Factor Models and Dynamic Stochastic General Equilibrium models: a forecasting evaluation

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Abstract

This paper aims to put dynamic stochastic general equilibrium (DSGE) forecasts in competition with factor models (FM) forecasts considering both static and dynamic factor models as well as regular and hybrid DSGE models. The empirical study shows three main conclusions. First, DSGE models are significantly outperformed by the generalized dynamic factor model (GDFM) in forecasting output growth in both short and long run, while the diffusion index (DI) model outperforms significantly DSGE models only in the short run. Second, the most surprising result of the paper, we discovered that only the hybrid DSGE model outperforms significantly all other competitive models in forecasting inflation in the long run. This evidence falls out with recent papers that found just regular DSGE models able to generate significant better forecasts for inflation in the long run as well as papers where hybrid DSGE models are found to forecast poorly. Third, in most cases, the unrestricted vector autoregressive (VAR) model represents the worse forecasting model. Although our results are consistent with the prevalent literature who gives to factor models the role to forecast output variables and to DSGE models the role to forecast monetary and financial variables, this research documents that exploiting more information on many macroeconomic time series, through hybrid DSGE models, is important not only to obtain more accurate estimates, but also to get significantly better forecasts.

Keywords: Diffusion Index (DI) model, Generalized Dynamic Factor Model (GDFM), Dynamic General Equilibrium (DSGE) model, Data-Rich DSGE (drDSGE) model, Equal Predictive Ability Tests.

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1 Introduction

Recent years have seen rapid growth in the availability of economic data. Statisticians, economists and econometricians now have easy access to data on many hundreds of variables that provide the information about the state of the economy. Coinciding with this growth in available data, two main new econometric models that exploit this wider information have been proposed: factor models and Dynamic Stochastic General Equilibrium (DSGE) models. Factor models have been successfully applied when we have to deal with: construction of economic indicators (Altissimo et al. (2010)), business cycle analysis (Gregory et al. (1997) and Inklaar et al. (2003)), forecasting (Stock and Watson (2002a,b) and Forni et al. (2000)), monetary policy (Bernanke and Boivin (2003) and Bernanke et al. (2005)), stock market returns (Ludvigson and Ng (2007)) and interest rates (Lippi and Thornton (2004)), while DSGE models have been successfully applied for: forecasting (Smets and Wouters (2002) and Smets and Wouters (2007)), estimation accurancy (Boivin and Giannoni (2006) and Kryshko (2009)), credit and banking (Gerali et al. (2008)), interest term of structure analysis (Amisano and Tristani (2010)) and monetary policy (Boivin and Giannoni (2008)).

Among all these applications, the recent economic global crisis has pointed out how forecasting well is central. For this reason, the main objective of the paper is to provide a detailed forecasting evaluation between these two econometric models taking into account of the recent developments in both factor and DSGE modelling. The novel of this study is the expanded range of forecasting models treated. Infact, our forecasting competition considers not only static factor models and regular DSGE models but also dynamic factor models, such as, the so-called Generalized Dynamic Factor Model (GDFM) of Forni et al. (2000) and hybrid DSGE models, such as, the so-called Data-Rich DSGE (drDSGE) following Boivin and Giannoni (2006) and Kryshko (2009). The paper is motivated by the fact that although there are some forecasting discussions on both dynamic factor model and regular DSGE individually, there is no attempt in the literature, to carry out a strong forecasting evaluation between dynamic factor model and hybrid DSGE models. In particular, what is missing is a forecasting comparison between the GDFM and the drDSGE.

The empirical study shows three main conclusions. First, DSGE models are significantly outperformed by the GDFM in forecasting output growth in both short and long run, while the static factor model outperforms significantly DSGE models only in the short run. Second, the most surprising result of the thesis, we discovered that only the drDSGE outperforms significantly all other competitive models in forecasting inflation in the long run. This evidence falls out with both Wang (2009) who found that a regular DSGE was able to generate significant better forecasts for inflation in the long run, and Paccagnini (2011) where hybrid models are found to forecast poorly. Therefore, the drDSGE outperforms significantly the regular DSGE in forecasting both output growth and inflation, confirming that exploiting more information on many macroeconomic time series, through the drDSGE, is important not only to obtain more accurate estimates, but also to get significant better forecasts. Third, in most cases, the unrestricted VAR represents the worse forecasting model.

This work is closely related with Wang (2009), but while we share some of the features of his study, our analysis is greatly expanded. First, we do not use the simple DSGE model of Del Negro and Schorfheide (2004) but the most elaborated DSGE model of Smets and Wouters (2007). Second, among factor models, we consider also the GDFM of Forni et al. (2000) whose forecasting performance is documented to be superior than the static factor model of Stock and Watson (2002a,b). Third, among DSGE models, we put side by side the regular DSGE model of Smets and Wouters (2007) with its representation in terms of drDSGE of Boivin and Giannoni (2006).

The remainder of the paper is organized as follows. The next section presents the forecasting models in competition. *Section (3)* describes the out-of-sample forecasting experiment as well as the estimation techniques and the test of equal predictive ability used. *Section (4)* discusses the forecasting results and *Section (5)* concludes.

2 Forecasting models

This section presents the forecasting models used in our out-of-sample forecasting experiment. We open presenting the autoregressive forecasting model, then we discuss how forecasts have been generated using the vector autoregressive model, the diffusion index model, the generalized dynamic factor model, the regular DSGE model and the Data-Rich DSGE model. We have considered also the unconditional mean of the series of interest as point forecast predictor, but being this case quite straightforward, we prefer discussing directly the other forecasting models.

2.1 Forecasting with the AR model

Let y_t be our observed stationary time series at time t. The most simple way to forecast y_t is assuming that it admits an **autoregressive process** of order p (hereafter AR(p)):

$$y_T = \alpha + \delta(L)y_T + \epsilon_T \tag{1}$$

where y_T denotes the time series of interest at the end of the estimation sample, α denotes the constant, $\delta(L) = 1 - \delta_1 L - \ldots - \delta_p L^p$ denotes the autoregressive lag polynominal of order p fixed using the Bayesian Information Criterion (BIC) that loads the past history of y_T, while ε_T is the stochastic error term. The **autoregressive forecasting model** becomes:

$$y^{AR}_{T+h} = \alpha + \delta_h(L)y_T + \epsilon_{T+h} \tag{2}$$

where $\delta_h(L) = 1 - \delta_1 L^{-h} - \ldots - \delta_p L^{p-h}$ denotes the autoregressive lag polynominal $\delta(L)$ shifted h-steps ahead, while ε_{T+h} denotes the stochastic error term shifted h-steps ahead. The AR forecasts has been generated by estimating the previous equation by OLS for each forecasting horizon. What we get is:

$$\hat{y}^{AR}_{T+h|T} = \hat{\alpha} + \hat{\delta}_h(L)y_T$$

where $\hat{y}^{AR}_{T+h|T}$ is the desidered point forecast predictor used in *Equation (27)*

2.2 Forecasting with the VAR model

Let $\mathbf{y}_{nt}$ be the n-dimensional vector of observed stationarity time series variables. If $\mathbf{y}_{nt}$ admits a **vector autoregressive process** of order p (hereafter VAR(p)), we have:

$$\mathbf{y}_{nT} = \sum_{j=1}^{p} \mathbf{A}_j \mathbf{y}_{n;T-j} + \boldsymbol{\epsilon}_T \qquad\qquad \boldsymbol{\epsilon}_T \;\sim\; iid\, \mathcal{N}(\mathbf{0}; \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}) \tag{3}$$

where y_{nT} denotes our observed time series variables at the end of the estimation sample, $\mathbf{A}_j$ are $(n \times n)$ matrices of parameters and $\boldsymbol{\epsilon}_T$ is the n-dimensional white noise process at the end of the estimation sample. Being our time series of interest into the set of observed time series variables, indeed $y_t \in \mathbf{y}_{nt}$, the **VAR forecasting model** is:

$$y_{T+h}^{VAR} = \alpha + \delta_h(L) y_T + \boldsymbol{\gamma}_h'(L)\tilde{\mathbf{y}}_T + \epsilon_{T+h} \tag{4}$$

where $\tilde{\mathbf{y}}_T$ denotes the vector of other observed time series variables in $\mathbf{y}_{nt}$ and $\boldsymbol{\gamma}_h'(L) = 1 - \gamma_1 L^{-h} - \ldots - \gamma_p L^{p-h}$ denotes the autoregressive lag polynominal shifted h steps ahead that loads the past hystory of $\tilde{\mathbf{y}}_T$. The VAR forecasts have been generated by estimating the previous equation by OLS for each forecasting horizon. What we obtain is:

$$\hat{y}_{T+h|T}^{VAR} = \hat{\alpha} + \hat{\delta}_h(L) y_T + \hat{\boldsymbol{\gamma}}_h'(L)\tilde{\mathbf{y}}_T$$

where $\hat{y}_{T+h|T}^{VAR}$ is the desidered point forecast predictor used in *Equation (27)*.

2.3 Forecasting with the Diffusion Index Model

Let $\mathbf{x}_{Nt} = (x_{1t}, x_{2t}, \ldots, x_{Nt})'$ be the N-dimensional vector (with N large) of all observed stationary time series variables in our data-set. Under the so-called **Diffusion Index (DI) model** or **static factor model** of Stock and Watson (2002a,b), $\mathbf{x}_{Nt}$ can be decomposed as:

$$\underbrace{\mathbf{x}_{NT}}_{(N\times 1)} = \underbrace{\boldsymbol{\Lambda}}_{(N\times r)} \underbrace{\mathbf{F}_{NT}}_{(r\times 1)} + \underbrace{\boldsymbol{\xi}_{NT}}_{(N\times 1)} = \boldsymbol{\chi}_{NT} + \boldsymbol{\xi}_{NT} \tag{5}$$

$$\mathbf{F}_{NT} = \mathbf{A}\mathbf{F}_{N;T-1} + \boldsymbol{\epsilon}_{NT} \qquad \boldsymbol{\epsilon}_{Nt} \sim iid\ \mathcal{N}(\mathbf{0}; \mathbf{Q}_{\boldsymbol{\epsilon}}) \tag{6}$$

$$\boldsymbol{\xi}_{NT} = \boldsymbol{\Psi}\boldsymbol{\xi}_{N;T-1} + \mathbf{v}_{NT} \qquad \mathbf{v}_{NT} \sim iid\ \mathcal{N}(\mathbf{0}; \mathbf{R}_{\mathbf{v}}) \tag{7}$$

where $\mathbf{x}_{NT}$ denotes the dataset at the end of the estimation sample, $\mathbf{F}_{NT}$ denotes the vector of r static common factors, $\boldsymbol{\Lambda}$ denotes the matrix of static factor loadings, $\boldsymbol{\chi}_{NT} = \boldsymbol{\Lambda}\mathbf{F}_{NT}$ denotes the vector of static common components while $\boldsymbol{\xi}_{NT}$ denotes the vector of idiosyncratic components. We assumed diagonal variance-covariance matrices $\mathbf{Q}_{\boldsymbol{\epsilon}}$ and $\mathbf{R}_{\mathbf{v}}$. The **DI forecasting model** can be written as:

$$y^{DI}_{T+h} = \alpha + \boldsymbol{\beta}'\hat{\mathbf{F}}_{NT} + \delta_h(L)y_T + \epsilon_{T+h} \qquad h = 1;\ldots;12 \tag{8}$$

where $\hat{\mathbf{F}}_{NT}$ are the estimated *static principal components* factors while $\boldsymbol{\beta}'$ denotes a properly chosen row of the matrix $\boldsymbol{\Lambda}_N$. The DI forecasts have been generated by estimating the previous equation using OLS for each forecasting horizon:

$$\hat{y}^{DI}_{T+h|T} = \hat{\alpha} + \hat{\boldsymbol{\beta}}'\hat{\mathbf{F}}_{N;T} + \hat{\delta}_h(L)y_T \qquad h = 1;\ldots;12$$

where $\hat{y}^{DI}_{T+h|T}$ is the desidered point forecast predictor used in *Equation (27)*.

2.4 Forecasting with the GDFM

Following Forni et al. (2000), if $\mathbf{x}_{Nt}$ admits a generalized dynamic factor model (GDFM), the **measurement equation** takes the following form:

$$\underbrace{\mathbf{x}_{NT}}_{(N\times 1)} = \underbrace{\boldsymbol{\Lambda}(L)}_{(N\times q)}\underbrace{\mathbf{f}_{NT}}_{(q\times 1)} + \underbrace{\boldsymbol{\xi}_{NT}}_{(N\times 1)} = \tilde{\boldsymbol{\chi}}_{NT} + \boldsymbol{\xi}_{NT} \tag{9}$$

where $\boldsymbol{\Lambda}(L) = \Lambda_0 + \Lambda_1 L + \ldots + \Lambda_s L^S$ denotes the matrix of dynamic factor loadings, $\mathbf{f}_{NT}$ denotes the vector of q dynamic factors with $r = q(s+1)$, $\tilde{\boldsymbol{\chi}}_{NT} = \boldsymbol{\Lambda}(L)\mathbf{f}_{NT}$ denotes the vector of dynamic commmon components while $\boldsymbol{\xi}_{NT}$ denotes the vector of idiosyncratic components. The **GDFM forecasting model** can be written as:

$$y^{GDFM}_{T+h} = \alpha + \underline{\boldsymbol{\beta}}'(L)\hat{\mathbf{f}}_{NT} + \delta_h(L)y_T + \varepsilon_{T+h} \qquad h = 1;\ldots;12 \tag{10}$$

where $\hat{\mathbf{f}}_{NT}$ are the estimated *dynamic principal components* factors using the two step estimation procedure of Forni et al. (2005), while $\underline{\boldsymbol{\beta}}'(L)$ a properly chosen row of $\boldsymbol{\Lambda}_N(L)$. The GDFM forecasts have been generated by estimating the previous equation by OLS for each forecasting horizon:

$$\hat{y}^{GDFM}_{T+h|T} = \hat{\alpha} + \underline{\hat{\boldsymbol{\beta}}}'(L)\hat{\mathbf{f}}_{N;T} + \hat{\delta}_h(L)y_T$$

where $\hat{y}^{GDFM}_{T+h|T}$ is the desidered point forecast predictor used in *Equation (27)*

2.5 Forecasting with the regular DSGE

The DSGE of Smets and Wouters (2007) is a medium-scale New Keynesian model with price and wage rigidities, capital accumulation, investment adjustment cost, and habit formation. This model, as every DSGE model, delivers a **linearized solution** which is a VAR process for DSGE state variables:

$$\underbrace{\mathbf{y}_{nt}}_{(n\times 1)} = \underbrace{\mathbf{D}(\boldsymbol{\vartheta})}_{(n\times r)}\underbrace{\mathbf{s}_t}_{(r\times 1)} \tag{11}$$

$$\underbrace{\mathbf{s}_t}_{(r\times 1)} = \underbrace{\mathbf{G}(\boldsymbol{\vartheta})}_{(r\times r)}\underbrace{\mathbf{s}_{t-1}}_{(r\times 1)} + \underbrace{\mathbf{H}(\boldsymbol{\vartheta})}_{(r\times r_e)}\underbrace{\mathbf{e}_t}_{(r_e\times 1)} \qquad \mathbf{e}_t \sim \mathcal{N}(\mathbf{0};\mathbf{Q_e}(\boldsymbol{\vartheta})) \tag{12}$$

where: $\mathbf{y}_{nt}$ denotes the n-dimensional vector of DSGE observed time series, $\mathbf{s}_t$ denotes the r-dimensional vector of DSGE state variables, $\boldsymbol{\vartheta}$ denotes the vector of DSGE *deep parameters* that we wish to estimate, $\mathbf{e}_t$ denotes the r_e-dimensional vector of DSGE exogenous shocks with diagonal variance-covariance matrix $\mathbf{Q_e}(\boldsymbol{\vartheta})$, while $\mathbf{D}(\boldsymbol{\vartheta})$, $\mathbf{G}(\boldsymbol{\vartheta})$ and $\mathbf{D}(\boldsymbol{\vartheta})$ denote matrices of parameters as a function of the deep parameters vector $\boldsymbol{\vartheta}$. As in Kryshko (2009), in order to interpret the r unobserved static factors as r state variables, we assumed that $\mathbf{s}_t$ has the same dimension of $\mathbf{F}_t$.

The regular DSGE forecasts have been generated using the state space representation given in *Equation (11)* and *Equation (12)* with a measurement error. The point forecast predictors has been formed by iterating on the last estimate of the unobserved state using the state equation *Equation (12)* and then backing out the corresponding value for the observable using the measurement equation *Equation (11)*. We do this using Bayesian estimation under the Metropolis-Hastings algorithm as described from **Step 1a** to **Step 2a** of *Section (3.2.3)*. The mean of the posterior forecast distributions is taken as the point forecast of the relevant variable. The Brooks and Gelman (1998) test has shown that all Markov chains for each estimation sample have converged nicely.

2.6 Forecasting with the drDSGE

This section describes the so-called Data-Rich DSGE (drDSGE) of Boivin and Giannoni (2006) used in our out-of-sample forecasting experiment. We begin with its representation theory, then we discuss its newness respect to regular DSGE, and finally we present our forecasting evaluation.

2.6.1 Representation Theory

The idea of drDSGE is to extract the common factor vector $\mathbf{F}_t$ from large panel of macroeconomic time series $\mathbf{x}_{Nt}$ and to match the state variable vector $\mathbf{s}_t$ of the model to the extracted common factor $\mathbf{F}_t$ (this matching generates the so-called *Data-Rich Environment*), where the law of common factors $\mathbf{F}_t$ is governed by the DSGE linearized solution. The key assumption of their approach is the separation betweeen *observed* or *data indicators* and *theoretical* or *model concepts*:

- the *data indicators* or simply *indicators* are the observed time series variables in $\mathbf{x}_{Nt}$;
- the *theoretical concepts* are time series variables in the vector $\mathbf{x}_{Nt}$ observed by econometricians or central banks, such as: employment, inflation or productivity shocks, that are assumed to be not properly measured by a single data series, but they are merely imperfect indicators of the observed time series. For example, the employment is imperfectly measured because there are discrepancies between its two main sources: one obtained from the establishment survey and the other from the population survey.

Their approach allows: first, to explore a richer amount of information by combining a DSGE model with a static factor model; second, to introduce imperfect information on DSGE estimation which is particular useful to characterize the desirable monetary policy (Boivin and Giannoni(2008)); third, to interpret structurally the latent factors; fourth, to avoid the so-called *Lucas critique*.

The drDSGE forecasts have been generated iterating its state space representation. Let $\bar{\mathbf{s}}_t =$

$[\mathbf{y}'_{nt} \;\; \mathbf{s}'_t]'$ be the vector collecting all variables in a given DSGE model, by definition:

$$\bar{\mathbf{s}}_t \equiv \begin{bmatrix} \mathbf{y}_{nt} \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \mathbf{D}(\boldsymbol{\vartheta}) \\ \mathbf{I} \end{bmatrix} \mathbf{s}_t \tag{13}$$

Representing the vector of common factors $\mathbf{F}_t$ as a subset of the variables in $\bar{\mathbf{s}}_t$, we can define:

$$\mathbf{F}_t \equiv \mathbf{F}\bar{\mathbf{s}}_t = \mathbf{F} \begin{bmatrix} \mathbf{D}(\boldsymbol{\vartheta}) \\ \mathbf{I} \end{bmatrix} \mathbf{s}_t \tag{14}$$

where $\mathbf{F}$ is a matrix that generates the common factors $\mathbf{F}_t$ from the vector $\bar{\mathbf{s}}_t$ of all DSGE variables. Now, by substituting *Equation (14)* into *Equation (5)*, we obtain the **static drDSGE observation equation**:

$$\underbrace{\mathbf{x}_t}_{(N\times1)} = \underbrace{\boldsymbol{\Lambda}}_{(N\times r)} \underbrace{\mathbf{F}_t}_{(r\times1)} + \underbrace{\boldsymbol{\xi}_t}_{(N\times1)} \quad \Rightarrow \quad \underbrace{\mathbf{x}_t}_{(N\times1)} = \underbrace{\boldsymbol{\Lambda}(\boldsymbol{\vartheta})}_{(N\times r)} \underbrace{\mathbf{s}_t}_{(r\times1)} + \underbrace{\boldsymbol{\xi}_t}_{(N\times1)} \tag{15}$$

Then, the **drDSGE state space representation** is:

$$\begin{aligned} \mathbf{x}_t &= \boldsymbol{\Lambda}(\boldsymbol{\vartheta})\mathbf{s}_t + \boldsymbol{\xi}_t && && (16) \\ \mathbf{s}_t &= \mathbf{G}(\boldsymbol{\vartheta})\mathbf{s}_{t-1} + \mathbf{H}(\boldsymbol{\vartheta})\mathbf{e}_t && \mathbf{e}_t \sim \mathcal{N}(\mathbf{0}; \mathbf{Q}_\mathbf{e}(\boldsymbol{\vartheta})) && (17) \end{aligned}$$

where $\boldsymbol{\xi}_t$ can be interpreted as serially correlated measurement errors. Adding their law of motion, as we did in *Equation (7)*, we obtain the **drDSGE static representation**:

$$\begin{aligned} \mathbf{x}_t &= \boldsymbol{\Lambda}(\boldsymbol{\vartheta})\mathbf{s}_t + \boldsymbol{\xi}_t && && (18) \\ \mathbf{s}_t &= \mathbf{G}(\boldsymbol{\vartheta})\mathbf{s}_{t-1} + \mathbf{H}(\boldsymbol{\vartheta})\mathbf{e}_t && \mathbf{e}_t \sim \mathcal{N}(\mathbf{0}; \mathbf{Q}_\mathbf{e}(\boldsymbol{\vartheta})) && (19) \\ \boldsymbol{\xi}_t &= \boldsymbol{\Psi}\boldsymbol{\xi}_{t-1} + \mathbf{v}_t && \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}; \mathbf{R}_\mathbf{v}) && (20) \end{aligned}$$

where $\boldsymbol{\Lambda}(\boldsymbol{\vartheta})\mathbf{s}_t$ can be interpreted as the static DSGE common component of $\mathbf{x}_t$ since the state variables $\mathbf{s}_t$ are loaded in $\mathbf{x}_t$ just in a contemporaneoulsy way. Otherwise, we might assume that the structural shocks contained $\mathbf{s}_t$ may impact the data in the present and in the past. In this case, the associated **drDSGE representation** becomes a dynamic representation:

$$\mathbf{x}_t = \underline{\mathbf{B}}(L)\begin{bmatrix}\mathbf{u}_t \\ \boldsymbol{\zeta}_t\end{bmatrix} + \boldsymbol{\xi}_t = \underline{\mathbf{B}}(L)\underline{\mathbf{s}}_t + \boldsymbol{\xi}_t \tag{21}$$

$$\boldsymbol{\xi}_t = \boldsymbol{\Psi}\boldsymbol{\xi}_{t-1} + \mathbf{v}_t \qquad \text{where: } \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}; \mathbf{R}_\mathbf{v}(\boldsymbol{\vartheta})) \tag{22}$$

where $\underline{\mathbf{B}}(L)$ are one-sided filters in the lag operator L, and $\underline{\mathbf{s}}_t = [\mathbf{u}_t\ \boldsymbol{\zeta}_t]'$ can be interpreted as the dynamic (primitive) factors associated to the state variables or static factors $\mathbf{s}_t$. This representation is not used by Boivin and Giannoni (2006) and it remains an open part of the empirical research.

2.6.2 Regular DSGE versus drDSGE

In the drDSGE representation, the matrix $\boldsymbol{\Lambda}(\boldsymbol{\vartheta})$ in *Equation (18)* plays the key role. Infact, in a regular DSGE model, the model concepts in $\mathbf{s}_t$ are assumed to be perfectly measured by a single data indicator in $\mathbf{x}_{Nt}$, so that the matrix $\boldsymbol{\Lambda}(\boldsymbol{\vartheta})$ is a $(r \times r)$ identity matrix, while the drDSGE model allows **many-to-many relations** between data indicators and theoretical concepts, so that the matrix $\boldsymbol{\Lambda}(\boldsymbol{\vartheta})$ becomes $(N \times r)$ with $(N >> r)$. It permits to brige the gap between data indicators and theoretical concepts. Therefore, to separate key DSGE variables from no-key DSGE variables, Boivin and Giannoni (2006) have proposed a partition of the data indicators in $\mathbf{x}_{Nt}$ into two groups of variables:

- the *core series* $\mathbf{x}_t^F \in \mathbf{x}_{Nt}$ which correspond to only one model concept;
- the *no-core series* $\mathbf{x}_t^S \in \mathbf{x}_{Nt}$ which are related linearly with more than one model concept.

In other words, the *core series* are time series in $\mathbf{x}_{Nt}$ that cannot be expressed as a linear combination of model concepts in $\mathbf{s}_t$, while the *no-core series* are time series in $\mathbf{x}_{Nt}$ that can be expressed as a linear combination of more than one model concept in $\mathbf{s}_t$. The **state space**

representation becomes:

$$\underbrace{\begin{bmatrix} \mathbf{x}_t^F \\ --- \\ \mathbf{x}_t^S \end{bmatrix}}_{\mathbf{x}_t\ (N\times 1)} = \underbrace{\begin{bmatrix} \boldsymbol{\Lambda}(\boldsymbol{\vartheta})^F \\ --- \\ \boldsymbol{\Lambda}(\boldsymbol{\vartheta})^S \end{bmatrix}}_{\boldsymbol{\Lambda}(\boldsymbol{\vartheta})\ (N\times r)} \underbrace{\mathbf{s}_t}_{(r\times 1)} + \underbrace{\begin{bmatrix} \boldsymbol{\xi}_t^F \\ --- \\ \boldsymbol{\xi}_t^S \end{bmatrix}}_{\boldsymbol{\xi}_t\ (N\times 1)} \tag{23}$$

$$\underbrace{\mathbf{s}_t}_{(r\times 1)} = \underbrace{\mathbf{G}(\boldsymbol{\vartheta})}_{(r\times r)} \underbrace{\mathbf{s}_{t-1}}_{(r\times 1)} + \underbrace{\mathbf{H}(\boldsymbol{\vartheta})}_{r\times r_{\mathbf{e}})} \underbrace{\mathbf{e}_t}_{(N_{\mathbf{e}}\ \text{x}\ 1)} \qquad \mathbf{e}_t \sim \mathcal{N}(\mathbf{0}; \mathbf{Q}_{\mathbf{e}}(\vartheta)) \tag{24}$$

where $\boldsymbol{\Lambda}(\boldsymbol{\vartheta})^F$ is the core matrix loadings that contains just one non-zero element for each row, while the matrix $\boldsymbol{\Lambda}(\boldsymbol{\vartheta})^S$ is the no-core matrix loadings that contains more than one non-zero element for each row. The measurement errors are assumed to follow:

$$\underbrace{\boldsymbol{\xi}_t}_{(N\times 1)} = \underbrace{\boldsymbol{\Psi}}_{(N\times N)} \underbrace{\boldsymbol{\xi}_{t-1}}_{(N\times 1)} + \underbrace{\mathbf{v}_t}_{(N\times 1)} \qquad \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}; \mathbf{R}_{\mathbf{v}}) \tag{25}$$

where the matrices $\mathbf{Q}_{\mathbf{e}}(\vartheta)$, $\mathbf{R}_{\mathbf{v}}$ and $\boldsymbol{\Psi}$ are assumed to be diagonal. The essential feature of the drDSGE is that the panel dimension of data set N is much higher than the number of DSGE model states r (with: $N >> r$). Respect to the static factor model representation (from *Equation (5)* to *Equation (7)*) now: the law of motion of the unobserved factors is now governed by a DSGE model solution and some factor loadings are restricted by the economic meaning of the DSGE model concepts.

2.6.3 Forecasting with the drDSGE

The drDSGE forecasts have been generated by iterating on the last estimate of the unobserved state using *Equation (24)* and then backing out the corresponding value for the observable using the measurement equation *Equation (23).* We do this using Bayesian estimation under the Metropolis-within-Gibbs algorithm as described from **Step 1b** to **Step 5b** of *Section (3.2.4).* The mean of the posterior forecast distributions is taken as the point forecast of the relevant variable. The Brooks and Gelman (1998) test has shown that all Markov chains for each estimation sample have converged nicely.

3 The empirical application

This section discusses our empirical application. We describe the experimental design of our out-of-sample forecasting experiment as well as the estimation techniques and the test of equal predictive ability used. All time series variables are transformed in a similar way to Stock and Watson (2002b) as reported in Appendix A. The forecasting results are reported in the next section.

3.1 The experimental design and the forecasting metric

The out-of-sample forecasting experiments are organized as follows. We use rolling regressions with sample size fixed at $R = 80$ observations to generate forecasts up to $h = 12$ quarters ahead for two key US time series varaibles: the output growth and inflation. The models in competition are: the unconditional mean of the time series, the autoregressive process (AR(p)), the vector autoregressive process (VAR(p)), the static factor model or diffusion index (DI(r)) model of Stock and Watson (2002a), the generalized dynamic factor model (GDFM(r;q)) of Forni et al. (2000) and Forni et al. (2005), the regular DSGE of Smets and Wouters (2007) and its Data-Rich Environment form following Boivin and Giannoni (2006).

The orders p, r and q has been estimated using different ways. The autoregressive order p has been estimated using the Bayesian Information Criterion (BIC), the number of static factor model r has been estimated using the procedure of Alessi et al. (2007), while the number of dynamic factors q has been estimated using the procedure of Hallin and Liška (2007). The first estimation sample starts from 1959:1 and ends in 1978:4 so that the first forecasting date is 1979:1. Earlier observations are used to compute the initial growth rates. After all models have been estimated, the first set of out-of-sample forecasts is computed. Then, sample range shifts one-step forward to 1959:2-1979:1 in order to compute the second set of forecasts. All models are fully re-estimated for each rolling sample with estimation procedures described in the previous chapter. The estimation is performed $S = 96$ times to obtain a series of forecasts for each forecast horizon and each model. The last sample is 1973:1-2001:4 and the last forecasting date is 2004:4.

The metric used to evalute the forecasting performance of alternative forecasting models is the relative mean squared forecast error ($rMSFE$), defined as:

$$rMSFE(m;n)_{|h} = 1 - \frac{MSFE_{m|h}}{MSFE_{n|h}} \tag{26}$$

where $MSFE_{m|h}$ and $MSFE_{n|h}$ denote the mean squared forecast error generated from two different alternative forecasting models at forecasting horizon h. This metric can be interpreted as gain (or loss) in $MSFE$ of model m relatively to the model n when it it positive (or negative). The model m forecast is considered as informative if its $rMSFE$ is larger than zero. The $MSFEs$ have been constructed in the following way. Let $\mathbf{x}_{NT}$ be the finite dataset of N stationary time series up to time T used in the empirical out-of-sample forecasting experiment where $T = R + s - 1$ is the end of each rolling sample s of size $R = 80$. If y_t is our time series of interest in $\mathbf{x}_{NT}$, the mean square forecast error ($MSFE$) of y_t repect to the i-th forecasting model has been worked out as:

$$MSFE_{i|h} = S^{-1} \sum_{i=1}^{S} (y_{T+h} - \hat{y}^{i,s}_{T+h|T})^2 \tag{27}$$

where y_{T+h} denotes the observed stochastic process y_t at time $T + h$, and $\hat{y}^{i,s}_{T+h|T}$ denotes its unknown point forecast predictor using the i-th forecasting model for the s-th rolling sample. This metric represents an appropriate tool to measure the forecasting perfomance of DSGE models as documented by Smets and Wouters (2003), Smets and Wouters (2007), Wang(2009), Edge et al. (2010) and Edge et al. (2011).

3.2 The Estimation

3.2.1 Diffusion Index estimation

The estimation of *Equation (8)* requires the estimation of the static factors $\mathbf{F}_T$. Stock and Watson (2002b) have proposed to estimate $\mathbf{F}_T$ as the r largest *static principal components* (SPC) of $\mathbf{x}_{NT}$:

$$\hat{\mathbf{F}}_{NT} = \hat{\mathbf{S}}_T \mathbf{x}_{NT} \tag{28}$$

where $\hat{\mathbf{S}}_T$ is the $(r \times N)$ matrix of eigenvectors corresponding to the r largest eigenvalues of the estimated contemporaneous variance covariance matrix of $\mathbf{x}_{NT}$, indeed $\hat{\mathbf{\Gamma}}_0^{\mathbf{x}} = T^{-1} \sum_{t=1}^{T} \mathbf{x}_{NT,t}\mathbf{x}_{NT,t}^{'}$. The estimator $\hat{\mathbf{F}}_{NT}$ of $\mathbf{F}_{NT}$ is proved to be consistent even in the presence of time variation in the factor model. We provide the forecasting results up to $r = 7$ as well as using an automatic selection with the BIC.

3.2.2 Generalized Dynamic Factor model estimation

Differently from Stock and Watson(2002b), Forni et al. (2000) have proposed a dynamic estimation method based on the spectral density of $\mathbf{x}_{NT}$ rather than on its contemporaneous variance covariance matrix that has: on the one hand, the advantage of exploring the dynamic structure of the data and needs few dynamic aggregates to approximate the space spanned by the common factors, and on the other, the drawback of producing a twosided filter of the observations that makes the method inappropriate for forecasting. This problem was solved successively by Forni et al. (2005) where they have proposed the one-sided version of their two-sided filter, which retains the advantages of their dynamic approach but allows observed variables to be related only with current and past value of the factors.

Their one-sided estimation and forecasting method consists of two steps: in the **first step**, they follow Forni et al. (2000) getting estimates of the variance covariance matrices for the commmon and the idiosyncratic components as the inverse Fourier transform of the spectral density matrix of the common and idiosyncratic component respectively, then in the **second step**, they use these estimates to construct r contemporaneous linear combination of the observations with the smallest idiosyncratic common variance ratio. In other words, they compute the eigenvalues and the eigenvectors of the couple $(\hat{\mathbf{\Gamma}}_{N0}^{\boldsymbol{\chi}}(\theta); \hat{\mathbf{\Gamma}}_{N0}^{\boldsymbol{\xi}}(\theta))$, then, ordering the eigenvalues in descending order and taking the eigenvectors corresponding to the r largest ones, they obtain the so-called *generalised principal components* that allow efficient estimates and forecasts of the common component of $\mathbf{x}_{NT}$ without the need of future values. Practically, the generalized eigenvalues are the solutions of $det(\hat{\mathbf{\Gamma}}_{N0}^{\boldsymbol{\chi}} - v_j \hat{\mathbf{\Gamma}}_{N0}^{\boldsymbol{\xi}}) = 0$ for $j = 1, 2, \ldots, r$, while the corresponding generalized eigenvectors are the weights $\hat{\mathbf{z}}_j$ that must satisfy:

$$\hat{\mathbf{z}}_j \hat{\mathbf{\Gamma}}_{N0}^{\boldsymbol{\chi}} = \hat{v}_j \hat{\mathbf{z}}_j \hat{\mathbf{\Gamma}}_{N0}^{\boldsymbol{\xi}} \qquad \text{for } j = 1; 2; \ldots; r \tag{29}$$

under the normalization conditions: $\hat{\mathbf{z}}_l \hat{\mathbf{\Gamma}}^{\boldsymbol{\chi}}_{N0} \hat{\mathbf{z}}'_j = 1$ for $l = j$ and $\hat{\mathbf{z}}_l \hat{\mathbf{\Gamma}}^{\boldsymbol{\chi}}_{N0} \hat{\mathbf{z}}'_j = 0$ for $l \neq j$. Then ordering the eigenvalues $\hat{v}_j$ in descending order and taking the eigenvectors corresponding to the r largest eigenvalues, they estimate the dynamic factors as:

$$\hat{\mathbf{f}}_{NT} = \hat{\mathbf{Z}}_T \mathbf{x}_{NT} \tag{30}$$

where $\hat{\mathbf{Z}}_T$ is the $(q \times N)$ matrix of generalized eigenvectors corresponding to the r largest generalized principal components. The order r is fixed using the Alessi et al. (2009) criteria, while the order q is fixed using the Hallin and Liška (2007) criteria.

3.2.3 Regular DSGE estimation

Let $\mathbf{x}^T = \{\mathbf{x}_1, \ldots, \mathbf{x}_T\}$ be the data-set up to time $t = T$ and let $\mathbf{s}^T = \{\mathbf{s}_1, \ldots, \mathbf{s}_T\}$ the states up to time $t = T$. As suggested by the literature, we estimate the system formed by *Equation (11)* and *Equation (12)* plus a measurement error for the unknown parameter vector $\boldsymbol{\vartheta}$ using **Bayesian estimation**. Because of the normality of the structural shocks $\mathbf{e}_t$, the system is a **linear Gaussian state space model** where the likelihood function of data $p(\mathbf{x}^T|\boldsymbol{\vartheta})$ can be evaluated using the Kalman filter. The estimation procedure is organized as follows:

step 1a: Set the prior distribution $p(\boldsymbol{\vartheta})$, which is the distribution of $\boldsymbol{\vartheta}$ that the researcher have in mind before observing the data.

step 2a: Convert the prior distribution to the posterior distribution $p(\boldsymbol{\vartheta}|\mathbf{x}^T)$, which is the distribution of $\boldsymbol{\vartheta}$ conditional on the data $\mathbf{x}^T$, using the Bayes theorem:

$$p(\boldsymbol{\vartheta}|\mathbf{x}^T) = \frac{p(\mathbf{x}^T|\boldsymbol{\vartheta})p(\boldsymbol{\vartheta})}{\int p(\mathbf{x}^T|\boldsymbol{\vartheta})p(\boldsymbol{\vartheta})\ d\boldsymbol{\vartheta}} \tag{31}$$

where $p(\mathbf{x}^T|\boldsymbol{\vartheta})$ denotes the likelihood function of the data given the deep parameter vector.

3.2.4 Data-Rich DSGE estimation

Following Boivin and Giannoni (2006), the state space representation (from *Equation (23)* to *Equation (25)*) represents the starting point to estimate the drDSGE, where the observation equation can be obtained just by adding observable time series variables to the vector $\mathbf{y}_{nt}$ as *core series* and/or *no-core series*. In this paper, we used **Case C** of Boivin and Giannoni (2006), where 21 time series are added as *core series* and 7 are added as *no-core series*. This resulting system is estimated using Bayesian methods under Markov Chain Monte Carlo (MCMC) algorithm. For convenience, we divide parameters of the model into two types: the first type are the *deep parameters* in vector $\boldsymbol{\vartheta}$, and the second type are the parameters collected by the state space representation of the model as $\Xi = \{\boldsymbol{\Lambda}(\boldsymbol{\vartheta}), \boldsymbol{\Psi}, \mathbf{R_v}\}$. Because of the normality of the structural shocks $\mathbf{e}_t$ and the measurement error innovations $\mathbf{v}_t$, the system from *Equation (23)* to *Equation (25)* is a **linear Gaussian state space model** and the likelihood function of data $p(\mathbf{x}^T|\boldsymbol{\vartheta}, \Xi)$ can be evaluated using the Kalman filter.

Differently from regular DSGE estimation, now the aim is to estimate the couple $(\boldsymbol{\vartheta}, \Xi)$, rather than just one single unknown vector. The posterior distribution of the couple is:

$$p(\boldsymbol{\vartheta}, \Xi|\mathbf{x}^T) = \frac{p(\mathbf{x}^T|\boldsymbol{\vartheta}, \Xi)p(\boldsymbol{\vartheta}, \Xi)}{\int p(\mathbf{x}^T|\boldsymbol{\vartheta}, \Xi)p(\boldsymbol{\vartheta}, \Xi)\ d\boldsymbol{\vartheta}\ d\Xi} \tag{32}$$

where $p(\boldsymbol{\vartheta}, \Xi)$ denotes its prior distribution, while $p(\boldsymbol{\vartheta}, \Xi|\mathbf{x}^T)$ denotes its likelihood function. In order to generate draws from the posterior distribution $p(\boldsymbol{\vartheta}, \Xi|\mathbf{x}^T)$, since it is not directly tractable, we divide it into the following four conditional posterior distributions:

$$p(\Xi|\boldsymbol{\vartheta}, \mathbf{x}^T) \quad p(\mathbf{s}^T|\Xi, \boldsymbol{\vartheta}; \mathbf{x}^T) \quad p(\Xi|\mathbf{s}^T, \boldsymbol{\vartheta}; \mathbf{x}^T) \quad p(\boldsymbol{\vartheta}|\Xi, \mathbf{x}^T)$$

and we adopt the **Metropolis-within-Gibbs** algorithm, wherethe Gibbs sampler generats draws from joint posterior distribution $p(\boldsymbol{\vartheta}, \Xi|\mathbf{x}^T)$ by repeating iteratively generation of draws from conditional posterior distributions $p(\Xi|\boldsymbol{\vartheta}, \mathbf{x}^T)$ and $p(\boldsymbol{\vartheta}|\Xi, \mathbf{x}^T)$. To be precise, the main steps of **Metropolis-within-Gibbs** algorithm used in drDSGE estimation are:

step 1b: Specify initial values of parameters $\boldsymbol{\vartheta}^{(0)}$ and $\Xi^{(0)}$. And set the iteration index g at $g = 1$.

step 2b: Solve the DSGE model numerically at $\boldsymbol{\vartheta}^{(g-1)}$ based on Sims (2002)' method and obtain $\mathbf{G}(\boldsymbol{\vartheta}^{(g;1)})$, $\mathbf{H}(\boldsymbol{\vartheta})$, and $\mathbf{Q}(\boldsymbol{\vartheta})$ in *Equation (24)*.

step 3b: Draw $\Xi^{(g)}$ from $p(\Xi|\boldsymbol{\vartheta}^{(g-1)}, \mathbf{x}^T)$.

(3.1b) Generate unobserved state variables $\mathbf{s}_t^{(g)}$ from $p(\mathbf{s}^T|\Xi^{(g-1)}, \boldsymbol{\vartheta}, \mathbf{x}^T)$ using simulation smoother by DeJong and Shephard (1995).

(3.2b) Generate parameters $\Xi^{(g)}$ from $p(\Xi|\mathbf{s}^T, \boldsymbol{\vartheta}, \mathbf{x}^T)$, using the sampled draw $\mathbf{s}^{T(g)}$.

step 4b: Draw deep parameters $\boldsymbol{\vartheta}^{(g)}$ from $p(\boldsymbol{\vartheta}|\Xi^{(g)}, \mathbf{x}^T)$ using Metropolis step:

(4.1b) Sample from proposal density $p(\boldsymbol{\vartheta}|\boldsymbol{\vartheta}^{(g-1)})$ and, using the sampled draw $p(\boldsymbol{\vartheta}^{(proposal)})$, calculate the acceptance probability ap as follows:

$$ap = \left[\frac{p(\boldsymbol{\vartheta}^{(proposal)}|\Xi^{(g)}, \mathbf{x}^T)\ p(\boldsymbol{\vartheta}^{(g-1)}|\boldsymbol{\vartheta}^{(proposal)})}{p(\boldsymbol{\vartheta}^{(g-1)}|\Xi^{(g)}, \mathbf{x}^T)\ p(\boldsymbol{\vartheta}^{(proposal)}|\boldsymbol{\vartheta}^{(g-1)})}\ ;\ 1\right]$$

(4.2b) Accept $\boldsymbol{\vartheta}^{(proposal)}$ with probability ap and reject it with probability $1 - ap$. Set $\boldsymbol{\vartheta}^{(g)} = \boldsymbol{\vartheta}^{(proposal)}$ when accepted and $\boldsymbol{\vartheta}^{(g)} = \boldsymbol{\vartheta}^{(g-1)}$ when rejected.

step 5b: Set the iteration index $g = g+1$, return to Step 2 up to $g = G$ where G is the number of MCMC iterations.

Step 4b of this algorithm plays an essential role. Infact, it is important to make the acceptance probability ap as close to one as possible especially around the mode of the posterior density $p(\boldsymbol{\vartheta}|\Xi, \mathbf{x}^T)$ because the same values are sampled consecutively if ap is low. To achieve this purpose, we should choose the proposal density $p(\boldsymbol{\vartheta}^{(proposal)}|\boldsymbol{\vartheta}^{(g-1)})$ that mimics the posterior density $p(\boldsymbol{\vartheta}|\Xi, \mathbf{x}^T)$ especially around its mode. This is why we firstly run *regular DSGE* model estimation and compute the posterior mode of the DSGE model parameters to obtain initial value $\boldsymbol{\vartheta}^{(0)}$ of Step 1. Then, we generate smoothed state variables $\mathbf{s}_t^{(0)}$ using $\boldsymbol{\vartheta}^{(0)}$ and obtain initial value $\Xi_t^{(0)}$ from OLS regressions of $\mathbf{x}_t$ on $\mathbf{s}_t^{(0)}$. The previous literature sugggest to use the so-called *random-walk MH algorithm* (see An and Schorfheide (2007)) as Metropolis step in Step 4b, where the proposal density $\boldsymbol{\vartheta}^{(proposal)}$ is sampled from the random-walk model:

$$\boldsymbol{\vartheta}^{(proposal)} = \boldsymbol{\vartheta}^{(g-1)} + \boldsymbol{\tau}_t \qquad\qquad \boldsymbol{\tau}_t \sim i.i.d\ \mathcal{N}(\mathbf{0}; c\mathsf{H})$$

where H is the Hessian matrix of the logarithm of the posterior distribution, indeed, $-l_p''^{-1}(\hat{\boldsymbol{\vartheta}})$ where $l_p(\boldsymbol{\vartheta}) = \ln(p(\boldsymbol{\vartheta}|\Xi, \mathbf{x}^T))$, while c is a scalar called the *adjustment coefficient*, whose choice will be explained below.

The merit of using this *random-walk proposal* is that $p(\boldsymbol{\vartheta}^{(g-1)}|\boldsymbol{\vartheta}^{(proposal)}) = p(\boldsymbol{\vartheta}^{(proposal)}|\boldsymbol{\vartheta}^{(g-1)})$, so that the acceptance probability ap collapses to:

$$ap = \min\left[\frac{f(\boldsymbol{\vartheta}^{(proposal)}}{f(\boldsymbol{\vartheta}^{(g-1)})}\ ;\ 1\right]$$

which does not depend on the proposal density $p(\boldsymbol{\vartheta}|\boldsymbol{\vartheta}^{(g-1)})$. We must, however, be careful for $p(\boldsymbol{\vartheta}^{(proposal)})$ not to deviate from $p(\boldsymbol{\vartheta}^{(g-1)}$ so much because the acceptance probability ap may be low when those deviate far from each other. This may be achieved by making c low, but $p(\boldsymbol{\vartheta}^{(proposal)}$ may be sampled only from the narrow range if c is too low. In random walk sampler, the optimal acceptance rate according to Roberts et al. (1997) and Neal and Roberts (2008) is around 25%, ranging from 0.23 for large dimensions to 0.45 for univariate case. Following the previous literature, we simply use this random-walk MH algorithm with $\mathsf{H} = -l_p''^{-1}(\hat{\boldsymbol{\vartheta}})$.

For the prior densities, we follow the general approach used for DSGE modelling. We assume that the exogenous shocks $\mathbf{e}_t$ such as technology shock, preference shocks or monetary shocks are persistent for their past shocks and their law of motions follow an AR(1) process, such that: $u_t = \rho u_{t-1} + \varsigma_t$ where the error term ς_t is *i.i.d.* Since the coefficient ρ must be between zero and one to satisfy the stationary property, their prior densities are assumed to follow *beta distributions*, while the variances of the error term ς_t are setted up on *inverted gamma distributions*. For the other parameters we assumed *normal distributions*.

3.3 Tests of equal predictive ability

The mean squared forecast error analysis suffers from the lack of significance. For this reason, we need to quantify the significancy of the observed differences in the $rMSFEs$ between alternative forecasting models using statistical tests called tests of equal predictive ability. Among all these tests, we used the conditional predictive ability test of Giacomini and White (2006) (hereafter GW test), because respect unconditional predictive ability tests can be used with Bayesian estimation and it has higher power with finite samples.

3.3.1 Test of equal conditional predictive ability

Let T be the end of the estimation sample. Let $\hat{y}_{T+h,1}$ and $\hat{y}_{T+h,2}$ be two alternative forecasts formulated at time T for the time series of interest h steps ahead. The GW test evaluates the sequence of out-of-sample forecasts using a loss function in the form $L_{T+h}(y_{T+h}, \hat{y}^i_{T+h})$ with $i = 1, 2$. The null hypothesis is:

$$H_0: \quad \mathbb{E}[L_{T+h}(y_{T+h}, \hat{y}^1_{T+h}) - L_{T+h}(y_{T+h}, \hat{y}^2_{T+h})|\mathcal{I}_T] = \mathbb{E}[\Delta L_{T+h|T}|\mathcal{I}_T] = 0 \tag{33}$$

$\Delta L_{T+h|T}$ denotes the loss differential series and $\mathcal{I}_T$ denotes the information set at time T. When $\mathcal{I}_t$ is the σ-field $\mathcal{I}_t = (\emptyset, \Omega)$ and $h \geq 1$, the null hyphotesis can be viewed as the Diebold and Mariano (1995) and West (1996) statistics and the test statistic is:

$$t^{GW}_{\tau,h} = \frac{\Delta \bar{L}_\tau}{\hat{\sigma}_\tau / \sqrt{\tau}} \tag{34}$$

where $\Delta \bar{L}_\tau$ denotes the sample mean of the loss differential $\Delta L_{T+h|T}$, indeed, $\Delta \bar{L}_\tau = \tau^{-1} \sum_{\tau=T}^{T_1-h} \Delta L_{\tau+h|T}$, and $\hat{\sigma}^2_\tau$ denotes a consistent estimator of the asymptotic variance of the difference in the squared forecast errors σ^2_τ. The GW statistic is a two-sided test statistic with a standard t-distribution. Positive (negative) values indicate that the $MSFE$ generated from model 1 is significantly lower (higher) than the $MSFE$ generated from model 2. If α is the level of significance, the test rejects the null hypothesis of equal unconditional predictive ability whenever $|t_{\tau,h}| > z_{\alpha/2}$ where $z_{\alpha/2}$ is the $z_{\alpha/2}$ quantile of a standard normal distribution. We used a quadratic loss function with the variance σ^2_τ estimated using the heteroskedasticity and autocorrelation consistent (HAC) estimator proposed by Newey and West (1987).

4 The forecasting results

This section concludes the paper presenting the forecasting results. First, we provide the relative mean square forecasting error ($rMSFE$) analysis, then to asses the significance of the observed differences among $MSFEs$ we apply the test of equal predictive ability of Giacomini and White (2006) explained before.

4.1 Mean squared forecast error analysis

Figure (1) plots the $rMSFE$ of forecasting models respect to the unconditional mean of the series of interest. The observed values are reported in *Table (1)*, where the better $rMSFEs$ for any forecasting horizon h are denoted in bold. These values depend critically upon the choice of: the number of lags p, the number of static factors r, and the number of dynamic factors q. The order p has been estimated using the Bayesian Information Criterion (BIC), the number of static factors r has been estimated using the Alessi et al. (2007) criterion, while the number of dynamic factors q has been estimated using the Hallin and Liška (2007). These criteria, have suggested to estimate the GDFM with $r^* = 5$ static factors and $q^* = 3$ dynamic factors.

In terms of forecasting output growth, we found that factor models yield lower $rMSFEs$ respect to the all other competitive models in both short and long run. In particular, the DI(r^*) model produces lower $MSFEs$ in the short run (up to 3 quarters ahead), while the GDFM(r^*,q^*) yields lower $MSFEs$ in the long run (from 4 quarter up to 12 quarters ahead). Therefore, the AR(p^*), the VAR(p^*), the DSGE, and the drDSGE do not provide informative forecasts (only the AR(p^*) model has a positive $rMSFE$ at $h = 10$), meaning that the unconditional mean should be used instead.

With regard of inflation, we found that the GDFM(r^*,q^*) yields lower $MSFEs$ in the short run (up to 5 quarters ahead), while the drDSGE produces lower $MSFEs$ in the long run (from 6 quarter up to 12 quarters ahead). Therefore, we discovered that the DI(r^*) is able to produce lower $MSFEs$ than the GDFM(r^*,q^*) in the long run (from 8 quarter ahead up to 12 quarters ahead). This results is against the prelevant literature who gives to the GDFM better accurancy in forecasting time series variables than DI especially in the long run (Forni et al. (2000) and Forni et al. (2005)).

Figure (2) plots the $rMSFE$ of diffusion index model with altenative number of static factors, respect to the autoregressive model with the optimal lag p^* fixed using the BIC. The observed values are reported in *Table (2)*, where the better $rMSFEs$ for any forecasting horizon h are denoted in bold.

For both output growth and inflation we see that only few static factors are needed to outperform the AR(p^*) model. Infact, we need just 2 factors to outperforme the AR(p^*) model for any forecasting horizon. It confirms the findings of Stock and Watson (2002b) where their DI model was found superior in $MSFE$ than an autoregressive process.

With regard to output growth, there are considerable forecasting gains when we pass from 2 to 4 factors especially when the forecasting horizon increases. At 4 quarters ahead, the DI(4) yields 15.1% higher $rMSFE$ than DI(2), at 6 quarters ahead, the DI(4) yields 11.55% higher $rMSFE$ than DI(2), while at 12 quarters ahead the DI(4) yields 7.23% higher $rMSFE$ than DI(2).

With regard to inflation, there are considerable gains when we consider a larger number of factors, 6 or 7, at least for the short and medium run. At 1 quarter ahead, the DI(7) yields 36.94% higher $rMSFE$ than DI(1), and 6.97% higher $rMSFE$ than DI(6). At 6 quarters ahead, the DI(7) yields 25.79% higher $rMSFE$ than DI(1), and 1.58% higher $rMSFE$ than DI(6). But at 12 quarter ahead, the DI(1) yields 4.38% higher $rMSFE$ than DI(7), and 10.42% higher $rMSFE$ than DI(6).

Figure (3) plots the $rMSFE$ of DSGE model respect to the VAR(p) model with alternative number of lags p. The observed values are reported in *Table (3)*, where we have denoted in italic the values of $rMSFEs$ for which the underlying VAR(p) loses less respect to the regular DSGE of Smets and Wouters (2007).

For both time series, the table shows that there are no cases where the the VAR(p) model is able to produce lower $MSFE$ than the the DSGE of Smets and Wouters (2007). This result is in line with the findings of Del Negro and Schorfheide (2004), where a VAR(4) is used as the benchmark. Here, we find that the DSGE model is able to outperform not only the VAR(4) but all the VAR models considerared. For the output growth, the VAR(1) is the VAR model that loses less respect to the DSGE at 1 quarter ahead, while in the long run is the VAR(5) the model that minimize the loses respect to the regular DSGE. The results of inflation are quite similar. The only difference is that now is the VAR(2) the model that loses less respect to the regular DSGE.

Figure (4) plots the $rMSFE$ of DI(r) model with alternative number of static factors r respect to the GDFM(r^*, q^*). The observed values are reported in *Table (4)*, where we have denoted in bold the cases where forecasting using a DI(r) is superior than forecasting with the GDFM(r^*, p^*), and in italic the values of $rMSFEs$ for which the underlying DI(r) loses less respect to the GDFM(r^*, p^*).

Most of the values contained in the table are negative, meaning that there are few occasions in which the DI model yield a lower $MSFE$. Regard to output growth, there are few cases where the GDFM is outperformed by the DI, while for inflation these cases are increased. For the output growth, the DI model with 5 and 4 factors tend to outperform the GDFM(r^*, q^*) in both short term (up to 2 quarters ahead) and long term (from 10 to 12 quarters ahead). For inflation, there are no cases where the DI model is able to produce informative forecasrs in the short run, while in the medium run and in the long run the DI(7) and the DI(1) are able to produce lower $MSFEs$ respectively.

The *Figure (5)* plots the $rMSFE$ of DSGE models respect to the GDFM(r^*, q^*). The observed values are reported in *Table (5)*, where we have denoted in bold the cases where a DSGE model is able to aoutperform the GDFM(r^*, q^*) in term of $rMSFE$.

About the output growth, the GDFM yields lower $MSFEs$ than the DSGE models for any forecasting horizon. It confirms the results of *Table (1)* where the DSGE models was found to generate higher $rMSFEs$ than the GDFM.Differently, when we have to forecast inflation, we find that DSGE models are able to produce lower $MSFEs$ than the GDFM only in the long run (from 7 to 12 quarters ahead).

The interesting result is the MSFE performance gap between the DSGE and the drDSGE. This gap, as shown in *Figure (5)* increases when the forecasting horizon is increased as a pair of open scissors. Regarding the output growth, at 1 quarter ahead, the drDSGE loses 6.01% less (in absolute value) than the DSGE, at 6 quarters ahead, the drDSGE loses 44.49% less than the DSGE, and at 12 quarters ahead, the drDSGE loses 103.09% less than the DSGE. The same situation happens for inflation. At 1 quarter ahead, the drDSGE loses 2.02% less (in absolute value) than the regular DSGE, at 6 quarters ahead, the drDSGE loses 3.09% less than the regular DSGE, and at 12 quarters ahead, the drDSGE loses 40.06% less than the regular DSGE. This result is in line with the findings of Boivin and Giannoni (2006), who show that more accurate estimates implies better forecasts at least one step ahead.

Concluding, the $rMSFE$ analysis has pointed out that output growth is not forecasted informatively by DSGE models, while factor models yield lower and informative $MSFEs$ for any forecasting horizon. Simmetrically, for the inflation, DSGE models tend to produce lower $MSFEs$ than factor models especially in the long run. We could take these results as definitive, but since the $MSFE$ analysis has not significance power, we have to work on forecasting inference implementing predictive ability tests. Among these tests, we interpreted the conditional predictive ability test of Giacomini and White (2006).

4.2 Equal predictive ability results

Table (6) reports the test statistic of equal conditional predictive ability test for $h = 1, 4, 8, 12$ quarters ahead. These statistics have the following interpretation: *plus signs* indicate that the forecating model in rows have *lower* mean squared forecasting errors than the corresponding forecating model in columns, then the model in row outperforms significantly the model in column. Simmetrically, `negative signs` indicate that the forecating model in rows have `higher` mean squared forecast errors than the corresponding forecasting model in columns, then the model in column outperforms significantly the model in column. Entries denoted in bold are significant at 5% level, while entries denoted in underlined bold are significant at 1% level. Critical levels of test statistics are fixed as suggested by Giacomini and White (2006).

Regarding the output growth, the test reveals on one hand that the GDFM is able to generate significantly better forecasts than DSGE models in both short and long run. On the other side, the DI model is able to outperforms significantly the DSGE models only in the short run. Therefore, is confirmed the superiority of the drDSGE in outperforming significantly the regular DSGE in the short, medium and long run.

Regarding the inflation, we discovered the most important result of the dissertation: only the drDSGE outperforms significantly all other competitive models in forecasting inflation in the long run. In other words, in the long run significant forecasts can be obtained only by combining a DSGE model with a static factor model. It means that exploiting more information on many macroeconomic time series, through the drDSGE, is important not only to obtain more accurate estimates, but also to get significant better forecasts.

5 Conclusions

We conducted several out-of-sample forecasting experiments to assess the forecasting power of factor models relatively to DSGE models. We found three main conclusions. First, DSGE models are significantly outperformed by the GDFM in forecasting output growth in both short and long run, while the static factor model outperforms significantly DSGE models only in the short run. Second, the most surprising result of the paper, we discovered that only the drDSGE outperforms significantly all other competitive models in forecasting inflation in the long run. This evidence falls out with both Wang (2009) who found that the regular DSGE of Del Negro and Schorfheide (2004) was able to generate significant better forecasts for inflation in the long run, and Paccagnini (2011) where hybrid models are found to forecast poorly. Therefore, the drDSGE outperforms significantly the regular DSGE in forecasting both output growth and inflation, confirming that exploiting more information on many macroeconomic time series, through the drDSGE, is important not only to obtain more accurate estimates, but also to get significant better forecasts. Third, in most cases, the unrestricted VAR represents the worse forecasting model, suggesting that this model should not be used as benchmark model in forecasting comparisons.

Given the wide variety of DSGE models in the literature, this paper should not be understood as a final research into the relative predictive ability of DSGE models relatively to factor models, but it should encourage further research in this topic. Our results raise several issues for future research. In our view four issues are preminent. First, we have shown that forecasting results vary according to the type of DSGE considered, then future research should consider a wider range of DSGE models with alternative structural restrictions. Second, being the drDSGE a static model, it would be useful to generalize its representation allowing state variables to be loaded with leads and lags. It might raise further forecasting gains. Third, we have estimated factor models assuming linearity but linearity is often not prevalent in the data-set. Then, it would be useful to introduce nonlinear dynamic factor models. Fourth, throughout the paper we assumed weakly stationarity time series. Although data-set differentiation and standardization achieve stationary in most cases, this is a strong assumption that should be relaxed.

References

[1] Alessi L., M. Barigozzi and M. Capasso (2007): "A Robust Criterion for Determining the Number of Static Factors in Approximate Factor Models", *LEM Papers Series 2007/19*, Sant'Anna School of Advanced Studies, Pisa, Italy.

[2] Altissimo F., R. Cristadoro, M. Forni, M. Lippi and G. Veronese (2001): "Eurocoin: a Real Time Coincident Indicator of the Euro Area Business Cycle", *CEPR Discussion Papers*, **3108**.

[3] Altissimo F., R. Cristadoro, M. Forni, M. Lippi and G. Veronese (2010): "New Eurocoin: Tracking Economic Growth in Real Time", *The Review of Economics and Statistics*, **92**, 1024-1034.

[4] Amisano G. and O. Tristani (2010): "A DSGE Model of the Term Structure with Regime Shifts", *European Central Bank Working Paper Series.*

[5] An S. and F. Schorfheide (2007): "Bayesian Analysis of DSGE Models", *Econometric Review*, **26**, 113-172.

[6] Bai, J. and S. Ng (2002): "Determining the Number of Factors in Approximate Factor Models", *Econometrica* **70**, 191-221.

[7] Bai J. and S. Ng (2006): "Confidence Intervals for Diffusion Index Forecasts and Inference for Factor-Augmented Regressions", *Econometrica* **74** 1133-1150.

[8] Bai J. and S. Ng (2007): "Determining the Number of Primitive Shocks in Factor Models", *Journal of Business and Economic Statistics*, **25**, 52-60.

[9] Banbura M., D. Giannone and L. Reichlin (2010): "Large Bayesian Vector AutoRegressions", *Journal of Applied Econometrics* **25**, 71-92.

[10] Bernanke B.S. and Boivin J. (2003): "Monetary Policy in a Data-Rich Environment", *Journal of Monetary Economics*, **50** 525-546.

[11] Boivin J. and M.P. Giannoni (2006): "DSGE Models in a Data-Rich Environment", *NBER Working Paper* n°12772.

[12] Boivin J. and M.P. Giannoni (2008): "Optimal Monetary Policy in a Data-Rich Environment", *NBER Working Paper.*

[13] Canova F. and L. Sala (2009): "Back to Square One: Identi.cation Issues in DSGE Models", *Journal of Monetary Economics* **56**, 431-449.

[14] Chamberlain G. (1983): "Funds, Factors, and Diversification in Arbitrage Pricing Models", *Econometrica* **51**, 1281-1304.

[15] Chamberlain G. and M. Rothschild (1983): "Arbitrage, Factor Structure and Mean-Variance Analysis in Large Asset Markets", *Econometrica* **51**, 1305-1324.

[16] Christoffel K., Coenen G. and A. Warne (2010): "Forecasting with DSGE Models", *European Central Bank Working Paper*, n°1185, may.

[17] Clark, T. and M. McCracken (2001): "Tests of Equal Forecast Accuracy and Encompassing for Nested Models", *Journal of Econometrics*, **105**, 85-110.

[18] Clark, T. and M. McCracken (2010): "Testing for Unconditional Predictive Ability", Forthcoming: *Oxford Handbook on Economic Forecasting.*

[19] De Jong P. and P. Shephard (1995): "The Simulation Smoother for Time Series" *Biometrika* **82**, 339-350.

[20] Del Negro M. and F. Schorfheide (2004): "Priors from General Equilibrium Models for VARs", *International Economic Review*, **45**, 643.673.

[21] Del Negro M. and F. Schorfheide (2008): "Forming priors of DSGE Models (and how it Affects the Assessment of Nominal Rigidities)", *Journal of Monetary Economics* **55** 1191-1208.

[22] Del Negro M. and F. Schorfheide (2009): "Monetary Policy Analysis with Potentially Misspecified Models", *American Economic Review* **99**, 1415-1450.

[23] Del Negro M., F. Schorfheide, F. Smets and R. Wouters (2007): "On the Fit and Forecasting Performance of New Keynesian Models"; *Journal of Business and Economic Statistics* **23**, 123-143.

[24] Diebold F. X. and R. S. Mariano (1995): "Comparing Predictive Accuracy", *Journal of Business and Economic Statistics* **13**, 253-263.

[25] Edge R. M. and R. S. Gürkaynak (2011): "How Useful are Estimated DSGE Model Forecasts?", *Federal Reserve Bank of St Louis Working Paper.*

[26] Forni, M., M. Hallin, M. Lippi and L. Reichlin (2000): "The generalized dynamic factor model: identification and estimation", *The Review of Economics and Statistics* **82**, 540-554.

[27] Forni, M., M. Hallin, M. Lippi and L. Reichlin (2005): "The generalized factor model: one-sided estimation and forecasting", *Journal of the American Statistical Association* **100**, 830-840.

[28] Forni M., D. Giannone, M. Lippi and L. Reichlin (2009): "Opening the Black Box: Structural Factor Models with Large Cross-Sections", *Econometric Theory* **25**, 1319-1347.

[29] Forni M. and M. Lippi (2001): "The generalized dynamic factor model: representation theory", *Econometric Theory* **17**, 1113-41.

[30] Gerali A., Neri S., Sessa L. and F. Signoretti (2008): "Credit and Banking in a DSGE model", mimeo.

[31] Geweke J. (1977): "The dynamic factor analysis of economic time series", in D.J. Aigner and A.S. Goldberger, Eds., *Latent Variables in Socio-Economic Models*. Amsterdam: North Holland.

[32] Giacomini R. and H. White (2006): "Tests of conditional predictive ability", *Econometrica* **74**, 1545-1578.

[33] Giannoni D., Reichlin L. and L. Sala (2006): "Monetary Policy in Real Time", *NBER Macroeconomics Annual 2004*, 161-200.

[34] Gregory A., Head A. and J. Raynauld (1997): "Measuring World Business Cycles" *International Economic Review*, **38**, 677-701.

[35] Hallin M. and R. Liška (2007): "Determining the Number of Factors in the Generalized Dynamic Factor Model", *Journal of the American Statistical Association* **102**, 603-617.

[36] Inklaar R.J., Jacobs, J. and W. Romp (2003): "Business Cycle Indexes: does a Heap of Data Help?", *Working Paper*, University of Groningen.

[37] Kocherlakota N. (2007): "Model Fit and Model Selection", Federal Reserve Bank of St. Louis.

[38] Kryshko M. (2009): "Data-Rich DSGE and Dynamic Factor Models", mimeo.

[39] Lippi M. and D.L. Thornton (2004): "A Dynamic Factor Analysis of the Response of U.S. Interest Rates to News", *Federal Reserve Bank of St Louis Working Paper.*

[40] Ludvigson S.C. and S.Ng (2007): "The Empirical Risk-Return Relation: A Factor Analysis Approach", *Journal of Financial Economics* **83**, 171-222.

[41] Neal P. and Roberts, G. (2008): "Optimal Scaling for Random Walk Metropolis on Spherically Constrained Target Densities", *Methodology and Computing in Applied Probability*, **10**, 277-297.

[42] Onatski A. (2009): "Testing Hypotheses About the Number of Factors in Large Factor Models", *Econometrica* **77**, 1447-1479.

[43] Paccagnini A. (2011): "Model Evaluation in the DSGE Approach: A Comparison of Hybrid Models", *European University Institute EUI, Working Paper Series*, November.

[44] Roberts G. O., A. Gelman and W. R. Gilks (1997): "Weak Convergence and Optimal Scaling of Randam Walk Metropolis Algorithms", *Annals of Applied Probability*, **7**, 110-120.

[45] Rubaszek M. and P. Skrzypczynski (2008): "On the Forecasting Performance of a Small-Scale DSGE Model", *International Journal of Forecasting*, **24**, 498-512.

[46] Sargent T.J. and C.A. Sims (1977): "Business Cycle Modelling without Pretending to have too much a Priori Economic Theory", in C.A. Sims, Ed., *New Methods in Business Research*, Minneapolis: Federal Reserve Bank of Minneapolis.

[47] Schorfheide F. (2010): "Estimation and Evaluation of DSGE Models: Progress and Challenges", University of Pennsylvania.

[48] Schorfheide F., Sill K., and M. Kryshko (2010): "DSGE Model-Based Forecasting of Non-Modelled Variables" *International Journal of Forecasting*, **26**, 348-373.

[49] Sims C. (2002): "Solving Linear Rational Expectations Models", *Computational Economics* **20**, 1-20.

[50] Smets F. and R. Wouters (2003): An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area, *Journal of the European Economic Association* **1**, 1123-1175.

[51] Smets F. and R. Wouters (2007): Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, *American Economic Review* **97**, 586-606.

[52] Stock J.H. and M.W. Watson (2002a): "Macroeconomic Forecasting Using Diffusion Indexes", *Journal of Business and Economic Statistics* **20**, 147-162.

[53] Stock J.H. and M.W. Watson (2002b): "Forecasting Using Principal Components from a Large Number of Predictors", *Journal of the American Statistical Association* **97**, 1167-1179.

[54] Stock J.H. and M.W. Watson (2005): "Implications of Dynamic Factor Models for VAR Analysis", *NBER Working Papers* no. 11467.

[55] Stock J.H. and M.W. Watson (2010): "Dynamic Factor Models", *Oxford Handbook of Economic Forecasting* chapter 2.

[56] Wang M. (2009): "Comparing DSGE model with factor model: An out-of-sample forecasting experiment", *Journal of Forecasting* **28**. 167-182.

[57] West K. D. (1996): "Asymptotic Inference about Predictive Ability", *Econometrica* **64**. 1067-1084.

Tables

$rMSFE$ of models versus the unconditional mean						
			Output Growth			
	AR(p^*)	DI(r^*)	VAR(p^*)	DSGE	GDFM(r^*,q^*)	drDSGE
$h=1$	-0.0320	**0.3037**	-0.6299	-0.3653	0.2226	-0.3553
$h=2$	-0.0496	**0.3694**	-0.9077	-0.3223	0.3634	-0.3011
$h=3$	-0.0718	**0.3718**	-1.1813	-0.3192	0.3594	-0.3002
$h=4$	-0.0494	0.2700	-1.4518	-0.3386	**0.3984**	-0.3156
$h=5$	-0.0580	0.0801	-1.6694	-0.3409	**0.3854**	-0.2811
$h=6$	-0.0469	0.2318	-1.8954	-0.3166	**0.3843**	-0.3011
$h=7$	-0.0234	0.2512	-2.1720	-0.3473	**0.3863**	-0.3173
$h=8$	-0.0034	0.0762	-2.4567	-0.4047	**0.3897**	-0.3247
$h=9$	-0.0061	0.1225	-2.9116	-0.5428	**0.3472**	-0.3328
$h=10$	0.0015	0.2055	-3.5283	-0.7684	**0.2474**	-0.3384
$h=11$	-0.0028	0.1399	-4.2084	-1.0366	**0.2258**	-0.3401
$h=12$	-0.0158	0.0304	-5.1195	-1.3826	**0.1580**	-0.3446
			Inflation			
$h=1$	0.3940	0.5637	0.3876	0.4020	**0.6738**	0.4195
$h=2$	0.4558	0.5388	0.4094	0.4564	**0.6653**	0.4694
$h=3$	0.4350	0.5225	0.3514	0.4763	**0.7058**	0.4998
$h=4$	0.3819	0.4906	0.2620	0.4854	**0.5956**	0.5094
$h=5$	0.3448	0.4899	0.2015	0.5001	**0.5907**	0.5321
$h=6$	0.3068	0.3882	0.1590	0.5255	0.5401	**0.5615**
$h=7$	0.2659	0.4245	0.1077	0.5443	0.4861	**0.5943**
$h=8$	0.2360	0.3953	0.0533	0.5699	0.3673	**0.6196**
$h=9$	0.2008	0.3998	0.0082	0.5939	0.2992	**0.6459**
$h=10$	0.1725	0.4098	-0.0287	0.6108	0.2427	**0.6608**
$h=11$	0.1334	0.4104	-0.0618	0.6258	0.1377	**0.6958**
$h=12$	-0.0623	0.3863	-0.0913	0.6456	-0.0042	**0.7096**

Table 1: The entries in the table are the $rMSFEs$ of alternative forecasting models relatively to the time series unconditional mean. A positive entry indicates model informative forecasts. A negative entry indicates noninformative model forecasts. The entries in bold indicate the most informative model forecasts for any forecasting horizon h. For example, for output growth at one step ahead, the most informative forecasts are produced by the DI model with r^*=3 static factors, while the AR(p^*), the VAR(p^*), the DSGE and the drDSGE yield noninformative forecasts that are outperformed by the unconditional mean of the series.

$rMSFE$ **of** $DI(r)$ **with** $r = BIC, 1, 2, \ldots, 7$ **versus** $AR(p^*)$								
Output Growth								
	DI(BIC)	DI(1)	DI(2)	DI(3)	DI(4)	DI(5)	DI(6)	DI(7)
$h = 1$	0.3253	0.1936	**0.3313**	0.2950	0.2588	0.2773	0.3258	0.3181
$h = 2$	0.3992	0.0895	0.4150	0.3906	0.3869	**0.4171**	0.4131	0.4093
$h = 3$	0.4138	0.0774	0.3727	0.3685	0.4108	**0.4232**	0.3623	0.3785
$h = 4$	0.3043	0.0462	0.2391	0.3312	**0.3901**	0.3538	0.3223	0.3436
$h = 5$	0.1305	0.0043	0.1185	0.2928	0.3188	**0.3874**	0.3349	0.3527
$h = 6$	0.2662	0.0016	0.2499	0.3430	**0.3654**	0.3576	0.3191	0.3171
$h = 7$	0.2684	-0.0934	0.2534	0.3289	**0.3748**	0.2443	0.2804	0.3095
$h = 8$	0.0793	-0.1250	0.2714	0.2843	**0.3395**	0.0351	0.1949	0.2662
$h = 9$	0.1278	-0.0303	0.2565	0.2461	**0.3267**	0.0493	0.1002	0.2052
$h = 10$	0.2043	-0.0390	0.2162	0.2182	**0.2732**	0.1217	0.1468	0.1341
$h = 11$	0.1423	-0.0319	0.2467	0.2486	**0.3122**	0.1593	0.1325	0.1761
$h = 12$	0.0454	-0.0948	0.2354	0.2422	**0.3077**	0.0820	0.0599	0.1173
Inflation								
$h = 1$	0.2801	0.0342	0.3882	0.2882	0.2736	0.3903	0.3339	**0.4036**
$h = 2$	0.1525	-0.0400	0.2362	0.1276	0.0659	0.2611	0.3557	**0.3691**
$h = 3$	0.1549	-0.0304	0.2723	0.2169	0.1616	0.1729	0.3634	**0.4275**
$h = 4$	0.1758	0.0463	0.2602	0.1270	0.2031	0.0936	0.3006	**0.4248**
$h = 5$	0.2215	0.1014	0.1706	0.1015	0.2194	0.1008	0.2826	**0.3728**
$h = 6$	0.1174	0.1647	0.2509	0.2991	0.2522	0.1686	0.4068	**0.4226**
$h = 7$	0.2160	0.2336	0.1077	0.1929	0.2917	0.1485	0.3864	**0.4372**
$h = 8$	0.2085	0.2723	0.2289	0.0921	0.2356	0.2148	0.4084	**0.4577**
$h = 9$	0.2491	0.3044	0.0892	0.0129	0.2073	0.1974	**0.4005**	0.3624
$h = 10$	0.2868	0.3375	0.0526	-0.0504	0.2020	0.1767	**0.3795**	0.3711
$h = 11$	0.3197	**0.3602**	0.1173	-0.0077	0.2374	0.1362	0.2382	0.3576
$h = 12$	0.4223	**0.4321**	0.2312	0.1315	0.3115	0.1961	0.3279	0.3883

Table 2: The entries in the table are the $rMSFEs$ of diffusion index (DI(r)) models with an alternative number of static factors $r = BIC, 1, 2, \ldots, 7$ relatively to the autoregressive model (AR(p)) with the lag p fixed using the BIC. A positive entry indicates DI informative forecasts, while a negative entry indicates DI noninformative forecasts. The entries in bold indicate the most informative forecasts for any forecasting horizon h. For example, for output growth at one step ahead, the most informative forecasts are produced by the DI model with two static factors.

$rMSFE$ **of** $DSGE$ **versus** $VAR(p)$ **with** $p = BIC, 1, 2, \ldots, 5$						
			Output Growth			
	VAR(BIC)	VAR(1)	VAR(2)	VAR(3)	VAR(4)	VAR(5)
$h = 1$	0.2602	*0.2602*	0.3210	0.3158	0.4055	0.4592
$h = 2$	0.2840	0.2840	0.3525	*0.2656*	0.3219	0.4187
$h = 3$	0.3430	0.3430	0.3849	*0.2430*	0.2895	0.4129
$h = 4$	0.3923	0.3923	0.4191	*0.2833*	0.3126	0.4003
$h = 5$	0.4357	0.4357	0.4434	*0.3419*	0.3643	0.4433
$h = 6$	0.4833	0.4833	0.4725	*0.4088*	0.4157	0.4728
$h = 7$	0.5141	0.5141	0.4851	0.4557	*0.4461*	0.4951
$h = 8$	0.5310	0.5310	0.4902	0.4887	*0.4593*	0.4843
$h = 9$	0.5423	0.5423	0.4924	0.5147	*0.4622*	0.4806
$h = 10$	0.5451	0.5451	0.4889	0.5347	*0.4686*	0.4701
$h = 11$	0.5426	0.5426	0.4815	0.5487	0.4755	*0.4701*
$h = 12$	0.5431	0.5431	0.4798	0.5632	0.4823	*0.4623*
			Inflation			
$h = 1$	0.0399	0.0399	*0.0013*	0.1074	0.1603	0.1856
$h = 2$	0.1227	0.1227	*0.0886*	0.2087	0.2496	0.2726
$h = 3$	0.2271	0.2271	*0.1857*	0.2765	0.3246	0.3607
$h = 4$	0.3230	0.3230	*0.2705*	0.3477	0.3770	0.3992
$h = 5$	0.3947	0.3947	*0.3251*	0.4161	0.4250	0.4379
$h = 6$	0.4529	0.4529	*0.3688*	0.4745	0.4697	0.4806
$h = 7$	0.5048	0.5048	*0.4147*	0.5241	0.5103	0.5182
$h = 8$	0.5614	0.5614	*0.4726*	0.5816	0.5611	0.5568
$h = 9$	0.6068	0.6068	*0.5232*	0.6311	0.6039	0.5907
$h = 10$	0.6395	0.6395	*0.5623*	0.6706	0.6384	0.6192
$h = 11$	0.6673	0.6673	*0.5961*	0.7047	0.6699	0.6489
$h = 12$	0.6961	0.6961	*0.6311*	0.7391	0.7031	0.6827

Table 3: The entries in the table are the $rMSFEs$ of the dynamic stochastic general equilibrium (DSGE) model of Smets and Wouters (2007) relatively to the vector autoregressive model (VAR(p)) with an alternative number of lags $p = BIC, 1, 2, \ldots, 5$. A positive entry indicates DSGE informative forecasts, while a negative entry indicates a noninformative DSGE forecasts. The entries in italic indicate the VAR model that loses less respect to the regular DSGE.

$rMSFE$ **of DI**(r) **versus GDFM**(r^*,q^*) **with** $r = BIC, 1, 2, \ldots, 7$								
Output Growth								
	DI(BIC)	DI(1)	DI(2)	DI(3)	DI(4)	DI(5)	DI(6)	DI(7)
$h = 1$	0.1044	-0.0705	**0.1123**	0.0641	0.0160	0.0406	0.1050	0.0947
$h = 2$	0.0095	-0.5011	0.0355	-0.0046	-0.0108	**0.0390**	0.0325	0.0261
$h = 3$	0.0193	-0.5435	-0.0495	-0.0565	0.0142	**0.0350**	-0.0668	-0.0398
$h = 4$	-0.2134	-0.6637	-0.3273	-0.1665	*-0.0639*	-0.1272	-0.1821	-0.1450
$h = 5$	-0.4967	-0.7139	-0.5174	-0.2173	-0.1727	*-0.0545*	-0.1449	-0.1142
$h = 6$	-0.2477	-0.6976	0.5925	-0.1172	*-0.0790*	-0.0923	-0.1577	-0.1612
$h = 7$	-0.2200	-0.8233	-0.2449	-0.1190	*-0.0425*	-0.2601	-0.1999	-0.1515
$h = 8$	-0.5137	-0.8496	-0.1979	-0.1767	*-0.0859*	-0.5864	-0.3237	-0.2065
$h = 9$	-0.3442	-0.5880	-0.1460	-0.1620	*-0.0377*	-0.4653	-0.3867	-0.2250
$h = 10$	-0.0557	-0.3785	-0.0400	-0.0373	**0.0357**	-0.1652	-0.1320	-0.1489
$h = 11$	-0.1110	-0.3366	0.0242	0.0268	**0.1092**	-0.0890	-0.1236	-0.0672
$h = 12$	-0.1516	-0.3208	0.0775	0.0858	**0.1648**	-0.1075	-0.1342	-0.0649
Inflation								
$h = 1$	-0.3373	-0.7942	-0.1365	-0.3223	-0.3495	-0.1327	-0.2374	*-0.1080*
$h = 2$	-0.3779	-0.6909	-0.2419	-0.4184	-0.5187	-0.2014	-0.0475	*-0.0258*
$h = 3$	-0.6231	-0.9789	-0.3976	-0.5038	-0.6102	-0.5884	-0.2226	*-0.0995*
$h = 4$	-0.2596	-0.4576	-0.1307	-0.3342	-0.2179	-0.3853	-0.0689	**0.1209**
$h = 5$	-0.2462	-0.4384	-0.3278	-0.4383	-0.2495	-0.4395	-0.1484	*-0.0041*
$h = 6$	-0.3303	-0.2590	-0.1718	-0.0564	-0.1272	-0.2532	0.1059	**0.1297**
$h = 7$	-0.1198	-0.0946	-0.0699	-0.1528	-0.0117	-0.2162	0.1236	**0.1961**
$h = 8$	0.0443	0.1214	0.0689	-0.0963	0.0771	0.0519	0.2857	**0.3452**
$h = 9$	0.1436	0.2067	-0.0387	-0.1257	0.0960	0.0847	0.3164	**0.2729**
$h = 10$	0.2207	0.2761	-0.0352	-0.1478	0.1280	0.1004	**0.3220**	0.3129
$h = 11$	0.3162	**0.3569**	0.1129	-0.0128	0.2336	0.1318	0.2343	0.3544
$h = 12$	0.3889	**0.3992**	0.1867	0.0812	0.2716	0.1495	0.2890	0.3529

Table 4: The entries in the table are the $rMSFEs$ of the diffusion index model (DI(r)) with an alternative number of static factors $r = BIC, 1, 2, \ldots, 7$ relatively to the generalized dynamic factor model (GDFM(r, q)) with the number of static factors r fixed using Alessi et al. (2007) criterion and the number of dynamic factors q fixed using the Hallin and Liška (2007) criterion. We found $r^* = 5$ e $q^* = 3$. A positive entry indicates DI informative forecasts, while a negative entry indicates noninformative DI forecasts. The entries in italic indicate the DI model that loses less respect to the GDFM. The entries in bold indicate the most informative forecasts for any forecasting horizion h. For example, for inflation at one step ahead, there are no cases in which a DI yields informative forecasts and the DI(7) is the model that loses less, while at four step ahead the DI(7) is able to produce informative forecasts.

$rMSFE$ of DSGE models versus GDFM(r^*, q^*)				
	Output Growth		**Inflation**	
	DSGE	drDSGE	DSGE	drDSGE
$h = 1$	-0.7573	-0.6963	-0.8331	-0.8129
$h = 2$	-1.0791	-0.8770	-0.6239	-0.5754
$h = 3$	-1.0595	-0.8643	-0.7801	-0.6705
$h = 4$	-1.2252	-0.6249	-0.2724	-0.1309
$h = 5$	-1.1810	-0.7016	-0.2213	-0.0215
$h = 6$	-1.1334	-0.6885	-0.0328	-0.0019
$h = 7$	-1.1926	-0.6752	0.1142	**0.3131**
$h = 8$	-1.3087	-0.5316	0.3287	**0.6204**
$h = 9$	-1.3632	-0.5034	0.4203	**0.6396**
$h = 10$	-1.3477	-0.4597	0.4849	**0.8861**
$h = 11$	-1.6358	-0.5606	0.5658	**0.9660**
$h = 12$	-1.8296	-0.7987	0.6465	**1.0471**

Table 5: The entries in the table are the $rMSFEs$ of the dynamic stochastic general equilibrium models relatively to the generalized dynamic factor model (GDFM(r, q)) with $r^* = 5$ e $q^* = 3$. A positive entry indicates DSGE informative forecasts, while a negative entry indicates noninformative DSGE forecasts. The entries in bold indicate the most informative DSGE forecasts. For example, for output growth there are no cases in which DSGE models yield informative forecasts, while for inflation at eight step ahead both DSGE and drDSGE produce informative forecasts but the drDSGE forecasts are more informative.

Test of equal conditional predictive ability (GW test)

	Output Growth						Inflation					
If $h=1$:	Mean	AR(p^*)	DI(r^*)	VAR(p^*)	DSGE	GDFM(r^*,q^*)	Mean	AR(p^*)	DI(r^*)	VAR(p^*)	DSGE	GDFM(r^*,q^*)
AR(p^*)	**8.2512**	0	0	0	0	0	**15.7459**	0	0	0	0	0
DI(p^*)	5.1260	5.0767	0	0	0	0	**9.5378**	0.7863	0	0	0	0
VAR($p*$)	3.7613	3.2106	**-7.5801**	0	0	0	**12.0002**	5.8369	0.7711	0	0	0
DSGE	2.1322	1.5354	**-8.2113**	5.2770	0	0	**13.8996**	4.3562	0.5602	5.8350	0	0
GDFM(r^*,q^*)	1.6440	1.8884	0.6445	**8.6058**	**9.2396**	0	**21.6968**	4.6374	3.0911	4.9811	4.0713	0
drDSGE	2.1523	1.6877	**-8.1921**	**8.5967**	**8.1265**	**-7.9321**	**12.2570**	**6.0129**	0.6032	4.8701	5.8033	-3.9034
If $h=4$:												
AR(p^*)	0.5407	0	0	0	0	0	2.9308	0	0	0	0	0
DI(p^*)	2.3150	2.8183	0	0	0	0	**8.0435**	1.4455	0	0	0	0
VAR(p^*)	3.2082	2.5118	-4.0286	0	0	0	4.8213	1.7167	1.0193	0	0	0
DSGE	2.9597	2.2769	-2.6651	3.4087	0	0	4.8167	2.4049	0.9501	1.5464	0	0
GDFM(r^*,q^*)	3.2138	3.3985	1.9478	4.2912	3.0909	0	**20.9087**	2.0743	1.1859	1.2578	3.6244	0
drDSGE	3.2021	2.5847	-2.4217	4.2517	**6.0154**	-2.9321	**6.2033**	**7.4279**	4.5611	**6.6037**	**6.5534**	5.9851
If $h=8$:												
AR(p^*)	2.6710	0	0	0	0	0	**15.3335**	0	0	0	0	0
DI(p^*)	1.3636	1.3212	0	0	0	0	**13.5552**	**7.1261**	0	0	0	0
VAR(p^*)	3.6685	4.1270	-3.6573	0	0	0	3.5745	0.2973	2.3745	0	0	0
DSGE	3.1501	3.4670	-3.8994	2.1355	0	0	**7.4830**	3.5709	2.7996	1.2484	0	0
GDFM(r^*,q^*)	0.6531	0.4712	2.1853	2.3374	3.7045	0	**23.4710**	4.4193	1.3812	2.3919	2.1791	0
drDSGE	5.2544	4.8821	-5.8621	**6.9063**	**6.4113**	-4.9321	**9.7838**	**8.1772**	**5.9918**	**11.5446**	**7.9253**	**8.7679**
If $h=12$:												
AR(p^*)	5.2311	0	0	0	0	0	**10.0696**	0	0	0	0	0
DI(p^*)	3.9721	1.7715	0	0	0	0	**11.7056**	**7.8748**	0	0	0	0
VAR(p^*)	**8.9385**	**9.3304**	**-8.3023**	0	0	0	**15.1905**	**11.9503**	2.1421	0	0	0
DSGE	**6.0661**	5.9044	-5.7588	**7.4603**	0	0	**11.5110**	5.6169	0.8479	1.2841	0	0
GDFM(r^*,q^*)	0.3319	1.4539	1.2591	**8.3659**	**6.2057**	0	**8.0811**	1.0432	1.8838	1.8486	4.2154	0
drDSGE	**6.4923**	5.9872	**-6.5821**	**6.0567**	**8.3445**	**-6.9355**	**18.8901**	**9.0332**	**7.0105**	**20.3044**	**8.2211**	**11.6031**

Table 6: This table contains the results of pairwise tests of equal conditional predictive accuracy of alternative forecasting models using a quadratic loss function. The entries in the table are the test-statistic of equal conditional predictive ability for the methods in the corresponding row and column. A positive (negative) entry indicates that the model in row is able to produce a significant lower (higher) mean squared forecast error than the corresponding model in column. The entries in bold indicate test-statistics that are significant at 5% level. The entries in underlined bold indicate test-statistics that are significant at 1% level. For example, for inflation at one step ahead, the drDSGE forecasts outperforms significantly the AR(p^*) forecasts.

Figures

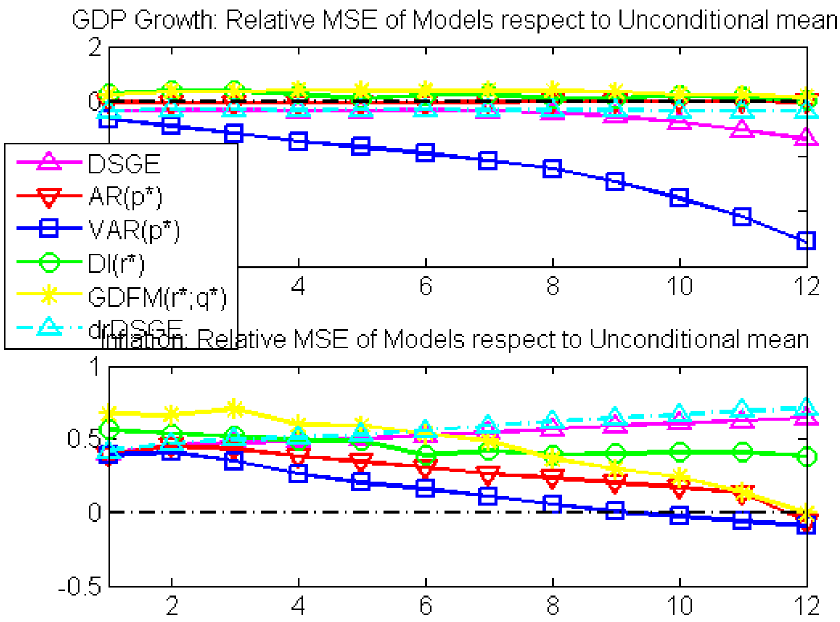

Figure 1: The figure plots the $rMSFEs$ of forecasting models relatively to the time series unconditional mean. The corresponding values are reported in *Table (1)*.

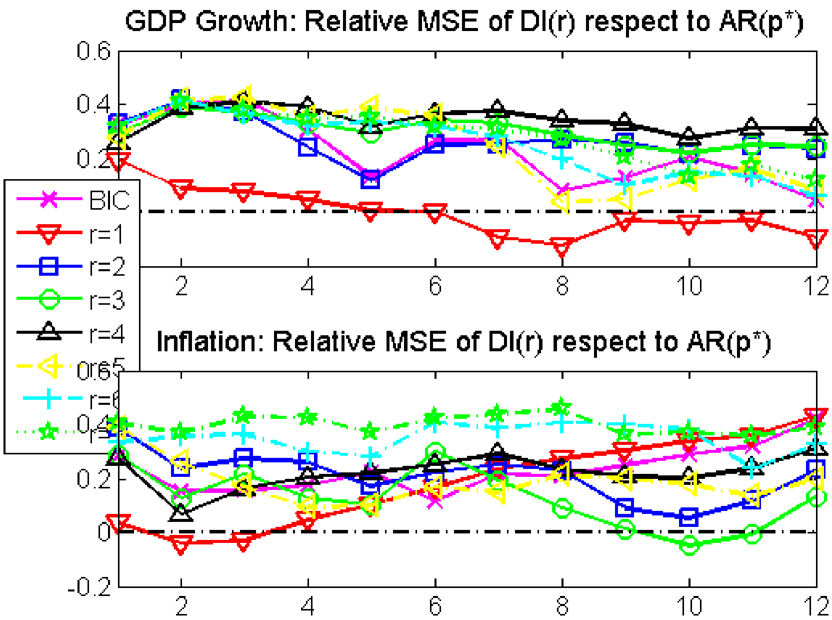

Figure 2: The figure plots the $rMSFEs$ of diffusion index models with an alternative number of static factors $r = BIC, 1, 2, \ldots, 7$ relatively to the autoregressive model (AR) with the lag p fixed using the BIC. The corresponding values are reported in *Table (2)*.

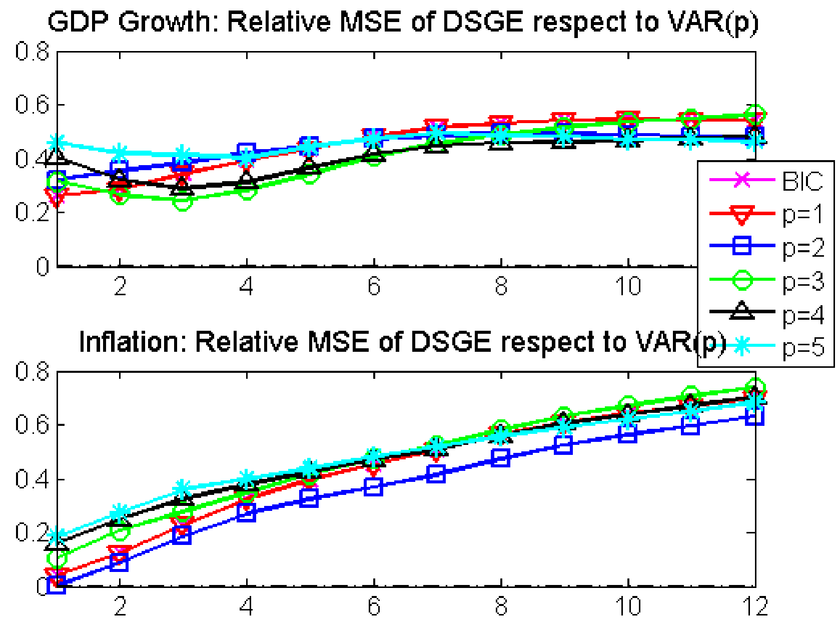

Figure 3: The figure plots the $rMSFEs$ of the dynamic stochastic general equilibrium (DSGE) model of Smets and Wouters (2007) relatively to the vector autoregressive model (VAR) with an alternative number of lags $p = BIC, 1, 2, \ldots, 5$. The corresponding values are reported in *Table (3)*.

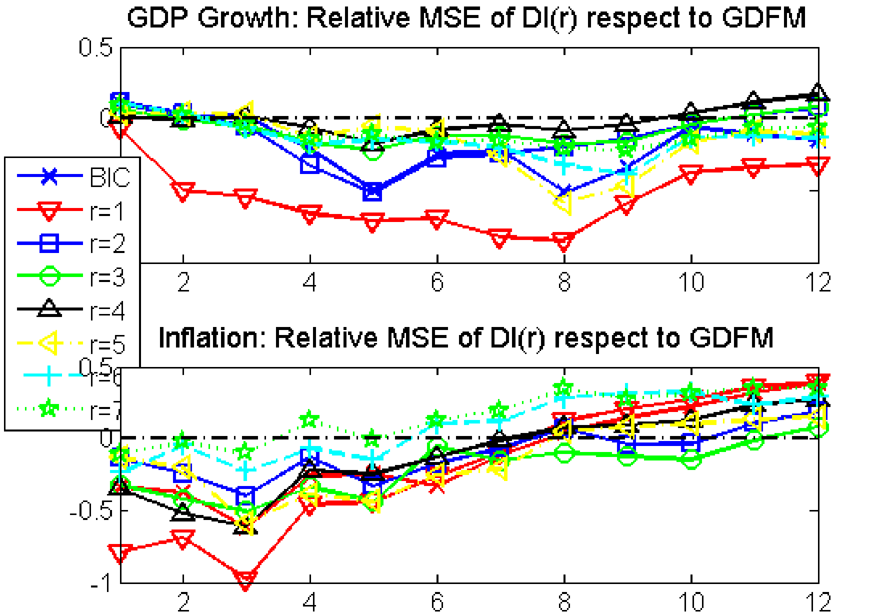

Figure 4: The figure plots the $rMSFEs$ of the diffusion index model (DI(r)) with an alternative number of static factors $r = BIC, 1, 2, \ldots, 7$ relatively to the generalized dynamic factor model (GDFM(p, q)) with the number of static factors r fixed using Alessi et al. (2007) criterion and the number of dynamic factors q fixed using the Hallin and Liška (2007) criterion. The corresponding values are reported in *Table (4)*.

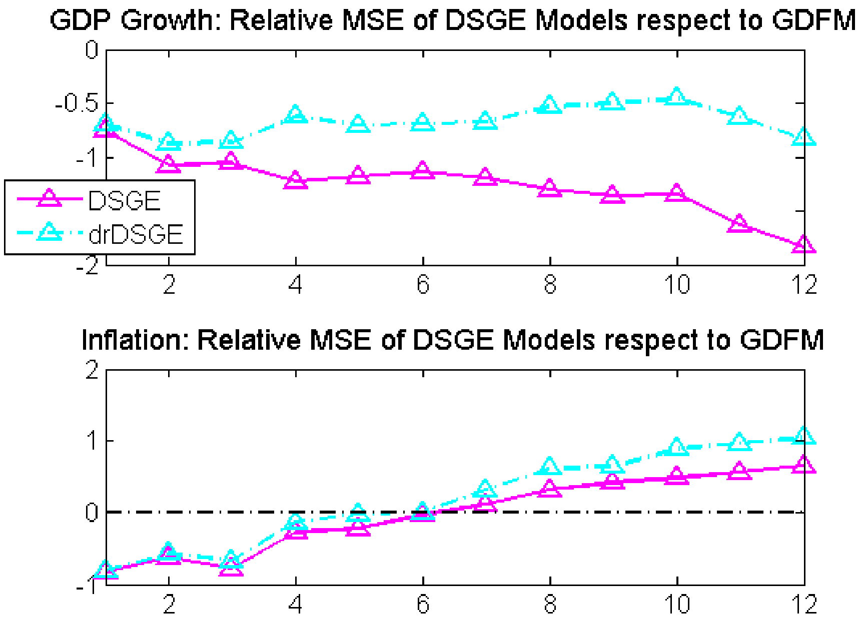

Figure 5: The figure plots the $rMSFEs$ of the dynamic stochastic general equilibrium models relatively to the generalized dynamic factor model (GDFM(r, q)) with $r^* = 5$ e $q^* = 3$. The corresponding values are reported in *Table (5)*.

Appendix A

This appendix gives an overview of the dataset used to construct the factors. The data are presented in the following ordering: series number, series mnemonic, series description and transformation code. The transformation codes are 1 = no transformation, 2 = first difference, 3 = first difference of logs, 4 = second difference of logs. All price series are obtained from Moody's Economy and all other series are obtained from `Datastream`. The series mnemonics and descriptions are taken directly from the associated sources. The interest rate spreads are calculated using the average federal funds rate obtained from Moody's Economy. The abbreviations appearing in the series descriptions are sa/sadj = seasonally adjusted, cura = current prices, seasonally adjusted, vola = volumn index, seasonally adjusted.

Table 7: The data-set used

Mnemonic	Description	Transformation
Prices		
1 cpiuaa_us	cpi: urban consumer apparel, (1982-84=100, sa)	4
2 cpiuac_us	cpi: urban consumer commodities, (1982-84=100, sa)	4
3 cpiuad_us	cpi: urban consumer durables, (1982-84=100, sa)	4
4 cpiuam_us	cpi: urban consumer medical care, (1982-84=100, sa)	4
5 cpiuas_us	cpi: urban consumer services, (1982-84=100, sa)	4
6 cpiuat_us	cpi: urban consumer transportation, (1982-84=100, sa)	4
7 cpiul1_us	cpi: urban consumer all items less food, (1982-84=100, sa)	4
8 cpiul2_us	cpi: urban consumer all items less shelter, (1982-84=100, sa)	4
9 cpiul5_us	cpi: urban consumer all items less medical care, (1982-84=100, sa)	4
10ppisp1000_us	ppi: stage of processing crude materials, (index 1982=100, sa)	4
11ppisp2000_us	ppi: stage of processing intermediate materials, (index 1982=100, sa)	4
12ppisp3000_us	ppi: stage of processing finished goods, (index 1982=100, sa)	4
13ppisp3100_us	ppi: stage of processing finished consumer goods, (index 1982=100, sa)	4
Consumption		
14uscdtan_b	pce durables, new autos (ar) cura	3
15uscondurb	personal consumption expenditures durables (ar) cura	3
16usconndrb	personal consumption expenditures nondurables (ar) cura	3
17usconsrvb	personal consumption expenditures services (ar) cura	3
18usperconb	personal consumption expenditures (ar) cura	3
Employment		
19usem21_o	employed mining vola	3
20usem23_o	employed construction vola	3
21usem42_o	employed wholesale trade vola	3
22usem81_o	employed otherservices vola	3
23usemig_o	employed government vola	3
24usemimd_o	employed durable goods vola	3
25usemip_o	employed totalprivate vola	3

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Table 7 ... *continued from previous page*

Mnemonic	Description	Transformation
26usemir_o	employed retail trade vola	3
27usemit_o	employed trade, transportation, utilities vola	3
28usempallo	employed nonfarm industries total (payroll survey) vola	3
29usempg_o	employed goodsproducing vola	3
30usempmano	employed manufacturing vola	3
31usemps_o	employed serviceproviding vola	3
32usemptoto	total civilian employment vola	3
33ushlpwadq	help wanted proportion of labor markets w/rising wantad vola	1
34usun_totq	unemployment rate sadj	2
35usundurne	average durationof unemployment (weeks) vola	1
36usunw14_q	unemployed distribution 5 to 14 weeks sadj	1
37usunw15_q	unemployed distribution 15 weeks over sadj	1
38usunw26_q	unemployed distribution 15 to 26 weeks over sadj	1
39usunw5_q	unemployed distribution less than 5 weeks sadj	1
40usvactoto	index of help wanted advertising vola	3
Housing		
41ushbrm_o	housing started midwest (ar) vola	3
42ushbrn_o	housing started northeast (ar) vola	3
43ushbrs_o	housing started south (ar) vola	3
44ushbrw_o	housing started west (ar) vola	3
45ushous_o	new private housing units started (ar) vola	3
Hours and Earnings		
46ushkim_o	avg wkly hours manufacturing vola	3
47ushxpmano	avg overtime hours manufacturing vola	3
48uswr23_b	avg hrly earn construction cura	4
49uswrim_b	avg hrly earn manufacturing cura	4
Output and Income		
50usipmbuqg	indl prod business equipment vola	3
51usipmcogg	indl prod consumer goods vola	3
52usipmducg	indl prod durable consumer goods vola	3
53usipmfgsg	industrial production manufacturing (sic) vola	3
54usipmfing	indl prod final products, total vola	3
55usipmmatg	indl prod materials, total vola	3
56usipmnocg	indl prod nondurable consumer goods vola	3
57usipmprog	indl prod final products nonindustrial supplies vola	3
58usiptot_g	industrial production total index vola	3
59usiumfgsq	indl utilizationmanufacturing (sic) sadj	1
60uspdispib	disposable personal income (ar) cura	3
61uspersinb	personal income (ar) cura	3
Interest Rates		
62uscrbbaa	corporate bond yield moody's baa, seasoned issues	2
63uscrbyld	corporate bond yield moody's aaa, seasoned issues	2
64ustrb3av	treasury bill secondary market rate on discount basis 3 month	2
65ustrcn10	treasury yield adjusted to constant maturity 10 year	2
66ustrcn1_	treasury yield adjusted to constant maturity 1 year	2
67ustrcn5_	treasury yield adjusted to constant maturity 5 year	2
68usytb6sm	treasury bill secondary market rate on discount basis 6 month	2

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Table 7 ... *continued from previous page*

Mnemonic	Description	Transformation
69ussfycrbyld	spread uscrbyld federal funds	1
70ussfycrbbaa	spread uscrbbaa federal funds	1
71ussfytrb3av	spread ustrb3av federal funds	1
72ussfyytb6sm	spread usytb6sm federal funds	1
73ussfytrcn1_	spread ustrcn1_ federal funds	1
74ussfytrcn10	spread ustrcn10 federal funds	1
75ussfytrcn5_	spread ustrcn5_ federal funds	1
Other Time Series		
76usm0_b	monetary base cura	4
77usnbrrsab	nonborrowed reserves of depository institutions cura	3
78uspmchin	chicago purchasingmanager diffusion indexinventories(sa)	1
79uspmchlt	chicago purchasingmanager diffusion indexdeliveries(sa)	1
80uspmchp_	chicago purchasingmanager diffusion indexprodn. (sa) sadj	1
81ustotrsab	total reserves of depository institutions cura	3
82usexpgdsb	exports f.a.s. cura	3
83uscnfbusq	ism purchasing managers index (mfg survey) sadj	1