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# Optimal Income Taxation in Models with Endogenous Fertility

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**Abstract**

This paper studies the efficient taxation of factor income in infinite-lived models with elastic fertility choices. Two models are considered, one with physical capital only, and one with physical and human capital. In the model with physical capital only, capital income should be subsidized, while labor income taxed. In the model with two types of capital, instead, Ramsey optimality prescribes that the tax on physical capital is zero (negative), if effective labor is constant (decreasing) returns to scale in human capital and market goods, while the tax on human capital is negative and the tax on effective labor positive. Our findings depart from those obtained in immortal models with an endogenous labor supply and constant population growth, because physical and human capital affect the demand for fertility.

*JEL classification*: E62; H22; J22; O41.

*Keywords*: Factor Income Taxes; Second-best Analysis; Endogenous Population Growth; Physical Capital; Human Capital.

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# 1 Introduction

In this paper, we study how the optimal factor income tax structure is influenced by families' decisions on fertility in intertemporal optimizing models of economic growth. The analysis focuses on infinitely lived models of endogenous fertility and investment in human capital and physical capital, which can be viewed as overlapping-generations models with parental altruism and bequests; see, for example, Nerlove and Raut (1997) and Barro and Sala-i-Martin (2003). The analysis focuses on explicit family decision-making models in which optimal fertility choices are made in a utility-maximizing framework. As endogenous fertility enters the utility function of consumers, along with consumption, —see, among the others, Becker and Lewis (1973), Nerlove (1974), Razin and Ben-Zion (1975), Barro and Becker (1988)—, from a formal standpoint, it plays the role of a time-consuming good, whose price is affected by the stock of per capita capital.

The issue of optimal income taxation is studied here by looking at two economies: one in which there is only physical capital formation, and one in which both physical and human capital are accumulated. The consideration of different types of capital has repercussions on the forms in which wealth is intertemporally transferred and on the way in which labor as a factor of production is measured. Regarding the latter aspect, when physical capital is the only factor that can be accumulated, labor is given by working hours. When instead human capital is accumulated along with physical capital, effective labor is a combination of human capital, market goods used to supply labor and working hours, as assumed, for example, by Jones, Manuelli and Rossi (1993 and 1997).

The analysis of optimal factor tax policy in models with endogenous fertility is interesting for the following reasons.[1] First, although infinite-lived

[1] The epistemological appeal of models with elastic labor-offspring choices, that are abundantly used to study different economic problems, generally comes from the following facts: they generate endogenous growth without imposing constant returns to scale with respect to accumulable factors of production; they are able to explain cases of persistent income differentials between poor and rich countries; and they have empirical support. See, for example, Wang, Yip and Scotese (1994), Palivos (1995), Nerlove and Raut (1997), Tamura (2000), Barro and Sala-i-Martin (2003), and Acemoglu (2008).

models with endogenous fertility are apparently similar to infinite-lived models incorporating elastic labor-leisure choices, these two types of models differ substantially, since the different time-allocation mechanisms have disparate implications. In fact, because of child-rearing time costs, unlike leisure fertility enters the consumer budget constraint not only through the time allocation constraint, but also through a dilution effect, which implies that population growth reduces capital per capita.

Second, the fertility choice aspect of optimal taxation, studied here, is important as child rearing needs time; this implies that time devoted to work and to accumulating human capital is reduced. Furthermore, population growth impacts on physical and human capital per capita through the dilution effect.[2] Because of these aspects, the implications of fertility on optimal taxation (especially when human capital is considered) may be profound.[3]

Third, the analysis of a model with endogenous population growth is potentially richer than it may appear at a first glance as the variable associated with fertility can have other interpretations. It can be interpreted, for example, as a time-using pleasurable activity, whose pecuniary costs, entering the consumer's budget constraint, depend on the capital stock.[4]

Fourth, the hypothesis of elastic fertility choices has been generally neglected by the copious literature on efficient income taxation based on intertemporal optimizing models.[5]

Given the highly distortionary nature of capital taxes among factor income taxes, any discussion on the desiderable fiscal policy structure in dynamic settings ends up with the question of alleviating capital, at least physical capital, from the burden of taxation.

The idea of a zero tax on income from physical capital income originates

[2]See, for example, Chu. Cozzi and Liao (2013) on the dilution effect with human capital.

[3]Note that the analysis of endogenous fertility with human capital has been largely considered in literature. Becker (1988), Becker and Barro (1988), Barro and Becker(1989), and Becker *et al.* (1990), for example, seek to establish a connection among fertility, bequests which may be in the form of human capital formation, parental altruism and economic growth.

[4]Home production is an application one can think about.

[5]One exception is given by Spataro and Renstrom (2012), who study positive and normative tax policies in a model of physical capital accumulation with endogenous fertility in the special case of "critical-level utilitarian preferences".

from Judd (1985) and Chamley (1986). In a representative agent economy with labor endogenously supplied, Chamley (1986) argues that the optimal dynamic tax configuration is one in which capital income should be exempted from taxation, while labor income should bear the tax burden required to finance a given stream of government expenditure.[6]

Lucas (1990), Jones, Manuelli and Rossi (1993 and 1997), Atkeson, Chari and Kehoe (1999), and Judd (1999), among others, find that the optimality of the zero physical capital tax carries over a wide variety of setups that incorporate human capital accumulation, perpetual growth, perfect capital mobility and overlapping-generations.

The second-best principle of capital taxation established by Judd (1985) and Chamley (1986), however, is not an ineluctable law of dynamic public finance. The optimal tax on physical capital differs from zero in the following cases: restrictions on the tax code (Correia, 1996, and Jones, Manuelli and Rossi, 1997), monopolistic competition in the product market (Judd, 2002), divergence between public and private rates of time preference (Arrow and Kurz, 1970), private borrowing constraints (Aiyagari, 1995, and Chamley, 2001), pure profits (Jones, Manuelli and Rossi, 1993 and 1997), lack of a commitment mechanism (Benhabib and Rustichini, 1997), heterogeneity and uncertainty (Golosov, Kocherlakota and Tsyvinski, 2003, and Kocherlakota, 2010), and capital investment expensing (Abel, 2007).

[6]Judd (1985) discovers that, in a neoclassical growth model with Kaldorian heterogeneity, the capital income tax has no redistributive potential since setting capital income taxes to zero in the long-run is optimal from any agent's standpoint.

The second-best optimality of the zero tax on physical capital income can be justified on the basis of two classic principles of public finance: i) intermediate goods should be exempted from taxation as taxes are to be levied only on final goods (Diamond and Mirrlees, 1971); and ii) all commodities should be taxed at a uniform rate (Atkinson and Stiglitz, 1972). A tax on capital income generates an intertemporal distortion between the marginal rate of substitution of consumption at two different dates and the corresponding marginal rate of transformation; such a distortion increases exponentially in time. Taxing capital income entails taxing consumption at different dates differently, thus violating the normative principle of uniform taxation of consumption goods. A labor income tax instead only distorts the static consumption-leisure decisions. Therefore, productive efficiency requires that the capital stock should be untaxed, while labor should be taxed. See Chari and Kehoe (1999), and Judd (1999).

In models with human capital accumulation, the normative results on the tax treatment of human capital depend on the way in which labor, which has both a stock and a flow component, is measured.[7] When human capital is introduced in the Chamley (1986) setup, effective labor is simply the product of human capital and working hours — like, for example, in the analysis of Lucas (1990), and Jones and Manuelli (2001)—, it is optimal to tax effective labor, which implies also taxing human capital. When instead the composite nature of effective labor is taken into account —this is because effective labor combines human capital, market goods used to supply labor and worker hours—, the zero capital tax result extends to human capital and labor taxes if the accumulation technologies are constant returns to scale. See Jones, Manuelli and Rossi (1997). If effective labor is not constant returns to scale in human capital and market goods, and human capital is a final good, the optimal tax on human capital may be positive, zero or negative, while the labor income tax is strictly positive; see Judd (1999).

The key findings of the present paper are the following. In an economy with physical capital only, we discover that the optimal tax configuration prescribes, in the limit, capital subsidization and labor taxation.[8] These results on the efficient capital taxation can be explained as follows. When the government has to collect a given amount of resources through distortionary factor taxation in a model with endogenous fertility, efficiency makes it necessary to tax labor (as in the standard model with elastic labor-leisure choices). By reducing the after tax-wage and hence the opportunity cost of children, the labor tax drives fertility up. Differently from the standard optimal tax analysis, this has intertemporal implications as fertility enters the 'Keynes-Ramsey rule'. In fact, by increasing the social return on capital of the 'modified golden rule', the rise in fertility raises the marginal product of capital and pulls the capital stock down. It is then optimal to subsidize capital income with the scope of correcting the distortionary effect of labor

[7]See Jones, Manuelli and Rossi (1993 and 1997), and Jones and Manuelli (2001) for the discussion of such an aspect.

[8]The result of a negative capital income tax rate is not new, having also been obtained by Correia (1996), when the additional untaxed factor of production is Edgeworth substitutable with capital, and Judd (2002), when monopolistically competitive firms are considered.

taxation on the demand for children, capital formation and therefore the intertemporal allocation of resources.

In the model with human capital, the assumption of endogenous population growth implies that the efficient physical capital income tax strictly depends on the properties of the effective labor function. If such a function is linearly homogeneous in human capital and market goods employed to provide labor, the Chamley-Judd result for physical capital taxation holds; when there are decreasing returns to scale in human capital and market goods, instead, Ramsey optimality involves subsidizing physical capital. A fiscal levy on labor income should be imposed, irrespective of the functional properties of effective labor. The distortion of such a tax is alleviated by subsidizing human capital, also when it is a final good, and the resources necessary to provide labor.

The paper is structured as follows. Section 2 presents a model of physical capital formation with endogenous fertility choices, and analyzes its normative implications for factor income taxation. Section 3 extends the previous analysis to an economy with physical and human capital, and studies the associated second-best tax policy. Section 4 concludes.

# 2 An economy with physical capital only

## 2.1 The setup

Consider a real economy peopled by immortal consumers, who decide on consumption, fertility, and saving on an intertemporal basis.[9] Following Becker and Lewis (1973), Nerlove (1974), Razin and Ben-Zion (1975), Barro and Becker (1988), Palivos (1995) and Barro and Sala-i-Martin (2003), we assume that the fertility rate enters, along with consumption, the momentary

[9] We use an infinite-horizon model, which implicitly accounts for altruistic parents, because it makes the comparison with the standard neoclassical growth model easier. In the same spirit as Chamley (1986), a continuous-time setup is employed, that allows for a simple characterization of the second-best tax analysis and an immediate understanding of the departure from the standard tax results.

utility function of the representative agent.[10] The fertility rate corresponds to population growth as the mortality rate is zero and the economy is closed (i.e., there is no immigration from abroad).

As child-rearing is assumed to be a time-consuming activity, the agent's fixed time endowment is divided between time spent on raising children, which depends on fertility, and working. In order to preserve a parallelism with Chamley's (1986) optimal factor tax analysis, in which leisure is not subject to taxation, time used for child-rearing (and hence fertility) is untaxed in the present analysis.

The representative consumer of this economy maximizes the following integral utility

$$\int_0^\infty U(c,n)\mathrm{e}^{-\rho t}dt, \tag{1}$$

where $c$ is per capita consumption, $n$ the fertility rate, and $\rho$ the exogenous rate of time preference. The instantaneous utility function $U(\,\cdot\,)$, which is strictly increasing in its arguments, satisfies the usual properties of regularity.[11] $c$ and $n$ are assumed to be normal goods.

Two constraints must be respected when (1) is maximized. One is the flow budget constraint, given by

$$\dot{k} = [(1-\tau_k)(r-\delta_k)-n]k + (1-\tau_l)wl - c, \tag{2}$$

where $k$ is the stock of physical capital per capita, $r$ the before-tax interest rate, $\delta_k$ the constant physical capital depreciation rate, $w$ the wage rate and $l$ labor hours. $\tau_k$ and $\tau_l$ indicate *ad valorem* capital and labor income tax rates, respectively; capital depreciation allowances are permitted.

Moreover, the time allocation constraint

[10] Nerlove and Raut (1997) provide a comprehensive survey of intertemporal optimizing models with endogenous fertility.

[11] The consideration of $N$, the population size —equal to the size of the family (because of the representative consumer paradigm)—, whose dynamics are given by $\frac{\dot{N}}{N} = n$ (as the mortality rate is zero), in the utility function $U(\,\cdot\,)$ —as assumed by Wang, Yip and Scotese (1994), and others— will not qualitatively modify the normative results obtained below.

$$l + T(n) = 1, \tag{3}$$

must also be considered in addition to (2), when (1) is maximized. According to (3), the fixed time endowment (normalized to one) can be used either for working or for raising children. $T(\cdot)$, which denotes the amount of time devoted to child-rearing, satisfies the following properties: $T(0) = 0$, $T' > 0$ and $T'' > 0$.

The maximization of (1) subject to (2) and (3) yields the following first-order conditions

$$U_c = \lambda, \tag{4a}$$

$$U_n = \lambda[(1 - \tau_l)wT' + k], \tag{4b}$$

$$\rho - \frac{\dot{\lambda}}{\lambda} = (1 - \tau_k)(r - \delta_k) - n, \tag{4c}$$

where $\lambda$ is the Lagrange multiplier associated with the flow budget constraint (2).

Equation (4b) asserts that the marginal rate of substitution of consumption for fertility must equal the opportunity cost of one unit of fertility. The latter is given by the after-tax wage times the marginal time-cost of child-rearing plus the per capita capital stock.[12] Equation (4c) is the Euler equation which ensures that in the intertemporal equilibrium the rate of return on consumption —i.e., the LHS— is equal to the after-tax return on per capita capital —namely, the RHS.

Production is carried out by many competitive firms. The technology, given by $y = F(k, l)$, satisfies the usual neoclassical properties of regularity and is linearly homogeneous in $k$ and $l$.

[12]This is because there are two costs associated with an additional unit of fertility. An increase in $n$ reduces productive market work as child-rearing is time consuming, thus entailing a reduction of welfare through the implied fall in per capita consumption. Moreover, a rise in $n$ implies that some resources must be devoted to the maintainance of the per capita capital stock, which 'depreciates' faster with higher fertility, leading to a reduction in per capita consumption.

Maximum profits require

$$F_k(k,l) = r, \tag{5a}$$

$$F_l(k,l) = w. \tag{5b}$$

The resource constraint is given by

$$F(k,l) = c + \dot{k} + (\delta_k + n)k + g, \tag{6}$$

where $g$ denotes the exogenous per capita government consumption expenditure.

Finally, the government balances its budget by financing public expenditures through factor income taxation

$$\tau_k(r - \delta_k)k + \tau_l wl = g. \tag{7}$$

Having presented the full analytical description of the economy, we are now ready to develop the normative analysis of taxation.

## 2.2 Normative analysis

The problem of efficient taxation, known as the 'Ramsey problem', is studied by using the so-called 'primal method' in the version developed by Lucas and Stokey (1983). Such a method is based on the concept of implementability constraint, which is obtained from the households' intertemporal budget constraint by expressing prices and taxes in terms of quantities through the marginal conditions (4). Second-best optimal taxation is analyzed under the assumption that government spending is fixed.

By integrating the dynamic budget constraint forward (2) and incorporating the condition precluding "Ponzi game", we obtain the intertemporal budget constraint, which is given by

$$\int_0^\infty [c - (1-\tau_l)wl]\mathrm{e}^{-\int_0^t [(1-\tau_k)(r-\delta_k)-n]ds} dt = k_0, \tag{2'}$$

where $k_0$ is the per capita capital stock $k$ at time 0.

From (4a) and (4b), the following expression for the after tax-wage can be easily obtained

$$w(1-\tau_l) = \frac{(U_n - kU_c)}{\lambda T'}.$$

Plugging this equation along with (3), (4a) and $\lambda = \lambda_0 \mathrm{e}^{-\int_0^t [(1-\tau_k)(r-\delta_k)-n-\rho]ds}$ —obtained by integrating (4c) forward (where $\lambda_0$ denotes the marginal utility of wealth a time 0)—, after rearranging, one obtains the implementability constraint; that is,

$$\int_0^\infty [cU_c - \frac{(1-T)}{T'}(U_n - kU_c)]\mathrm{e}^{-\rho t} dt = k_0\lambda_0. \tag{8}$$

The efficient taxation of factor income is found by maximizing the utility functional (1) subject to the implementability constraint (8) and the feasibility constraint (6), once the time allocation constraint (3) is brought in. Such a normative analysis is based on the assumption that $g$ is exogenously given.

Define the pseudo-welfare function as

$$W(c, n, k, \Phi) = U(c, n) + \Phi[cU_c - (U_n - kU_c)\frac{(1-T)}{T'}],$$

where $\Phi$ is the Lagrange multiplier associated with (8). $\Phi$ is positive with distortionary taxation of labor income.[13]

The second-best problem can be formulated in a formal way as follows:

$$\max \int_0^\infty W(c, n, k, \Phi)\mathrm{e}^{-\rho t} dt \tag{9a}$$

subject to

$$\dot{k} = F[k, 1 - T(n)] - c - (\delta_k + n)k - g. \tag{9b}$$

We show that:

In an infinite-lived model of physical capital accumulation with endogenous population growth, tax efficiency requires the subsidization of capital

[13] See footnote 14 below.

in the long-run; this implies that it is optimal to tax labor income in order to finance a given stream of government spending and the capital subsidy.
**Proof**. The first-order conditions for the "Ramsey optimum" (9) are

$$W_c = \Gamma, \tag{10a}$$

$$W_n = \Gamma(F_l T' + k), \tag{10b}$$

$$-\dot{\Gamma} + \Gamma\rho = W_k + \Gamma(F_k - \delta_k - n), \tag{10c}$$

where $\Gamma$ is the co-state variable on the feasibility constraint, $W_c \equiv U_c[1 + \Phi(1+\varepsilon_c)]$, $W_n \equiv U_n[1+\Phi(1+\varepsilon_n)]$, and $W_k \equiv \Phi\dfrac{(1-T)}{T'}U_c$. $\varepsilon_c$ and $\varepsilon_n$ represent general equilibrium elasticities for consumption and fertility, respectively, defined as

$$\varepsilon_c = (c-q)\frac{U_{cc}}{U_c} - \frac{(1-T)}{T'}\frac{(U_{nc} - kU_{cc})}{U_c};$$
$$\varepsilon_n = (c-q)\frac{U_{cn}}{U_n} + \frac{(1-T)T''}{T'^2} - k\frac{U_c}{U_n}[1 + \frac{(1-T)T''}{T'^2}] - \frac{(1-T)}{T'}\frac{(U_{nn} - kU_{nc})}{U_n}.$$

In the steady state, (4c) and (5a) imply that $(1-\tau_k)(F_k - \delta_k) = \rho + n$. Combining this equation with the long-run version of (10c), one gets the optimal capital tax rate $\tau_k^*$; that is,

$$\tau_k^* = -\frac{W_k}{(F_k - \delta_k)W_c} < 0.$$

Normatively speaking, physical capital should be subsidized in the long-run as $W_k$ is positive; the efficient labor income tax rate, obtained by the government budget constraint after using $\tau_k^*$, should, instead, be positive.[14] $\square$

[14] $W_k$ is positive as $\Phi$ is positive. Distortionary labor taxation implies that $\Phi > 0$. This can be demonstrated as follows. By combining (10a) and (10b), once the expressions for $W_c$ and $W_n$ are taken into account, one obtains

$$\frac{U_n[1+\Phi(1+\varepsilon_n)]}{U_c[1+\Phi(1+\varepsilon_c)]} = F_l T' + k.$$

Plugging $F_l T' + k$ from this equation into (4b'), once the relationship $w = F_l$ is used, These results can be explained as follows. In order to raise a given amount of revenue, the government should tax labor from a second-best perspective, as such a type of taxation does not directly impact on the intertemporal margin.[15] By reducing the after-tax wage, labor taxation reduces the opportunity cost of children and hence stimulates fertility. The fertility rate, however, affects the intertemporal margin —that is, the 'modified golden rule'— as it enters the social return on saving. In fact, the rise in $n$, by increasing the marginal product of capital through the 'Keynes-Ramsey rule', lowers the capital-labor ratio.[16] It is then optimal to correct the distortion of labor taxation on the marginal product of capital and capital formation by subsidizing capital income.[17]

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and rearranging, we get

$$\Phi = \tau_l \frac{[1 + \Phi(1 + \varepsilon_c)]F_l T'}{(\varepsilon_n - \varepsilon_c)} \frac{U_c}{U_n};$$

this equation, with the aid of (10a), can be written, after considering the optimal labor tax rate, as

$$\Phi = \tau_l^* \frac{F_l T'}{(\varepsilon_n - \varepsilon_c)} \frac{W_c}{U_n}.$$

In this equation, the expression $(\varepsilon_n - \varepsilon_c)$ is positive since $c$ and $n$ are normal goods. Therefore, $\Phi > 0$ as $\tau_l^* > 0$.

[15]This is because taxes that affect the intratemporal margins minimize allocative distortions as highlighted by Diamond and Mc Fadden (1974).

[16]Note that also the reduction of labor, caused by the higher fertility rate, leads to a fall in the capital-labor ratio.

[17]There is also an explanation of the normative results, based on the analytical physiognomy of the model. It stems from the fact that the per capita capital stock enters the demand for fertility. This wealth effect is obtained because the opportunity cost of fertility depends on nonhuman wealth as population growth erodes its stock in per capita terms. Since the static efficiency condition for fertility is incorporated into the implementability constraint (8) and hence the pseudo-welfare function of the social planner, the capital stock appears directly in the maximand function of the 'Ramsey problem', thus altering the Chamley-Judd optimal tax rule.

# 3 An economy with physical and human capital

## 3.1 The setup

The purpose of this section is to study the question of optimal taxation in a model of endogenous fertility with physical and human capital. By combining elastic fertility choices with endogenous human capital accumulation, we analyze how child quality (human capital of descendants) is traded-off with child quantity (the number of children).

Elastic fertility choices impact directly on human capital formation in two ways. First, child rearing, which is time consuming, causes a reduction of time devoted to human capital accumulation. Second, a rise of the fertility rate dilutes human capital per capita over time.

For this end, elastic fertility choices are introduced in a one-sector model of human and physical capital accumulation, as in Judd (1999, section 7). This model, that combines in a simplified way the Lucas (1988) approach with that of Jones, Manuelli and Rossi (1993 and 1997), is very useful in order to grasp the essence of the problem under investigation here since it conveys all the relevant information contained in more comprehensive and articulated models of human capital.[18]

The representative consumer maximizes

$$\int_0^\infty U(c, n, h)\mathrm{e}^{-\rho t} dt, \tag{11}$$

where $h$ is the stock of human capital per capita and the other variables have the same meaning as before. $U(\ \cdot\ )$, which depends positively on $c$, $n$ and $h$, satisfies the usual properties of regularity. $h$ enters the utility function,

[18]Since Judd (1999) does not consider a sector producing human capital (i.e., his model does not have an equation of human capital accumulation), the crowding-out effect of child-rearing on households' time endowment and hence on human capital accumulation is absent; in this case only the dilution effect of fertility on the accumulation of human capital is at work. This simplification does not have a substantial effect on the main results of this paper. The same qualitative results would be obtained if an equation of human capital accumulation *à la* Jones, Manuelli and Rossi (1993 and 1997) were employed.

because it can play the role of a final good, as in Judd (1999). We assume, for the sake of simplicity, that $U(\,\cdot\,)$ is strongly separable in $c$, $n$ and $h$; that is, $U_{cn} = U_{ch} = U_{nh} = 0$.

The flow budget constraint faced by the representative consumer is

$$\dot{k} + \dot{h} = [(1-\tau_k)(r-\delta_k) - n]k + (1-\tau_l)wL(h,z,l) +$$

$$-(1+\tau_z)z - (n+\delta_h+\tau_h)h - c, \tag{12}$$

where $L(\,\cdot\,)$ denotes the effective labor function, $z$ per capita market goods used to provide labor, $\tau_z$ the tax rate on market goods spent for supplying labor, $\tau_h$ a specific tax on human capital per capita and $\delta_h$ the human capital depreciation rate. The other variables have the same meaning as before. Effective labor is seen as the combined result of human capital, market goods employed to supply labor and time spent working.[19] The properties of the $L(\,\cdot\,)$ function, which is positively affected by its arguments, are crucial for the normative results that will be obtained below.[20] To simplify matters, we postulate that $L(\,\cdot\,)$ takes the Cobb-Douglas form; i.e., $L = h^\alpha z^\beta l^\gamma$, where $\alpha$, $\beta$ and $\gamma$ are non-negative parameters and $\alpha + \beta \leqslant 1$.[21]

Since time is allocated between working and child-rearing, as before, equation (3) still applies.

The first-order conditions for the consumer's maximization problem are

$$U_c = \lambda, \tag{13a}$$

$$U_n = \lambda[(1-\tau_l)wL_l T' + k + h], \tag{13b}$$

$$(1-\tau_l)wL_z = (1+\tau_z), \tag{13c}$$

[19]The composite role of effective labor has been put forward by Jones, Manuelli and Rossi (1997). Their specification of the effective labor function is employed here.

[20]In the Judd (1999) model, effective labor is given by the function $L = L(h,l)$.

[21]Note that the results would remain unaltered if the function $L(\,\cdot\,)$ were assumed to be homogeneous of degree $q$ in $h$ and $z$, with $q \leqslant 1$.

$$-\dot{\lambda} + \lambda\rho = \lambda[(1-\tau_k)(r-\delta_k) - n], \tag{13d}$$

$$-\dot{\lambda} + \lambda\rho = U_h + \lambda[(1-\tau_l)wL_h - (n+\delta_h+\tau_h)], \tag{13e}$$

where $\lambda$ is the Lagrange multiplier associated with (12).

Output per capita is produced by using the neoclassical technology $y = F[k, L(h, l, z)]$, which is linearly homogeneous in $k$ and $L(\,\cdot\,)$.

The feasibility constraint, once expressed in per capita terms, is described by

$$F[k, L(h, z, l)] = c + \dot{k} + \dot{h} + (\delta_k + n)k + (\delta_h + n)h + g + z. \tag{14}$$

The government budget, which is kept balanced period by period, is now given by

$$\tau_k(r-\delta_k)k + \tau_l wL(h, z, l) + \tau_z z + \tau_h h = g. \tag{15}$$

### 3.2 Normative analysis

The combination of (13d) and (13e) yields

$$(1-\tau_k)(r-\delta_k) - n = \frac{U_h}{\lambda} + (1-\tau_l)wL_h - (\delta_h + \tau_h + n). \tag{16}$$

This equation ensures the equality between the return obtained by holding physical capital and the return obtained by accumulating human capital.

By plugging (13c) and (16) into the flow budget constraint (12), integrating the relationship obtained forward and incorporating the condition preventing 'Ponzi games', the consumer's intertemporal budget constraint is obtained

$$\int_0^\infty \left\{ c + \frac{hU_h}{\lambda} - (1-\tau_l)w(L - zL_z - hL_h) \right\} e^{-\int_0^t [(1-\tau_k)(r-\delta_k)-n]ds} dt = a_0, \tag{17}$$

where $a_0 = k_0 + h_0$

From (13a) and (13b), we get the following expression for the after tax-wage

$$(1-\tau_l)w = \frac{[U_n - (k+h)U_c]}{\lambda L_l T'}. \tag{18}$$

After substituting (18), the time allocation constraint (3) and the relationship $\lambda = \lambda_0 e^{-\int_0^t [(1-\tau_k)(r-\delta_k)-n-\rho]ds}$, derived from (13d), into (17), we have the implementability constraint

$$\int_0^\infty \left\{ cU_c + hU_h - [U_n - (k+h)U_c] \frac{[L - (hL_h + zL_z)]}{L_l T'} \right\} e^{-\rho t} dt = a_0 \lambda_0. \tag{19}$$

The optimal tax structure is obtained by maximizing the representative consumer utility integral (11) subject to the implementability constraint (19) and the resource constraint (14).

By using the Cobb-Douglas functional form for $L(\,\cdot\,)$, presented above, we can express the pseudo-welfare function of the Ramsey problem as follows

$$W(c,n,h,k;\Phi) = U(c,n,h) + \Phi \left\{ cU_c + hU_h - [U_n - (k+h)U_c] \frac{(1-\alpha-\beta)(1-T)}{\gamma T'} \right\},$$

where $\Phi$ is the Lagrange multiplier associated with the implementability constraint (19).

The optimal social planner problem entails

$$W_c \equiv U_c[1 + \Phi(1+\eta_c)] = \Gamma, \tag{20a}$$

$$W_n \equiv U_n(1 + \Phi\eta_n) = \Gamma(F_L L_l T' + k + h), \tag{20b}$$

$$F_L L_z = 1, \tag{20c}$$

$$W_k \equiv \frac{(1-\alpha-\beta)\Phi l U_c}{\gamma T'} = -\dot{\Gamma} + \Gamma[(\rho+n+\delta_k) - F_k], \tag{20d}$$

$$W_h \equiv U_h[1+\Phi(1+\eta_h)] = -\dot{\Gamma} + \Gamma[(\rho+n+\delta_h) - F_L L_h], \tag{20e}$$

where $\eta_c$, $\eta_n$ and $\eta_h$ are general equilibrium elasticities, given by

$$\eta_c = \frac{cU_{cc}}{U_c} + \frac{(1-\alpha-\beta)l(k+h)U_{cc}}{\gamma T' U_c};$$

$$\eta_n = -\frac{(1-\alpha-\beta)}{\gamma T'}\left\{\frac{lU_{nn}}{U_n} - [1-(k+h)\frac{U_c}{U_n}]\frac{[T'^2+(1-T)T'']}{T'}\right\};$$

$$\eta_h = \frac{hU_{hh}}{U_h} + \frac{(1-\alpha-\beta)lU_c}{\gamma T' U_h}.$$

Consider the steady state equilibrium. By substituting the relationship $\rho = [(1-\tau_k)(F_k-\delta_k)-n]$, obtained from (13d), into (20d), the optimal capital tax rate is derived; that is,

$$\tau_k^* = -\frac{(1-\alpha-\beta)l\Phi}{\gamma T'(F_k-\delta_k)[1+\Phi(1+\eta_c)]}. \tag{21a}$$

The optimal labor tax rate, obtained by contrasting the optimality conditions for the private and Ramsey problems regarding $c$ and $n$, is given by

$$\tau_l^* = \frac{(\eta_n-\eta_c-1)\Phi U_n}{[1+\Phi(1+\eta_c)]F_L L_l T' U_c}. \tag{21b}$$

After using the optimal condition $F_L L_z = 1$, obtained from (20c), together with (13c), we have that

$$\tau_z^* = -\tau_l^*. \tag{21c}$$

Finally, the optimal tax rate on human capital, that can be recovered by jointly considering (13e) and (20e), is

$$\tau_h^* = \frac{(\eta_c-\eta_h)\Phi U_h}{[1+\Phi(1+\eta_c)]U_c} - \tau_l^* F_L L_h. \tag{21d}$$

In the discussion of the optimal tax results, we consider the following two cases:

a) $\alpha+\ \beta = 1$ —i.e., the function $L(\ \cdot\ )$ is homogeneous of degree one in human capital and market goods;[22]

b) $\alpha+\ \beta < 1$ —i.e., the function $L(\ \cdot\ )$ is homogeneous of degree less than one in $h$ and $z$.[23]

Consider each case in turn.

### 3.3 Optimal fiscal policy when $\alpha+\ \beta = 1$

In this case, the conceptual characterization of the efficient tax structure is as follows:

In an infinite-lived model of physical and human capital accumulation with endogenous fertility, in which effective labor is homogenous of degree one in human capital and market goods used to provide labor (i.e., $\alpha+\ \beta = 1$), the optimal factor tax structure makes it necessary to exempt physical capital from taxation as well as to subsidize human capital and market resources employed to supply labor. Effective labor bears the burden of taxation necessary to finance government outlays.

**Proof**. When $\alpha+\ \beta = 1$, $W_k = 0$, $\eta_c = -\theta < 0$ (where $\theta > 0$ is the reciprocal of the elasticity of intertemporal substitution; that is, the inverse of the elasticity of demand for $c$), $\eta_n = 0$, and $\eta_h = -\sigma < 0$ (where $\sigma > 0$ is the inverse of the elasticity of demand for $h$ as consumption goods, taken in absolute value).[24]

From (21a), $\tau_k^* = 0$ as the pseudo-welfare function is independent of $k$.

Considering that $\eta_n = 0$ and $\eta_c = -\theta$, from (21b), we have

$$\tau_l^* = \frac{(\theta - 1)\Phi U_n}{[1 + \Phi(1-\theta)]F_L L_l T' U_c}. \tag{21b'}$$

[22]This case is considered by Jones, Manuelli and Rossi (1993 and 1997). The common formulation $L = hl$ (used, for example, by Lucas, 1990, Trostel, 1993, and Jones and Manuelli, 2001) — that is obtained if we assume that $\alpha = \gamma = 1$ and $\beta = 0$— can be associated with this case.

[23]This hypothesis is considered, among the other cases studied, by Judd (1999).

[24]It is assumed that $U(\ \cdot\ )$, strongly separable, is isoelastic in its arguments.

Since $\theta > 1$ from an empirical viewpoint,[25] $\tau_l^* > 0$. This implies that $\tau_z^* < 0$.

Finally, human capital should be unambiguously subsidized even if it is a final good as plausibly $\theta \gtrless \sigma$ (as $\theta$ is extremely high empirically).[26] $\square$

The findings of this case, that differ substantially from those obtained without human capital, can be explained as follows.

By reducing the after-tax wage, labor taxation lowers the opportunity cost of fertility, stimulates fertility and reduces time for working. This implies a rise of the marginal product of capital because the 'modified golden rule', which drives the capital to effective labor ratio down.

The subsidies on human capital and market goods employed to provide labor are necessary to offset the distortionary effect of labor taxation. The capital income tax rate is zero in this circumstance as the negative taxes on human capital and market goods are sufficient to compensate the labor tax distortions on an efficiency ground.

The subsidy on human capital can be viewed as a form of positive bequests that is left by parents to their children.

## 3.4 Optimal fiscal policy when $\alpha + \beta < 1$

In this circumstance, we can state that

In an immortal model of endogenous fertility with physical and human capital accumulation, when effective labor is homogeneous of degree less than one in human capital and market resources employed for working —i.e., $\alpha + \beta < 1$—, the optimal tax rate on physical capital is negative, while the optimal tax rate on effective labor is positive. Human capital should be subsidized too, independently of whether it is a final good or not, as market goods used to supply labor should be.

[25]According to Hall (1988), the intertemporal elasticity of substituion is moderately positive, being close to zero.

[26]Note that in the very special theoretical case in which $\theta = 1$ —i.e., the utility function is logarithmic in consumption— the optimal labor tax rate is zero, and if also $\sigma = 1$ —i.e., the utility function is logarithmic also in human capital— the optimal human capital tax rate is zero too. This case confirms the Jones, Manuelli and Rossi (1993 and 1997) results, although obtained from an endogenous fertility perspective.

**Proof.** When $\alpha+\ \beta<1$, $W_k>0$, $\eta_c<0$, $\eta_n>0$ and $\eta_h \lesseqgtr 0$. Therefore, from (21c), labor taxation is positive and higher than in the case in which $\alpha+\ \beta=1$. From (21a) and (21c), we get that $\tau_k^*$ and $\tau_z^*$ are negative; also $\tau_h^*$ is plausibly negative. □

The rationale for the above results (that extend beyond those obtained in an economy with physical capital only) is as follows.

A tax on labor, required to finance government expenditure, pulls the fertility rate and the marginal product of capital up. The distortionary effects of labor taxation can be corrected through the subsidization of $h$ and $z$. But, since $L(\ \cdot\ )$ is limited responsive to human capital and market goods employed to supply labor changes (as $\alpha+\beta<1$), capital should be subsidized too in order to constrast the distortive effect of the rise of labor taxation and avoid capital being underaccumulated. Physical and human capital subsidies reduce the increase of fertility (because they increase its opportunity costs through a positive effect on the marginal product of effective labor), thus contrasting the adverse effects of labor taxation.

Now the subsidies on both physical and human capital represent a way of increasing transfers to descendants.

In terms of analytical formalism, as in the case with physical capital only, the appearence of the stock of human and physical capital in the implementability constraint — due to an effective labor function that is decreasing returns to scale in $h$ and $z$— and the fact that per capita wealth enters the opportunity cost of fertility, are at the base of the violation of the Chamley (1986) and Jones, Manuelli and Rossi (1993 and 1997) results.

# 4 Concluding remarks

This paper has investigated the question of the optimal factor income taxation in intertemporal optimizing models with endogenous population growth. The analysis has considered infinitely-lived models of capital formation in which fertility, that enters the households' utility, is endogenously chosen and the agent's time is allocated between working and raising children. Two models of economic growth are studied: one with physical capital only, and one with physical and human capital. The model with human capital, which takes into account the trade-off between quality and quantity of descendants, has allowed us also to investigate the implications of the composite nature of labor on optimal taxation — which is the combination of human capital, market goods used to supply labor and working hours.

When only capital is present in the economy, the consideration of elastic fertility choices may invalidate the Chamley-Judd normative prescription of a zero tax on physical capital found in a neoclassical growth model with endogenous labor-leisure choices and a constant population growth rate. In fact, welfare maximization implies that capital should be subsidized in the steady state.

These general findings are in some way surprising as infinitely-lived models with endogenous fertility are similar to the corresponding models incorporating elastic labor-leisure choices and exogenous population growth. The results obtained here can be explained, for example, in the case of an economy with physical capital only, as follows. The division of time between working and looking after children, as supposed here, is basically the same as that in the standard neoclassical growth model, where time is divided between working and leisure. The analogy applies only to the instantaneous division of time between the two available activities in each scenario. The long-run implications of such a time-allocation choice are, instead, very different in the two models. In the standard model with elastic labor-leisure choice, increasing or decreasing time spent on leisure has no impact on population growth and hence on the intertemporal margin. In the model of this paper, instead, a change of the time spent on child-rearing, which is an indication of a variation of fertility, impacts on the 'modified golden rule'. Therefore, an increase in non-working time (leisure in the standard analysis and time used for child-rearing in the present one), induced by labor taxation, leaves the capital/labor ratio invariant in the traditional model, but lowers it in the model of this paper. The need for a subsidy on capital arises here precisely in order to offset such an effect of labor taxation.

In mathematical terms, unlike leisure fertility enters the consumer flow budget constraint not only through the time allocation constraint, but also through a nonlinear term (given by the population growth rate times capital per capita) that reflects the reduction in the capital-labor ratio due to population growth (i.e., the so-called dilution effect). This element generates a demand for fertility that depends on the per capita capital stock. Since the static efficiency condition for fertility enters the implementability constraint and hence the pseudo-welfare function of the social planner, the capital stock appears directly in the maximand function of the 'Ramsey problem', thus altering the Chamley-Judd optimal capital income tax rule.

When an economy with both physical and human capital is considered, the Jones, Manuelli and Rossi (1993 and 1997) results are modified. In fact, physical capital income should be exempted from taxation when effective labor is homogeneous of degree one in human capital and market resources used to provide labor. In this case, the tax treatment of physical and human capital is asymmetric as the latter should be subsidized (also when human capital is considered a final good). When effective labor is decreasing returns to scale in human capital and market goods, instead, the tax rates on physical and human capital should be both negative. The burden of taxation necessary to finance government spending should in any case fall on effective labor.

The above observations regarding the mathematical aspects and motivation of the results are also valid in the model with human and physical capital except for the case in which effective labor is linearly homogeneous in human capital and market goods. In this case, as physical capital no longer enters the pseudo-welfare function of the Ramsey planner, it should be exempted from taxation.

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**Highlights**

- ▶ Optimal income taxation is studied in immortal models with endogenous fertility.
- ▶ The analysis considers economies with physical and human capital accumulation.
- ▶ With physical capital only, optimality prescribes to subsidize capital and tax labor.
- ▶ With physical and human capital, optimality requires a subsidy on human capital.
- ▶ With human capital, physical capital should not necessarily be subsidized.