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Optimal inventory model in which the imperfect items will be repaired after the screening process

Milad Farhangi^a, Iraj Mahdavi^b, Behzad Maleki Vishkai^c, Yousef Rahmati Nodehi^d

^a Young Researchers and Elite Club, Qazvin Branch, Islamic Azad University, Qazvin, Iran

Phone: (+98) 281 3665275, Fax: (+98) 281 3665277, E-mail: m.Farhangi@qiau.ac.ir

^b Department of Industrial Engineering, Mazandaran University of Science & Technology, Babol, Iran

Phone: (+98) 11 32191205, Fax: (+98) 11 32191205, E-mail: rajarash@rediffmail.com

^c Department of Industrial Engineering, Mazandaran University of Science & Technology, Babol, Iran

Phone: (+98) 11 32191205, Fax: (+98) 11 32191205, E-mail: b.maleki.v@ustmb.ac.ir

^d Department of Management, Rahbord Shomal University, Rasht, Iran, E-mail: Yousef.zf@gmail.com

Abstract:

The aim of this paper is defining an optimal model for a retailer who uses a 100% screening process to reveal defective items which are received from the supplier. The demand rate is lower than the screening rate and the defective items will be sent to a repairing shop to be repaired. According to the fixing rate, the products will be sent back to the retailer after a while. During the fixing process the retailer may face shortages. The goal is finding the ordering amount to maximize the retailer's profit during the planning horizon. The mathematical model will be described and renewal-reward theorem will be used to solve the proposed model considering a numerical example.

Keywords: Economic order quantity, screening process, repairing process, shortage.

1. Introduction

Discovering the imperfect items in an inventory model can be mentioned as one of the most important assumptions that can help the inventory

owners to have a more realistic mathematical model. After that, finding a proper solution about managing the amount of defective items to avoid shortage and balancing the inventory level seems to be a great factor in reducing the cost of the system.

Defective items which are sold at the end of 100% screening process was studied by Salameh et al (2000). Chan et al (2003) developed a model by categorizing products into three different groups: good quality, good quality after reworking, imperfect quality and scrap. Chang (2004) extended Salameh's model considering fuzzy calculations. Wen Kai et al (2009) considered discount for Salameh's model and gained the optimal order size. Moreover, the model was extended by Wee et al (2007) via adding a shortage backordering assumption to it. There was an error in their model which was corrected by Hsu et al (2012). Chang et al (2010) used renewal-reward theorem to reach a new expected net profit per unit in Wee's model. Maddah and Jaber (2008) defined unreliable supply and random fraction of defective items and used screening process for it. Khan et al (2011) studied a model similar to Salameh's one considering an

imperfect inspection process according to Raouf et al (1983) to describe the defective proportion of the received lot. Haidar et al (2013) developed a model in which each order contains a random proportion of defective items. They used acceptance sampling plan and the lot was screened through a 100% screening only if the number of imperfect items were not below an acceptance number.

Vishkaei et al (2014) extended Hsu's model considering that the defective items will be stored until the end of each cycle and will be sent back to the supplier when the ordering vehicle arrives. Later some other assumptions like discount, permissible delay in payment and warehouse constrain were added to the model by Vishkaei et al (2014). Farhangi et al (2015) studied an optimal model for exchangeable imperfect items in which the delay time for the exchange process depends on the quantity of imperfect items.

In this paper, after finding the imperfective items through 100% screening, they will be repaired in supplier's repair shop and will be sent back to retailer's warehouse to be sold. In the next section the model will be completely defined and the mathematical formulation will be developed. It will be shown that the model can be studied in two cases: with shortage and without it. For each one of them, the optimal order size will be gained using renewal-reward theorem. It will be proved that the order quantities are unique. A numerical example will be solved for more illustration. Finally, conclusion and future studies will be discussed.

2. Model description

Consider a retailer who purchases his products from a supplier and after using 100% screening process, he will send defined defective items to a repair shop to fix them. The fixing time hinges on the repairing rate. If the repairing process lasts too much, the

retailer may face shortage. Otherwise, there is no backordering cost and the holding cost will be increased. The fixing cost will be paid by the supplier and the retailers pays nothing for it. The goal is determining the order size to maximize the total profit. The replenishment is instantaneously and the number of defective items in ordering batches follows a probabilistic distribution. Therefore, the expected profit value will be considered to solve the model.

2.1. Parameters and decision variables

The model parameters and decision variables are described as follows:

2.1.1. Parameters

D: The demand rate

p: The defective percentage in which $f(p)$ is its density function

X: The screening rate, $x > D$

r: The fixing rate

d: The screening cost per unit

b: The shortageing cost per unit per unit of time

h: The holding cost per unit per unit of time

S: The selling price per unit

C: The purchasing cost per unit

K: The ordering cost per order

T: The cycle time

t₁: The time that it lasts to screen the received orders in each cycle

t₂: The time it lasts to fix the defective items in repair shop in each cycle



t_3 : The remaining time of each cycle when the repaired items are sent back

t_4 : Part of the cycle time which there is no inventory in the warehouse

2.1.2. Decision variables:

Q : Order size of product

2.2. Inventory model without shortage

Figure 1 illustrates the inventory model when the repaired items are sent back to the retailer before that the inventory level reaches zero. In other words, in this case the retailer doesn't face shortage and the backordering cost equals zero. According to figure 1, $t_1 = \frac{Q}{x}$, $t_2 = \frac{PQ}{r}$ and $T = \frac{Q}{D}$ therefore, $t_3 = \frac{Q}{D} - (\frac{Q}{x} + \frac{PQ}{r})$.

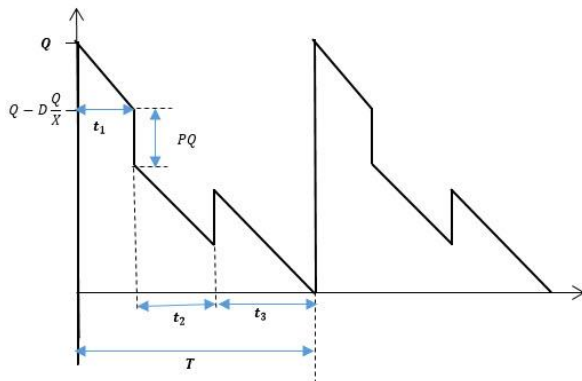


Fig.(1). Behavior of the proposed inventory model when there is no shortage

The holding cost of defective items during t_1 , the holding cost of perfect items during t_1, t_2 and the holding cost during t_3 can be formulated as $PQ \cdot \frac{Q}{x}$, $[(1-P) \cdot Q - \frac{1}{2}D \cdot (\frac{Q}{x} + \frac{PQ}{r})] (\frac{Q}{x} + \frac{PQ}{r})$ and $\frac{1}{2}[(1-P)Q - D(\frac{Q}{x} + \frac{PQ}{r})] + PQ \cdot (\frac{Q}{D} - \frac{Q}{x} - \frac{PQ}{r})$

respectively. So the holding cost of each period can be calculated as

$$Th = PQ \cdot \frac{Q}{x} + [(1-P) \cdot Q - \frac{1}{2}D \cdot (\frac{Q}{x} + \frac{PQ}{r})] (\frac{Q}{x} + \frac{PQ}{r}) + \frac{1}{2}[(1-P)Q - D(\frac{Q}{x} + \frac{PQ}{r})] + PQ \cdot (\frac{Q}{D} - \frac{Q}{x} - \frac{PQ}{r}) \quad (1)$$

The ordering cost equals K , the purchasing cost, the screening process cost and the gross profit of each cycle equals cQ , dQ and sQ respectively. The net profit that is gained during each cycle by the retailer can be gained via the following formulation

$$TP(Q) = SQ - K - cQ - dQ - h [PQ \cdot \frac{Q}{x} + [(1-P) \cdot Q - \frac{1}{2}D \cdot (\frac{Q}{x} + \frac{PQ}{r})] (\frac{Q}{x} + \frac{PQ}{r}) + \frac{1}{2}[(1-P)Q - D(\frac{Q}{x} + \frac{PQ}{r})] + PQ \cdot (\frac{Q}{D} - \frac{Q}{x} - \frac{PQ}{r})] \quad (2)$$

Now, the expected profit value per cycle will be considered to gain the order size. According to the Renewal Theorem, $\frac{E[TP(Q)]}{E(T)}$ is the expected profit per unit time. As the expected cycle time $E[T]$ equals to $\frac{Q}{D}$, the expected profit per time, $ETP(Q)$ is

$$ETP(Q) = \frac{E[TP(Q)]}{E[T]} = SD - \frac{KD}{Q} - CD - dD - h \left(\frac{E[P]DQ}{x} + QD \left((1 - E[P]) - \frac{1}{2}D \left(\frac{1}{x} + \frac{E[P]}{r} \right) \right) \left(\frac{1}{x} + \frac{E[P]}{r} \right) + \frac{1}{2}QD \left((1 - E[P]) - D \left(\frac{1}{x} + \frac{E[P]}{r} \right) + E[P] \right) \left(\frac{1}{D} - \frac{1}{x} - \frac{E[P]}{r} \right) \right) \quad (3)$$

Equation 3, can be revised as

$$ETP(Q) = SD - \frac{KD}{Q} - CD - dD - h \left(\frac{E[P]QD}{x} + QD \left(\frac{2Xr^2 + 2X^2E[P]r - 2Xr^2E[P] - 2X^2E[P]^2r - Dr^2 - 2DrE[P]X - DX^2E[P]^2}{r^2X^2} \right) + \frac{1}{2}QD \left(\frac{X^2r^2 + D^2r^2 + D^2E[P]^2X^2 - 2XD r^2 - 2DE[P]rX^2 + 2D^2rE[P]X}{DX^2r^2} \right) \right) \quad (4)$$

To gain the optimal order size the first and second derivatives of $ETP(Q)$ with respect to Q are gained respectively. By simplifying equation $\frac{\sigma[ETP(Q)]}{\sigma Q} = 0$, the optimal order quantity will be gained through the following equation

$$\frac{DX^2r^2 - 2Xr^2D^2E[P] - 2rX^2D^2E[P]^2 + 2XD^2r^2E[P]}{2DX^2r^2} = \frac{KD}{Q^2} \quad (5)$$

Therefore, the optimal order size equals to equation 6.

$$Q^* = \sqrt{\frac{2KD}{h(1 - \frac{2D}{r} E[P^2])}} \quad (6)$$

As it is shown in equation 7, the second derivative of $ETP(Q)$ is negative which proves there is a unique Q for the model to maximize the net profit.

$$\frac{\sigma^2[ETP(Q)]}{\sigma^2 Q} = -\frac{2KD}{Q^3} \quad (7)$$

Obviously, if there is no defective items in the model which means $E[p]=0$, then the model converts to an

$$EOQ \text{ model and } Q^* = \sqrt{\frac{2KD}{h}}$$

2.3 Inventory model with shortage

Figure 2, shows the behavior of the proposed inventory model when the retailer faces shortage because the repaired items are sent back to him after that the inventory level reaches zero. Note that there are shortages in the system when $\frac{Q}{X} + \frac{PQ}{r} \geq \frac{(1-P)Q}{D}$ which can be implied as $\frac{1}{X} + \frac{E[P]}{r} \geq \frac{(1-E[P])}{D}$. In this case, the holding cost of defective items during t_1 , the holding cost of perfect items during t_1, t_2 and the holding cost during t_3 can be formulated as $PQ \cdot \frac{Q}{X}$, $\frac{(1-P)^2 Q^2}{2D}$ and $\frac{1}{2} \left[PQ - D \left(\frac{PQ}{r} + \frac{Q}{X} - \frac{(1-P)Q}{D} \right) \right] \left(\frac{Q}{D} - \frac{PQ}{r} - \frac{Q}{X} \right)$ respectively.

So the holding cost of each period can be calculated as

$$Th = PQ \cdot \frac{Q}{X} + \frac{(1-P)^2 Q^2}{2D} + \frac{1}{2} \left(PQ - D \left(\frac{PQ}{r} + \frac{Q}{X} - \frac{(1-P)Q}{D} \right) \right) \left(\frac{Q}{D} - \frac{PQ}{r} - \frac{Q}{X} \right) \quad (8)$$

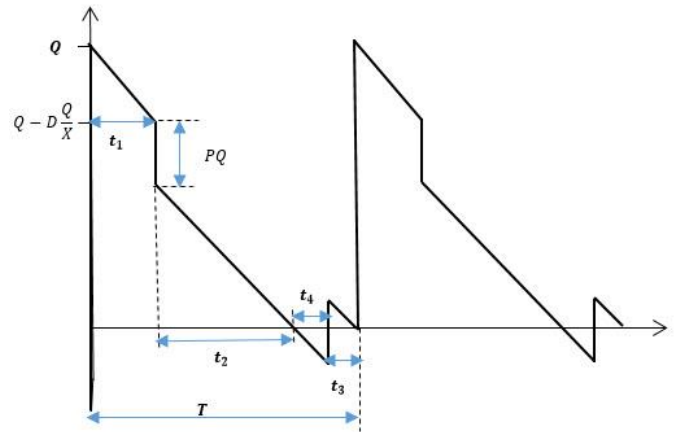


Fig.(2).Behavior of the proposed inventory model when there is shortage

The shortage cost per cycle which occurs during t_4 is given as

$$Tb = \frac{1}{2} bD \left(\frac{PQ}{r} + \frac{Q}{X} - \frac{(1-P)Q}{D} \right) \left(\frac{PQ}{r} + \frac{Q}{X} - \frac{(1-P)Q}{D} \right) \quad (9)$$

Other costs can be exactly formulated just like the previous section. Therefore, the net profit can be formulated as follows

$$TP(Q) = SQ - K - CQ - dQ - h \left(PQ \cdot \frac{Q}{X} + \frac{(1-P)^2 Q^2}{2D} + \frac{1}{2} \left(PQ - D \left(\frac{PQ}{r} + \frac{Q}{X} - \frac{(1-P)Q}{D} \right) \right) \left(\frac{Q}{D} - \frac{PQ}{r} - \frac{Q}{X} \right) \right) - \frac{1}{2} bD \left(\frac{PQ}{r} + \frac{Q}{X} - \frac{(1-P)Q}{D} \right) \left(\frac{PQ}{r} + \frac{Q}{X} - \frac{(1-P)Q}{D} \right) \quad (10)$$

$E[T]$ equals to $\frac{Q}{D}$ and the expected profit per time is

$$ETP(Q) = \frac{E[TP(Q)]}{E[T]} = SD - \frac{KD}{Q} - CD - dD - hQD \left(\frac{E[P]}{X} + \frac{(1-E[P])^2}{2D} + \frac{1}{2} [E[P] - D \left(\frac{E[P]}{r} + \frac{1}{X} - \frac{1-E[P]}{D} \right)] \left(\frac{1}{D} - \frac{E[P]}{r} - \frac{1}{X} \right) \right) - \frac{1}{2} bQD^2 \left(\frac{E[P]}{r} + \frac{1}{X} - \frac{1-E[P]}{D} \right) \left(\frac{E[P]}{r} + \frac{1}{X} - \frac{1-E[P]}{D} \right) \quad (11)$$

Equation 11, can be revised as

$$ETP(Q) = SD - \frac{KD}{Q} - CD - dD - h \left(\frac{E[P]DQ}{X} + \frac{(1-E[P])^2Q}{2} + \frac{1}{2} QD \left(\frac{(rX-rD-E[P]DX)(rX-rD-E[P]DX)}{Dr^2X^2} \right) \right) - \frac{1}{2} bQD^2 \left(\frac{E[P]^2}{r^2} + \frac{1}{X^2} + \frac{1-2E[P]+E[P]^2}{D^2} + \frac{2P}{rX} - \frac{2E[P](1-E[P])}{rD} - \frac{2(1-E[P])}{XD} \right) \quad (12)$$

By simplifying equation $\frac{\sigma[TPU(Q)]}{\sigma Q} = 0$ and considering $R_1 = \frac{1}{2} + \frac{1}{2} E[P]^2$, $R_2 = E[P] - 1$, $R_3 = 1 - E[P] + \frac{1}{2} E[P]^2$ and $R_4 = E[P]^2 - E[P]$ the optimal order quantity will be gained through the following equation

$$\frac{KD}{Q^2} \cdot h \left\{ \frac{D^2}{r^2} \left(E[P] \left(\frac{r}{X} - \frac{r}{D} \right) + R_1 \right) + \frac{D}{X} \left(R_2 + \frac{1}{2} \cdot \frac{D}{X} \right) + R_3 + \frac{1}{2} \right\} \cdot b \left\{ \frac{D^2}{r^2} \left(\frac{E[P]^2}{2} + \frac{r}{X} E[P] + \frac{rR_4}{D} \right) + \frac{D}{X} \left(\frac{1}{2} \cdot \frac{D}{X} + R_2 \right) + R_3 \right\} = 0 \quad (13)$$

Therefore, the optimal order size equals to

$$Q^* = \sqrt{\frac{KD}{h \left(\frac{D^2}{r^2} \left(E[P] \left(\frac{r}{X} - \frac{r}{D} \right) + R_1 \right) + \frac{D}{X} \left(R_2 + \frac{D}{2X} \right) + R_3 + \frac{1}{2} \right) + b \left(\frac{D^2}{r^2} \left(\frac{E[P]^2}{2} + \frac{r}{X} E[P] + \frac{rR_4}{D} \right) + \frac{D}{X} \left(\frac{D}{2X} + R_2 \right) + R_3 \right)}} \quad (14)$$

According to equation 15, the second derivative of $ETP(Q)$ is a negative quantity which proves there is a unique Q for the model to maximize the net profit.

$$\frac{\sigma^2[ETP(Q)]}{\sigma^2 Q} = -\frac{2KD}{Q^3} \quad (15)$$

3. Numerical example

For more illustration a numerical example is studied. Table 1 includes the parameters.

Table(1). Parameters of the numerical example.

h	910
D	200000
d	250
b	3000
X	480000
r	300000
C	4550
S	60000
K	1500000

The percent of defective items follows a uniform distribution in the interval $[0.01, 0.05]$. So, $E[P] = \frac{0.01+0.05}{2} = 0.03$. To find the type of the model, inequality $\frac{1}{X} + \frac{E[P]}{r} \geq \frac{(1-E[P])}{D}$ should be checked. As $2 \times 10^{-6} \ll \frac{1 - \frac{200.000}{480.000}}{1 - \frac{200.000}{300.000}} = 1.73$, there is no shortage and to gain the optimal order size equation (6) will be used.

$$Q^* = \sqrt{\frac{2(1.500.000)(200.000)}{(910)(1 - \frac{2(200.000)}{300.000})(0.00104)}} = 25695 \quad (16)$$



4. Conclusion

This paper discussed a retailer's inventory model in the goal of maximizing its profit by determining a proper order size. All the received items from the supplier are screened through a 100% screening process and the defective items are sent to a repair shop to get fixed. The retailer pays nothing for the repairing process and the repair shop is related to the supplier. Shortage occurs when during the repairing process the inventory level reaches zero. Both of the situations: inventory model with no shortage and inventory model with shortage are discussed and renewal-reward theorem is used to gain optimal order sizes. It is proved that the gained optimal order sizes are unique answers for the proposed models. Future research may focus primarily on the following points:

- Considering more than one supplier with different defective rate.
- Considering more than one repair shop with different repairing rate.
- Considering warehouse and budget constraints

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