International Journal of Applied Operational Research
Vol. 1, No. 2, pp. 57-64, Autumn 2011

Journal homepage: www.ijorlu.ir

Increasing the Reliability and the Profit in a Redundancy Allocation Problem

M. EbrahimNezhad ∗ , B. Maleki Vishkaei, H. R. Pasandideh, J. Safari

Received: May 24, 2011 ; **Accepted:** September 17, 2011

Abstract This paper proposes a new mathematical model for multi-objective redundancy allocation problem (RAP) without component mixing in each subsystem when the redundancy strategy can be chosen for individual subsystems. Majority of the mathematical model for the multi-objective redundancy allocation problems (MORAP) assume that the redundancy strategy for each subsystem is predetermined and fixed. In general, active redundancy has received more attention in the past. However, in practice both active and cold-standby redundancies may be used within a particular system design and the choice of the redundancy strategy becomes an additional decision variable. The proposed model for MORAP simultaneously maximizes the reliability and the net profit of the system. And finally, to clarify the proposed mathematical model a numerical example will be solved.

Keywords Redundancy Allocation Problem, Serial-Parallel System, Redundancy Strategies, MORAP.

1 Introduction

The primary goal of reliability engineering is to improve the reliability of system. In the initial design activity, the redundancy allocation is a direct way of enhancing system reliability. The redundancy allocation problem involves the simultaneous selection of components and a system-level design configuration, which can collectively meet all design constraints in order to optimize some objective functions such as system cost and/or reliability. RAP can be categorized into RAP without component mixing [1, 2] and RAP with mix of components [3-9]. Some researchers studied the RAP when the redundancy strategy can be chosen for individual subsystems [10-12].

MORAP is studied by many researchers. Basacca et al. [13] used a multi-objective genetic algorithm which allows the decision maker to identify set of pareto optimal solutions. They present a new model for maximizing the net profit and system reliability. Coit et al. [14] studied a MORAP to maximize system reliability and minimize the estimation of reliability. Coit et al. [15] used a multi-objective genetic algorithm to find pareto optimal solutions with three different goals (maximum system reliability and minimum cost and weight). Finally, the Pareto optimal solutions are prioritized based on the decision maker's objective function preferences. Coit et al. [16] considered a MORAP to maximize the reliability of each

∗ **Corresponding Author.** (✉)
E-mail: M.Ebrahimnazhad@gmail.com (M. EbrahimNezhad)

M. EbrahimNezhad, B. Maleki Vishkaei, H. R. Pasandideh
Department of Industrial Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran.

J. Safari
Department of Industrial Engineering, Karaj Branch, Islamic Azad University, Karaj, Iran.

individual subsystem. Konak et al. [17] considered a problem to determine the optimal design configuration to maximize system reliability and minimize system cost, considering weight constraint and constraint for the number of dedicated components to each subsystem. They used a multinomial Tabu Search to solve this problem. Konak et al. [18] considered a multi-objective RAP to maximize the reliability and minimize the cost. They used Tabu Search approach to initially find the entire Pareto set and then pruned the set of Pareto solutions by a Monte-Carlo simulation based on the decision marker's predefined objective function preferences. Taboada et al. [19] solved the same problem by using a new multiple objective evolutionary algorithm (MOEA) which mainly differs from other MOEAs. Taboada et al. [20] studied MORAP to maximize the overall system reliability and minimize the system cost and weight. It was initially solved by using the Non-dominate Sorting Genetic Algorithm (NSGA). They proposed two pruning methods to reduce the size of the Pareto optimal set. Ling et al. [21] developed a multi-objective variable neighborhood search (MOVNS) algorithm for solving MORAP. They verified the performance of the proposed algorithm by testing it on three sets of complex instances with different subsystems. Lins et al. [22] studied a MORAP subject to imperfect repairs and system availability. They used a multi-objective genetic algorithm to solve this problem and compared result with multi-objective Ant colony algorithm. In all these introduced multi-objective papers, the active strategy is predetermined for subsystems.

Based on the single objective model which is presented by Coit [10], in this paper a new MORAP will be proposed. The main difference of this study with other MORAP is considering each subsystems strategy as a decision variable.

The structure of this paper is organized as follows; Sections 2 and 3 present the problem definition and the problem modeling, respectively. In Section 4 a numerical example is solved. Finally, conclusion and future research is presented.

2 Problem definition

The problem that is studied in this paper is the result of adding a new objective function to the RAP without component mixing that is presented by Coit [10]. This new objective function calculates the pure profit that is gained during a limit period, considering different costs like purchasing, penalties during downtime and damage cost of the components. The objective is to determine the strategy and choose the element and redundancy-level for each subsystem to maximize system reliability and profit of system subject to cost and weight constraints.

3 Problem modeling

To formulate the problem, the parameters first are defined in section 3.1. The new objective function is then derived in section 3.2. Finally, the mathematical model is developed in section 3.3.

3.1 Parameters

The parameters of the model are defined as follows.

A set of all subsystems with active redundancy,

S	set of all subsystems with cold-standby redundancy,
N	set of all subsystems with no redundancy.
s	number of subsystems
n_i	number of components used in subsystems i(i=1,2,3,…,s)
n	set of $\mathrm{n_i}\,(n_1, n_2, ..., n_s)$
$\mathrm{z_i}$	index of the component that is dedicated to subsystem i
z	set of $\mathrm{z_i}\ (z_1, z_2, ..., z_s)$
T	mission time
$\mathrm{R(t, z_i)}$	system reliability at time t
$\mathrm{r_{ij}(t)}$	reliability at time t for component j in subsystem i
λ_{ij}, k_{ij}	scale and shape parameters for the Erlang distribution; $\mathrm{f_{ij}}(t) = (\lambda_{ij}^{\kappa_{ij}} t^{\kappa_{ij}-1} e^{-\lambda_{ij} t}) / \Gamma(\kappa_{ij})$
C, W	system constraint limits for cost and weight
$\mathrm{c_{ij}}, w_{ij}$	purchasing cost and weight for the jth available component for the subsystem i
$\rho_{\mathrm{i}}(t)$	failure switching reliability at time t
$\mathrm{p_t}$	the amount of money per unit time paid by the customer for the plan service
TP	total profit from plant operation
$\mathrm{B_i}$	installation cost per each component
$\mathrm{TC_p}$	total purchase and installation costs
$\mathrm{c_{NS}}$	penalty cost during downtime, due to missed delivery of agreed service
$\mathrm{Tc_{NS}}$	total penalties during downtime for period T
$\mathrm{c_d}$	damage cost per each component
$\mathrm{Tc_d}$	total damaged cost during period T

3.2 Objective functions

The profit function contains the plant profit, purchasing and installation cost, penalties during downtime and the damage cost.

$$\mathrm{TP} = \mathrm{P_t} . \int_{\circ}^{\mathrm{T}} \mathrm{R(t)} \tag{1}$$

Equation 1 is the plant profit in which p_t represents the amount of money per until time paid by the customer for the plant service, and R(t) is the instantaneous plant reliability.

$$\mathrm{TC_p} = \sum_{\mathrm{i=1}}^{\mathrm{s}} \mathrm{n_i} (\mathrm{C_{iz_i}} + \mathrm{B_{iz_i}}) \tag{2}$$

Equation 2 is the purchasing and installation cost of the s nodes in which the $\mathrm{i^{th}}$ of them constituted of n_i components.

$$\mathrm{TC_{NS}} = \mathrm{C_{NS}} . \int_{\circ}^{\mathrm{T}} (1 - \mathrm{R(t)}) \tag{3}$$

Equation 3 is the amount of money to be paid to the customer because of missed delivery of the agreed service when the plant is unavailable.

$$TC_d = \sum_{i=1}^{S} \min\left\{n_i, \lambda_{iz_i} K_{iz_i} T\right\} C_s = \sum_{i=1}^{S} U_i \tag{4}$$

Equation 4 is the total damage cost during period T. The component time – to-failure is distributed according to Erlang distribution, so $\lambda_{iz_i}\kappa_{iz_i}T$ is the average failed components during period T for the ith subsystem. As all the dedicated components may fail before finishing period T, the number of damaged components for subsystem i would be $\min\{n_i, \lambda_{iz_i}\kappa_{iz_i}T\}$

Equation (4) can be changed as equation(5):

$$TC_d = \sum_{i=1}^{S} U_i \tag{5}$$

In which u_i can be shown as:

$$U_i \le \lambda_{iz_i}\kappa_{iz_i}Tc_s \ \ \& \ U_i \le n_i.c_s \tag{6}$$

The net profit objective function can be then written as follows:

$$G = P_t \int_{\circ}^{T} R(t) - (\sum_{i=1}^{s} n_i(C_{iz_i} + B_{iz_i}) + C_{NS}\int_{\circ}^{T}(1 - R(t)) + \sum_{i=1}^{S} \min\left\{n_i, \lambda_{iz_i} K_{iz_i} T\right\} C_s) \tag{7}$$

According to equations (5):

$$G = P_t \int_{\circ}^{T} R(t) - (\sum_{i=1}^{s} n_i(C_{iz_i} + B_{iz_i}) + C_{NS}\int_{\circ}^{T}(1 - R(t)) + \sum_{i=1}^{S} U_i)$$

St :

$$U_i \le \lambda_{iz_i}.k_{iz_i}.T.c_s \qquad \forall i$$

$$U_i \le n_i.c_s \qquad \forall i$$

3.3 The mathematical model

The proposed mathematical model is described as follow:

$$\text{Max} \quad R(T) = \prod_{i\in S}(r_{iz_i}(T) + \rho(T)e^{-\lambda_{iz_i}T} \times \sum_{l=k_{iz_i}}^{k_{iz_i}n_i-1} \frac{(T\times\lambda_{iz_i})^l}{l!}) \times \prod_{i\in A}(1-(1-r_{iz_i}(T))^{n_i} \times \prod_{i\in N} r_{iz_i}(T) \tag{1}$$

$$\text{Max} \quad G = P_t \int_{\circ}^{T} R(t) - (\sum_{i=1}^{s} n_i(C_{iz_i} + B_{iz_i}) + C_{NS}\int_{\circ}^{T}(1 - R(t)) + \sum_{i=1}^{S} U_i) \tag{2}$$

s.t.

$$\sum_{i=1}^{S} C_{iz_i} n_i \le C, \tag{3}$$

$$\sum_{i=1}^{S} w_{iz_i} n_i \le W, \tag{4}$$

$$U_i \le n_i C_s, \qquad i = 1,2,....,S, \tag{5}$$

$$U_i \le C_s \lambda_{iz_i} K_{iz_i} T, \qquad i = 1,2,....,S. \tag{6}$$

(1): Maximize the reliability of the system in which the first, second and third term of the equation denotes the reliability for the subsystem with cold redundancy strategy, active strategy and no redundant Coit [10].
(2): Maximize the net profit value.
(3): Purchasing cost constraint.
(4): Weight constraints.
(5),(6): $u_i = \min\{ n_i, \lambda_{iz_i} \kappa_{iz_i} T \} C_s$ is replaced with the two constraint
$u_i \geq n_i C_s, u_i \geq \lambda_{iz_i} \kappa_{iz_i} T . C_s$

This model belongs to Np-hard class and solving it by exact methods is not possible, a small example considering two subsystems will be solved whit exact method in the next section. All possible amounts for the variables are considered and the problem will be solved for all the possible combination of these variable amounts. Finally, the set of pareto optimal solution will be calculated among these possible solutions and a unique solution will select among the set of solution using normalize method.

4 Numerical example

In this example, a series-parallel system whit two parallel subsystems is considered. In each subsystem three or four components type can be assigned. Component cost, weight and Erlang distribution parameters (λ_{ij}, k_{ij}) are shown in table 1. The objectives are maximization system reliability and profit at 100 hours, subject to purchase cost constraint (C=230max) and the system weight constraint (W=270max). Active or cold-standby redundancy can be used for each subsystem. Reliability to switch to non-failed components is 0.99 for all subsystems whit cold-standby redundancy. Maximum number of components which can be assigned in each subsystem are 2. $p_t = 2000000\$, c_{NS} = 100\$$ and installation costs are shown in Table 2. The small example is solved for different possible solutions. Table 3 shows the 81 possible solutions. A normalize method is used to select a unique solution from the set of Pareto optimal solutions. Considering F_1 as the best gained reliability and F_2 as the best gained net profit value, the set of Pareto optimal solutions can be normalized by; $\sqrt{(x_i - F_1)^2 + (y_i - F_2)^2}$ in which the x_i and y_i are the amounts of the first and second objective for the *i*th optimal solution, respectively. The best solution has the lowest normalized amount. In the proposed example, 41 in table 3, contains the biggest amounts for both reliability and net profit value in contrast with other possible solutions.

Table 1 Input of the small example

i	Choice(j)											
	1				2				3			
	λ_{ij}	k_{ij}	c_{ij}	w_{ij}	λ_{ij}	k_{ij}	c_{ij}	w_{ij}	λ_{ij}	k_{ij}	c_{ij}	w_{ij}
1	0.00532	2	1	3	0.000726	1	1	4	0.00499	2	2	2
2	0.00818	3	2	8	0.000619	1	1	10	0.00431	2	1	9

Table 2 Installation costs for each subsystem

Subsystem	Choice(j)		
	1	2	3
	β_{iz_i}	β_{iz_i}	β_{iz_i}
1	82	21	17
2	15	93	13

Table 3 Possible solutions and the objective amounts of them for the proposed small example

solution	Subsys1			Subsys2			Reliability	Net Profit	solution	Subsys1			Subsys2			Reliability	Net Profit
	ARS	z_1	n_1	ARS	z_2	n_2				ARS	z_1	n_1	ARS	z_2	n_2		
1	N	1	1	N	1	1	0.3363496	672332.8	42	A	2	2	N	3	1	0.3363496	672460.8
2	N	2	1	N	2	1	0.3363496	672083.8	43	A	3	2	N	3	1	0.99995	1999828
3	N	3	1	N	3	1	0.999998	1999202	44	S	1	2	N	3	1	0.3363496	672460.8
4	N	1	1	N	2	1	0.3363496	672083.8	45	S	2	2	N	3	1	0.3363496	672460.8
5	N	1	1	N	3	1	0.3363496	671834.8	46	S	3	2	N	3	1	0.99994	1999828
6	N	1	1	A	2	2	0.9399769	1879218	47	A	1	2	A	1	2	0.9399769	1879710
7	N	2	1	N	1	1	0.3363496	672332.8	48	A	1	2	A	2	2	0.9399769	1879685
8	N	2	1	N	3	1	0.3363496	671834.8	49	A	1	2	A	3	2	0.9399769	1879660
9	N	1	1	A	3	2	0.9399769	1879047	50	A	1	2	S	1	2	0.6043351	1208534
10	N	3	1	N	1	1	0.99997	1999700	51	A	1	2	S	2	2	0.6043351	1208491
11	N	3	1	N	2	1	0.999995	1999451	52	A	1	2	S	3	2	0.6043351	1208449
12	N	1	1	S	1	2	0.6043351	1208313	53	A	2	2	A	1	2	0.9963972	1992556
13	N	1	1	A	1	2	0.9399769	1879389	54	A	2	2	A	2	2	0.9963972	1992531
14	N	1	1	S	3	2	0.6043351	1207838	55	A	2	2	A	3	2	0.9963972	1992506
15	N	1	1	S	2	2	0.6043351	1208075	56	A	2	2	S	1	2	0.8730956	1746082
16	N	2	1	A	1	2	0.9963972	1992235	57	A	2	2	S	2	2	0.8730956	1746039
17	N	2	1	A	2	2	0.9963972	1992064	58	A	2	2	S	3	2	0.8730956	1745997
18	N	1	1	S	3	2	0.6043351	1207838	59	A	3	2	A	1	2	0.9975796	1994921
19	N	2	1	A	3	2	0.9963972	1991893	60	A	3	2	A	2	2	0.9975796	1994896
20	N	2	1	S	1	2	0.8730956	1745861	61	A	3	2	A	3	2	0.9975796	1994871
21	N	2	1	S	3	2	0.8730956	1745386	62	A	3	2	S	1	2	0.98801	1975922
22	N	3	1	A	1	2	0.9975796	1994600	63	A	3	2	S	2	2	0.98801	1975880
23	N	3	1	A	2	2	0.9975796	1994429	64	A	3	2	S	3	2	0.98801	1975837
24	N	3	1	A	3	3	0.9969435	1992792	65	A	1	2	A	3	2	0.9399769	1879660
25	N	3	1	S	1	2	0.98801	1975701	66	A	2	2	A	3	2	0.9963972	1992506
26	N	3	1	S	2	2	0.98801	1975464	67	S	1	2	A	3	2	0.9399769	1879660
27	N	3	1	S	3	2	0.98801	1975226	68	S	2	2	A	3	2	0.9963972	1992506
28	A	1	2	N	1	1	0.3363496	672551.9	69	S	3	2	A	3	2	0.9975796	1994871
29	A	2	2	N	1	1	0.3363496	672551.9	70	S	2	2	A	1	2	0.9963972	1992556
30	S	1	2	S	3	2	0.6043351	1208449	71	S	2	2	A	2	2	0.9963972	1992531
31	S	1	2	N	1	1	0.3363496	672551.9	72	S	2	2	A	3	2	0.9963972	1992506
32	S	2	2	N	1	1	0.3363496	672551.9	73	S	2	2	S	1	2	0.8730956	1746082
33	S	3	2	N	1	1	0.99997	1999919	74	S	2	2	S	2	2	0.8730956	1746039
34	A	1	2	N	2	1	0.3363496	672506.4	75	S	2	2	S	3	2	0.8730956	1745997
35	A	2	2	N	2	1	0.3363496	672506.4	76	S	3	2	A	1	2	0.9975796	1994921
36	A	3	2	N	2	1	0.99996	1999874	77	S	3	2	A	2	2	0.9975796	1994896
37	S	1	2	N	2	1	0.3363496	672506.4	78	S	3	2	S	2	2	0.98801	1975880
38	S	2	2	N	2	1	0.3363496	672506.4	79	S	3	2	S	3	2	0.98801	1975837
39	S	3	2	N	2	1	0.999998	1999874	80	S	1	2	A	1	2	0.9399769	1879710
40	A	1	2	N	3	1	0.3363496	672460.8	81	S	1	2	A	2	2	0.9399769	1879685
41	A	3	2	N	1	1	0.999999	1999919									

5 Conclusion and future research

In this paper, a new MORAP with choice of redundancy strategies has been studied. We solved a small example considering all the possible solutions. After gaining the set of Pareto optimal solutions, we normalized them to filter the set and reach a unique solution. Solving the model for large examples with exact methods is not possible. Therefore, heuristic algorithms like NSGAII is a good method to solve the problem.

References

1. Coit, D. W., (2001). Cold Standby Redundancy Optimization for Non-repairable Systems. IEEE Transactions, 33, 471 – 478.
2. Coit, D. W., Liu, J., (2000). System Reliability Optimization with k – out – of – n Subsystems. International Journal of Reliability, Quality & Safety Engineering, 7, 129 – 143.
3. Yalaoui, A., Chu, C., Chatelet, E., (2005). Reliability Allocation Problem in Series – Parallel System. Reliability Engineering and System Safety, 90, 55 – 61.
4. Ramirez Marquez, J. E., Coit, D. W., (2004). A Heuristic for Solving the Redundancy Allocation Problem for Multi – State Series – Parallel Systems. Reliability Engineering and System Safety, 83, 341 – 349.
5. Coit, D. W., Smith, A., (1995). Optimization approaches to the redundancy allocation to the redundancy allocation problem for series – parallel systems. Proceedings of the fourth industrial engineering research conference.
6. Coit, D. W., Smith, A., (1996). Solving the Redundancy Allocation Problem Using a Combined Neural Network / Genetic Aalgorithm Approach. Ccomputers & Operations Research, 23, 515 – 526.
7. Kulturel Konak, S., Smith, A., Coit, D. W., (2003). Efficiently Solving the Redundancy Allocation Problem Using Tabu Search. IIE Transactions, 35, 515 – 526.
8. Liang, Y. C., Wu, C. C., (2005). A Variable Neighborhood Descent Algorithm for the Redundancy Allocation Problem. Industrial Engineering Manage System, 4(1), 109 – 116.
9. Onishi, J., Kimura, S., James, R. J. W., Nakagawa, Y., (2007). Solving the Redundancy Aallocation Problem with a Mix of Components Using the Iimproved Surrogate Constraint method. IEEE transactions on reliability, 56(1), 94 – 101.
10. Coit, D. W., (2003). Maximization of System Reliability with a Choice of Redundancy Strategies. IIE Transactions, 35(6), 535 – 544.
11. Tavakkoli – Moghaddam, R., Safari, J., Sassani, F., (2008). Reliability Optimization of Series – Parallel Systems with a Cchoice of Redundancy Strategies Using a Genetic Algorithm. Reliability Engineering and System Safety, 93, 550 – 556.
12. Tavakkoli – Moghaddam, R., Safari, J., (2007). A New Mathematical Model for a Redundancy Allocation Problem with Mixing Components Redundant and Choice of Redundancy Strategies. Applied Mathematical Sciences, 45(1), 2221 – 2230.
13. Busacca, P. G., Marseguerra, M., Zio, E., (2001). Multi-objective Optimization by Genetic Algorithm: Application to Safety Systems. Reliability Engineering and System Safety, 72, 59 – 74.
14. Coit, D. W., Jin, T., (2001). Multi Criteria Optimization: Maximization of a System Reliability Estimate and Minimization of the Estimate Variance. IEEE Congress on Evolutionary Computation.
15. Baheranwala, F., Konak, S. K., Coit, D. W., (2004). Solution of Stochastic Multi-objective System Reliability Design Problems Using Genetic Algorithm, IEEE Transaction on Reliability.
16. Coit, D. W., Konak, A., (2006). Multiple Weighted Objectives Heuristic for the Redundancy Allocation Problem. IEEE Transactions on Reliability, 55, 3.
17. Kulturel Konak, S., Smith, S., Norman, A., (2006). Multi-Objective Tabu Search Using a Multinomial Probability Mass Function. European Journal of Operational Research.
18. Kulturel Konak, S., Coit, D. W., (2007). Determination of Pruned Pareto Sets for the Multi-objective System Redundancy Allocation Problem. IEEE Transactions on Reliability, pp 390-394.
19. Taboada, H. A., Coit, D. W., 2008. Development of a New Multiple Objective Prioritized Genetic Algorithm. Proceedings of the 2008 Industrial Engineering Research Conference.

20. Taboada, H. A., Baheranwala, F., Coit, D. W., Wattanapongsakorn, N., (2007). Practical Solutions for Multi-objective Optimization: An Application to System Reliability Design Problems. Reliability Engineering and System Safety, 92, 314-322.
21. Liang, Y. C., LO, M. H., (2010). Multi Objective Redundancy Allocation Optimizationusing a Variable Neighborhood Search Algorithm. Journal of Heuristics, 16(3), 511-535.
22. Lins, I. D., Droguett, E. L., (2011). Redundancy Allocation Problems Considering System with Imperfect Repairs Using Multi-objective Genetic Aalgorithms and Discrete Event Simulation. Simulation Modelling Practice and Theory, 19, 362-381.