i(g)$, and the associated path for $c_t$, for all $t \geq t_0$ until the economy converges back to the steady state.

As mentioned, in the baseline analysis we assume that firms are not aware of the monetary shock until their first review occurs. Because of this, the time of their first review after $t_0$ is unaffected and no action is taken before then. Upon the first review the firm learns about the aggregate shock as well as about its own idiosyncratic productivity, and revises its beliefs so that the firm problem in Proposition 1 at any review date $\tau_{i,t} \geq t_0$ is based on perfect foresight of the path of $c_t$. In Section 5.1 we analyze the alternative setup in which all firms are perfectly informed about the realization and size of the monetary shock. The main result is that for small monetary shocks the results are not affected.

4. A Calibration for the U.S. Economy

This section presents a calibration of the model fundamental parameters that matches some key statistics on price setting behavior from the U.S. economy. We use this calibrated model in the next section to study how the aggregate economy responds to a monetary shock. The key part of the calibration concerns the parameters that govern the importance of information versus menu cost frictions. Many other parameters are informative about features that are common to other monetary models. In particular we set $\eta = 4$ so that the average price markup is roughly one third, that is, between the values used by Midrigan (2011) and Golosov and Lucas (2007). Following Golosov and Lucas (2007), we set $\varepsilon = 2$ in order to have an intertemporal elasticity of substitution of $1/2$, and $\xi = 6$ so that households allocate approximately $1/3$ of the unit time endowment to work in steady state. We set the yearly discount rate to $\rho = 0.02$. 

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4.1. Information versus Menu Cost Frictions

We choose the parameters, $\theta$, $\psi$ and $\sigma$, so that the steady moments from our model match some key U.S. statistics about the frequency and size of price adjustments, as well as on the frequency of price reviews. We target the average number of price adjustments (denoted by $n_a$) and reviews (denoted by $n_r$) per year implied by the estimates of Blinder et al. (1998) for a sample of U.S. firms reported in Table 1, that is, $n_a = 1.4$ and $n_r = 2$. Proposition 6 and equation (18) in Alvarez et al. (2011) show that the ratio between the frequency of price reviews and adjustments identifies the ratio of menu to observation costs: the larger the ratio of the frequency of price reviews to adjustments, the larger the ratio $\psi/\theta$. For a given value of $\psi/\theta$, the frequency and average size of price adjustments identify the levels of $\psi$ and $\sigma$. The target for the average size of price changes, measured by their mean absolute value, is given by the estimates of Nakamura and Steinsson (2008) on U.S. data and it is equal to $e_{\Delta p} = 0.085$. This procedure gives $\theta = 0.0075$, $\psi = 0.0027$ that are percentages of year profits, and a volatility of productivity shocks given by $\sigma = 0.11$. The value of $\psi = 0.0027$ implies that the yearly cost of price adjustments is about 0.1% of revenues, which is comparable to the menu cost estimated directly by Levy et al. (1997) on retailer data. The estimated value of $\theta$ implies that the yearly cost of reviews is about four times larger than that of the physical menu cost of price adjustment. This finding is consistent with estimates by Zbaracki et al. (2004) for a large U.S. manufacturer, who find that managerial (information processing costs) are about 6 times larger than the menu cost.

4.2. Lack of Price Plans

The growth rate of the money supply is chosen to target a yearly inflation (of the price index $P_t$) equal to 2%, implying $\mu = 0.02$. For a given value of $\mu$, the value of $\gamma$ determines the incentives of firms to use price plans between consecutive review dates. At the baseline calibration of the menu cost $\psi$, price adjustments occur only upon reviews for a large and empirically reasonable range of values of $\gamma$ and $\mu$, as implicitly conjectured in the firm problem of Proposition 1: at $\mu = 0.02$, $\gamma$ should be larger.

---

16. In Alvarez et al. (2011) we have also shown that $\psi/\theta$ can be identified from other moments of the distribution of price changes. In particular, the larger the fraction of small price changes, the smaller the ratio of menu to observation cost. We notice that our baseline model predicts that the fraction of price changes smaller than 5% (in absolute value) is equal to 25% of all price changes. This statistic is consistent with estimates by Eichenbaum et al. (2014a) obtained on U.S. microdata from the CPI (see their Table 1). As we are not targeting such moment, we interpret this finding as a sign of robustness of our estimate of the ratio $\psi/\theta$.

17. This statistic is computed as $e_{\Delta p} = \mathbb{E}[|\Delta \log(p)| | \Delta \log(p) \neq 0]$.

18. Revenues are measured in steady state and at the profit-maximizing price in absence of frictions in price setting, that is, revenues are equal to $\eta \bar{F}$. The yearly flow costs of adjustments and reviews are obtained by multiplying the cost of each adjustment and review with the average frequency of adjustments and reviews, respectively, that is, $\psi n_a$ and $\theta n_r$. 

---
than 10% for a price plan to be optimal. Moreover, the frequency of price adjustments and reviews have a near zero elasticity with respect to $\mu$ and $\gamma$ in that range. Thus the qualitative and quantitative results of the baseline model with observation and menu cost are not affected by the choice of $\gamma$, as long as $\gamma$ is not unrealistically large. As noticed in Section 2 this assumption is also consistent with the evidence that the frequency of price adjustments is smaller than the frequency of observation (i.e., absence of price plans) in several low inflation economies. Given the ample range of values of $\gamma$ consistent with no price plans, we set $\gamma = \mu + (2\eta - 1)\sigma^2/2$ so that price plans are not optimal in our model even if the menu cost $\psi$ is arbitrarily small. This choice has the advantage of making the polar case with positive observation cost and zero menu cost able to reproduce the same frequency of price changes of our baseline model.

4.3. The Calibration of the Polar Cases with Only One Friction

Our model nests the canonical menu cost and the canonical observation cost models, each with only one friction. These models have been studied extensively in the literature, and therefore offer an interesting benchmark of comparison against our baseline model where the two frictions coexist. As a disciplining device in comparing the different economies, we calibrate the parameters governing the frequency and size of price adjustments for each model to match the same average frequency and size of price changes of our baseline parametrization, that is, $n_{\text{st}} = 1.4$ and $e_{\Delta\rho} = 0.085$, respectively.

The menu cost model is obtained when $\psi > 0$ and $\theta = 0$ so that firms observe the state continuously. The firm’s problem in this case has been analyzed in the seminal papers by Barro (1972) and Dixit (1991), and its aggregate consequences in Danziger (1999) and Golosov and Lucas (2007) among others. In this case the posted price is adjusted whenever it is far enough from the optimal price, so that the price adjustment rule is a state-dependent one. The observation cost model is obtained when $\theta > 0$ and $\psi = 0$. If $\psi = 0$, firms adjust prices continuously, even between two consecutive reviews, as long as they expect a drift in the nominal marginal cost. Price changes between consecutive review dates are referred to in the literature as price plans: such price changes are based upon the information gathered in the last review and the law of motion of the relevant states. Price plans have been emphasized in the sticky information literature that developed after Mankiw and Reis (2002). Under the assumption that $\gamma = \mu + (2\eta - 1)\sigma^2/2$, the expected drift in inflation and productivity offset each other in our model, so that prices adjust only in response to shocks to the

---

19. In Table I of Alvarez et al. (2011) we show that the frequency of price adjustments and reviews has a near zero elasticity with respect to the drift (inflation) for values of the drift smaller than 10% (in absolute value).

20. When $\gamma = \mu + (2\eta - 1)\sigma^2/2$ expected profits are invariant to the time elapsed between consecutive review dates up to a second-order approximation. As no new information arrives between review dates, the optimal price is constant during this period. See Online Appendix C for more details.
idiosyncratic productivity. As firms review their idiosyncratic state infrequently, prices adjust infrequently. In particular, the frequency of price changes coincides with the frequency of reviews. Thus the assumption $\gamma = \mu + (2\eta - 1)\sigma^2/2$ makes price plans immaterial and allows this specification of the model to be consistent with the fact that prices change infrequently. In this environment, the sticky information model is equivalent to a sticky price model.21 In Table 3 we summarize the parameters of our calibrations for our baseline economy and for the two polar cases with observation and menu cost only, respectively.

### 5. The Propagation of the Monetary Shock

In this section we analyze the response of aggregate output to the once and for all monetary shock described in Section 3.4, using the parameter values of Section 4. We compare the predictions of our baseline model with both adjustment and observation costs to the impulse responses predicted by the menu-cost and the observation-cost models. We solve numerically for the equilibrium aggregate consumption as defined in Section 3.3 by discretizing the model to a one week period (see Online Appendix B for a detailed description of our algorithm).

Figure 1 displays the results of the calibration. The figure plots the output response to a monetary shock of size $\delta = 0.01$ in deviation from the steady state, for the three different economies specified in Table 3. Our baseline model with both frictions (tick solid line) predicts a significantly larger and more persistent real response than the menu-cost model, getting close to the response of the model with the observation cost only. Moreover, the shape of the impulse response in our model is quasi linear, thus more similar to the one predicted by the observation-cost model than by the menu-cost model.

21. A version of this model was first formulated by Caballero (1989), then extended by Reis (2006), whereas Bonomo and Carvalho (2004) studied a version where the observation cost is associated with an adjustment cost. As firms face the same cost/benefit of an observation in these models, the endogenous time between consecutive reviews is constant delivering *Taylor* type price adjustments. We notice that if the time between consecutive reviews of a firm were instead random and exponentially distributed as in Mankiw and Reis (2002), then the assumption $\gamma = \mu + (2\eta - 1)\sigma^2/2$ would deliver *Calvo* type price adjustments. In Alvarez, Lippi, and Paciello (2016a) we derive the mapping from random cost/benefit of observations to exponentially distributed times between reviews in the absence of a menu cost.

<table>
<thead>
<tr>
<th>Model specification</th>
<th>$\psi$</th>
<th>$\theta$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation + menu cost</td>
<td>0.0027</td>
<td>0.0075</td>
<td>0.11</td>
</tr>
<tr>
<td>Observation cost only</td>
<td>0.0000</td>
<td>0.0210</td>
<td>0.12</td>
</tr>
<tr>
<td>Menu cost only</td>
<td>0.0053</td>
<td>0.0000</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 3. Calibrated parameters in different specifications of the model.
The reason for the different behavior of these models, which are identical in their steady-state behavior concerning the frequency and size of price changes, stems from the different adjustment rules followed by the individual firms. The price setting rule is state-dependent in the menu cost model: firms adjust prices whenever the price gap \(g_t\) crosses the thresholds \(\{g_t^-, g_t^+\}\). A positive monetary shock reduces all price gaps on impact by the same amount. The larger the mass of firms that is moved outside of the inaction region on impact, the larger the increase of the aggregate price, and the smaller the output response to the monetary shock. As explained by Golosov and Lucas (2007) the response of the aggregate price to the monetary shock takes place through a selection effect by which firms with the highest price gap adjust first. In the model with observation cost only, instead, the price setting policy follows a time-dependent rule where firms observing the state and adjusting the price are selected as a function of the time elapsed since the last observation/adjustment, and not of the size of their current price gap. Thus, as noticed in the literature, models with time dependent price adjustment rules predict larger real effects than models with state dependent price adjustment rules because the former are characterized by a weaker selection effect than the latter.22

Our model with both frictions is characterized by a price adjustment policy that has both a time- and a state- dependent element. The time dependent element is due to the fact that firms adjust prices only upon observation of the state, which is a function of the time elapsed since the last observation. The state dependent element is due to the fact that, conditional on observing the state, firms decide whether to adjust the price and when to observe the state again as a function of the current price gap. We emphasize that, even though our model inherits both the time- and state-dependent elements of the polar cases, the shape of the predicted impulse response resembles those of models with only time dependent adjustment policies.

In order to isolate the role of observation and menu costs in accounting for the output impulse response to monetary shocks in the baseline economy of Figure 1, we consider a counterfactual exercise in which, starting from the parametrization of the baseline economy, we set the observation cost and the menu cost equal to zero one at the time, leaving all other parameters unchanged. In particular we first simulate impulse responses to a \(\delta = 0.01\) monetary shock in an economy where we set the menu cost to zero, that is, \(\psi = 0\). We find that the output impulse response in this counterfactual economy, represented by a dotted-dashed line, is very close to the output response in the baseline economy where the menu cost is instead \(\psi = 0.0027\) of profits, so that the cumulative output response is approximately 90% as large as in the baseline economy. Next, we simulate impulse responses to a \(\delta = 0.01\) monetary shock in an economy where we set the observation cost to zero, that is, \(\theta = 0\). This counterfactual

22. The statement that the output effect is larger in models with “time dependent” as opposed to “state dependent” rules hinges crucially on the observable moments that one conditions on. In Alvarez, Lippi, and Passadore (2016b) we show that for small monetary shocks the effect is identical in the two frameworks provided the models share the same frequency and kurtosis of the (size of) price changes. This equivalence does not apply with large shocks, an issue that we discuss below.
economy, represented by a dashed line, predicts a much smaller real effect than the baseline economy where instead the observation cost is \( \theta = 0.0075 \) of profits, with a cumulative output response that is about 30% of the cumulative effect of the baseline economy. Thus we conclude that the observation cost accounts for the most part of the real effects of the 1% monetary shock in the baseline economy.

Finally we quantify the size of the cumulative real effects of the monetary shock for different sizes of the monetary shock \( \delta \) in each of the three economies defined in Table 3. As a summary measure of the real effects of a monetary shock of a given size \( \delta \), we use the cumulative output response \( M(\delta) \), namely, the area under the output impulse response function, defined as

\[
M(\delta) = \int_{t_0}^{\infty} (\log(C_t(\delta)) - \log(\bar{c})) \, dt,
\]

where \( C_t(\delta) \) is the equilibrium path of consumption \( c_t \) for all \( t \geq t_0 \), after a monetary shock of size \( \delta \) at \( t = t_0 \). As consumption coincides with aggregate output in our economy, we will refer to \( C_t(\delta) \) as the response of output to a monetary shock of size \( \delta \). As shown in Figure 2, the cumulative output response \( M(\delta) \) predicted by the baseline economy is in between the values predicted by the observation cost only and menu cost only models for all values of \( \delta \): for a given positive inflation response, the baseline economy predicts higher output response than the menu-cost model but smaller than the observation-cost model. For instance, the baseline economy predicts a cumulative output response that is about 70% of the observation-cost only economy and about 1.5 times larger than predicted by the menu-cost model for shocks smaller than 0.05. The difference gets much larger for larger shocks since the impulse response function of the menu cost economy is highly sensitive to the size of the shock (due to the state dependent nature of adjustments) whereas the effect is linear, that is, proportional to the monetary shock, in the observation-cost only and in the baseline model. In the next section we show that such prediction depends crucially on the assumption that large monetary shocks are unobserved by firms.

5.1. Output Responses to Publicly Known Monetary Shocks

In this section we compute the impulse response in the baseline model with both observation and menu costs under the alternative assumption that the monetary shock is immediately observed by all firms upon impact. We continue to assume that the firm does not observe its own idiosyncratic cost unless it pays the observation cost. This model produces a richer set of behavior, in particular at the moment they learn about the aggregate shock some firms decide to adjust prices without paying the observation costs. Such price adjustments are implemented by the firm using partial information. We will show that the quantitative and qualitative predictions of the model are unchanged so long as the monetary shock is small (i.e., a small fraction of the norm or standard deviation of price changes). Interestingly, for large shocks the output effect
FIGURE 2. Cumulative output response, $M(\delta)$, as a function of the monetary shock $\delta$. All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, $n_p = 1.4$, and to the mean absolute size of price changes, $e_{\Delta p} = 0.085$.

is substantially smaller when the shock is public information. Next we illustrate the result and explain the mechanism behind it.

We start by considering a monetary shock of size $\delta = 0.01$ and a five times larger shock $\delta = 0.05$. We assume that all firms observe the realization of the monetary shock on impact but remain ignorant about their own idiosyncratic state, and reoptimize accordingly. Figure 3 plots the output response to the monetary shock for the case in which the monetary shock is known (dashed line) versus the baseline case in which the shock is not known until the first observation occurs (solid line). The output response to the small monetary shock is roughly identical in the two cases. In contrast, the bottom panel of the figure shows that the response to the large shock is much smaller on impact and much less persistent under the assumption that the monetary shock is known on impact. The results thus illustrate that whether the shock is public information or not only matter for modeling the propagation of large monetary shocks. Next we show that this happens since a large and publicly known shock induces a large mass of firms to revise their planned policy about observations and price adjustments, making the model behavior more similar to a state-dependent model.
FIGURE 3. Impulse response of log $c_t$ to an unexpected increase in $m_t$. Impulse responses of log $c_t$ to a one-time permanent increase in $m_t$, in percent deviation from steady state in our baseline model, and in the case where monetary shocks are perfectly and freely observed upon impact. All models are calibrated so that they are observationally equivalent with respect to the average frequency of price changes, $n_d = 1.4$, and to the mean absolute size of price changes, $e_{\Delta p} = 0.085$.

To understand the different impulse responses described above we must consider the firm’s optimal decisions over the state space of the problem. The tick solid line in Figure 4 denotes the contour of the steady state distribution of firms over the two dimensions of the state space: the expected price gap (horizontal axis), and the uncertainty about it, which is proportional to the time since the last observation (vertical axis). The boundary of the state space has an inverse-U shape due to the fact that firms with larger expected price gap (in absolute value) find it optimal to observe the idiosyncratic state relatively sooner.23 The consequence of a positive monetary shock that is observed by all firms is to shift the distribution to the left on impact, as it reduces the expected price gap equally for all firms, and the more so the larger the shock. The figure displays how a $\delta = 0.01$ shock (left panel) and a $\delta = 0.05$ shock (right panel)

23. The reason why the shape of this contour, that is, the support of the state space, is slanted to the right is the presence of drift, given by inflation net of productivity, in the law of motion of the price gaps.
FIGURE 4. Distribution of firms upon impact of the observed monetary shocks. Distribution of firms over the expected price gap and the time elapsed since the last observation of the idiosyncratic state before and after the realization of the 1% (left panel) and 5% (right panel) monetary shocks; the tick solid line represents the contour of the steady state distribution; the dashed line represents the contour of distribution of firms that neither observe the idiosyncratic state nor adjust the price; the area filled with dots in the top-left corner represents the mass of firms that observe the idiosyncratic state (and eventually adjust the price); the area filled with circles in the bottom-left corner is the mass of firms that adjust the price without observing the idiosyncratic state.

The results are based on numerical solution of the firm’s optimal decision problem.24

We distinguish three sets of firms depending on their behavior in response to the monetary shock on impact, and denote them by different markers in Figure 4. The dense black area filled with circles denotes the firms that decide to adjust the price without observing the idiosyncratic state. This area is situated in the bottom-left corner of the distribution and features firms that have both high incentives to adjust the price due to the small (negative) price gap, and a precise estimate of the idiosyncratic state since they made their last observation recently. The area filled with black dots denotes the firms that decide to observe the idiosyncratic state on impact. This area is situated in the top-left corner of the contour and features firms with high incentives to adjust the price because of the small price gap and, at the same time, with a noisy estimate of their own idiosyncratic state since they made the last observation a long time ago. The other firms, located within the area determined by the dashed black line, find it optimal to remain inactive after the 1% monetary shock. We notice that the latter group represents the vast majority of firms, as in the case of the undisclosed monetary shock. This explains why the output response to the 1% monetary shock displayed in Figure 3 is similar for the two cases considered; there is essentially no response of the aggregate price level on impact. Intuitively, the mass of firms that are distributed over the black and green area described above is second order, as can be inferred from the triangular shape of those areas. In contrast, the majority of firms either adjust the price without observing the state, or observe the state and eventually adjust their prices, on

24. See Online Appendix F for more details on this analysis and the computations.
impact after a 5% monetary shock. This large shock triggers a first order reaction by firms, which explains why the real effects are much smaller than in the case in which the shock is unknown to firms.

6. Concluding Remarks

We extended the general equilibrium model of Golosov and Lucas (2007) to include both an information friction in the form of an observation cost, and a price adjustment friction in the form of a menu cost. We exploited survey data on the firms’ frequencies of price adjustments and reviews to measure the relative importance of menu and observation costs to generate price stickiness, following the identification scheme laid out in Alvarez et al. (2011). We have used this economy as a laboratory to evaluate the role of the two types of frictions for the propagation of monetary shocks. We have found that the observation cost friction is non-negligible, and that its presence significantly increases the real effects of monetary shocks compared to the canonical menu cost model.

The role of the observation cost is important and implies that the output response to a monetary shock is akin to the one predicted by a pure time-dependent model, provided that the monetary shock is either small or not known by firms when it occurs. The timing of price adjustments and the timing of observations are highly correlated, and mostly depend on the time elapsed since the last observation, so that the response of the aggregate price level resembles a linear function of the time elapsed since the monetary shock. This is true even when the monetary shock is known to firms, provided that the shock size is small (relative to the size of idiosyncratic shocks). Since the vast majority of monetary shocks are small, this property illustrates the robustness of the approach proposed by Mankiw and Reis (2002). In particular, this robustness to the observability of the aggregate shock provides a response to the common criticism that easily observable monetary shocks would substantially change the importance of the imperfect information hypothesis.

For a large monetary shock, instead, the effect on output depends on whether the shock is known to the firms. When the monetary shock is large and observed by all firms the state dependent features of the firms’ decision rules become relevant and many firms adjust their price when the shock hits, independently of the time elapsed since the last observation. Thus it is important to take a stand on which assumption is more appropriate for the modeling of large shocks. We stress that the nature of the evidence used in Section 2 to identify the importance of observation cost is cross-sectional, and hence relevant for information about idiosyncratic shocks as opposed to information about aggregate shocks. Using direct evidence on large aggregate shocks is a potentially viable strategy to identify the nature of the role of observation cost, although issues of identification and exogeneity are nontrivial. Some preliminary cross-country evidence using aggregate shocks is given in Alvarez et al. (2016b): it is found that the response of aggregate prices to nominal exchange rate devaluations is nonlinear, so that larger shocks trigger a response of the economy with more aggregate price flexibility. A
similar result is obtained by Bonadio, Fischer, and Saure (2016) in their case study of high-frequency data following the large Swiss appreciation of January 2015. Both of these studies thus show that the economy displays more price flexibility when faced with large shocks. Interpreted through the lens of our model, these findings suggest that such large shocks are “observed” by firms.

The prominent role for the observation friction that we end up estimating is not inherent to the model we propose but follows from matching the model with features observed in firm-level survey data. In Alvarez et al. (2012) we study a household portfolio problem in which households face both an observation cost and an adjustment (transaction) cost, along the lines of Abel et al. (2013). Although the context is different the nature of the problem is similar, and the investors’ frequency of review and adjustment depends, as in this paper, on the relative costs of each of these actions. In contrast with the firms’ data, Alvarez et al. find that large transactions costs and small observation costs are needed to account for the investors’ portfolio-review and portfolio-adjustment frequencies.

Although the results in the main body of the paper rely on several simplifying assumptions and on the calibration for the United States, some extensions were explored confirming the robustness of the findings. Online Appendix E shows that alternative parametrizations for other developed economies deliver results that are similar to those discussed for the U.S. economy. Our baseline model assumed perfectly correlated productivity and preference shocks to simplify the analytics of the firm optimal decision problem. Online Appendix G solves the model relaxing this assumption in a model with productivity shocks only. Although the solution of this problem is more involved, the results on the propagation of the monetary shocks are virtually unchanged. We have abstracted from studying the role of higher-order beliefs in price setting for the propagation and amplification of monetary shocks to output since, for simplicity, we have chosen a setup where there is no strategic complementarity in pricing. We see this topic as an interesting avenue for future research given that it is not easy to conjecture how strategic complementarity in price setting would affect the relative importance of observation and menu costs for the propagation of monetary shocks.

References


**Supplementary Data**

Supplementary data are available at *JEEA* online.