ESSAYS ON FISCAL RULES

Ph.D Dissertation

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Abstract

This dissertation discusses the design of optimal fiscal rules in a dynamic setting in which national governments with quasi-hyperbolic preferences are subject to privately observed idiosyncratic shocks. In this context, fiscal rules aim at striking a balance between flexibility to react to shocks, and commitment to avoid excessive government spending.

Chapter 1 derives and compares optimal rules in two different environments: one in which a supranational authority is allowed to transfer resources across countries (i.e., a fiscal union) and one in which transfers are forbidden. I find that optimal fiscal rules can be implemented as deficit limits and are complemented with a combination of grants and loans in a fiscal union. All instruments are debt-contingent: higher public debt contemporaneously tightens deficit limits and reduces the entity of both transfers and credits. The chapter includes a calibration of the model using EU data.

In Chapter 2, which is joint work with Facundo Piguillem, we study the effect of stochastic government turnover on fiscal rules' design. The model decomposes governments' present-bias in different components: the fundamental political friction – captured by hyperbolic discounting; the overall uncertainty in the economy; and the relative relevance of political turnover versus business cycle fluctuations. Fiscal rules, both in a national and in a supranational setting, are found to be stricter when insurance needs are low, the present bias is high and government turnover is frequent.

Chapter 3, which is joint work with Facundo Piguillem and Liyan Shi, analyzes the role of sovereign default in the design of fiscal rules. We build a continuous-time model and derive the optimal fiscal rules, which tun out to be debt-dependent only when default is possible. Depending on the severity of the spending bias and the cost of default, optimal fiscal rules range from strict debt limits – complemented by strong deficit limits – to the absence of all rules. In intermediate cases, debt-dependent deficit limits must be complemented with default rules, with some areas where default is prohibited and others where default is mandatory.

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A short introduction

As extensively documented in the recently compiled Global Debt Database, worldwide debt ratios have been on a continuously increasing trend since the early 1980s, especially in advanced economies (Mbaye et al., 2018). Although private debt is the main culprit of this ascent, public obligations have also soared: sovereign liabilities decreased up to the mid-1970s – mainly due to rapid growth, inflation, and financial repression (Reinhart and Sbrancia, 2015) – but steeply and almost uninterruptedly reversed their drift from then on and had already reached a plateau exceeding the post-World War II levels (at above 100 percent of GDP) before the COVID-19 pandemic. Figure 0.1 plots (left panel) general government debt as a percentage of GDP, singling out the United Stated, United Kingdom and Japan, as three examples of the most indebted areas of the world: North America, Europe and Asia Pacific accounted for a striking 93 percent of global debt in 2016, well above their share of GDP.[1]

Political economy models are among the most promising for explaining advanced economies' rising trend in government debt. As argued in Yared (2019), normative macroeconomic theories can only account for temporary, shock-induced, increases in the level of debt, but do not provide an adequate explanation for its sustained growth rate. Moreover, normative theories are at odds with the growing body of empirical evidence showing that larger deficits are associated with countries having more ideological polarization (Woo, 2003) or political fragmentation (Crivelli et al., 2015) and with a proportional (rather than majoritarian) election system (Persson and Tabellini, 2004).

The main idea behind political economy models is that, although debt might be an effective instrument to smooth consumption across time and countries, governments have a tendency to systematically exceed the socially optimum level of spending since they have time-inconsistent, present-biased preferences. This bias arises naturally from the interaction of rational agents driven by political self-interest, and we can

[1] North America, Europe and Asia Pacific owed, respectively, 33, 25 and 35 percent of global debt in 2016.

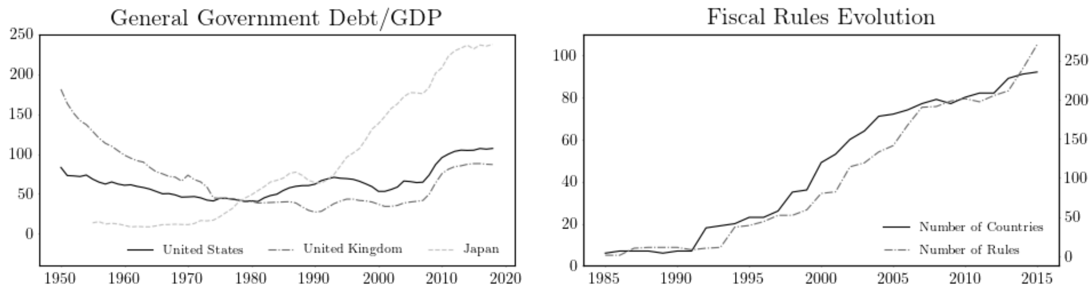

Figure 0.1 Debt and Rules' Evolution in Advanced Economies.

Data from the IMF Global Debt Database (left) and the IMF Fiscal Rules Dataset (right). The left panel plots the time series of general government debt to GDP (percentage) for the United States, United Kingdom and Japan. The right panel plots the number of countries having at least one fiscal rule in place (left axis) and the total amount of fiscal rules in place (right axis). The IMF Fiscal Rules Dataset defines fiscal rules as long-term binding constraints on fiscal policy and classifies them in the following categories: expenditure rules, revenue rules, budget balance rules and debt rules.

divide this class of models in three categories, depending on the specific mechanism that gives rise to this political friction: *(i)* heterogeneous discounting, *(ii)* pecuniary externalities and *(iii)* political turnover models.

In heterogeneous discounting models, agents have time-consistent preferences but discount the future differently. As shown in Jackson and Yariv (2015), a democracy in which policy is chosen sequentially and without long-standing fiscal rules (so without a commitment device) this heterogeneity necessarily results in a present-biased and dynamically-inconsistent government.[2] This is the case, for instance, in Tabellini (1991) and Cukierman and Meltzer (1989), where heterogeneity in preferences arises, respectively, as a result of demography and ability differentials.

The second class of models features agents who, in one way or another, fail to internalize the full financing cost of the chosen government projects. In a national context, a pecuniary externality can arise because of the redistributive nature of political interventions. If costs are shared, but benefits only accrue to the decision making group, a tragedy-of-the-commons type of situation ensues, as in Weingast et al. (1981), where government benefits are geographically concentrated, or in Velasco (2000) where different interest groups all pay the financing costs of projects but do no equally participate in their gains. At an international level, a similar externality can originate either as a consequence of shared inflation (as in Beetsma and Uhlig

[2] More precisely, the authors show that any non-dictatorial aggregation method respecting unanimity is time-inconsistent and, if the method also is time-separable, it delivers a present bias.

(1999), Chari and Kehoe (2007) and Aguiar et al. (2015)) or shared sovereign debt interest rates (see, for example Halac and Yared (2018) or Azzimonti et al. (2014)).

Finally, political turnover models feature groups with different preferences over the composition of public expenditures. Since the currently ruling party knows that the opposition may take control of the government in the future, he tries to constrain the opponent by leaving large deficits. In this type of models, then, a political bias arises because of the temporary nature of power and because of the inability of parties to make binding commitments to each other. Early examples are Alesina and Tabellini (1990), Persson and Svensson (1989) and, more recently, Amador et al. (2006) and Battaglini and Coate (2008).

Although the actual source of the political bias is an interesting topic in itself, analyzing it is beyond the scope of this work and would go at the expenses of clarity and simplicity. The mentioned models are all isomorphic in that they deliver a government with quasi-hyperbolic preferences, which is why a reduced form version of the political friction – that can accommodate any of those micro-founded settings – will be used throughout the dissertation.

What is important is that this political friction, coupled with imperfect information, creates a trade-off between allowing authorities the flexibility in spending required to react to macroeconomic shocks and the commitment society would like to impose on them to dampen biased expenditures. As a consequence, this class of models also manages to make sense of the existence of fiscal rules: on the one hand they do reduce the ability to smooth consumption at the national level, but on the other hand, they impose predetermined fiscal constraints that narrow the gap between socially optimal and actual policy.

Understanding what fiscal rules are useful for seems trivial, but is actually not so common an endeavor, as most models do not feature frictions that could justify their use. Further, policymakers typically consider the models that do as either too stylized or yielding excessively complex prescriptions to provide any useful guidance. In a IMF note on "How to select fiscal rules", Eyraud et al. (2018) write that "The choice of rules is generally based on ad hoc criteria rather than theoretical considerations", and, in fact, the guide focuses on qualitative rather than quantitative criteria. One such criterion, for example, is long-term sustainability of debt, but no unambiguous definition of it is given: the quantitative threshold for sustainability is chosen on a country by country basis as a function of its historical record of debt, default and similarly relevant economic variables.[3] Yet, from a theoretical perspective, sustainability of debt is either ensured by definition as the inability to run Ponzi

[3] For more details, see the companion note on "How to calibrate fiscal rules" (Baum et al., 2018).

schemes, or, like in most models featuring strategic default à la Eaton and Gersovitz (1981), debt repudiation is sometimes optimal and pursuing sustainability can even be counterproductive.

Given that the use of fiscal rules has sky-rocketed in the past thirty years, both on the extensive and on the intensive margin, it seems pivotal that we try to close the gap between theory and practice. As documented in Figure 0.1 (right panel), before the 90's less than 10 countries had a fiscal rule in place, they became more than 90 in 2015. At the same time, the number of fiscal rules in place went from less than 10 to more than 250 (IMF Fiscal Rules Dataset, Schaechter et al. (2012)).

Research on the optimal design of fiscal rules has typically taken one of two routes. One possibility is to assume a specific structure for a fiscal rule and evaluate it: Azzimonti et al. (2016) and Stockman (2001), for example, assess balanced budget rules through simulation. The alternative is to use mechanism design to simultaneously characterize the specific form and level of the rule. The seminal work of Amador et al. (2006) shows that optimal fiscal rules are of the threshold kind. More specifically, they assume that governments have a political bias and private information about the actual spending needs of the nation (i.e. on the business cycle). They then find that – when cross-country subsidies are not allowed – allowing complete flexibility to governments having current needs below a certain threshold, while restraining the rest, is the best among all possible policies, including taxes.[4] Depending on the specifics of the model, this threshold has been shown to vary with, among other things, the extent of the political friction (Amador et al., 2006), the persistence of shocks (Halac and Yared, 2014), the framework in which rules are imposed, whether national or supranational (Halac and Yared, 2018).

The research presented here extends this strand of the literature in several ways. First, I recast the basic framework in fiscal rules' mechanism design in continuous time and provide a sample calibration of the model (to assess it against existing fiscal frameworks). The model structure gains in flexibility as compared to the discrete time version: realistic features (e.g. default) can be incorporated in a simple manner and, when closed form solution are not available, the algorithm from Achdou et al. (2017) can be adapted to generate fast and efficient simulations.

Second, I relax the assumption on transfers between types and solve for the optimal mechanism in two distinct environments: one in which the central authority is allowed to transfer resources across countries and one in which cross-subsidies are forbidden. I therefore characterize optimal rules in a fiscal union and quantify the

[4] Under some weak conditions on the shocks' distribution. Amador et al. (2006) discuss a more general model and the application to governments' self-control issues is one among others.

magnitude of welfare gains deriving from setting up a transfer arrangement. This is extremely relevant, for one, in the European Union, where discussions on whether cross-country insurance would be beneficial to the union itself or whether it would only provide further governmental incentives to overspend have been going on since the union's creation.

Third, while the literature usually assumes deterministic changes in governments which are perfectly correlated with economic shocks (or, more precisely, with preference shocks), I introduce stochastic government duration and disentangle the two sources of volatility. This allows me to study the effects of political uncertainty in the determination of optimal fiscal rules and provide a clearer analysis of the trade-off between commitment and flexibility.

Fourth, I introduce default in the standard setting and ask how the existence of risky sovereign-bonds alters the design of optimal rules.

One of the main conclusions that can be drawn from this dissertation is that the existence of a political friction (and its combination with private information) is not enough to justify the imposition of fiscal thresholds that require convergence to some specific debt level. As we shall see in Chapter 1, optimal rules take the form of limits to the *speed* of accumulation/decumulation of assets (i.e. debt-contingent deficit limits), they cannot be implemented as fixed debt/GDP ratios. Another friction is needed to rationalize the type of fiscal rules we currently have in place, and, in particular, the presence of default can, under certain conditions, deliver a strict debt limit (Chapter 3). As an interesting avenue for future research, a multi-country model with heterogeneous, not contractible political bias could also render the use of an additional instrument welfare-improving.

Further, in Chapter 1, I highlight that under certain conditions debt-dependent transfers can be added to the set of instruments available to the central authority without altering fiscal rules. Welfare gains from transfers are sizable: they are estimated to be around 10% of GDP in a calibrated version of the model using European Union data. However, they are also diminishing in the degree of political bias and vanish when governments are completely myopic (i.e. they only value consumption when in charge). Similarly to the more commonly used moral hazard problems, grants generate a trade-off between welfare improving risk-sharing and increased incentives to overspend for the 'undisciplined'national governments.

A second important point, explored in Chapter 2, is that disentangling economic shocks from variability due to political turnover is quite crucial in the design of rules. When the two sources of uncertainty are perfectly correlated, an increase in turnover simultaneously strengthens the insurance needs of the member countries and exacerbates the political bias by shortening average government duration.[5] In this case, insurance motives are always found to dominate overconsumption concerns, and optimal rules loosen in response to an increase in turnover. When, instead, business cycle variations follow a distinct process from government turnover, the central authority response can be tailored to the appropriate source of uncertainty.

Finally, the possibility of default alters optimal fiscal rules in several important ways. First, the threshold below which governments should be unconstrained in their spending decisions is debt dependent if default optimally happens in equilibrium. Second, strict debt limits are part of the set of instrument of the central authority. Third, debt dependent deficit limits must, under certain conditions, be complemented with default rules that identify debt thresholds after which default either becomes compulsory or is banned.

The dissertation is structured as follows: Chapter 1 is devoted to exploring the role of transfers in a set of countries adopting coordinated fiscal rules. Optimal fiscal rules are characterized both under the assumption that transfers are forbidden and under the alternative. Further, both an implementation of the rules and a calibration of the model are provided. In Chapter 2, fiscal rules are characterized when government duration is stochastic but shocks to government composition and preferences are not perfectly correlated. Finally, Chapter 3 introduces default, solves for the optimal fiscal rules and derives policy implications.

[5] Governments discount exponentially while in charge but apply an extra-discount to future governments' consumption. In this sense, they care more about consumption while in charge. In the limit in which a government remains in charge forever, the political bias disappears.

Chapter 1

Transfers in the design of fiscal rules

(Job Market Paper)

I study dynamic optimal fiscal rules in a supranational setting in which national governments with quasi-hyperbolic preferences are subject to privately observed idiosyncratic shocks. In this context, fiscal rules aim at striking a balance between flexibility to react to shocks, and commitment to avoid excessive government spending. I compare optimal rules in two different environments: one in which the supranational authority is allowed to transfer resources across countries (i.e. a fiscal union) and one in which transfers are forbidden. I find that optimal fiscal rules can be implemented as deficit limits and are complemented with a combination of grants and loans in a fiscal union. All instruments are debt-contingent: higher public debt contemporaneously tightens deficit limits and reduces the entity of both transfers and credits. Welfare gains from setting up a transfer system are positive, but vanish in the limit case in which governments only care about their own consumption. Further, gains from transfers are found to be diminishing in the degree of political bias. I present a sample calibration of the model using EU data. Optimal deficit limits are not far from Maastricht 3%; member countries under extreme distress receive help in the form of grants and loans; grants account for 30% of the overall financial help and are at most 4.5% of GDP.

1.1 Introduction

Advanced economies' debt has been on a continuously increasing trend since the early 1980s, and a growing body of empirical evidence shows that larger deficits are associated with countries having short-lived governments, more ideological polarization or political fragmentation and with a proportional (rather than majoritarian) electoral system.[1] This evidence hints at the political nature of the bias behind advanced economies' sky-rocketing liabilities. Governments and international institutions have tried to limit sovereign debt growth by imposing a multiplicity of fiscal rules: at the beginning of the 90's less than 10 countries had a fiscal rule in place, for a grand-total of a dozen rules; the corresponding numbers for 2015 are over 90 nations and more than 250 rules.[2] Yet, design of fiscal rules "is generally based on ad hoc criteria rather than theoretical considerations" (Eyraud et al., 2018), and debt, deficit or expenditure limits are typically chosen on a country by country basis as a function of historical records of default and similarly relevant economic variables.[3] Given the impressive proliferation of rules imposed by national or international fiscal institutions, it seems pivotal to provide a theory-based approach to rule selection that can have practical and empirical validity.

This work falls within the commitment versus flexibility literature, pioneered by Amador et al. (2006), which formalizes the common rationale for having fiscal rules.[4] Rules are necessary to offset a spending bias in fiscal policy: they provide a *commitment* device trough which governments can avoid overspending. Yet, rules also come at the cost of reducing fiscal policy's *flexibility* in reacting to adverse economic conditions. My research extends this strand of the literature in several ways, but its main contribution is to characterize optimal rules in a fiscal union, namely, under the assumption that the central authority can provide cross-country subsidies. The model, which is calibrated on European data, supplies a useful framework to discuss whether financial assistance should be provided to members of a union and, if so, whether it should be in the form of grants or credits – a point of disagreement among EU member countries in the recent design of the Recovery Fund.

Transferring resources across countries subject to idiosyncratic shocks provides advantages in terms of insurance, yet, when union members have private information

[1] See Mbaye et al. (2018), Roubini and Sachs (1989), Woo (2003), Crivelli et al. (2015), Persson and Tabellini (2004). For a comprehensive discussion see Yared (2019).

[2] IMF Fiscal Rules Dataset, Schaechter et al. (2012).

[3] For more details, see Baum et al. (2018).

[4] See, for instance, Vitor Gaspar's keynote address delivered during the recent workshop on "Fiscal rules in Europe" organized by the Directorate General of Economic and Financial Affairs (DG ECFIN).

on the shock and have a tendency to overspend, the additional funds might result in increased wasteful spending, which is detrimental to welfare. In response to this trade-off, a wide variety of arrangements offering different bundles of rules and grants has been set-up across the globe. While the U.S. combines sizable federal transfers with a requirement that state budgets balance in the medium or short run; the withholding of regional subsidies can be used as punishment for fiscal rules' breaching in Argentina.[5] Europe, on the other hand, mostly relies on deficit and debt rules and is reluctant to set-up risk-sharing arrangements: the European Union (EU) is grounded in a legal framework that explicitly forbids cross-country bailouts or joint debt liability, it enforces a balanced budget rule, and its emergency credit institution (the ESM) can normally only extend credit lines under strict conditionality.[6]

Set up & Main Results. This paper focuses on the design of optimal fiscal rules at a supranational level and, in particular, considers two distinct environments: one in which transfers across union members are not allowed and one in which they are.[7] I set up a mechanism design problem, in which a planner, who is unrestricted in her instrument selection, chooses allocations ensuring that *(i)* union members reveal their information truthfully and *(ii)* either each member's budget constraint (in the no-transfer set-up), or a union-wide resource constraint (in the alternative set-up) are satisfied. I consider a continuous time, infinite horizon model, with a continuum of identical governments having stochastic duration. I make two key assumptions in the model. First, governments have time-inconsistent, politically-biased preferences. Second, preferences over the value of public spending are governments' private information. Present biased preferences have been shown to arise naturally from the interaction of rational agents driven by political self-interest as a consequence of heterogeneous discounting, pecuniary externalities or political turnover; while private information over the value of spending is meant to capture the familiar argument that it is difficult to foresee or verify all possible contingencies.[8]

I find that optimal fiscal rules are of the threshold kind in both environments,

[5] For a comprehensive survey of fiscal rules around the globe see Lledó et al. (2017).

[6] See the *Treaty on the Functioning of the EU*, Art.125 and Art.310 and the *European Stability Mechanism Treaty.*

[7] Although the terminology of the paper refers to a supranational setting, the model can equivalently be applied to a supraregional environment where nations are substituted by regions, national governments by local ones and the international planner by the central government.

[8] For micro-funded models featuring a present bias see, among others Tabellini (1991), Cukierman and Meltzer (1989) (heterogeneous discounting); Weingast et al. (1981), Beetsma and Uhlig (1999), Halac and Yared (2018) (pecuniary externalities); Alesina and Tabellini (1990), Battaglini and Coate (2008) (political turnover).

namely, governments having current needs below a certain threshold should be given complete flexibility, while the rest should be restrained. However, rules are weakly more stringent when transfers are allowed. Further, in a fiscal union, debt-dependent transfers complement the set of rules. Resources are redistributed towards the countries most in need, but the more indebted they are, the fewer resources they receive. Since transfers simultaneously provide insurance and strengthen government's incentives to overspend, welfare gains derived from setting up a transfer system vanish in the limit case in which governments exclusively care about their own consumption.

Fiscal rules can be implemented as deficit limits and complemented with a combination of grants and loans when cross-country subsidies are allowed. All instruments are debt-contingent: higher public debt contemporaneously tightens deficit limits and reduces financial assistance. Further, the proposed transfer implementation has a very simple form and can easily be added to preexisting fiscal rules. When a new government is formed, it is tasked with the preparation of a budgetary document detailing its spending needs for the next subsequent years. Based on this document, the union grants an initial transfer to the country and opens a credit line from which the government can draw at any time. The only condition on this credit is that loans will automatically decrease the entity of the next transfer. In other terms, the cost of the credit line is paid in the form of decreased insurance opportunities in the future.

Maastricht, Recovery Fund & Policy Implications. One of the main policy implications of this paper is that uniform, constant thresholds across countries, like the Maastricht 3% deficit limit, are sub-optimal. Fiscal constraints contingent on preexisting debt-levels, like some of the ones detailed in the more recent fiscal compact, are much closer to the derived optimal rule. Further, the model details the optimal transfer system that should be set-up in a fiscal union. It shows under which conditions the addition of transfers should be complemented with a tightening of the fiscal rules. It can be used to frame the discussion on the entity of the overall financial help member countries should have access to, and on how this assistance should be divided between grants and credits. Lastly, while the general consensus is that current rules, and in particular European ones, are too complicated, this paper provides simple, easily enforceable rules having a single operational target.

I present a calibration of the model using EU data which shows how optimal deficit limits (as a percentage of GDP) in Europe – although debt-contingent – are not far from Maastricht 3% . Under extreme distress, member countries are entitled to transfers ranging between 3% and 4.5% of GDP depending on the level of previously accumulated debt. For example, a country having a 90% debt-to-GDP ratio and a 35% revenue-to-GDP ratio would receive a grant amounting to 3.9% of

GDP when hit with the worst possible shock realization. Further, transfers should represent about 30% of the overall financial help (including the credit-line) provided by the union. For a quick comparison, consider that, under the European pandemic relief program Next Generation EU (NGEU), grants amount to 52% of the total available resources (750 billion Euros), with considerable cross-country variations. In Italy, which is one of the worst hit nations in the union, the percentage of grants is around 39% (209 billion Euros ca., of which 81.4 in grants and 127.4 in loans). Moreover, notice that the Recovery and Resilience Facility (part of NGEU) amounts to 672.5 billion, 70% of which will be distributed in the next two years. Back of the envelope calculations reveal that the planned yearly disbursement is around 1.7% of European GDP.[9]

1.1.1 Related Literature

This paper is closest to the work in Amador et al. (2006) and Halac and Yared (2014, 2018), which falls within the mechanism design literature in self-control settings. When governments have private information on the state of the economy and a tendency to systematically exceed the socially optimum level of consumption, a trade-off arises between allowing authorities the flexibility in spending required to react to macroeconomic shocks and the commitment society would like to impose on them to dampen biased expenditures. Fiscal rules, in this setting, reduce the ability to smooth consumption at the national level, but also impose predetermined fiscal constraints that narrow the gap between socially optimal and actual policy. In Amador et al. (2006), the same trade-off between commitment and flexibility arises and the authors show that optimal fiscal rules are of the threshold kind. Depending on the specifics of the model, this threshold has been shown to vary with, among other things, the extent of the political friction (Amador et al., 2006), the persistence of shocks (Halac and Yared, 2014) and the framework in which rules are imposed (Halac and Yared, 2018).[10]

The modeling approach of this paper is akin to Amador et al. (2006), in that I assume a reduced form political bias and focus on normative prescriptions of the set-up. However, I analyze a setting in which transfers are allowed, recast the framework in continuous time and introduce random government duration, which is a tractable

[9] NGEU resources are in 2018 prices. The 2018 EU-27 countries GDP is slightly lower than 1.3518 billions, while the Italian one is around 1.771 billion Euros (Eurostat).

[10] More specifically, Halac and Yared (2018) show that when interest rates are an equilibrium object, the supranational planner can account for the pecuniary externality generated by governments' accumulation strategies. Amador et al. (2006) frame the discussion around a general principal-agent problem.

way to capture political turnover.[11] Although stylized, the framework detailed in here gains enough flexibility as to incorporate a range of additional features, including the possibility of sovereign default, which I explore in a companion paper. Further, it allows me to construct a viable implementation of fiscal rules, in addition to their characterization.

This work also contributes to the extensive literature on fiscal unions, including Sibert (1992), Dixit and Lambertini (2001), Cooper and Kempf (2004) and Aguiar et al. (2015), who focus on the conflicts between fiscal and monetary authorities; Von Hagen and Eichengreen (1996) who explore the possible determinants of fiscal rules; Evers (2012) and Azzimonti et al. (2016) who evaluate specific fiscal constraints; Abrahám et al. (2018) and Ferrari et al. (2020) who study insurance provision within a fiscal or monetary union.[12] Perhaps most related to this paper are Chari and Kehoe (2007) and Dovis and Kirpalani (2020), in which the need to impose fiscal constraints in a union arises from a time-inconsistency problem.[13] Differently from them, however, fiscal rules are not here meant to solve a lack of commitment on the part of the central authority. Rather, they are designed to mitigate member governments' political bias – the institutions' common rationale for having fiscal rules – as in, among others, Aizenman (1998) and Beetsma and Uhlig (1999). While most of the above-mentioned literature in this strand assumes *a priori* restrictions on the set of instruments available to the supranational fiscal authority, I solve a more general, mechanism design problem.[14] Farhi and Werning (2017) present a comprehensive analysis of policy instruments available in the context of a fiscal union, finding that state-contingent transfers provide larger benefits the more asymmetric the shocks affecting the members of a union, the more persistent these shocks, and the less open the member economies. The authors set-up a New Keynesian environment with the aim of isolating the effects of aggregate demand externalities on optimal risk sharing, explicitly setting aside concerns arising from incentive provision. This paper complements their analysis by abstracting from nominal considerations and

[11] The mentioned papers feature a two-period model in which the incumbent values spending in the first period (while he is in charge) more than in the second period, the assumption being that some other government will be in charge in the future with probability one.

[12] For policy, rather than rule coordination at a supranational level, see, among others, Chari and Kehoe (1990), Persson and Tabellini (1995), Cooley and Quadrini (2003), Alesina and Barro (2002).

[13] In Chari and Kehoe (2007) a union-wide central bank is tempted to increase inflation when member countries have sizable debts, while in Dovis and Kirpalani (2020) it is the fiscal authority who is assumed not to have commitment.

[14] For instance, Beetsma and Uhlig (1999) limit their analysis to the European Stability and Growth Pact, while Dovis and Kirpalani (2020) set-up a Ramsey problem.

focusing, instead, on the design of incentives when members have private information on the state of the economy.

Finally, the paper relates to the vast literature on the political economy of fiscal policy, including Alesina and Tabellini (1990), Krusell and Rios-Rull (1999), Persson and Svensson (1989), Battaglini and Coate (2008) and Azzimonti (2011). As in Acemoglu et al. (2008) and Yared (2010), I study the provision of dynamic incentives to self-interested politicians, but I concentrate on an international context, rather than on the conflict between citizens and their own national government.[15] Yet, contrary to the general-equilibrium set-ups in Song et al. (2012) and Halac and Yared (2018), where supranational coordination results in an endogenously determined interest rate, unions of countries are here solely characterized by their joint fiscal constraints. I abstract form debt pricing considerations (i.e. interest rates are exogenous) to focus on optimal mechanisms and the debate on whether transfers should be part of the fiscal instruments in a union.[16]

Broadly speaking, the paper also relates to the literature on hyperbolic discounting and commitment devices à la Phelps and Pollak (1968).[17] In particular, the model presented here converges to the quasi-hyperbolic preferences set-up in Harris and Laibson (2012) for extreme values of government turnover.

The paper is organized as follows: sections 1.2 and 1.3 provide, respectively, the model set-up and the characterization of the resulting optimal allocations. Section 1.4 provides an implementation of optimal rules; while Section 1.5 presents a calibrated version of the model using European data. Concluding remarks are in Section 1.6.

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[15] Acemoglu et al. (2008) show, for instance, that when elected officials are as patient as their citizens, no additional distortions arise, other than those implied by their incentive compatibility constraints.

[16] More generally, this work also contributes to the literature on international or inter-regional risk-sharing, including Atkeson and Bayoumi (1993) and Bucovetsky (1998). Persson and Tabellini (1996a) explore the effectiveness of different fiscal agreements in a theoretical model comprising moral-hazard, while Persson and Tabellini (1996b) investigate how different fiscal constitutions shape insurance provision. This paper is closest to Lockwood (1999), who also sets-up a mechanism design problem in an environment in which regional authorities have private information on their idiosyncratic shocks. However, I do not model externalities in the public good provision and provide, instead, an extension focusing on political bias. Finally, for some empirical work on cross-country or cross-regional risk-sharing see, among others, Asdrubali et al. (1996), Canova and Ravn (1996) Mélitz and Zumer (2002), Afonso and Furceri (2008).

[17] See also Laibson (1997), Barro (1999), Krusell and Smith (2003), Krusell et al. (2010), Bisin et al. (2015), Lizzeri and Yariv (2017).

1.2 Environment

There is a unit mass of countries, ruled over time by a series of governments having a stochastic duration and indexed with $n \in N = \{0, 1, 2...\}$. Time is continuous and infinite, but each government n has a finite life: is formed at time $t = \tau_n$ and dissolved at time $t = \tau_{n+1}^-$, where the time of dissolution is ex-post observable and assumed to be stochastic. I denote with $F(\cdot\,; \lambda)$ the cdf of such exponentially distributed random variable. The arrival rate λ captures in a tractable way the frequency with which governments undergo radical transformations, thus providing a proxy for political (in)stability.[18]

The arrival of a new incumbent determines a preference change. Depending on the value they attribute to public spending, governments can be of different types θ. Types with high θ place more weight on spending than low types, who have low marginal utility of current consumption. Government preferences can be interpreted as arising from the underlying constituency's opinions on the social value of spending, which can change over time and determine an alteration of the country's stance on fiscal policy.[19] Another interpretation is that demographic changes in the constituency's composition or power struggles between different parties induce the preference shock.[20]

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a filtered probability space described as follows: the sample space Ω is such that $\{\omega : \mathbb{R}_+ \to \Theta | \omega$ is right-continuous with a finite number of jumps in any interval $[0, t]\}$; $\mathcal{F}$ is a *σ-field* on Ω; $\{\mathcal{F}_t\}_{t\geq 0}$ is the filtration denoting information up to time t; and P is the probability measure of a Poisson process such that jumps arrive with intensity λ and – conditional on a jump occurring at time t – the value of the process at t, $\omega(t)$, is drawn from from a continuous distribution function $H(\theta)$, within a bounded set $\Theta \equiv [\,\underline{\theta}, \bar{\theta}\,] \in \mathbb{R}_+$. Preference shocks are distributed independently over time and across governments and, without loss of generality, are normalized so as to have mean one. A key assumption is that the realization of θ is *privately observed* by the current government.

[18] In parliamentary systems the uncertain duration of the governments is built into the system. In presidential systems, it can be interpreted as changes in the ruling majority after midterm elections. Alternatively, stochastic duration can be though of as the risk of anticipated elections due to a political crisis.

[19] One possibility, for example, is that preference shocks capture responses to the actual economic conditions of the country: people may think that a lean-against-the-wind type of policy is more effective during crises. Notice that in this set-up, if utility is exponential, taste shocks are equivalent to income shocks.

[20] Think, for example, about the proportion of young and old citizens in a country, or see the entrepreneurs/workers conflict in Azzimonti et al. (2014).

Every instant $t \geq 0$ governments receive a fixed portion κ of their country's endowment y, so that κy can be thought of as tax revenues. Utility is logarithmic and all governments suffer from a *political bias*, which I model with quasi-hyperbolic preferences (see Laibson (1997) or Harris and Laibson (2012)). Namely, although all governments discount the future exponentially at rate γ, they value spending less when they are not in office or, in other terms, they discount utility by the extra-term β whenever they are not in power. This political friction is such that the bias is stronger for lower values of the parameter β, where $0 < \beta \leq 1$. Quasi-hyperbolic preferences of this kind can be micro-funded by appealing to the interaction between turnover and political polarization as in the seminal work by Alesina and Tabellini (1990), or invoking "pork barrel" spending, as in Battaglini and Coate (2007). According to this interpretation, introducing the discount term β is a reduced form way of capturing disagreement within a country over the composition of public spending, rather than over its level. A second interpretation, is that the preference structure arises naturally from the aggregation of time consistent preferences with heterogeneous discount rates (see Jackson and Yariv (2014, 2015)).[21]

I consider the problem of a benevolent planner who can be thought of as a supranational authority and allocates governments' consumption under incomplete information about their types. Formally, I set up a direct mechanism problem in which, after observing its type, a newly formed government n provides a report $\widehat{\theta}_n$. The path of government reports is given by $\widehat{\omega} : \mathbb{R}_+ \to \Theta$ defined as $\widehat{\omega}(t) \equiv \omega(t) - \theta_n + \widehat{\theta}_n$, for $\tau_n \leq t < \tau_{n+1}$. I let $\{\widehat{\mathcal{F}}_t\}_{t\geq 0}$ be the filtration generated by $\widehat{\omega}(\cdot)$. Note that $\{\widehat{\mathcal{F}}_t\}_{t\geq 0}$ contains the public information up to time t, including past government reports and times of formation. Let $\sigma_n : \mathcal{F}_{\tau_n} \times \widehat{\mathcal{F}}_{\tau_n^-} \to \Theta$ be the reporting strategy of government n. I denote with σ_n^* the truthful-reporting strategy, i.e. the strategy such that $\widehat{\theta}_n = \theta_n$, for all histories. Since the planner does not observe governments' types, consumption can only depend on reports, that is, $g_t : \widehat{\mathcal{F}}_t \to \mathbb{R}_+$. Let g denote the entire sequence of consumption g_t.

Take any time $\tau_n \leq t < \tau_{n+1}$ such that government n is in power. Given consumption sequence g and the reporting strategies of all governments different from n, denoted with σ_{-n}, utility of the incumbent government is given by

$$U_t(\sigma_n|g,\sigma_{-n}) \equiv \mathbb{E}_t \left[\int_t^{\tau_{n+1}} e^{-\gamma(s-t)} \theta_n \log(g_s) ds + \beta \sum_{j=n+1}^{\infty} e^{-\gamma \tau_j} \theta_j \left(\int_{\tau_j}^{\tau_{j+1}} e^{-\gamma(s-\tau_j)} \log(g_s) ds \right) \right], \quad (1.1)$$

[21] As it is well known, both Alesina and Tabellini (1990) and Battaglini and Coate (2007) are isomorphic to the standard quasi-hyperbolic discounting set-up in Laibson (1997). Indeed, I show that when $\lambda \Rightarrow \infty$ our model maps to a continuous time equivalent of the quasi-hyperbolic discounting framework in Harris and Laibson (2012). The possibility of achieving this mapping implies that the assumed political friction can arise from the aggregation of time consistent preferences with heterogeneous discount rates.

where I omitted explicit dependence on the history $\mathcal{F}_{\tau_n} \times \widehat{\mathcal{F}}_{\tau_n^-}$ to simplify notation. Preferences of the benevolent planner, instead, are described by the expected present value of governments' consumption

$$V(g,\sigma) \equiv \mathbb{E}_- \left[\sum_{n=0}^{\infty} e^{-\gamma\tau_n} \theta_n \int_{\tau_n}^{\tau_{n+1}} e^{-\gamma(t-\tau_n)} \log(g_t) dt \right]. \tag{1.2}$$

Notice that there are two key differences between individual governments' and planner's preferences: the latter *(i)* has no information on governments' true types θ_n and *(ii)* places equal weights on countries' consumption irrespective of which specific government is in place (i.e. there is no β). Finally, truthful reporting is incentive compatible, given the sequence g, if, for all histories and governments,

$$U_{\tau_n}(\sigma_n^*|g,\sigma_{-n}^*) \geq U_{\tau_n}(\sigma_n|g,\sigma_{-n}^*), \text{ for all } \sigma_n. \tag{IC}$$

The incentive-compatibility condition (IC) restricts the set of allocations available to the planner by requiring that, after any history, governments must be better off truthfully reporting their type rather than lying.

I consider two separate environments, depending on whether the planner can transfer resources across countries. When such transfers are allowed, the planner must satisfy an aggregate resource constraint requiring that the present value of total allocated consumption does not to exceed the present value of collectively available resources. Formally, allocation g must satisfy the resource constraint

$$\mathbb{E}_- \left[\int_0^{\infty} e^{-rt} g_t dt \right] \leq \mathbb{E}_- \left[\int_0^{\infty} e^{-rt} \kappa y dt \right]. \tag{RC}$$

On the contrary, when transfers across countries are forbidden, the planner must ensure that each country avoids consuming more than its own resources, so constraint (RC) must hold individually for every history and for every country. In particular, given a sequence of government consumption g, let a country's wealth at any time t be defined as $a_t \equiv \int_0^t e^{r(t-s)} (\kappa y - g_s)\, ds + \kappa y/r$, and initial wealth a_0 be given by the present value of future tax income, $a_0 = \kappa y/r$. In the no-trasfer set-up, the planner must satisfy the following budget constraint for each country:

$$\dot{a}_t = r a_t - g_t, \quad a_t \geq 0, \forall t. \tag{BC}$$

Summarizing, the planner's problem when transfers are allowed is to maximize welfare subject to incentive compatibility and the resource constraint

$$v^{tr} \equiv \max_g V(g,\sigma^*), \text{ s.t. } (IC), (RC). \tag{$\mathcal{P}_{TR}$}$$

while, in the alternative environment (without transfers), the planner maximizes welfare subject to incentive compatibility and each country's budget constraint

$$v^{nt} \equiv \max_{g} V(g, \sigma^*), \text{ s.t. } (IC), (BC). \tag{$\mathcal{P}_{NT}$}$$

Notice that Problem ($\mathcal{P}_{TR}$) is considerably more relaxed than Problem ($\mathcal{P}_{NT}$). The planner has to provide incentives according to (IC) in both setting. However, the only other limitation she has when transfers are allowed is the aggregate resource constraint (RC), which requires expected consumption across histories to be equal to the total available resources in the union. Without transfers, instead, the planner has to insure that the collection of budget constraints summarized by (BC) holds in every possible history and for all countries. It is then intuitive that the planner could do better in the relaxed problem, namely when transfers are allowed. However, this begs the question as to whether the presence of transfers interacts with incentive provision and, if so, how it alters optimal allocations.

1.3 Optimal Allocations

Two frictions prevent the attainment of the first best allocation in this model: (i) the fact that preference shocks θ are private information and (ii) the presence of a political bias for $\beta < 1$. Absent private information, both in the no-transfer and in the transfer set-up, the the mechanism design problem is a relaxed one, in which the incentive constraint (IC) can be dropped. If government types were observable, no incentives would have to be provided to governments for truthful revelation since the information would be public.

When governments do not have biased preferences (i.e. $\beta = 1$) their objective function coincides with the planner's one. As a result, when transfers are forbidden, planner's and governments' preferred allocations coincide, and the incentive constraint is trivially satisfied. In the transfer setting, instead, governments may still be tempted to use their private information to exploit the insurance system. The model, then, collapses to an incomplete-information insurance problem à la Atkeson and Lucas (1992).

I will refer to the *full information* allocations under the transfer and no-transfer assumption as the solutions to, respectively, problems ($\mathcal{P}_{TR}$) and ($\mathcal{P}_{NT}$) when the incentive compatibility constraint (IC) is slack. It is easy to show that, under full information, the planner only manages to deliver perfect insurance when transfers are allowed.[22] Intuitively, this is because whenever cross-subsidies between different

[22] Unless $\lambda = 0$, in which case there is no uncertainty in the first place: governments are in charge

countries are forbidden, intertemporal allocation of resources remains the only available tool to provide insurance, so the planner is unable to equate marginal utility across government types.

1.3.1 No Transfers

In this section, I recast the sequential problem ($\mathcal{P}_{NT}$) in its recursive formulation and characterize the solution to the planner's problem under the no-transfer assumption. Generally speaking, the planners' problem can be written as a mechanism that is recursive in promised utilities whenever preferences are standard and shocks i.i.d. – even if planner and agents have differing degrees of patience.[23] In this setting, however, although the political friction does, in some sense, make agents relatively more impatient, governments' preferences are quasi-hyperbolic.[24]

I exploit the fact that government formation is observable to solve the problem in two separate steps. In the first step, the planner determines consumption allocations for any given promised utility. In the second step, the planner chooses the overall level of expected utility and continuation utility for, respectively, the period in which the incumbent remains in charge and the time after its dissolution. The first step, then, is devoted to choosing consumption *within* the current government's tenure. The second step, instead, is the one concerned with providing the right incentives by selecting expected utility levels *across* governments.

Recursive problem. Given initial wealth $\bar{a}$, let $\mathcal{V}_0(\bar{a})$ be the set of planner's payoffs such that for all $v_0 \in \mathcal{V}_0(\bar{a})$ there exists a sequence of spending g and an associated wealth process $\{a_t\}$, which *(i)* satisfy the governments' budget constraint (BC) with initial assets $\bar{a}$, *(ii)* are such that truthful reporting is incentive compatible (i.e. constraint (IC) is satisfied) and *(iii)* deliver utility $v_0 = V(g, \sigma^*)$. Define v_n, the utility promised to government n at the time of formation τ_n, as

$$v_n = \mathbb{E}_{\tau_n^-}\left[\sum_{s=n}^{\infty} e^{-\gamma(\tau_s - \tau_n)}\,\theta_s\,\mathbb{E}_{\tau_s}\left[\int_{\tau_s}^{\tau_{s+1}} e^{-\gamma(t-\tau_s)}\log(g_t)dt\right]\right],$$

and the set $\mathcal{V}_n(\bar{a})$ analogously to $\mathcal{V}_0(\bar{a})$. Standard properties of logarithmic preferences imply that, for any $\widetilde{v}_n \in \mathcal{V}_n(\bar{a})$, we can find some $v_n \in \mathcal{V}_n(1)$ such that

and keep their type forever.

[23] See Green (1987), Sleet and Yeltekin (2006), Farhi and Werning (2007).

[24] Preferences are quasi-hyperbolic in Amador et al. (2006) too, but the authors solve a static problem.

$\widetilde{v}_n = v_n + \log(\bar{a})/\gamma$. It is thus sufficient to only characterize the set $\mathcal{V}_n(1)$. Moreover, the combination of exponential discounting and the assumption that new governments are formed according to a standard Poisson process imply that the set $\mathcal{V}_n(1)$ is independent of time. We can thus simplify notation by dropping the subscript n.

The value of the sequential problem ($\mathcal{P}_{NT}$), v^{tr}, equals $\overline{v} + \log(\kappa y/r)/\gamma$, where $\overline{v}$ is the highest payoff in $\mathcal{V}(1)$. In the appendix, I show that $\overline{v}$ is the solution to a simple recursive problem. Formally, the planner chooses policies $\widehat{g} : \Theta \times [0, \infty) \to \mathbb{R}_+$, $\widehat{a} : \Theta \times [0, \infty) \to \mathbb{R}_+$, $u, w : \Theta \to \mathbb{R}$ so as to solve the following problem

$$\overline{v} = \max_{u,\widehat{g},\widehat{a},w \in \mathcal{V}(1)} \mathbb{E}_- \left[\theta u(\theta) + e^{-\gamma\tau} w(\theta) + \frac{1}{\gamma} e^{-\gamma\tau} \log(\widehat{a}_\tau)\right], \qquad (\mathcal{P}_{NT:Rec})$$

$$\text{s.t.} \quad \theta \in \arg\max_{\tilde{\theta} \in \Theta} \left\{ \frac{\theta}{\beta} u(\tilde{\theta}) + \frac{\lambda}{\gamma + \lambda} w(\tilde{\theta}) + \frac{1}{\gamma} \mathbb{E}\left[e^{-\gamma\tau} \log(\widehat{a}_\tau) | \tilde{\theta}\right] \right\}, \tag{1.3}$$

$$\int_0^\tau e^{-rt}\, \widehat{g}_t dt + e^{-r\tau}\, \widehat{a}_\tau = 1, \tag{1.4}$$

$$u(\theta) = \mathbb{E}\left[\int_0^\tau e^{-\gamma t} \log(\widehat{g}_t) dt\right], \tag{1.5}$$

where, in the recursive formulation, the planner faces one generic government at a time.[25] Notice that the planner now chooses expected utility u and processes $(\,\widehat{g},\, \widehat{a}\,)$ of spending and wealth for the incumbent while in charge, together with its continuation value w after dissolution.

In general, to find $\overline{v}$ and the the optimal allocation that supports it, we would first need to characterize the entire set $\mathcal{V}(1)$, from which continuation values are chosen. It turns out, however, that $\overline{v}$ satisfies a simple self-generating property, namely that the continuation value following $\overline{v}$ is also equal to $\overline{v}$, independently of the government's reports. In other terms, when utility is logarithmic, shocks are i.i.d and transfers are forbidden, the mechanism is static: incentives are automatically provided through the budget constraint since spending more today directly translates in having fewer disposable resources tomorrow. To see this, first notice that standard arguments imply that the incentive constraint (1.3) is equivalent to

$$\frac{\theta}{\beta} u(\theta) + \frac{\lambda}{\gamma + \lambda} w(\theta) + \frac{1}{\gamma} \mathbb{E}\left[e^{-\gamma\tau} \log(\widehat{a}_\tau) | \theta\right] \geq \frac{1}{\beta} \int_{\underline{\theta}}^{\theta} u(z) dz + \frac{\theta}{\beta} u(\underline{\theta}) + \underline{w}, \tag{1.6}$$

[25] It is possible to drop time indexes, so this generic government is formed at time $t = 0$ and dissolved at a random date τ^-. Notice that I use "hats" to denote spending and wealth *within* the incumbents' tenure.

where $(\underline{\theta}u(\underline{\theta})/\beta + \underline{w})$ is the lifetime utility of the government with the lowest type; plus a monotonicity constraint on $u(\cdot)$, which must be non-decreasing. We can thus rewrite problem $(\mathcal{P}_{NT:Rec})$ by replacing constraint (1.3) with (1.6).

Consider now the choice of continuation values. By choosing a higher continuation, the planner can increase the objective function and, at the same time, relax the incentive constraint. It is then immediate that the optimal continuation values must be such that $w(\theta) = \overline{v}$ for all $\theta \in \Theta$. Therefore, problem $(\mathcal{P}_{NT:Rec})$ becomes

$$\overline{v} = \max_{u,\widehat{g},\widehat{a},\underline{w}} \frac{1}{1-\delta}\mathbb{E}_{-}\left[\theta u(\theta) + \frac{1}{\gamma}e^{-\gamma\tau}\log(\widehat{a}_\tau)\right] \qquad (\mathcal{P}'_{NT:Rec})$$
$$\text{s.t. } (1.4), (1.5), (1.6).$$

Step 1. (Within Government) The problem can be further simplified by noticing that the incentive constraint does not directly depend on $\widehat{g}$, and that a higher end-of-life wealth $\widehat{a}_\tau$ increases the objective function while contemporaneously relaxing the incentive constraint. As a result, ceteris paribus, the planner will want to choose the highest possible $\widehat{a}_\tau$. We can then characterize the optimal instantaneous consumption for any given (expected) utility level $\overline{u}$ by solving the first step of the planner's problem, namely by choosing $(\widehat{g}, \widehat{a})$ such that

$$\max_{\widehat{g},\widehat{a}} \mathbb{E}_{-}\left[\frac{1}{\gamma}e^{-\gamma\tau}\log(\widehat{a}_\tau)\right], \qquad (\mathcal{P}_{NT:S1})$$
$$\text{s.t. } \overline{u} = \mathbb{E}\left[\int_0^\tau e^{-\gamma t}\log(\widehat{g}_t)dt\right] \text{ and } (1.4).$$

The following lemma contains the solution to sub-problem $(\mathcal{P}_{NT:S1})$ and characterizes the optimal consumption and wealth allocated by the planner to the government currently in charge, for a given utility level $\overline{u}$.

Lemma 1.1 *Let $k(\overline{u})$ be the solution to $\overline{u}(\gamma+\lambda)^2 = \log(k(\overline{u}))(\gamma+\lambda) + (r - k(\overline{u}))$. Then, for all θ, $\overline{u}$, t, the solution to $(\mathcal{P}_{NT:S1})$ is given by*

$$\widehat{g}_t = k(\overline{u})e^{(r-k(\overline{u}))t}$$

with associated wealth process $\widehat{a}_t = e^{(r-k(\overline{u}))t}$.

Proof. In the appendix. ■

Notice that, once the type of the current government is revealed, consumption follows a deterministic path. Yet, the incumbent's time of dissolution is random, and so is the wealth left at the end of its tenure.

Step 2. (Across Governments) We now turn to the second step of the planner's problem, namely the one of choosing utility u and promised utility w subject to the governments' incentive constraint. If we substitute the allocations described in Lemma 1.1 into the maximization problem ($\mathcal{P}'_{NT:Rec}$) and use the incentive constraint (1.6) to rewrite the objective function, we obtain that the optimal choice of $(\underline{w}, u)$ must be the solution to

$$\max_{\underline{w},u\in\Phi} \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}}(1-M(\theta))u(\theta)d\theta + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w}, \qquad (\mathcal{P}_{NT:S2})$$
$$\text{s.t. } \frac{\theta}{\beta}u(\theta) + W(u(\theta)) \geq \frac{1}{\beta}\int_{\underline{\theta}}^{\theta} u(z)dz + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w},$$

where

$$M(\theta) \equiv H(\theta) + \theta(1-\beta)h(\theta),$$
$$W(\overline{u}) \equiv (r - k(\overline{u}))\mathbb{E}[e^{-\gamma\tau}\tau]/\gamma + \mathbb{E}[e^{-\gamma\tau}]\overline{w},$$
$$\Phi = \{\underline{w}, u \mid \underline{w} \in W(\mathbb{R}), u : \Theta \to \mathbb{R}, u \text{ non-decreasing}\}.$$

Notice that bunching types in the upper tail of the shock distribution is always feasible, and in particular is incentive compatible under the monotonicity of u. The following lemma shows that such bunching is, in fact, optimal.

Lemma 1.2 *The optimal allocation $(\underline{w}^{nt}, u^{nt})$ satisfies $u^{nt}(\theta) = u^{nt}(\theta^\star)$ for all types $\theta > \theta^\star$, where $\theta^\star$ is the smallest value such that*

$$\int_{\widetilde{\theta}}^{\overline{\theta}}(1-M(x))dx \leq 0,$$

for all $\widetilde{\theta} \geq \theta^\star$.

Proof. In the appendix. ∎

The proof follows the lines of Amador et al. (2006) and has a very intuitive interpretation. Since shocks are multiplicative, the planner is generally unable to distinguish between a non-biased government subject to shock $\theta = \widetilde{\theta}/\beta$ and a biased one with type $\widetilde{\theta}$. However, government types having a sufficiently high marginal utility (i.e. $\theta > \beta\overline{\theta}$) cannot disguise themselves because there exists no value of the shock such that their preferences are equivalent to those of an unbiased government with a higher type. Separating high types would require allocating a consumption that is increasing in θ. High types, however, are already over-consuming and increasing their consumption can only reduce welfare, so the planner is better off bunching them. This result is in line with the previous literature: optimal fiscal rules feature bunching at the top of the shock distribution.

Following Amador et al. (2006), I restrict attention to shock distributions that satisfy

Assumption 1.1 *$M(\theta)$ is nondecreasing.*

Assumption 1.1 is a relatively weak requirement on the shock process which is satisfied by all log-concave distributions.[26] For differentiable densities, this is equivalent to a lower bound on the distribution's elasticity

$$\frac{\theta h'(\theta)}{h(\theta)} \geq -\frac{2-\beta}{1-\beta}.$$

When Assumption 1.1 is satisfied, the threshold $\theta^\star$ is implicitly defined by the following equation

$$\beta \mathbb{E}\left[\theta | \theta \geq \theta^\star\right] = \theta^\star. \tag{1.7}$$

Notice that the threshold above which utility becomes constant depends on the degree of political bias β. In particular, when $\beta \leq \underline{\theta}$ national governments' political bias is extremely severe and all types are allocated the same utility since $\theta^\star = \underline{\theta}$. At the same time, no type is bunched when there is no political friction at all: for $\beta = 1$, $\theta^\star = \overline{\theta}$ and each θ is offered a type-specific utility level.

I now turn to the full characterization of problem ($\mathcal{P}_{NT:S2}$) solution. The following proposition presents the main result of this section and characterizes optimal allocations in the economy without transfers.

Proposition 1.1 *Under Assumption 1.1, optimal spending and associated wealth process for $\tau_n \leq t < \tau_{n+1}$ are given by*

$$g_t^{nt} = a_t \cdot \begin{cases} k^{nt}(\theta_n) & for\, \theta < \theta^\star \\ k^{nt}(\theta^\star) & for\, \theta \geq \theta^\star \end{cases} \quad and \quad a_t = a_{\tau_n} \cdot \begin{cases} \exp\left((r - k^{nt}(\theta_n)(t-\tau_n)\right) & for\, \theta < \theta^\star \\ \exp\left((r - k^{nt}(\theta^\star)(t-\tau_n)\right) & for\, \theta \geq \theta^\star \end{cases}$$

where

$$k^{nt}(\theta) \equiv k(u(\theta)) = \frac{\gamma\theta(\gamma+\lambda)}{\gamma\theta + \lambda\beta} \quad and \quad a_{\tau_n} = \frac{\kappa y}{r} \ \exp\left(\sum_{i=0}^{n-1} (r - k^{nt}(\theta_i))\tau_{i+1}\right).$$

[26] See Halac and Yared (2018), Amador et al. (2006).

Proof. In the appendix. ■

Proposition 1.1 characterizes governments' current consumption. It shows that the planner allocates a type-dependent portion of wealth to consumption. Further, this proportion is increasing in θ, meaning that optimal spending is higher when its social worth increases. Dependence on types disappears when uncertainty about the value of future spending vanishes. This uncertainty is captured by the political turnover parameter λ. Thus, when $\lambda = 0$, there is no uncertainty about subsequent spending needs and the planner allocates a constant proportion γ of wealth to current spending (i.e. $k^{nt}(\theta) = \gamma$, for all θ).

Assumption 1.1 guarantees that, for all types $\theta \leq \theta^\star$, optimal spending coincides with what the government would choose in a consumption-saving problem.[27] This feature yields the following simple interpretation of the planner's solution. Governments are granted full flexibility over spending decisions as long as their spending is below a certain level. Due to hyperbolic preferences, they always allocate a greater fraction of their wealth to current spending than what would be optimal for the planner. To counteract governments' desire for excessive spending, the planner limits their flexibility by introducing a bound on spending. More specifically, the optimal mechanism consists in allowing types below the threshold $\theta^\star$ to make their unconstrained choice – i.e. to enjoy full flexibility when choosing spending – and in bunching all the others. What is more, the threshold becomes tighter (i.e. more government types are constrained) when the political bias is stronger (lower β). In fact, when there is no bias at all ($\beta = 1$) full flexibility ($\theta^\star = \bar{\theta}$) is optimal, while when the bias is very high ($\beta \leq \underline{\theta}$) all types should be constrained ($\theta^\star = \underline{\theta}$). If $\beta \in (\underline{\theta}, 1)$ the threshold is monotonically decreasing in β under Assumption 1.1.

Remark. (Full Information) A useful benchmark is the *full information* case, which is defined as the solution to the relaxed problem obtained by dropping the incentive constraint from problem ($\mathcal{P}_{NT}$). In this benchmark, the planner obtains the preferred level of spending, which equals $g_t^f = k^f(\theta)a_t$, where $k^f(\theta) \equiv \gamma\theta(\gamma + \lambda)(\gamma\theta + \lambda)^{-1}$, and invests the remaining wealth. Comparing g^f with g^{nt}, we see that asymmetric information has a bite, that is, it distorts allocations relative to full information only when hyperbolic discounting is present (i.e. $\beta < 1$).

1.3.2 Transfers

I now characterize the solution to the planner's problem when transfers among countries are allowed. First, I rewrite the sequential problem of Section 1.2 recursively.

[27] I show this in the implementation in Section 1.4.

Similarly to the no-transfer case in Section 1.3.1, the problem of choosing optimal allocations can be solved in two steps. In the first step, the planner takes as given the current utility and the continuation value that must be delivered to the government in charge and chooses the optimal sequence of instantaneous spending. In the second step, the planner chooses current utility and continuation value optimally.

Recursive Problem. I here focus on the recursive version of the planner's dual problem, that is, the problem of minimizing the expected resources of delivering a certain lifetime utility to a given country. Standard arguments, which I present in the appendix, imply that the history of a country until the formation of a new government can be summarized by the country's continuation utility.

Let $K(v)$ be the expected amount of resources that are necessary to deliver lifetime utility v to a country, when the n-th government is formed. It satisfies the recursion

$$K(v) = \min_{\widehat{g},u,w} \mathbb{E}_{-}\left[\int_0^{\tau} e^{-rt}\widehat{g}_t dt + \frac{\lambda}{r+\lambda}K(w(\theta))\right], \tag{$\mathcal{P}_{TR:Rec}$}$$

$$\text{s.t. } \theta \in \arg\max_{\tilde{\theta}\in\Theta}\left\{\frac{\theta}{\beta}u(\tilde{\theta}) + w(\tilde{\theta})\right\}, \tag{1.8}$$

$$(\gamma+\lambda)v = \mathbb{E}_{-}\left[\theta u(\theta) + \lambda w(\theta)\right], \tag{1.9}$$

$$u(\theta) = \mathbb{E}\left[\int_0^{\tau} e^{-\gamma t}\log(\widehat{g}_t)dt\right], \tag{1.10}$$

where $u, w : \Theta \to \mathbb{R}$ and $\widehat{g} : \Theta \times [0,\infty) \to \mathbb{R}_{+}$. Notice that the planner minimizes resources subject to the constraint of delivering lifetime utility v to the country and subject to truthful reporting by the current government.

Step 1. (Within Government) Maximization problem ($\mathcal{P}_{TR:Rec}$) can be simplified by noticing that government spending g only features in the objective function and in the constraint on the incumbent expected utility (1.10). We can therefore characterize the optimal instantaneous consumption by minimizing the cost of delivering a given (expected) utility level $\overline{u}$.

Let $G(\overline{u})$ be the expected resources of delivering current utility $\overline{u}$ to a generic government formed at time 0 and remaining in charge until the random time τ.

Then G is given by

$$G(\overline{u}) \equiv \min_{\widehat{g}} \mathbb{E}_{-}\left[\int_0^\tau e^{-rt}\widehat{g}_t dt\right], \qquad (\mathcal{P}_{TR:S1})$$
$$\text{s.t. } \overline{u} = (\gamma+\lambda)\mathbb{E}\left[\int_0^\tau e^{-\gamma t}\log(\widehat{g}_t)dt\right].$$

For simplicity, I will assume in what follows, that the interest rate is equal to the rate of time preference, $r = \gamma$. It is immediate to see that, under this assumption, the solution to the problem above entails a constant consumption over the government's lifetime.

Lemma 1.3 *For all θ, $\overline{u}$, t, the solution to ($\mathcal{P}_{TR:S1}$) is given by*

$$\widehat{g}_t = e^{\overline{u}}.$$

As a result, $G(\overline{u}) = \exp(\overline{u})(\gamma+\lambda)^{-1}$.

Notice that government consumption would grow (fall) deterministically over time if, instead, $r > \gamma$ ($r < \gamma$).

Step 2. (Across Governments) I now characterize the planner's choice of incentive compatible current utilities and continuation values across governments. Incorporating the results from Step 1 in the recursive problem we have that

$$K(v) = \min_{u,w} \mathbb{E}_{-}\left[G(u(\theta)) + \frac{\lambda}{\gamma+\lambda}K(w(\theta))\right], \qquad (\mathcal{P}_{TR:S2})$$
$$\text{s.t. } (1.8), (1.9).$$

First, notice that, using the homotheticity properties of logarithmic preferences, it is immediate to verify that $K(v) = K(0)\exp(\gamma v)$. Similarly, if we denote with (u_v^{tr}, w_v^{tr}) the solution to problem ($\mathcal{P}_{TR:S2}$) for some lifetime utility v, then $u_v^{tr}(\theta) = u_0^{tr}(\theta) + \gamma v$ and $w_v^{tr}(\theta) = w_0^{tr}(\theta) + v$. Therefore, it is sufficient to characterize the solution for $v = 0$.

Second, to facilitate the comparison with the no-transfer case, I recast the problem in its primal form and, proceeding as in Section 1.3.1, replace the incentive constraint (1.8) with a a monotonicity condition on u and a more convenient constraint, featuring only the continuation utility of the lowest type

$$
\begin{aligned}
0 = \max_{u,w,\underline{w}} & \mathbb{E}_{-}\left[\theta u(\theta) + \lambda w(\theta)\right], \\
& \text{s.t. } \mathbb{E}_{-}\left[G(u(\theta)) + \frac{\lambda}{\gamma+\lambda}K(0)\exp(\gamma w(\theta))\right] \leq K(0), \\
& \frac{\theta}{\beta}u(\theta) + \lambda w(\theta) \geq \frac{1}{\beta}\int_{\underline{\theta}}^{\theta} u(z)dz + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \lambda \underline{w},
\end{aligned}
$$

with u non-decreasing.

Third, I perform a simple change of variable which emphasizes the fact that shifting resources across types is possible in this setting. In particular, notice that $K(0)$ is the expected resources needed to deliver the desired overall utility level (i.e. $v = 0$) to the incumbent before its type is revealed. Once the shock is realized, the planner uses resources $G(u(\theta))$ and $K(0)\exp(\gamma w(\theta))$ for, respectively, current and continuation utility. I then let transfers $T(\cdot)$ capture the difference between the initial expectation and actual realization of employed resources, as a function of the incumbent's type. Formally, let $T(\theta) \equiv G(u(\theta)) - K(0) + \lambda(\gamma+\lambda)^{-1}K(0)\exp(\gamma w(\theta))$, and recast the planner's choice in terms of transfers T instead of continuation utility w:

$$
\begin{aligned}
0 = \max_{(\underline{w},u,T)\in\Phi'} & \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} (1 - M(\theta))u(\theta)d\theta + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \lambda\underline{w}, \qquad (\mathcal{P}'_{TR:S2}) \\
& \text{s.t. } \mathbb{E}[T(\theta)] \leq 0, \\
& \frac{\theta}{\beta}u(\theta) + \lambda W\Big(K(0) - G(u(\theta)) + T(\theta)\Big) \geq \frac{1}{\beta}\int_{\underline{\theta}}^{\theta} u(z)dz + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \lambda\underline{w},
\end{aligned}
$$

where

$$
\begin{aligned}
& W(x) \equiv \frac{1}{\gamma}\log\left(\frac{\gamma+\lambda}{\lambda K(0)}x\right), \\
& \Phi' = \Big\{\underline{w}, u, T \mid \underline{w} \in W(\mathbb{R}), u : \Theta \to \mathbb{R}, u \text{ non-decreasing}, T : \Theta \to (-K(0), \infty)\Big\}.
\end{aligned}
$$

and $M(\cdot)$ is defined as in Section 1.3.1.

Problem ($\mathcal{P}'_{TR:S2}$) bears a close resemblance with the no-transfer problem ($\mathcal{P}_{NT:S2}$). The crucial difference is that here the planner is allowed to transfer resources across countries. Such transfers must be zero in the aggregate, as specified by the first constraint. To solve problem ($\mathcal{P}'_{TR:S2}$), I use the common approach of ignoring the monotonicity constraint on u, and then showing that the solution to this relaxed problem verifies monotonicity.

I begin the solution characterization by showing that the problem with transfers also features bunching at the top.

Lemma 1.4 *Under Assumption 1.1, the policy functions u_v^{tr}, T_v^{tr} satisfy $u_v^{tr}(\theta) = u_v^{tr}(\theta^\star)$ and $T_v^{tr}(\theta) = T_v^{tr}(\theta^\star)$ for all types $\theta > \theta^\star$ and for all continuation values v.*

Proof. In the appendix ■

This lemma is the analogue of Lemma 1.2 and shows that the solution to problem $(\mathcal{P}'_{TR:S2})$ is constant for types above the same threshold $\theta^\star$. Notice, however, that the lemma does not exclude the possibility that solutions become constant for types below $\theta^\star$, as it will be the case for some type distributions.

Further, to obtain a sharp solution characterization, I restrict attention to shock distributions satisfying the following assumption.

Assumption 1.2 *The shock distribution $h(\theta)$ is such that*

$$\text{if} \quad \tilde{\theta}\,\frac{\varphi'(\tilde{\theta})}{\varphi(\tilde{\theta})} < -\frac{\lambda\beta}{\gamma\tilde{\theta} + \lambda\beta} \quad \text{for some } \tilde{\theta} < \theta^\star,$$

$$\text{then} \quad \theta\,\frac{\varphi'(\theta)}{\varphi(\theta)} < -\frac{\lambda\beta}{\gamma\theta + \lambda\beta} \quad \text{for all } \theta \in [\,\widetilde{\theta}, \theta^\star\,],$$

where $\varphi(\cdot)$ is a non-negative function defined as

$$\varphi(\theta) \equiv \frac{1 - M(\theta^\star)}{h(\theta)} + \frac{\gamma\theta + \lambda\beta}{\gamma\beta} \cdot \frac{m(\theta)}{h(\theta)},$$

and $m(\cdot)$ is the derivative of $M(\cdot)$.

This is a weak requirement on the distribution of types, whose role will be clear once I provide the characterization of optimal allocations.[28] Intuitively speaking, this assumption guarantees that the relaxed problem delivers a policy u that is a single-peaked function of types θ. This, in turn, implies that the solution to problem $(\mathcal{P}'_{TR:S2})$ will still be characterized by a single threshold rule on spending.

The following proposition characterizes optimal spending in a fiscal union, namely when the planner is allowed to transfer resources across countries.

[28] Assumption 1.2 is always satisfied when the shock distribution is uniform. It is also satisfied for the calibration in Section 1.5. In fact, the requirement was never violated in all the simulations using a normal or log-normal distribution. If Assumption 1.2 is not satisfied, the solution is obtained by the "*bunching and ironing*" method described in Bolton et al. (2005), Section 2.3.3.3, p.88.

Proposition 1.2 *Suppose assumptions 1.1 and 1.2 are satisfied. Then, there exists a threshold $\theta^{\star\star} \leq \theta^{\star}$, such that optimal government consumption and transfers are given by*

$$g_t^{tr} = K(v_n) \cdot \begin{cases} k^{tr}(\theta_n) & \text{for } \theta < \theta^{\star\star} \\ k^{tr}(\theta^{\star\star}) & \text{for } \theta \geq \theta^{\star\star} \end{cases} \quad \text{and} \quad T_{v_n}^{tr} = K(v_n) \cdot \begin{cases} (\alpha(\theta_n) - 1) & \text{for } \theta < \theta^{\star\star} \\ (\alpha(\theta^{\star\star}) - 1) & \text{for } \theta \geq \theta^{\star\star} \end{cases}$$

for $\tau_n \leq t < \tau_{n+1}$, where $\{v_n\}$ is the sequence of government's continuation values constructed recursively using the policy function w_v^{tr} obtained in problem ($\mathcal{P}'_{TR:S2}$); $k^{tr}(\theta) \equiv \alpha(\theta)\, k^{nt}(\theta)$; and

$$\alpha(\theta) \equiv \frac{\varphi(\theta)}{\int_{\underline{\theta}}^{\theta^{\star\star}} \varphi(\theta) h(\theta) d\theta + \varphi(\theta^{\star\star}) \int_{\theta^{\star\star}}^{\overline{\theta}} h(\theta) d\theta}.$$

Proof. In the appendix. ∎

Proposition 1.2 shows that optimal spending takes a particularly simple form. In particular, at any point in time, the planner guarantees that the resources given to each country are enough to achieve its promised continuation utility (i.e. $K(v_n)$). Each government will then consume a type-dependent fraction k^{tr} of such resources. What is more, this fraction coincides with its counterpart in the no-transfer case (i.e. k^{nt}) rescaled using weights α – where $\mathbb{E}[\alpha] = 1$ and $\alpha > 0$. Below, in the decentralization of the optimal allocation, I show that such resources can be interpreted as a country's wealth at the moment the previous government is dissolved. I will thus use this interpretation to compare optimal spending with its counterpart in the no-transfer environment. However in the full information case (i.e. $\beta = 1$) the comparison is immediate.

Remark. (Full Information) The type distribution only matters insofar as $\beta < 1$. When $\beta = 1$ weights are $\alpha(\theta) = (\gamma\theta + \lambda)(\gamma + \lambda)^{-1}$, so $\alpha(\theta)k(\theta) = \gamma\theta$ and marginal utility is constant, meaning that the planner provides full insurance. Whenever marginal utility is not constant, this is due to the bite of incomplete information.

In contrast, full insurance was not achievable without transfers, even under complete information (i.e. $\beta = 1$). In fact, marginal utility turned out to be increasing in θ in Section 1.3.1.

When transfers are available, they do not necessarily alter the optimal bunching threshold. More specifically,

Proposition 1.3 *Suppose assumptions 1.1 and 1.2 are satisfied. The bunching threshold $\theta^{\star\star}$ is such that*

$$\begin{aligned} &\theta^{\star\star} = \theta^{\star} \qquad \textit{if} \qquad \theta\,\frac{\varphi'(\theta)}{\varphi(\theta)} \geq -\frac{\lambda\beta}{\gamma\theta+\lambda\beta} \quad \textit{for all } \theta \leq \theta^{\star}, \\ &\theta^{\star\star} < \theta^{\star} \qquad \textit{otherwise.} \end{aligned}$$

Proof. In the appendix. ■

To understand the proposition, remember that, by Proposition 1.2, government spending in the transfer set-up is a weighted average of its no-transfer counterpart. In addition, weights α are proportional to φ. The first condition of the proposition, then, expresses a lower bound on the rate of change of weights as a function of types θ. When it is satisfied, the relative importance of high types is sufficient to guarantee that the monotonicity constraint does not bind. On the contrary, when weights fall sufficiently fast, the planner would like to transfer resources away from very high types in a way that violates incentive compatibility. As a result, she lowers the bunching threshold to ensure consumption monotonicity and, thus, truthful revelation.

Closed-form Solution. (Uniform Distribution) When shocks have a uniform distribution, $\varphi(\theta)$ turns out to be an increasing function of types θ, so the first condition in Proposition 1.3 is always satisfied, meaning that the transfer and no-transfer case have the same threshold $\theta^{\star\star} = \theta^{\star}$. Further, the expression in (1.7) can be used to obtain the closed-form solution for the threshold $\theta^{\star} = \overline{\theta}\beta/(2-\beta)$.

Consumption weights $\alpha(\theta)$ take a particularly simple form, since the term $m(\theta)/h(\theta)$ simplifies to $(2-\beta)$, and are a linear, increasing function of types θ. As a result, it is immediate to show that transfers as a proportion of resources satisfy

$$\frac{T_v^{tr}(\theta)}{K(v)} = \overline{T}_0(\lambda,\beta)\left(\theta - 1 + \overline{T}_1(\beta)\right),$$

for some non-negative functions $\overline{T}_0(\cdot,\cdot)$, $\overline{T}_1(\cdot)$ given in the appendix. Furthermore, $\partial\overline{T}_0(\lambda,\beta)/\partial\lambda < 0$.

Transfers induce low types (i.e. for $\theta < 1-\overline{T}_1(\cdot)$) to spend less and, consequently, high types (i.e. for $\theta > 1-\overline{T}_1(\cdot)$) to spend more relative to the economy in Section 1.3.1. The reason is intuitive. The planner faces a trade-off between granting greater flexibility to governments – which have superior information about their preferences – and suffering from their excessive spending desire – which stems from hyperbolic discounting. When transfers are not allowed, the planner solves this trade-off by

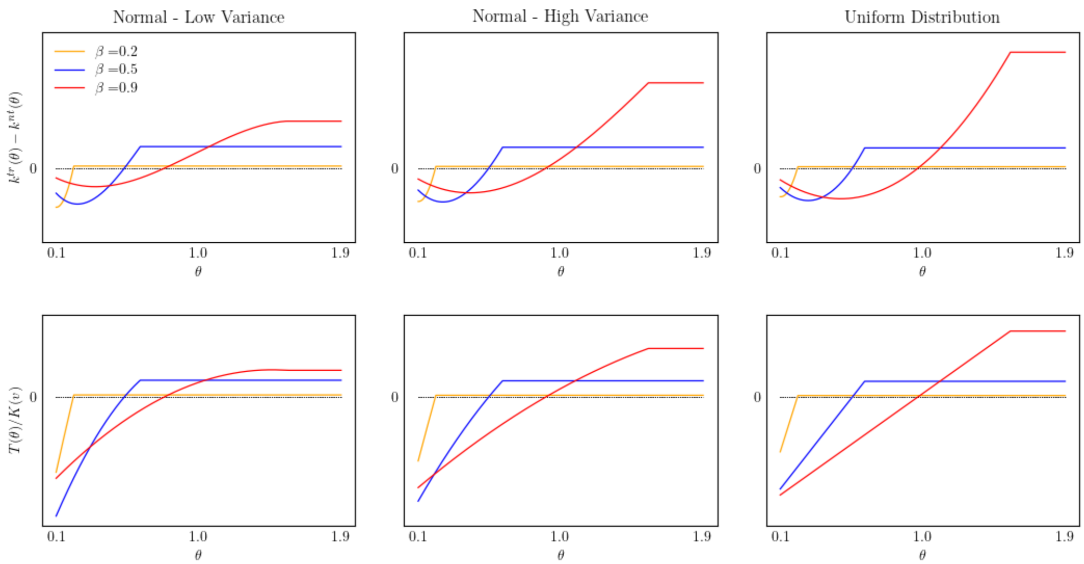


Figure 1.1 Consumption Comparison & Transfers.

Upper Panels: Difference in consumption in the transfer vs. no-transfer case as a function of θ for two governments having equal initial resources. The figures plot $(k^{tr} - k^{nt})$ for different values of β and different shock distributions. *Lower Panels*: Transfers as a percentage of resources for different values of β and different shock distributions.

imposing a cap on spending. The resulting allocation is imperfect: low types spend excessively while high types spend too little. In a fiscal union, instead, the planner has an extra tool to insure governments against fluctuations in their spending needs and, at the same time, limit their excessive spending. More specifically, by transferring resources from low types to high types, the planner makes consumption more sensitive to the shock realization and, by doing so, reduces the volatility of governments' marginal utility.

At the same time, however, increasing a biased government's wealth with further resources exacerbates its incentives to lie, so the planner has to carefully balance insurance and incentives provision in its choice of transfers. When the political friction worsens (β decreases), the planner tightens fiscal rules by constraining more government types ($\partial\theta^\star/\partial\beta > 0$) while contemporaneously diminishing the entity of their subsidies ($\partial T_v^{tr}(\theta^\star)/\partial\beta > 0$). Intuitively, longer government duration relaxes this trade-off between insurance and incentives by extending the period of time in which governments discount exponentially. Formally, since $\overline{T}_0$ increases as λ diminishes (i.e. for infrequent turnover), transfers become larger in absolute value. As governments last for longer periods of time, the political friction becomes less and less important, until it completely vanishes in the limit case in which one government lasts forever (i.e. for $\lambda \to 0$). Accordingly, incentive provision also becomes less relevant, thereby allowing for more substantial cross-country subsidies.

Finally, we can compute welfare gains from transfers as the amount of additional resources the planner would require in order to give up the possibility of making transfers. The present value of initial resources in the no-transfer case is $\kappa y/r$ and delivers utility v^{nt}. To deliver the same utility in the transfer case, the planner would need resources $K(v^{nt})$. Therefore, welfare gains as a proportion of the country's endowment y are

$$\Psi = \left(\frac{\kappa}{r} - \frac{K(v^{nt})}{y}\right) = (1 - \psi(\beta))\,\frac{\kappa}{r}, \tag{1.11}$$

where $\psi(\beta)$ is defined in the appendix. Two things are worth noticing. First, in the limit case in which current governments do not care about the future at all, for $\beta \to 0$, welfare gains completely vanish since $\lim_{\beta\to 0}\psi(\beta) = 1$. On the contrary, welfare gains are strictly positive when the present bias is not strong: $\lim_{\beta\to 1}\psi(\beta) > 1$ (proof in the appendix). Although this is not the typical moral hazard set-up, then, there still is a sense in which grants generate a trade-off between welfare improving risk-sharing and an increase of the temptation to overspend for the 'undisciplined' national governments.[29] Second, welfare gains are proportional to tax revenues.

Numerical Illustration. Figure 1.1 plots the difference in consumption between two governments who happen to have the same wealth in the transfer and no-transfer scenario, for different values of β and different distributions – where, anticipating the implementation results, I let wealth be $a = K(v^t)$ in this set-up. The numerical simulations suggest that the conclusions of the closed-form case extend to a different shock distribution and, in particular, to normally distributed types. In addition, transfers seem to be larger (in absolute value) when the distribution of types is more spread out, especially for high values of β.[30] Again, the reason has to do with the value of insurance: when governments' types are likely to be very different, insurance becomes more valuable, thus, the planner relies on transfers more heavily.

[29] The set up does not feature moral hazard because member government actions do not alter the shock distribution.

[30] Notice that the uniform distribution is obtained as the limit of a sequence of Normal distributions with variance growing to infinity.

1.4 Implementation

The direct mechanisms described in this paper already has a natural interpretation in terms of rules and redistribution: a central institution is tasked with collecting tax-revenues on behalf of its members and allocates resources optimally. Yet, while the existence of such an arrangement seems palatable, and is indeed frequently enforced in inter-regional contexts, it might be difficult to see the practical applications of this model in a supranational environment since sovereign nations are generally reticent to surrender fiscal authority.

This section aims at describing a set of rules which implement the optimal allocation under the assumption that governments choose spending autonomously. To do so, I consider a Markov game among current and future governments within a same country, and solve for individual governments' optimal choice under the fiscal rules described in the previous section. I then focus on the possible rule implementations such that equilibrium allocations are recursive in governments' accumulated wealth. This is because, as in Albanesi and Sleet (2006), wealth turns out to be a sufficient statistic to summarize the national history: it plays the same role promised utility performs in the recursive formulation of the direct mechanism.[31]

Since government preferences are quasi-hyperbolic, their decisions are, in general, not time consistent. In addition, forward-looking governments understand, and take into account, that their actions will affect the choices of future governments.[32] I thus follow the literature on hyperbolic discounting and characterize the equilibrium of a game in which each government takes future governments' best responses as given and selects spending subject to the threshold rule described in (1.7), and a budget constraint (either including or excluding transfers).

Formally, I consider a Markov equilibrium such that, at any instant, the state of the economy is summarized by the country's wealth a and government type θ. Governments make a simple saving-spending choice: every instant they receive tax revenues κy and can either consume or invest in a risk-free market at interest rate r. I denote with a the present value of future wealth $a \equiv x + \kappa y/r$, where x are net assets of the country.[33] Assets follow a diffusion process $dx = (rx+y-g)dt$. Further, I denote with $J(a, \theta)$ the government's equilibrium payoff when the state is (a, θ), but I will omit the dependency on the state in what follows to simplify notation. The

[31] Albanesi and Sleet (2006) show that, when types are i.i.d. and utility functions are separable between consumption and labor, it is possible to decentralize constrained-efficient allocations through a tax system in which taxes depend only on current period's labor income and on individual wealth.

[32] See, for example, Harris and Laibson (2012).

[33] I use the convention that positive values of x are a credit, while negative ones are a debt.

government's payoff and optimal spending are the solution to the following system of Hamilton-Jacobi-Bellman equations (proof in the appendix)

$$\gamma\, J = \max_{g} \Big\{ \theta \log(g) + J_a\,(\,rx + \kappa y - g\,) \Big\} + \lambda\,\big(\,\beta \mathbb{E}[\,\Upsilon\,] - J\,\big), \tag{1.12}$$

$$\gamma\, \Upsilon = \theta \log(g^\star) + \Upsilon_a\,(\,rx + \kappa y - g^\star\,) + \lambda\,\big(\,\mathbb{E}[\,\Upsilon\,] - \Upsilon\,\big), \tag{1.13}$$

$$\text{s.t. } g \leq k^{nt}(\theta^\star)(x + \kappa y / r). \tag{1.14}$$

Equation (1.14) features the optimal threshold rule derived in the previous section, presented here as a cap on spending. The first equation characterizes the current government's payoff J as a function of the state (a, θ) and of the value function Υ, which captures the continuation payoff of the incumbent after its replacement by a new government (J_a and Υ_a denote the derivatives with respect to wealth). Notice that the government currently in charge discounts the future at rate γ, both during the periods in which it is in power and during the periods in which it is not. When the incumbent is replaced, there is a once-and-for-all change in discounting, represented by the additional term β in equation (1.12). Further, since the incumbent can only make spending decisions when in power, the maximization operator shows up exclusively in equation (1.12) and not in equation (1.13). Finally, notice that the current government takes future governments' behavior as given, hence, the term $g^\star$ in equation (1.13).[34]

Equations (1.12) and (1.13) have a very intuitive interpretation. The term $\gamma\, J$ in equation (1.12) is the expected value of instantaneous changes in J arising from the exponential discounting. The first term of the right hand side, $\theta \ln(g)$, is the flow utility derived from government spending, while $J_a \dot{a}$ is the expected value of instantaneous changes in J arising from the (deterministic) returns process. Finally, the term $\lambda\,[\,\beta \mathbb{E}[\Upsilon] - J\,]$ represents the expected value of the instantaneous change in J due to the possible dissolution of the current government. Equations (1.12) and (1.13) are almost identical, with one important exception. From the point of view of the incumbent, future streams of consumption obtained after its replacement are discounted at rate β. Yet, once control has passed to a new government, any subsequent transition is not further discounted. In fact, the two equations are identical when $\beta = 1$.

Notice that, in the absence of political instability, the problem collapses to a standard saving/spending model. In fact, for $\lambda = 0$ the current value function J is

[34] Formally, the first order condition gives $g^\star(a, \theta) = \theta / J_a(a, \theta)$ for unconstrained types.

equal to the continuation value function Υ: the incumbent remains in charge forever and the type never changes. For intermediate levels of instability, $\lambda \in (0, \infty)$, this model generalizes the micro-founded set-ups in Alesina and Tabellini (1990) and Battaglini and Coate (2007) by letting the social value of spending be uncertain. Finally, with extreme political instability, $\lambda \to \infty$, a new government is formed every instant, and the set-up converges to a quasi-hyperbolic discounting model in continuous time that can be considered a generalization of the two period model in Halac and Yared (2018), thus, the model with $0 < \lambda < \infty$ provides a generalization of their policy prescription.

As it turns out, the solution to this problem is recursive in the present value of wealth a and generates a government policy function closely replicating allocations in Proposition 1.1. The implication, then, is that countries should, within a predetermined range, be allowed complete flexibility in their spending choices. Formally, the solution to this problem is that countries with low enough spending needs – types $\theta < \theta^\star$ – are free to select $g^\star(a, \theta) = k^{nt}(\theta)a$, even if they end up overspending with respect to their unbiased (i.e. if $\beta = 1$) choice; while incumbents with larger spending needs – types $\theta \geq \theta^\star$, are limited by a binding cap $g^\star(a, \theta) = k^{nt}(\theta^\star)a$, even if this implies a loss in terms of insurance. We can then interpret the results in Section 1.3.1 in the following way: the threshold separating unconstrained from constrained government types was chosen as to balance the costs of limiting insurance at the top of the distribution with those deriving from overspending at the bottom.

1.4.1 Fiscal Rules

I here show that the threshold rule can be implemented as either an upper bound on the growth rate of debt or a debt-contingent deficit limit. For simplicity, I focus on the case without transfers since the alternative case is analogous. Remember that at the optimum countries spend a proportion $k^{nt}(\theta)$ of their wealth a, which is bounded by $k^{nt}(\theta^\star)$. Thus, countries with types above $\theta^\star$ do not have the discretion to spend as much as they desire. Instead, they can only spend up to proportion $k^{nt}(\theta^\star)$ of their assets. Although the fiscal rule is framed in terms of a threshold on the government's type, it is immediate to see that it can be implemented as a limit on the rate at which governments borrow. In particular, recall that only wealth a and spending g are observable; and that financial assets are defined as $x = a - \kappa y/r$, where negative values of x denote a debt. Debt evolution can be inferred from wealth evolution since $\dot{x} = \dot{a}$. It follows that the threshold rule can be implemented as an upper bound on the percentage growth rate of debt given by:

$$\frac{\dot{x}}{x} \leq \left(r - k^{nt}(\theta^\star) \right) \left(1 + \frac{\kappa y}{rx}\right).$$

This upper bound is debt dependent, namely it becomes tighter as debt grows.[35] In the extreme case in which $x = -\kappa y/r$ (i.e. at the "natural debt limit"), the right-hand side of the equation above becomes zero, so that no further debt is allowed.

Equivalently, the same threshold rule can be implemented using deficit limits rather than caps on the debt growth rate. Since (primary) deficit is defined as $d \equiv g - \kappa y$, the optimal mechanism would require an upper bound on the deficit/GDP ratio:

$$\frac{d}{y} \leq k^{nt}(\theta^\star) \left(\frac{x}{y} + \frac{\kappa}{r}\right) - \kappa.$$

Under this alternative implementation, the rule becomes a deficit limit that is contingent on the debt/GDP ratio. Notice that, insofar as $k^{nt}(\theta^\star)$ depends on political instability, the implementation of the threshold rule varies with λ. More specifically, since $\partial k(\theta^\star)/\partial\lambda \propto (\theta^\star - \beta)$ and $\theta^\star \geq \beta$ for any value of β, higher political instability (i.e. shorter average government duration) calls for looser fiscal rules.[36]

Policy Implications. (Fiscal Rules) The optimal commitment device can be implemented using a single operational target, may it be spending, debt growth rates or deficits.[37] One key feature of the optimal fiscal rule is that it tightens with a country's indebtedness: past obligations are important in that heavily indebted governments are more constrained in their spending choices.

Clearly, this implies that a uniform threshold across countries (i.e. the Maastricht requirement) is a sub-optimal instrument. However, rules that try to link fiscal constraints with preexisting debt-levels, like some of the most recent ones, are strikingly similar to the optimal mechanism implementation derived in this section. For example, under the so called "fiscal compact", European Union member states'

[35] The inequality follows from the fact that $x < 0$. When $x > 0$ the inequality must be reversed.

[36] The optimal upper bound on the percentage growth rate of debt and on the deficit as a percentage of GDP are, respectively, decreasing and increasing in λ. At the optimal threshold $\theta^\star = \beta\mathbb{E}[\theta|\theta \geq \theta^\star] \geq \beta$ since $\mathbb{E}[\theta|\theta \geq \theta^\star] \geq 1$.

[37] The specific choice of operational target is unimportant as long as all target variables are perfectly observed by the central institution. In the model, I assume that everything – except governments' types – is public knowledge. A more realistic assumption, is that the quantification of some target variables might be more precise, or have fewer measurement errors than others, in which case, targets would not be equivalent.

borrowing is constrained to be lower than either 1% or 0.5% of GDP depending on whether the country's debt-to-GDP ratio is below or above 60%.[38]

A second interesting conclusion, is that limits should only be imposed on the *speed* of debt accumulation, not on debt levels. In other terms, there is no debt anchor towards which member countries should strive in this model. The result is in contrast with many existing frameworks, including the European one, which requires long-run convergence to a 60% debt-to-GDP ratio. The implication, then, is that the existence of a political bias is not enough, from a theoretical perspective, to understand why or when debt anchors might be useful. Another friction is needed to rationalize the type of fiscal rules currently in place, and, in particular, I show in a companion paper that the introduction of sovereign default does, under certain conditions, deliver the imposition of a strict debt limit.

Finally, it is worth noticing that the frequency of government turnover turns out to be relevant in rules selection. In particular, shorter average government duration implies that, since shocks are more frequent in the economy, there is more scope for insurance and rules should loosen up to allow it.

1.4.2 Transfers

The most immediate way to implement the transfer allocation is to simply let the central authority choose consumption levels. However, in an international setting, this might be politically unfeasible. An alternative, is to use a combination of transfers and loans that are conditional on countries' wealth and on government's consumption, in the spirit of Albanesi and Sleet (2006).

More specifically, let a_- be the present value of wealth of the country at the time of a generic government's dissolution, before a new incumbent is elected. Further, let the continuation value promised to the previous government in the mechanism design formulation v be summarized by wealth a_- in this implementation. That is, v is such that $a_- = K(0)\exp(\gamma v)$. In this arrangement, the planner will infer the new government type from its spending choice at the moment of formation. One possible interpretation of this process is that the government is required to prepare a document detailing future expenditure plans from which the planner can guess the government type.[39] Formally, if the government consumes $\widetilde{g}$ in the first instant

[38] The fiscal compact is part of the Treaty on Stability, Coordination and Governance in the Economic and Monetary Union (TSCG), signed by all EU countries, except the Czech Republic and the UK in March 2012 and entered into force in January 2013. More precisely, the rule constraints countries' Medium Term Objective (MTO) not to be below a structural balance of -1% or -0.5% of GDP. For more information, see the Vade Mecum on the Stability and Growth Pact.

[39] For example, EU members that adhere to the Stability and Growth Pact have to submit a

of its life, the planner infers that its type is the $\widetilde{\theta}$ satisfying $\widetilde{g} = (\gamma + \lambda)G(u_v(\widetilde{\theta}))$. By incentive compatibility, the type inferred by the planner will coincide with the government's true type, that is, $\widetilde{\theta} = \theta$.

At the time of its formation, the incumbent receives a subsidy equal to

$$\chi(\theta, a_-) = (\alpha(\theta) - 1)a_-. \tag{1.15}$$

Let $a \equiv a_- + \chi(\theta, a_-)$ be the present value of wealth, including transfers χ, at the time of formation, then $a = \alpha(\theta)a_-$. Similarly, let a_t denote wealth at time $0 < t < \tau$. In addition to the above transfer, the central authority provides a savings account with a credit line which works as follows. At any point in time, as long as it is in charge, the government can draw (deposit) an amount b_t from the credit line (savings account). When a new government is formed, an amount $P = \lambda^{-1} b_-$ is repaid to (received from) the central authority in the form of lower (higher) transfers, where b_- is the amount used at the time of dissolution.[40]

Under this arrangement, the government's decision problem is described by the analogue of HJB equations (1.12) and (1.13) where governments choose both consumption g and the entity of the loans (savings) b; and assets follow a jump-diffusion process

$$dx = (rx + \kappa y + b - g)dt + \left(-\lambda^{-1}b_- + \chi(\theta', a'_-)\right) dN,$$

where N_t is the jump process and a'_- is the country's wealth at the time of dissolution net of the payment $-\lambda^{-1}b_-$.

In the appendix, I prove that governments choose $b_t^\star = \bar{b}(\theta)a_t$, with $\bar{b}(\theta) = \gamma\lambda(\theta - \beta)(\gamma\theta + \lambda\beta)^{-1}$. Notice that types with higher marginal value of consumption ($\theta > \beta$) choose a positive b_t but the next government will have to pay in the form of lower transfers, while the opposite is true for low types. In addition, the amount $b_t^\star$ is such that, at the optimum, country's wealth is constant, thus, wealth at the time of formation is equal to wealth at the time of dissolution. Since spending is a constant fraction of wealth, the latter also implies that government consumption is constant throughout its life and, in particular, is equal to $k^{nt}(\theta)a$.

Finally, the reduction (increase) in the future transfer P is such that the average balance on the account is zero:

$$\mathbb{E}\left[e^{-\gamma\tau}P - \int_0^\tau e^{-\gamma t}b_t^\star dt\right] = \left[\frac{\bar{b}(\theta)}{\gamma + \lambda} - \frac{\bar{b}(\theta)}{\gamma + \lambda}\right] a = 0.$$

Stability and Convergence Programme detailing the country's public finance plans.

[40] Notice that, although the final payment is proportional only to the last amount b'_-, since the time of dissolution is random, governments understand that they can be called to make such a payment at any time, thus, the optimal value of b_t is finite for all t.

The savings account is necessary to implement the optimal allocation because governments are subject to the risk of sudden termination, which requires a financial instrument whose payoff is contingent on such an event. In other terms, it provides an insurance against the observable shocks of the model. Although I followed the literature in linking the formation of a new government with the draw of a new shock (the two events always happen simultaneously), a more realistic assumption would be that the two are not perfectly correlated. This alternative is explored in a companion paper focusing on the role of political uncertainty in the selection of optimal fiscal rules.

Policy Implications. (Transfers) Let me conclude the section with a few comments on the interpretation of this implementation. First, notice that, although in this implementation transfers can be negative, it is easy to rescale the problem in such a way that transfers are always positive. What is needed, is to levy a tax on the union members such that the sum of tax and transfers is equal to zero on average. This tax would be akin to, for instance, the EU budget contributions, except for the fact that –instead of being proportional to the countries' revenues– it would be proportional to members' accumulated wealth.[41]

Secondly, the proposed implementation has a very simple form and can easily be introduced in addition to preexisting fiscal rules. The idea can be summarized as follows. When a new government is formed, it is tasked with the preparation of a budgetary document detailing its spending needs for the next subsequent years. Based on this document, the union grants an initial transfer to the country and opens a credit line from which the government can draw at any time. The only condition on this credit is that loans will automatically decrease the entity of the next transfer. In other terms, the cost of the credit line is paid in the form of decreased insurance opportunities in the future. When a new government, with different spending needs is formed, the process starts anew, with the caveat that the bargaining process for grants will take into consideration the resources previously drawn from the credit line.

The described credit line would not be very different from some existing programs. The European unemployment insurance scheme SURE, for example, extends loans to member states with the aim of mitigating sudden increases in public expenditure related to employment protection.[42] The main difference would be that, rather than

[41] EU funding comes mainly from customs duties, sugar levies and a portion of value added tax (VAT) collected on behalf of the EU by the Member States. Additional contributions are made in proportion to the members' gross national income.

[42] SURE stands for "Support to mitigate Unemployment Risks in an Emergency".

being conditional on pre-specified consumption items or reforms, loans would only be conditioned on wealth: the more indebted the country is, the less loans it could receive. Further, loan repayment would decrease the entity of the next transfer rather than being repaid at a set date.

1.5 Quantitative Application

One of the main concerns that prevents practitioners from adopting theory-inspired models in the actual selection of fiscal rules, is that theory often produces excessively stylized set-ups.[43] Although the one presented here also is an extremely simple model, this section aims at showing that it can effectively be taken to the data. Moreover, as shown in a companion paper, the set-up is flexible enough as to accommodate other practical concerns (e.g. the existence of default) and represents, as such, a first step toward the integration of policy and theory oriented literature strands.

The question we are going to ask in this section is: what should have been the fiscal rule adopted by Maastricht signatories in 1993 according to this model? The idea behind this exercise is to *(i)* use data on average government duration to estimate the political uncertainty parameter λ, *(ii)* identify the polarization parameter β and the shock distribution $H(\theta)$ through the model, *(iii)* compute the optimal thresholds $\theta^{\star}, \theta^{\star\star}$ and their corresponding fiscal rules, *(iv)* quantify optimal transfers across union members.

Data Sources. Data on debt and GDP have been obtained from the recently compiled Global Debt Database (GDD), while the series of government revenue and government expenditures as a percentage of GDP are from the Macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs AMECO.[44] Historical information on government duration has been obtained from The Party Government Data Set (PGDS). All the original signatories of the Maastricht treaty have been included in the analysis, except for Denmark and the United Kingdom.[45] As previously mentioned, optimal threshold rules can be implemented in several ways. However, to allow for a more immediate comparison with

[43] The complaint is expressed, for example, in Eyraud et al. (2018).

[44] Data in GDD are in nominal terms. To compute gross debt real growth I subtract inflation from nominal growth using the World Development Indicators database of the World Bank (WDI).

[45] The two countries have been excluded because the opt-outs they managed to obtain render comparison with the other EU countries problematic. Nonetheless, results are robust to their inclusion.

Table 1.1: Parameters values and estimates.

Parameter	*Value*	*Source or Target*
Discount rate	$\gamma = 0.05$	Set 5% yearly interest rate.
Turnover frequency	$\lambda = 0.68$	Estimate from government duration.
Political bias	$\beta = 0.36$	Estimate from debt growth.
Standard deviation of shocks	$\sigma = 0.96$	Estimate from debt growth.

Maastricht 3% deficit rule, I will focus on debt-contingent deficit limits.[46]

Political Uncertainty, Polarization & Shock Distribution. In this model, the political uncertainty parameter λ simply is the arrival rate of a new government, so it can be easily estimated as the inverse of average government's duration. An interesting issue, however, concerns the definition of "government" itself. Is it enough to change a few key figures or the ruling party coalition to establish that a new government has been formed? Or does a country also need to select a new prime minister? The answer is likely to change on a country by country basis, depending on the institutional peculiarities of the various nations.[47] Yet, to ensure consistent estimates across different national states, government duration should be defined in the same manner. I adopt the loosest possible definition of government, according to which any change is a government change. More specifically, in accordance with the PGDS definition, any official government resignation and any change in prime minister or party composition of the cabinet is considered a government switch.[48] Let $Dur_{i,n}$ be the number of years government n in country i lasts. The political uncertainty parameter in country i is the inverse of government duration's sample

[46] Maastricht Treaty actually requires countries to keep debt to GDP levels below 60%, deficits lower than 3% of GDP, or to reduce debt in excess of the 60% threshold by $1/20^{\text{th}}$ of the distance every year. As mentioned before, however, there is no reason, in this model, to constraint debt levels, just debt growth, so only the deficit rule will be considered in this application. Further, I here only consider the corrective arm of EU rules for clarity purposes, but an interesting extension would be to compare optimal rules to the new, post 2008 financial crisis, prescriptions of the EU framework's preventive arm. Medium term objectives (MTO), for example, are generally stricter than Maastricht deficit limit.

[47] To give you an example, a head-of-the-state change is extremely relevant in presidential systems like France, less so in parliamentary ones such as Italy.

[48] Including cases in which, after resigning, a government with the same prime minister and party composition of the previous one is formed. The number and time-span of observations is variable in the PGDS, as every country had its own beginning of democratic life and number of governments.

average $\lambda_i = N(\sum_n \mathrm{Dur}_{i,n})^{-1}$. The introduction of fiscal rules should not, according to the model, have any effect on government duration. Nonetheless, to avoid endogeneity problems, the sample only includes governments formed before 1993 (the year Maastricht entered into force).

The polarization parameter β is meant to capture the present government discount of future governments' spending. Since there is no clear consensus on how polarization in democratic systems should be measured, one advantage of this estimation strategy is that β can be identified through the model.[49] Assume that the preference rate γ is equal to the interest rate r and use (i) the unconstrained asset growth equation $\dot{a}/a = \left[(r-\gamma)\gamma\theta + (r\beta - \gamma\theta)\lambda\right](\gamma\theta + \lambda\beta)^{-1}$, and (ii) the fact that preference shocks have mean one. Taking expectations and rearranging we have that, for $\gamma = r$

$$\beta = \frac{\gamma}{\lambda}\mathbb{E}\left[\frac{\gamma - \dot{x}/x}{\lambda + \dot{x}/x}\right].$$

Let $Debt_{i,t}$ be the real debt of country i in year t, I compute the sample growth rate of debt as the first difference in the log series of debt: $g^d_{i,t} = log(Debt_{i,t+1}) - log(Debt_{i,t})$. We then have that the polarization parameter of country i is $\beta_i = \gamma\,(\lambda_i T)^{-1}\sum_t(\gamma - g^d_{i,t})(\lambda_i + g^d_{i,t})^{-1}$ where I set $\gamma = 5\%$. The assumption on interest rates being equal to the time preference rate substantially simplifies exposition, but is in no way necessary.

Notice that, in the model, x represents net assets, rather than gross debt. However, measurement issues in the quantification of national assets are such that debt data are to be preferred; especially considering that, when government assets are relatively stable over time, net asset and debt growth are one and the same.[50] One possible alternative would be to construct a series for government net debt by subtracting financial assets from gross debt. Growth rates estimated using general government net financial assets instead of debt are quite similar, but rules, being debt-dependent, are substantially looser when computed with net assets. The results of the calibration then, can be seen as an upper bound on how strict rules should be.

An objective interpretation of the distribution of shocks implies that, given a utility function, the distribution $H(\theta)$ can be identified from unrestricted behavior. In fact, if countries have full flexibility, the observed growth rate of debt identifies the distribution of preference shocks, given the utility functions and the polarization

[49] An alternative, could be to use a measure of disagreement within the government (e.g. the Partisan Conflict Index of Azzimonti (2018)) to proxy for β.

[50] It is unclear, for example, if illiquid assets should be included in the assessment of debt sustainability, and if yes how to value them.

and political uncertainty parameters β and λ.[51] Namely, we can compute preference shocks θ from

$$\theta = \frac{\beta\lambda}{\gamma}\left(\frac{\gamma - \dot{x}/x}{\lambda + \dot{x}/x}\right).$$

Let $\theta_{i,t}$ be the preference shock experienced by country i at time t, then we have that $\theta_{i,t} = \beta_i\lambda_i(\gamma - g^d_{i,t})(\gamma\lambda_i + \gamma g^d_{i,t})^{-1}$. The country specific shocks are then pulled together, and a truncated normal distribution is fitted to the data. Finally, to have a union-wide political uncertainty and polarization parameter estimates, I compute a weighted average of the individual countries' parameters, where the weights w_i for country i are given by its relative contribution to the 1995 GDP of the union, namely $w_i = GDP_{i,1995}/\sum_i GDP_{i,1995}$, and the union-wide parameters $\widehat{\lambda} = I^{-1}\sum_i w_i\lambda_i$ and $\widehat{\beta} = I^{-1}\sum_i w_i\beta_i$.

Thresholds & Optimal Fiscal Rules. We now have all the information needed to compute both the optimal threshold $\theta^\star$ and the optimal rule each individual country should have imposed according to the model. I use equation (1.7) to compute the optimal threshold for the union $\theta^\star$, and the deficit rule in Section 1.4.1 to calculate the minimum allowable net lending of the European nations for the period 1995-2018.[52] Remember that fiscal rules can be implemented as debt-contingent deficit limits, so the net-lending limits from 1995 to the present are a function of the countries' realized debt to GDP ratio in that particular year. Let $Debt\_Ratio_{i,t}$ and $Rev\_Ratio_{i,t}$ be, respectively, the debt to GDP and revenue to GDP ratio for country i at time t. Then the country's net lending limit (as a percentage of GDP) $\bar{L}_{i,t}$ is computed as

$$\bar{L}_{i,t} = \left[1 - \frac{\gamma\widehat{\theta}^\star(\gamma+\widehat{\lambda})}{\gamma\widehat{\theta}^\star + \widehat{\lambda}\widehat{\beta}}\left(\frac{Debt\_Ratio_{i,t}}{Rev\_Ratio_{i,t}} + \frac{1}{r}\right) - \frac{Debt\_Ratio_{i,t}}{Rev\_Ratio_{i,t}}\, r\right] Rev\_Ratio_{i,t}.$$

I compute the net lending limit $\bar{L}_{i,t}$ for each country i and time $t \in [1995, 2018]$ using the realized revenue and debt to GDP ratio for that particular year. Figures 1.2 and 1.3 summarize the results.

It is important to notice that, since debt is an endogenous variable of the model, and we are using the realized debt and revenue series, the net lending limit $\bar{L}_{i,t}$ does not provide an estimate of how countries' debt would have evolved had they implemented the model's rules. Rather, it shows what the optimal net-lending limit

[51] Remember that $\dot{a}/a = \dot{x}/x$ in the model.

[52] In Section 1.4.1 we characterized the primary deficit limit. The net lending limit can be easily inferred by adding interest repayment and changing the sign.

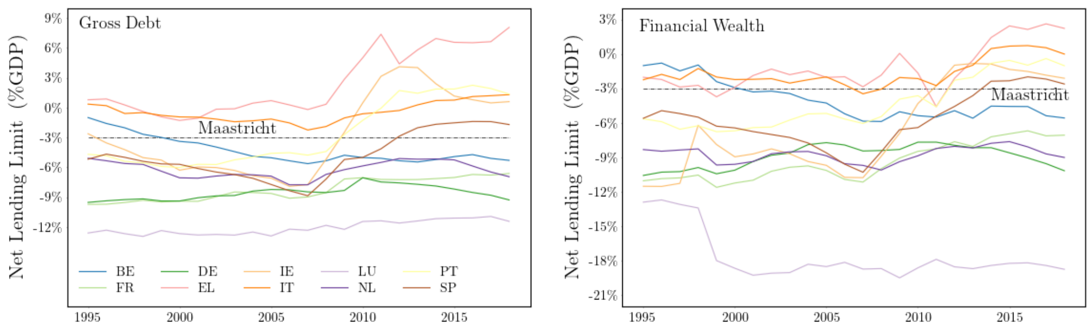


Figure 1.2 Optimal Net Lending Limit

Both graphs have net lending on the vertical assets: positive numbers are a surplus while negative numbers are a deficit. Each line is a country specific net-lending limit. To comply with the rule, countries should choose net lending above their line. *Left Panel*: Net lending limit computed with general government debt data (AMECO). *Right Panel*: Net lending limit computed with financial wealth data (World Bank).

would have been, had they decided to introduce a fiscal rule in that specific year. The left panel in Figure 1.2, for example, shows that had Europe, at the beginning of the century, decided to institute fiscal rules, the optimal choice would have been to allow France to run deficits up to 9% of GDP (given its 2000 debt level); while Italy and Greece should have been required to run a balanced budget. Overall, the 3% deficit limit imposed by the Maastricht Treaty was, according to this model, too loose for about half of the union members and too strict for the other half, suggesting that the political bargaining process in fiscal constraint selection managed to find a middle ground on which all member could agree.[53] However, as previously pointed out, the limit should be estimated using net assets. The right panel in Figure 1.2 presents a robustness check in which net financial assets, rather than gross debt are used in the computation of lending limits $\bar{L}_{i,t}$. It shows rules that are considerably looser with respect to the ones in the right panel. If one where to include illiquid public capital (i.e. public buildings), optimal rules would become even looser, indicating that a 3% deficit limit is too tight for the union.

Figure 1.3 depicts, in the upper-left panel, the net lending limit as a function of debt for the estimated union threshold $\widehat{\theta}^{\star}$ and for different revenue levels. The higher the revenue to GDP ratio, the higher the permissible deficit for a given debt to GDP ratio. For example, a 75% debt to GDP ratio corresponds to a maximum deficit of around 5% if revenues account for 40% of GDP, but only allows for a

[53] The extremely loose fiscal constraint in Luxembourg is due to its historically low debt to GDP ratio.

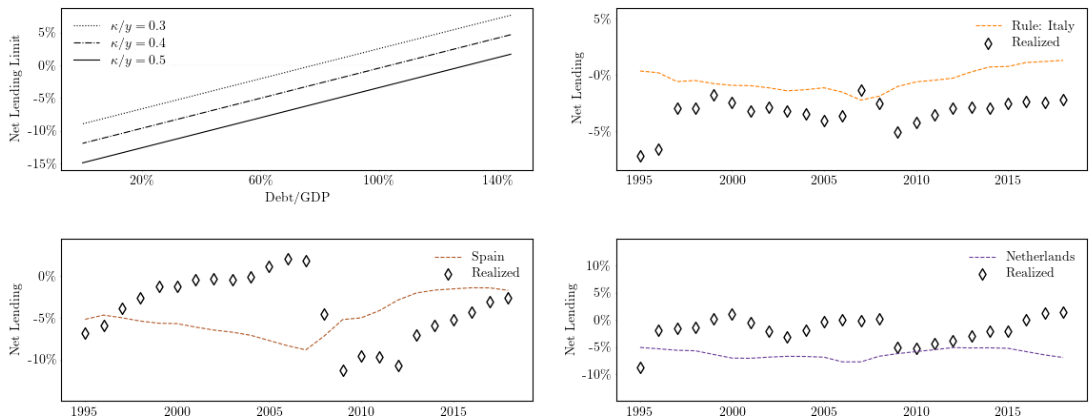


Figure 1.3 Optimal Lending Limit vs. Realized Lending

Upper Left Panel: Optimal fiscal rule as a function of Debt/GDP ratio for different Revenue/GDP levels ($\kappa/y \in [0.3, 0.4, 0.5]$). *Other Panels:* Comparison between the prescribed minimum net lending (colored line) and realized primary deficit/surplus as a percentage of GDP (diamonds) for Italy, Spain and the Netherlands (AMECO) .

balanced budget with a revenue to GDP ratio of 30%. The other three panels, instead, compare realized net lending in Italy, Spain and the Netherlands against the country-specific net lending limit $\bar{L}_{i,t}$ for the years between 1995 and 2018. Countries are not compliant with the rule whenever the realized surplus/deficit is below the country's minimum net-lending $\bar{L}_{i,t}$ (colored line). While the Netherlands has almost always managed to sustain large enough surpluses (or low enough deficits) to comply with its optimal deficit limit, Spain has over-borrowed after the financial crisis.[54] Italy, on the other hand, would have been required to keep a balanced budget (if not a small surplus) under this rule, but has almost always run a deficit.

Grants, Credit & Welfare Gains. To quantify optimal transfers, the first step is to find consumption shares $\widehat{k}^{nt}(\theta)$ and weighs $\widehat{\alpha}(\theta)$ using Proposition 1.2 and the previously estimated parameters, including the shock distribution. As it turns out, $\widehat{\theta}^{\star}$ is such that the first condition in Proposition 1.3 is satisfied. In other terms, since the estimated share of consumption for the transfer case $\widehat{k}^{tr}(\theta)$ is non-decreasing in θ, the no-transfer threshold coincides with the transfer one $\widehat{\theta}^{\star\star} = \widehat{\theta}^{\star}$. If the European Union implemented the optimal fiscal rules, then, it could add transfers to its instrument

[54] Yet, it could be argued that the shock distribution has changed after the financial crisis and that its variance should be re-estimated for the post-2008 period.

set without altering deficit limits. Let $Spend\_Ratio_{i,t}$ be the planned government expenditure as a proportion of GDP for country i in year t. The planner can infer $\theta_{i,t}$ from optimal spending in Proposition 1.2 as

$$\widehat{k}(\theta_{i,t})\widehat{\alpha}(\theta_{i,t}) = \frac{r\, Spend\_Ratio_{i,t}}{r\, Debt\_Ratio_{i,t} + Rev\_Ratio_{i,t}},$$

and compute the due (annualized value of) transfers, $T\_Ratio_{i,t}$, as a percentage of GDP from, equivalently, Proposition 1.2:

$$T\_Ratio_{i,t} = (\gamma + \widehat{\lambda})(\widehat{\alpha}(\theta_{i,t}) - 1)\left(Debt\_Ratio_{i,t} + \frac{Rev\_Ratio_{i,t}}{r}\right).$$

As an example, to get a sense of the transfers' entity, we can consider the case of Spain in 2012. Had the European Union decided to start implementing fiscal rules from that year on, Spain would have been allowed to run deficits up to 3% of GDP, a rule that coincides with the actual Maastricht requirement. Using data for spending, debt and revenue ratios, it turns out that Spain should have received a positive transfer of 3.02% of GDP.[55]

Further, we can compute what the maximum grant would be under an extreme shock realization as a function of the previously accumulated debt, for a given revenue-to-GDP ratio. Assuming that revenues are 35% of GDP, under the worst case scenario (i.e for $\theta = \overline{\theta}$), member countries would be entitled to transfers between 3% and 4.5% of GDP depending on the level of previously accumulate debt. Having lower debt or higher revenues would increase the transfer's entity. For example, a country having a 90% debt-to-GDP ratio and a 35% revenue-to-GDP ratio would receive a grant amounting to 3.9% of GDP when hit with the worst possible shock realization, while a union member with debts for 150% of GDP would only be entitled to a 3.5% transfer. Using the credit-line equation in the implementation Section 1.4.2:

$$Credit\_Ratio_{i,t} = \frac{\gamma\widehat{\lambda}(\theta_{i,t} - \widehat{\beta})}{\gamma\theta_{i,t} + \widehat{\lambda}\widehat{\beta}}\widehat{\alpha}(\theta_{i,t})\left(Debt\_Ratio_{i,t} + \frac{Rev\_Ratio_{i,t}}{r}\right),$$

we find out that transfers should represent about 30% of the overall financial help (including the credit-line) under extreme financial distress.

For a quick comparison, consider that, under the European pandemic relief program Next Generation EU (NGEU), grants amount to 52% of the total available

[55] Spending in Spain in 2012 was 41.5% of GDP (AMECO).

resources (750 billion Euros), with considerable cross-country variations. In Italy, which is one of the worst hit nations in the union, the percentage of grants is around 39% (209 billion Euros ca., of which 81.4 in grants and 127.4 in loans). Further, notice that the Recovery and Resilience Facility (part of NGEU) amounts to 672.5 billion, 70% of which will be distributed in the next two years. Back of the envelope calculations reveal that the planned yearly disbursement is around 1.7% of European GDP.

Finally, we can quantify welfare gains accruing to the EU from setting up a transfer system. More specifically, evaluating equation (1.11) with the previously estimated parameters and shock distribution, and data on the average revenue ratio for the union, I obtain that welfare gains are between 10% and 11% of European GDP.[56] This should be read as an upper bound on welfare gains, considering that the model only features a simplified asset market (non-contingent risk-free-bonds) with little availability of insurance against macroeconomic shocks.

1.6 Conclusions

The common rationale for having fiscal rules is that they are necessary to offset biases in fiscal policy. Yet, most rules comprise escape clauses, reflecting the fact that spending flexibility might be needed in adverse economic conditions to conduct counter-cyclical fiscal policies. The theory articulated in this paper formalizes this two key elements in a model with present biased governments having private information on the idiosyncratic state of the economy.

I have focused on the design of optimal fiscal rules at a supranational level in two distinct environments: one in which transfers across union members are not allowed and one in which they are. Optimal rules were found to be of the threshold kind in both environments, but weakly more stringent when transfers are allowed. In a fiscal union, debt-dependent transfers complement the set of rules. All instruments are debt-contingent: higher public debt contemporaneously tightens deficit limits and reduces financial assistance. One of the main policy implications of this paper, then, is that uniform, constant thresholds across countries, like the Maastricht 3% deficit limit, are sub-optimal. Fiscal constraints contingent on preexisting debt-levels, like some of the ones detailed in the more recent fiscal compact, are much closer to the derived optimal rule.

[56] Variability is due to changes in the average revenue ratio for the union over the years, so welfare gains depend on when, exactly, transfers are assumed to be introduced. However, since the average revenue ratio is quite stable, welfare gains do not heavily depend on the year in which the transfer system is set-up.

The described optimal rules can be implemented as simple deficit limits and complemented with a combination of grants and loans when cross-country subsidies are allowed. This paper details the optimal transfer system in a fiscal union, and shows under which conditions the introduction of transfers should be matched with a tightening of the fiscal rules. The recent creation of a European Recovery Fund in response to the Covid pandemic, has spurred new disagreements between EU members over the entity of financial assistance provided and over the relative proportion of grants and credits it should entail. This work supplies a useful benchmark to frame this now salient discussion.

Chapter 2

Turnover in the design of fiscal rules

(Joint with Facundo Piguillem)

We study the optimal trade-off between commitment and flexibility in a model in which governments are present-biased toward spending and have private information on the state of the economy. Importantly, we introduce stochastic government turnover. The model decomposes the present-bias in different components: the fundamental political friction – captured by hyperbolic discounting; the overall uncertainty in the economy; and the relative relevance of political turnover versus business cycle fluctuations. Fiscal rules, both in a national and in a supranational setting, are found to be stricter when insurance needs are low, the present bias is high and government turnover is frequent.

2.1 Introduction

The recent debt crisis in European countries revived the debate about the effectiveness of fiscal rules in debt accumulation prevention. In the past 30 years, there has been a sharp increase in the number of countries adopting fiscal rules, even though not all sovereign debt accumulation is necessary inefficient, as it may allow governments to decouple spending needs from available revenues. Moreover, currently implemented rules all over the world are largely based on rules of thumb, mostly because the theoretical literature on optimal fiscal rules' design considers models that – while providing key insights – are too abstract and simplified for policymaking.

This paper makes a step forward in narrowing the gap between theory and practice of fiscal rules by modeling political uncertainty in a less reduced-form way while retaining tractability. We show that the extent and nature of sound fiscal rules should depend on the underlying friction distorting debt's optimal use and, in particular, that they should target the political source of inefficiency rather than the normal insurance needs of an economy in response to its physiological business cycles. In fact, countries having unstable governments – with a high degree of political turnover – should be subjected to strict rules; while countries who experience frequent economic shocks – and therefore would benefit from insurance – require looser constraints on their fiscal policy.

The narrative on myopic politicians has a long tradition, yet, it wasn't until Persson and Svensson (1989) and Alesina and Tabellini (1990) that the link between political turnover and preference misalignment across policymakers was formally established.[1] The mechanism through which political frictions distort governments' budget choices in the literature is, in almost all cases, a variation on the so-called "tragedy of the commons". Current policymakers do not, or not fully, value the spending choices of future office holders. As a result, the lower-than-socially-optimal weight placed on future government spending incentivizes policymakers to overspend in the present. As it turns out, this effect is stronger than additional intertemporal discounting (i.e., modeled through a lower discount factor): it resembles quasi-hyperbolic discounting. In this context, Halac and Yared (2018) show that – among all possible policy instruments – spending limits are optimal tools when governments are subject to shocks that are neither observable nor verifiable.[2]

[1] Later, Battaglini and Coate (2007) showed a similar mechanism in a richer and more sophisticated environment of policy making.

[2] This insight links prescriptions for fiscal rules with the normative implications solving the optimal trade-off between commitment and flexibility with time inconsistent spenders as in Amador et al. (2006).

This paper is closest to the work in Amador et al. (2006) and Halac and Yared (2014, 2018), which falls within the mechanism design literature in self-control settings. When governments have private information on the state of the economy and a tendency to systematically exceed the socially optimum level of consumption, a trade-off arises between allowing authorities the flexibility in spending required to react to macroeconomic shocks and the commitment society would like to impose on them to dampen biased expenditures. Fiscal rules, in this setting, reduce the ability to smooth consumption at the national level, but also impose predetermined fiscal constraints that narrow the gap between socially optimal and actual policy. In Amador et al. (2006), the same trade-off between commitment and flexibility arises and the authors show that optimal fiscal rules are of the threshold kind. Depending on the specifics of the model, this threshold has been shown to vary with, among other things, the extent of the political friction (Amador et al., 2006), the persistence of shocks (Halac and Yared, 2014) and the framework in which rules are imposed, whether national or supranational (Halac and Yared, 2018).[3]

The paper is akin to Amador et al. (2006), in that we assume a reduced form political bias and focus on the normative prescriptions of the set-up. Importantly, however, we analyze a setting in which *i)* government duration is stochastic and *ii)* idiosyncratic shocks to the economy are not necessarily related to political turnover. The approach provides two important extensions to the current literature. First, we go a step further endogenizing the intertemporal friction, thereby allowing for a better identification of the inefficiency sources in the model and potentially providing a richer mapping to the data. Second, we disentangle the effects of economic shocks (that call for efficient insurance) from those of the political friction (which instead generate over-spending) on the design of fiscal rules.

As in Halac and Yared (2018), we consider two frameworks: a national one in which the interest rate at which countries can borrow is fixed and a supranational one in which, instead, the interest rate is an endogenous object.

First, in a national environment in which political and economic shocks are perfectly correlated, we extend Amador et al. (2006) and Halac and Yared (2018) static results on the relation between political bias and optimal spending threshold to a dynamic setting: under mild conditions on the shock distribution, thresholds are monotonically decreasing in the degree of political bias β, meaning that tighter rules are required for more myopic governments. Having closed-form solutions, we can

[3] More specifically, Halac and Yared (2018) show that when interest rates are an equilibrium object, the supranational planner can account for the pecuniary externality generated by governments' accumulation strategies. Amador et al. (2006) frame the discussion around a general principal-agent problem.

also show that the optimal fiscal rule can be implemented as a limit to the speed of debt accumulation.

We further add that, when political and economic shocks are *not* perfectly correlated, optimal spending-thresholds in a national setting respond to the specific type of uncertainty in the economy: rules should be looser when economic uncertainty (i.e., business cycle shocks) is relatively more prominent and tighter when political volatility is high (i.e., frequent government turnover).

In fact, we establish that shock frequency is almost always relevant for the spending threshold determination except in the special case in which economic and political shocks are always concomitant. Further, even in this special case, overall uncertainty in the economy still alters the *implementation* of fiscal rules: countries are allowed to accumulate debt at a faster pace when uncertainty is high.

Secondly, in a supranational (or coordinated) environment – as in the national case – rules should be permissive when economic uncertainty (i.e., business cycle shocks) is relatively more prominent and restrictive when political volatility is high (i.e., frequent government turnover). Finally, we show that rules become less effective when turnover is the main source of volatility in the economy and that, when political and economic shocks are uncorrelated, coordinated rules tend to be stricter than uncoordinated ones, regardless the strength of the political friction (i.e., how myopic governments are).

2.1.1 Related Literature

This work relates to the vast literature on the political economy of fiscal policy, including Alesina and Tabellini (1990), Krusell and Rios-Rull (1999), Persson and Svensson (1989), Battaglini and Coate (2007, 2008) and Azzimonti (2011). In particular, we provide an alternative, continuous time, representation of the positive literature in which the political bias is micro-funded: our setup can be used to nest the different interpretations in, for instance, Persson and Svensson (1989), Alesina and Tabellini (1990) and Battaglini and Coate (2007). As in Acemoglu et al. (2008) and Yared (2010), we study the provision of dynamic incentives to self-interested politicians, but we concentrate on an international context, rather than on the conflict between citizens and their own national government.[4]

More generally, this work also contributes to the literature on international or inter-regional risk-sharing, including Atkeson and Bayoumi (1993) and Bucovet-

[4] Acemoglu et al. (2008) show, for instance, that when elected officials are as patient as their citizens, no additional distortions arise, other than those implied by their incentive compatibility constraints.

sky (1998). Persson and Tabellini (1996a) explore the effectiveness of different fiscal agreements in a theoretical model comprising moral-hazard, while Persson and Tabellini (1996b) investigate how different fiscal constitutions shape insurance provision. Within this literature, the paper is closest to Lockwood (1999), who also sets-up a mechanism design problem in an environment in which regional authorities have private information on their idiosyncratic shocks. However, we do not model externalities in the public good provision and provide, instead, an extension focusing on political bias.

Finally, broadly speaking, the paper also relates to the literature on hyperbolic discounting and commitment devices à la Phelps and Pollak (1968), including Laibson (1997), Barro (1999), Krusell and Smith (2003), Krusell et al. (2010), Bisin et al. (2015), Lizzeri and Yariv (2017). In particular, the model presented here converges to the quasi-hyperbolic preferences set-up in Harris and Laibson (2012) when shocks to the economy and political turnover are contemporaneous and extremely frequent.

The paper is organized as follows: Sections 2.2 provides both the model set-up and the characterization of allocations when no fiscal rules are imposed on the economy. Section 2.3 derives optimal rules both in a national and in a supranational context; while concluding remarks are presented in Section 2.4.

2.2 Environment

This section develops a dynamic model incorporating the standard frictions of spending bias and incomplete information. Contrary to the previous literature however, we distinguish between shocks to preferences (which are private information) and stochastic government turnover. We first briefly describe the model's fundamentals, then we characterize the equilibrium without fiscal rules.

2.2.1 Model

There is a unit mass of infinitely lived countries that may differ, ex-ante, only in their initial savings $x(t)$, where we use the convention that $x(t) < 0$ represents a debt. Time is continuous and every instant $t \geq 0$ countries receive and exogenous source of revenue τ. In every country, the *incumbent* government faces a saving/spending choice, subject to its budget constraint. Savings today generate a risk-free return r, which is taken as given by governments. Since the debt is considered risk-free, a government cannot borrow more than the present value of future revenues (natural debt limit) $\phi = \tau/r$, so that $x(t) \geq -\phi$, for all t. The law of motion of financial wealth is:

$$dx = (\tau + rx - g)dt. \tag{2.1}$$

To simplify notation we change the state variable. Define total wealth a as the current financial assets plus the present value of all future resources $a(t) \equiv x(t) + \phi$. Then, for any stochastic sequence g, total wealth follows a Brownian motion with drift $(ar - g)$ and zero variance [5]

$$da = (ra - g)dt. \tag{2.2}$$

Depending on the value they attribute to public spending, governments can be of different types θ. In particular, we assume that the instanteneous flow of utility for type θ is:

$$u(g; \theta) = \theta \log(g). \tag{2.3}$$

Types with high θ place more weight on spending than low types, who have low marginal utility of current consumption $g(t)$. Government preferences can be interpreted as arising from the underlying constituency's opinions on the social value of spending, they can change over time and determine an alteration of the country's stance on fiscal policy. One possible interpretation is that preferences vary in response to the business cycle: the government may asses that a lean-against-the-wind type of policy is more effective during downturns.[6] In the rest of the paper, we will refer to a type θ change as either a preference or business cycle shock.

A key assumption is that the realization of θ is privately observed by the government and therefore it is not possible to write contracts contingent on it. The important component of this assumption is that θ is non-contractible, so it applies even in the case in which shocks are either measurable or only ex-post verifiable, and captures the idea that it may be (politically or technically) unfeasible for a rule-designer to write a specific policy prescription for each possible contingency or shock in the economy. Without loss of generality, governments' types θ are drown from a bounded set $\Theta \equiv [\underline{\theta}, \bar{\theta}]$, with distribution function $H(\theta)$, normalized so that $\mathbb{E}[\,\theta\,] = 1$.

Countries, however, are not only subject to *preference* changes, but also to *political* ones, as different governments alternate in power. Differently from the previous

[5] Notice that the requirement that current resources are above the natural debt limit $x(t) \geq -\phi$ translates in the requirement that wealth is non-negative: $a(t) \geq 0$. To derive the "cash-on-hand" formulation of the budget constraint is enough to substitute $x = a - \phi$ in (2.1) and rearrange to obtain $da = y - g + ra - r\phi$. Finally substitute $\phi = y/r$ to obtain (2.2)

[6] Amador et al. (2006) show that if utility is exponential, taste shocks are equivalent to income shocks.

literature, the random government changes might occur either separately or concomitantly with a preference shock. More specifically – within a country – a shock occurs with arrival rate λ and, conditional on the shock occurring, one of three things can happen:

i) with probability p_θ the incumbent government stays in power but is subject to a preference shock, namely, a new type θ is drawn from the distribution $H(\theta)$. This is meant to capture instances in which the governing party's (or coalition's) marginal value of spending changes. For example, if we interpret preferences shocks as a response to the business cycle, λp_θ would represent the frequency with which the incumbent government alters its fiscal policy in response to an idiosyncratic shock to the economy. An economy in which $p_\theta = 1$ (e.g., all shocks are preference shocks), is not subject to political turnover and only displays *economic uncertainty*: there only are business cycle (or preference) shocks;

ii) with probability p_β the current government falls and a new one, of the same type θ is formed. This second possibility arises whenever there is a reshuffling of the government absent a substantial preference change. The important thing here, is not whether the new governing coalition is of the same "color" of the previous one, rather, whether it maintains the same view on the marginal value of public spending. An economy in which $p_\beta = 1$ (e.g., all shocks are political shocks), is not subject to business cycle shocks and only displays *political uncertainty*: government turnover is the only shock;

iii) with probability $p_{\beta\theta} = (1 - p_\theta - p_\beta)$ the incumbent looses power and a new government of a different type θ is elected. In this case, the arrival of a new incumbent also determines a preference change, capturing *the interaction* between economic shocks and political unrest. The last possibility captures the frequency with which changes to the marginal value of spending occur because the government changed or, alternatively, cases in which a shock to the economy determines a government change. An economy in which $p_{\beta\theta} = 1$ (e.g., all shocks determine both a preference and a government change) is not subject to economic or political uncertainty in isolation. Rather, business cycle and turnover shocks always occur concomitantly as is the case in Amador et al. (2006), Halac and Yared (2018) and the existing literature.

Summing up, contingent on the arrival of a shock, the overall probability of a preference change $P_\theta \equiv (1 - p_\beta)$ captures the *economic uncertainty* in the model; while the overall probability of government turnover $P_\beta \equiv (1 - p_\theta)$ is a proxy for the amount of *political (in)stability* in the economy.

All governments, whether they are the incumbent or the opposition, discount the future exponentially at rate γ. However, incumbents value spending less when they are not in office. To be precise, every unit of spending transforms into one unit of consumption when the government is in power, but it only delivers $0 < \tilde{\beta} < 1$ units of consumption when the government is out of office. To simplify the analysis, we recast this political friction in terms of utility. Namely, we assume that governments discount utility by the extra-term β whenever they are not in power.[7] Thus, introducing the discount term β is a reduced form way of capturing disagreement within a country over the composition of public spending, rather than over its level.[8]

We study a Markovian equilibrium. Each government takes future governments' actions as given and chooses spending subject to the budget constraint (2.2). Let $w(a, \theta)$ be the government's present value of utility for a given level of wealth a and current type θ, and let $v(a, \theta)$ be the present value of utility discounted at rate γ. Thus, w_a and v_a are, respectively, their derivatives with respect to assets, and we indicate with $\bar{w}$ and $\bar{v}$ their expected value over the support of theta - i.e. for a generic function $f(\theta)$: $\bar{f} = \int_{\underline{\theta}}^{\bar{\theta}} f(\theta)h(\theta)d\theta$. Asset holdings a and current preference shock θ completely describe the state of the economy, so that the value functions w, v and equilibrium policy functions depend on the pair (a, θ). However, with a slight abuse of notation, we omit in what follows the dependency on assets a. The equilibrium can be characterized as the solution to the following system of Jacobi-Bellman equations[9]:

$$\gamma w(\theta) = \max_{g \ \text{s.t.}(2.2)} \left\{ \theta \ln(g) + w_a(\theta)(ra - g) \right\} + \lambda \left(p_\theta \, \bar{w} + p_\beta \, \beta v(\theta) + p_{\beta\theta} \, \beta \bar{v} - w(\theta) \right), \quad (2.4)$$

$$\gamma v(\theta) = \theta \ln(g^\star(\theta)) + v_a(\theta)(ra - g^\star(\theta)) + \lambda \left(p_\theta \, \bar{v} + p_\beta \, v(\theta) + p_{\beta\theta} \, \bar{v} - v(\theta) \right), \quad (2.5)$$

[7] This type of preferences can be micro-funded by appealing to the interaction between turnover and political polarization as in the seminal work by Alesina and Tabellini (1990), or (more recently) invoking "pork barrel" spending, as in Battaglini and Coate (2007).

[8] A second interpretation, is that the preference structure arises naturally from the aggregation of time consistent preferences with heterogeneous discount rates (see Jackson and Yariv (2014, 2015)). Both Alesina and Tabellini (1990) and Battaglini and Coate (2007) are isomorphic to the standard quasi-hyperbolic discounting set-up in Laibson (1997). Indeed, we show that when $\lambda \Rightarrow \infty$ our model maps to a continuous time equivalent of the quasi-hyperbolic discounting framework in Harris and Laibson (2012). The possibility of achieving this mapping implies that the assumed political friction can arise from the aggregation of time consistent preferences with heterogeneous discount rates.

[9] See the appendix for a derivation of the Jacobi-Bellman equations from the sequential problem

where $g^\star(a,\theta) \equiv \arg\max_g\{w(a,\theta)\}$ such that the budget constraint in (2.2) holds. Notice that the incumbent always discounts the future at rate γ, both during periods in which she is in power and during the ones in which she is not. However, when she loses control, there is a once-and-for-all change in discounting, represented by the additional term β in the second and third components of the last term in equation (2.4). This additional discounting, not present in equation (2.5), generates the gap between w and v. Moreover, since the incumbent can only make spending decisions when in power, the maximization operator shows up exclusively in equation (2.4), not in equation (2.5). When out of power, she takes future governments' choice $g^\star$ as given.

This representation has a very intuitive interpretation. The term $\gamma\, w$ in equation (2.4) is the expected value of instantaneous changes in w arising from the exponential discounting. The first term of the right hand side – $\theta\ln(g)$ – is the flow utility derived from government spending, while $w_a\dot{a}$ is the expected value of instantaneous changes in w arising from the returns process. The next three terms represent the expected value of the instantaneous change in w due to a possible transition – with arrival rate λ – from the current situation with value $w(a,\theta)$, to one of the three possible future states. With probability p_θ the incumbent government stays in charge but preferences change, in which case the change in present value is $\bar{w} - w(\theta)$. With probability p_β the incumbent looses power but θ remains the same. Since a new government is formed but preferences remain unaltered the change in utility in value is $\beta v(\theta) - w(\theta)$. Finally, with probability $p_{\beta\theta}$ a new government with different preferences is elected. The last term of equation (2.4) represents the instantaneous change in value which is $\beta\bar{v} - w(\theta)$. Equations (2.4) and (2.5) are almost identical, with one important exception: from the point of view of the current incumbent, future streams of spending after loosing power are discounted at rate β. Yet, once control has passed to a new government, any subsequent transition has already been discounted, which is why the term β appears in equation (2.4), but not in equation (2.5).

Notice that when there are no shocks, the problem collapses to a standard saving-spending problem. In fact, for $\lambda = 0$ the current value function w is equal to the continuation value function v: the incumbent government keeps its type and remains in office forever. In this case countries spend a constant portion γ of their wealth a every period.

For $\lambda > 0$, our model adds to two streams of literature. First, it provides an alternative continuous time representation of the *positive literature* with micro-funded setups in Persson and Svensson (1989), Alesina and Tabellini (1990) and Battaglini and Coate (2007) nesting different interpretations. For instance, $\beta < 1$ could be thought of as the probability that the current politician has access to "pork" in the following legislatures as in Battaglini and Coate (2007). A government with $\theta < 1 = E(\theta)$ could be interpreted as the stubborn politician who is expecting to face a more lefty government in the future as in Persson and Svensson (1989) and so on.

Second, it provides a link and enriches the *normative literature* on commitment vs. flexibility discussion à la Amador et al. (2006). We do so in two ways. First, we go a step further endogeneizing the intertemporal friction. When preferences and government changes only happen contemporaneously and with certainty, namely for $p_{\beta\theta} = 1$ and $\lambda \to \infty$, the set-up converges to a continuous-time version of the two period model in Halac and Yared (2018). This generalization allows us to better identify the sources of inefficiencies and potentially provides a richer mapping to the data. Second, we disentangle the effects of preference change θ, that call for efficient insurance, from those of the political friction β, which instead generate over-spending, on the design of fiscal rules. Characterizing the solution under the possibility of having two separate shocks, and/or under stochastic political turnover ($p_\theta, p_\beta \neq 0$, $\lambda < \infty$) provides a generalization of their policy prescription.

As in Halac and Yared (2018), we consider two types of problems: a national and a supranational one. In the national, or uncoordinated framework we characterize the equilibrium under the assumption that both the government and a social planner live in a small open economy, meaning that they take the interest rate r as given. In the supranational framework we instead assume that the social planner sees the group of different countries as a closed economy and can therefore internalize the effect government borrowing has on the interest rate; while single governments are price takers.

2.2.2 Rules-Free Solutions

Two frictions prevent the attainment of the first best allocation in this model: (i) the fact that preference shocks θ are private information, (ii) the presence of a political conflict for $\beta < 1$. The combination of these two frictions leads to a trade-off between commitment and flexibility.

Planner Benchmark. The first informative benchmark for this set-up is the spending choice of a social planner who can perfectly observe the realization of the preference shock θ and only cares about the efficient inter-temporal allocation of resources, independently of the identity of the government in power. This planners' spending choice is equivalent to a state contingent policy that maximizes the ex-ante expected value of utility v: $g^\circ(a, \theta) \equiv \{\arg\max(v)\}$ subject to (2.2). The implied first-best spending is:

$$g^{\circ}(a,\theta) = \frac{\theta\,(\gamma+\lambda)}{\theta\,(\gamma+\lambda\Lambda_\beta)+\lambda(1-\Lambda_\beta)}\ \ \gamma a, \tag{2.6}$$

where

$$\Lambda_\beta \equiv \frac{\gamma p_\beta}{\gamma+\lambda(1-p_\beta)}.$$

We interpret the variable $\Lambda_\beta \in [0,1]$ as the portion of uncertainty in the economy that is *not due* to preference shocks.[10] The terminology might seem confusing since we could simply say that Λ_β captures uncertainty *due* to political shocks. However, this would be slightly misleading. Notice that when $p_\beta = 1$, namely when there is only political uncertainty, $\Lambda_\beta = 1$, meaning, in fact, that none of the uncertainty in the economy is due to the business cycle or, equivalently, that all the uncertainty is of a purely political nature. However, the fact that $p_\beta = 0$ only implies that all shocks determine a preference change, but they might (i.e., $p_{\beta\theta} = 1$) or might not (i.e., $p_\theta = 1$) also feature government turnover. Then, when $\Lambda_\beta = 0$, what is true is that no uncertainty is *not due* to the business cycle, not that all uncertainty is due to political shocks.

The first best option for the planner would be to spend a type-dependent portion of wealth that is increasing in θ and linear in a, meaning that the planner reacts to business cycle shocks by expanding the government's balance when its marginal utility is higher. Spending flexibility is valuable in this model as it allows to run countercyclical fiscal policy, delivering insurance against the idiosyncratic shocks.

Moreover, the amount of insurance the planner would like to provide depends on both the shock frequency and on the portion of volatility in the economy that is attributable to preference shocks. In fact, as shown in the right-hand panels of Figure 2.1, when shocks are extremely frequent ($\lambda \to \infty$), spending is linear in θ and the planner restraints herself in good times (when the marginal utility of spending is low), while she splurges in bad times. On the other hand, if $\lambda P_\theta = 0$, meaning either that there are no shocks in the economy ($\lambda = 0$), or that shocks are not due to economic uncertainty ($\lambda > 0$ and $P_\theta = 0 \Rightarrow \Lambda_\beta = 1$), there is no need for insurance and the planner allocates a constant proportion γ of wealth to current spending ($g^{\circ} = \gamma a$ for any possible level of assets a and type θ).

In general, when $\lambda \in (0,\infty)$, the planners' policy is concave in the share of wealth spent as a function of θ, and the more so the lower is the portion of uncertainty that is not due to the business cycles (e.g., for low values of Λ_β). If preferences are relatively stable (i.e., long business cycles) and the marginal utility of spending is currently

[10] Alternatively, one can interpret Λ_β as a discounted odds-ratio: it measures the relative likelihood of keeping one's type θ against the likelihood of changing it.

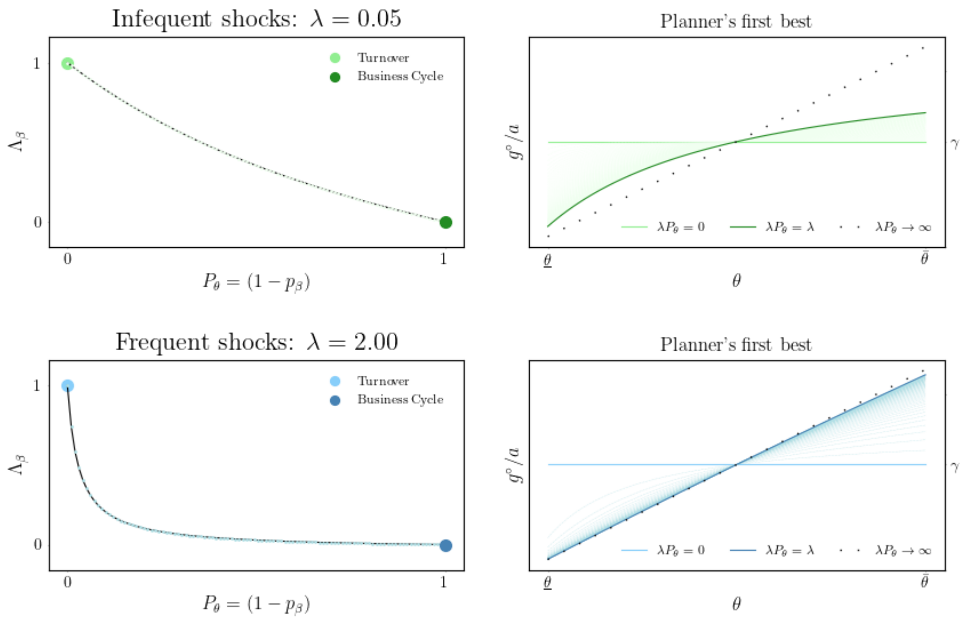


Figure 2.1 First Best

Upper panels consider an economy with infrequent shocks, while the opposite is true in bottom panels. A low value of the rate λ implies that shocks are infrequent (if $\lambda = 0.05$, then $1/\lambda = 20$, meaning shocks hit on average once every 20 years, while shocks hit twice a year when $\lambda = 2$).The left panels show the portion of volatility in the economy that is not due to preference shocks Λ_β as a function of the (conditional) overall probability of a preference change P_θ. The right panels plot the share of wealth consumed by the planner as a function of the current preference θ. The colored lines show the share consumed for $P_\theta \in [0, 1]$. As a reference, we also plot the optimal share of wealth consumed in the limit with instantaneous shocks, namely for $\lambda \to \infty$ (dotted line).

high ($\theta > 1$), the planner spends less as compared to the case in which preference shocks are more frequent. This is because insuring today by increasing g° reduces her ability to provide insurance in the future by depleting resources. On the contrary, when $\theta < 1$ the knowledge that good times are going to last for a relatively long period of time allows the planner to provide more insurance by spending a higher portion of wealth (as compared to the case in which business cycles are short). Further, the higher is the portion of volatility in the economy attributable to preference shock, the stronger is the insurance motive.

Unrestricted Government Benchmark. The second informative benchmark is the spending policy of a government when no restriction is placed on its debt choice.

The problem solves (2.2)-(2.5), generating an *unrestricted* spending policy function:

$$g^{un}(a,\theta) = \frac{\theta\,(\gamma+\lambda)}{\theta\,(\gamma+\lambda\beta\Lambda_\beta)+\lambda\beta(1\,-\,\Lambda_\beta)\,+\,\lambda(1-\beta)\Lambda_\theta}\;\gamma a,$$

where

$$\Lambda_\theta \equiv \frac{\gamma p_\theta}{\gamma+\lambda(1-p_\theta)}.$$

We interpret the variable Λ_θ as the portion of volatility in the economy that is *not due* to political turnover. Analogously to Λ_β, notice that when there are only preference shocks without any government change (i.e., $p_\theta = 1$) then none of the uncertainty in the economy is of a political nature ($\Lambda_\theta = 1$).

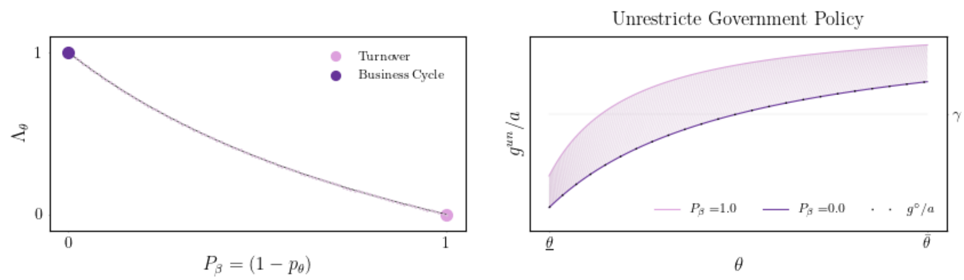


Figure 2.2 Unrestricted Government Policy

The left panel shows the portion of volatility in the economy that is not due to political shocks, Λ_θ, as a function of the (conditional) overall probability of a government change P_β. The right panel plots the share of wealth consumed by an unrestricted government as a function of the current preference θ. The colored lines show the share consumed for $P_\beta \in [0,1]$. As a reference, we also plot the planner's first best (dotted line). The arrival rate of the shock is $\lambda = 0.05$.

If the incumbent preferences are not biased ($\beta = 1$) or the incumbent remains in charge forever ($p_\theta = 1$), the unrestricted policy coincides with the optimal ex-ante spending of the planner, rendering any additional intervention inefficient.[11] However, as long as $\beta < 1$ and $p_\theta < 1$, countries overspend with respect to their ex-ante optimum – i.e., $g^{un}(a,\theta) > g^\circ(a,\theta)$ for all γ, a, θ and r[12]. If they could commit to spend g° rather than g^{un}, thereby limiting their flexibility, welfare would improve.

[11] To see this, remember that when $p_\theta = 1$, p_β must be zero, so $\Lambda_\beta = 0$. Further, notice that no intervention is required also in the trivial case in which there are no shocks to the economy, namely for $\lambda = 0$.

[12] To see this, notice that: *(i)* g°,g^{un} have the same numerator, *(ii)* using the definitions for Λ_θ,Λ_β, it is easy to prove that the denominator for the planner's policy function g° is larger than the unconstrained government's one – g^{un} – for every possible type θ.

If the rule designer could observe θ and write verifiable enforceable rules, she would simply impose g° as the state-contingent maximum spending limit. However, since preference shocks are private information, the rule-making body cannot write contracts contingent on θ and the optimal mechanism is non-trivial.

2.3 Fiscal Rules and Implementations

One intuitive way to correct the friction implied by β would be to tax debt accumulation and rebate its proceeds to the countries. However, since we are thinking about sovereign states, the redistributive consequences implied by such a policy could make it non-politically viable (i.e., transfers among European countries are a controversial subject). Alternatively, one could allow for taxation but exclude transfers among countries, which is, indeed, the approach taken by Amador et al. (2006) and Halac and Yared (2018). Yet, they show that the optimal mechanism restricts types above some threshold to choose the same saving/spending bundle (i.e., is akin to a deficit/spending limit). We follow Halac and Yared (2018) in restricting the analysis to *threshold rules* that bunch high types and limit their spending behavior while leaving the others unrestricted. This section focuses on how optimal thresholds are selected both in a small open economy and in a closed economy. We discuss the effect of political instability on threshold selection and on concrete policy prescriptions.

Fiscal Rules. We define a fiscal rule as a cut-off $\theta^{\star} \in \Theta$ such that governments of type $\theta < \theta^{\star}$ are left free to chose spending $g(a,\theta) = g^{un}(a,\theta)$; while types $\theta \geq \theta^{*}$ must choose spending $g(a,\theta) = g^{un}(a,\theta^{*})$. We later show that this rule can be implemented as an upper bound on the growth rate of debt or a deficit limit.

Remember that spending maximises (2.4) subject to the budget constraint in (2.2), for a given threshold $\theta^{\star}$ and future selves' spending behaviour. In other terms, when a fiscal rule is in place, spending must satisfy the optimality condition:

$$g = \begin{cases} \theta \,/\, w_a(a,\theta) & \text{for } \theta < \theta^{\star}, \\ \theta^{\star}/w_a(a,\theta^{\star}) & \text{for } \theta \geq \theta^{\star}. \end{cases} \tag{2.7}$$

We solve the individual problem by guessing a functional form for the value functions w, v and verifying that the guess is a solution to the system in (2.2) - (2.5),(2.7). Since preferences are logarithmic, a reasonable guess is that value functions will also be logarithmic.

Proposition 2.1 *Given a threshold $\theta^\star$, all types $\theta < \theta^\star$ will exert full discretion and thus choose spending $g(a,\theta) = C(\theta)a$, while types $\theta \geq \theta^\star$ will have no discretion and thus choose $g(a,\theta) = C(\theta^\star)a$, where*

$$C(\theta) = \frac{\gamma\theta\,(\gamma+\lambda)}{\theta\,(\gamma+\lambda\beta\Lambda_\beta)+\lambda\,[\,\beta(1\,-\,\Lambda_\beta)\,+\,(1-\beta)\Lambda_\theta\,]}.$$

Countries with discretion spend a proportion of their wealth a which is type-dependent and constant for a given type θ. Countries without discretion can only spend a fixed portion of their assets, determined by the threshold $\theta^\star$.

Implementation As explained in the first chapter of this dissertation, although the solution of the problem seems to depend on a particular type-realization, it is immediate to see that it can be implemented as a limit on the rate at which governments reduce their financial assets. To see this, recall that $a \equiv (x + \frac{y}{r})$ and suppose that the regulator only observes x and g. The definition of a immediately implies that $\dot{a} = \dot{x}$. Then, using equation (2.2), the fiscal rule can be implemented with an upper bound on the percentage growth rate of debt given by:

$$\frac{\dot{x}}{x} \leq (\,r - C(\theta^*)\,)\left(1+\frac{y}{rx}\right). \tag{2.8}$$

This upper bound is debt dependent, it becomes tighter as debt grows.[13] In the extreme where $x = -\phi$, the upper bound is zero, so that no further debt is allowed. In short, *the fiscal rule can be implemented as a debt dependent limit in the percentage growth rate of debt.*

Alternatively, one can think about deficit rules as a proportion of GDP. Since the deficit is defined as $d \equiv (g - y)$ the threshold rule would impose:

$$\frac{d}{y} \leq C(\theta^*)\left(\frac{x}{y}+\frac{1}{r}\right) - 1. \tag{2.9}$$

This fiscal rule expresses a deficit limit *contingent* on the debt/GDP ratio.

2.3.1 Small Open Economy

We focus here on the selection of the optimal threshold $\theta^\star$ when the social planner takes the interest rate as given. To have a well defined level of assets we assume

[13] The inequality follows from the fact that $x < 0$. When $x > 0$ the inequality in equation (2.8) must be reversed.

that $r \leq \gamma$. In the next subsection we relax this assumption and study an environment with endogenous interest rate.[14] The planner chooses $\theta^\star$ to maximize ex-ante expected welfare: $\int_{\underline{\theta}}^{\bar{\theta}} v(a, \theta; \theta^*) h(\theta) d\theta$, taking the optimal government choice (2.7) as given. Thus, the planner's problem in a small open economy solves

$$\max_{\theta^* \in [\underline{\theta}, \bar{\theta}]} \left\{ \int_{\underline{\theta}}^{\theta^*} v(a, \theta; \theta^*)\ h(\theta)\, d(\theta) + \int_{\theta^*}^{\bar{\theta}} v(a, \theta; \theta^*)\ h(\theta)\, d(\theta) \right\}, \tag{2.10}$$

where, since the types $\theta \geq \theta^*$ are currently constrained while the remaining types are not, we write the welfare function separating the two cases. Then we have,

Proposition 2.2 *For any given interest rate r, the optimal uncoordinated cut-off $\theta_u^\star$ satisfies*

$$\frac{\mathbb{E}_\theta[\theta | \theta \geq \theta_u^\star]}{\theta_u^\star} = \frac{(1 - \Lambda_\beta)}{\beta(1 - \Lambda_\beta) + (1 - \beta)(\Lambda_\theta - \theta_u^\star \Lambda_\beta)}. \tag{2.11}$$

To interpret this result it is important to bear in mind the interpretation of each component. Λ_β captures the *political* portion of uncertainty or, more precisely, the variability that is not due to business cycles (without shocks to θ only). Similarly, Λ_θ captures the *economic* portion of uncertainty or, more precisely, variability that is not due to government turnover (without changes in government only). Therefore, their values are affected by the correlation between business cycle shocks and political shocks. If the correlation between them is one, then both shocks happen simultaneously generating $\Lambda_\beta = \Lambda_\theta = 0$. In this special case the optimal threshold satisfies:

$$\frac{\mathbb{E}_\theta[\theta | \theta \geq \theta_u^\star]}{\theta_u^\star} = \frac{1}{\beta}.$$

This value is exactly the same as in Halac and Yared (2018), who assume that governments change every period with certainty and that, in addition, every period governments have a new type θ. From this point of view our setup provides a more general characterization of the optimal threshold, which in turn allows for a better understanding. For instance, notice that government turnover is the fundamental driver of the overaccumulation of debt, driven by λ, without it the outcome would be efficient. Yet, the optimal threshold is independent of it. This happens because both shocks are perfectly correlated. More uncertainty, i.e. larger λ, worsens the inefficiency due to turnover, which calls for tighter rules, but at the same time it

[14] If $r > \gamma$ there could be histories after which the government ends up with infinite positive assets.

also increases the variability of θ, which calls for more insurance and therefore looser rules. Both effects cancel each other and as a result the optimal threshold is invariant to political uncertainty when $p_{\beta\theta} = 1$.

Another way to interpret our result is to rewrite equation (2.11) as

$$\frac{\theta_u^\star}{\mathbb{E}_\theta[\theta|\theta \geq \theta_u^\star]} = \beta + (1-\beta)\frac{\Lambda_\theta - \theta_u^\star \Lambda_\beta}{1-\Lambda_\beta},$$

and think about its right-hand side as the micro-foundation of the usually assumed hyperbolic discounting factor. To be more precise, let β^{HY} be the exogenous hyperbolic discounting factor in Halac and Yared (2018), then the setups are equivalent if

$$\beta^{HY} = \beta + (1-\beta)\frac{\Lambda_\theta - \theta_u^\star \Lambda_\beta}{1-\Lambda_\beta}.$$

Thus, our model decomposes their friction in two different components: the fundamental political friction β, and a second term summarizing how the he overall uncertainty in the economy λ depends on the relative degree of political turnover Λ_β and the relative relevance of business cycle fluctuations Λ_θ. Further, this decomposition highlights how the reduced-form bias β^{HY} is endogenously affected by the fiscal rules in place θ_u^*. This decomposition is helpful bringing the sharp theoretical results closer to practical quantitative policy recommendations.

Further, notice that fiscal rules could be loose even when the fundamental political friction β is extremely low (meaning politicians have a strong present bias). Suppose for instance that current governments do not value at all the spending choices of future governments, so that $\beta = 0$. One could think that in this situation the fiscal rule should be very tight, however equation (2.11) makes it clear that rules could be substantially looser if business cycle fluctuations are sufficiently more important than governments turnover, such that $\Lambda_\theta - \theta_u^\star \Lambda_\beta$ is positive and large. Could it be that is optimal to allow for full discretion even when $\beta = 0$? Full discretion arises when $\theta_u^* = \bar{\theta}$, so that $\frac{\mathbb{E}_\theta[\theta|\theta \geq \theta_u^\star]}{\theta_u^\star} = \frac{\mathbb{E}_\theta[\theta|\theta \geq \bar{\theta}]}{\bar{\theta}} = 1$. Although this appears to be a possibility the following results discards it:

Corollary 2.1 ***(Full discretion is never optimal.)*** *For any distribution of θ and for any $\beta < 1$ and $p_\beta > 0$, $\theta_u^* < \bar{\theta}$.*

Proof: Suppose not, and assume $\theta_u^* = \bar{\theta} > 1$, because $\frac{\mathbb{E}_\theta[\theta|\theta \geq \bar{\theta}]}{\bar{\theta}} = 1$, it must be the case that

$$1 = \beta + (1-\beta)\frac{\Lambda_\theta - \bar{\theta}\Lambda_\beta}{1-\Lambda_\beta},$$

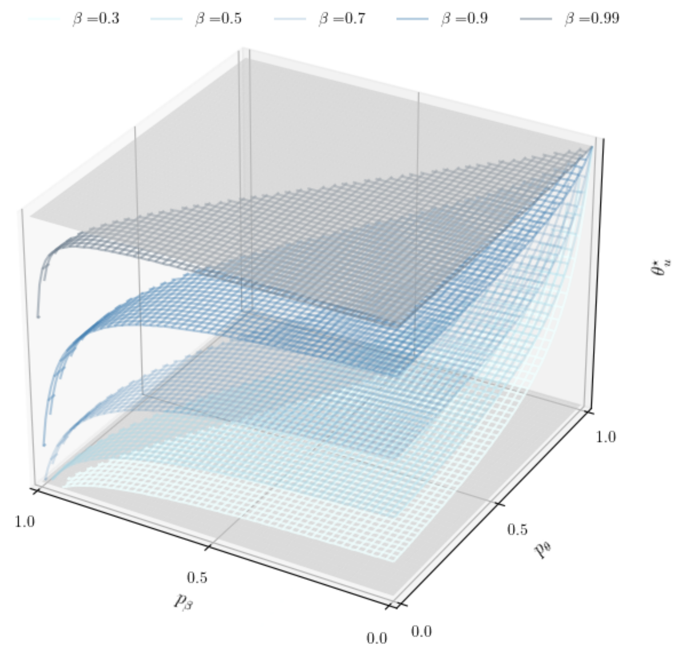


Figure 2.3 Uncoordinated Fiscal Rules

The plot shows the optimal threshold $\theta^\star_u$ in a small open economy (value on the z axis) as a function of the conditional probability of pure government change (p_β) and pure preference change (p_θ). Notice that the lower gray surface ($\theta^\star_u = \underline{\theta}$) corresponds to the case in which all government types are constrained, while the upper gray surface ($\theta^\star_u = \overline{\theta}$) corresponds to the case in which no government type is constrained. For example, full discretion is always optimal ($\theta^\star_u = \overline{\theta}$) for $p_\theta = 1$.

but then, for all $\beta \neq 1$,

$$1 - \Lambda_\theta = \Lambda_\beta(1 - \bar{\theta}),$$

which is not possible because $1 \geq \Lambda_\theta$, $\Lambda_\beta > 0$ and $\bar{\theta} > 1$. ■

In general, doing straightforward comparative statics it is possible to show that θ^*_u is increasing in Λ_θ, so that the larger the insurance needs the looser the fiscal rule; and decreasing in both β and Λ_β: the larger the political friction and higher the turnover, the stricter the fiscal rule.

Figure 2.3 shows that fiscal rules are more stringent (lower threshold) when the present bias is strong (for low β) and when either pure turnover (i.e., without preference change) is relatively more frequent ($p_\beta \to 1$) or pure business cycles shocks (i.e., without government change) are relatively less frequent ($p_\theta \to 0$). As can be seen from the graph, full discretion is never optimal in a small open economy except for the degenerate case in which only economic shocks are present (i.e., $p_\theta = 1$, there is no turnover).

In addition, depending of the actual implementation, there could be other effects of uncertainty on fiscal rules: recall that the rule is implemented with either a debt contingent limit or a deficit contingent limit, as in equations (2.8) and (2.9), which will generate additional effects through the endogenous speed of accumulation $C(\theta)$ and the current state of the economy captured by r and x. For example, notice that in case the political and business cycle shocks always happen contemporaneously (i.e., $\Lambda_\theta = \Lambda_\theta = 0$ as in Halac and Yared (2018)), we have that $C(\theta) = \gamma\theta(\gamma+\lambda)/(\gamma\theta+\lambda\beta)$ is increasing in λ. Since in this case the threshold $\theta^\star$ does not depend on the overall uncertainty in the economy λ, increasing the shock frequency (higher λ) would make the implemented fiscal rules looser: from (2.8) it is immediate to see that the higher $C(\theta^\star)$ is, the faster the allowed debt accumulation is. We then have the counter-intuitive result that more uncertainty justifies laxer rules, thereby implying that countries with high government turnover should be less restrained. However, this is only due to the fact that gains from insurance always outweigh losses from biased debt accumulation when economic and political shocks are perfectly correlated. It is enough to decouple the two kinds of shocks to see that only increases in *economic* uncertainty (i.e., shorter business cycles) call for looser fiscal constraints.

2.3.2 Closed Economy

Even though individual countries are price takers with respect to the interest rate r, a group of countries designing a fiscal rule can be sufficiently large to affect prices. In the section, we consider this possibility by focusing on the extreme case in which the fiscal union as a whole is a closed economy. We then study the problem of a planner who must design an optimal fiscal rule knowing that, in equilibrium, the interest rate will be a function of the rule. This problem coincides with the one that countries would solve, should they have the ability to coordinate on an optimal fiscal rule; we will therefore refer to the resulting optimal rule as the *coordinated* rule.

Formally, we require that the interest rate is such that the resource constraint:

$$\mathbb{E}[\, g\,] = y, \tag{2.12}$$

is satisfied for all t. This constraint states that aggregate spending must equal the aggregate amount of resources available in every period in the union.

The planner solves:

$$\max_{\theta^* \in [\underline{\theta},\bar{\theta}]} \left\{ \int_x \left(\int_{\underline{\theta}}^{\theta^*} v(a,\theta; r(\theta^*),\theta^*)\ h(\theta)\, d(\theta) + \int_{\theta^*}^{\bar{\theta}} v(a,\theta; r(\theta^*),\theta^*)\ h(\theta)\, d(\theta) \right) m(x) dx \right\}, \tag{2.13}$$

such that (2.12) is satisfied and where $m(x)$ is the asset distribution across countries (remember that wealth a depends on assets x). Since each country takes prices as given, their problem is not altered by the assumption that the union is a closed economy. We can thus substitute the policy functions in the resource constraint (2.12) to derive the equilibrium interest rate.

For clarity purposes, let us explicitate the dependency of wealth a on assets x. Given a threshold $\theta^\star$, we also let $r(\theta^\star)$ denote the equilibrium interest rate. Since policy functions are linear in wealth, aggregate spending is simply the product between the *average* spending share, $\bar{C}(\theta^\star) \equiv \int_{\underline{\theta}}^{\theta^\star} C(\theta) h(\theta) d\theta + \int_{\theta^\star}^{\bar{\theta}} C(\theta^\star) h(\theta) d\theta$, and aggregate wealth, $\bar{a}(x) \equiv \int_x a(x)\, m(x) dx$. What is more, by using the definition of wealth $a(x) = x + y/r(\theta^\star)$ together with the fact that, in a closed economy, debt must be zero in the aggregate, we conclude that $\bar{a}(x) = y/r(\theta^\star)$. Aggregate spending is thus $\bar{C}(\theta^\star) y/r(\theta^\star)$ and the resource constraint (2.12) becomes:

$$r(\,\theta^\star) = \bar{C}(\theta^\star). \tag{2.14}$$

The equilibrium interest rate, therefore, coincides with the average spending share

The following proposition contains the characterization of the optimal coordinate threshold in a closed economy.

Proposition 2.3 *The optimal coordinated cut-off $\theta^\star_c$ with associated interest rate $r = r(\theta^\star_c)$ satisfies:*

$$\frac{\mathbb{E}[\theta | \theta \geq \theta^\star_c]}{\theta^\star_c} - \frac{1-\Lambda_\beta}{\beta(1-\Lambda_\beta) + (1-\beta)(\Lambda_\theta - \theta^\star_c \Lambda_\beta)} =$$

$$\frac{\gamma(\gamma+\lambda)\lambda^{-1}}{\beta(1-\Lambda_\beta) + (1-\beta)(\Lambda_\theta - \theta^\star_c \Lambda_\beta)} \left(\frac{y}{r(\theta^\star_c)} \int_x \frac{1}{r(\theta^\star_c) x + y} m(x) dx - \frac{1}{\gamma} \right). \tag{2.15}$$

To gain intuition on the coordinated rule, notice that, by Proposition 2.2, the left-hand side of equation (2.15) equals zero under the uncoordinated rule $\theta^\star_u$. It follows that coordination among countries leads to a different rule if and only if the right-hand side of equation (2.15) differs from zero.

Coordination is beneficial as it enables the planner to manipulate the interest rate, which is increasing in $\theta_c^\star$ by the resource constraint (2.14).[15] The interest rate enters the right-hand side of equation (2.15) in two places. First, it determines the discounted value of country's income, y/r. All other things equal, a lower interest rate makes countries wealthier, allowing them to increase their consumption for every type θ. Since higher interest rates partly offset countries' tendency to overspend, Halac and Yared (2018) calls this the *disciplining* effect of the interest rate.

Second, a lower interest rate reduces inequality across countries. The reason is the following. As time passes, countries experience different histories of shocks. They thus accumulate different amounts of financial wealth x and, as a result, make different consumption choices, even conditional on the same type θ. A lower interest rate makes countries more equal by reducing creditors' income, while increasing debtors' income. Halac and Yared (2018) calls this the *redistributive* effect of interest rates.

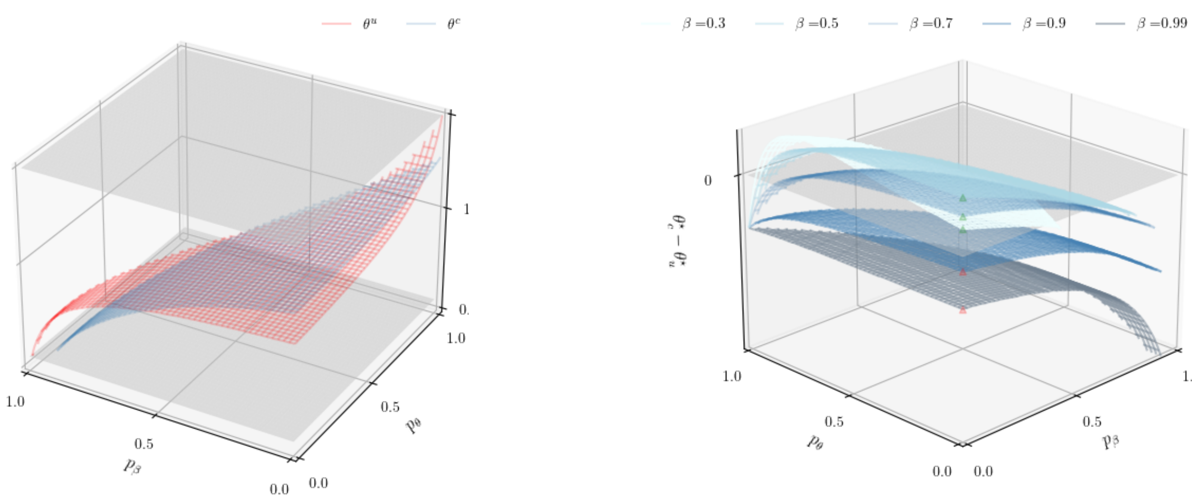
Depending on which of the two effects dominates, the planner's fiscal rule may be stricter or looser than the uncoordinated one. Intuitively, the redistributive effect should dominate – resulting, therefore, in a stricter coordinated rule – when β is close to one, that is, when overspending is less of a concern. On the contrary, the disciplining effect becomes more relevant – and, therefore, the coordinated rule should be looser – when countries have a strong desire to overspend (i.e. β is low).

Figure 2.4 confirms our intuition. Consider first the case in which shocks to preferences and government changes are concomitant, i.e. $p_\beta = p_\theta = 0$. The figure shows that $\theta_c^\star - \theta_u^\star < 0$, i.e. the coordinated rule is stricter that the uncoordinated one, when β is close to one, whereas the opposite is true for low values of β. This is the same result delivered in Halac and Yared (2018), but our paper generalizes their conclusions to a dynamic setting: there always exists a value for β low (high) enough such that coordinated rules become looser (stricter) than uncoordinated ones. This result, however, only holds in the special case in which economic and political shocks are perfectly correlated. As one can easily see from panel (b) in Figure 2.4, coordinated rules can be (weakly) stricter than uncoordinated ones for any value of the bias β, when either pure political shocks or pure preference shocks are the main source of uncertainty (i.e., for p_β or p_θ high enough).[16]

Consider the case in which $p_\beta = 0$, that is, when the government change, preferences always change as well. Figure 2.4 shows that, as p_θ increases, the coordinated rule becomes stricter than the uncoordinated one. This conclusion can be made formal by noticing that equation (2.11) implies $\theta_u^\star \to \bar{\theta}$ as $p_\theta \to 1$, whereas the co-

[15] This follows from the definition of $\bar{C}$, together with fact that C is an increasing function.

[16] Note that, where not plotted, the two thresholds are $\theta_u^\star = \theta_c^\star = \underline{\theta}$.



(a) Threshold Comparison, $\beta = 0.7$ **(b)** Threshold Comparison, changing β

Figure 2.4 Coordinated Fiscal Rules

Left: The plot shows the coordinated and uncoordinated optimal threshold $\theta_c^\star, \theta_u^\star$ as a function of the conditional probability of pure government change (p_β) and pure preference change (p_θ). Notice that the lower gray surface ($\theta^\star = \underline{\theta}$) corresponds to the case in which all government types are constrained, while the upper gray surface ($\theta^\star = \overline{\theta}$) corresponds to the case in which no government type is constrained. *Right:* The plot shows the difference between the coordinated and uncoordinated optimal threshold $\theta_c^\star - \theta_u^\star$ in a small open economy (value on the z axis) as a function of the conditional probability of pure government change (p_β) and pure preference change (p_θ). Notice that on the gray surface $\theta_c^\star = \theta_u^\star$. The green and red triangles are the points at which $p_\beta = p_\theta = 0$, as in Halac and Yared (2018). When coordinated rules are looser (as in the green triangle points), the difference is positive.

ordinated threshold $\theta_c^\star$ will in general be strictly lower than $\bar{\theta}$. The reason is simple. When p_θ increases (keeping $p_\beta = 0$), pure political shocks become more rare; as a result, countries' desire to overspend becomes weaker. The only remaining concern for the planner is consumption inequality across countries. The planner, therefore, chooses a stricter rule to take advantage of the redistributive effect of the interest rate and reduce inequality.

Consider now the extreme case with $p_\theta = 0$, that is, when a business cycle shock hits, the incumbent always falls. Similarly to the previous case, Figure 2.4 shows that, as p_β increases, the coordinated rule becomes stricter than the uncoordinated one. To understand this result, notice that equation (2.11) implies $\theta_u^\star \to \underline{\theta}$ as $p_\beta \to 1$. As changes in preferences become less likely, there is less need for spending flexibility;

in the limit, the uncoordinated rule calls for every type to be constrained. Turning to the coordinated rule, observe that p_β has a direct effect on the spending share $C(\theta)$ and, thus, on the equilibrium interest rate (which is just the average spending share), for any given threshold.[17] In particular, the interest rate tends to increase with p_β. As a result, the planner must lower the coordinated threshold more than the uncoordinated one to counteract the effect on the interest rate.

Finally, notice that fiscal rules tend to become, in some sense, less effective when pure turnover is high ($p_\beta \to 1$). This, again, results form the unrestricted governments' spending choices: as p_β increases, $C(\theta)$ becomes a flatter function of θ. In fact, in the limit in which $p_\beta = 1$, consumption shares do not depend on types θ and are equal to $C(\theta) = \bar{C}(\theta_c^\star) = \gamma(\gamma + \lambda)/(\gamma + \lambda\beta)$. By the resource constraint, the equilibrium interest rate is equal to the average (common in this case) spending share and is therefore independent of the optimal threshold. It follows that each country spends the amount $C(\theta)(y/r + x) = y + rx$, namely they spend their endowment net of (plus) any interest payment on previously accumulated debt (assets). Since, in the first best the equilibrium interest rate would be lower (and equal to γ), the presence of a political bias results in greater disparities in spending across governments: debtors, with $x < 0$, must allocate more of their endowment to pay creditors interest on the accumulated debt. This greater consumption inequality – which follows from the greater wealth inequality – also explains why, away from the limit (i.e. $p_\beta < 1$), the coordinated planner chooses a tighter fiscal rule that depresses the interest rate and redistributes wealth.

2.4 Conclusions

In this paper, we have shown how modeling the source of uncertainty in the economy is crucial in the design of fiscal rules. Long-standing fiscal constraint – both in a national and in a supranational setting – should be more restrictive when either insurance needs are low, governments are very myopic, or turnover is frequent. Further, when political and economic uncertainty are not correlated, coordinated rules (i.e., in a supranational setting) tend to be stricter than uncoordinated ones (i.e., in a national setting) since the gains from redistributing resources from creditors to debtors by depressing the sovereign interest rate outweigh the corresponding loss in market discipline. An interesting avenue for future research would be to endogenize, within a normative model, the link between political bias and government turnover frequency as in the positive theory of Alesina and Tabellini (1990).

[17] We have that $\partial C/\partial \Lambda_\beta = \beta\gamma(1-\theta)\theta\lambda(\gamma+\lambda)/(\beta\lambda((\theta-1)\Lambda_\beta + 1) + \gamma\theta)^2$.

Chapter 3

Default in the design of fiscal rules

(Joint with Facundo Piguillem and Liyan Shi)

It is widely believed that governments tend to over-accumulate debt, which gives rise to the need for fiscal rules. This chapter studies the optimal fiscal and default rules when governments can default on their debt obligations. We build a continuous-time model that encompasses the standard rationale for debt over-accumulation: hyperbolic discounting and political economy frictions. In addition, governments are subject to taste shocks, which makes spending optimally random. Since shocks are private information, there is a trade-off between rules and discretion. We derive the optimal fiscal rules which are debt-dependent only when default is possible. Depending on the severity of the spending bias and the cost of default, the optimal fiscal rules range from strict debt limits, complemented by strong deficit limits, to the absence of all rules. In intermediate cases, debt-dependent deficit limits must be complemented with default rules, with some areas where default is banned and others where default is mandatory.

3.1 Introduction

Over the past few decades, sovereign debt has substantially increased in many developing and advanced economies. As a result, fiscal rules have become increasingly prevalent (Halac and Yared, 2019). The driving force behind this wave of rules is the concern about debt sustainability and the implied risk of default. However, the implemented rules are in general ad hoc and not based on sound theories—and when they are, they mostly abstract from the interaction with risk of default.[1] How do fiscal rules change when a sovereign can default? Should there also be "default rules"? In this paper we show that depending on the economic environment many possibilities can arise, ranging from replacement of fiscal rules with default rules to the imposition of constitutional borrowing limits with hard spending limits.

When analyzing fiscal rules a question naturally arises: what is the underlying friction generating the need to impose rules? One of the commonly accepted reasons for imposing fiscal rules is rooted in political economy. In a nutshell, political turnover together with political polarization creates incentives for incumbents to overspend at the expense of future governments: there is a *spending bias*. This friction by itself is simple to deal with. If a rule-maker could perfectly observe current and future spending needs, a rule limiting the ability of governments to spend would easily ensure that only optimal spending decisions were possible. However, spending needs are affected by random and unpredictable events that render it necessary to endow policy makers with the discretion to optimally adjust spending and, thus, to smooth out the consequences of these shocks. Still, if the shocks were fully observable and contractible a contingent fiscal rule using this information would again solve the problem. In reality, information is imperfect, but even when it is relatively precise, it is hardly contractible. This creates a meaningful trade-off between discretion, to allow governments to respond to shocks, and rules to prevent them from overspending.

Starting from this premise there is a literature, originated by Amador et al. (2006), that analyzes the optimal trade-off between commitment and flexibility when agents discount the future quasi-hyperbolically. This approach has been extended by Halac and Yared (2014), who applied it to governments and interpreted the outcomes as fiscal rules. However, this literature abstracts from the possibility of debt repudiation. Thus, the prescriptions are only about spending or deficit limits and are independent of the level of debt. Moreover, the possibility of default brings about new dimensions to the trade-off between discretion and rules. Since this possibility increases

[1] See Eyraud et al. (2020). There are a few exceptions, e.g., Hatchondo et al. (2015), Adam and Grill (2017) and Alfaro and Kanczuk (2017), which we describe in Section 3.1.1.

welfare when the financial markets are incomplete (Dovis, 2019), many questions arise. Should default be restricted? That is, should there be default rules? If so, in which scenarios? Does the possibility of default affect fiscal rules in the states in which the default option is not exercised?

To analyze this problem, we develop a continuous-time model with *spending-biased governments*. Consider an environment where spending needs are random. These genuine needs represent the real social value of spending, but they are observed only by the incumbent government. Thus, it is desirable to endow governments with some discretion to react to them. At every instant, with some probability, a change of government can occur: the incumbent is replaced by a new one who draws a new spending need. Governments are forward-looking, but they value the decisions made by other governments less. To be precise, the incumbent discounts any allocation chosen by any future government by a factor $\beta \leq 1$. This factor captures the extent of political polarization. If $\beta < 1$, political turnover generates a *spending bias* resembling hyperbolic discounting. Even though the needs are genuine, because of the spending bias, the incumbent has incentives to overstate them and overspend. Hence, imposing fiscal rules could be instrumental for restoring efficiency.

Governments can save and borrow using a noncontingent short bond, which is supplied by a continuum of risk-neutral international lenders. Since at any instant the government can default, the interest rate charged on the loans endogenously reflects the default risk. When the government defaults, it is excluded from the financial markets, as in Eaton and Gersovitz (1981), and it suffers a proportional loss in resources, as in Arellano (2008). This status does not need to be permanent. With some probability, the government regains access to the financial markets and the resource loss vanishes. Despite the potential loss in output and future insurance, default could be welfare-improving, because it helps complete the markets. However, due to the spending bias, the incumbent's decisions may be inefficient, which creates another impetus for rules.

To design the optimal regulation we study a mechanism-design problem. We take the perspective of a noncommitted, benevolent planner who chooses spending and default allocations subject to the truthful revelation of spending needs. We then show that these optimal choices can be implemented as a Markov equilibrium between current and future governments, when they are subject not only to *fiscal rules* but also to *default rules*. Our perspective has two appealing features. First, the mechanism-design approach does not constrain the set of instruments available to the rule-writer. The sufficiency of both fiscal and default rules *arises endogenously*. Second, the planner's lack of commitment ensures that rules are *sustainable*: future rule-writers have no incentive to change them ex post.

Our first contribution is to show that default rules are necessary complements to the standard fiscal rules. Default rules can take many forms. They can imply mandatory default after a certain level of debt or forbid default when the debt level is not high enough. In between, governments would be endowed with full discretion to default, depending on their spending needs. A special case is a hard borrowing limit (the intermediate area is degenerated) coupled with mandatory default beyond the limit. For instance, the constitution could state that any debt level above a certain percentage of GDP would be illegal.

To understand the intervention on both ends, it is important to keep in mind that the spending bias makes it unclear whether there is too much or too little default. One may think a myopic government would tend to default too much, but that is not necessarily the case. The default decision arises from comparing the present value of the nondefault benefits with the present value of the default costs. The government's "myopic" discounting affects both costs and benefits. Hence, the intertemporal distribution of costs and benefits could lead to either over-default or under-default. This last possibility arises when the probability of reentry into the financial markets is large enough. *Reentering is a benefit* that happens in a potentially distant future. Thus, an incumbent, mostly concerned about its own term, may not fully internalize it. A rule-writer with an undistorted, intertemporal view would consider this benefit and force the current government to default.

Our second contribution is to show how the possibility of default affects fiscal rules. The presence of default risk and the default rules also affect the fiscal rules in the states of nature in which default does not happen. For debt levels for which there is no default risk, we find that the optimal fiscal rule is of the threshold type, as in Amador et al. (2006), independent of the debt level. To be precise, let θ be the reported spending needs, then there exists a threshold θ^{s*} such that all governments reporting $\theta \leq \theta^{s*}$ are unconstrained on their spending decisions, while all those reporting $\theta > \theta^{s*}$ must choose the same spending as a government reporting θ^{s*}. This optimal fiscal rule can easily be implemented with a spending cap or a deficit limit. In other words, θ^{s*} separate the areas between those governments that are endowed with discretion, low θ types, and those that are committed and must abide by the rule, high θ types.

For debt levels for which default risk is strictly positive, additional elements start to act. First, we show that the optimal fiscal rule is still of the threshold type, but now *dependent on the debt*. Let a be the government's financial position, so that $a < 0$ is debt, then there exist a threshold $\theta^s(a)$ such that all types $\theta \leq \theta^s(a)$ are unconstrained, while all $\theta > \theta^s(a)$ spend no more than $\theta^s(a)$. Again, this allocation can be implemented with either a spending cap or a deficit limit, but now the fiscal rule can be tightened or loosened as debt rises.

To uncover the interaction between risk of default and fiscal rules one must consider several factors. First, the possibility of default may render the spending/deficit constraint innocuous, because the planner may prefer a high spending type to default rather than constraining its spending choice. This indeed happens when political polarization is sufficiently low (β is close to 1). In this case the planner optimally chooses that all types $\theta \geq \theta^d(a)$ must default. Since $\theta^d(a) \leq \theta^s(a)$ there is no need to impose spending limits. On the equilibrium path this outcome would look like a roughly constant deficit or spending limit as long as there is no default risk, and all these restrictions would be lifted as soon as the default risk becomes positive. The task of imposing discipline is allocated entirely to the market.

Second, if the planner deems it optimal not to default, i.e., $\theta^d(a) > \theta^s(a)$, the fiscal rule imposed in the absence of default risk must be modified. To this end, they must consider the effect of the rule on the interest rate, through the risk premium, and on the government's incentives to default. The interest-rate effect brings out some elements reminiscent of Halac and Yared (2018). An interest-rate hike is good for imposing discipline on myopic governments, but unlike in Halac and Yared (2018), it has a negative income effect, since interest payments are transfers to foreign lenders. As a result, as debt grows, so does the risk premium; therefore, the planner chooses to reduce discretion by tightening up the spending or deficit limits. In addition, by manipulating the spending threshold, the planner alters the default threshold, which in turn changes the government's incentives to overspend.

Finally, our paper bridges the theoretical and quantitative literature, analyzing all these instruments simultaneously. Partial answers to these questions can be found in previous works. Some works are highly theoretical and stylized while others focus on fully quantitative models incorporating one policy instrument at a time. Our framework provides sharp theoretical characterizations of the optimal rules under a rich set of political and economic environments, while allowing us to meaningfully evaluate its quantitative implications.

To this end, we calibrate the model economy to characterize three types of economy: 1) one with a low debt capacity and a risk premium highly sensitive to debt accumulation, which we call the Greece-like regime; 2) one with a high debt capacity and a sensitive risk premium, that we call the Italy-like regime; and 3) one with a high debt capacity combined with a mildly sensitive risk premium. Here we emphasize two important findings. First, the interaction between fiscal rule and default rule is sizeable. Absent any fiscal rule, the optimal default rule follow closely the governments' preferred decisions; when complemented with a fiscal rule, the intervention greatly modifies the governments' choices. Second, the combination of fiscal and default rules implies a very large debt capacity. This outcome resembles an otherwise seemingly unsustainable debt burden, together with extreme austerity measures. These two elements together, which are on occasions regarded as irrational, arise naturally as optimal rules in our setup.

3.1.1 Related Literature

We relate to three strands of literature: on dynamically inconsistent preferences, and their relation to political economy; on the optimal trade-off between commitment and flexibility; and the rich, growing body of work on sovereign default.

The literature closest to our work was originated by Amador et al. (2006), who analyze the optimal trade-off between commitment and flexibility when agents discount the future quasi-hyperbolically.[2] The main premise in this literature is that agents (or governments) are tempted to overspend, so ideally it would be optimal to limit their possibilities to accumulate debt. What makes the problem nontrivial is that agents are subject to random spending needs, which are either not observable or not contractible. Thus, it would be desirable to endow them with some discretion. More recently this approach has been extended by Halac and Yared (2014), allowing for persistence shocks when preferences are logarithmic, and Halac and Yared (2018) who consider the endogeneity of the interest rate. With respect to them we cast the problem in a continuous-time framework, and we add the possibility of default. We show that the optimal fiscal rule can be debt-dependent and is complemented by default rules.

We also build on the rich literature analyzing environments with quasi-hyperbolic discounting, originated by Strotz (1955) and augmented by Laibson (1997). These environments are in general analyzed in discrete-time frameworks.[3] However, as Chatterjee and Eyigungor (2016) pointed out, this approach generates many technical challenges when the economic agents are subject to borrowing limits. In particular, all Markov equilibria generate discontinuous decision rules. Since default decisions imply de facto borrowing limits, we avoid this difficulty by modeling decisions in continuous time similarly to Harris and Laibson (2012). They consider a limit behavior which they termed "instant gratification." Our modeling strategy is instead more closely related to Cao and Werning (2016).[4] They analyze the savings

[2] See Ambrus and Egorov (2013) for corrections to the original results and Athey et al. (2005) for an alternative approach with endogenous time inconsistency.

[3] See for instance Krusell and Smith (2003) and Cao and Werning (2018).

[4] For another application of quasi-hyperbolic discounting in continuous time see Laibson et al. (2020).

behavior for a more general "disagreement index," while we maintain it constant, interpreting it as political polarization. With respect to them, we allow for the possibility of default that endogenizes both the interest rate and the type-contingent borrowing limits.

Combining these two approaches allows us to build a bridge between the highly theoretical and the quantitative analysis, looking for more precise answers to actual rules that should be implemented in real life. By imposing additional structure in the friction generating over-accumulation of debt, the model allows for a meaningful mapping to the data. To be precise, it maps neatly to the standard political economy models á la Persson and Svensson (1989), Alesina and Tabellini (1990), and Battaglini and Coate (2008). Thus, we can decompose the hyperbolic discounting factor into *political turnover* and *polarization*. This is along the line of recent quantitative work by, for example, Azzimonti et al. (2016), who assess the effect of imposing a balanced budget.

Finally, we contribute to the theoretical and quantitative literature analyzing sovereign default. We build on the seminal contributions of Eaton and Gersovitz (1981) and Arellano (2008). However, our default's modeling approach is more closely related to Bornstein (2020), who analyzes the case of exponential discounting. Beyond the abundant positive, mostly quantitative, literature studying sovereign default, there are few studies focusing on normative issues. Dovis (2019) introduces private information and lack of commitment to debt repayment as we do, but all agents discount the future consistently. Thus, he derives prescriptions for optimal lending agreements, not fiscal rules. Instead, Hatchondo et al. (2015) analyze how committing to future decisions, imposing fiscal rules, could improve current outcomes. With respect to them, we introduce the present bias and consider the interaction between fiscal and default rules, even though we do not incorporate long term debt.[5] To the best of our knowledge, Adam and Grill (2017) is the only paper studying the possibility of incorporating default rules. They study a Ramsey equilibrium with perfect information and geometric discounting. Nevertheless, they find that when default is costly, it is only optimal to default when shocks are of "rare disasters" type. Finally, Alfaro and Kanczuk (2017) analyze a discrete-time environment similar to ours. They build a quantitative model and evaluate the welfare properties of some selected rules, whereas we derive and characterize theoretically what rules are optimal for different environments.

This chapter is organized as follows. Section 3.2 describes the environment under which we generate our results, including the equilibrium in the absence of any rule.

[5] See also Chatterjee and Eyigungor (2015) and Hatchondo et al. (2016) for normative prescriptions to improve lending contracts.

Section 3.3 contains our main results, delivering both the optimal fiscal and the optimal default rules. Section 3.4 generalizes the results. Section 3.5 evaluates the quantitative implications applied to selected European countries. Section 3.6 concludes.

3.2 Environment

This section develops a dynamic model incorporating the standard frictions of spending bias and default choice. We first briefly describe the model's fundamentals, then we characterize the equilibrium without fiscal rules.

3.2.1 Model

Time is continuous and infinite, $t \in [0, \infty)$. At every instant $t \geq 0$, the economy is governed by an incumbent government. A *political turnover* event occurs with Poisson arrival rate λ: the incumbent government loses power and is replaced by a new one. Each incumbent government receives an exogenous source of tax revenue τ and faces a spending choice g_t. Different governments attribute different values to their spending needs. This value is determined by their "taste" type θ, which is an *i.i.d.* random variable, drawn from a bounded set $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ according to the cumulative distribution function $F(\cdot)$ and expected value $\mathbb{E}[\theta] = 1$. The change in preferences can be interpreted as arising from the underlying constituency's opinions on the social value of spending, changing over time and determining the alteration of the country's stance on fiscal policy.[6] The preferences for spending flows are

$$\theta u(g),$$

where $u(\cdot)$ is strictly increasing and strictly concave, $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Thus, types with high θ experience a larger marginal utility from spending than low types.

Spending bias. All governments, whether incumbent or opposition, discount the future exponentially at rate ρ. However, the incumbent government values spending

[6] One possible interpretation is that preferences vary in response to the business cycle. For instance, Amador et al. (2006) show that if utility is exponential, taste shocks are equivalent to income shocks. Another interpretation is that demographic changes in the constituency's composition or power struggles between different parties induce a preference shock. See, for example, the entrepreneur-worker conflict in Azzimonti et al. (2014). Nevertheless, in Section 3.4.2 we provide an environment where we make the revenue τ random.

by future governments less. To be precise, every unit of spending transforms into one unit of consumption when the government is in power, but it delivers only $0 < \beta < 1$ units of consumption when the government is out of office. To simplify the analysis, we recast this political friction in terms of utility. Namely, we assume that governments discount utility by the extra-term β whenever they are not in power. Thus, introducing the discount term β is a reduced-form way of capturing disagreement within a country over the composition of public spending, rather than over its level. For this reason, we refer to β as the *political polarization* parameter. In Section 3.2.2, we explain in detail how β shapes the outcomes and we further discuss our interpretation, providing additional ones.

Information friction. The realization of spending needs θ is privately observed by the government, which renders it impossible to write contracts contingent on it. The important part of this assumption is that θ is noncontractible. There are many reasons why the government could be better informed about its spending needs. Moreover, even when the shocks were observable or ex post verifiable, they may not be contractible. For instance, one may think that it would be politically infeasible to write a policy rule that constrains a specific political party. Still, the shocks are assumed to *represent genuine spending needs*. This is what creates a meaningful trade-off between discretion, to smooth out the shocks, and rules to contain the spending bias.

Default. At any instant, the government can default on its debt obligations, upon which the government is excluded from financial markets, as in Eaton and Gersovitz (1981). While in default, tax revenues are reduced to $\kappa\tau$, with $\kappa \in [0, 1]$. This captures output loss due to financial exclusion, as in Arellano (2008). Financial access is regained at Poisson rate $\phi \geq 0$: upon reaccess previously defaulted debt is fully discharged, and the government returns to the market with a zero-asset position. While how much debt is discharged for defaulting countries could be important, we show in Section 3.4.1, where we extend the analysis to partial discharge, that it has a minor qualitative impact.

Denote the default decision by $\delta(\theta, a) \in \{0, 1\}$, with value 1 if the government of type θ defaults at assets position a. Then,

Definition 3.1 (Allocation) *An allocation* $\{g(\theta, a), \delta(\theta, a)\}_{\theta\in\Theta, a\in\mathbb{R}}$ *specifies the government spending and default decisions for all types and asset levels.*

Interest rate. There is a continuum of competitive risk-neutral lenders with access to risk-free rate $r_f \leq \rho$. The risky interest rate charged to governments adjusts for the expected default probability:

$$r(a) = r_f + \lambda \mathbb{E}[\delta(\theta, a)]. \tag{3.1}$$

Given the interest rate, the asset or debt accumulation process follows:

$$\dot{a}(\theta, a) = r(a)a + \tau - g(\theta, a). \tag{3.2}$$

It is worth mentioning the lending-market features that lead to equation (3.1). Despite the government's type θ being private information, the lenders could observe its actions and recover the true type. Thereby the lenders could potentially charge an interest rate depending not only on how much debt the government has accumulated but also on its specific type. However, the equilibrium interest rate depends *only* on the asset level a, not on the type θ.

To understand why, note that because time is continuous; therefore, the accumulation process in equation (3.2) is smooth and the interest-rate adjustment is instantaneous. Consider a borrower that has accumulated enough debt to be on the verge of default. The lenders would not extend additional funds to her, because the debt would be defaulted for sure. Alternatively, one may think that lenders would charge an infinitely high interest rate, preventing any borrowing. Thus, at the default threshold, further debt accumulation doesn't occur, i.e., $\dot{a}(\theta, a) \geq 0$. Default happens only when there is a discontinuous jump in θ.[7] For this reason the risk premium is the jump probability λ times the expected default rate of new types, $\mathbb{E}[\delta(\theta, a)]$. Still, the default probability depends on debt because, even if the current type would not default, by accumulating debt the current agent enlarges the set of future types who would default. By charging the appropriate interest rate, the lenders can make sure that the current government will not default, but they remain afraid that the future, still unknown, government would default at the given level of debt.

To fine tune the business-cycle properties of spending, in Section 3.4.2 we consider an extension where τ follows a *Browning motion, which is contractible.* Since

[7] Alternatively, the interest rate schedule can be written as

$$r(\theta, a) = \begin{cases} r_f + \lambda \mathbb{E}[\delta(\theta, a)] & \text{if } \delta(\theta, a) = 0, \\ \infty & \text{if } \delta(\theta, a) = 1. \end{cases}$$

Bornstein (2020), which studies a continuous-time version of the sovereign default model by Arellano (2008), also notes that it is critical to have stochastic jumps in order to observe defaults on the equilibrium path.

Brownian motions move continuously over time, they *do not have a direct impact on default*. For instance, if we were to abstract from the "jump" θ shock and we allow for only the Brownian motion, there would be no default on equilibrium and the risk premium would be zero at all debt levels, except the default threshold where it would be infinity. For this reason, we develop the main results of the paper assuming that τ is constant and we leave for Section 3.4.2 the implications of additional business-cycle features.

Our choice of a continuous-time setup is not arbitrary—it's a key element that helps us to overcome technical challenges. We just mentioned how this choice helps us to simplify the default decision. In addition, the possibility of default generates endogenous "wealth limits" that, in combination with spending-biased agents, render the framework intractable. As Chatterjee and Eyigungor (2016) show, in a discrete-time environment there would be no equilibrium with continuous decision functions. The jumping actions that characterize the solutions in discrete time are no longer present in continuous time, which allows us to focus on equilibria that are differentiable almost everywhere.

3.2.2 Rules-Free Equilibrium

Before proceeding to the analysis of optimal rules it is instructive to discuss the equilibrium in the absence of rules. Moreover, a slight modification of this equilibrium, adding the constraints implied by the rules, is the benchmark used in the implementation. Let $w^j(\theta, a)$ be the value function of an incumbent that has spending needs θ, financial position a and is in state $j = n, d$, where d stands for default and n for non-defaulting status. Let $v^j(\theta, a)$ be the analogous value function from the perspective of a subject that values all governments' decisions equally, i.e., as if $\beta = 1$. When the economy is not in default, these two value functions solve the following system of Hamilton-Jacobi-Bellam (HJB) equations:

$$\rho w^n(\theta, a) = \max_g \{\theta u(g) + (r(a)a + \tau - g)w_a^n(\theta, a)\} + \lambda \left(\beta \mathbb{E}[v(\theta', a)] - w^n(\theta, a)\right), \quad (3.3)$$

$$\rho v^n(\theta, a) = \theta u(g^*) + (r(a)a + \tau - g^*)v_a^n(\theta, a) + \lambda \left(\mathbb{E}[v(\theta', a)] - v^n(\theta, a)\right), \quad (3.4)$$

where $\mathbb{E}[v(\theta', a)]$ also embodies the expectation over future default decisions. Thus, if default were not possible $\mathbb{E}[v(\theta', a)] = \mathbb{E}[v^n(\theta', a)]$. To understand these equations, it's useful to start by assuming that default is not possible. Equation (3.3) makes clear that as long as $\lambda = 0$, this is a standard HJB equation for a continuous-time savings problem. When $\lambda > 0$, the political friction starts to play a role. The last term in equation (3.3) captures the effect of turnover. With arrival intensity λ, the incumbent loses its position, which implies a loss in value of $-\lambda w^n(\theta, a)$, and it is replaced by a new government, generating a value $\lambda\beta\mathbb{E}[v(\theta', a)]$. Here, two components are important. First, the future government's spending needs are not yet known; that is why there is an expectation over the future θ'. Second, and more importantly, the incumbent discounts ex ante the continuation value by the additional factor β.

For each θ, the continuation value satisfies the HJB represented by equation (3.4). It is clear from it that after the incumbent loses power, it discounts all future allocations at the same rate ρ, independently of the identity or type of the eventual government. The current government doesn't care who will be in power, as long as it is not itself. All non-me governments are equally discounted by β. Equation (3.4) also makes clear that the incumbent takes as given that future governments will spend (and accumulate debt) following their own optimal choices. That is the reason for the presence $g^* = g^*(\theta, a)$. The incumbent correctly assesses that, when it is not in control, whichever government in power would maximize its own utility.

In the absence of default, equations (3.3) and (3.4) would characterize the equilibrium. Instead, when default is possible, there is an additional decision and the continuation value must take into account that future governments may default. Let $\delta^A(\theta, a)$ be an indicator function denoting when a government defaults, so that:

$$\delta^A(\theta, a) = \begin{cases} 1 & \text{if } w^n(\theta, a) < w^d(\theta), \\ 0 & \text{if } w^n(\theta, a) \geq w^d(\theta). \end{cases} \tag{3.5}$$

Note that, as discussed in Section 3.2.1, even though default could happen at any instant, on the equilibrium path it only happens in the next instant after a change of government. This happens because the lenders can observe, or uncover, the incumbent's true type and would never lend an amount that would lead with certainty to default. Thus, the continuation value is given by

$$v(\theta, a) = (1 - \delta^A(\theta, a))v^n(\theta, a) + \delta^A(\theta, a)v^d(\theta).$$

It remains to describe what is the value functions' evolution when in default. Recall that when a government defaults, it is forced into financial autarky and suffers a loss of resources, which leaves only $\kappa\tau$ to spend. The analogous to (3.3)-(3.4) are:

$$\rho w^d(\theta) = \theta u(\kappa\tau) + \phi\left(w^n(\theta, 0) - w^d(\theta)\right) + \lambda\left(\beta\mathbb{E}[v^d(\theta')] - w^d(\theta)\right), \tag{3.6}$$

$$\rho v^d(\theta) = \theta u(\kappa\tau) + \phi\left(v^n(\theta, 0) - v^d(\theta)\right) + \lambda\left(\mathbb{E}[v^d(\theta')] - v^d(\theta)\right). \tag{3.7}$$

One important difference between (3.3)-(3.4) and (3.6)-(3.7) is that optimization is no longer possible. Since countries cannot participate in the financial markets, it is impossible to smooth out spending. For the same reason, the HJBs in (3.6)-(3.7) do not display the derivative with respect to a. Both equations also make clear that when a government has the chance to reenter the financial markets, it makes it for sure. This is because it can return with zero debt as the terms $w^n(\theta, 0)$ and $v^n(\theta, 0)$ make clear. In Section 3.4, we relax this assumption and show how the default HJBs must be modified. Finally, notice that the political friction parameter β enters in the default status in the same way as in (3.3)-(3.4). The political friction distorts not only the present value of not defaulting but also the default value. As countries could regain access to financial markets in the future ($0 < \phi < 1$), and potentially only by future governments, the incumbent could heavily discount this benefit. This will have important implications in Section 3.3.5 where the possibility of under-default appears.

Definition 3.2 (Markov Equilibrium) *An equilibrium is a collection of decision functions* $\left\{g^*(\theta, a), \delta^A(\theta, a)\right\}$*, and value functions* $\left\{w(\theta, a), w^n(\theta, a), w^d(\theta), v(\theta, a), v^n(\theta, a), v^d(\theta)\right\}$*, such that given the interest-rate process* (3.1)*, equations* (3.3) *to* (3.7) *are satisfied for all* $\theta \in \Theta, a \in \mathbb{R}$*.*

Alternative interpretations. Although we have focused the discussion interpreting λ as political turnover and β as political polarization, this framework lends itself to multiple interpretations. Our main interpretation is based on the seminal paper by Alesina and Tabellini (1990). Suppose that the incumbent selects the attributes of a public good that forms the basis of total consumption. If parties disagree (are polarized) on the desirable attributes of the spending good, the utility stemming from a given level of spending will be greater for the party in power. In this case β would capture the loss in utility due to the suboptimal allocation supplied by an alternative government. One can also think about a political environment with legislative bargaining where members of the governing coalition have access to "pork," while those not in the coalition do not, as in Battaglini and Coate (2008). If q is the probability that the current legislators in power remain in the governing coalition, after a change of government a current legislator receives pork only with probability q. Under this interpretation, we could set $\beta = q$.[8]

We could also appeal to the extensive literature on quasi-hyperbolic discounting. In general, following Strotz (1955) and Laibson (1997), it is customary to assume

[8] Another interpretation related to political economy is that the preferences arise naturally from the aggregation of time-consistent preferences with heterogeneous discount rates. See Jackson and Yariv (2014).

that individuals, besides the standard geometric discounting, placed an additional discount factor, β, between today and all future periods. The only difference with that framework is that we do it in continuous time and the additional discounting is placed randomly rather than deterministically, similar to the approach by Cao and Werning (2016). Indeed, it is straightforward to show that when $\lambda \to \infty$ our model maps to a continuous-time equivalent of instantaneous quasi-hyperbolic discounting framework in Harris and Laibson (2012) with random taste shocks. From this point of view, the results of this paper can be interpreted as optimal regulation of individuals borrowing decisions when default is possible.

Definition of discretion. In the next section we endow the planner with the possibility of choosing allocations other than the one implied by the rules-free equilibrium. Bear in mind some optimality conditions that shape our terminology. The first-order condition with respect to spending in equation (3.3) generates[9]

$$\theta u'(g^*(\theta, a)) = w_a(\theta, a). \tag{3.8}$$

While the default decision is characterized by equation (3.5) or alternatively by the unique default threshold $a^A(\theta)$, satisfying:

$$w^n(\theta, a^A(\theta)) = w^d(\theta). \tag{3.9}$$

In what follows, whenever the planner chooses to respect equation (3.8), it is endowing the governments with *discretion to spend*; whenever the planner respects equation (3.5), or equivalently equation (3.9), it is allowing *discretion to default*.

Before analyzing the efficiency of the equilibrium, it is instructive to roughly characterize the equilibrium. Some patterns emerge that are instrumental for understanding the next section's results. In the next three lemmas, we show how spending is expected to vary with the different parameters and what the main pattern for default is, including the possibility of an endogenous borrowing limit.

Lemma 3.1 (Spending pattern) *Suppose $u(g) = (g^{1-\gamma} - 1)/(1-\gamma)$. When not in default, the spending growth rate is given by:*

$$\frac{\dot{g}(\theta, a)}{g(\theta, a)} = \frac{1}{\gamma}\left(\frac{\partial(r(a)\,a)}{\partial a} - \rho - \lambda + \lambda\beta\frac{\mathbb{E}\left[v_a(\theta', a)\right]}{w_a^n(\theta, a)}\right). \tag{3.10}$$

[9] The fact that time is continuous allows us to use the first-order condition overcoming the difficulties present in Krusell and Smith (2003), as pointed out by Chatterjee and Eyigungor (2016). See Cao and Werning (2016) for similar arguments.

Proof: See Appendix C.1.2.

The key component determining the impact of the friction is $\lambda\beta\frac{\mathbb{E}[v_a(\theta',a)]}{w_a^n(\theta,a)}$. Since both $v_a(\cdot)$ and $w_a(\cdot)$ are positive, this term is also positive. It captures the incumbent's precautionary savings motive. The ratio $\frac{\mathbb{E}[v_a(\theta',a)]}{w_a^n(\theta,a)}$ determines the value of future shocks relative to the current spending needs. The larger the relative value of current wealth, $w_a(\cdot)$, the lower the spending growth. The factor $\lambda\beta$ is the arrival rate of future shocks multiplied by the valuation of the future; the larger the β, the more the savings.

There are two main differences with respect to what a planner would do. First, there is a direct effect because the planner, who does not penalize future governments, would have $\beta = 1$. Second, there is also a dynamic indirect effect because the planner would value the future with $v_a(\cdot) > w_a(\cdot)$. The optimal fiscal rules must deal with these two distortions to correct spending decisions.

Since the marginal value of assets $w_a(\theta, a)$ is increasing in type θ, equation (3.10) suggests that the growth rate of spending is decreasing in θ. The dissaving types (high θ) disaccumulate assets and decrease their spending over time. In contrast, the saving types (low θ) accumulate assets and increase their spending over time.

To understand the default incentive let:

$$\underline{a}^A \equiv \inf_{\theta\in\Theta} a^A(\theta) \quad \text{and} \quad \bar{a}^A \equiv \sup_{\theta\in\Theta} a^A(\theta).$$

Lemma 3.2 (Default pattern) *When financial exclusion is permanent, i.e., $\phi = 0$, the discretionary default threshold $a^A(\theta)$ is monotone increasing in spending needs θ: $\frac{\partial a^A(\theta)}{\partial\theta} \geq 0$. For those types that are savers, i.e., $\dot{a}(\theta, a^A(\theta)) > 0$, the default threshold is strictly increasing: $\frac{\partial a^A(\theta)}{\partial\theta} > 0$.*

Proof: See Appendix C.1.3.

Lemma 3.2 states that we should in general expect that higher θ types are more prone to default. This is intuitive, since governments with higher spending needs are more likely to default when they have to spend part of their resources to cover debt services. As intuitive as this may appear, this is not always true. We can only prove it when the punishment for default is permanent exclusion from the financial markets. In Section 3.5, we provide numerical examples that the assumption that $\phi = 0$ is not innocuous.

The previous lemma implies the existence of a nondegenerate area where there is positive default risk. However, this is not always the case; in some situations, default risk does not emerge on the equilibrium path. This happens when the potential default risk premium is so high that it renders the debt burden too heavy for all government types. Thus, any government would be tempted to default if such area existed. As a result, the market imposes a debt limit. The default risk is zero until that limit is reached, then jumps to infinity after that. To be precise:

Lemma 3.3 (High default risk) *There exists $\bar{\lambda}$ such that if turnover is sufficiently high $\lambda > \bar{\lambda}$, for any β, the market imposes a debt limit a^*. Moreover, if $\phi = 0$, then $\bar{\lambda} = \frac{r_f \kappa}{1-\kappa}$.*

Proof: See Appendix C.1.4.

This is informative because even though there is no default risk, a rule-writer may have incentives to intervene. This borrowing limit may not be optimal from the point of the planner. The rule-writer may choose to make the debt limit tighter, defaulting before the government would, or looser, committing not to default when the government would. We analyze this possibility in Proposition 3.3.

3.3 Constrained Efficiency

In this section, we develop our main results. We start by showing that the constrained efficient allocation is simply characterized by fiscal and default rules that constrain government's actions, allowing for discretion when these rules do not apply. Then, we theoretically characterize the rules for some combinations of parameters, leaving for Section 3.5 the general cases.

3.3.1 The Mechanism-Design Problem

We study a planner, or rule-writer, who maximizes the ex ante social welfare, i.e., before the information about types is revealed, for each financial position. Referring to the system of equations (3.3)-(3.4), instead of leaving the governments to choose spending and default, the government chooses $\mathcal{M}(a) = \{g(\theta, a), \delta(\theta, a)\}$ to maximize $\mathbb{E}[v(\theta, a)]$ for each a. Thus, the planner is unaffected by the political distortion β: it weights all current and future governments decisions equally.

If the planner could observe θ, only the analogous equations to (3.4) and (3.7) would be necessary. The planner would choose the optimal allocations and force governments to implement them. However, since we assume that θ is either not observable or not contractible, the planner must induce the governments to truthfully reveal their realized θ. Thus, we set up the problem as a principal-agent problem: the principal chooses the allocation contingent on $\{\theta, a\}$, restricted to truthful revelation. Hence, there are no restrictions on the set of instruments that can be used: they arise endogenously from the problem's solution.

Implementing this approach requires some modifications to (3.3)-(3.4). When the planner proposes the allocation $\mathcal{M}(a)$, any government can lie and report that it has observed an alternative value $\tilde{\theta}$, in which case the government obtains a value $w(\tilde{\theta}, \theta, a)$ given by:

$$w\left(\tilde{\theta}, \theta, a\right) = \left(1 - \delta\left(\tilde{\theta}, a\right)\right) w^n\left(\tilde{\theta}, \theta, a\right) + \delta\left(\tilde{\theta}, a\right) w^d\left(\tilde{\theta}, \theta\right),$$

where the nondefault and default value functions are slight modifications of (3.3)-(3.4) satisfying:

$$\rho w^n(\tilde{\theta}, \theta, a) = \theta u(g(\tilde{\theta}, a)) + \dot{a}(\tilde{\theta}, a) w_a^n(\tilde{\theta}, \theta, a) + \lambda\left(\beta\mathbb{E}[v(\theta', a)] - w^n(\tilde{\theta}, \theta, a)\right), \tag{3.11}$$

$$\rho w^d(\tilde{\theta}, \theta) = \theta u(\kappa\tau) + \phi\left(w^n(\tilde{\theta}, \theta, 0) - w^d(\tilde{\theta}, \theta)\right) + \lambda\left(\beta\mathbb{E}[v^d(\theta')] - w^d(\tilde{\theta}, \theta)\right). \tag{3.12}$$

The main difference between (3.3)-(3.6) and (3.11)-(3.12) is that the latter are evaluated at the planner's proposed allocations rather than the optimal choice of each government; and that each government can misrepresent its type to an alternative $\tilde{\theta}$. Hence the additional first argument in the value functions. At the solution, each agent takes the truthful reporting of future governments as given. For that reason, the HJBs for v, v^n and v^d remain unaltered with respect to (3.4)-(3.7), with the exception that they are now evaluated at $\mathcal{M}(a)$ rather than the government's optimal choices. To be precise, the value functions are conditional on the mechanism, $w(\tilde{\theta}, \theta, a; \mathcal{M})$, $v(\theta, a; \mathcal{M})$. For convenience, we drop the notation $\mathcal{M}$, but the reader should bear in mind that the values are determined by the allocation choice.

As a result, for each a, the planner chooses $\mathcal{M}(a) = \{g(\theta, a), \delta(\theta, a)\}$ to maximize the expected social value:

$$\max_{\mathcal{M}(a)} \int_{\underline{\theta}}^{\bar{\theta}} v(\theta, a; \mathcal{M}) dF(\theta), \tag{3.13}$$

subject to:

$$w(\theta, \theta, a; \mathcal{M}) \geq w(\tilde{\theta}, \theta, a; \mathcal{M}), \forall \theta, \tilde{\theta} \in \Theta, \forall a \tag{3.14}$$

$r(a)$ is given by equation (3.1).

Equation (3.14) is the truth-telling condition: a type θ prefers to tell its true type rather than imitating any other type. For convenience, after imposing truth-telling, we adopt the notation $w(\theta, a) = w(\theta, \theta, a)$, $w^n(\theta, a) = w^n(\theta, \theta, a)$, and $w^d(\theta) = w^d(\theta, \theta)$.

Remarks on the welfare function. Several implications of problem (3.13) that are worth mentioning. First, since the planner is choosing allocations before the realization of θ, it is implicitly allowed to transfer utility across types. It can do this by manipulating the interest rate. This is especially important because of the market incompleteness. When a type θ defaults, it does not consider that its actions affect the interest rate that other types must pay at the same level of assets. Thus, even when $\beta = 1$ the planner would like to alter the individual decisions. We discuss this effect in detail in Lemma 3.5.

Second, the planner chooses a mechanism that is optimal at each a, thus it does not have incentives to change it ex post. In other words, *the mechanism is sustainable*. One could consider different solution strategies. A planner could choose, with commitment, contingent spending and default future paths given an initial assets position a_0. By committing to future potential nonoptimal allocations, the planner could manipulate future interest rates in ways that were beneficial from the initial perspective. This raises many questions about the sustainability of such policies that could render them impractical. Moreover, the impact of such a price-manipulation strategy could be minimal depending on the utility function. For instance, with a logarithmic utility function, the income and substitution of interest-rate changes cancel out, leaving allocations unaffected.

Finally, the proposed contract is history-independent. This is likely to be irrelevant due to the *i.i.d* nature of the shocks and the fact that the budget constraint must hold at every period. Allowing for monetary transfers across types could add an important role to history even when the shocks are *i.i.d.*

3.3.2 The Optimality of Fiscal and Default Rules

In this section, we characterize the optimal allocations. We start by characterizing the set of incentive-compatible allocations and show that they could be implemented with both fiscal and default rules. We then provide some sharper results about the optimal rules. Throughout the rest of the paper, we maintain the following two assumptions:

Assumption 3.1 *$f(\theta)$ is differentiable and satisfies:*

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{2-\beta}{1-\beta}; \quad \forall \theta.$$

This assumption is the same as in Amador et al. (2006). It is a *sufficient* condition ensuring that the threshold defined in Proposition 3.1, equation (3.15), is unique. A quick inspection of the condition reveals that the left-hand side is akin to the elasticity of the density function as θ increases, which must be bounded below. We later discuss potential implications of its failure, one of which would be the possibility of "money burning" on the equilibrium path.[10]

Assumption 3.2 *$g(\theta, a)$ is differentiable almost everywhere.*

First and foremost, Assumption 3.2 is *necessary* to make sure that all the differential equations are well-defined. Moreover, we use the first-order approach to characterize the problem.

3.3.3 Incentive-Compatible Allocations

Armed with Assumptions 3.1 and 3.2, we can provide the key proposition of this paper. This result sharply characterizes the set of incentive-compatible allocations to three thresholds delimiting areas where discretion must be maintained and other areas where discretion is banned. We then show, when analyzing the implementation, how these thresholds map in a straightforward fashion into fiscal and default rules.

Proposition 3.1 (Incentive-compatible allocation) *There exist maximum and minimum debt levels, $\underline{a}$ and $\bar{a}$, with $-\frac{\tau}{r_f} < \underline{a} \leq \bar{a} \leq 0$, and a debt-dependent threshold $\theta^s(a) \in \Theta$, $\forall a \geq \underline{a}$, such that*

i) The spending rule $g(\theta, a)$ allows for discretion below the spending threshold and imposes rules above:

$$g(\theta, a) = \begin{cases} u'^{-1}(\frac{1}{\theta} w_a^n(\theta, a)) & \text{for } \theta \leq \theta^s(a), \ \forall a \geq \underline{a}, \\ g(\theta^s(a), a) & \text{for } \theta \geq \theta^s(a), \ \forall a \geq \underline{a}; \end{cases} \tag{3.15}$$

[10] See Ambrus and Egorov (2013) for the precise conditions under which money burning could arise.

ii) The default rule imposes mandatory default on high debt levels and forbids default for low debt levels, allowing for discretionary default for intermediate debt levels:

$$\delta(\theta, a) = \begin{cases} 1 & \textit{for } a \leq \underline{a},\ \forall\theta \\ \delta^A(\theta, a) & \textit{for } a \in [\underline{a}, \bar{a}]\ \forall\theta \\ 0, & \textit{for } a \geq \bar{a},\ \forall\theta. \end{cases} \tag{3.16}$$

Proof: See Appendix C.1.5.

Proposition 3.1 is simple and powerful. It states that all that the planner can do to improve outcomes is to determine the areas in which the governments are free to choose their preferred policies, while in the remaining areas they must abide by the imposed rule. Item *i)* states the space of θ governments can be split into two well-defined areas. Note that the first line of equation (3.15) resembles equation (3.8). Thus, incumbents claiming sufficiently low spending needs are endowed with discretion: they can optimally choose the desired level of spending and debt. Instead, if the incumbent claims larger than allowed spending needs, it is bound to spend no more than a predetermined amount, as shown in the second line of (3.15). This is true regardless of whether they are in the default area. Of course, the threshold depends on the debt level, hence the planner could tighten or loosen the allowed degree of discretion as debt is piling up. We analyze these possibilities in Section 3.3.4, but a priory everything is possible.

Item *ii)* states that, similarly, the space of financial assets can be split into regions where default is restricted and another where governments are free to choose. Now, the space is divided into three well-defined areas. If the debt level is neither too low nor too high, governments can discretionally decide whether to default, and they do so by following the rule in equation (3.5). Again, this is true independently of the spending limit in place, which changes the value functions but not the nature of the decision.

Discussion about implementation. The allocations in Proposition 3.1 have two components that deserve a discussion about its implementation. The first one is straightforward. To implement $\theta^s(a)$, there are several alternatives are often found in the observed fiscal rules. A debt-contingent spending cap or a debt-contingent deficit limit will easily do it. In what follows, we will call this component the *fiscal rule*. Keeping Section 3.2.2 in mind, adding the constraint $g \leq g(\theta^s(a), a)$, $\forall\theta, a$ to the maximization involved in equation (3.3) would achieve the desired outcome.[11] Low types would be unconstrained, choosing their preferred spending, while high θ types would meet the constraint and would only spend $g(\theta^s(a), a)$.

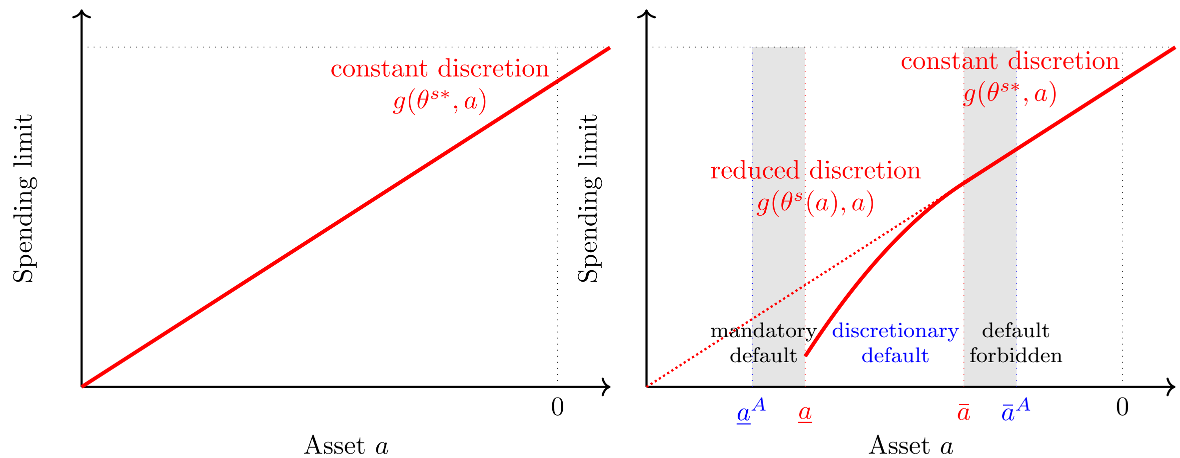
The implementation of the second component raises more questions. Taken literally, it implies that either the constitution or some special law, requiring a supermajority to be overturned, forbids default when the debt level is below $\bar{a}$, or forces it when $a \leq \underline{a}$. These kinds of rules are rare, if not completely absent, in the currently observed set of "fiscal rules." Their unusual existence should not be a deterrent to future implementation. Moreover, the rule-writer could use indirect mechanisms without explicitly stating forbidden or mandatory default. For instance, a fiscal rule could mandate that whenever the debt level is below, say, 25% of GDP, the payments of debt services should have absolute priority in the budget. Once the threshold is exceeded, the incumbent could freely reallocate spending, including the possibility of not paying the debt obligations. This feature could be interpreted as a *relaxation* of fiscal rules when the sovereign enters the default risk region.

To impose the upper bound on debt ($\underline{a}$) the constitution or a special law could state that any debt level above this threshold, say 125% of GDP, would not be recognized as a legitimate obligation, rendering it outright illegal. Under this circumstance, the lenders would not be willing to extend additional funding, creating a de facto hard borrowing limit. For debt levels between 25% and 125% of GDP the government would be allowed to borrow and freely default when necessary, subject to the risk premium imposed by the financial markets.

Proposition 3.1 also implies *the optimal convergency path* when the debt happens to be, for any reason, outside the "desired" range. As an example, the European Fiscal Compact states that the debt-to-GDP ratio of the member countries cannot exceed 60% of GDP and that the deficit should be no more than 3% of GDP. When a country exceeds the 60% threshold a "Debt-Break-Rule" is triggered, which essentially tightens the deficit limit to at least a 1% surplus. Since many European countries are currently well above the 60% mark, the agreement has triggered a growing literature that, taking the target as given, studies the optimal debt path toward it. We believe this literature is faulty by conception or partial at best. It is not possible to study the optimal convergency path without incorporating into the framework the reason that gave rise to the threshold. They go hand in hand.

Finally, from the perspective of our theory, the European Fiscal Compact is "incomplete," in the sense that it does not provide guidelines regarding the course of action when facing default decisions. Is it implicit in the rule that governments

[11] The same fiscal rule can be implemented with a deficit limit imposing $\dot{a} \geq \dot{a}(\theta^s(a), a)$, for all θ and a.



(a) Fiscal rule without default

(b) Fiscal rule and default rule

Figure 3.1 Implementation

with a debt ratio below 60% of GDP cannot default? How far above the 60% mark is tolerable? Out theory dictates that at a certain level of indebtedness countries should be forced to default and, thus, converge instantaneously to a sound financial position.

3.3.4 Fiscal (Spending) Rules

In Proposition 3.1, we argue that $\theta^s(a)$ could be debt-dependent. There are only two elements that create this dependency. One is the presence of default risk: the planner may want to set rules that manipulate the interest rate in the right direction. The second element is the possibility of affecting the government's default decisions: different deficit/spending limits could change the government's incentives to default.

To clarify this point, it is useful to consider a benchmark economy when there is no default risk, either by taking away the possibility of default or by making the default cost so high, i.e., $\kappa = 0$ and $u(0) \to -\infty$, that no government would ever find it appealing to default for any debt level. Indeed, the government can now borrow up to the natural debt limit. The following lemma characterizes the optimal allocation, which specifies a constant level of discretion regardless of how much debt the government has accumulated.

Lemma 3.4 (No default) *When default is not possible, the debt's lower bound is the natural debt limit, $\underline{a} = -\frac{\tau}{r_f}$. The exists a unique spending threshold, θ^{s*}, inde-*

pendent of debt, characterized by:

$$\theta^{s*} = \beta \mathbb{E}\left[\theta | \theta \geq \theta^{s*}\right]. \tag{3.17}$$

Proof: See Appendix C.1.6.

The threshold solving equation (3.17) is identical to the one by Amador et al. (2006). Under Assumption 3.1, θ^{s*} is increasing in the present-bias parameter β: the less present-biased the governments are, the more discretion is allowed. In one extreme, when there is no present bias, i.e., $\beta = 1$, the planner allows full discretion to spend: $\theta^{s*} = \bar{\theta}$. In the other extreme, with a severe present bias, i.e., $\beta \leq \underline{\theta}$, the planner bans all discretion: $\theta^{s*} = \underline{\theta}$.

It may appear puzzling that equation (3.17) does not involve λ. After all, political turnover is what generates the need for rules. Larger values of λ increase the frequency at which the political friction β impacts the economy, calling for stricter rules, but also increases the frequency of the genuine spending shocks, requiring better insurance and thus looser rules. All in all, both effects cancel out rendering the threshold independent of λ.

When we allow for default risk the characterization of the optimal threshold becomes cumbersome, not allowing for a closed form solution. Nevertheless, we are able to implicitly characterize it:

Proposition 3.2 (Spending threshold) *For any $a \in [\underline{a}, \infty)$, the optimal spending rule satisfies:*

$$\begin{aligned}
\theta^s(a) \quad &= \quad \beta \underset{\theta \in \Theta^n(a)}{\mathbb{E}} [\theta | \theta \geq \theta^s] + \chi(a) \left[\underbrace{\int_{\underline{\theta}}^{\theta^s} [w_a^n(\theta) - v_a^n(\theta)] \frac{\partial g(\theta, a)}{\partial r(a)} dF(\theta)}_{\textit{Discipline effect} \geq 0} \right. \\
&+ \quad \left. \underbrace{a \mathbb{E}[v_a^n(\theta)]}_{\textit{Income effect} \leq 0} + \underbrace{\int_{\underline{\theta}}^{\theta^s} [g(\theta^s, a) - g(\theta, a)] \frac{\partial v_a^n(\theta)}{\partial r(a)} dF(\theta)}_{\textit{Insurance effect} \geq 0} \right] \frac{\partial r(a)}{\partial \theta^s} \\
&+ \quad \chi(a) \underbrace{\left[\theta^d [u(g(\theta^s, a)) - u(\kappa y)] + \lambda[\mathbb{E}v(\theta') - \mathbb{E}v^d(\theta')]\right] f(\theta^d) \frac{\partial \theta^d}{\partial \theta^s}}_{\textit{Default manipulation} \lesseqgtr 0}.
\end{aligned}$$

Proof: See Appendix C.1.7.

There are five components determining the $\theta^s(a)$. The first component is akin to equation (3.17), but now the conditional expectation is over the nondefaulters, while in (3.17) it is unrestricted. Absent the other effects, this would generate a threshold smaller than θ^{s*}. Since the planner knows that only less tempted types remain among the nondefaulters, it can tighten the spending limit without much loss in efficiency.

The second and third components are similar to those in Halac and Yared (2018). The discipline effect tends to increase $\theta^s(a)$. Because $\frac{\partial g}{\partial r} < 0$, an increase in the interest rate reduces the overspending, which is welfare-improving as long as $w_a^n < v_a^n$. The next component is what we term the income effect, while Halac and Yared (2018), correctly, call it the redistribution effect. In their environment the interest rate affects outcomes through an asset-market clearing condition, thus changes in the interest rate have redistributive effects across types. In our environment, instead, changes in the risk premium increase the debt burden for the country as a whole, which must transfer more resources to the lenders. Also as in their paper, the net contribution of these two effects is ambiguous, depending on the value of β. However, in their environment the asset is in constant (zero) net supply, thus there is not "aggregate" debt effect. In our case, instead, as the debt level increases the negative impact tends to tighten the spending limit by reducing the discretion region.

The last two components do not have an equivalence in literature. What we call the *insurance effect* arises because of the change in the planner's marginal value of wealth, which is absent in Halac and Yared (2018) since they focus on a two-period economy. This provides an additional incentive to the planner to increase $\theta^s(a)$, with a positive impact on the interest rate. The larger interest rate allows all types with discretion to obtain a larger continuation value for the same.

The final component is the default manipulation. Unfortunately, we are not able to sign this component. This happens because, as we show in the next section, it is not clear whether the economy experiences excessive or insufficient default. Both cases are possible, which could lead the planner to either relax or tighten the constraint. We analyze this effect quantitatively in Section 3.5.

It is clear from Proposition 3.2 that these additional effects depend on the debt level and appear only when there is risk of default. Thus, for debt levels for which the risk of default is zero the optimal fiscal rule resembles that in equation (3.17). In addition, for the fiscal rule to be meaningful, it must apply to governments that actually have a spending choice. Let $\theta^d(a)$ be the type's default threshold for each a. In Section 3.3.5, we provide conditions under which it is unique and monotone increasing, in which case all types $\theta \geq \theta^d(a)$ default. It is evident that $\theta^s(a) > \theta^d(a)$ would render the fiscal rule innocuous, since all constrained governments would default. Moreover, when the present bias is strong enough, the direct effect due to (3.17) dominates, then we have:

Corollary 3.1 (General characterization) *For any combination of parameters:*

i) Severe present bias. If $\beta \leq \underline{\theta}$, there is no discretion to spend $\theta^s(a) = \underline{\theta}$, $\forall a$.

ii) Mild present bias. If $\beta > \underline{\theta}$, there is discretion. In the nondefault area, the threshold is constant. In the default risk area, there could be increased or reduced discretion.

$$\theta^s(a) \begin{cases} = \theta^{s*} & \text{for } a \geq \bar{a}, \\ \leq \theta^d(a) & \text{for } a < \bar{a}. \end{cases}$$

Loosely speaking, Corollary 3.1, part *ii)* implies that as the debt level rises there is a race between the solution to the threshold in Proposition 3.2 and the default threshold. If the income effect and the default manipulation effect are not powerful enough, it may be optimal to lift the fiscal limits and allow the governments to discretionarily default, if they have large spending needs, or to freely choose spending, if their needs are moderate or small. On the equilibrium path, this could be implemented with either unaltered fiscal rules as the debt level increases (but ineffective) or complete elimination of all fiscal rules.

3.3.5 Default Rules

Before characterizing the constrained efficient default policies, it is informative to analyze what the planner would do if she had perfect information about θ. From now on we denote by $\delta^P(\theta, a)$, the optimal default policy under a *perfect-information* benchmark.

Lemma 3.5 (Unconstrained optimal default) *If the principal had perfect information, it would choose to default $\delta^P(\theta, a) = 1$ if and only if*

$$v^n(\theta, a) + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta', a)}{\partial r(a)} \left(1 - \delta^P(\theta', a)\right) dF(\theta') \leq v^d(\theta). \tag{3.18}$$

Correspondingly, there is a desired default threshold for the principal $a^P(\theta)$ at which equation (3.18) *holds with equality.*

Proof: See Appendix C.1.8.

Equation (3.18) makes clear that even when the planner has perfect information, it chooses $\delta^P(\theta, a) \neq \delta^A(\theta, a)$. To see this, note that when $\beta = 1$, $v^n(\theta, a) = w^n(\theta, a)$. Since a government would default when $w^d(\theta, a) \geq w^n(\theta, a)$, it implies $v^d(\theta, a) \geq v^n(\theta, a)$. This happens because the principal takes into consideration the interest-rate effect for all possible θ's. If one type defaults, it increases the interest rate that lenders charge to all types. In this sense, the implications for default rules are different than for spending rules. Recall that in Lemma 3.4, we show that the equilibrium converges to the first best as $\beta \to 1$, so that fiscal rules are not needed. However, when default is possible the equilibrium may be inefficient even when there is no present bias. While the principal does not need to impose spending limits, its default incentive differs from that of the agent.

We now turn to the optimal intervention. We start by showing that even when there is no default risk, as long as $\beta < 1$ the planner would like to alter the default decisions. When analyzing the rules-free equilibrium, we show, in Lemma 3.3, that one possible outcome is a market-imposed endogenous borrowing limit. One implication of it is that there is no default risk on the equilibrium path. Still, the borrowing limit imposed by the market may not be optimal from the point of view of the planner. Now we show how the fiscal rules can also generate an endogenous borrowing constraint. When this happens, we say that the government takes away all the discretion to default and imposes a *debt limit rule*. The rule states that a government can borrow only up to the limit and never default. If the government happens to start with a debt level beyond the limit, then it must default. The following describes conditions under which a debt limit is optimal.

Proposition 3.3 (Optimal debt limit) *If the present bias is severe $\beta \leq \underline{\theta}$ or the risk premium is high $\lambda \geq \bar{\lambda}$, the optimal default rule is a debt limit a^*. Moreover,*

i) If exclusion is permanent $\phi = 0$, the debt limit coincides with the rules-free equilibrium:
$$a^* = a^A = \frac{1}{r_f}(\kappa - 1)\tau. \tag{3.19}$$

ii) If there is no default cost, either because there is no revenue loss $\kappa = 1$ or reaccess is instantaneous $\phi = \infty$, the debt limit coincides with the rules-free equilibrium, $a^* = 0$.

iii) Otherwise, if $\kappa < 1$ and $0 < \phi < \infty$, the planner imposes a tighter limit and defaults sooner, $a^* > a^A$.

iv) Whenever $\beta \leq \underline{\theta}$, there is no discretion, neither to spend nor to default.

Proof: See Appendix C.1.9.

There are two conditions under which a debt limit is optimal, when the present bias is severe, $\beta \leq \underline{\theta}$, and when the risk premium is large, $\lambda \geq \bar{\lambda}$. Since the mechanisms are different, we discuss them separately.

When $\beta \leq \underline{\theta}$, regardless of the incumbent's spending needs, even the least-tempted type $\underline{\theta}$ is too myopic to save on behalf of future governments with higher spending needs. As a result, all types are dissaving, $\dot{a}(\theta, a) \leq 0$, $\forall\theta$. In this situation, Lemma 3.4 applies, the planner takes away all discretion to spend and imposes the same spending to all types. As they approach their default threshold, they stop accumulating debt and stay at the threshold. Their spending is exactly equal to their net income after interest payments, $r_f a + \tau$, which does not depend on θ. Therefore, all types have the same default threshold, $a^{*A} = \underline{a}^A = \bar{a}^A$. Thus, due to the fiscal rule, a borrowing limit endogenously arises: the market stops lending to all types, knowing that they would default. There is no discretionary default region.

Is this market-imposed limit optimal? Recall that the discrepancy in the default incentives between the planner and the agent due to present bias manifests in two ways: the agent not only discounts the continuation value of not defaulting but also the continuation value of defaulting. So, it is possible that the agent defaults too much too early or too little too late. In case *i)*, with permanent financial exclusion, it turns out that the default incentives between the planner and the agent are exactly aligned. Intuitively, once in default the economy falls into permanent autarky with spending permanently fixed to $\kappa\tau$. At the borrowing limit the government also faces a "permanent" constant spending $r_f a^* + \tau$. The two must equate so that the government is indifferent to defaulting or not, which generates the borrowing limit in equation (3.19). This also makes clear why the incentives are aligned. Since both are comparing constant streams of consumption, the excess discounting by the government becomes irrelevant. It has the same effect on the default and nondefault states.

It follows from the borrowing limit that when $\kappa = 1$, the market would not be willing to lend and the borrowing limit is $a^* = 0$. The same outcome also occurs when there is immediate financial reaccess. Since the government can restart instantly with a clean slate with zero debt burden, it will default on any amount of debt. Thus, case *ii)* resembles a "rainy day fund": the government cannot borrow but can save up for rainy days.

Whenever default is costly, $\kappa < 1$ and financial exclusion is temporary $0 < \phi < \infty$, the debt capacity is positive. In this case, perhaps counterintuitively, the agent defaults too little too late. What happens is that the benefits of default happen after the costs, so the myopic discounting has a larger effect on the default value. One of the main benefits of default arrives later: upon reentry, the government starts with no debt. A government that mostly cares about its own term overweights the immediate costs under financial exclusion, and does not internalize the future benefits that can be enjoyed by another government. The planner who weights all governments alike wants to default sooner, and before the agent. As a result, the planner imposes a tighter debt limit and *forces default* whenever the government is holding on to too much debt.

When the debt limit arises because $\lambda > \bar{\lambda}$ the outcome is similar, but the underlying mechanism is different. As shown in Lemma 3.3, the market discipline induces an endogenous borrowing limit. The high risk premium demanded by the market makes it impossible for the late-default types to separate themselves from the early-default ones. For ease of exposition, suppose financial exclusion is permanent, i.e., $\phi = 0$. With an endogenous borrowing limit, the debt capacity would be $a^A = \frac{(\kappa-1)\tau}{r_f}$. If there is a type contemplating borrowing slightly more and delaying default, the market would demand an interest rate $r_f + \lambda$. Thus, the total interest payments would be $\frac{(1-\kappa)\tau}{r_f}(r_f + \lambda)$, which if $\lambda > \bar{\lambda}$ would generate negative spending. This implies, for instance, that in environments with instantaneous gratification as in Harris and Laibson (2012), where $\lambda \to \infty$, there cannot be default in equilibrium and a planner should impose a tighter debt limit than the market.

The endogenous borrowing limit results are instructive about the directions on which the planner wants to move the default thresholds, but abstract from the risk-premium effect. In realit, the risk premium is prevalent and the main subject of policy makers' concern. It only arises when neither the present bias nor the uncertainty is too large. It is cumbersome to sharply characterize all the possibilities, however, we can state the following result:

Proposition 3.4 (General characterization) *Suppose the present bias is mild, $\beta > \underline{\theta}$, and there is low turnover $\lambda < \bar{\lambda}$. Then, depending on the spending needs heterogeneity,*

i) (Some savers). If the taste distribution is dispersed, i.e., the lowest type satisfies:

$$\underline{\theta} < \frac{\lambda\beta}{(\rho + \lambda - r_f)} \frac{\mathbb{E}\left[v_a\left(\theta, a^A\right)\right]}{u'\left(r_f a^A + \tau\right)}, \tag{3.20}$$

there exists θ such that $\dot{a}(\theta, a) > 0$, for each a. Default optimally happens on the equilibrium path. Moreover, when $\phi = 0$, the planner sets $\underline{a} = \underline{a}^A$ and $\bar{a} < \bar{a}^A$.

ii) *(All dissavers). Otherwise, all types dissave, $\dot{a}(\theta, a) \leq 0$, $\forall \theta, a$. There is an endogenous borrowing constraint a^A; hence, the optimal default rule is a debt limit a^*.*

Proof: See Appendix C.1.10.

The condition in equation (3.20) provides a necessary condition for the existence of risk premium on the equilibrium path. Unfortunately, we can only go as far as to characterize it as a function of the (endogenous) value $v_a(\theta, a)$. Nevertheless, it is useful to interpret the condition under which a government saves or dissaves. When equation (3.20) is violated, all possible governments want to reduce their asset positions, even the less tempted would not be willing to reduce debt. As a result, the market again imposes a borrowing limit. Intuitively, lenders are willing to lend today only if someone in the future would be willing to pay back. The outcome then, as stated in *ii)* is similar to Proposition 3.3, with the exception that in this case the fiscal rule could allow for some discretion to spend.

Instead, when condition (3.20) is satisfied both savers are dissavers co-exist in the same environment, which gives rise to the possibility of lending with potential for repudiation. There is a nondegenerate area of assets $[\underline{a}, \bar{a}]$, where sometimes default happens and others times it doesn't. Who are the defaulters and who are the out-of-default-region savers depends on the probability of regaining access to the financial markets. When there is permanent exclusion, $\phi = 0$, we can characterize the type-dependent default threshold $a^A(\theta)$ and show that it is strictly increasing in θ. In other words, the high-need types default for lower level of assets, while the lowest-need type, $\underline{\theta}$, is the last willing to default.

Due to Proposition 3.1 we know that the rule-writer cannot alter the default decisions on the interior of $[\underline{a}, \bar{a}]$, it can only change the borders. But then Proposition 3.3, *i)* also applies to the determination of $\underline{a}$: if all types want to default when the assets position is below $\underline{a}$ and $\phi = 0$, the planner does not want to distort that decision. However, in the upper bound things are different. Only a measure zero of agents would find it optimal to default, which triggers some inefficient effects. If the upper bound were $\bar{a} = a^A(\bar{\theta})$, two inefficiencies would play an important role. First, because $\beta < 1$, the high types initially defaulting do not accurately internalize that by defaulting they are forcing other less-tempted but still high-need types to financial autarky, imposing on them low consumption. Second, the sole possibility that this upper-bound type can default increases the risk premium to all other governments, which, as we explain after Lemma 3.5, is not internalized by any government, irrespective of β. Both effects point to the same policy intervention:

the planners can increase welfare by imposing an upper bound $\bar{a} < a^A(\bar{\theta})$. This is what we call the forbidding default region in Proposition 3.1.

When $\phi > 0$, the outcome and optimal policy are difficult to characterize. The main problem arises because the default threshold may no longer be monotone in θ. This creates some problems that make a formal proof difficult. Nevertheless, in the next section, we quantitatively evaluate the optimal problem, and we show analogous results.

3.4 Extensions

3.4.1 Partial Forgiveness and Suspension of Payments

So far, we have assumed full debt forgiveness in the analysis. That is, upon reaccessing the market, previously defaulted debt is fully discharged. However, as documented by Arellano et al. (2019), default is often partial and only a portion of the debt is discharged. In these situations, upon reaccessing the market, the government may need to repay some of the previously defaulted debt. In this section, we adopt an alternative assumption. Suppose that financial exclusion is temporary $\phi > 0$ and that upon reentry the government must repay a proportion $\alpha \in [0, 1]$ of the defaulted debt. Thus, if the government or any future government regains access to the market, it starts with debt $b = \alpha a$.[12] When $\alpha = 1$, default leads to *suspension of payments*. When $\alpha = 0$, default leads debt being fully discharged as in the baseline model.

Under this assumption, when not in default, the value functions are identical to the ones before, with the exception that the risk premium must be modified to incorporate the recovery value. The most substantial difference arises on the computation on the default value functions. Now upon reaccessing the market the values depend on the past defaulted debt. We denote the default value functions by $w^d(\theta, b)$ and $v^d(\theta, b)$. They satisfy the following HJB equations:

$$\rho w^d(\theta, b) = \theta u(\kappa\tau) + \phi\left(w^n(\theta, b) - w^d(\theta, b)\right)(1 - \delta_d(\theta, b)) + \lambda\left(\beta\mathbb{E}[v^d(\theta', b)] - w^d(\theta, b)\right), \quad (3.21)$$

$$\rho v^d(\theta, b) = \theta u(\kappa\tau) + \phi\left(v^n(\theta, b) - v^d(\theta, b)\right)(1 - \delta_d(\theta, b)) + \lambda\left(\mathbb{E}[v^d(\theta', b)] - v^d(\theta, b)\right). \quad (3.22)$$

In equations (3.21) and (3.22), the changes in value in the event of financial reaccess are multiplied by $(1 - \delta_d(\theta, b))$. When $\alpha = 0$, as in the baseline model, all debt is discharged, thus whenever a government has the chance to reenter the financial markets, it does it with certainty. But when $\alpha > 0$, an incumbent could disregard the opportunity and remain in default. This could be interpreted as immediate default,

[12] For simplicity, we assume that, when a government is in default, interest payments do not accumulate.

as prescribed by the equilibrium default decision or the default rule. In this case, reentry into the financial markets happens only when a government with sufficiently low spending needs is willing to resume the debt payments. Note that we have defined this rejection to reenter function with a subscript d, $\delta_d(\cdot)$, to stress that it may not coincide with the previous function $\delta(\cdot)$.

When the country is in a nondefault state, the default condition is analogous to the previous condition (3.9) now modified to:

$$w^n(\theta, a^A(\theta)) = w^d(\theta, \alpha a^A(\theta)). \tag{3.23}$$

Regarding the reentering condition we assume that the haircut $(1-\alpha)$ is not applied again whenever the government rejects the option to enter. Then, $\delta_d(\cdot)$ satisfies:

$$\delta_d(\theta, b) = \begin{cases} 1 & \text{if } w^n(\theta, b) < w^d(\theta, b), \\ 0 & \text{if } w^n(\theta, b) \geq w^d(\theta, b). \end{cases} \tag{3.24}$$

This equation points out to two special cases. In the baseline model $\alpha = 0$, then because it is always true that $w^n(\theta, 0) > w^d(\theta, 0)$, we have $\delta_d(\theta, 0) = 0$ for all θ. Another special case is when there is suspension of payments. Since in this case $\alpha = 1$ we have that $\delta_d(\theta, a) = \delta(\theta, a)$ for all θ and all a.

In addition, introducing some recovery value changes the risk premium charged on loans. Since lenders could recover some of the funds lent, the risk premium must take into account this additional benefit. To be precise, now the interest rate must satisfy:

$$r(a, b) = r_f + \lambda \mathbb{E}\left[\delta(\theta, a)(1 - R(\theta, b))\right],$$

where $R(\theta, b; \delta_d)$ is the per unit recovery value of a defaulted loan when a type-θ borrower is required to, conditional on reentry, pay back b dollars per each a borrowed. In Appendix C.1.11, we show that $R(\cdot)$ is linear in b and therefore the risk premium per unit of loan can be written as:

$$r(a) = r_f + \lambda \mathbb{E}\left[\delta(\theta, a)(1 - \alpha \hat{R}(\theta, \alpha a))\right]. \tag{3.25}$$

This equation also makes clear that because of the linearity of the recovery value on the amount to be repaid, it can be decomposed on the haircut, $1 - \alpha$, and the present expected value of the probability of reentry, $\hat{R}(x) = R(1, x)$. In Appendix C.1.11, we show that:

$$\hat{R}(\theta, b) = \begin{cases} \frac{(1-\mathbb{E}[\delta_d(\theta,b)])\lambda\phi}{\lambda\phi(1-\mathbb{E}[\delta_d(\theta,b)])+r_f(r_f+\lambda+\phi)} & \text{if } \delta_d(\theta, b) = 1, \\ \frac{(\lambda+r_f)}{(\lambda+r_f+\phi)} \frac{(1-\mathbb{E}[\delta_d(\theta,b)])\lambda\phi}{\lambda\phi(1-\mathbb{E}[\delta_d(\theta,b)])+r_f(r_f+\lambda+\phi)} & \text{otherwise.} \end{cases} \tag{3.26}$$

Under suspension of payments, i.e., $\alpha = 1$, the last equations simplify even further since $\delta_d(\cdot) = \delta(\cdot)$, which also ensures that the government initially defaulting would not reenter, even when it has the chance. This implies that only the first line of (3.26) applies. As a result, the default incentives are analogous to the ones under full debt forgiveness with permanent exclusion ($\phi = 0$). To see why, consider a type θ who defaulted the amount a, then $\delta(\theta, a) = \delta_d(\theta, b) = 1$. Since it must repay the same amount of debt, this type would never return to the market even if given the opportunity. Thus, the direct effect on default incentives is akin to permanent exclusion. Therefore, the results in Lemma 3.2 and 3.3 still apply: default $\delta(\theta, a)$ is monotonously increasing in type θ and the debt limit due to high risk premium, with the same $\bar{\lambda}$, follows without modifications. It then follows in a straightforward way that:

Lemma 3.6 (Suspension of Payments) *When $\alpha = 1$, for any $\phi > 0$, the monotone default pattern in Lemma 3.2 holds as in the case when $\phi = 0$. Correspondingly, the qualitative characterization of Propositions 3.2, 3.3 and 3.4 still hold conditional on an alternative $\phi = 0$.*

The previous lemma states that an economy with suspension of payments can be, *at least qualitatively*, analyzed as an economy with full debt forgiveness and permanent exclusion upon default. Of course, the *quantitative results are not the same*. There are additional indirect effects through the continuation values and the level of the risk premium that would generate different quantitative rules. Lemma 3.6 stresses the potential substitutability between ϕ and the haircut. With suspension of payments, this substitutability is as close to being perfect as possible. However, when $\alpha < 1$ and $\phi > 0$ the substitution is not perfect, which can generate further quantitative implications. We leave that analysis for the quantitative part of Section 3.5.

3.4.2 Business Cycles

In this section, we relax the assumption that τ is constant, allowing for a mean-reverting tax revenue. To ensure that τ is always positive, we assume that it follows a Cox–Ingersoll–Ross process:

$$d\tau_t = \nu(\bar{\tau} - \tau_t) + \sigma_\tau \sqrt{\tau_t} dW_t, \tag{3.27}$$

where W_t is a Wiener process. We further assume that τ is perfectly observable and contractible. One may wonder about the informational asymmetry between τ and

θ. The main premise here is that the availability of resources to a government can be measured, if not perfectly, almost certainly, so it is possible to write contracts contingent on it.[13] While θ can be interpreted as the needs generated by the revenue shock. These needs even though real can arise from subjective assessments of the economic situation or could be difficult to verify. Alternatively, one can think that the business cycle is driven by a unique process that has an observable component, τ, and a nonmeasurable component and is therefore noncontractible.

This shock's addition affects the baseline environment in two main ways. First, the allocation, the interest rate, and all value functions will now depend on the additional state τ. Thus, an allocation now will be $\{g(\theta, a, \tau), \delta(\theta, a, \tau)\}$. The interest rate is:

$$r\left(a, \tau\right) = r_f + \lambda \mathbb{E}\left[\delta\left(\theta, a, \tau\right)\right].$$

To write down the last equation we have used the arguments of Section 3.2.1 to show that the interest rate is still independent of θ, because only a jump in the type can generate a default. The revenue process is a smooth one, which never triggers default directly. However, now the default rate does depend on this additional source of uncertainty, but only indirectly as it changes the regions of defaulters and nondefaulters.

Finally, the "continuously" moving shock to τ requires a modification to the value functions. Here we present only the difference with Section 3.2.2, but the analogous changes for the mechanism-design problem should be clear to the reader. The set of HJB equations (3.3)-(3.6) and (3.4)-(3.7) for the value functions are replaced by:

$$\begin{aligned}(\rho+\lambda)w^n(\theta,a,\tau) &= \max_g \left\{\theta u(g) + (r(a,\tau)a + \tau - g)w^n_a(\theta,a,\tau)\right\} + \lambda\beta\mathbb{E}[v(\theta',a,\tau)] \\ &\quad + \nu(\bar{\tau}-\tau)w^n_\tau(\theta,a,\tau) + \frac{1}{2}\sigma_\tau^2\tau w^n_{\tau\tau}(\theta,a,\tau),\end{aligned} \tag{3.28}$$

$$\begin{aligned}(\rho+\lambda)v^n(\theta,a,\tau) &= \theta u(g^*) + (r(a,\tau)a + \tau - g^*)v^n_a(\theta,a,\tau) + \lambda\mathbb{E}[v(\theta',a,\tau)] \\ &\quad + \nu(\bar{\tau}-\tau)v^n_\tau(\theta,a,\tau) + \frac{1}{2}\sigma_\tau^2\tau v^n_{\tau\tau}(\theta,a,\tau),\end{aligned} \tag{3.29}$$

[13] This may not be true in countries with weak institutions. For instance, Argentina from 2011 to 2015 was consistently misreporting not only inflation but also GDP. Still, because tax revenue must be shared with independent provinces, the Federal government had to report the true tax revenue.

$$\begin{aligned}(\rho+\lambda+\phi)w^d(\theta,\tau) &= \theta u(\kappa\tau)+\phi w^n(\theta,0,\tau)+\lambda\beta\mathbb{E}[v^d(\theta',\tau)] \\ &\quad + \nu(\bar{\tau}-\tau)w^d_\tau(\theta,\tau)+\frac{1}{2}\sigma_\tau^2\tau w^d_{\tau\tau}(\theta,\tau),\end{aligned} \tag{3.30}$$

$$\begin{aligned}(\rho+\lambda+\phi)v^d(\theta,\tau) &= \theta u(\kappa\tau)+\phi v^n(\theta,0,\tau)+\lambda\mathbb{E}[v^d(\theta',\tau)] \\ &\quad + \nu(\bar{\tau}-\tau)v^d_\tau(\theta,\tau)+\frac{1}{2}\sigma_\tau^2\tau v^d_{\tau\tau}(\theta,\tau).\end{aligned} \tag{3.31}$$

The main difference with respect to Section 3.2.2 is the extra terms in the second line of each equation. These "drift" terms capture the effect of the movements on and uncertainty about τ on the value functions. They clearly affect their levels and shape but do not change the fundamental structure of the problem and hence their optimal conditions. Following similar steps to proof of Proposition 3.1, it readily follows that:

Lemma 3.7 (Random Revenues) *If the government's revenue follows the process in equation* (3.27)*, the optimal intervention has the same pattern as Proposition 3.1. There exist asset thresholds* $\underline{a}(\tau)$ *and* $\bar{a}(\tau)$*, with* $-\frac{\tau}{r_f} < \underline{a}(\tau) \leq \bar{a}(\tau) \leq 0$*, and a state-dependent threshold* $\theta^s(a,\tau) \in \Theta$*,* $\forall a \geq \underline{a}(\tau)$*, such that:*

i) all types $\theta \leq \theta^s(a,\tau)$ *have discretion to spend, while those above abide by the rule;*

ii) if $a < \underline{a}(\tau)$ *all types are forced to default, while if* $a > \bar{a}(\tau)$*, default is banned. In between governments have discretion to default.*

Proof: Online Appendix.

Lemma 3.7 is an extension of Proposition 3.1. It shows that the main properties shown in the previous sections remain, adding a dependency of the optimal rules on the observable state of the economy. As one can see, the notational burden grows considerably by the addition of the extra state. For this reason, we avoid the characterization of the thresholds and we rely on numerical results. It is interesting, though, that this section adds a requirement that the optimal fiscal rules be dependent on the current state of the economy. Regarding our discussion in Section 3.3.3 about implementation, this result adds the requirement that the rules should be contingent on the observable state of the economy. This dependency is, for instance, currently absent in the European Fiscal Compact.

3.5 Quantitative Application

In this section, we aim to understand some quantitative implications of our theory. Our goal is general, not focused on a particular country, but it is relevant to understand the quantitative prescriptions of the theory for alternative "standard" cases. For this reason, we quantitatively analyze three cases that we named: Germany, Greece, and Italy. This naming is motivated by some broad empirical patterns, but by no means should the naming be interpreted as accurate representations of the countries. We broadly see Greece as having a relatively low debt capacity and a risk premium sensitive to debt accumulation. We see Germany as having a large debt capacity and a risk premium only mildly sensitive to debt accumulation. And we characterize Italy as having both a large debt capacity and a high risk premium sensitivity. In this spirit, we calibrate these three "countries."

3.5.1 Calibration

We specify a constant relative risk aversion (CRRA) utility function for spending with risk aversion parameter γ. The spending needs follow a truncated lognormal distribution, i.e., $\theta \sim LN\left(-\frac{1}{2}\sigma^2, \sigma\right)$ in the domain $\left[\underline{\theta}, \bar{\theta}\right]$. To ensure that its expected value is normalized to 1, we set the upper bound $\bar{\theta} = 1/\underline{\theta}$. Given the functional forms, the model consists of a set of eleven parameters $\{\gamma,\, \rho,\, r_f,\, \tau,\, \lambda,\, \beta,\, \sigma,\, \underline{\theta},\, \bar{\theta},\, \kappa,\, \phi\}$.

Since none of these countries had fiscal rules before 1993, we calibrate the model-generated moments of the rules-free equilibrium to the pre-1993 data wherever possible. We select the data sample prior to the year the Maastricht Treaty came into force to ensure that no fiscal rules were imposed. Appendix C.2.1 includes details on data sources and measurements. All level variables are normalized relative to GDP. The model is calibrated at an annual frequency.

Table 3.1 reports the calibrated parameters and the corresponding data moments. We set the risk aversion parameter γ to 1. The risk-free rate r_f is calibrated to match an annual risk-free interest rate of 4%. The discount rate ρ is set to equal to the risk-free rate, which avoids dissaving due to any gap between the interest rate and the discount rate. The tax revenue parameter τ corresponds to the size of government revenue (percentage of GDP). For the three European countries we study, the average tax revenue is around 45%.

We calibrate the reaccess rate following empirical work on sovereign default and subsequent financial exclusion. Cruces and Trebesch (2013) document that the duration of financial exclusion depends on the extent of haircut in the event of sovereign default, with restructuring involving higher haircuts associated with longer periods

Table 3.1: Parameters and moments

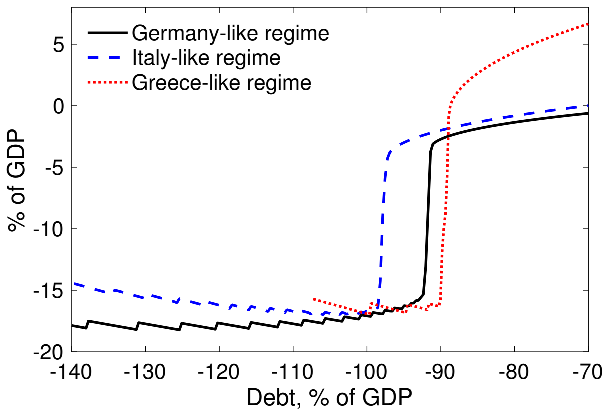
Parameter	Symbol	Value			Moment
		Germany	Italy	Greece	
CRRA	γ		1		preset
Discount rate	ρ		0.04		interest rate
Risk-free rate	r_f		0.04		interest rate
Tax revenue	τ		0.45		government revenue (% of GDP)
Spending needs					
Turnover rate	λ	1/15	1/8	1/8	corr(default risk, debt level)
Present bias	β	0.75	0.75	0.5	average debt growth
Distribution std dev	σ		0.5		dispersion in debt growth
Type lower bound	$\underline{\theta}$		0.2		default risk debt upper bound
Type upper bound	$\bar{\theta}$		5		inverse of $\underline{\theta}$ to normalize mean
Cost of default					
Output loss	κ		0.89		default risk debt lower bound
Reaccess rate	ϕ		0.02		Cruces and Trebesch (2013)

of exclusion. They estimate that, in a default scenario involving a haircut of 60% or above, the probability of remaining excluded after 10 years is slightly over 50%. This estimate implies a reaccess rate of 0.03. Given our baseline model assumption of full default, we set a slightly lower reaccess rate ϕ at 0.02.

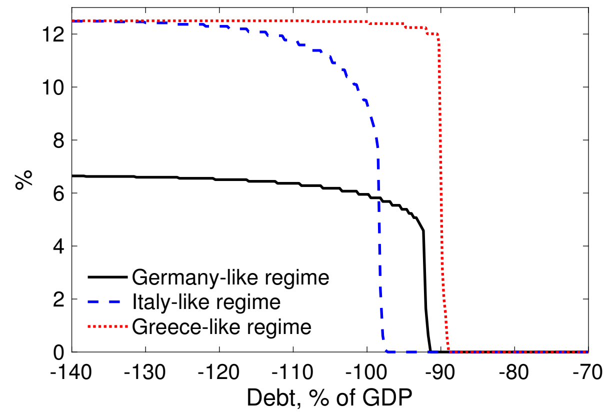
We estimate the remaining five parameters by taking advantage of the spending and default risk patterns in the rules-free equilibrium. First, regarding the spending pattern, the present-bias parameter β can be identified from the average debt growth, as it directly corresponds to the overspending tendency. Instead, the variance of taste distribution σ maps directly to the dispersion in debt growth. Second, regarding the default risk, the output loss parameter κ affects the minimum level of debt $\bar{a}^A$, at which default risk starts to emerge. Further, the spending-needs bounds are calibrated to ensure that the maximum level of debt $\underline{a}^A$ is at least above the one observed in the data. Finally, the correlation between debt level and default risk premium $\text{corr}(r(a), a)$ allows us to identify the turnover rate λ.

3.5.2 Quantitative Regimes

To differentiate the three different regimes of interest, we rely on two parameters, political turnover and political polarization. The first regime that we study, Ger-



(a) Expected debt growth, $\mathbb{E}\dot{a}\,(\theta, a)$



(b) Default risk premium, $r(a) - r_f$

Figure 3.2 Equilibrium Default-risk area

many, is characterized by low turnover and mild present bias. The second one is the Italy-like regime, which has high turnover and mild present bias. The last one, Greece (or an Argentina-like regime), has high turnover and severe present bias. All the calibrated parameters can be seen in Table 3.1.

While all three regimes fit into case *i)* of Proposition 3.4 where condition (3.20) is satisfied, the range of default risk area differs quantitatively. Panel (b) of Figure 3.2 plots the default risk premium at different levels of government debt. In all three regimes, when government debt exceeds around 90% of GDP, default risk starts to emerge. Both the Germany-like and the Italy-like regimes exhibit a wide range of debt levels with default risk. This is because the mild present bias leads to many saving types. However, the default risk premium still reacts very differently depending on the turnover rate: in the Italy-like regime, given that the spending shocks associated with turnover occur at a higher frequency, the default risk premium jumps up by a larger magnitude as debt accumulates beyond a certain threshold, while the Germany-like regime experiences a milder increase in default risk premium. In contrast, the Greece-like regime has a very narrow band of debt levels with default risk. This is because when present bias becomes more severe, fewer types save. In this regime, compounded by the high turnover risk, the default risk premium jumps up sharply when debt increases.[14]

[14] The quantitative magnitude of the default risk premium is much larger than the empirical levels. The extension incorporating partial debt forgiveness in Section 3.4.1 would be able to match better the empirical magnitude.

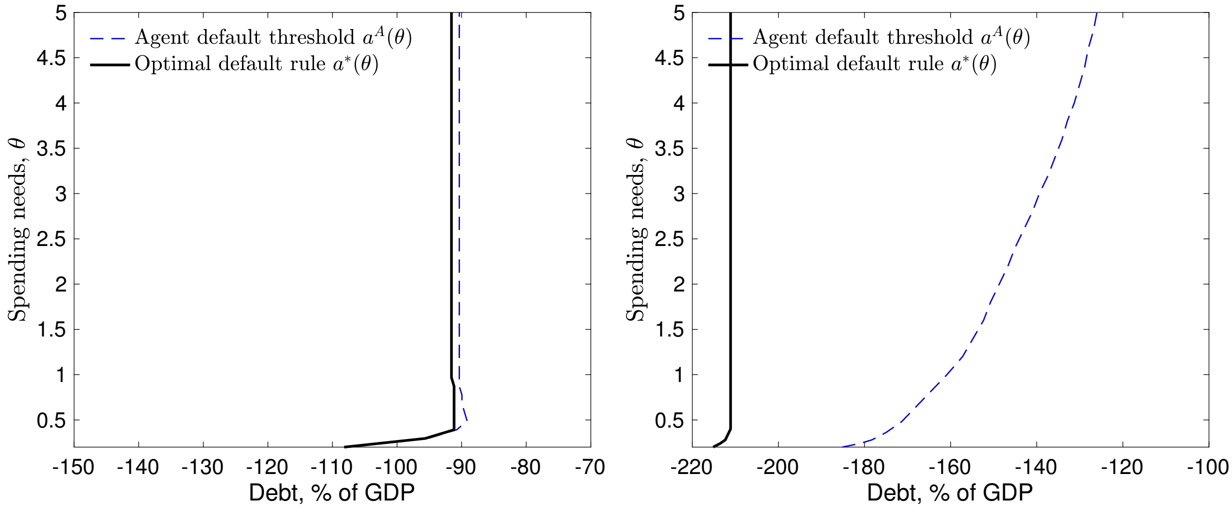
3.5.3 Policy Analysis

We start by analyzing the optimal default rules in the Greece-like regime. In our classification, this is a country with high turnover and high political polarization. Thus, in the absence of any rules, it is characterized by a low debt capacity with a highly sensitive risk premium. We depict this situation with the blue dashed line in Panel (a) of Figure 3.3. For levels of debt above 90% of GDP, most θ-types would default, generating the almost vertical line around 90%. The few types that do not default are depicted along the positively sloped line between 90% and 110% of GDP.

The first exercise we perform is to analyze what the optimal default rule should be when the planner does not impose any fiscal rule. This exercise is depicted by the black solid line in Panel (a) of Figure 3.3. The optimal default rule in this case has the pattern described in Proposition 3.1. It forbids default for debt levels below 92.5% of GDP and then coincides with the desired default thresholds of the governments. This small deviation from the governments' desired default thresholds is partially due to the interest-rate effect. By banning default, the planner makes sure that the nondefaulting types bear a lower burden by the debt services. The extent of intervention is so mild that one may even think that is not worth bothering. However, the situation is substantially different when the *default rules is complemented with the fiscal rule*.

In the second exercise we compute the optimal default rule when the planner **also** imposes a fiscal rule, which is depicted in Panel (b) of Figure 3.3. To understand this figure, it is important to bear in mind that the fiscal rule drastically reduces the spending capacity of each government. Thus, the immediate effect of the fiscal rule is to build debt capacity. The financial markets are more confident that the country would repay its debt and thus willing to lend a larger amount. In the absence of a default rule, the governments would follow a default strategy depicted by the dashed blue line in Panel b) of Figure 3.3. This by itself expands the risk-free area from 0 to 90% of GDP when the fiscal rule is absent to 0-125% of GDP when the fiscal rule is imposed. In addition, it also increases the maximum debt capacity from 115% of GDP to more than 180%. But that is not the end of the optimal intervention. Given the fiscal rule, the planner also set different default rules, which is depicted in the black solid line. Now the extent of intervention is sizeable. The rule-writer forbids default of any debt level below 215% of GDP, while the area with discretionary default is drastically reduced. Note that the government would have a strong incentive to default for any debt level above 130%, but the default rule **optimally forbids it**.

Although the spirit of our calibration is not to accurately reflect any country in particular, the pattern that we called Greece-like generates some interesting analogies to the observations after the 2011 European debt crisis. In the intervention after



(a) Default rule alone

(b) Default rule complemented with fiscal rule

Figure 3.3 Optimal rules: Greece-like regime

the Greek default, the European troika imposed fiscal rules that were considered draconian from many viewpoints, and validated a haircut that left Greece with a debt-to-GDP ratio barely short of 200%, which is considered unsustainable. In the light of our model, these highly criticized decisions are perfectly consistent with the optimal rules imposed by an unbiased planner given the political environment of the country.

3.6 Conclusions

Sovereign debt accumulation has long been a subject of controversial debate. Although the possibility of borrowing is accepted as an important tool to efficiently smooth adverse shocks, government debt is also widely regarded as exploited by self-interested governments for their own benefits. Hence, fiscal rules appear as fundamental components of every healthy democracy. This concern triggers hefty debates, especially when default is a possibility, reflecting the potential unsustainability of honoring past obligations, making the debate unavoidable. When the financial markets are incomplete, the possibility of defaulting on past obligations does not necessarily reflect an inefficiency; it could also be a welfare-improving tool. Thus, to analyze the need and optimality of fiscal rules, one must incorporate these three key elements: the need to smooth spending, political bias, and risk of default. In this paper, we have approached the problem based on this premise.

We have extended previous insights showing that limits on deficits or debt growth, together with discretion to respond to spending needs, are a necessary component of sound fiscal constraints in environments with risk of default. In addition, we have shown that an analogous principle applies to the default decision. Default decisions must also be regulated. Even though default rules are unusual, it does not mean that they are unnecessary. We have shown that sometimes, when debt is low, governments defaults too much too early, which calls for the need of banning default for low levels of debt. We also show that, especially when debt is large, governments default too little too late, so forcing default by imposing a "hard" debt limit would be optimal. Defaulting can be optimal, and governments concerned only about the cost borne by their own administration inefficiently avoid it. Similarly, for intermediate levels of debt, defaulting is an optimal tool that can be welfare-improving. Regulating this decision whenever debt is neither too low nor too high is too costly, so governments should default at their discretion, using the information available to them.

We see this paper as a first step toward developing a theory that can help provide precise quantitative prescriptions for real-life case studies. At this stage, we have compromised on omitting important features of reality to derive clear theoretical prescriptions. Nevertheless, there are many dimensions in which this theory could be enriched. Among many other, the debt maturity structure and the endogeneity of political turnover appear as key elements that must be studied in follow-ups to this paper.

Appendix A

Appendix to Chapter 1

A.1 Proofs

A.1.1 Recursive Formulation

I begin by rewriting the sequential problem ($\mathcal{P}_{NT}$) recursively. Remember that, given an initial level of wealth $\bar{a}$, $\mathcal{V}_0(\bar{a})$ is the set of planner's payoffs such that, for all $v_0 \in \mathcal{V}_0(\bar{a})$, there exists a sequence of spending g and an associated wealth process that *(i)* satisfy the government's budget constraint (BC) with initial assets $\bar{a}$, *(ii)* are such that truthful reporting is incentive compatible (i.e. constraint (IC) is satisfied) and *(iii)* deliver utility $v_0 = V(g, \sigma^*)$. Also, v_n is the continuation utility of government n at the time of its formation τ_n.

Now, take a sequence of spending g and associated wealth process $\{a_t\}$, with $a_0 = \kappa y/r$, satisfying incentive compatibility (1.1), the budget constraint (BC) and delivering utility $v_0 = V(g, \sigma^*)$. Standard properties of logarithmic preferences imply that the sequence $\widehat{g} \equiv g/a_0$ and the associated wealth process $\widehat{a}_t \equiv a_t/a_0$, with $\widehat{a}_0 = 1$, satisfy incentive compatibility (1.1), the budget constraint (BC) and deliver utility to the planner:

$$\begin{aligned}
\widehat{v}_0 &= V(\widehat{g}, \sigma^*) \\
&= \mathbb{E}_- \left[\sum_{n=0}^{\infty} e^{-\gamma \tau_n} \theta_n \int_{\tau_n}^{\tau_{n+1}} e^{-\gamma(t-\tau_n)} \log(\widehat{g}_t) dt \right] \\
&= v_0 - \mathbb{E}_- \left[\sum_{n=0}^{\infty} e^{-\gamma \tau_n} \theta_n \int_{\tau_n}^{\tau_{n+1}} e^{-\gamma(t-\tau_n)} \log(a_0) dt \right] \\
&= v_0 - \frac{1}{\gamma} \log(a_0).
\end{aligned}$$

Exactly the same arguments show that, for any value $v_n \in \mathcal{V}_n(\bar{a})$, there is a corresponding value in $\widehat{v}_n \in \mathcal{V}_n(1)$ such that $\widehat{v}_n = v_n - \log(\bar{a})/\gamma$. As a result, it is sufficient to characterize the problem for $a_0 = 1$.

Consider now a sequence g with associated wealth process $\{a_t\}$, $a_{\tau_n} = 1$, delivering utility $v_n \in \mathcal{V}_n(1)$ to the planner. Let a_{τ_n} the country's wealth at the time the n-th government is formed. Using the law of iterated expectations and the above results, the expected utility of government n at time τ_n is:

$$\begin{aligned}
&U_{\tau_n}(\sigma_n^* | g, \sigma_{-n}^*) \\
&= \mathbb{E}_{\tau_n} \left[\int\limits_{\tau_n}^{\tau_{n+1}} e^{-\gamma(s-t)} \theta_n \log(g_s) ds + \beta \sum_{j=n+1}^{\infty} e^{-\gamma \tau_j} \left(\int\limits_{\tau_j}^{\tau_{j+1}} e^{-\gamma(s-\tau_j)} \theta_j \log(g_s) ds \right) \right] \\
&= \mathbb{E}_{\tau_n} \left[\int\limits_{\tau_n}^{\tau_{n+1}} e^{-\gamma(s-t)} \theta_n \log(g_s) ds + \beta e^{-\gamma \tau_{n+1}} \mathbb{E}_{\tau_{n+1}^-} \sum_{j=n+1}^{\infty} e^{-\gamma(\tau_j - \tau_{n+1})} \left(\int\limits_{\tau_j}^{\tau_{j+1}} e^{-\gamma(s-\tau_j)} \theta_j \log(g_s) ds \right) \right] \\
&= \mathbb{E}_{\tau_n} \left[\int\limits_{\tau_n}^{\tau_{n+1}} e^{-\gamma(s-t)} \theta_n \log(g_s) ds + \beta e^{-\gamma \tau_{n+1}} v_{n+1} \right] \\
&= \mathbb{E}_{\tau_n} \left[\int\limits_{\tau_n}^{\tau_{n+1}} e^{-\gamma(s-t)} \theta_n \log(g_s) ds + \beta e^{-\gamma \tau_{n+1}} \left(\widehat{v}_{n+1} + \frac{1}{\gamma} \log\left(a_{\tau_{n+1}}\right) \right) \right], \qquad \text{(A.1)}
\end{aligned}$$

where $\widehat{v}_{n+1}$ in the last equality is such that $\widehat{v}_{n+1} \in \mathcal{V}_{n+1}(1)$. By the same arguments, planner's utility equals:

$$v_n = \mathbb{E}_{\tau_n^-} \left[\int\limits_{\tau_n}^{\tau_{n+1}} e^{-\gamma(s-t)} \theta_n \log(g_s) ds + e^{-\gamma \tau_{n+1}} \widehat{v}_{n+1} + e^{-\gamma \tau_{n+1}} \frac{1}{\gamma} \log\left(a_{\tau_{n+1}}\right) \right].$$

I now consider the planner's problem at time 0, when the first government is formed (the other formation periods τ_n, $n = 1, 2...$, are analogous). By definition, the value of such a problem – which I denoted with v^{nt} – corresponds to the point in $\mathcal{V}_0(a_0)$ such that $v^{nt} \geq v_0$, for all $v_0 \in \mathcal{V}_0(a_0)$. In addition, the arguments above imply that there exists a point in $\mathcal{V}_0(1)$ – which I denoted with $\overline{v}$ – such that $\overline{v} = v^{nt} - \log(a_0)/\gamma$ and $\overline{v} \in \mathcal{V}_0(1)$. Since $\overline{v} \in \mathcal{V}_0(1)$, it must be that:

$$\overline{v} = \max_{g,a,w \in \mathcal{V}(1)} \mathbb{E}_{-} \left[\int_0^{\tau_1} e^{-\gamma t}\theta_0 \log(g_t)dt + e^{-\gamma\tau_1} w(\theta) + e^{-\gamma\tau_1}\frac{1}{\gamma}\log\left(a_{\tau_1}\right)\right],$$

subject to the incentive compatibility constraint (IC), which, using (A.1), can be equivalently expressed as:

$$\begin{aligned} &\mathbb{E}\left[\int_0^{\tau_1} e^{-\gamma t}\theta_0 \log(g_t)dt + \beta e^{-\gamma\tau_1} w(\theta_0) + \beta\frac{1}{\gamma}e^{-\gamma\tau_1}\log\left(a_{\tau_1}\right)\right] \qquad (A.2)\\ \geq &\mathbb{E}\left[\int_0^{\tau_1} e^{-\gamma t}\theta_0 \log(g_t)dt \middle| \tilde{\theta}\right] + \beta w(\tilde{\theta})\mathbb{E}\left[e^{-\gamma\tau_1}\right] + \beta\frac{1}{\gamma}\mathbb{E}\left[e^{-\gamma\tau_1}\log\left(a_{\tau_1}\right)\middle| \tilde{\theta}\right], \end{aligned}$$

and the budget constraint (BC) with initial wealth equal to 1. The latter can be equivalently written as:

$$\int_0^{\tau} e^{-rt} g_t dt + e^{-r\tau}a_\tau = 1,$$

which is (1.4). Finally, to obtain problem ($\mathcal{P}_{NT:Rec}$), I add the constraint (1.5), together with the extra choice variable $u : \Theta \to \mathbb{R}$, and use it to rewrite (A.2) as:

$$\mathbb{E}\left[\theta_0 u(\theta_0) + \beta e^{-\gamma\tau_1} w(\theta_0) + \frac{\beta}{\gamma}e^{-\gamma\tau_1}\log\left(a_{\tau_1}\right)\right] \geq \theta_0 u(\tilde{\theta}) + \beta w(\tilde{\theta})\mathbb{E}\left[e^{-\gamma\tau_1}\right] + \frac{\beta}{\gamma}\mathbb{E}\left[e^{-\gamma\tau_1}\log\left(a_{\tau_1}\right)\middle| \tilde{\theta}\right],$$

or, using $\mathbb{E}[e^{-\gamma\tau_1}] = \lambda/(\gamma+\lambda)$,

$$\mathbb{E}\left[\theta_0 u(\theta_0) + \beta\frac{\lambda}{\gamma+\lambda} w(\theta_0) + \beta\frac{1}{\gamma}e^{-\gamma\tau_1}\log\left(a_{\tau_1}\right)\right] \geq \theta_0 u(\tilde{\theta}) + \beta\frac{\lambda}{\gamma+\lambda} w(\tilde{\theta}) + \beta\frac{1}{\gamma}\mathbb{E}\left[e^{-\gamma\tau_1}\log\left(a_{\tau_1}\right)\middle| \tilde{\theta}\right],$$

which is equivalent to (1.5).

A.1.2 Proof of Lemma 1.1

Consider the subproblem ($\mathcal{P}_{NT:S1}$). Integrating by parts, I can rewrite the constraint on current utility as follows:

$$\begin{aligned} \overline{u} &= \mathbb{E}\left[\int_0^{\tau} e^{-\gamma t}\log(\widehat{g}_t)dt\right] \\ &= \int_0^{\infty} \lambda e^{-\lambda t}\int_0^{t} e^{-\gamma s}\log(\widehat{g}_t)dsdt \\ &= \int_0^{\infty} e^{-(\lambda+\gamma)t}\log(\widehat{g}_t)dt. \end{aligned}$$

The Lagrangian associated to the problem is then:

$$\mathcal{L} = \int_0^\infty \lambda e^{-\lambda t} \log(\widehat{g}_t)dt - \int_0^\infty \left(\int_0^t e^{-r\cdot s}\widehat{g}_s ds + e^{-rt}\widehat{a}_t - 1 \right) d\Phi_t + \mu \int_0^\infty e^{-(\lambda+\gamma)t} \log(\widehat{g}_t)dt,$$

where μ and Φ are Lagrange multipliers on, respectively, the current-utility constraint above and the budget constraint (1.4). Integrating by parts the second term:

$$\begin{aligned}\mathcal{L} =& \frac{1}{\gamma}\int_0^\infty \lambda e^{-(\lambda+\gamma)t} \log(\widehat{a}_t)dt \\ & - \int_0^\infty (e^{-rt}\widehat{a}_t - 1)d\Phi_t - \left(\Phi_t \int_0^t e^{-r\cdot s}\widehat{g}_s ds \Big|_0^\infty - \int_0^\infty \Phi_t e^{-r\cdot t}\widehat{g}_t dt \right) \\ & + \mu \int_0^\infty e^{-(\lambda+\gamma)t} \log(\widehat{g}_t)dt,\end{aligned}$$

or

$$\begin{aligned}\mathcal{L} =& \frac{1}{\gamma}\int_0^\infty \lambda e^{-(\lambda+\gamma)t} \log(\widehat{a}_t)dt - \int_0^\infty (e^{-rt}\widehat{a}_t - 1)d\Phi_t \\ & - \int_0^\infty (\Phi_\infty - \Phi_t)e^{-r\cdot t}\widehat{g}_t dt + \mu \int_0^\infty e^{-(\lambda+\gamma)t} \log(\widehat{g}_t)dt,\end{aligned}$$

where $\Phi_\infty \equiv \lim_{t\to\infty} \Phi_t$. The first-order conditions with respect to $\widehat{a}_t$ and $\widehat{g}_t$ are, respectively,

$$\frac{1}{\gamma}\lambda e^{-(\lambda+\gamma)t}\frac{1}{\widehat{a}_t} - \dot{\Phi}_t e^{-rt} = 0$$

and

$$\mu e^{-(\lambda+\gamma)t}\frac{1}{\widehat{g}_t} - (\Phi_\infty - \Phi_t)e^{-r\cdot t} = 0.$$

Conjecture $\widehat{a}_t = e^{(r-\Delta)t}$ and $\widehat{g}_t = \Gamma e^{(r-\Delta)t}$, for some positive scalars Δ, Γ, with $\Delta > \lambda + \gamma$. Then, the first condition yields:

$$\frac{1}{\gamma}\lambda e^{-(r+\lambda+\gamma-\Delta)t} - \dot{\Phi}_t e^{-rt} = 0$$

and, thus,

$$\Phi_t = \frac{1}{\gamma}\lambda\frac{1}{\Delta - (\lambda+\gamma)}e^{-(\lambda+\gamma-\Delta)t}.$$

Also, since $\Delta > \lambda + \gamma$, $\Phi_\infty = 0$, the second condition becomes:

$$\mu e^{-(\lambda+\gamma)t}\frac{1}{\Gamma}e^{-(r-\Delta)t} + \frac{1}{\gamma}\lambda\frac{1}{\Delta - (\lambda+\gamma)}e^{-(\lambda+\gamma-\Delta)t}e^{-r\cdot t} = 0$$

or

$$\mu\frac{1}{\Gamma}+\frac{1}{\gamma}\lambda\frac{1}{\Delta-(\lambda+\gamma)}=0. \tag{A.3}$$

Also, $\{\widehat{a}_t,\widehat{g}_t\}_{t\geq 0}$ must satisfy the budget constraint (1.4), which given the conjectures above is:

$$\Gamma\int_0^{\tau} e^{-\Delta t}dt+e^{-\Delta\tau}=1.$$

The latter is true for all possible realizations of τ if and only if $\Gamma=\Delta$. Combining the latter result with (A.3) yields:

$$\Gamma=\frac{\mu(\lambda+\gamma)}{\frac{1}{\gamma}\lambda+\mu}.$$

The Lagrange multiplier μ must be chosen to satisfy the current-utility constraint:

$$\begin{aligned}\overline{u}=&\int_0^{\infty}e^{-(\gamma+\lambda)t}\log(\widehat{g}_t)dt\\ =&\log(\Gamma)\int_0^{\infty}e^{-(\gamma+\lambda)t}dt+(r-\Gamma)\int_0^{\infty}e^{-(\gamma+\lambda)t}tdt\\ =&\log\left(\frac{\mu(\lambda+\gamma)}{\frac{1}{\gamma}\lambda+\mu}\right)\frac{1}{\gamma+\lambda}+\left(r-\frac{\mu(\lambda+\gamma)}{\frac{1}{\gamma}\lambda+\mu}\right)\frac{1}{(\gamma+\lambda)^2}.\end{aligned}$$

Thus, the conjecture is verified.

Finally, by denoting with $k(\overline{u})$ the solution to

$$\overline{u}=\log(k(\overline{u}))\frac{1}{\gamma+\lambda}+(r-k(\overline{u}))\frac{1}{(\gamma+\lambda)^2},$$

the optimal levels of spending and wealth are, respectively, $\widehat{g}_t=k(\overline{u})e^{(r-k(\overline{u}))t}$ and $\widehat{a}_t=e^{(r-k(\overline{u}))t}$.

A.1.3 Proof of Lemma 1.2

The proof follows from the arguments in Proposition 2 in Amador et al. (2006). In particular, notice that the objective function can be written as:

$$\frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}}(1-M(\theta))u(\theta)d\theta+\frac{\underline{\theta}}{\beta}u(\underline{\theta})+\underline{w}=\frac{1}{\beta}\int_{\underline{\theta}}^{\theta^\star}(1-M(\theta))u(\theta)d\theta+\frac{1}{\beta}\int_{\theta^\star}^{\overline{\theta}}(1-M(\theta))u(\theta)d\theta+\frac{\underline{\theta}}{\beta}u(\underline{\theta})+\underline{w}.$$

Also, using $u(\theta) = \int_{\theta^\star}^{\theta} du + u(\theta^\star)$, for $\theta \geq \theta^\star$, after integrating by parts, the second term becomes:

$$\frac{1}{\beta}\int_{\theta^\star}^{\overline{\theta}}\int_{\theta}^{\overline{\theta}}(1-M(x))dxdu + \frac{1}{\beta}\int_{\theta^\star}^{\overline{\theta}}(1-M(\theta))u(\theta^\star)d\theta.$$

Since u is non-decreasing it must be that $du \geq 0$. Also, since $\int_{\theta}^{\overline{\theta}}(1-M(x))dx \leq 0$ for all $\theta \geq \theta^\star$, the term above is maximized for $du = 0$ or, equivalently, for $u(\theta) = u(\theta^\star)$ for all $\theta \geq \theta^\star$. Finally, notice that bunching types in the upper tail is always incentive compatible since the incentive constraint

$$\frac{\theta}{\beta}u(\theta) + W(u(\theta)) \geq \frac{1}{\beta}\int_{\underline{\theta}}^{\theta} u(z)dz + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w}$$

is satisfied for all $\theta > \theta^\star$ if it is satisfied for $\theta \leq \theta^\star$.

A.1.4 Proof of Proposition 1.1

Consider the Lagrangian associated to problem ($\mathcal{P}_{NT:S2}$):

$$\begin{aligned}\mathcal{L} =& \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}}(1-M(\theta))u(\theta)d\theta + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w} \\ &+ \int_{\underline{\theta}}^{\overline{\theta}}\left(\frac{1}{\beta}\theta u(\theta) + W(u(\theta)) - \int_{\underline{\theta}}^{\theta}\frac{1}{\beta}u(x)dx - \frac{\underline{\theta}}{\beta}u(\underline{\theta}) - \underline{w}\right)d\Lambda(\theta),\end{aligned}$$

for some non-decreasing function $\Lambda(\cdot)$. Integrating by parts yields:

$$\mathcal{L} = \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}}(\Lambda(\theta)-M(\theta))u(\theta)d\theta + \left(\frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \underline{w}\right)\Lambda(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}}\left(\frac{1}{\beta}\theta u(\theta) + W(u(\theta))\right)d\Lambda(\theta).$$

Following Amador et al. (2006), I set $\Lambda(\underline{\theta}) = 0$, $\Lambda(\theta) = M(\theta)$, for $\theta \leq \theta^\star$, and $\Lambda(\theta) = 1$, for $\theta > \theta^\star$. Notice that, by Assumption 1.1, $\Lambda(\cdot)$ is non-decreasing, as required. The arguments in Amador et al. (2006) then imply that the value of u that maximizes the Lagrangian with this particular choice of $\Lambda(\cdot)$ is the solution to problem ($\mathcal{P}_{NT:S2}$). In particular, the first-order condition with respect to $u(\theta)$, for $\theta \leq \theta^\star$, gives:

$$\frac{1}{\beta}\theta + W'(u(\theta)) = 0.$$

Since $W(x) = (r - k(x))\mathbb{E}[e^{-\gamma\tau}\tau]/\gamma + \mathbb{E}[e^{-\gamma\tau}]\overline{w}$ and $\mathbb{E}[e^{-\gamma\tau}\tau] = \lambda/(\gamma+\lambda)^2$, then $W'(x) = -k'(x)\lambda/[\gamma(\gamma+\lambda)^2]$ and the condition above implies:

$$\frac{1}{\beta}\theta = \frac{\lambda k'(u(\theta))}{\gamma(\gamma+\lambda)^2}. \tag{A.4}$$

Also, from Lemma 1.1, $k(x)$ is the solution to the equation

$$(\gamma+\lambda)x = \log(k(x)) + (r - k(x))\frac{1}{\gamma+\lambda}.$$

Differentiating both sides gives $-k'(x)/(\gamma+\lambda) = \gamma+\lambda - k'(x)/k(x)$ which, combined with (A.4), yields:

$$-\frac{1}{\beta}\theta\frac{\gamma}{\lambda} = 1 - \frac{\frac{1}{\beta}\theta\gamma(\gamma+\lambda)}{\lambda k(u(\theta))}$$

or

$$k(u(\theta)) = \frac{\gamma\theta(\gamma+\lambda)}{\gamma\theta+\lambda\beta} \equiv k^{nt}(\theta).$$

The statement then follows directly from Lemma 1.1 with initial wealth $a_0 = \kappa y/r$.

I begin by rewriting the sequential problem ($\mathcal{P}_{TR}$) recursively. It is equivalent, but more convenient to work with the dual problem of minimizing expected resources of delivering a given lifetime utility. Formally, let $K(v_0)$ be the minimum amount of resources necessary to delivery utility v_0:

$$K(v_0) \equiv \min_g \mathbb{E}_- \left[\int_0^\infty e^{-rt} g_t dt\right], \tag{A.5}$$

subject to (IC) and

$$v_0 = \mathbb{E}_- \left[\sum_{n=0}^\infty e^{-\gamma\tau_n}\theta_n \int_{\tau_n}^{\tau_{n+1}} e^{-\gamma(t-\tau_n)} \log(g_t)dt\right].$$

The value of ($\mathcal{P}_{TR}$) (i.e. v^{tr}) is then given by $K(v^{tr}) = \kappa y/r$.

I write the recursive version of (A.5) at time 0, that is, the time at which the first government is formed (the other cases are analogous). Take a sequence g and let $v_1(\theta_0)$ be the associated continuation value at time τ_1 (i.e. the time at which the next government is formed), as a function of the current government's type. It is equal to

$$v_1(\theta_0) = \mathbb{E}_{\tau_1^-} \left[\sum_{n=1}^\infty e^{-\gamma(\tau_n-\tau_1)}\theta_n \int_{\tau_n}^{\tau_{n+1}} e^{-\gamma(t-\tau_n)} \log(g_t)dt\right]. \tag{A.6}$$

As a result,

$$
\begin{aligned}
v_0 &= \mathbb{E}_- \left[\sum_{n=0}^{\infty} e^{-\gamma \tau_n} \theta_n \int_{\tau_n}^{\tau_{n+1}} e^{-\gamma(t-\tau_n)} \log(g_t) dt \right] \\
&= \mathbb{E}_- \left[\theta_0 \int_0^{\tau_1} e^{-\gamma t} \log(g_t) dt + e^{-\gamma \tau_1} v_1(\theta_0) \right] \\
&= \mathbb{E}_- \left[\theta_0 \int_0^{\tau_1} e^{-\gamma t} \log(g_t) dt + \frac{\lambda}{\gamma + \lambda} v_1(\theta_0) \right],
\end{aligned}
\tag{A.7}
$$

where the last line uses $\mathbb{E}_- \left[e^{-\gamma \tau_1} v_1(\theta_0) \right] = \mathbb{E}_- \left[e^{-\gamma \tau_1} \mathbb{E}_{\tau_1^-} [v_1(\theta_0)] \right] = \lambda \mathbb{E}_-[v_1(\theta_0)]/(\gamma + \lambda)$. The same arguments imply that the incentive constraint (IC) at time 0 can be rewritten as:

$$
\mathbb{E}\left[\int_0^{\tau_1} e^{-\gamma t} \theta_0 \log(g_t) dt \right] + \beta \frac{\lambda}{\gamma+\lambda} v_1(\theta_0) \geq \mathbb{E}\left[\int_0^{\tau_1} e^{-\gamma t} \theta_0 \log(g_t) dt \middle| \tilde{\theta} \right] + \beta \frac{\lambda}{\gamma+\lambda} v_1(\tilde{\theta}). \tag{A.8}
$$

Therefore, problem (A.5) can be equivalently stated as:

$$
K(v) \equiv \min_g \mathbb{E}_- \left[\int_0^{\infty} e^{-rt} g_t dt \right],
$$

subject to (A.8), the incentive constraint (IC) from time τ_1 onward, (A.7), and (A.6). In addition, the value function $K(v)$ then satisfies the following property:

$$
\begin{aligned}
K(v) &\equiv \min_g \mathbb{E}_- \left[\int_0^{\infty} e^{-rt} g_t dt \right] \\
&= \min_g \mathbb{E}_- \left[\int_0^{\tau_1} e^{-rt} g_t dt + e^{-r\tau_1} \mathbb{E}_{\tau_1^-} \left[\sum_{n=1}^{\infty} e^{-r(\tau_n - \tau_1)} \int_{\tau_n}^{\tau_{n+1}} e^{-r(t-\tau_n)} g_t dt \right] \right] \\
&= \min_{\{g_t\}_0^{\tau_1}} \mathbb{E}_- \left[\int_0^{\tau_1} e^{-rt} g_t dt + \frac{\lambda}{r+\lambda} \min_{g|_{\tau_1}} \mathbb{E}_{\tau_1^-} \left[\sum_{n=1}^{\infty} e^{-r(\tau_n - \tau_1)} \int_{\tau_n}^{\tau_{n+1}} e^{-r(t-\tau_n)} g_t dt \right] \right],
\end{aligned}
$$

subject to (A.8), the (IC) at $t \geq \tau_1$, (A.7), and (A.6), where $g|_{\tau_1}$ is a short-hand notation for the sequence of spending starting from time τ_1. Since constraints (A.8) and (A.7) depend only on spending until time τ_1, the latter is equivalent to

$$
K(v) = \min_{\{g_t\}_0^{\tau_1}} \mathbb{E}_- \left[\int_0^{\tau_1} e^{-rt} g_t dt + \frac{\lambda}{r+\lambda} K(v_1(\theta_0)) \right],
$$

subject to (A.8) and (A.7).

Finally, to obtain problem ($\mathcal{P}_{TR:Rec}$), I add the constraint (1.10), together with the extra choice variable $u : \Theta \to \mathbb{R}$, and use it to rewrite (A.7) as (1.9) and (A.8) as:

$$\theta_0 u(\theta_0) + \beta\frac{\lambda}{\gamma+\lambda}v_1(\theta_0) \geq \theta_0 u(\tilde{\theta}) + \beta\frac{\lambda}{\gamma+\lambda}v_1(\tilde{\theta}),$$

which is equivalent to (1.8).

A.1.5 Proof of Lemma 1.3

The Lagrangian associated to problem ($\mathcal{P}_{TR:S1}$) is

$$\mathcal{L} = \int_0^\infty \lambda e^{-(\lambda+r)t}\widehat{g}_t dt - \mu(\gamma+\lambda)\int_0^\infty e^{-(\lambda+\gamma)t}\log(\widehat{g}_t)dt,$$

for some Lagrange multiplier μ. The first-order condition with respect to g_t is

$$\lambda e^{-(\lambda+r)t} - \mu(\gamma+\lambda)e^{-(\lambda+\gamma)t}\frac{1}{\widehat{g}_t} = 0.$$

As a result,

$$\widehat{g}_t = \mu(\gamma+\lambda)e^{(r-\gamma)t}. \tag{A.9}$$

Finally, the multiplier μ is chosen so as to satisfy the current-utility constraint. Condition (A.9) shows that the planner minimizes the resources to deliver a given utility level $\overline{u}$ by allocating government spending which is increasing or decreasing depending on whether $r > \gamma$ or $r < \gamma$. In the special case that $r = \gamma$, condition (A.9) becomes

$$\widehat{g}_t = \mu(\gamma+\lambda),$$

where μ satisfies

$$\overline{u} = (\gamma+\lambda)\log(\mu(\gamma+\lambda))\mathbb{E}\left[\int_0^\tau e^{-\gamma t}dt\right].$$

Properties of the Poisson process imply $\mathbb{E}\left[\int_0^\tau e^{-\gamma t}dt\right] = 1/(\gamma+\lambda)$, thus,

$$\mu = \frac{1}{\gamma+\lambda}e^{\overline{u}}.$$

As a result, the optimal amount of resources necessary to deliver utility $\overline{u}$ is

$$\begin{aligned} G(\overline{u}) &\equiv \min_g \mathbb{E}\left[\int_0^\tau e^{-\gamma t}\widehat{g}_t dt\right] \\ &= \frac{1}{\gamma+\lambda}e^{\overline{u}}. \end{aligned}$$

Derivation of problem ($\mathcal{P}'_{TR:S2}$). As discussed in the main text, I conjecture that the value function $K(\cdot)$ satisfies $K(v) = K(0)\exp(Dv)$, for some scalar D. With this conjecture, we obtain

$$K(0)\exp(Dv) = \min_{u,w} \mathbb{E}_{-}\left[\frac{1}{\gamma+\lambda}\exp(u) + \frac{\lambda}{\gamma+\lambda}K(0)\exp(Dw(\theta))\right],$$

subject to (1.8) and

$$(\gamma+\lambda)v = \mathbb{E}_{-}\left[\theta u(\theta) + \lambda w(\theta)\right].$$

Consider the change of variables: $\widetilde{u}(\theta) = u(\theta) - \gamma v$, $\widetilde{w}(\theta) = w(\theta) - v$, for some scalars A, B. Notice that the incentive constraint is not affected by this change of variables. As a result,

$$K(0)\exp(Dv) = \min_{\widetilde{u},\widetilde{w}} \mathbb{E}_{-}\left[\frac{1}{\gamma+\lambda}\exp(\widetilde{u}(\theta)+\gamma v) + \frac{\lambda}{\gamma+\lambda}K(0)\exp(D(\widetilde{w}(\theta)+v))\right],$$

subject to (1.8) and $0 = \mathbb{E}_{-}\left[\theta\widetilde{u}(\theta) + \lambda\widetilde{w}(\theta)\right]$. The conjecture is therefore verified by letting $D = \gamma$.

Finally, the same steps as those for the case with transfers imply that the recursive problem can be equivalently rewritten as ($\mathcal{P}'_{TR:S2}$).

A.1.6 Proof of Lemma 1.4

The proof is analogous to the one of Lemma 1.2 and follows the arguments of Proposition 2 in Amador et al. (2006).

A.1.7 Proof of Proposition 1.2 & 1.3

To prove the proposition, I consider the relaxed problem obtained by dropping the monotonicity constraint on u. I then solve the resulting problem in two steps. In the first step, I find optimal utility given transfers. Let $P(\cdot)$ the value of this problem:

$$P(T) \equiv \max_{u,U(\underline{\theta})} \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} (1-M(\theta))u(\theta)d\theta + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \lambda\underline{w}, \tag{A.10}$$

subject to

$$\frac{\theta}{\beta}u(\theta) + \lambda W\left(K(0) - G(u(\theta)) + T(\theta)\right) \geq \frac{1}{\beta}\int_{\underline{\theta}}^{\theta} u(x)dx + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \lambda\underline{w}.$$

In the second step, I find optimal transfers:

$$\max_T P(T), \tag{A.11}$$

subject to $\mathbb{E}[T(\theta)] \leq 0$.

Consider the Lagrangian associated to problem (A.10):

$$\mathcal{L} = \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} (1 - M(\theta))u(\theta)d\theta + \frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \lambda\underline{w}$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{1}{\beta}\theta u(\theta) + \lambda W\left(K(0) - G(u(\theta)) + T(\theta)\right) - \int_{\underline{\theta}}^{\theta} \frac{1}{\beta}u(x)dx - \frac{\underline{\theta}}{\beta}u(\underline{\theta}) - \lambda\underline{w}\right) d\Lambda(\theta),$$

for some non-decreasing function $\Lambda(\cdot)$. Integrating by parts yields:

$$\begin{aligned}\mathcal{L} = &\frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} (\Lambda(\theta) - M(\theta))u(\theta)d\theta + \left(\frac{\underline{\theta}}{\beta}u(\underline{\theta}) + \lambda\underline{w}\right)\Lambda(\underline{\theta}) \\ &+ \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{1}{\beta}\theta u(\theta) + \lambda W\left(K(0) - G(u(\theta)) + T(\theta)\right)\right) d\Lambda(\theta).\end{aligned}$$

Follows the arguments in Amador et al. (2006), we set $\Lambda(\underline{\theta}) = 0$, $\Lambda(\theta) = M(\theta)$, for for $\theta \leq \theta^\star$ and $\Lambda(\theta) = 1$, for $\theta > \theta^\star$. In particular, the first-order condition with respect to $u(\theta)$, for $\theta \leq \theta^\star$, gives:

$$\frac{1}{\beta}\theta - \lambda G'(u(\theta))W'\left(K(0) - G(u(\theta)) + T(\theta)\right) = 0.$$

Since $G(u) = \exp(u)(\gamma + \lambda)^{-1}$ and $W(x) = \log\left((\gamma + \lambda)x/\lambda K(0)\right)/\gamma$, the latter gives

$$u(\theta) = \log\left(k^{nt}(\theta)(K(0) + T(\theta))\right) \equiv U(T(\theta), \theta).$$

For $\theta > \theta^\star$, current utility and transfers must be constant by Lemma 1.4. As a result, $u(\theta) = U(T(\theta^\star), \theta^\star)$. Clearly, it must be that $T(\theta) > -K(0)$, for all $\theta \in \Theta$.

I now turn to problem (A.11). Using the results just derived, the objective function becomes:

$$\begin{aligned}P(T) &= \frac{1}{\beta}\int_{\underline{\theta}}^{\overline{\theta}} (1 - M(\theta))u(\theta)d\theta \\ &= \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^\star} (1 - M(\theta))u(\theta)d\theta + \frac{1}{\beta}u(\theta^\star)\int_{\theta^\star}^{\overline{\theta}} (1 - M(\theta))d\theta.\end{aligned}$$

By definition of $\theta^\star$ the last integral is zero, thus,

$$P(T) = \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^\star}(1-M(\theta))u(\theta)d\theta$$

or, integrating by parts,

$$\begin{aligned}P(T) &= \frac{1}{\beta}(1-M(\theta))\int_{\underline{\theta}}^{\theta}u(x)dx\Big|_{\underline{\theta}}^{\theta^\star} + \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^\star}m(\theta)\left(\int_{\underline{\theta}}^{\theta}u(x)dx\right)d\theta\\ &= \frac{1}{\beta}(1-M(\theta^\star))\int_{\underline{\theta}}^{\theta^\star}u(\theta)d\theta + \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^\star}m(\theta)\left(\int_{\underline{\theta}}^{\theta}u(x)dx\right)d\theta.\end{aligned}$$

Finally, combining the latter with the incentive constraint,

$$\begin{aligned}P(T) =& \frac{1}{\beta}(1-M(\theta^\star))\int_{\underline{\theta}}^{\theta^\star}u(\theta)d\theta\\ &+ \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^\star}m(\theta)\left[\frac{1}{\beta}\theta u(\theta) + \lambda W\left(K(0)-G(u(\theta))+T(\theta)\right) - \frac{\underline{\theta}}{\beta}u(\underline{\theta}) - \lambda\underline{w}\right]d\theta,\end{aligned}$$

where, in addition, $u(\theta) = U(T(\theta),\theta)$. The Lagrangian associated to the second-step problem (A.11) is then

$$\begin{aligned}\mathcal{L} =& \frac{1}{\beta}(1-M(\theta^\star))\int_{\underline{\theta}}^{\theta^\star}u(\theta)d\theta\\ &+ \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^\star}m(\theta)\left[\frac{1}{\beta}\theta u(\theta) + \lambda W\left(K(0)-G(u(\theta))+T(\theta)\right) - \frac{\underline{\theta}}{\beta}u(\underline{\theta}) - \lambda\underline{w}\right]d\theta\\ &- \mu\mathbb{E}[T(\theta)],\end{aligned}$$

with $u(\theta) = U(T(\theta),\theta)$, where $\mu \geq 0$ is the Lagrange multiplier on the constraint $\mathbb{E}[T(\theta)] \leq 0$. The first-order condition with respect to $T(\theta)$ is

$$\frac{1}{\beta}(1-M(\theta^\star))\frac{1}{K(0)+T(\theta)} + \frac{1}{\beta}m(\theta)\frac{\theta}{\beta}\frac{1}{K(0)+T(\theta)} + \frac{1}{\beta}m(\theta)\frac{\lambda}{\gamma}\cdot\frac{1}{K(0)+T(\theta)} - \mu h(\theta) = 0.$$

Rearranging yields

$$\begin{aligned}K(0)+T(\theta) &= \frac{1}{\mu}\cdot\frac{1}{\beta}\cdot\frac{1}{h(\theta)}\left(1-M(\theta^\star)+m(\theta)\left(\frac{1}{\beta}\theta+\frac{\lambda}{\gamma}\right)\right).\\ &\equiv \frac{1}{\mu}\cdot\frac{1}{\beta}\varphi(\theta),\end{aligned}$$

where I have used the definition of $\varphi(\cdot)$ in Assumption 1.2. Therefore, using $u(\theta) = U(T(\theta), \theta)$ and the definition of $U(T(\theta), \theta)$,

$$\begin{aligned} u(\theta) &= \log\left(k^{nt}(\theta)(K(0) + T(\theta))\right) \\ &= \log\left(k^{nt}(\theta)\frac{1}{\mu}\cdot\frac{1}{\beta}\varphi(\theta)\right). \end{aligned}$$

Also, from the sub-problem ($\mathcal{P}_{TR:S1}$), we know that instantaneous spending is constant throughout the tenure of a government and equals $(\gamma+\lambda)G(u(\theta)) = \exp(u(\theta))$. To compute μ, we take the average of transfers and set it equal to zero:

$$K(0) + \mathbb{E}[T(\theta)] = \frac{1}{\mu}\cdot\frac{1}{\beta}\mathbb{E}\left[\varphi(\theta)\right],$$

hence, $\mu = \mathbb{E}\left[\varphi(\theta)\right]/(\beta K(0))$. The latter can be used to replace μ in the expression for $u(\theta)$. As a result, instantaneous government spending becomes:

$$\begin{aligned} \exp(u(\theta)) &= k^{nt}(\theta)\frac{1}{\mu}\cdot\frac{1}{\beta}\varphi(\theta) \\ &= k^{nt}(\theta)\frac{\varphi(\theta)}{\mathbb{E}\left[\varphi(\theta)\right]}K(0). \end{aligned} \tag{A.12}$$

Finally, transfers are given by

$$\begin{aligned} T(\theta) &= \frac{1}{\mu}\cdot\frac{1}{\beta}\varphi(\theta) - K(0) \\ &= \left(\frac{\varphi(\theta)}{\mathbb{E}\left[\varphi(\theta)\right]} - 1\right)K(0). \end{aligned} \tag{A.13}$$

I am left to verify the monotonicity constraint on u. Simple calculation gives

$$\begin{aligned} \frac{d}{d\theta}u(\theta) &= \frac{d}{d\theta}\log\left(k^{nt}(\theta)\frac{\varphi(\theta)}{\mathbb{E}\left[\varphi(\theta)\right]}K(0)\right) \\ &= \frac{1}{k^{nt}(\theta)}\frac{d}{d\theta}k^{nt}(\theta) + \frac{\varphi'(\theta)}{\varphi(\theta)} \\ &= \frac{1}{\theta}\cdot\frac{\lambda\beta}{\gamma\theta+\lambda\beta} + \frac{\varphi'(\theta)}{\varphi(\theta)}. \end{aligned}$$

By Assumption 1.2, the latter is either positive for all $\theta \leq \theta^\star$ or, if it becomes negative for some $\widetilde{\theta}$, then it will be negative for all $[\widetilde{\theta}, \theta^\star]$. In the former case, utility is non-decreasing, thus, the solution to the relaxed problem satisfies the monotonicity constraint. In particular, optimal government spending and transfers are, respectively, given by (A.12) and (A.13), for all $\theta \leq \theta^\star$, and are constant thereafter. In the latter case, instead, there must exist a threshold $\theta^{\star\star} < \theta^\star$ such the solution coincides with the one of the relaxed problem for all $\theta \leq \theta^{\star\star}$ and it is constant thereafter. In particular, optimal government spending and transfers are, respectively, given by (A.12) and (A.13), for all $\theta \leq \theta^{\star\star}$, and are constant thereafter. This also proves Proposition 1.3.

Finally, notice that government spending (A.12) and transfers (A.13) correspond to the case in which $v = 0$. By replacing $K(0)$ with $K(v)$, we obtain the their counterparts for any v.

Uniform Distribution. I now provide an explicit solution for the special case in which shocks are uniformly distributed. Let $H(\theta) = \theta/(\overline{\theta} - \underline{\theta})$, for $\theta \in [\underline{\theta}, \overline{\theta}]$. The equation $\beta\mathbb{E}\left[\theta|\theta \geq \theta^\star\right] = \theta^\star$ immediately implies the threshold:

$$\theta^\star = \frac{\overline{\theta}\beta}{2 - \beta}.$$

In addition, $M(\theta) \equiv H(\theta) + \theta(1-\beta)h(\theta) = (2-\beta)\theta/(\overline{\theta} - \underline{\theta})$, hence, $m(\theta) = (2 - \beta)/(\overline{\theta} - \underline{\theta})$. I can then compute the weights α in Proposition 1.2 explicitly. First,

$$\begin{aligned}\varphi(\theta) &\equiv \frac{1}{h(\theta)}\left(1 - M(\theta^\star) + m(\theta)\left(\frac{1}{\beta}\theta + \frac{\lambda}{\gamma}\right)\right)\\ &= \kappa_0 + (2-\beta)\frac{1}{\beta}\theta,\end{aligned}$$

where $\kappa_0 \equiv \overline{\theta}(1-\beta) - \underline{\theta} + (2-\beta)\lambda/\gamma$. It is immediate to see that $\varphi(\cdot)$ is increasing, thus, Assumption 1.2 is verified. By Proposition 1.3, $\theta^{\star\star} = \theta^\star$. Also,

$$\begin{aligned}\int_{\underline{\theta}}^{\theta^\star} \varphi(\theta)h(\theta)d\theta + \varphi(\theta^\star)\int_{\theta^\star}^{\overline{\theta}} h(\theta)d\theta &= \kappa_0 + (2-\beta)\frac{1}{\beta}\left[\int_{\underline{\theta}}^{\theta^\star} \theta h(\theta)d\theta + \theta^\star \int_{\theta^\star}^{\overline{\theta}} h(\theta)d\theta\right]\\ &= \kappa_0 + (2-\beta)\frac{1}{\beta}\kappa_1,\end{aligned}$$

where, using $\int_{\underline{\theta}}^{\theta^\star} \theta h(\theta) d\theta = 1 - \int_{\theta^\star}^{\overline{\theta}} \theta h(\theta) d\theta$,

$$\begin{aligned}
\kappa_1 &\equiv 1 - \frac{1}{\overline{\theta} - \underline{\theta}} \left[\frac{1}{2} \left((\overline{\theta})^2 - (\theta^\star)^2 \right) - \theta^\star \left(\overline{\theta} - \theta^\star \right) \right] \\
&= 1 - \frac{1}{2} \frac{(\overline{\theta} - \theta^\star)^2}{\overline{\theta} - \underline{\theta}} \\
&= 1 - \frac{2\overline{\theta}^2}{\overline{\theta} - \underline{\theta}} \left(\frac{1-\beta}{2-\beta} \right)^2,
\end{aligned}$$

where the last line uses the definition of $\theta^\star$. Therefore, for $\theta \leq \theta^\star$,

$$\begin{aligned}
\frac{T_v^{tr}(\theta)}{K(v)} &= \alpha(\theta) - 1 \\
&= \frac{\varphi(\theta)}{\int_{\underline{\theta}}^{\theta^\star} \varphi(\theta) h(\theta) d\theta + \varphi(\theta^{\star\star}) \int_{\theta^\star}^{\overline{\theta}} h(\theta) d\theta} - 1 \\
&= (2-\beta)\frac{1}{\beta} \cdot \frac{1}{\kappa_0 + (2-\beta)\frac{1}{\beta}\kappa_1} \left(\theta - 1 + \frac{2\overline{\theta}^2}{\overline{\theta} - \underline{\theta}} \left(\frac{1-\beta}{2-\beta} \right)^2 \right)
\end{aligned}$$

and the decomposition in the main text follows by letting

$$\overline{T}_0(\lambda, \beta) \equiv (2-\beta)\frac{1}{\beta} \cdot \frac{1}{\kappa_0 + (2-\beta)\frac{1}{\beta}\kappa_1}$$

and

$$\overline{T}_1(\beta) \equiv \frac{2\overline{\theta}^2}{\overline{\theta} - \underline{\theta}} \left(\frac{1-\beta}{2-\beta} \right)^2.$$

Finally, since κ_0 is increasing in λ while κ_1 is independent of λ, it follows immediately that $\partial \overline{T}_0(\lambda, \beta)/\partial\lambda < 0$, as claimed in the main text.

A.1.8 Welfare Gains

To compare welfare in the transfer and no transfer scenario we *(i)* compute welfare in the no-transfer case, for given resource $\kappa y/r$, namely $v^{nt}(\kappa y/r)$ *(ii)* compute the amount of resources necessary to deliver the same utility in the transfer case $K(v^{nt})$, define welfare gain as the difference between $K(v^{nt})$ and $\kappa y/r$ divided by GDP.

In the no transfer problem, we know that the planner's value function is

$$v^{nt}(a) = \bar{A} + \frac{1}{\gamma} \log(a),$$

where

$$\bar{A} = \frac{1}{\gamma}\mathbb{E}\left[\theta\log(k^{nt}(\theta)) - k^{nt}(\theta)\frac{\gamma\theta+\lambda}{\gamma(\gamma+\lambda)} + 1\right],$$

and we set initial amount of resources to $a = \kappa y/r$. Further, we know that in the transfer case $K(v^{nt}) = K(0)\exp(\gamma\bar{A})(\kappa y/r)$, where

$$K(0) = \exp\left(-\mathbb{E}\Big[\theta\log(k^{nt}(\theta)\alpha(\theta)) + \lambda\log\left((\gamma+\lambda-k^{nt}(\theta))\frac{\alpha(\theta)}{\lambda}\right)\Big]\right).$$

Now define welfare gains as the difference in resources needed to make the planner indifferent between making transfers or not, as a proportion of the endowment:

$$\Psi(\beta) \equiv \left(\frac{\kappa y}{r} - K(v^{nt})\right)\frac{1}{y} = \left(1 - K(0)\exp(\gamma\bar{A})\right)\frac{\kappa}{r}.$$

Welfare gains clearly depend on all the parameters of the model, but I make explicit the dependency on β to emphasize the following. Define tha function $\psi(\beta)$ as:

$$\begin{aligned}\psi(\beta) &\equiv K(0)\exp(\gamma\bar{A})\\ &= \exp\left(-\mathbb{E}\left[\theta\log(\alpha(\theta)) + \lambda\log\left((\gamma+\lambda-k^{nt}(\theta))\frac{\alpha(\theta)}{\lambda}\right) + k^{nt}(\theta)\frac{\gamma\theta+\lambda}{\gamma(\gamma+\lambda)} - 1\right]\right),\end{aligned}$$

then welfare gains are

$$\Psi(\beta) = (1-\psi(\beta))\frac{\kappa}{r}.$$

Notice that when everybody is constrained (for β very low), weights and consumption share are constant, namely $\alpha(\theta) = 1$; $k^{nt}(\theta) = \gamma\ \forall\theta$. Substituting those values in the expression for welfare gains we get that $\psi(\beta) = 1$, namely there is no gain in setting-up a transfer system when the political friction is extreme. When $\beta = 1$, instead, we have $\alpha(\theta) = (\gamma\theta+\lambda)(\gamma+\lambda)$ and $k^{nt}(\theta) = \gamma\theta(\gamma+\lambda)(\gamma\theta+\lambda)^{-1}$, meaning that $\psi(1) = \exp(-\mathbb{E}[\theta\log(\alpha(\theta))])$, which is grater than one by Jensen's inequality. In other terms, welfare gains are positive when there is no political friction and vanish when governments' exclusively care about their own consumption.

I provide a heuristic derivation of the HJB system (1.12), (1.13). Let us start with the equation for the value function Υ in (1.13). This function represents the value of a sequence of spending, after the government has been dissolved, generated by policy function $g^\star(a,\theta)$ which prescribes government spending as a function of current wealth a and type θ. Formally, take any time $t_0 < \tau_0$ (the case with τ_n, $n > 0$, is analogous) and suppose country's wealth is $a_{t_0} = a$ and current government's type is $\theta_0 = \theta$. Then,

$$\Upsilon(a,\theta) = \mathbb{E}_{t_0}\left[\sum_{n=0}^{\infty} e^{-\gamma\tau_n}\theta_n \int_{\tau_n}^{\tau_{n+1}} e^{-\gamma(t-\tau_n)}\log(g^\star)dt\right],$$

where wealth evolves according to the process $\dot{a}_t = ra_t - g^\star(a_t,\theta_t)$.

At time t_0, the government enjoys flow utility of $\theta\log(g^\star(a,\theta))$. Moreover, only two things can happen in the next instant of time t_0+dt. First, with probability $e^{-\lambda dt}$, the type θ remains unchanged and wealth increases by a deterministic amount da. When this occurs, the expected discounted payoff becomes $e^{-\gamma dt}\Upsilon(a+da,\theta)$. Second, with probability $(1-e^{-\lambda dt})$, a new taste shock θ will be drawn from the distribution $H(\theta)$. When this occurs, the expected discounted payoff becomes $\mathbb{E}[e^{-\gamma dt}\Upsilon(a+da,\widetilde{\theta})]$. Putting the pieces together, and weighting them by their respective probabilities, I obtain:

$$\Upsilon(a,\theta) = \theta\log(g^\star(a,\theta))dt + e^{-\lambda dt}e^{-\gamma dt}\Upsilon(a+da,\theta) + (1-e^{-\lambda dt})\mathbb{E}\left[e^{-\gamma dt}\Upsilon(a+da,\widetilde{\theta})\right].$$

Using the approximations,

$$\begin{aligned} e^{-\gamma dt} &= 1-\gamma dt + O(dt^2),\\ e^{-\lambda dt} &= 1-\lambda dt + O(dt^2), \end{aligned}$$

and ignoring higher-order terms, the expression above can be rewritten as:

$$\Upsilon(a,\theta) = \theta\log(g^\star(a,\theta))dt + (1-\lambda dt-\gamma dt)\Upsilon(a+da,\theta) + \lambda dt\mathbb{E}\left[\Upsilon(a+da,\widetilde{\theta})\right].$$

Since changes in wealth are deterministic, the term $\Upsilon(a+da,\theta)$ is simply $\Upsilon(a,\theta) + \Upsilon_a(a,\theta)da$, where Υ_a is the derivative of the value function with respect to its first argument, and $da = (ra - g^\star(a,\theta))dt$. As a result,

$$\begin{aligned} \Upsilon(a,\theta) =& \theta\log(g^\star(a,\theta))dt + (1-\lambda dt-\gamma dt)\left[\Upsilon(a,\theta) + \Upsilon_a(a,\theta)(ra-g^\star(a,\theta))dt\right]\\ &+ \lambda dt\mathbb{E}\left[\Upsilon(a,\widetilde{\theta}) + \Upsilon_a(a,\widetilde{\theta})(ra-g^\star(a,\widetilde{\theta}))dt\right]. \end{aligned}$$

Ignoring second-order terms, subtracting $\Upsilon(a,\theta)$ from both sides, and dividing through by dt (letting $dt \longrightarrow 0$) yields:

$$(\lambda+\gamma)\Upsilon(a,\theta) = \theta\log(g^\star(a,\theta)) + \Upsilon_a(a,\theta)(ra-g^\star(a,\theta)) + \lambda\mathbb{E}\left[\Upsilon(a,\widetilde{\theta})\right],$$

which is (1.13).

The proof for (1.12) follows analogous arguments. There are two differences though. First, in this case the policy function $g^\star(a,\theta)$ is not taken as given, but chosen by the government in charge, hence, the maximization operator in (1.12). In particular, the maximization problem is subject to the fiscal rule (1.14). Second, when the shock causing a type change occurs, the government is dissolved. As a result, this event is discounted by $\beta e^{-\gamma dt}$, instead of the standard discount $e^{-\gamma dt}$ and, in addition, the value function switches from J to Υ.

To find a solution to the HJB system (1.12), (1.13), I guess and verify that the value functions take the following form:

$$\Upsilon(a,\theta) = \overline{\Upsilon}(\theta,\theta^\star) + A(\theta)\log(a),$$
$$J(a,\theta) = \overline{J}(\theta,\theta^\star) + B(\theta)\log(a),$$

for some functions $\overline{\Upsilon}$, A, $\overline{J}$ and B. The first-order condition of the maximization problem in (1.12) is then

$$g^\star(a,\theta) = \frac{\theta}{J_a(a,\theta)} = \frac{\theta}{B(\theta)}a,$$

for $\theta < \theta^\star$ and simply $g^\star(a,\theta) = k^{nt}(\theta^\star)a$, otherwise. Substituting $g^\star(a,\theta)$, together with the conjectures above, into (1.12) and rearranging gives:

$$\begin{aligned}(\gamma+\lambda)&\left(\overline{J}(\theta,\theta^\star) + B(\theta)\log(a)\right)\\ &= \theta\log\left(\frac{\theta}{B(\theta)}a\right) + B(\theta)\left(r - \frac{\theta}{B(\theta)}\right) + \lambda\beta\mathbb{E}\left[\overline{\Upsilon}(\widetilde{\theta},\theta^\star) + A(\widetilde{\theta})\log(a)\right],\end{aligned}$$

for $\theta < \theta^\star$ and an analogous expression for $\theta \geq \theta^\star$. Similarly, equation (1.12) becomes:

$$\begin{aligned}(\gamma+\lambda)&\left(\overline{\Upsilon}(\theta,\theta^\star) + A(\theta)\log(a)\right)\\ &= \theta\log\left(\frac{\theta}{B(\theta)}a\right) + A(\theta)\left(r - \frac{\theta}{B(\theta)}\right) + \lambda\mathbb{E}\left[\overline{\Upsilon}(\widetilde{\theta},\theta^\star) + A(\widetilde{\theta})\log(a)\right],\end{aligned}$$

for $\theta < \theta^\star$, and an analogous expression for $\theta \geq \theta^\star$.

Consider the case with $\theta < \theta^\star$. Equalizing terms multiplying $\log(a)$, the second equation immediately gives

$$(\gamma+\lambda)A(\theta) = \theta + \lambda\mathbb{E}\left[A(\widetilde{\theta})\right],$$

hence, taking expectations of both sides,

$$\mathbb{E}\left[A(\widetilde{\theta})\right] = \frac{1}{\gamma}$$

and

$$A(\theta) = \frac{\gamma\theta + \lambda}{\gamma(\gamma+\lambda)}.$$

Using the latter in the first equation and equalizing terms multiplying $\log(a)$, I obtain:

$$(\gamma+\lambda)B(\theta) = \theta + \lambda\beta\frac{1}{\gamma}$$

and, thus,

$$B(\theta) = \frac{\gamma\theta + \lambda\beta}{\gamma(\gamma+\lambda)}.$$

Spending is, therefore,

$$g^{\star}(a,\theta) = \frac{\theta}{B(\theta)}a = k^{nt}(\theta)a.$$

The latter, together with the law of motion for wealth, generates the optimal spending in Proposition 1.1. Finally, the functions $\overline{\Upsilon}(\theta,\theta^{\star})$, $\overline{J}(\theta,\theta^{\star})$ can be obtained by solving the system of equations

$$(\gamma+\lambda)\overline{J}(\theta,\theta^{\star}) = \theta\log\left(\frac{\theta}{B(\theta)}\right) + B(\theta)\left(r - \frac{\theta}{B(\theta)}\right) + \lambda\beta\mathbb{E}\left[\overline{\Upsilon}(\widetilde{\theta},\theta^{\star})\right],$$

$$(\gamma+\lambda)\overline{\Upsilon}(\theta,\theta^{\star}) = \theta\log\left(\frac{\theta}{B(\theta)}\right) + A(\theta)\left(r - \frac{\theta}{B(\theta)}\right) + \lambda\mathbb{E}\left[\overline{\Upsilon}(\widetilde{\theta},\theta^{\star})\right],$$

for $\theta < \theta^{\star}$, and analogous expressions for $\theta \geq \theta^{\star}$. The latter also verify our original conjecture.

Transfers Consider now the implementation with transfers. The derivation of the HJB equations is analogous to the no-transfer case, so I will not repeat it here. There are two main differences. First, at the time a new government is formed, the country's assets change discontinuously due to the payment for the credit line and the new transfer. Formally, assets now evolve according to

$$dx_t = (rx_t + \kappa y + b_t - g_t)dt + \left(-\lambda^{-1}b_{t^-} + \chi(\widetilde{\theta}_t, a'_{t^-})\right)dN_t,$$

where N_t is the jump process and a'_{t^-} is wealth at the time of dissolution net of the payment $-\lambda^{-1}b_{t^-}$. In particular, notice that, when the Poisson jump occurs, assets x_t jump by the amount $-\lambda^{-1}b_{t^-}+\chi(\widetilde{\theta}_t, a'_{t^-})$. Second, now the government in charge has two choice variables, spending and credit-line drawdown.

Using (1.15), after the government is dissolved and a new government with type $\widetilde{\theta}$ is formed, the country's wealth becomes $a_t = \alpha(\widetilde{\theta})\left(a_{t^-} - \frac{1}{\lambda}b(\theta, a_{t^-})\right)$. As a result, the system of HJB equations is

$$(\lambda+\gamma)J(\theta,a) = \max_{g,b}\{\theta\log(g(\theta,a)) + J_a(\theta,a)(ra+b(\theta,a)-g(\theta,a))\} \\ + \beta\lambda\mathbb{E}\left[\Upsilon\left(\widetilde{\theta}, \alpha(\widetilde{\theta})\left(a - \frac{1}{\lambda}b(\theta,a)\right)\right)\right],$$

$$(\lambda+\gamma)\Upsilon(\theta,a) = \theta\log(g^\star(\theta,a)) + \Upsilon_a(\theta,a)(ra+b^\star(\theta,a)-g^\star(\theta,a)) \\ + \lambda\mathbb{E}\left[\Upsilon\left(\widetilde{\theta}, \alpha(\widetilde{\theta})\left(a - \frac{1}{\lambda}b^\star(\theta,a)\right)\right)\right],$$

where the maximization problem in the first equation is subject to the constraint $g \le k^{tr}(\theta^{\star\star})a$. As for the no-transfer case, I conjecture

$$\Upsilon(a,\theta) = \overline{\Upsilon}(\theta,\theta^\star) + A(\theta)\log(a),$$
$$J(a,\theta) = \overline{J}(\theta,\theta^\star) + B(\theta)\log(a),$$

for some functions $\overline{\Upsilon}$, A, $\overline{J}$ and B. The first-order condition with respect to g is exactly the same as in the no-transfer case. Also, the same arguments for the no-transfer case yield $\mathbb{E}[A(\widetilde{\theta})] = 1/\gamma$ and $B(\theta) = (\gamma\theta+\lambda\beta)\gamma^{-1}(\gamma+\lambda)^{-1}$. As a result, the first-order condition with respect to b is

$$\frac{\gamma\theta+\lambda\beta}{\gamma(\gamma+\lambda)}\cdot\frac{1}{a} - \beta\lambda\frac{1}{\gamma}\cdot\frac{\frac{1}{\lambda}}{a-\frac{1}{\lambda}b^\star(\theta,a)} = 0.$$

Straightforward algebra gives:

$$b^\star(\theta,a) = \gamma\lambda\frac{\theta-\beta}{\gamma\theta+\lambda\beta}a \equiv \bar{b}(\theta)a.$$

Finally, by replacing the expressions for $g^\star(\theta,a)$ and $b^\star(\theta,a)$ into the HJB equations, I obtain two equations that can be used to solve for $\overline{\Upsilon}(\theta,\theta^\star)$, $\overline{J}(\theta,\theta^\star)$, thus verifying our original conjecture.

Appendix B

Appendix to Chapter 2

B.1 Proofs

B.1.1 Recursive Formulation

We begin with a heuristic derivation of the HJB equations, which must be satisfied by the problem's solution. Let $w(\hat{\theta}, \hat{a})$ be lifetime expected utility of a government with current type $\hat{\theta}$ and current wealth $\hat{a}$. It is given by

$$w(\hat{\theta}, \hat{a}) = \max_{g} \mathbb{E}\left[e^{-\gamma(s-t)} \int_{t}^{\tau} \theta_s \ln(g_s) ds + \beta e^{-\gamma(\tau-t)} v(\theta_\tau, a_\tau)\right],$$

subject to $\dot{a} = ra - g$, with $\theta_t = \hat{\theta}$, $a_t = \hat{a}$, where τ is the stochastic time of government change and $v(\hat{\theta}, \hat{a})$ is the continuation value, that is, lifetime expected utility from the date in which the current government is replaced. This function is given by

$$v(\hat{\theta}, \hat{a}) = \mathbb{E}\left[\int_{t}^{\infty} e^{-\gamma(s-t)} \theta_s \ln(g^\star_s) ds\right],$$

subject to $\dot{a} = ra - g^\star$, with $\theta_t = \hat{\theta}$, $a_t = \hat{a}$, where $g^\star$ is the optimal policy chosen by the following governments.

Notice that, by standard arguments, value functions will be independent of time. In addition, when a fiscal rule is present, the spending choice must be subject to the further constraint that $g(\theta_t, a_t) \leq g(\theta^\star, a_t)$.

Consider now a small interval of time dt. Conditional on a Poisson shock, there are three possibles events: a change of preferences alone—with probability p_θ—a change of government alone—with probability p_β—and a change of both preferences and government—with probability $p_{\beta\theta} = 1 - p_\theta - p_\beta$. Then, by standard properties of Poisson processes, unconditional probabilities in the interval of time dt are as follows:

- a change of preferences alone occurs with probability $1 - e^{-\lambda p_\theta dt}$;
- a change of government alone occurs with probability $1 - e^{-\lambda p_\beta dt}$;
- a change of both preferences and government occurs with probability $1 - e^{-\lambda p_{\beta\theta} dt}$;
- no even occurs with probability $e^{-\lambda dt}$.

Therefore,

$$w(\hat{\theta}, \hat{a}) = \max_{g \text{ s.t. } \dot{a} = ra - g} \theta \ln(g) dt + e^{-\gamma dt} \Big[(1 - e^{-\lambda p_\theta dt}) \mathbb{E}_\theta [w(\theta, \hat{a})] + (1 - e^{-\lambda p_\beta dt}) \beta v(\hat{\theta}, \hat{a}) + (1 - e^{-\lambda p_{\beta\theta} dt}) \beta \mathbb{E}_\theta [v(\theta, \hat{a})] + e^{-\lambda dt} w(\hat{\theta}, \hat{a} + da) \Big].$$

Using the approximation $e^{-\varphi dt} \simeq 1 - \varphi dt$, for some scalar φ, and disregarding terms in $(dt)^2$, we can rewrite the latter expression as:

$$w(\hat{\theta}, \hat{a}) = \max_{g \text{ s.t. } \dot{a} = ra - g} \theta \ln(g) dt + (\lambda p_\theta dt) \mathbb{E}_\theta [w(\theta, \hat{a})] + (\lambda p_\beta dt) \beta v(\hat{\theta}, \hat{a}) + (\lambda p_{\beta\theta} dt) \beta \mathbb{E}_\theta [v(\theta, \hat{a})] + (1 - (\gamma + \lambda) dt) w(\hat{\theta}, \hat{a} + da).$$

In addition, since $w(\hat{\theta}, \hat{a} + da) \simeq w(\hat{\theta}, \hat{a}) + w_a(\hat{\theta}, \hat{a}) da = w(\hat{\theta}, \hat{a}) + w_a(\hat{\theta}, \hat{a})(ra - g) dt$, the latter becomes

$$((\gamma + \lambda) dt) w(\hat{\theta}, \hat{a}) = \max_{g \text{ s.t. } \dot{a} = ra - g} \theta \ln(g) dt + (\lambda p_\theta dt) \mathbb{E}_\theta [w(\theta, \hat{a})] + (\lambda p_\beta dt) \beta v(\hat{\theta}, \hat{a}) + (\lambda p_{\beta\theta} dt) \beta \mathbb{E}_\theta [v(\theta, \hat{a})] + w_a(\hat{\theta}, \hat{a})(ra - g) dt.$$

We conclude that the value function w must satisfy the following HJB equation:

$$(\gamma + \lambda) w(\hat{\theta}, \hat{a}) = \max_{g \text{ s.t. } \dot{a} = ra - g} \theta \ln(g) + w_a(\hat{\theta}, \hat{a})(ra - g) + \lambda \big(p_\theta \mathbb{E}_\theta [w(\theta, \hat{a})] + p_\beta \beta v(\hat{\theta}, \hat{a}) + p_{\beta\theta} \beta \mathbb{E}_\theta [v(\theta, \hat{a})] \big).$$

Analogous steps imply that the continuation value function $v(\hat{\theta}, \hat{a})$ must satisfy the following HJB equation:

$$(\gamma + \lambda) v(\hat{\theta}, \hat{a}) = \theta \ln(g^\star) + v_a(\hat{\theta}, \hat{a})(ra - g) + \lambda \big(p_\theta \mathbb{E}_\theta [v(\theta, \hat{a})] + p_\beta v(\hat{\theta}, \hat{a}) + p_{\beta\theta} \mathbb{E}_\theta [v(\theta, \hat{a})] \big).$$

B.1.2 Proof of Proposition 2.1

The HJB equations are:

$$\gamma w(\theta) = \max_{g \text{ s.t. } \dot{a}=ra-g} \left\{ \theta \ln(g) + w_a(\theta)(ra-g) \right\} + \lambda \bigg(p_\theta \bar{w} + p_\beta \, \beta v(\theta) + p_{\beta\theta} \beta \bar{v} - w(\theta) \bigg),$$

$$\gamma v(\theta) = \theta \ln(g^\star) + v_a(\theta)(ra-g) + \lambda \bigg(p_\theta \bar{v} + p_\beta v(\theta) + p_{\beta\theta} \bar{v} - v(\theta) \bigg),$$

where, to simplify notation, we omitted explicit dependence on a and used bars to denote averages over types.

If fiscal rules are present, then spending must be such that $g(\theta, a) \leq g(\theta^\star, a)$, for all types and levels of wealth. Below, we show that policy functions are increasing in θ. It follows that optimal spending for unconstrained and constrained types will satisfy:

$$g^\star = \begin{cases} g_u^\star = \theta / w_a(\theta) \text{ for } \theta \leq \theta^\star \\ g_c^\star = \theta^\star / w_a(\theta^\star) \text{ for } \theta > \theta^\star. \end{cases} \tag{B.1}$$

To find a solution to the HJB equations, we guess and later verify a specific functional form for the value functions. In particular, we guess $v = A(\theta, \theta^\star) + B(\theta) \ln(a)$ and $w = D(\theta, \theta^\star) + F(\theta) \ln(a)$. Notice that our guesses are such that only the constant term depends on the threshold rule.

Using our guesses, $w_a(\theta) = F(\theta) a^{-1}$, therefore, policy functions are $g_u^\star(\theta) = \theta a / F(\theta)$ and $g_c^\star(\theta) = \theta^\star a / F(\theta^\star)$. To simplify exposition, let us define $C(\theta) \equiv \theta / F(\theta)$, so that $g_u^\star(\theta) = C(\theta) a$ and $g_c^\star(\theta) = C(\theta^\star) a$.

Continuation value function

Let us indicate with v_u the continuation value function for $\theta \leq \theta^\star$ (unconstrained), and with v_c the continuation value function for $\theta > \theta^\star$ (constrained). As a result, $\bar{v} = \int_{\underline{\theta}}^{\theta^\star} v_u + \int_{\theta^\star}^{\bar{\theta}} v_c$.

To verify our conjectures, we plug them into the second HJB equation:

$$\overbrace{(\gamma+\lambda) A_u(\theta, \theta^\star) + (\gamma+\lambda) B(\theta) \ln(a)}^{(\gamma+\lambda) v_u(\theta)} = \overbrace{\theta \ln(C(\theta)) + \theta \ln(a)}^{\theta \ln(g_u^\star)} + \overbrace{B(\theta)(r - C(\theta))}^{v_a(\theta)(ra - g_u^\star)}$$
$$+ \lambda \Bigg(p_\beta \underbrace{(A_u(\theta, \theta^\star) + B(\theta) \ln(a))}_{v_u(\theta)} + \underbrace{(p_\theta + p_{\beta\theta})}_{=(1-p_\beta)} \underbrace{\left(\int_{\underline{\theta}}^{\theta^\star} A_u(\theta, \theta^\star) + \int_{\theta^\star}^{\bar{\theta}} A_c(\theta, \theta^\star) + \bar{B} \ln(a) \right)}_{\bar{v}} \Bigg)$$

for the unconstrained, and

$$\overbrace{(\gamma+\lambda)A_c(\theta,\theta^\star)+(\gamma+\lambda)B(\theta)\ln(a)}^{(\gamma+\lambda)v_c(\theta)} = \overbrace{\theta\ln(C(\theta^\star))+\theta\ln(a)}^{\theta\ln(g_c^\star)} + \overbrace{B(\theta)(r-C(\theta^\star))}^{v_a(\theta)(ra-g_c^\star)}$$
$$+\lambda\Bigg(\ p_\beta\underbrace{\left(A_c(\theta,\theta^\star)+B(\theta)\ln(a)\right)}_{v_c(\theta)} + \underbrace{(p_\theta+p_{\beta\theta})}_{=(1-p_\beta)}\underbrace{\left(\int_{\underline{\theta}}^{\theta^\star}A_u(\theta,\theta^\star)+\int_{\theta^\star}^{\bar\theta}A_c(\theta,\theta^\star)+\bar B\ln(a)\right)}_{\bar v}\ \Bigg)$$

for the constrained.

Terms in a: Notice that the terms in $\ln(a)$ are the same for constrained and unconstrained types:

$$(\gamma+\lambda)B(\theta)\ln(a) = \left(\theta+\lambda p_\beta B(\theta)+\lambda(1-p_\beta)\bar B\right)\ln(a).$$

Taking the mean of both sides, we have $\bar B = 1/\gamma$, which can then be used into the above equation to obtain

$$B(\theta) = \frac{\gamma\theta+\lambda(1-p_\beta)}{\gamma(\gamma+\lambda(1-p_\beta))}. \tag{B.2}$$

Intercept: The terms that are independent of a for the unconstrained and constrained types are, respectively,

$$\begin{aligned}(\gamma+\lambda)A_u(\theta,\theta^\star) =& \theta\ln(C(\theta))+B(\theta)(r-C(\theta))+\\ &\lambda\left[\ p_\beta A_u(\theta,\theta^\star)+(1-p_\beta)\left(\int_{\underline{\theta}}^{\theta^\star}A_u(\theta,\theta^\star)+\int_{\theta^\star}^{\bar\theta}A_c(\theta,\theta^\star)\right)\ \right],\end{aligned}$$

and

$$\begin{aligned}(\gamma+\lambda)A_c(\theta,\theta^\star) =& \theta\ln(C(\theta^\star))+B(\theta)(r-C(\theta^\star))+\\ &\lambda\left[\ p_\beta A_c(\theta,\theta^\star)+(1-p_\beta)\left(\int_{\underline{\theta}}^{\theta^\star}A_u(\theta,\theta^\star)+\int_{\theta^\star}^{\bar\theta}A_c(\theta,\theta^\star)\right)\ \right].\end{aligned}$$

Taking the average yields:

$$\begin{aligned}(\gamma+\lambda)\left[\int_{\underline{\theta}}^{\theta^\star}A_u(\theta,\theta^\star)+\int_{\theta^\star}^{\bar\theta}A_c(\theta,\theta^\star)\right] &= \int_{\underline{\theta}}^{\theta^\star}\Big(\theta\ln(C(\theta))+B(\theta)(r-C(\theta))\Big)\\ &+\int_{\theta^\star}^{\bar\theta}\Big(\theta\ln(C(\theta^\star))+B(\theta)(r-C(\theta^\star))\Big)+\lambda\left(\int_{\underline{\theta}}^{\theta^\star}A_u(\theta,\theta^\star)+\int_{\theta^\star}^{\bar\theta}A_c(\theta,\theta^\star)\right).\end{aligned}$$

Also, by letting $\bar{A}(\theta^\star) = \left(\int_{\underline{\theta}}^{\theta^\star} A_u(\theta, \theta^\star) + \int_{\theta^\star}^{\bar{\theta}} A_c(\theta, \theta^\star) \right)$, we obtain:

$$\gamma \bar{A} = \int_{\underline{\theta}}^{\theta^\star} \Big(\theta \ln(C(\theta)) + B(\theta)(r - C(\theta)) \Big) + \int_{\theta^\star}^{\bar{\theta}} \Big(\theta \ln(C(\theta^\star)) + B(\theta)(r - C(\theta^\star)) \Big).$$

Finally, we can substitute the latter back to get both the unconstrained intercept

$$A_u(\theta, \theta^\star) = \frac{1}{(\gamma + \lambda(1 - p_\beta))} \theta \ln(C(\theta)) + B(\theta)(r - C(\theta)) \tag{B.3}$$

$$+ \frac{\lambda(1 - p_\beta)}{\gamma(\gamma + \lambda(1 - p_\beta))} \left[\int_{\underline{\theta}}^{\theta^\star} \Big(\theta \ln(C(\theta)) + B(\theta)(r - C(\theta)) \Big) + \int_{\theta^\star}^{\bar{\theta}} \Big(\theta \ln(C(\theta^\star)) + B(\theta)(r - C(\theta^\star)) \Big) \right],$$

and the constrained one

$$A_c(\theta, \theta^\star) = \frac{1}{(\gamma + \lambda(1 - p_\beta))} \theta \ln(C(\theta^\star)) + B(\theta)(r - C(\theta^\star)) \tag{B.4}$$

$$+ \frac{\lambda(1 - p_\beta)}{\gamma(\gamma + \lambda(1 - p_\beta))} \left[\int_{\underline{\theta}}^{\theta^\star} \Big(\theta \ln(C(\theta)) + B(\theta)(r - C(\theta)) \Big) + \int_{\theta^\star}^{\bar{\theta}} \Big(\theta \ln(C(\theta^\star)) + B(\theta)(r - C(\theta^\star)) \Big) \right].$$

Value function

We proceed in the same way we verify w. Substituting our guesses into the first HJB equation gives:

$$\begin{aligned}
&\overbrace{(\gamma + \lambda) D_u(\theta, \theta^\star) + (\gamma + \lambda) F(\theta) \ln(a)}^{(\gamma+\lambda) w_u(\theta)} = \overbrace{\theta \ln(C(\theta)) + \theta \ln(a)}^{\theta \ln(g_u^\star)} + \overbrace{F(\theta)(r - C(\theta))}^{w_a(\theta)(ra - g_u^\star)} \\
&+ \lambda \Bigg(p_\theta \underbrace{\left(\int_{\underline{\theta}}^{\theta^\star} D_u(\theta, \theta^\star) + \int_{\theta^\star}^{\bar{\theta}} D_c(\theta, \theta^\star) + \bar{F} \ln(a) \right)}_{\bar{w}} + p_\beta \beta \underbrace{(A_u(\theta, \theta^\star) + B(\theta) \ln(a))}_{v_u(\theta)} \\
&+ p_{\beta\theta} \beta \underbrace{\left(\int_{\underline{\theta}}^{\theta^\star} A_u(\theta, \theta^\star) + \int_{\theta^\star}^{\bar{\theta}} A_c(\theta, \theta^\star) + \bar{B} \ln(a) \right)}_{\bar{v}} \Bigg),
\end{aligned}$$

for the unconstrained, and

$$\overbrace{(\gamma+\lambda)D_c(\theta,\theta^\star)+(\gamma+\lambda)F(\theta)\ln(a)}^{(\gamma+\lambda)w_c(\theta)} = \overbrace{\theta\ln(C(\theta^\star))+\theta\ln(a)}^{\theta\ln(g_c^\star)} + \overbrace{F(\theta)(r-C(\theta^\star))}^{w_a(\theta)(ra-g_c^\star)}$$
$$+\lambda\Bigg(p_\theta \underbrace{\left(\int_{\underline{\theta}}^{\theta^\star} D_u(\theta,\theta^\star)+\int_{\theta^\star}^{\bar{\theta}} D_c(\theta,\theta^\star)+\bar{F}\ln(a)\right)}_{\bar{w}} + p_\beta\beta \underbrace{\left(A_c(\theta,\theta^\star)+B(\theta)\ln(a)\right)}_{v_c(\theta)}$$
$$+ p_{\beta\theta}\beta \underbrace{\left(\int_{\underline{\theta}}^{\theta^\star} A_u(\theta,\theta^\star)+\int_{\theta^\star}^{\bar{\theta}} A_c(\theta,\theta^\star)+\bar{B}\ln(a)\right)}_{\bar{v}} \Bigg)$$

for the constrained.

Terms in a: Notice that the terms in $\ln(a)$ are the same for constrained and unconstrained types:

$$(\gamma+\lambda)F(\theta) = \theta + \lambda p_\theta \bar{F} + \lambda\beta p_\beta B(\theta) + \lambda\beta p_{\beta\theta}\bar{B}. \tag{B.5}$$

Taking the mean and using $B=\gamma^{-1}$, $(p_\beta+p_{\beta\theta})=(1-p_\theta)$ gives:

$$\bar{F} = \frac{\gamma+\lambda\beta(1-p_\theta)}{\gamma(\gamma+\lambda(1-p_\theta))}.$$

Substituting $\bar{F}$ back into equation (B.5) and remembering that $p_{\beta\theta}=(1-p_\theta-p_\beta)$ gives:

$$(\gamma+\lambda)F(\theta) = \theta + \frac{\lambda p_\theta}{\gamma}\frac{\gamma+\lambda\beta-\lambda\beta p_\theta}{(\gamma+\lambda-\lambda p_\theta)} + \frac{\lambda\beta p_\beta}{\gamma}\frac{\gamma\theta+\lambda-\lambda p_\beta}{(\gamma+\lambda-\lambda p_\beta)} + \frac{\lambda\beta(1-p_\theta-p_\beta)}{\gamma}.$$

Using $C(\theta)=\theta/F(\theta)$, simple steps of algebra imply that:

$$C(\theta) = \frac{\gamma\theta(\gamma+\lambda)}{\gamma\theta+\lambda\beta\ +\ \lambda(1-\beta)\frac{\gamma p_\theta}{\gamma+\lambda-\lambda p_\theta}\ +\ \lambda\beta(\theta-1)\frac{\gamma p_\beta}{\gamma+\lambda-\lambda p_\beta}}.$$

To simplify the latter expression further, we define $\Lambda_\beta \equiv \gamma p_\beta/(\gamma+\lambda(1-p_\beta))$ and $\Lambda_\theta \equiv \gamma p_\theta/(\gamma+\lambda(1-p_\theta))$. We thus obtain:

$$C(\theta) = \frac{\gamma\theta(\gamma+\lambda)}{\theta(\gamma+\beta\lambda\Lambda_\beta)+\beta\lambda(1-\Lambda_\beta)+(1-\beta)\lambda\Lambda_\theta}. \tag{B.6}$$

Intercept: The same steps as those for the continuation value function deliver an expression for the intercepts D_u, D_c, thus, verifying our guess. We do not report these derivations here since the exact expressions of such intercepts are not relevant for the analysis.

B.1.3 Proof of Proposition 2.2

The uncoordinated planner chooses the threshold $\theta^\star$ so as to maximize the country's ex-ante welfare:

$$\int_x \left(\int_{\underline{\theta}}^{\theta^\star} v_u(\theta,\theta^\star)h(\theta)d\theta + \int_{\theta^\star}^{\bar{\theta}} v_c(\theta,\theta^\star)h(\theta)d\theta \right) m(x)dx. \tag{B.7}$$

Remember that the value function takes the form $v_i(\theta,\theta^\star) = A_i(\theta,\theta^\star) + B(\theta)\ln(a)$, where $i = \{u, c\}$. Namely, only the intercepts depend on $\theta^\star$. We can thus disregard the dependency on x ant the problem is equivalent to maximizing

$$\int_{\underline{\theta}}^{\theta^\star} A_u(\theta,\theta^\star)h(\theta)d\theta + \int_{\theta^\star}^{\bar{\theta}} A_c(\theta,\theta^\star)h(\theta)d\theta.$$

Let us define Z_u, Z_c as:

$$Z_u(\theta) \equiv \theta\ln(C(\theta)) + B(\theta)(r - C(\theta)),$$

$$Z_c(\theta,\theta^\star) \equiv \theta\ln(C(\theta^\star)) + B(\theta)(r - C(\theta^\star)).$$

Using (B.3) and (B.4), we can then rewrite A_u, A_c as:

$$A_u(\theta,\theta^\star) = \frac{Z_u(\theta)}{\gamma+\lambda(1-p_\beta)} + \frac{\lambda(1-p_\beta)}{\gamma(\gamma+\lambda(1-p_\beta))}\left(\int_{\underline{\theta}}^{\theta^\star} Z_u(\theta)h(\theta)d\theta + \int_{\theta^\star}^{\bar{\theta}} Z_c(\theta,\theta^\star)h(\theta)d\theta \right), \tag{B.8}$$

$$A_c(\theta,\theta^\star) = \frac{Z_c(\theta,\theta^\star)}{\gamma+\lambda(1-p_\beta)} + \frac{\lambda(1-p_\beta)}{\gamma(\gamma+\lambda(1-p_\beta))}\left(\int_{\underline{\theta}}^{\theta^\star} Z_u(\theta)h(\theta)d\theta + \int_{\theta^\star}^{\bar{\theta}} Z_c(\theta,\theta^\star)h(\theta)d\theta \right). \tag{B.9}$$

Further, notice that at $\theta^\star$, $A_u(\theta^\star,\theta^\star) = A_c(\theta^\star,\theta^\star)$ (and $Z_u(\theta^\star) = Z_c(\theta^\star,\theta^\star)$). As a result, the optimal threshold must satisfy the first-order condition

$$\int_{\underline{\theta}}^{\theta^\star} \frac{\partial}{\partial\theta^\star} A_u(\theta,\theta^\star)h(\theta)d\theta + \int_{\theta^\star}^{\bar{\theta}} \frac{\partial}{\partial\theta^\star} A_c(\theta,\theta^\star)h(\theta)d\theta = 0,$$

with

$$\frac{\partial}{\partial\theta^\star} A_u(\theta,\theta^\star) = \frac{\lambda(1-p_\beta)}{\gamma(\gamma+\lambda(1-p_\beta))} \int_{\theta^\star}^{\bar{\theta}} \frac{\partial}{\partial\theta^\star} Z_c(\theta,\theta^\star)h(\theta)d\theta,$$

$$\frac{\partial}{\partial\theta^\star} A_c(\theta,\theta^\star) = \frac{\partial}{\partial\theta^\star}\frac{Z_c(\theta,\theta^\star)}{\gamma+\lambda(1-p_\beta)} + \frac{\lambda(1-p_\beta)}{\gamma(\gamma+\lambda(1-p_\beta))} \int_{\theta^\star}^{\bar{\theta}} \frac{\partial}{\partial\theta^\star} Z_c(\theta,\theta^\star)h(\theta)d\theta,$$

and

$$\frac{\partial}{\partial\theta^\star} Z_c(\theta,\theta^\star) = \Big(\theta - B(\theta)C(\theta^\star)\Big)\frac{C'(\theta^\star)}{C(\theta^\star)}.$$

Since $\int \frac{\partial}{\partial\theta^\star} Z_c(\theta,\theta^\star)h(\theta)d\theta$ is independent of θ, we can further rewrite the first-order condition as:

$$\frac{1}{\gamma+\lambda(1-p_\beta)}\int_{\theta^\star}^{\bar\theta}\frac{\partial}{\partial\theta^\star} Z_c(\theta,\theta^\star)h(\theta)d\theta + \frac{\lambda(1-p_\beta)}{\gamma(\gamma+\lambda(1-p_\beta))}\int_{\theta^\star}^{\bar\theta}\frac{\partial}{\partial\theta^\star} Z_c(\theta,\theta^\star)h(\theta)d\theta = 0.$$

The latter implies that the optimal threshold must be such that

$$\int_{\theta^\star}^{\bar\theta}\frac{\partial}{\partial\theta^\star} Z_c(\theta,\theta^\star)h(\theta)d\theta = 0.$$

Finally, if we substitute $\partial Z_c/\partial\theta^\star$ into the above condition, we obtain the following first-order condition for the planner's problem:

$$\int_{\theta^\star}^{\bar\theta}\Big(\theta - B(\theta)C(\theta^\star)\Big)h(\theta)d\theta \;\; \frac{C'(\theta^\star)}{\gamma C(\theta^\star)} = 0. \tag{B.10}$$

Since $C'(\theta^\star)$ is non-zero, we are left to study $\int_{\theta^\star}^{\bar\theta}(\theta - B(\theta)C(\theta^\star))\,h(\theta)d\theta = 0$. Remember that $B(\theta)$ and $C(\theta)$ are given by (B.2) and (B.6), respectively. Consider first the product $B(\theta)C(\theta^\star)$:

$$B(\theta)C(\theta^\star) = \frac{\frac{\theta^\star(\gamma+\lambda)(\gamma\theta+\lambda-\lambda p_\beta)}{\gamma+\lambda-\lambda p_\beta}}{(\gamma\theta^\star+\lambda\beta)\;+\;\lambda(1-\beta)\Lambda_\theta\;+\;\lambda\beta(\theta^\star-1)\Lambda_\beta}.$$

We now take the common denominator for $\theta - B(\theta)C(\theta^\star)$ and focus on the numerator of the resulting expression (we can disregard the denominator because we are looking for a zero of $\theta - B(\theta)C(\theta^\star)$):

$$\int_{\theta^\star}^{\bar\theta}\left(\theta\big[\gamma\theta^\star+\lambda\beta+\lambda(1-\beta)\Lambda_\theta+\lambda\beta(\theta^\star-1)\Lambda_\beta\big]-\frac{\theta^\star(\gamma+\lambda)(\gamma\theta+\lambda-\lambda p_\beta)}{(\gamma+\lambda-\lambda p_\beta)}\right)h(\theta)d\theta = 0.$$

Let us group together the terms that depend on θ and move all the remaining terms to the right-hand side:

$$\begin{aligned}\int_{\theta^\star}^{\bar\theta}\left(\gamma\theta^\star+\lambda\beta+\lambda(1-\beta)\Lambda_\theta+\lambda\beta(\theta^\star-1)\Lambda_\beta-\frac{\gamma\theta^\star(\gamma+\lambda)}{(\gamma+\lambda-\lambda p_\beta)}\right)\theta h(\theta)d\theta \\ = \int_{\theta^\star}^{\bar\theta}\left(\frac{\theta^\star(\gamma+\lambda)(\lambda-\lambda p_\beta)}{(\gamma+\lambda-\lambda p_\beta)}\right)h(\theta)d\theta\end{aligned}$$

or

$$\lambda\Bigg(-\theta^\star\Lambda_\beta+\beta+(1-\beta)\Lambda_\theta+\beta(\theta^\star-1)\Lambda_\beta\Bigg)\int\limits_{\theta^\star}^{\bar\theta}\theta h(\theta)d\theta=\Bigg(\frac{(\gamma+\lambda)(\lambda-\lambda p_\beta)}{(\gamma+\lambda-\lambda p_\beta)}\Bigg)\theta^\star\int\limits_{\theta^\star}^{\bar\theta}h(\theta)d\theta.$$

Rearranging yields:

$$\underbrace{\frac{\int_{\theta^\star}^{\bar\theta}\theta h(\theta)d\theta}{\theta^\star\int_{\theta^\star}^{\bar\theta}h(\theta)d\theta}}_{\frac{\mathbb{E}[\theta|\theta\geq\theta^\star]}{\theta^\star}}=\Bigg(\underbrace{-\theta^\star\Lambda_\beta+\beta+(1-\beta)\Lambda_\theta+\beta(\theta^\star-1)\Lambda_\beta}_{-\theta^\star(1-\beta)\Lambda_\beta+\beta(1-\Lambda_\beta)+\Lambda_\theta(1-\beta)}\Bigg)^{-1}\underbrace{\Bigg(\frac{(\gamma+\lambda)(\lambda-\lambda p_\beta)}{\lambda(\gamma+\lambda-\lambda p_\beta)}\Bigg)}_{(1-\Lambda_\beta)}.$$

We conclude that the uncoordinated rule satisfies:

$$\frac{\mathbb{E}[\theta|\theta\geq\theta^\star]}{\theta^\star}=\frac{1-\Lambda_\beta}{\beta(1-\Lambda_\beta)+(1-\beta)(\Lambda_\theta-\theta^\star\Lambda_\beta)}. \tag{B.11}$$

B.1.4 Proof of Proposition 2.3

From the uncoordinated planner's problem, the value function v equals $A(\theta,\theta^\star,r)+B(\theta)\ln(x+y/r)$, where, with a slight abuse of notation, we have made explicit the dependence on r. In addition, the arguments in the text show that, in equilibrium, the interest rate is a function of the threshold, $r(\theta^\star)$, which satisfies the aggregate resource constraint:

$$\bar C(\theta^\star)\equiv\int_{\underline\theta}^{\theta^\star}C(\theta)d\theta+\int_{\theta^\star}^{\bar\theta}C(\theta)d\theta=r(\theta^\star).$$

When choosing the optimal threshold, the coordinated planner must then take into account the effect of $\theta^\star$ on r. The planner solves:

$$\max_{\theta^\star}\int_x\Bigg(\bar A(\theta^\star,r(\theta^\star))+\gamma^{-1}\ln(x+y/r(\theta^\star))\Bigg)m(x)dx,$$

where $\bar A(\theta^\star,r)\equiv\int_{\underline\theta}^{\theta^\star}A_u(\theta,\theta^\star,r)h(\theta)d\theta+\int_{\theta^\star}^{\bar\theta}A_c(\theta,\theta^\star,r)h(\theta)d\theta$ and where we used $B(\theta)=\gamma^{-1}$.

The first-order condition for the optimal threshold is then the sum of the direct derivative of the objective function—which is exactly the same as the one in the uncoordinated problem—plus the indirect derivative through interest rate. We consider this latter derivative. From (B.8) and (B.9), we have that $\partial A_u/\partial r=\partial A_c/\partial r=\gamma^{-2}$, thus,

$$\frac{\partial\bar A(\theta,\theta^\star,r)}{\partial r}=\gamma^{-2}.$$

The derivative of $\gamma^{-1}\ln(x+y/r)$ is simply $-y/[\gamma r(rx+y)]$. Adding them together, we obtain that the derivative of the objective function with respect to r is

$$\frac{1}{\gamma^2}-\frac{y}{r^2\gamma(x+y/r)}=\frac{1}{r\gamma^2}\left(\frac{r^2x+(r-\gamma)y}{rx+y}\right).$$

From the resource constraint, the derivative of r with respect to $\theta^\star$ is

$$r'(\theta^\star)=C'(\theta^\star)\int_{\theta^\star}^{\bar\theta}h(\theta)d\theta.$$

We conclude, therefore, that the indirect derivative of the objective function with respect to $\theta^\star$ is

$$\frac{C'(\theta^\star)\int_{\theta^\star}^{\bar\theta}h(\theta)d\theta}{r\gamma^2}\int_x\left(\frac{r^2x+(r-\gamma)y}{rx+y}\right)m(x)dx.$$

Since the direct derivative, from (B.10) was

$$\int_{\theta^\star}^{\bar\theta}\Big(\theta-B(\theta)C(\theta^\star)\Big)h(\theta)d\theta\ \ \frac{C'(\theta^\star)}{\gamma C(\theta^\star)}.$$

Combining direct and indirect derivatives, we obtain the first-order condition for the coordinated planner:

$$\int_{\theta^\star}^{\bar\theta}\Big(\theta-B(\theta)C(\theta^\star)\Big)h(\theta)d\theta\ \ \frac{1}{C(\theta^\star)}+\frac{\int_{\theta^\star}^{\bar\theta}h(\theta)d\theta}{r\gamma}\underbrace{\int_x\left(\frac{r^2x+(r-\gamma)y}{rx+y}\right)m(x)dx}_{\equiv M}=0. \quad \text{(B.12)}$$

From the uncoordinated planner's problem,

$$\theta-B(\theta)C(\theta^\star)=\lambda\frac{\theta\beta(1-\Lambda_\beta)+\theta(1-\beta)(\Lambda_\theta-\theta^\star\Lambda_\beta)-(1-\Lambda_\beta)\theta^\star}{\gamma\theta^\star+\lambda\beta\ +\ \Lambda_\theta(1-\beta)\ +\ \Lambda_\beta\beta(\theta^\star-1)}.$$

Also, from (B.6),

$$\frac{1}{C(\theta^\star)}=\frac{\theta^\star(\gamma+\beta\lambda\Lambda_\beta)+\beta\lambda(1-\Lambda_\beta)+(1-\beta)\lambda\Lambda_\theta}{\gamma\theta^\star(\gamma+\lambda)}.$$

Thus, the first-order condition (B.12) can be rewritten as:

$$\lambda\frac{\beta(1-\Lambda_\beta)+(1-\beta)(\Lambda_\theta-\theta^\star\Lambda_\beta)}{\gamma\theta^\star(\gamma+\lambda)}\int_{\theta^\star}^{\bar\theta}\theta h(\theta)d\theta=\frac{\lambda(1-\Lambda_\beta)}{\gamma\theta^\star(\gamma+\lambda)}\theta^\star\int_{\theta^\star}^{\bar\theta}h(\theta)d\theta-\frac{M}{r\gamma}\int_{\theta^\star}^{\bar\theta}h(\theta)d\theta.$$

Rearranging, we get:

$$\underbrace{\frac{\int_{\theta^\star}^{\bar{\theta}} \theta h(\theta)d\theta}{\theta^\star \int_{\theta^\star}^{\bar{\theta}} h(\theta)d\theta}}_{\frac{\mathbb{E}[\theta|\theta\geq\theta^\star]}{\theta^\star}} = \frac{1-\Lambda_\beta}{\beta(1-\Lambda_\beta)+(1-\beta)(\Lambda_\theta-\theta^\star\Lambda_\beta)} - \frac{(\gamma+\lambda)\lambda^{-1}}{\beta(1-\Lambda_\beta)+(1-\beta)(\Lambda_\theta-\theta^\star\Lambda_\beta)}M.$$

Using the definition of M, we conclude that the optimal coordinated threshold satisfies:

$$\begin{aligned}\frac{\mathbb{E}[\theta|\theta\geq\theta^\star]}{\theta^\star} - &\frac{1-\Lambda_\beta}{\beta(1-\Lambda_\beta)+(1-\beta)(\Lambda_\theta-\theta^\star\Lambda_\beta)} \\ &= \frac{\gamma(\gamma+\lambda)\lambda^{-1}}{\beta(1-\Lambda_\beta)+(1-\beta)(\Lambda_\theta-\theta^\star\Lambda_\beta)}\left(\frac{y}{r}\int_x \frac{1}{rx+y}m(x)dx - \frac{1}{\gamma}\right).\end{aligned}$$

Appendix C

Appendix to Chapter 3

C.1 Proofs

C.1.1 Sequential Representation

We formulate the problem in sequential representation. When not in default, the value functions $w^n(\theta, a)$ and $v^n(\theta, a)$ satisfy:

$$w^n(\theta, a) = \int_0^T e^{-\rho t}\theta u(g_t)\,dt + e^{-\rho T}\beta\mathbb{E}\left[v(\theta', a_t)\right],$$
$$v^n(\theta, a) = \int_0^T e^{-\rho t}\theta u(g_t)\,dt + e^{-\rho T}\mathbb{E}\left[v(\theta', a_t)\right],$$

where $\{g_t\}_{t\geq 0}$ denotes the path of spending plan for the agent at state (θ, a) at time $t = 0$ and T denotes the time at which political turnover occurs for the first time.

Adjusting for the Poisson rate of the turnover shock, the value functions transform into

$$w^n(\theta, a) = \int_0^\infty e^{-(\rho+\lambda)t}\left(\theta u(g_t) + \lambda\beta\mathbb{E}\left[v(\theta', a_t)\right]\right) dt, \tag{C.1}$$
$$v^n(\theta, a) = \int_0^\infty e^{-(\rho+\lambda)t}\left(\theta u(g_t) + \lambda\mathbb{E}\left[v(\theta', a_t)\right]\right) dt. \tag{C.2}$$

Lemma C.1 *Under the incentive-compatible spending plan $g(\theta, a)$,*

i) The marginal values of asset satisfy $w_a^n(\theta, a) < v_a^n(\theta, a)$.

ii) The cross-partial of the agent's value function is bounded by:

$$0 < w^n_{\theta a}(\theta, a) \leq \frac{1}{\theta} w^n_a(\theta, a), \forall \theta, a. \tag{C.3}$$

When $\beta > 0$, the inequality is strict: $w^n_{\theta a}(\theta, a) < \frac{1}{\theta} w^n_a(\theta, a)$.

Proof. To show item i), we differentiate equations (C.1) and (C.2) with respect to a:

$$w^n_a(\theta, a) = \int_0^\infty e^{-(\rho+\lambda)t} \left(\theta u'(g_t) \frac{\partial g_t}{\partial a} + \lambda \beta \mathbb{E} \left[v_a(\theta', a_t) \frac{\partial a_t}{\partial a} \right] \right) dt,$$
$$v^n_a(\theta, a) = \int_0^\infty e^{-(\rho+\lambda)t} \left(\theta u'(g_t) \frac{\partial g_t}{\partial a} + \lambda \mathbb{E} \left[v_a(\theta', a_t) \frac{\partial a_t}{\partial a} \right] \right) dt.$$

When the agent is endowed with more initial asset, the spending plan under a lower level of asset is feasible. However, we can improve the outcome by increasing spending at every point of time. Thus, it must be the case that $\frac{\partial g_t}{\partial a} > 0$. To afford higher future spending, it also must be that $\frac{\partial a_t}{\partial a} > 0$. Since $v_a(\theta, a) > 0$ and $\frac{\partial a_t}{\partial a} > 0$, it must be that $\mathbb{E}\left[v_a(\theta', a_t) \frac{\partial a_t}{\partial a}\right] > 0$. Thus we obtain that $w^n_a(\theta, a) < v^n_a(\theta, a)$.

For item ii), the Envelope condition for equation (C.1) implies that

$$w^n_\theta(\theta, a) = \int_0^\infty e^{-(\rho+\lambda)t} u(g_t)\, dt.$$

Note that the condition above holds under the incentive-compatible spending. It also holds under the spending in the Markov equilibrium. Further differentiating with respect to a:

$$w^n_{\theta a}(\theta, a) = \int_0^\infty e^{-(\rho+\lambda)t} u'(g_t) \frac{\partial g_t}{\partial a} dt.$$

Since $u'(g_t) > 0$ and $\frac{\partial g_t}{\partial a} > 0$, we obtain that $w^n_{\theta a}(\theta, a) > 0$. Given that $\mathbb{E}\left[v_a(\theta', a_t) \frac{\partial a_t}{\partial a}\right] > 0$, we have $w^n_{\theta a}(\theta, a) \leq \frac{1}{\theta} w^n_a(\theta, a)$. It only holds with equality when there is extreme present bias, i.e., $\beta = 0$. ■

Lemma C.1 holds under the incentive-compatible spending plan $g(\theta, a)$. It also holds under the spending in the Markov equilibrium.

C.1.2 Proof of Lemma 3.1

Differentiating the first-order condition (3.8) with respect to a:

$$\theta u''(g(\theta, a))\, g_a(\theta, a) = w_{aa}(\theta, a). \tag{C.4}$$

Differentiating the HJB equations (3.11) for $w^n(\theta, a)$ with respect to a:

$$\left(\rho+\lambda-\frac{\partial\left(r\left(a\right)a\right)}{\partial a}\right)w_a^n\left(\theta,a\right)=\left(\theta u'\left(g\left(\theta,a\right)\right)-w_a^n\left(\theta,a\right)\right)g_a\left(\theta,a\right)$$
$$+\dot{a}\left(\theta,a\right)w_{aa}^n\left(\theta,a\right)+\lambda\beta\mathbb{E}\left[v_a\left(\theta',a\right)\right]. \qquad \text{(C.5)}$$

Using the first-order condition (3.8) and equation (C.4) in the Envelope condition (C.5) and incorporating that $\dot{a}(\theta, a)\, g_a(\theta, a) = \dot{g}(\theta, a)$, we obtain:

$$\left(\rho+\lambda-\frac{\partial(r\left(a\right)a)}{\partial a}\right)\theta u'\left(g\left(\theta,a\right)\right)=\dot{g}\left(\theta,a\right)\theta u''\left(g\left(\theta,a\right)\right)+\lambda\beta\mathbb{E}\left[v_a\left(\theta',a\right)\right].$$

Reorganizing the last equation to obtain:

$$\frac{\dot{g}\left(\theta,a\right)}{g\left(\theta,a\right)}=\left(\rho+\lambda-\frac{\partial(r\left(a\right)a)}{\partial a}\right)\frac{u'\left(g\left(\theta,a\right)\right)}{g\left(\theta,a\right)u''\left(g\left(\theta,a\right)\right)}-\lambda\beta\frac{\mathbb{E}\left[v_a\left(\theta',a\right)\right]}{\theta g\left(\theta,a\right)u''\left(g\left(\theta,a\right)\right)}.$$

Consider a CRRA utility with risk aversion parameter denoted by γ. We have $\gamma = -\frac{gu''(g)}{u'(g)}$. Further, $\theta g(\theta, a)\, u''(g(\theta, a)) = -\gamma\theta u'(g(\theta, a)) = -\gamma w_a^n(\theta, a)$. We obtain equation (3.10). Since the cross-partial $w_{\theta a}^n(\theta, a) > 0$, $w_a^n(\theta, a)$ is strictly increasing in θ. Therefore the growth rate of spending $\frac{\dot{g}(\theta,a)}{g(\theta,a)}$ is decreasing in θ.

The effect due to the interest rate

$$\frac{\partial(r\left(a\right)a)}{\partial a}=r'\left(a\right)a+r\left(a\right)\geq r_f.$$

The larger β is, the larger the precautionary savings. When there is extreme present bias, i.e., $\beta = 0$, the growth rate of spending doesn't depend on type. As we show later that there is endogenous borrowing constraint, $\frac{\partial(r(a)a)}{\partial a} = r_f$, for $a > a^A$. All types would spend the same, and the path of spending grows at rate $\frac{1}{\gamma}(r_f - \rho - \lambda) < 0$.

C.1.3 Proof of Lemma 3.2

To understand how the agent's default threshold changes with type, we differentiate the indifference condition (3.9) with respect to θ and obtain:

$$\frac{\partial a^A\left(\theta\right)}{\partial\theta}=\frac{w_\theta^d\left(\theta\right)-w_\theta^n\left(\theta,a^A\left(\theta\right)\right)}{w_a^n\left(\theta,a^A\left(\theta\right)\right)}. \qquad \text{(C.6)}$$

Differentiating the HJB equations (3.11) and (3.12) for $w^n(\theta,a)$ and $w^d(\theta)$ with respect to θ:

$$(\rho+\lambda)\,w_\theta^n(\theta,a) = u(g(\theta,a)) + (\theta u'(g(\theta,a)) - w_a(\theta,a))\,g_\theta(\theta,a) + \dot{a}(\theta,a)\,w_{a\theta}^n(\theta,a)$$
$$(\rho+\lambda)\,w_\theta^d(\theta) = u(\kappa\tau) + \phi\left(w_\theta^n(\theta,0) - w_\theta^d(\theta)\right).$$

In the Markov equilibrium, the first-order condition (3.8) for spending holds. In the incentive-compatible allocation, the IC constraint (C.11) holds. In either case, the first of the two equations above reduces to

$$(\rho+\lambda)\,w_\theta^n(\theta,a) = u(g(\theta,a)) + \dot{a}(\theta,a)\,w_{a\theta}^n(\theta,a).$$

Substituting the expressions above in equation (C.6):

$$\frac{\partial a^A(\theta)}{\partial\theta} = \frac{\phi\left(w_\theta^n(\theta,0) - w_\theta^d(\theta)\right) - \left(u\left(g\left(\theta,a^A(\theta)\right)\right) - u(\kappa\tau) + \dot{a}\left(\theta,a^A(\theta)\right)w_{a\theta}^n\left(\theta,a^A(\theta)\right)\right)}{(\rho+\lambda)w_a^n\left(\theta,a^A(\theta)\right)}.$$

At the discretionary default threshold, the indifference condition (3.9) implies that

$$\begin{aligned}&\theta\left(u\left(g\left(\theta,a^A(\theta)\right)\right) - u(\kappa\tau)\right) + \dot{a}\left(\theta,a^A(\theta)\right)w_a^n\left(\theta,a^A(\theta)\right) \\ =\ & -\lambda\beta\left(\mathbb{E}\left[v\left(\theta',a^A(\theta)\right)\right] - \mathbb{E}\left[v^d(\theta')\right]\right) + \phi\left(w^n(\theta,0) - w^d(\theta)\right).\end{aligned} \tag{C.7}$$

Consider when the exclusion from financial market is permanent, i.e., $\phi = 0$. The difference in the expected continuation values:

$$\mathbb{E}[v(\theta,a)] - \mathbb{E}\left[v^d(\theta)\right] = \mathbb{E}\left[\left(v^n(\theta,a) - v^d(\theta)\right)(1-\delta(\theta,a))\right] \geq 0.$$

This is because, at the maximum debt level, all types default: $\delta\left(\theta',\underline{a}^A\right) = 1$, $\forall\theta$. Therefore, $\mathbb{E}\left[v\left(\theta,\underline{a}^A\right)\right] - \mathbb{E}\left[v^d(\theta)\right] = 0$. Since $\mathbb{E}[v(\theta,a)]$ is strictly increasing in a, we obtain that $\mathbb{E}[v(\theta,a)] - \mathbb{E}\left[v^d(\theta)\right] > 0$, $\forall a > \underline{a}^A$.

According to Lemma C.1, we have $w_{a\theta}^n(\theta,a) \leq \frac{1}{\theta}w_a^n(\theta,a)$. In addition, at the default threshold, $\dot{a}\left(\theta,a^A(\theta)\right) \geq 0$. Therefore,

$$\begin{aligned}&u\left(g\left(\theta,a^A(\theta)\right)\right) - u(\kappa\tau) + \dot{a}\left(\theta,a^A(\theta)\right)w_{a\theta}^n\left(\theta,a^A(\theta)\right) \\ \leq\ & u\left(g\left(\theta,a^A(\theta)\right)\right) - u(\kappa\tau) + \dot{a}\left(\theta,a^A(\theta)\right)\frac{1}{\theta}w_a^n\left(\theta,a^A(\theta)\right) \leq \phi\frac{1}{\theta}\left(w^n(\theta,0) - w^d(\theta)\right).\end{aligned}$$

Therefore, the default threshold is increasing in type: $\frac{\partial a^A(\theta)}{\partial\theta} \geq 0$.

When there exists savers, the lowest type $\underline{\theta}$ must be a saving type. Given that default threshold is increasing in type, we have that the debt lower bound $\underline{a}^A = a^A(\underline{\theta})$. Then $w_{a\theta}^n(\underline{\theta},a) < \frac{1}{\theta}w_a^n(\underline{\theta},a)$ and $\dot{a}\left(\underline{\theta},\underline{a}^A\right) > 0$. For the lowest type, the first inequality in the equation above is strict. For all other types, the second inequality is strict. Therefore the default threshold is strictly increasing everywhere: $\frac{\partial a^A(\theta)}{\partial\theta} > 0$, for all θ.

C.1.4 Proof of Lemma 3.3

Consider an economy with an endogenous borrowing limit, $a^A(\theta) = a^A$. The highest type $\bar{\theta}$ is always a dissaver at any level of asset, $\dot{a}(\bar{\theta}, a) \leq 0$. Therefore, at the borrowing limit, the highest type has zero net saving, $\dot{a}(\bar{\theta}, a^A) = 0$. According to equation (C.7), the borrowing limit satisfies

$$\bar{\theta}\left(u\left(r_f a^A + \tau\right) - u\left(\kappa\tau\right)\right) = \phi\left(w^n\left(\bar{\theta}, 0\right) - w^d\left(\bar{\theta}\right)\right),$$

which implies that

$$a^A = \frac{1}{r_f}\left(u^{-1}\left(u\left(\kappa\tau\right) + \phi\frac{w^n\left(\bar{\theta}, 0\right) - w^d\left(\bar{\theta}\right)}{\bar{\theta}}\right) - \tau\right). \tag{C.8}$$

The equilibrium interest rate is discontinuous at the borrowing limit,

$$r\left(a\right) = \begin{cases} r_f, & \text{if } a \geq a^A \\ \infty, & \text{if } a < a^A. \end{cases}$$

Now suppose there is a type that would like to default at a slightly lower asset level $a < a^A$. At that asset level, the interest it would be charged by the market is $r(a) = r_f + \lambda$. If it leads to an interest payment above the tax revenue, even if the government incurs zero spending, it would not be able to cover the interest payment:

$$-\left(r_f + \lambda\right) a > -\left(r_f + \lambda\right) a^A \geq \tau.$$

Thus it wouldn't be possible for any type to borrow more. When the tax revenue is just able to cover the interest payment, we obtain an expression for $\bar{\lambda}$:

$$\left(r_f + \bar{\lambda}\right)\frac{1}{r_f}\left(u^{-1}\left(u\left(\kappa\tau\right) + \phi\frac{w^n\left(\bar{\theta}, 0\right) - w^d\left(\bar{\theta}\right)}{\bar{\theta}}\right) - \tau\right) + \tau = 0.$$

When the financial exclusion is permanent, i.e., $\phi = 0$, the debt capacity is $a^A = \frac{1}{r_f}\left(\kappa - 1\right)\tau$. The threshold turnover rate has a simple analytical expression:

$$\lambda \geq \bar{\lambda} = \frac{r_f \kappa}{1 - \kappa}.$$

C.1.5 Proof of Proposition 3.1

Given a mechanism $\mathcal{M}$, the agent chooses the report x to maximize its value:

$$\max_x w\left(x, \theta, a; \mathcal{M}\right), \forall a, \theta.$$

Default rule. We start by examining the default rule $\delta(\theta, a)$. To do so, we fix a candidate spending rule $g(\theta, a)$.

First, we show that, in the partial default area, the only possible default rule that can induce truth-telling is one that allows for discretionary default, $\delta(\theta, a) = \delta^A(\theta, a)$. For a given level of asset a, we partition the set of reports into two subsets: the nondefault set and the default set,

$$\Theta^n(a) = \{\theta \in \Theta : \delta(\theta, a) = 0\} \quad \text{and} \quad \Theta^d(a) = \{\theta \in \Theta : \delta(\theta, a) = 1\}.$$

If the principal wishes to implement partial default, $\mathbb{E}(\delta(\theta, a)) \in (0, 1)$, the sets Θ^n and Θ^d are both nonempty. The agent will report in the nondefault set if and only if

$$\max_{x \in \Theta^n(a)} w^n(x, \theta, a; \mathcal{M}) > \max_{x \in \Theta^d(a)} w^d(x, \theta; \mathcal{M}).$$

The binary nature of the default choice makes it impossible for the principal to alter the agent's default behavior. The types that can obtain a higher value defaulting would pretend to be in the default set Θ^d. Likewise, the types that can obtain a higher value not defaulting would pretend be the type that maximizes its value in the nondefault set Θ^n.

Given we consider truth-telling mechanisms, if the agent is told not to default according to its report, its type must be in the nondefault set, $x = \theta \in \Theta^n(a)$. Imposing truth-telling, the IC constraint can be written as $\delta(\theta, a) = 0$ if and only if $w^n(\theta, a) \geq w^d(\theta)$. And it's only relevant in the partial default area.

Next, we show that, under the optimal rule, the default decision $\delta(\theta, a)$ must be decreasing in a. We set up the Lagrangian for the problem in (3.13). Let $\psi(\theta, a)$ be the Lagrange multiplier for the truth-telling constraint for type θ at asset a:

$$\mathcal{L} = \int_{\underline{\theta}}^{\bar{\theta}} \left(v^n(\theta, a)(1 - \delta(\theta, a)) + v^d(\theta)\,\delta(\theta, a) + \psi(\theta, a)\left(w^n(\theta, a) - w^d(\theta)\right)\right) dF(\theta).$$

If type θ does not default at asset a, in comparison to default, the Lagrangian changes by

$$\left[v^n(\theta, a) - v^d(\theta) + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta', a)}{\partial r(a)} \left(1 - \delta(\theta', a)\right) dF(\theta')\right] f(\theta) + \psi(\theta, a)\left(w^n(\theta, a) - w^d(\theta)\right). \tag{C.9}$$

Since the IC constraint is only relevant in the partial default area, $\psi(\theta, a) = 0$ in the zero default area and the full default area. In the expression in (C.9), the third term $\lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta',a)}{\partial r(a)} (1-\delta(\theta',a))\, dF(\theta')$ captures the interest-rate effect of default decision of θ on other types, which is why it is multiplied by the density $f(\theta)$. However, the value of this term is unaffected by the default choice for this specific type. Fixing the default choices for all other types, $\delta(\theta',a)$, $\forall\theta' \neq \theta \in \Theta$, $\forall a$, the default choice for type θ doesn't affect the interest rates $r(a)$. Therefore, the expression in (C.9) is strictly increasing in a. Thus we conclude that the optimal default decision $\delta(\theta,a)$ is decreasing in a. There exists a unique threshold $a^*(\theta)$ such that

$$\delta(\theta,a) = \begin{cases} 1, & \text{if } a < a^*(\theta) \\ 0, & \text{if } a \geq a^*(\theta). \end{cases}$$

This in turn implies that the default probability $\mathbb{E}[\delta(\theta,a)]$ is decreasing in asset, so is the interest rate $r(a)$.

Further, at the natural limit $-\frac{\tau}{r_f}$, the principal would always want to default. When the asset position is zero, the principal would never want to default. There exists a maximum debt level $\underline{a} > \frac{\tau}{r_f}$ such that beyond this debt level there is full default, $\delta(\theta,a) = 1$, $\forall\theta$, for all $a \leq \underline{a}$. There also exists a minimum debt level $\bar{a} \leq 0$ such that within this debt level there is zero default, $\delta(\theta,a) = 0$, $\forall\theta$, for all $a \geq \bar{a}$. The following relations hold: $-\frac{\tau}{r_f} < \underline{a} \leq \bar{a} \leq 0$. In the intermediate debt level, $\forall a \in (\underline{a}, \bar{a})$, the principal allows for discretionary default: $\delta(\theta,a) = \delta^A(\theta,a)$, $\forall\theta$.

Spending rule. In the nondefault area, the optimality condition with respect to the report:

$$w_x^n(x,\theta,a;\mathcal{M}) = 0, \forall a \geq a^*(\theta), \forall\theta.$$

Differentiating the optimal condition with respect to a, we obtain that $w_{ax}^n(x,\theta,a) = 0$. The condition above holds with strict equality. Otherwise if $\delta(x,a) = 1$,

$$w_x^d(x,\theta;\mathcal{M}) = \frac{\phi}{\rho+\phi+\lambda} w_x^n(x,\theta,0;\mathcal{M}) = 0.$$

When the asset position is zero, the agent will never default. If the agent reports truthfully when its asset position is zero, the agent will report truthfully when in default.

Without loss of generality, we restrict to spending $g(\theta,a)$ that is continuously differentiable almost everywhere. Differentiating the HJB equation (3.11) with respect to x, for $a \geq a^*(x)$,

$$(\rho+\lambda) w_x^n(x,\theta,a) = (\theta u'(g(x,a)) - w_a^n(x,\theta,a))\, g_x(x,a) + \dot{a}(x,a)\, w_{ax}^n(x,\theta,a). \tag{C.10}$$

There is no direct effect of $a^*(x)$ and any indirect effect is taken care of by $w^n_{ax}(x, \theta, a)$, which we have shown is zero:

$$(\rho + \lambda)\, w^n_x(x, \theta, a) = (\theta u'(g(x, a)) - w^n_a(x, \theta, a))\, g_x(x, a) = 0.$$

Imposing truth-telling, $x = \theta$, the IC constraint for spending in equation (C.10) becomes

$$(\theta u'(g(\theta, a)) - w^n_a(\theta, a))\, g_\theta(\theta, a) = 0. \tag{C.11}$$

We can see from equation (C.11) that there is limited scope for intervention: the first component is the same as without intervention, and the second term implies that intervention can only constrain changes in spending. Thus, same as in the economy without default, the optimal intervention either gives flexibility, allowing agents to spend at their discretion according to their first-order condition (3.8) or setting a rule where $g_\theta(\theta, a) = 0$ holds.

Before proceeding further, we show that the optimal spending is monotone.

Lemma C.2 *The optimal spending rule $g(\theta, a)$ is monotonically increasing in θ:*

$$g_\theta(\theta, a) \geq 0.$$

Proof. We only need to show that, in the discretionary spending area $g_\theta(\theta, a) \geq 0$, since in the area with rules $g_\theta(\theta, a) = 0$. In the discretionary spending area, the first-order condition (3.8) for spending holds. Differentiating it with respect θ:

$$u'(g(\theta, a)) + \theta u''(g(\theta, a))\, g_\theta(\theta, a) = w^n_{\theta a}(\theta, a). \tag{C.12}$$

Combining the expressions in equations (3.8) and (C.12), we obtain that

$$\theta u''(g(\theta, a))\, g_\theta(\theta, a) = w^n_{\theta a}(\theta, a) - \frac{1}{\theta} w^n_a(\theta, a). \tag{C.13}$$

According to Lemma C.1, the right-hand side of equation (C.13) is negative. Further, the utility function is strictly concave $u''(\cdot) < 0$. Thus it must be that spending is increasing in type $g_\theta(\theta, a) \geq 0$. ■

Given the monotonicity result for spending in Lemma C.2, it implies that incentive-compatible allocations features a spending threshold, denote by $\theta^s(a)$. All types with $\theta \leq \theta^s(a)$ have flexibility and all types $\theta > \theta^s(a)$ are bunched and spend the same as type $\theta^s(a)$.

C.1.6 Proof of Lemma 3.4

Lemma C.3 *For any threshold $\theta^s(a)$, in the area with rules, the marginal values of asset satisfy*

$$w_a^n(\theta, a) = \beta v_a^n\left(\frac{\theta}{\beta}, a\right), \ \forall \theta \geq \theta^s(a), \forall a. \tag{C.14}$$

The value functions are affine in θ:

$$v^n(\theta, a) = v_1(a) + \theta v_2(a),$$
$$w^n(\theta, a) = \beta v_1(a) + \theta v_2(a).$$

Proof. Differentiating the HJB equation (3.4) for $v^n(\theta, a)$ with respect to a:

$$\left(\rho + \lambda - \frac{\partial(r(a)a)}{\partial a}\right) v_a^n(\theta, a) = (\theta u'(g(\theta, a)) - v_a^n(\theta, a)) g_a(\theta, a) + \dot{a}(\theta, a) v_{aa}^n(\theta, a) + \lambda \mathbb{E}[v_a(\theta', a)]. \tag{C.15}$$

Consider a spending threshold θ^s. For a type $\theta > \theta^s$, its spending is bunched to $g(\theta^s, a)$ and its asset position evolves according to $\dot{a}(\theta^s, a)$. The same applies to type $\frac{\theta}{\beta}$. Evaluating equation (C.15) at $v_a^n\left(\frac{\theta}{\beta}, a\right)$ and multiplying both sides by β, comparing with equation (C.5), one can see that (C.14) holds.[1].

We guess and verify that the value functions are affine in θ. Substituting the guesses in the HJB equations and (3.11) and (3.4),we obtain the following ordinary-differential equations:

$$(\rho + \lambda) v_1(a) = \dot{a}(\theta^s, a) v_1'(a) + \lambda \mathbb{E}[v(\theta', a)],$$
$$(\rho + \lambda) v_2(a) = u(g(\theta^s, a)) v_1'(a) + \dot{a}(\theta^s, a) v_2'(a).$$

■

Consider an economy in which the default punishment is extreme $\kappa = 0$. The agents would never want to default and thus they can borrow up to the natural debt limit $\frac{\tau}{r_f}$. In problem (3.13), we only need to design the spending rule $g(\theta, a)$, or equivalently the spending threshold $\theta^s(a)$. Formally, the planner chooses θ^s to maximize

$$\mathbb{E}[v^n(\theta, a)] = \int_{\underline{\theta}}^{\theta^s} v^n(\theta, a)\, dF(\theta) + \int_{\theta^s}^{\bar{\theta}} v^n(\theta, a)\, dF(\theta).$$

[1] For $\theta > \beta\bar{\theta}$, the corresponding value of θ/β is outside the domain Θ. In this case, we can extend the v_a^n to be also defined in the range $(\bar{\theta}, \bar{\theta}/\beta]$, where the density is zero.

subject to equation (3.15).

The optimality condition with respect to θ^s:

$$\frac{\partial \mathbb{E}\left[v^n(\theta,a)\right]}{\partial \theta^s} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v^n(\theta,a)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\bar{\theta}} \frac{\partial v^n(\theta,a)}{\partial \theta^s} dF(\theta) = 0. \tag{C.16}$$

Since the optimality condition (C.16) holds for all a, the cross-partial $\frac{\partial \mathbb{E}\left[v_a^n(\theta,a)\right]}{\partial \theta^s} = 0$.

We first examine types $\theta \leq \theta^s$. According to the HJB equations, we have:

$$(\rho+\lambda)\frac{\partial w^n(\theta,a)}{\partial \theta^s} = \dot{a}(\theta,a)\frac{\partial w_a^n(\theta,a)}{\partial \theta^s} + \lambda\beta\frac{\partial \mathbb{E}\left[v_a(\theta,a)\right]}{\partial \theta^s},$$

$$(\rho+\lambda)\frac{\partial v^n(\theta,a)}{\partial \theta^s} = \left(\theta u'(g(\theta,a)) - v_a^n(\theta,a)\right)\frac{\partial g(\theta,a)}{\partial \theta^s} + \dot{a}(\theta,a)\frac{\partial v_a^n(\theta,a)}{\partial \theta^s} + \lambda\frac{\partial \mathbb{E}\left[v_a^n(\theta,a)\right]}{\partial \theta^s}.$$

We guess and verify that $\frac{\partial w^n(\theta,a)}{\partial \theta^s} = \frac{\partial w_a^n(\theta,a)}{\partial \theta^s} = 0$, which implies that $\frac{\partial g(\theta,a)}{\partial \theta^s} = 0$. We also guess and verify that $\frac{\partial v^n(\theta,a)}{\partial \theta^s} = \frac{\partial v_a^n(\theta,a)}{\partial \theta^s} = 0$. Therefore the optimality condition (C.16) simplifies to

$$\int_{\theta^s}^{\bar{\theta}} \frac{\partial v^n(\theta,a)}{\partial \theta^s} dF(\theta) = 0,$$

Now we examine types $\theta > \theta^s$, we have:

$$(\rho+\lambda)\frac{\partial v^n(\theta,a)}{\partial \theta^s} = \left(\theta u'(g(\theta^s,a)) - v_a^n(\theta,a)\right) g_\theta(\theta^s,a) + \dot{a}(\theta^s,a)\frac{\partial v_a^n(\theta,a)}{\partial \theta^s}.$$

The optimality condition turns into

$$g_\theta(\theta^s,a)\int_{\theta^s}^{\bar{\theta}} \left(\theta u'(g(\theta^s,a)) - v_a^n(\theta,a)\right) dF(\theta) + \dot{a}(\theta^s,a)\int_{\theta^s}^{\bar{\theta}} \frac{\partial v_a^n(\theta,a)}{\partial \theta^s} dF(\theta) = 0.$$

Since $g_\theta(\theta^s,a) > 0$ and the cross-partial $\int_{\theta^s}^{\bar{\theta}} \frac{\partial v_a^n(\theta,a)}{\partial \theta^s} dF(\theta) = 0$, we obtain:

$$\int_{\theta^s}^{\bar{\theta}} \left(\theta u'(g(\theta^s,a)) - v_a^n(\theta,a)\right) dF(\theta) = 0.$$

Assessing the marginal utility for spending at the threshold according to the first-order condition (3.8) and using the affine result in Lemma C.3, we have $\theta u'(g(\theta^s,a)) = \frac{\theta}{\theta^s} w_a^n(\theta^s,a) = \frac{\theta}{\theta^s}\beta v_1'(a) + \theta v_2'(a)$ and $v_a^n(\theta,a) = v_1'(a) + \theta v_2'(a)$. Replacing in the last equation, we have

$$\int_{\theta^s}^{\bar{\theta}} (\beta\theta - \theta^s)\, dF(\theta) = 0,$$

which can be rewritten as equation (3.17). It shows that the spending threshold is independent of a. Assumption 3.1 ensures that $\beta\mathbb{E}\left[\theta|\,\theta > \theta^s\right] - \theta^s$ is strictly decreasing in θ^s. Given that $\beta\bar{\theta} - \bar{\theta} \leq 0$, if $\beta > \underline{\theta}$, equation (3.17) has a unique root.

C.1.7 Proof of Proposition 3.2

Consider asset level a. If all types default, $\delta\ (\theta, a) = 1$, there is no spending rule to be made since all types are in autarky. If no type defaults , $\delta\ (\theta, a) = 0$.

Given a default rule $\delta\,(\theta, a)$, it's without loss of generality to consider the bunching threshold in the nondefault set, $\theta^s \in \Theta^n\,(a)$.

The nondefault value is unaffected by the spending threshold: $\frac{\partial v^d(\theta)}{\partial \theta^s} = 0$. Also,

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta, a)}{\partial \theta^s} \left(1 - \delta(\theta, a)\right) dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta, a)}{\partial r(a)} \left(1 - \delta(\theta, a)\right) dF(\theta) \frac{\partial r(a)}{\partial \theta^s} = 0. \tag{C.17}$$

Therefore,

$$\mathbb{E}\left[v(\theta, a)\right] = \int_{\underline{\theta}}^{\bar{\theta}} \left(v^n(\theta, a)\left(1 - \delta\,(\theta, a)\right) + v^d\,(\theta)\,\delta\,(\theta, a)\right) dF(\theta).$$

or

$$\mathbb{E}\left[v(\theta, a)\right] = \int_{\underline{\theta}}^{\theta^s} v^n(\theta, a) dF(\theta) + \int_{\theta^s}^{\bar{\theta}} v^d(\theta) dF(\theta).$$

C.1.8 Proof of Lemma 3.5

We build on the arguments in the proof for Proposition 3.1. When there is no information friction, the IC constraint for default becomes irrelevant everywhere. The expression in (C.9) immediately implies equation (3.18).

the lower and upper bounds $\underline{a}^P$ and $\bar{a}^P$. When $\phi = 0$, default decision $\delta\,(\theta, a)$ is monotonously increasing in θ. If the lowest type should default,

$$v^n(\underline{\theta}, \underline{a}^P) = v^d(\underline{\theta}) \tag{C.18}$$

If the highest type shouldn't default,

$$v^n(\bar{\theta}, \bar{a}^P) + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta, \bar{a}^P)}{\partial r(\bar{a}^P)} dF(\theta) = v^d(\bar{\theta}). \tag{C.19}$$

C.1.9 Proof of Proposition 3.3

For the planner, the borrowing limit it would like to implement is such that

$$\bar{\theta}\left(u\left(r_f a^* + \tau\right) - u\left(\kappa\tau\right)\right) = \phi\left(v^n\left(\bar{\theta}, 0\right) - v^d\left(\bar{\theta}\right)\right),$$

which implies that

$$a^* = \frac{1}{r_f}\left(u^{-1}\left(u\left(\kappa\tau\right) + \phi\frac{v^n\left(\bar{\theta},0\right) - v^d\left(\bar{\theta}\right)}{\bar{\theta}}\right) - \tau\right). \tag{C.20}$$

Taking the difference between the HJB equations for the nondefault values and the default values and evaluating at $a = 0$, we obtain that

$$\begin{aligned}
(\rho+\lambda+\phi)\left(w^n\left(\theta,0\right) - w^d\left(\theta\right)\right) &= \theta\left(u\left(g\left(\theta,0\right)\right) - u\left(\kappa\tau\right)\right) + \dot{a}\left(\theta,0\right) w_a^n\left(\theta,0\right) \\
&\quad + \lambda\beta\left(\mathbb{E}\left[v\left(\theta',0\right)\right] - \mathbb{E}\left[v^d\left(\theta'\right)\right]\right), \\
(\rho+\lambda+\phi)\left(v^n\left(\theta,0\right) - v^d\left(\theta\right)\right) &= \theta\left(u\left(g\left(\theta,0\right)\right) - u\left(\kappa\tau\right)\right) + \dot{a}\left(\theta,0\right) v_a^n\left(\theta,0\right) \\
&\quad + \lambda\left(\mathbb{E}\left[v\left(\theta',0\right)\right] - \mathbb{E}\left[v^d\left(\theta'\right)\right]\right).
\end{aligned}$$

Define an auxiliary expression $\tilde{v}^n\left(\theta,0\right) - \tilde{v}^d\left(\theta,0\right)$ that satisfies below:

$$\begin{aligned}
(\rho+\lambda+\phi)\left(\tilde{v}^n\left(\theta,0\right) - \tilde{v}^d\left(\theta\right)\right) &= \theta\left(u\left(g\left(\theta,0\right)\right) - u\left(\kappa\tau\right)\right) + \dot{a}\left(\theta,0\right) \tilde{v}_a^n\left(\theta,0\right) \\
&\quad + \lambda\beta\left(\mathbb{E}\left[v\left(\theta',0\right)\right] - \mathbb{E}\left[v^d\left(\theta'\right)\right]\right).
\end{aligned}$$

It immediately follows that

$$v^n\left(\theta,0\right) - v^d\left(\theta\right) \geq \tilde{v}^n\left(\theta,0\right) - \tilde{v}^d\left(\theta\right) = w^n\left(\theta,0\right) - w^d\left(\theta\right). \tag{C.21}$$

It is also obvious from the sequential representation why $v^n\left(\theta,0\right) - v^d\left(\theta\right) \geq w^n\left(\theta,0\right) - w^d\left(\theta\right)$: starting at zero asset position, given any sequence of spending, the streams of utility from spending before the taste shock arrives are identical, the only difference is that the government discounts the future utilities after the shock by a factor β. Hence $a^* \geq a^A$.

In the following cases, the default incentives between the agent and the principal are exactly aligned. When the financial exclusion is permanent, i.e., $\phi = 0$, the borrowing limits in equations (C.8) and (C.20) simplify to

$$a^* = a^A = \frac{1}{r_f}\left(\kappa - 1\right)\tau.$$

When there is zero cost of default, either because there is no revenue loss, i.e., $\kappa = 1$, or financial reaccess happens instantly, i.e., $\phi = \infty$, the borrowing limits in equations (C.8) and (C.20) becomes

$$a^* = a^A = 0.$$

However, in general, in the presence of present bias $\beta < 1$, there is a gap in the value financial reaccess for the agent and the planner, $v^n\left(\theta,0\right) - v^d\left(\theta\right) > w^n\left(\theta,0\right) - w^d\left(\theta\right)$. If $\kappa < 1$ and $0 < \phi < \infty$, the planner would prefer to strictly default earlier,

$$a^* > a^A.$$

C.1.10 Proof of Proposition 3.4

The highest type $\bar{\theta}$ is always dissaving. The question is whether the lowest type $\underline{\theta}$ is a saver or dissaver. Suppose that there is an endogenous borrowing limit. At the borrowing limit a^A, which is characterized in equation (C.8), all types would spend $r_f a^A + \tau$ and $\dot{a}\left(\theta, a^A\right) = 0$. The interest rate is right-differentiable: $\frac{\partial(r(a)a)}{\partial a} = -r_f$. The envelope condition becomes

$$w_a^n\left(\theta, a^A\right) = \frac{\lambda\beta}{\rho + \lambda - r_f}\mathbb{E}\left[v_a\left(\theta', a^A\right)\right], \forall\theta.$$

If condition (3.20) is satisfied, the spending of amount $r_f a^A + \tau$ is not optimal for the lowest type $\underline{\theta}$ at the borrowing constraint. Instead, it has incentive to deviate and save at least a little bit.

C.1.11 Computation of Interest Rate

To compute the interest rate with partial default we need the recovery value of the defaulted asset for each incumbent θ. However, conditional on repayment, the payment is independent of θ, the type only matters to determine when the incumbent would reaccess the financial markets, i.e., whether $\delta_d(\theta, b)$ is 0 or 1. Let $R^1(b)$ be the value of the asset that is currently on default and there is government for which $\delta_d(\theta, b) = 1$. The value of this asset satisfies the recursion:

$$(r_f + \lambda)R^1(b) = 0 + \lambda\mathbb{E}[R(b)].$$

This happens because the flow payment is zero and even if the government has the chance to return, it will not do it.

Let $R^0(b)$ be the value of the asset that is currently on default and there is government for which $\delta_d(\theta, b) = 0$. This government if it had the chance it would return to the financial markets. Thus, the value satisfies the recursion:

$$(r_f + \lambda + \phi)R^0(b) = 0 + \phi b + \lambda\mathbb{E}[R(b)].$$

Where $R(b) = \delta_d(\theta, b)R^1(b) + (1 - \delta_d(\theta, b))R^0(b)$ is the ex ante expected value of the asset, which satisfies:

$$\mathbb{E}[R(b)] = \mathbb{E}[\delta_d(\theta, b)]R^1(b) + (1 - \mathbb{E}[\delta_d(\theta, b)])R^0(b)$$

Combining all the equations we obtain:

$$\mathbb{E}[R(b)] = \mathbb{E}[\delta_d(\theta, b)]\frac{\lambda\mathbb{E}[R(b)]}{r_f + \lambda} + (1 - \mathbb{E}[\delta_d(\theta, b)])\frac{(\phi b + \lambda\mathbb{E}[R(b)])}{r_f + \lambda + \phi},$$

whose solution is

$$\mathbb{E}[R(b)] = \frac{b(1-\mathbb{E}[\delta_d(\theta,b)])\phi(\lambda+r_f)}{\lambda\phi(1-\mathbb{E}[\delta_d(\theta,b)])+r_f^2+\lambda r_f+r_f\phi}.$$

Which confirms that the recovery value is linear in the amount defaulted. Then we have:

$$R^1(b) = \frac{b(1-\mathbb{E}[\delta_d(\theta,b)])\lambda\phi}{\lambda\phi(1-\mathbb{E}[\delta_d(\theta,b)])+r_f(r_f+\lambda+\phi)} \tag{C.22}$$

The last equation makes clear that the recovery value in linear in b. Thus, setting $\alpha = 1$, so that the first appearance of b in (C.22) is replaced by a, but keeping the dependency of δ_d on b we obtain (3.26), where the first line is R^1/b and the second R^0/b.

C.2 Data

C.2.1 Data Sources

Data on debt is obtained from IMF's Global Debt Database. Government debt is measured using central government debt (percentage of GDP). The time series is available for Germany during year 1961-2018 and for Greece, Italy and Argentina during year 1950-2018. There is an alternative measure using general government debt (percentage of GDP). The two measures have small discrepancies and generate remarkably similar debt growth profiles. We use the growth paths before 1993, the year the Maastricht Treaty came into force. Table C.1 reports the average and standard deviation of debt grow, for the sub-periods pre-1993 and post-1993. Debt accumulation slowed down by about a half after Maastricht.

Data on government revenue and expenditures is obtained from the annual macroeconomic database of the European Commission's Directorate General for Economic and Financial Affairs (AMECO). Government revenue is defined as total general government revenue (percentage of GDP). We use the available series for Germany during year 1991-2019 and for Greece and Italy during year 1995-2019. As shown in Table C.1, the government sizes have remained stable.

Information on government duration is obtained from the Party Government Data Set (PGDS). In the first chapter, we use data on average government duration to calibrate the political turnover rate λ. In Table C.1, we report two measures of government duration. One captures only prime minister change. Another captures all changes, including changes in prime minister, party ideology, party name, and

Table C.1: Data Moments

	Germany	Greece	Italy	Argentina
Debt growth, % of GDP (pre-1993)				
Mean	0.5%	1.7%	1.7%	0.2%
Std dev	1.0%	3.4%	2.6%	15.4%
Debt growth, % of GDP (post-1993)				
Mean	0.6%	4.3%	1.2%	2.2%
Std dev	2.6%	9.6%	3.8%	23.6%
Government revenue, % of GDP				
Mean	44.9%	42.3%	45.5%	
Std dev	1.0%	4.3%	1.4%	
Government duration				
Mean (prime minister change)	7.53	1.29	1.61	
Mean (all changes)	1.81	0.93	0.92	

prime minister ideology. From these statistics we can see that Germany is of much lower political turnover than Greece and Italy regardless of the exact measurement.

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