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Relationship Between *R&D* Spending and Education Level in the Economy

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Introduction

This work investigates the relationship between *R&D* expenditure, and the education level in the economy. Looking at the empirical data it is shown that the growth rate of education is positive to *R&D* expenditure and that the higher schooling level, the greater *R&D* expenditure is. Consequently, the higher *R&D* investments, the more efficient research and the higher patents production are. Technological advancements determine the efficiency of a new stock of equipment goods, and they are the key driver of business cycles. So, what's the best way to stimulate investments in *R&D*, and how an *R&D* development affects economic growth?

To this scope, I have built a DSGE model with endogenous growth. Simulation results demonstrate a high correlation between *R&D* investments, and a knowledge rate increase, in response to an investment-specific technological shock. The shock considered is temporary, but it shows a high persistence even in a long term. The simulation exercise is repeated twice, first time the shock is over the equipment investment variable, second time it concerns the schooling-education variable.
In the model, economic growth depends on innovation efficiency, and not on how many people work in the *R&D*-equipment production sector. That efficiency is supported by an increase in the schooling-education growth rate. Finally, the economic growth process is slow because of delays in the diffusion of new technologies, it is a *slow adoption process*.

The work is organized as follows. Chapter 1 reviews the growth literature, using the standard neoclassical growth model as a point of departure, and focusing on the research that has developed regarding the intersection of endogenous growth, and investment-specific technological change in the business cycle. It explores the relationship among four variables. On one hand, economic growth and capital accumulation, and on the other hand it treats strategic complementarities between knowledge and *R&D*, or better between schooling-education growth rate and technological shocks.
Chapter 2 supplies an empirical analysis focused on *R&D* data spending within the European Union (UE) in comparison with the rest of the world.
Chapter 3 builds a dynamic stochastic general equilibrium model (DSGE) of endogenous growth. I calibrate, and simulate, the model in order to analyze how the investment-specific technological shock affects the economy.
The main mathematical computations are provided in Appendix A.

This thesis basically, takes inspiration from these papers: the RBC model built in Greenwood Hercowitz and Krusell (1998), which was the first to suggest that investment-specific technological shocks could be an alternative to neutral technology shocks as a source of economic growth; and from the knowledge-based growth model as built in Romer (1990) and in Jones (1995), where it is asserted that the stock of knowledge determines the development of the economy.

Chapter 1

Literature Review

This first chapter aims to review the growth literature up to the analysis of the Greenwood Hercowitz and Krusell (1998), the paper I used as the main framework for my work.

1.1 Introduction

Over the long-run period the aggregate supply depends on the following factors which affect the potential output[1]:

1. Natural resources (land, fuel, climate, environmental quality)

2. Knowledge resources (human capital, labor supply, education, motivation)

3. Capital formation (factory, equipment, infrastructure)

4. Technology (science, engineering, management, entrepreneurship)

[1] Nordhaus and Samuelson, chapter 2

We all agree that all of these factors have a positive impact on growth. But what is the relative importance of each factor in determining long-run growth? Particularly, the chapter reviews the recent literature, regarding two of the most active fields in economics, in the past few years: *knowledge-based growth models*, and models with technological progress.
As Greenwood Hercowitz and Krusell (1997, 1998) demonstrate, the role of investment-specific technological change becomes fundamental in the interpretation of business cycle movements. That assumption permits us to explore not only the long-run implications of growth, as in the most parts of growth models, but also the short-run effects.
What Greenwood et. al (1997, 1998) does not treat, is how that technology is produced. For that reason I here analyze the possible intersection between endogenous growth literature, and an investment-specific technological change.
Looking at the empirical data we can observe that the European time series about *R&D* expenditure at a national level (GERD, Eurostat database), and about patent applications to EPO (European Patent Organization) are increasing over time. What is, then, the role of *R&D* dedicated investments as an economic growth engine?
That topic is not just an academic issue. To demonstrate it here, I have drawn up a list of the main decisions taken by the European Institutions about science, technology and innovation:
In 2000 the European Council launched the *Lisbon Strategy*, aimed at transforming the EU by 2010 into the most competitive and dynamic knowledge-based economy in the world, capable of sustainable economic growth with more and better jobs and greater social cohesion.
In 2002 in Barcelona, a further target was added, namely to spend at least 3 percentage of GDP on research by 2010. Two thirds of that expenditure should be financed by the business sector.
In 2005 the Lisbon Strategy was re-launched with the initiative Working together for growth and jobs.

In 2006 at a Council meeting in Brussels, it was recognised that Europe should invest more in knowledge and growth.
In 2006-07 at the Spring European Councils, one of the four priority areas agreed upon by the Member States was more investment in knowledge and innovation.
In 2007 the European Commission launched the *Green Paper about the European Research Area*: new perspectives, a broad institutional and public debate on what should be done to create a unified and attractive European Research Area.

The chapter is organized as follows: section 2 provides an overview about the Neoclassical Growth Model, section 3 presents the evolution up to the New Growth Theory. In section 4 I start speaking about the Business Cycle Theory, and in the following section I focus on RBC models. Section 6 introduces the paper I will use as a framework to construct the new model, and finally section 7 concludes the chapter.

1.2 The Neoclassical Growth Model

This chapter is devoted to growth theory review[2] it starts, as almost all analysis of growth, from the neoclassical Solow growth model. That model considers an exogenous growth, and it belongs to a class of long-run economic growth models. These models attempt to explain long-run growth by looking at productivity, capital accumulation, population growth and technological progress.
The principal conclusion of the Solow model is that, the accumulation of physical capital makes a direct contribution to an increase in production, for which it is paid its marginal cost. The model treats other potential sources

[2] Avoiding the very earlier economists who stressed the importance of natural resources as did Smith (1776)

of differences in real incomes as either exogenous, and thus not explained by the model (in the case of technological progress, for example), or absent altogether (in the case of positive externalities from capital, for example).
The model is set up on continuous time, there is a single good produced with a constant technology, and it takes as given the initial level of capital and labor. Solow model implies, regardless of its starting point, the economy converges to a balanced growth path; a situation where each variable of the model is growing at a constant rate. On the balanced growth path, the growth rate of output per worker is determined solely by the art of technological progress; all factors of production are fully employed, and the labor force grows at a constant rate. In the model there is no government and no international trade. Solow identifies two possible sources of variation in output per worker: differences in capital per worker, or differences in the effectiveness of labor. Analyzing the model it is possible to observe that only growth in the effectiveness of labor can lead to permanent growth, and that the impact of changes in capital per worker is modest. That assumption can be verified in two ways: directly, and indirectly. Directly, looking at capital-output ratios data, differences in capital per worker are far smaller than those needed to account for the differences in output per worker. Indirectly, we can observe that the model cannot account for a large variation in output per worker on the basis of differences in capital per worker because required differences in capital imply enormous differences in the rate of return to capital (Lucas, 1990). On the other hand, in the Solow model the description of the effectiveness of labor is highly incomplete. The model takes as given the behaviour of the variable. Both effectiveness of labor, and knowledge-technological progress are taken as exogenous and constant. Hence, even the saving rate results to be constant. The Solow model does not have optimization in it.
In order to study growth, it becomes necessary, indeed, relaxing the Solow's assumption about those constant variables. That does allow considering investment increase, labor specialization, technology progress, and also welfare

issues as an endogenous variable. Therefore in order to address central questions of growth theory, we must move beyond the Solow model.
Others models are based on the Solow's assumption, but they consider the dynamics of economic aggregates determined by decisions at a microeconomic level. They still consider growth rates of labor and knowledge as given, but they derive the evolution of the capital stock from the interaction of maximizing households and firms utilities in competitive markets. As a result, the saving rate is no longer exogenous, and it does not need to be constant. The first model was developed by Ramsey (1928), Cass (1965), and Koopmans (1965). It avoids all market imperfections, and all issues raised by heterogeneous household and links among generations. Competitive firms rent capital and hire labor to produce and sell output, and a fixed number of infinite households supply labor, hold capital, consume and save.
The second model is the overlapping generation model developed by Diamond, (1965). The key difference between the Diamond model and the Ramsey-Cass-Koopmans model is that the Diamond model assumes that there is continual entry of new households into the economy. With turnover, it turns out to be simpler to assume that time is discrete rather than continuous. Like Solow, these models are based on the assumption that for any positive level of capital there is a unique initial level of consumption. Anyway differently from Solow it has to be consistent with the household's intertemporal optimization, the dynamics of the capital stock, the household's budget constraint, and the requirement that the capital does not have to be negative. The function giving this initial consumption as a function of capital is known as the *saddle path*. For any initial value of capital, the initial consumption must be the value on the saddle path. The entire economy then moves along the saddle path. It is important to stress that the behaviour of the economy, once it has converged to the saddle path point, is identical to that of the Solow economy on the balanced growth path. Capital, output, and consumption per unit of effective labor, are constant again. At the saddle path point output y and consumption c are constant, so the saving rate $\frac{(y-c)}{y}$ is

also constant. Thus the central implications of the Solow model, concerning the driving forces of economic growth do not hinge on its assumption of a constant saving rate.
Even when saving is endogenous, growth in the effectiveness of labor remains the only source of persistent growth in output per worker. And since the production function is the same as in the Solow model, it is easy to demonstrate those significant differences in output per worker which can arise from differences in capital per worker only if, the differences in capital per worker, and in rates of return to capital, are enormous. Concluding, in these models, the saving rate is no longer exogenous and it does not need to be constant. We have taken a first step towards a more complete growth model explanation.

1.3 The New Growth Theory

As we have seen in the previous section Neoclassical growth models do not provide satisfying answers to central questions about economic growth. The model's principal result is a negative one: capital accumulation cannot account for economic growth.[3] The only determinant of income in these models is the variable that describes the effectiveness of labor whose exact meaning is still not specified, and whose behaviour is still taken as exogenous. Endogenous growth theory tries to overcome this limitation by building macroeconomic models out of microeconomic foundations. Since the mid-1980s, a group of growth theorists became increasingly dissatisfied with exogenous factors determining long-run growth. They favoured a model in which the key determinants of growth were governed within the model.[4]

[3] The long-run rate of growth is exogenously determined by either the saving rate (Harrod-Domar 1939, 1946), and the rate of capital, or better the technical progress (Solow model 1956). In these models the previous variables remain unexplained.

[4] The initial research was based on the work of Kenneth Arrow (1962), Hirofumi Uzawa (1965), and Miguel Sidrauski (1967). Then Romer (1986), Lucas (1988), and Rebelo (1991) focused their attention on investments in knowledge and on their spillover effects on the economy

The endogenous growth theory is further supported with models in which agents optimally determine consumption and saving. In these models households are assumed to maximize utility subject to budget constraints, while firms maximize profits. Crucial importance is usually given to the production of new technologies and to knowledge accumulation. The engine for growth can be as simple as a constant return to scale production function, the AK model,[5] or more complicated setups with spillover effects (positive externalities), increasing numbers of goods produced, increasing qualities, etc.
The endogenous growth theory assumes a constant marginal product of capital at the aggregate level, and usually it is possible to construct models with perfect competition. However, in many endogenous growth models the assumption of perfect competition is relaxed, and some degree of monopoly power is thought to exist. Romer (1987, 1990), significant contributions by Aghion and Howitt (1992), and by Grossman and Helpman (1991), incorporate imperfect markets and *R&D* production sector to the growth model. Generally monopoly power in these models comes from the holding of patents. These models have two sectors of production, final output and an *R&D* production sector (see, for example, Comin and Gertler, 2003). The *R&D* production sector develops new ideas, and for that reason *R&D* firms are assumed to be capable of making monopoly profits selling their inventions to final good production firms. Anyway, the free entry condition means that these profits are dissipated on *R&D* spending.[6] The endogenous growth theory considers knowledge investments, innovation, and research, as significant contributors to economic growth. Theory focuses on positive externalities and spillover effects derived from knowledge-based growth, analysing how it

[5]The AK model of economic growth is the simplest endogenous growth model. It considers a constant exogenous saving rate, and assumes a fixed level of technology. It shows the elimination of diminishing returns leading to endogenous growth. Rather than decreasing returns to capital, the model implies, by the usual parameterizations of a Cobb-Douglas production function, a linear model where output is a linear function of capital. Usually it is used in handbooks in order to introduce the endogenous growth theory.

[6]Model as Stiglistz Dasgupta, 1980; Peretto, 1999.

leads to the development of economies.
This class of models called *knowledge-based growth models*[7] investigate the question of growth theory focusing on the analysis of the accumulation of knowledge. These models treat capital accumulation and its role in production in ways that are similar to the earlier models. But they are different, from the previous, in interpreting the effectiveness of labor as knowledge, and in formally modelling its evolution over time. Here, the effectiveness of labor, represents knowledge advancement, or technology progress. Now knowledge accumulation is an endogenous variable. Various views have been considered concerning how knowledge is produced, and about what determines the allocation of resources necessary to knowledge production. These models are also called $R\&D$ models, and their main conclusions are basically two: first, knowledge accumulation is central to worldwide growth but not to cross-country income differences. Second, they start to consider knowledge accumulation, or knowledge capital, as well as physical capital.
Certainly it is plausible that technological progress is the reason that more output can be produced today, from a given quantity of capital and labor, than could be produced a century or two ago. To do this, we need to introduce an explicit research and development sector, and then model the production function of new technologies. We also need to model the allocation of resources between conventional goods production, and $R\&D$ production. In these models, usually, both $R\&D$ and final goods production functions are assumed to be generalized Cobb-Douglas functions.
These models, as the previous, involve four variables: labor, capital, technology, and output. They are set in continuous time. Since most of the time, as I said before, there are two sectors of production, labor force is divided into two fractions. The first one is used in the $R\&D$ production sector, and the second one is used in the final goods production sector. Both fractions are exogenous and constant. In some cases these models of knowledge accumula-

[7] In 1990s the most recent advancement of endogenous growth theory has been the emergence of $R\&D$-models of growth

tion are models without capital, but in order to have a better interpretation it is useful to add it. So finally we end up with models that have two endogenous stock variables: the structures capital and the equipment capital or *R&D*-capital, and also two different laws of motion. An important motivation for work on the new growth theory is to understand variations in long-term growth. Early new growth models focused on constant or increasing returns to produced factors, where changes in savings rates and resources devoted to *R&D* permanently change growth.
Jones (1995) points out an important problem of these models. Over the postwar period the forces, that these models consider important to affect the long-run growth, have all been increasing. Population has been rising steadily, saving rates have increased, the fraction of resources devoted to knowledge accumulation has risen considerably, and the fraction devoted to *R&D* appears to have increased sharply. Thus, new growth models with constant or increasing returns imply that growth should have increased considerably. But in fact, growth shows no positive trend. A crucial debate arose between Romer (1990) and Jones (1995) within the *R&D*-based growth literature. Specifically, the debate was focused on the fact that Romer in his work assumes that the stock of *human capital* determines the economic rate of growth. He states that economic development depends basically on how many workers devote their time to research. On the other hand Jones says that is not enough, what is really important is the efficiency of the inventions and not the quantity.
The simplest interpretation of Jones's results is that there are decreasing returns to product factors. However, several papers suggest another possibility. They continue to assume constant or increasing returns to produced factors, but adding a channel through which the overall expansion of the economy does not lead to a faster growth. Specifically they assume that is the effectiveness of *R&D* activity per sector that determines growth. As a result, growth is steady, despite the fact that the population is rising.
So these last models (see, for example, Peretto, 1998; Howitt, 1999) main-

tain the ability, as the previous new growth models, to explain variations in long-run growth. Anyway they do not imply that worldwide population growth leads to ever-increasing growth. More recently, Jones came back to that emphasizing the fact that the *R&D* share, rates of investments in physical capital, and knowledge accumulation have been rising as well without a corresponding increase in growth. Repeated rises in *R&D* lead not to increasing growth, but to an extended period of above-normal growth (Jones, 2002). Knowledge advancement can have many different forms. At one extreme knowledge is basic scientific such as the mathematic theory, on the other hand knowledge can be about a specific and tangible good. All these ideas and inventions have an important impact on growth and those effects occur with different lags.[8] Moreover ideas can be un-rival, and so open to everyone, or rival when the good is excludable. In the last case, obviously, the impact on growth will take more time. The degree of excludability is likely to have a strong influence on how the development and the allocation of knowledge departs from perfect competition. When knowledge is rival the producers of new knowledge can license the right to use the knowledge at a positive price, and hence hope to earn positive returns on their *R&D* efforts.

1.4 The Business Cycle Theory of Fluctuation

Another step in growth theory analysis is starting to consider the implications on the business cycle.

During the latter half of the 19th century, economists began to note recurrent booms and depressions of the industrial economy in which each trade cycle resembled the others in many respects. In 1935, John Maynard Keynes, totally transformed the analysis of the business-cycle with his *General Theory of Employment, Interest and Money*[9]. Keynes focused his attention on the role of deficient demand in generating and prolonging cyclical downturns. He

[8] see for example Osterloh and Frey, 2000

[9] John Maynard Keynes (1963)

believed that investment operating through its effect on aggregate demand, is the primary engine driving the business cycle. The dominance of Keynes's idea began to wane in the 1970s when the combination of inflation and oil-shocked-induced stagnation in production - stagflation - presented a situation that did not fit in with the traditional Keynesian theory. On the other hand Walsarian neoclassical macroeconomists try to understand the new events by building models of the business cycle based on rigorously specified set of microeconomic assumptions such as utility maximization.[10]. The families of theories that grow out of this first generation of "*microfoundation*" models can be represented by the Lucas model.[11] Later new Keynesian macroeconomists respond with an alternative set of theoretical explanations based on sticky wages and prices. They emphasize the presence of coordination failures that lead to inefficiencies in aggregate equilibrium.
Since the basic forms of the currently popular theories, business-cycle models have been though not the most relevant to represent empirical data. The principal reason is that various model are observationally equivalent. This means that similar outcomes are consistent with several theories, even if these theories are not equivalent. Theories that have very different implications for the optimal design of economic policy, may share many of the same predictions about observable relationship among variables. A second reason for multiplicity of models is that empirical evidence itself is subject to alternative method of measurement and interpretation. To take one example, it matters greatly whether one considers the cyclical behaviour of the price level or of the inflation rate.[12] Similarly, authors using different methods and data sets have found real wages to be procyclical, countercyclical and acyclical. Each of these possibilities is supported by one or more business

[10]see Dymsky (1988)

[11]That model includes imperfect information and market clearing.

[12]See for example Stock and Watson (1999) which shows that cyclical movements in the level of GDP price deflator are negatively correlated with output movements, while the inflation rate of deflator is positively correlated with output.

cycle models.[13]

Because of those reasons in some models non-Walrasian features have been added, which improve the model's fit with the data. This style of modelling, through empirical evaluation, has modified the basic RBC model with variations and extensions. One change to the model that has attracted considerable attention has been the addition of indivisible labor; see Rogerson, 1988; Hansen, 1985. A second extension is to include distortionary taxes; see Greenwood and Hoffman, 1991; Baxter and King, 1993; Campbell, 1994; Braun, 1994; and McGrattan, 1994. Another important extension in the model is the inclusion of multiple sectors, and sector-specific shocks. Long and Plosser (1983) developed a multisector model and investigated its implications for the transmission of shocks among sectors. Finally, Lilien (1982) proposesed a distinct mechanism through which sectoral technology shocks can cause employment fluctuation.

Classification of business-cycle models

Early time business cycle research is dominated by theories of endogenous business cycle, in which the economy follows a cyclical trajectory even in the absence of external shocks. In these theories the boom lays the seeds for its own demise and for the ensuing slump.[14]. Endogenous cycle models have fallen out of favour in recent decades and have largely been replaced by *impulse-propagation-models.* In those models business cycle results come from the response of the economy to an exogenous shock.[15].

There are many individual theories within the class of *impulse-propagation models* that vary in a number of ways. Anyway, we can divide two main groups:

[13]see Gali, Gertler and Lopez-Salido (2007) they underline that the marginal rate of substitution is likely to be procyclical, rigidities in the real wage resulting either from nominal or real rigidities, will give rise to countercyclical movements in the wage markup.

[14]Examples of endogenous-cycle models from the earlier literature include Goldwin, 1948 and Hicks, 1950

[15]Much of this history is discussed in Chatterjee, 2000

- Theories that retain the *Walrasian or Neoclassical* assumptions that prices and wages are perfectly flexible, and that supply equals demand in every market at all times. We analyse these models better later on.

- And *Keynesian theories* which feature markets that do not clear because of imperfect adjustment to wages and prices.

This last class of models emphasize the rigidities and the coordination failures which prevent markets from clearing. They often place greater importance on short-run outcomes, rather than on the long-run effects. As the New Classical approach, new Keynesian macroeconomic analysis, assumes that households and firms have rational expectations. But the two schools differ in that respect: new Keynesian analysis usually assumes a variety of market failures. Particularly, new Keynesians assume that there is imperfect competition in prices and wage setting. Prices and wages become "*sticky*", they do not adjust instantaneously to changes in economic conditions. That *stickiness*, and the other market failures present in new Keynesian models, imply that the economy may fail to attain full employment. Therefore, new Keynesians argue that macroeconomic stabilization by the government (using fiscal policy), or by the central bank (using monetary policy) can lead to a more efficient macroeconomic outcome than a *laissez faire* policy. As in the previous endogenous growth theory, new Keynesians assume that policy measures can have an impact on the long-run growth rate of an economy. For example, subsidies on research and development, or education increase, develop the growth rate by raising the incentives to invest in innovation.
Significant early contributions to the new keynesian theory are compiled in 1991 by Gregory Mankiw and David Romer. These papers focused mostly on microfoundations, including microeconomic ingredients that produce Keynesian macroeconomic effects.

1.5 The Real Business Cycle Theory

In this chapter we focus on "*real business cycle" models.* These models attempt to explain the business cycle entirely within the framework of an efficient competitive market equilibrium. They are a direct extension of the Ramsey growth model, however unlike the Ramsey model, the rate of technological progress is assumed to vary over time in response to shocks, which leads to fluctuations in the growth rate. Hence, RBC models require a source of disturbance, without which the economy converges to a balanced growth path and then grows smoothly. In addition to that, RBC models need to allow for variations in employment. Models in this literature allow for changes in employment by making household's utility depends not just on his consumption but also on the amount of labor supply and labor demand. As I said before, neoclassical macroeconomics view the Walrasian market clearing as an appropriate paradigm for economic analysis. In contrast with the new Keynesian assumptions, they argue that prices are relatively flexible, and those long-run considerations are more relevant than the short-run ones. Within the neoclassical theory, there are two main kinds of models:

- The first group follow the work of Lucas, Sargent, Wallace and others in the 1970s; these models are set up on continuous time, and based on market clearing. They consider an environment where agents have imperfect information.
- On the other hand, the second group follows the Kydland and Prescott model, they developed the first *real business cycle model* in the 1980s.[16].

Unlike the imperfetc-information models, the pure RBC model introduces no imperfections into the system and hence it assumes perfect competition, perfect information, and instantaneous market clearing. The RBC model is a stochastic growth model, and it allows for random fluctuations in the rate of growth. Under the Walrasian assumption, the level of output is always at

[16]Kidland and Prescott, 1983 is usually quoted as the seminal paper in RBC theory.

its natural level, the level which is consistent with full employment of labor and full utilization of capital, given the state of available technology.
The Walrasian theory would have to explain business cycle fluctuations in a natural level of output, rather than fluctuations of actual production around the natural level. The traditionally Walsarian view assumes that technological progress, changes in the labor force, and in capital stock, usually have a smooth trend. If the determinants of natural output move smoothly rather than cyclically, then the Walrasian model cannot explain the business cycle. Anyway two aspects have enabled RBC modellers to construct a Walrasian competitive equilibrium model with a business-cycle. First, the model recognizes that technological progress does not necessarily occur smoothly but it may instead have decreases and flows, perhaps even a period of regress (Greenwood, Hercowitz and Krusell, 1998). Second, RBC modellers consider the *endogenous propagation mechanism*[17]; it causes changes in the rate of technological progress, and affects other variables in a way that leads to co-movements that resemble those we observe in the business-cycle (Greenwood, Hercowitz and Krusell, 1998). The central question that RBC models emphasize is if it is possible to observe movements over the business-cycle without stepping outside the competitive Walrasian model? They claim that the ability to replicate real-life co-movements among major macroeconomic variables using a model, which is purely Walrasian, means that Keynesian concepts of wage and prices stickiness are not essential to explain the economic fluctuation.
Despite a large amount of attention that RBC models have received in the last three decades, most macroeconomics remain skeptical.
Another RBC's key characteristic is that they focus their attention on general equilibrium. A Keynesian macroeconomist is more interested in a single firm response to a one-time monetary disturbance, instead an RBC macroecominist is much more likely to build a dynamic model where the money supply follows a stochastic process, and examines the resulting general equi-

[17]The propagation mechanism is well described in Comin and Gertler, 2003

librium. Because the modern models in the RBC tradition focus on general equilibrium and fully specify the behaviour of the driving variables, they are often referred to as dynamic stochastic general equilibrium (or DSGE) models (these models are evaluated by calibration). The main difference between a RBC model and a Keynesian model is again a broad and narrow focus. A Keynesian modeller considers the microeconomic evidence. RBC models have moved away from the original idea of using microeconomic evidence, to tie down all the relevant parameters and functional forms. Given the model's wide variety of features, they have some flexibility in matching the data.
An RBC model recommends very different business-cycle policies than a Keynesian model. Keynesian models emphasize the inefficiency of cyclical fluctuations and especially the waste resulting from unemployed resources during a recession. RBC models, instead, claim that cyclical fluctuations are efficient responses of the economy to unavoidable variations in the rate of technological progress. Thus, RBC advocates argue that government action to stabilize the economy through aggregate demand is inappropriate. The major issue of the real business-cycle model is whether it is capable of explaining the pattern of movements that are characterized by the modern business-cycle. Opinions on the empirical performances of the RBC models vary.

Objections to the real-business-cycle model

There are four objections to the real-business-cycle model:

1. The first concerns technology shocks, the standard RBC model considers technology shocks with a standard deviation of 1 percent each quarter. Anyway, it is usually difficult to identify specific innovations associated with quarter-to quarter swings. Mankiw (1989) and Bernanke and Parkinson (1991) found the Solow residual moves may be a poor measure of economic changes especially in a short-run analysis. There

is significant evidence that short-run variations in the Solow residual reflect more than changes in the pace of technological innovation.

2. The second criticism of the model concerns not the shocks but one of its central propagation mechanisms, intertemporal substitution in labor supply. Variations in the incentives to work in different periods drive employment fluctuations in the model. Microecominic studies have found little support for this view of employment fluctuations: intertemporal elasticity of substitution is usually low, and changes in the quantity of labor happened through this channel are small. Moreover, a prediction of the model that changes in labor demand have an impact on wages, is rejected by the data (see MaCurdy, 1981; Altonji, 1986; and Ham and Reilly, 2002).

3. The third criticism is the omission in a basic RBC model of monetary disturbances. A central issue of the model is that fluctuations are due to real rather than monetary shocks. Anyway, there is strong evidence that monetary shocks have important real effects, this finding means that basic RBC models omit one source of output movements.

4. The fourth criticism is about the dynamic of the model, which usually does not look like the actual data. Rotemberg and Woodord (1996) demonstrate that predictable output movements in the basic RBC model are much smaller than what we observe in the data, and have very different characteristics.

It is definitely true that, since the Great Depression, the Keynesian view has dominated macroeconomic research with the idea that recession is a reduction in output below the natural level, and not the declines in the natural level itself. Anyway the intellectual combat between RBC modellers and

Keynesian theories obscures a crucially important point: it is entirely plausible that business-cycle fluctuations reflect both uneven movements in natural output, and movements in actual output, away from a natural view. Even if RBC models are capable of reproducing business-cycle movements, this does not mean that the source of fluctuations emphasized by these models, is the only source of business-cycle. Most macroeconomists agree that the oil-shock of 1970-s had a substantial macroeconomic impact, as predicted by the RBC models and verified constantly by much macroeconomic research. However, most macroeconomists also believe that monetary movements and other shifts in aggregate demand have a strong influence in short-run economic activity, as in Keynesian models.

Introducing the random technological shock

In this section I'm going to explain a fundamental element of the RBC model, the technological shock, as a source of disturbance.[18] RBC models are stochastic and that means they include random elements. The shock represents real, opposed to monetary or nominal, disturbance and it changes the amount that is produced from a given quantity of inputs. Without these random shocks the model would converge to a balanced-growth path, without considering the business cycle analysis. It is the shock that introduces cyclical behaviour in the model.
The log of the random variable is determined as the sum of a trend, and a shock. The shock is then decomposed into two parts: one, representing the tendency for past shocks to persist, and the other measuring the new innovation in the shock (it is the lowest-level random variable). At any point in time, productivity is subject to the shock that moves, above or below, its trend level. The easiest way to model the shock is to consider it a random disturbance which is totally independent over time. But if each period's shock

[18] Many models have treated this argument, see for example Gali, 1996; Cogley and Nason, 1995; Christiano Eichenbaum and Vigfusson, 2003

is independent from the ones before and the ones after, then we would expect to see productivity chaotically jumping back and forth around its trend line. This is not very realistic; evidence suggests that productivity tends to have sustained periods in which it is above, or below, its trend. So the random shock variable (z_t) follows a first-order autoregressive process, where innovation is represented by a *white noise random process* (ε_t), meaning that it cannot be predicted ahead of time.[19]

$$z_t = \rho_{t-1} + \varepsilon_t$$

and

$$\varepsilon_t \sim (0, \sigma)$$

An important goal of a business-cycle model is to explain the persistence of macroeconomic fluctuations, it assumes exogenous persistence rather than explaining persistence endogenously. These models determine how much of the persistence in business-cycle fluctuations is explained endogenously by the model, and how much is attributed to the persistence of shocks hitting the economy.[20]

The investment-specific technological change

How should investment-specific technological change, or capital-embodied technological change, be modelled? As Hulten (1992) has highlighted, two distinct accounting frameworks have been used to study this form of techno-

[19] see Romer, chapter 4

[20] It is interesting to note that the two papers that originated the literature on real business-cycle had a more elaborated propagation mechanism than those in much of the subsequent literature. Kydland and Prescott (1982) assumed that investment projects required several periods to complete (time to build), while Long and Plosser (1983) used an input-output structure where changes in demand take multiple periods to work their way through the purchasing of inputs to production.

logical change. The first was developed by Solow (1960),[21] and the second, which has dominated the practice of growth accounting (see Gordon 1990), is due to Domar (1963) and Jorgenson (1966). Here, I treat these two views using the notation of Greenwood Hercowitz and Krusell (1998). Both frameworks express the law of motion for equipment capital as:

$$k_{e,(t+1)} = (1 - \delta_e)k_{e,t} + i_{e,t}q_t$$

In both frameworks, δ_e represents the physical depreciation on equipment capital $k_{e,t}$; $i_{e,t}$ represents investments in equipment; and q_t represents the current state of the technology for producing equipment. The models differ in the way they express the resource constraint. Again, in line with the Greenwood et al's notations, the resource constraint for the Solow model reads:[22]

$$c_t + i_{e,t} = z_t F(k_{e,t}, l_t)$$

Where c_t represents the consumption variable, $i_{e,t}$ stands for the investments in equipment, z_t is the random shock, and $F(k_{e,t}, l_t)$ is the production function based on equipment capital $k_{e,t}$ and labor l_t. On the other hand the resources constraint in the Domar-Jorgenson model appears as:

$$c_t + i_{e,t}q_t = z_t F(k_{e,t}, l_t)$$

Thus, the sole difference between the two models is the inclusion of q_t in the resource constraint. Which model, then, is better suited to analyze the long-run effects of investment-specific technological change? The analysis

[21] That approach is similar to one used in Greenwood Hercowitz and Krusell (1997, 1998). I will come back to their model later.

[22] For simplicity structures capital, $k_{c,t}$, has been dropped from the analysis. Instead of $F(k_{c,t}, k_{e,t}, l)$ here we have $F(k_{e,t}, l_t)$.

of Greenwood et.al suggests that both, theory and data, speak clearly in favour of the Solow approach. When embedded into a fully specific general equilibrium setting, the Domar-Jorgenson specification does not allow for investment-specific technological change to operate as an engine for growth. This is easy to see using a simple change of variable. Define $x = i_{e,t}q_t$, so previous equations can be rewritten as:

$$k_{e,(t+1)} = (1 - \delta_e)k_{e,t} + x$$

and

$$c_t + x = z_t F(k_{e,t}, l_t)$$

Clearly, this is the conventional neoclassical growth model. Given that $i_{e,t}$ and q_t do not enter separately into the model, an optimal allocation for c_t, $k_{e(t+1)}$, and l_t is independent from the behaviour from q_t. Agents choose the same path for x regardless of the behaviour of q_t. To conclude, the Domar-Jorgenson framework does not allow for investment-specific technological changes to affect growth. It is possible to recast the model with investment-specific technological change, as represented in equation $k_{e,(t+1)} = (1 - \delta_e)k_{e,t} + i_{e,t}q_t$ and equation $c_t + i_{e,t} = z_t F(k_{e,t}, l_t)$, so that it appears as a conventional model with neutral technological change. Solow (1960), illustrates this fact for a vintage capital model with investment-specific technological change. Growth accounting could be done using this alternative formulation of investment-specific technological change. A key variable in the transformed model is the *economic rate of depreciation.* Investment-specific technological change can be measured by the spread between the economic and the physical rates of depreciation.[23]

[23]see Greenwood Hercovitz and Krusell, 1997

1.6 Understanding the Greenwood Hercowitz and Krusell model

The role of technology change in business cycle fluctuation attracted the attention of macroeconomics, particularly since the seminal work of Kydland and Prescott (1982), and Long and Plosser (1983). In these studies and in the literature that followed, technological change is modelled as an aggregate, sectoral-neutral, productivity shock.[24] The main result is the surprisingly high degree to which this type of shock, when incorporated into a stochastic growth model, can explain a set of business cycle phenomena. A characteristic for this setup, with sector-neutral productivity change, is that relative prices of different uses of output are assumed to be fixed. [25]
Greenwood Hercowitz and Krusell (1997, 1998) demonstrate that this setup is not equipped to address the evidence from the post-war U.S. period; they, instead, suggest an important link between relative prices and technological investments. As they prove, in both low and high frequencies, the relative price of new equipment goods declines at an annual rate. On the other hand, they demonstrate that investment in equipment goods increases substantially over time. They show a negative co-movement between price and quantity of new equipment goods at both, low and high, frequencies. That suggests the presence of investment-specific technological change - in contrast to the sector-neutral form referred to above - affecting the production of new equipment goods. Concrete examples of this type of technological change are well known: more powerful computers, faster and more efficient means of telecommunication infrastructure and transportation, etc. Technological advances have made equipment less expensive, triggering increases in the accumulation of the equipment demand curve. So the fall in the relative price of

[24] $y_t = z_t F(k_t, l_t) = z_t k_t^{\alpha} l_t^{1-\alpha}$; this is a typical example of a Cobb Douglas production function with a sectoral neutral productivity shock. Here the variable z_t is a measure of TFP, or neutral, shock.

[25] Many papers assume that U.S. post-war quarterly data about total factor productivity (TFP), and data about the relative price of the equipment investment are cointegrated.

new equipment goods is a direct, micro-based measure of investment-specific technological change.
The long-run implications of the investment-specific technological change are analyzed in Greenwood, Hercovitz and Krusell (1997). By analyzing the balanced growth path for their model, they conclude that the contribution of the investment-specific technological change to U.S. post-war economic growth, explains about 60 percent of the growth in output per hours worked. Residual coming from neutral productivity change, then accounts for the remaining 40 percent. In order to analyze the role of the investment-specific technological change as an engine for growth, they use a simple vintage capital model, embedded into a general equilibrium framework. The main feature of the model is that the production of equipment goods becomes increasingly efficient with the passage of time.
Later in the Greenwood Hercovitz and Krusell (1998), the focus is on the short-run implications. They analyze the quantitative role of investment-specific technological change in the generation of business cycles.[26].
The Greenwood Hercovitz and Krusell (1998) model is important because, unlike the standard real business cycle model, here the technology shock does not directly affect the production in the current period.
At the current period, instead, we have only an increase in equipment capital, and in labor, in response to changed investment opportunities. The transmission mechanism to current output described in the Greenwood Hercovitz and Krusell model, is the following: a positive shock raises the return on equipment investments. This entices equipment investments and hence a higher equipment goods stock in the next period. The resulting decline in the equipment goods' replacement value implies a lower marginal utilization cost. This promotes a more intensive utilization of the existing equipment goods, which leads to an increase in the employment of labor and to output expan-

[26]In both cases the models do not give an endogenous explanation of the equipment good production function, and they have just one sector of production. The consumption good sector.

sion. The increase in the rate of return on equipment investments stimulates production, however, at the same time it operates to dissuade consumption. Hence, it is a priori uncertainty whether consumption is procyclical in the model. An important aspect of the Greenwood, Hercovitz and Krusell (1998) analysis is that the process for investment-specific technological change is estimated using the equipment's price series. This has an advantage over the real business cycle literature, which emphasizes the *Solow residual* as the driving force underlying the business cycle. This residual may include other influences, besides technological change.[27]
The Greenwood Hercovitz and Krusell's model is related to the work written by Hulten (1992), which also stresses capital-embodied technological change as a key variable to long-run productivity movements. Both works use Gordon's (1990) price index, which was constructed precisely to capture the increased productivity in the production of new equipment goods. A key distinction between the two papers, however, is the adoption of a general equilibrium approach, into the Greenwood Hercovitz and Krusell's model.
In line with conventional growth account literature, Hulten (1992) uses an aggregate production function to decompose output growth into technological change and changes in input, in particular capital accumulation. A large part of capital stock growth reflects the endogenous response of capital accumulation to technological change. Considering a general equilibrium approach the current analysis can go one step further: inferences can be made about how much capital stock growth is due to investment-specific technological change, versus neutral productivity growth. As highlighted by Hulten (1992), there is a controversy in the growth account literature over whether or not GDP should be upwardly adjusted to reflect quality improvements in new equipment goods. The Greenwood, Hercovitz and Krusell model pro-

[27] Government spending, for instance, tends to be positively related to the Solow residual, and energy price negatively. Finn (1995) has explained these correlations by modelling the effect that such factors have on capacity utilization.

vides a decisive answer to that question: it should not be.[28]
The analysis in Greenwood Hercovitz and Krusell's is motivated by the negative co-movement between the relative price of new equipment goods and equipment investments growth rate. This evidence suggests that the investment-specific technological change may trigger equipment investment and be a source of economic fluctuation. The kind of technological change considered here is *embodied in the form of new equipment*. It represents phenomena such as advances in computer technology, robotization of assembly lines, faster and richer means of telecommunication, etc.
Although the contribution of the Greenwood Hercovitz and Krusell (1998) is relevant, it is important to highlight that their model does not explain how the new equipment goods are created, they just observe the relationship between new equipment goods produced and their price level. They do not introduce an endogenous explanation of growth, avoiding to explain the role of technological progress in the economic development process. Differently *R&D*-based models (Romer, 1990; Grossman and Helpman, 1991; and Aghion and Howitt, 1992) view technological progress as the primary determinant of growth and they treat it as an endogenous variable. At the heart of *R&D*-based growth models there is a knowledge technology production function that describes the evolution of knowledge creation. According to that function, the rate of production of new knowledge depends on the efficiency of the labor engaged in the *R&D* sector of production, and on the existing stock of knowledge available in the economy. In literature, there is a lack of studies about the integration between these two points of view. For that reason, that work tries to explain what Greenwood Hercovitz and Krusell have found about long-run growth using an increasing knowledge production function which depends on: the equipment labor force, the existing stock of knowledge, and the schooling-education growth rate in the economy.

[28]Note that equipment investment is only 7 percent of GDP with only 18 percent of the value of output derived from the use of equipment in production. Hence, differently from the standard RBC models, the fraction of GDP directly affected by the shock is quite small. Greenwood Hercovitz and Krusel (1998)

1.7 Conclusion

This chapter provides an overview of economic growth models, and concludes that these kind of models have to pay special attention to one of the four fundamental factors of potential growth in output: *the technological progress.*

Technological improvements lead to a capital deepening process and allow the output per worker to increase over time. On the other hand technological progress is usually linked to an increase in the education and schooling rate in the economy.

In chapter 3 I will present a DSGE model where technological progress is strictly connected to the knowledge growth rate. Some papers argue that an economy's deficiency in education and training may be intimately related to firms' investments in research and development (Redding, 1996). The two forms of investments exhibit externalities and are strategic complements; the incentives for both forms of investments are interdependent.

Using the Greenwood Hercovitz and Krusell (1998) framework as a starting point, I will analyze a basic two-sectors model. Where one sector produces consumption goods and structures, and the other manufactures equipment goods. The equipment production sector needs to be much more intensive in its use of knowledge than the consumption of goods. In the model, growth is driven explicitly by the accumulation of knowledge (following the theory related to the *R&D*-based models).

Most of the existing *R&D* models employ setups with monopolistic competition (afterwards Romer 1987). Hence, there would be a range of different types of equipment, each associated with a producer who makes a product-specific *R&D* decision. In this setup, new products are not priced at marginal cost, and so, relative price movements may also capture movements in markups. Therefore, this makes the identification of the rate of

relative price decline problematic, in case of the presence of an investment-specific technological change (Krusell 1992). For that reason I will present a model of centralized economy with perfect competition.
Before analyzing the DSGE model, let's have a look at the European empirical data about the relation between *R&D* expenditure, and the education rate in the economy.

Chapter 2

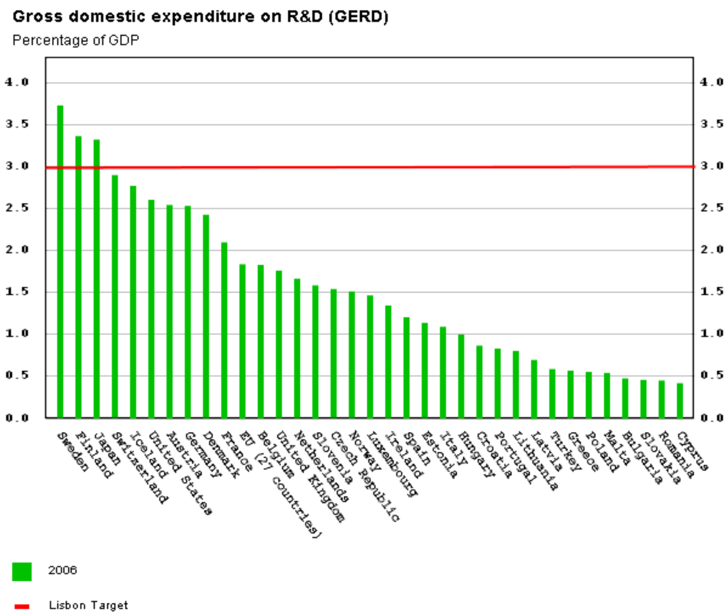
Empirical Analysis

This second chapter introduces an empirical analysis about the relationship between $R\&D$ expenditure, and the education growth rate in the economy.

2.1 Empirical Analysis on the European Data

As I said in the previous chapter the relation between $R\&D$ expenditure, or technological investments, and the education level in the economy is not just an academic issue. Most European research is funded at national level, by private and/or public sources. This chapter presents data on $R\&D$ spending within the European Union (EU), according to the sector performing the research and according to the source of funds. Framework programmes are the main instrument for funding $R\&D$, within the EU. The 7th framework programme (FP7) for research and technological development started in 2007 and is due to continue for a total of seven years. In the following figure the $R\&D$ expenditure at regional level is represented, in light of the Lisbon strategy (2006) .

Figure 2.1: *R&D* expenditure at regional level in light of the Lisbon target (2006)



The *Horizon 2020* is planned as the framework programme for research and innovation after 2013. The European Research Area (ERA) is composed of all research and development activities, programmes and policies in Europe which involve a transnational perspective. In December 2008, the Competitiveness Council adopted a 2020 vision for the ERA, which foresees the introduction of a 'fifth freedom' - namely, the free circulation of researchers, knowledge and technology.

Europe 2020, a strategy for jobs and smart, sustainable and inclusive growth, is based on five EU headline targets[1] which are currently measured by eight headline indicators[2].

The total gross domestic expenditure on research and development comprises of: business enterprise expenditure on *R&D*, higher education expenditure on *R&D*, government expenditure on *R&D* and private non-profit sector expenditure on *R&D*. The indicator measures the key *R&D* investments that support future competitiveness and result in higher GDP. *R&D* expenditure represents one of the major driving forces in economic growth in a knowledge-based economy. As such, trends in the *R&D* expenditure indicator provide key indications of the future competitiveness and wealth of the EU. Research and development spending is essential for making the transition to a knowledge-based economy as well as for improving production

[1] 1) 75 per cent of the population aged between 20-64 should be employed; 2) 3 per cent of the EU's GDP should be invested in *R&D*; 3) Reduction of greenhouse gas emissions by 20 per cent compared to 1990. Increase in the share of renewable energy sources in final energy consumption to 20 per cent. 20 per cent increase in energy efficiency; 4) The share of early school leavers should be under 10 per cent and at least 40 per cent of 30-34 year olds should have completed a tertiary or equivalent education; 5) Reduction of poverty by aiming to lift at least 20 million people out of the risk of poverty or its exclusion.

[2] 1) Employment rate by gender, age group between 20-64; 2) Gross domestic expenditure on *R&D* (GERD); 3) Greenhouse gas emissions, base year 1990; 4) Share of renewables in gross final energy consumption; 5) Energy intensity of the economy (proxy indicator for Energy savings, which is under development); 6) Early leavers from education and training by gender; 7) Tertiary educational attainment by gender, age group between 30-34; 8) People at risk of poverty or social exclusion (union of the three sub-indicators below). People living in households with very low work intensity. People at-risk-of-poverty after social transfers. Severely materially deprived people.

technologies and stimulating growth. Recognising the benefits of *R&D* for growth and being aware of the rapidly widening gap between Europe's *R&D* effort and that of the principal partners of the EU in the world, the Barcelona European Council (March 2003) set the EU a target for increasing *R&D* expenditure to 3 per cent of GDP by 2010, two thirds of which should come from the business enterprise sector. Investing 3 per cent of GDP is one of the headline targets in the new Europe 2020 strategy for developing an economy based on knowledge and innovation.[3]

Restriction of the indicator's relevance and other characteristics which may lead to restrictions in using it in monitoring and reporting:

GERD includes total intramural expenditure on *R&D* performed within a country, funded nationally and from abroad but excludes payments for *R&D* performed abroad. To complete the picture, information on international purchases of *R&D* performed abroad should be taken into account. Moreover, an emerging EU emphasis on encouraging international collaboration in *R&D* may not be fully revealed as recording each partner's actual (intramural) *R&D* expenditure only understates the investment, provided all parts have full access to the outcome of the project. In some countries, small enterprises with less than 10 employees or some economic activities where *R&D* activity is expected to be negligible, are excluded from the *R&D* surveys. However, this leads only to minor impact on the aggregates. For some countries which attract significant foreign direct investments, the use of GDP as a denominator restricts relevance whereas these investments are visible in GDP and high-tech export figures for countries where investments are made, *R&D* work may be performed in investor countries and they are not visible in *R&D* expenditure figures for the countries where the investments have been made. In these cases it would be better to use Gross National Income (GNI) as denominator, provided all transactions between *R&D* -exporting

[3]Data is collected from reliable sources applying high standards with regard to the methodology. Shortcomings, with regard to the comparability over time, are well documented.

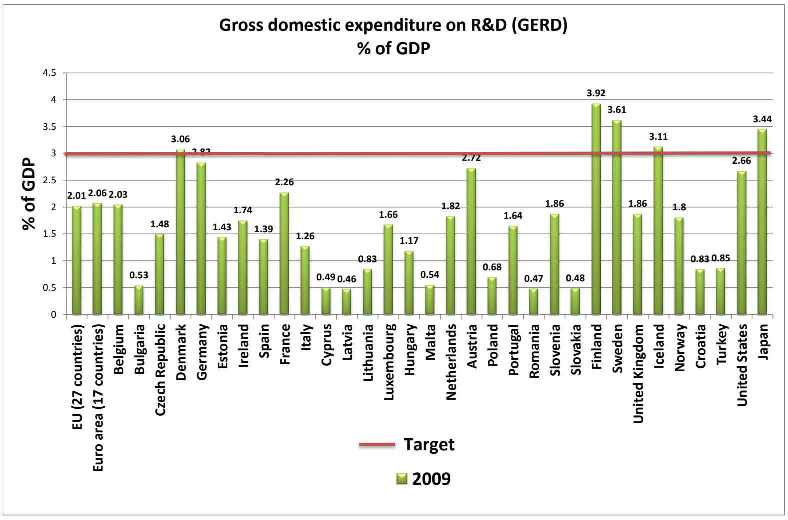
and importing countries are measured. Measurement problems may occur in the case of multi-national analysis.

2.2 Main statistical findings

Gross domestic expenditure on *R&D*, (GERD) stood at EUR 236.820 million in the EU-27 in 2009, which marked a 1.2 per cent decrease on the level of GERD in 2008, but was 50.3 per cent higher than ten years earlier (1999) - note that these rates of change are in current prices and so reflect price changes as well as real changes in the level of expenditure. In 2008, the level of expenditure on *R&D*, in the EU-27 was 88.5 per cent of that recorded by the United States, although slightly more than double the level of expenditure in Japan and considerably above *R&D*, expenditure levels in the emerging economies - for example, EU-27 expenditure was 5.3 times as high as in China.
In order to make figures more comparable, GERD is often expressed as relative to gross domestic product (GDP). The ratio of GERD compared to GDP increased marginally in the EU-27 during the period up to 2002, reaching a high of 1.87 per cent, before declining modestly through to 2005 (1.82 per cent), and climbing again to 1.92 per cent by 2008 and 2.01 per cent by 2009. The ratio of GERD to GDP increased between 2008 and 2009 despite a fall in the absolute level of expenditure; this can be explained by GDP falling even more than GERD during the financial and economic crisis. Nevertheless, the EU-27's *R&D* expenditure relative to GDP remains well below the corresponding shares recorded in Japan (3.44 per cent) and the United States (2.77 per cent) in 2008; this pattern has existed for a lengthy period. There was a far higher increase in the relative importance of GERD in the Japanese economy, as its share of GDP rose by 0.42 percentage points during the period from 1999 to 2008; note, however, that the Japanese economic growth was also subdued during this period.
One of the key objectives of the EU during the last decade has been to en-

courage increasing levels of investment, in order to provide a stimulus to the EU's competitiveness. Using this measure, the highest *R&D* intensity in 2009 was recorded in Finland (3.96 per cent), Sweden (3.62 per cent) and Denmark (3.02 per cent). While none of the other Member States reported GERD rising above 3 per cent of GDP at a national level, *R&D* intensity also rose to relatively high levels in a number of regions, for example in Baden-Württemberg and Berlin (Germany), the east of England (United Kingdom), and southern Austria. There were eight Member States that reported *R&D* expenditure accounting for less than 1 per cent of their GDP in 2009, with Latvia, Cyprus, Romania and Slovakia below 0.5 per cent. The regions with the lowest *R&D* intensity were generally in southern and eastern Europe. In the following figure is represented the *R&D* expenditure at regional level, in the light of the Europe 2020 strategy.

Figure 2.2: *R&D* expenditure in the light of the Europe 2020 strategy



The differences in the relative importance of *R&D* expenditure between countries are often explained by referring to the levels of investments within the business enterprise sector. An important indicator is also the education rate in the economy. Many authors have studied the relationship between *R&D* and knowledge, also recent works as: Machin and Van Reenen (1998); Griffith, Redding and Van Reenen (2004); De Rassenfosse, and Van Pottelsberghe (2009).

The Objectiveness and relevance of the indicator:

Combating early school leaving is an integral part of the new *Europe 2020 Strategy*, which is the successor to the *Lisbon Strategy* to enhance Europe's competitiveness. It was set a target of 10 percent, or less, of early school leavers by 2020. An operational objective of the renewed Sustainable Development Strategy is to ensure that at least 85 per cent of 22 year olds should have completed upper secondary education. Education is critical to promote sustainable development. It is essential that all people have a set of basic knowledge and skills in order to fully participate in society. This is crucial in social and political life but also for smoothly entering the labour market, and will enable young people to understand and adapt to our quick-evolving societies, especially in the context of globalisation. Reducing the number of early school-leavers is crucial in the European Union, because better educational levels help employability and progress in increasing the employment rate helps to reduce poverty.

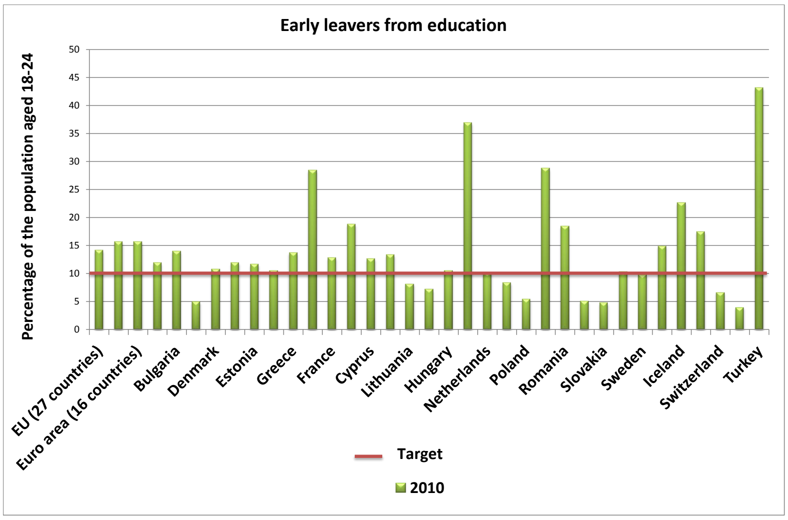
Restriction of the indicator's relevance and other characteristics which may lead to restrictions in using it in monitoring and reporting.

Students living abroad for one year or more and conscripts on compulsory military or community service are not covered by the EU Labour Force Survey, which may imply higher rates than those available at national level. This is especially relevant for Cyprus. The results do not cover persons living in institutional households neither.[4] In the following figure, an analysis about

[4]Data on early school leavers are collected from reliable sources applying high standards with regard to the methodology and ensuring high comparability across countries. Due to the heterogeneity of the implementation of certain concepts in the Labour Force Survey the comparability over time is restricted. Some tests are foreseen over the period between

early school leavers is presented, in light of the *Europe 2020 Strategy*.

Figure 2.3: Early school leavers, in light of the Europe 2020 strategy



A second indicator measures the share of the population aged between 30-34 who have successfully completed tertiary (or equivalent) education (ISCED 5-6). The Europe 2020 strategy for jobs, smart, sustainable and inclusive growth should help Europe to recover from the crisis and become stronger, both internally and at an international level, by boosting competitiveness, productivity, growth potential, social cohesion and economic convergence. The European Council gave its political endorsement on the 17 June 2010 to increase participation in tertiary education: the share of the 30-34 year olds having completed tertiary or equivalent education should be at least 40 per cent by 2020. Education has a central role in this important strategy in terms of fostering both societal and economic progress across the EU. It is crucial for young people's transition from education into the labour market and for their successful integration in society. Higher educational attainment levels increase employability and reduce poverty in the context

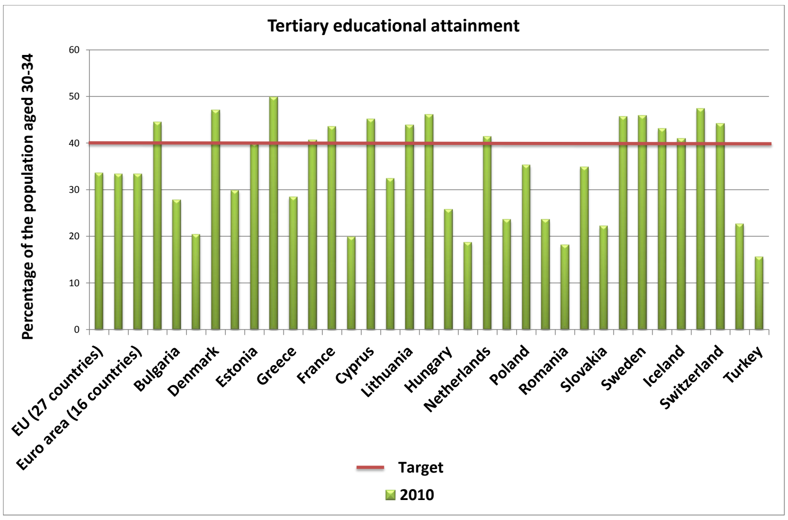
2010-2011 to improve the quality of the indicator.

of a knowledge-based economy.

Restriction of the indicator's relevance and other characteristics which may lead to restrictions in using it in monitoring and reporting.

Successful educational attainment of the younger generation already at upper secondary level is an important complementary indicator for monitoring the progress in the increase of the population's educational attainment. The selection of the age group (30-34 years) excludes people who complete tertiary education at a higher age (i.e. people returning to formal education in their thirties).[5] In the following figure is presented an analysis about tertiary educational attainment, in light of the Europe 2020 strategy.

Figure 2.4: Tertiary educational attainment, in light of the Europe 2020 strategy



Finally, evaluation of the data for the Member States confirms that those countries with relatively high shares of business enterprise expenditure on *R&D* and the higher education sector - namely, Finland, Sweden, Denmark,

[5]Data are collected from reliable sources applying high standards with regard to the methodology - European Union Labour Force Survey.

Austria and Germany - also reported relatively high levels of total research activities and patent production.

Triadic Patent Families are defined as a set of patents taken at the European Patent Office (EPO), the Japan Patent Office (JPO) and granted by the US Patent and Trademark Office (USPTO) to protect the same invention. Patent counts are based on the earliest priority date, the inventor's country of residence and use fractional counts. Data mainly derive from EPO Worldwide Statistical Patent Database (October 2007).
The patent production area has received considerable attention in recent years because of the structural difference in *R&D* funding between Europe and its main competitors. Policymakers in Europe have tried to increase *R&D* business expenditure so that it is more in line with relative contributions observed in Japan or the United States. In the following figures is represented the patent growth rate, source: OECD, Patent Database, June 2008.

Figure 2.5: Differences in $R\&D$ Expenditure between Europe, USA, and Japan.

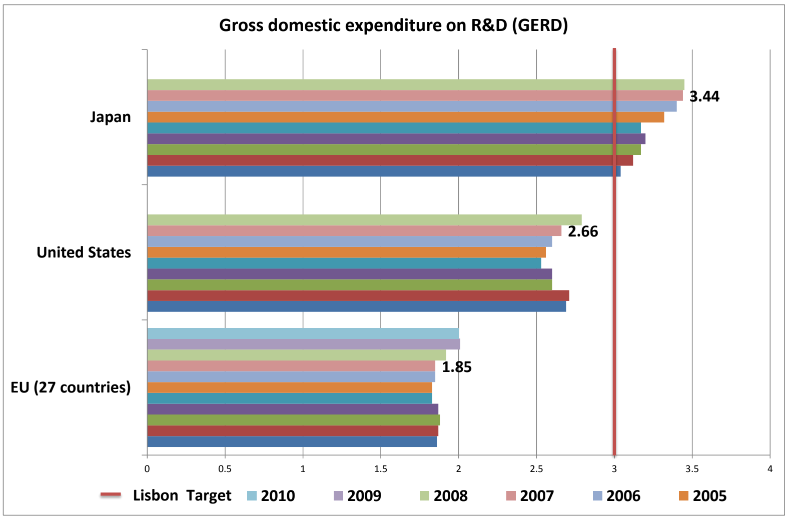
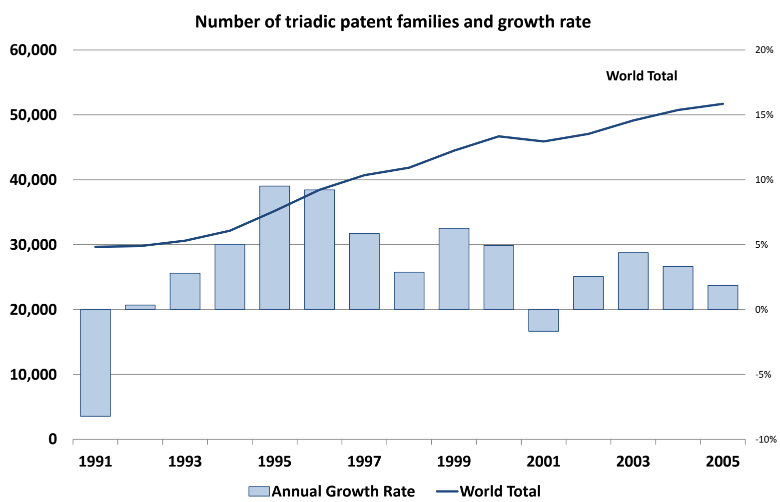


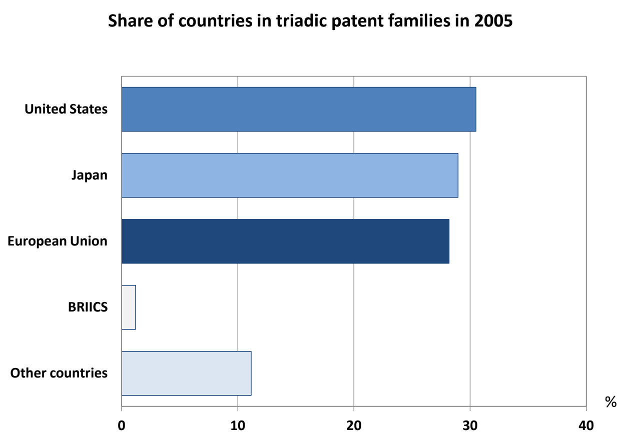
Figure 2.6: Patent Growth Rate in the World



2.3 Trends in triadic patent families

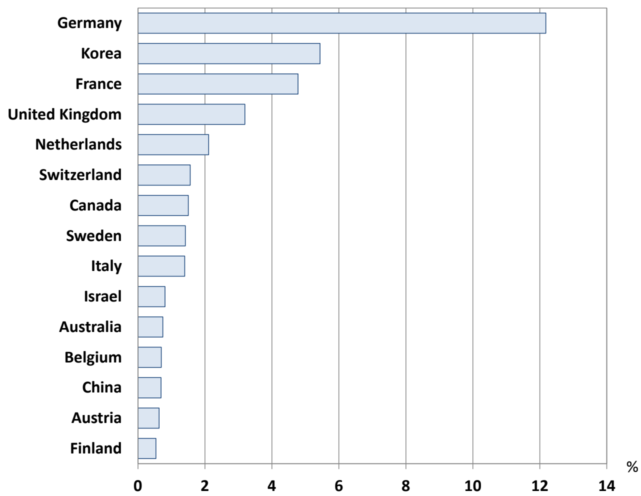
The European Commission has placed renewed emphasis on the conversion of Europe's scientific expertise into marketable products and services, through seeking to use public sector intervention to stimulate the private sector and to remove bottlenecks which stop such ideas reaching the market. Furthermore, the latest revision of the integrated economic and employment guidelines (revised as part of the Europe 2020 strategy for smart, sustainable and inclusive growth) includes a guideline to optimise support for *R&D* and innovation, strengthening the knowledge triangle and unleashing the digital economy potential. *The European Research Area (ERA)* is designed to overcome some of these barriers which are thought to have hampered European research efforts, for example, by addressing geographical, institutional, disciplinary and sectoral boundaries. The following figures show the share of countries in triadic patent families production.

Figure 2.7: Share of countries in triadic patent families in 2005



BRIICS refers to Brazil, China, India, Indonesia, the Russian Federation and South Africa.

Figure 2.8: Share of European countries in triadic patent families in 2005



In December 2008, the Competitiveness Council adopted a 2020 vision for the ERA (European Research Area). According to the opening statement of this vision, all players should benefit from: the *'fifth freedom'*, introducing the free circulation of researchers, knowledge and technology across the ERA; attractive conditions for carrying out research and investing in *R&D* intensive sectors; European-wide scientific competition, together with the appropriate level of cooperation and coordination. The 2020 vision for the ERA is part of a wider picture of *Europe's 2020 Strategy* for smart, sustainable and inclusive growth. As part of the EU's 7th framework programme for research and technological development, the European Commission announced in July 2011, nearly EUR 700 million of investment in research and innovation, with the aim to providing an economic stimulus expected to create around 174.000 jobs in the short-term. The following figures show the patent production trends in the main states.

Figure 2.9: Trends in Triadic Patent Families: USA, EU27, Japan

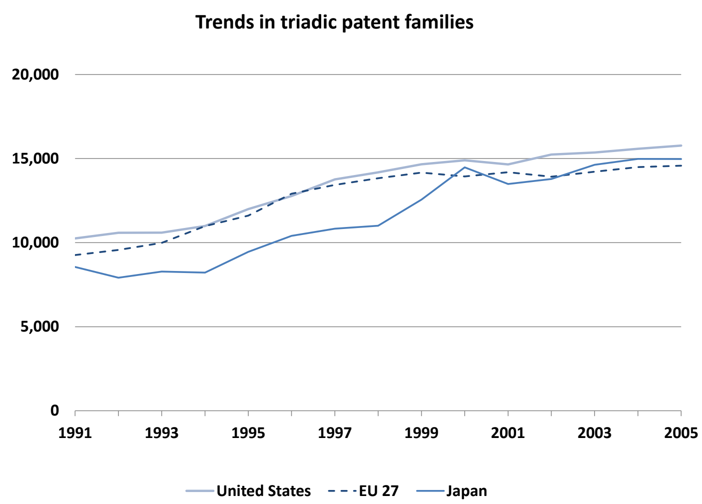


Figure 2.10: Trends in Triadic Patent Families: GE, FR, UK, Korea

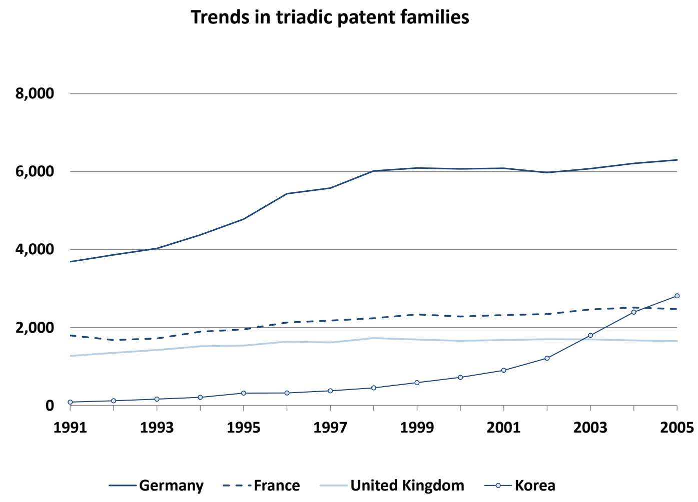
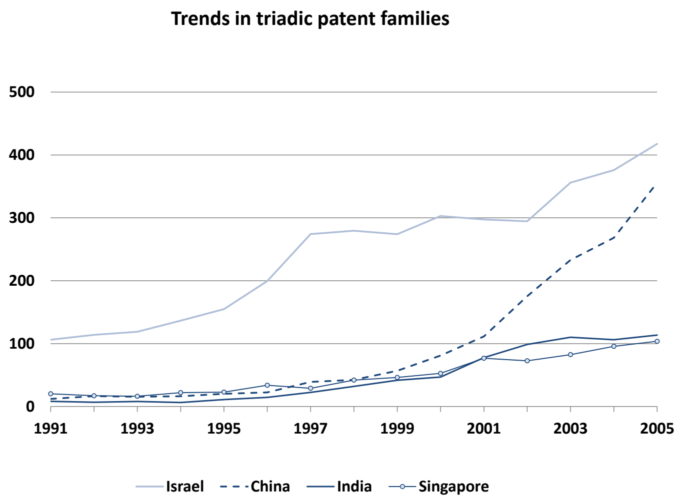


Figure 2.11: Trends in Triadic Patent Families: Israel, China, India, Singapore



Studies have been conducted in respect to European business enterprises' investments in an annual report. This presents information on the top 1000 research investors whose registered offices are in the EU and the top 1000 investors registered elsewhere. According to this source Volkswagen (Germany) and Nokia (Finland) were among the global top ten investors in 2010, a group that was led by Roche (Switzerland) and Pfizer (the United States), and also included Novartis (Switzerland).

2.4 Data sources and availability

Statistics on science, technology and innovation (STI statistics) are based on Decision 1608/2003/EC concerning the production and development of Community statistics on science and technology. In close cooperation with the Member States, this Decision was implemented by Eurostat in the form of legislative measures and other work. Regulation 753/2004 on statistics on science and technology was adopted in 2004 implementing Decision

1608/2003/EC. Eurostat's statistics on *R&D* expenditure are compiled using guidelines laid out in the Frascati manual, published in 2002 by the OECD. *R&D* expenditure is a basic measure that covers intramural expenditure, in other words, all expenditure for *R&D* that are performed within a statistical unit or sector of the economy. The European Commission compiles three levels of indicators to support research and innovation policymaking. These indicators are generally grouped together as: headline indicators; innovation union scoreboard (or core) indicators; and a comprehensive set of indicators. Within the headline indicators - also referred to as *Europe 2020 Strategy* indicators - is the research intensity measure (with a 3 per cent target for investment in research across the EU). The scoreboard (or core) indicators are designed to monitor research and innovation for the Competitiveness Council, while the comprehensive set of indicators are for in-depth economic analytical purposes and Commission services to produce a science, technology and competitiveness report.

Chapter 3

A Knowledge-Based DSGE Model with Endogenous Growth

Finally this last chapter presents the DSGE model with endogenous growth. I calibrate, and simulate, the model to analyze how the investment-specific technological shock affects the economy.

3.1 Introduction

The present chapter considers the previous empirical analysis' results, and presents a formal model of endogenous growth in which workers invest in knowledge accumulation, or in the acquisition of skills[1], while firms invest in quality-augmentation enhancing the $R\&D$-equipment sector of production. Incentives for both forms of investment are interdependent and determine the growth of the economy. The analysis is focused on the integration of these two sources of economic growth. On one side investments in $R\&D$ have been analyzed by many authors as Aghion and Howitt (1992). On the other side knowledge accumulation has also been treated by many authors,

[1] I make the standard assumption that the education, training and skills of a workforce in an economy may be represented by an aggregate stock of knowledge, H_t.

as in the case of Romer (1990),[2] Stokey (1991), and Jones (1995). In terms of empirical evidence, Lichtneberg (1992) and Coe, and Helpman (1993), have found that $R\&D$ has a significant effect on growth, while a number of authors including Mankiw et al. (1992) and Barro (1991) have found that knowledge and human capital variables are important for the economic development.
Here I will carry on with what, in the Greenwood Hercowitz and Krusell (1997,1998) papers, is briefly discussed as an interesting possibility for a model extension. The analysis starts considering a two-sector model with capital-embodied technological progress. The first sector produces a consumption good (or final good), while the other in the $R\&D$-equipment sector, produces equipment goods using dedicated labor and knowledge stock.[3] Labor is divided into two types: labor used in the consumption goods production sector and labor used in the equipment goods production sector.
Within this model, growth is driven by: the accumulation of the knowledge stock, which is an endogenous variable. Differently from Romer (1990), what is important here is not how many skilled workers devote their time operating in the $R\&D$-equipment sector, but rather, the efficiency of new ideas is important, as Jones (1995).[4] As I said the stock of knowledge-technology available in the economy is an endogenous variable (as described in Romer, 1990), and it can be taught as the accumulation of all ideas and inventions developed by researchers, and engineers. The model introduces a stochastic

[2]In his seminal paper Romer (1990) assumes a knowledge production function in which new knowledge is linear in the existing stock of knowledge, holding the amount of research labor constant. The implication of this strong form of knowledge spillovers is that the growth rate of the stock of knowledge is proportional to the amount of labor engaged in $R\&D$. Hence, policies such as subsidies to $R\&D$ that increase the amount of labor allocated to research will increase the growth rate of the stock of knowledge.

[3]There are two well-known endogenous growth models by Larry E. Jones and Manuelli (1990,1997) and Rebelo (1991), these researches introduce the model with two-sectors of production.

[4]As in Jones (1995) there is a keep connection between knowledge and equipment investment: the equipment investment sector needs to be much more intensive in its use of knowledge than the consumption sector.

temporary shock which hits the economy twice: first it concerns the equipment investment variable, and then it concerns the education-schooling variable. The shock is modeled as a capital embodied shock, as in Greenwood et. al (1998); for that reason it affects the stock of knowledge directly, but it has just an indirect impact over output, consumption, and investment rate variables.[5]

Methodology

The link between technology change and business cycle fluctuations has attracted the attention of macroeconomists particularly since the seminal work of Kydland-Prescott (1982) and Long-Plosser (1983). In these studies and in the literature that follows, technological change is modelled as a productivity shock. Kydland and Prescott's theoretical framework is based on the neoclassical growth model, and on the idea of capital accumulation. These models use stochastic rational expectations following Lucas' microfoundations research agenda. As in Lucas (1972, 1973), agent behaviour is governed by the optimization under uncertainty frameworks. Elements of this approach were foreshadowed by Robertson (1915), who noted that technical changes contributed to business fluctuations, and Frisch (1933), who studied business cycles within an optimizing framework.
Unfortunately, optimization in the neoclassical growth model has a non-linear structure. The common approach is to linearize the model around the steady state of the system and to consider an approximate solution.
In that section, I briefly analyze the debate between the *estimation* and *calibration approach*[6]. The traditional approach is to specify a model consisting of a set of equations having unknown parameters to be estimated, then use econometric techniques to estimate the parameters and test whether their sign and magnitude is consistent with the model's assumptions. That ap-

[5]In Greenwood Hercovitz and Krusell there are two shocks, the first one is over labor and the second one is over equipment investment rate variable.

[6]Calibration methods used in dynamic macro models since the Kydland-Prescott model.

proach on one hand provides a formal statistical test to the hypotheses of the model. On the other hand the weakness of this approach is that restrictive assumptions have to be made in order to estimate the model. Identifying restrictions include assumptions about which variables are exogenous and therefore which variables can be excluded from having a direct effect in the equation determining each other variable. RBC modellers have usually avoided econometric estimation in favour of a technique called *calibration*. This involves choosing parameters on the basis of long-run data properties, sometimes guided by microeconomic evidence. Hence, economists calibrate RBC models by choosing values for their behavioural parameters, such as the capital depreciation rate and marginal substation between productivity factors, and then comparing the correlations produced by repeated model simulations with corresponding correlations in real-world macroeconomic data. The advantage of the calibration approach is to avoid the restrictions that are necessary with the estimation process. Anyway, calibration has disadvantages as well, as there is no consensus about the parameters value.
More recently, many RBC practitioners have turned to Bayesian methods in order to check the empirical validity of models. Bayesian distributions are often estimated by Monte Carlo Markov Chain (MCMC) simulation techniques; by choosing suitable transition functions for Markov chains it's possible to show that the posterior distribution of a model parameter coincides with the stationary distribution of the chain. Consequently, posterior distribution can be approximated by sampling from a suitably long realization of the chain.[7]
Even if there is not unanimity about which approach is better to perform the economic fluctuations, here I will present a calibrated model, as is usual for DSGE models. Anyway, an estimation of such a model would be surely an interesting test.

[7] See Stern and Rubin (2003) and Koop (2003) about MCMC methods for Bayesian analysis.

A DSGE Snapshot

Real business-cycle theory has used different methods of empirical validation. Early RBC models were often inconsistent with the available empirical evidence, and for that reason subsequent models increased considerably in complexity to cope with the criticisms that arose. By the late 1990s the RBC literature produced models with, for example, multiple shocks, price rigidities, monetary and fiscal policies. Modern models in the RBC tradition are associated with real disturbances, they focus on general equilibrium and fully specify the behaviour of the driving variables, for that reason they are often referred to as dynamic stochastic general equilibrium models. These models are computationally-demanding and have multiple disturbances. Dynamic general equilibrium models have proved to be valuable tools in examining both economic growth and fluctuations. DSGE modelling is a branch of applied general equilibrium theory, and it is pretty influential in the study of contemporary macroeconomics. That methodology attempts to explain aggregate economic phenomena, such as economic growth, business cycles, monetary and fiscal policy effects. One of the main reasons why macroeconomists have begun to build DSGE models, is that, unlike more traditional macro forecasting models, they are not vulnerable to the Lucas critique (1976). DSGE models include elements from both, new keynesian paradigm, and the real business cycle approach (RBC). For that reason these models are known as the *new neoclassical synthesis.*

Over the last 15 years, there has been remarkable progress in the specification and estimation of dynamic stochastic general equilibrium models. Central banks, governments, and institutions have become increasingly interested in their use for policy analysis and their forecasting properties. These models help to identify sources of fluctuation, to answer research questions about structural changes, to forecast and predict the effect of policy change. Nevertheless, they are not yet ready to accomplish all that is being asked of them. They still need to incorporate relevant sectors of the economy, and several issues remain open on how to empirically validate them. Nowadays, DSGE models have important limitations.[8]

R&D and Endogenous Growth

Since the seminal papers of Romer (1990), Grossman-Helpman (1991a, 1991b), and Aghion-Howitt (1992) growth theory focused its attention on knowledge-based models. This class of models aims to explain the role of technological change in the growth process. Here, technology production function has a central role, it describes the evolution of knowledge creation. In that work, I will use a knowledge-based model framework in order to explain endogenous technological shock. Economic growth depends on innovation efficiency, and that efficiency is supported by an increase in the schooling-education growth rate. It is important to highlight that, differently from Romer (1990), I don't consider *human capital*[9] as the most important variable for economic growth. Rather, what is a real determinant for economic development here, is knowledge production which is directly affected by the schooling-education variable. Thereafter, with the stock of knowledge produced in each period of time, it will be possible to produce new and efficient equipment goods. Knowledge-based models focused their attention on the functional form of the knowledge production function; and on how strongly new knowledge depends on the existing stock of knowledge. Intuitively, the dependence of new knowledge on the existing stock is intended to capture an intertemporal spillover. Knowledge or ideas discovered in the past may facilitate the discovery or the creation of new ideas in the present.

[8]It has only been since the work of Smets and Wouters (2003) that some evidence was put together showing that a new keynesian model could track and forecast time series as well as a vector autoregression estimated with the Bayesian technique (BVAR).

[9]The main conclusion in Romer (1990) is that the stock of human capital, how many people devote their time to research, determines the rate of economic growth.

Chapter target

Greenwood, Hercowitz, and Krusell (1998) were the first to suggest that investment-specific technological shocks could be an alternative to neutral technology shocks in the business cycle environment.[10]. They suggest that this kind of shock is responsible for the major share of economic growth. Latter analysis shows, using structural VARs, that an investment-specific technological shock is responsible for permanent changes in output production and hours worked. In a recent paper Justiniano, Primiceri, and Tambalotti, (2009) highlight, as did Greenwood Hercowitz and Krusell (1998), that an investment-specific technological shock determines the efficiency of a new stock of equipment goods, and that is the key driver of business cycles.

The novelty of my contribution with respect to Greenwood's model, is to make the technological progress endogenous. I will introduce a second sector of production that requires dedicated labor and a stock of knowledge[11] to produce equipment goods. The stock of knowledge will directly depend on the schooling-education growth rate, hence the schooling-education variable is considered to be the most important engine for economic development. I define technology progress as capital embodied. For that reason the quantity of equipment capital ($k_{e,t}$) available in the economy strictly depends on technology advancement. The representative agent supplies labor to firms. Since I assume the existence of a *benevolent social planner* who makes all the decisions in the economy, the agent has no preferences on how to allocate his time: working in the consumption or equipment goods production sector, or increasing his education. The *social planner problem* has a close connection with the competitive equilibrium.[12]

[10]see, Greenwood, Hercowitz, Krusell (1997), and Fisher (2006)

[11]The stock of knowledge available in the economy can be thought of as the accumulation of all ideas that have been invented or developed at any given point in time.

[12]The first welfare theorem tells us that if an allocation and a set of prices constitute a competitive equilibrium, then the allocation will be Pareto efficient. The second wel-

This chapter contributes in answering the following research question: which is the quantitative role of an investments-specific technological change as an engine towards growth?

Finally the work is organized as follows: section 2 describes the model's theoretical explanation. Section 3 provides the model's analytical solution, and section 4 shows the main results obtained by calibration analysis. Section 5 concludes the chapter.

fare theorem, on the other hand, tells us that only a social planning problem can be decentralized as a competitive equilibrium.

3.2 A Theoretical Model

The Economic Environment

The economy is inhabited by a representative agent who maximizes the expected value of his lifetime utility, as:

$$E_t[\sum_{t=0}^{\infty} \beta^t U_t(c_t; l_t)]$$

with

$$U(c_t, l_t) = lnc_t + \psi ln[(1 - l_{c,t} - l_{e,t} - s_t)]$$

c_t and $l_t = (1 - l_{c,t} - l_{e,t} - s_t)$ represent period-t consumption and leisure marginal substitution, and s_t is the schooling-education variable which represents how much the agent wants to spend to improve his skills. The parameter ψ measures the marginal substitution between leisure and work. The higher ψ is, the more important leisure for the agent will be.[13]

Characterization of the two-sector economy

The two sectors are defined as follow: one sector produces equipment goods - *the equipment production sector* - and the other produces consumption goods - *the consumption production sector*.[14]

[13] In that case the representative agent has the same utility spending time in working in consumption or in the equipment goods production sector, or spending time in increasing his schooling-education level. They are substitute goods. A substitute good, in contrast to a complementary good, is a good with a positive cross elasticity of demand. This means a good's demand is increased when the price of another good is increased. Conversely, the demand for a good is decreased when the price of another good is decreased.

[14] Differently from Greenwood et. al (1998) where it is assumed that a key for a two-sector interpretation is given by the productivity parameter q_t, as the only difference

The production function of consumption goods y_t requires the service of $l_{c,t}$ labor force, and two types of capital: structures capital $k_{c,t}$ (such as machinery, tools, premises) and equipment capital (such as innovative technologies, research, software, patents) $k_{e,t}$.

The production function of consumption goods is described by:[15]

$$y_t = F[k_{c,t}; k_{e,t}; l_{c,t}] = k_{c,t}^{\alpha} k_{e,t}^{\gamma} l_{c,t}^{(1-\alpha-\gamma)} \quad 0 < 1-\alpha-\gamma < 1$$

The model stresses capital-embodied technological changes as a key to long-run productivity movements (Hulten, 1992).

The stock of structures capital evolves according to:

$$k_{c,(t+1)} = i_t + (1-\delta)k_{c,t}$$

The story is different for the equipment goods production sector. The accumulation for equipment capital is expressed as:

$$k_{e,(t+1)} = i_{e,t} + (1-\delta_e)k_{e,t}$$

As we can see from the law of motion of the equipment capital the technological progress, $i_{e,t}$, is capital embodied. Differently from the Greenwood Hercovitz Krusell (1998) original model, here, the technological progress is

between the consumption goods and the equipment goods production function, here the two sectors of production functions are totally different. Basically, the equipment goods production sector is introduced in order to explain the endogenous technological progress.

[15]Differently from a typical example of a Cobb Douglas production function with a sectoral neutral productivity shock as $y_t = z_t F(k_t, l_t) = z_t k_t^{\alpha} l_t^{1-\alpha}$, where the variable z_t is a measure of TFP, or a neutral shock.

an endogenous variable, and it is produced by the equipment goods production function $i_{e,t} = (H_t, l_{e,t})$. Technological progress changes are assumed to affect the equipment goods production sector only.[16]
Another important thing to highlight is the difference between the depreciation rate of structures capital (δ) and of the equipment capital (δ_e). The deprecation rate of equipment capital (δ_e) depends on the utilization rate, reflecting a "user cost". Thus δ_e, unlike δ, has variable rates of utilization and depreciation.[17] This is due to the more active role equipment capital plays in production, which is precisely why it is less durable than structures capital. It is natural, then, to model the *depreciation rate on equipment capital as an increasing convex function of its rate of utilization* (Greenwood, 1998).

On the other hand, the production function of the equipment goods $i_{e,t}$ requires the service of dedicated labor $l_{e,t}$ and the existing stock of knowledge H_t. The production function is described by:

$$i_{e,t} = [H_t, l_{e,t}] = H_t^{\theta} l_{e,t}^{(1-\theta)} \quad 0 < \theta < 1$$

The knowledge stock is defined recursively as:

$$H_{t+1} = (1 - \delta_s)H_t + F(s_t)H_t$$

The equation above represents the law of motion of knowledge, where H_t is the stock of knowledge at a given point in time, and δ_s is the knowledge

[16]The motivation for this is empirical. First, the relative price of structures capital appears to be stationary over time. Secondly casual observations suggest that there are less productivity changes in structures capital than in equipment capital (Greenwood et. al 1997).

[17]Equipment capital as patents, research or innovative ideas cannot have a constant rate of production and utilization. That depends on their deeply different nature from the structures capital. That assumption is also important in order to demonstrate that *R&D* development cannot be a constant trend in data analysis.

depreciation rate. The most important aspect of this equation is the inclusion of the factor s_t^ω that represents the current state of schooling-education level for producing equipment goods. That schooling-education level, together with the amount of knowledge at a given point in time, represent the new total amount of knowledge to sum to the previous one. That new surplus of knowledge determines the amount of equipment goods that can be purchased for one unit of final consumption output y_t. Changes in s_t^ω formalize the notion of investment-specific technological change. [18].
The schooling-education rate is driven by s_t^ω which represents the schooling-education production function. The knowledge variable is directly affected by s_t^ω, so we end up with the following relation:[19]

$$H_{t+1} \geq H_t \Longleftrightarrow H_t F(s_t) \geq \delta_s H_t$$

The future knowledge stock is given by the previous knowledge stock depreciated at δ_s, plus the schooling-education results multiplied the present knowledge stock.

The schooling-education production function measures the schooling-education level rate of utilization, it evolves according to:[20]

$$F(s_t) = s_t^\omega$$

and

[18]In Greenwood Hercowitz and Krusell (1997,1998) the factor which causes the investment-specific technological change is q_t and it defines the current state of technology for producing equipment goods.

[19]From the knowledge law motion we get s_t value at the steady state, $s_t = (\delta_s)^{\frac{1}{\omega}}$.

[20]This Formulation is used in Greenwood, Hercovitz and Huffman (1988). The role of a variable rate of utilization in business cycle fluctuations has been studied by Lucas (1970), Greenwood, Hercovitz and Huffman (1988), Kydland and Prescott (1988), Bils and Cho (1991), Finn (1992), Burnside and Eichenbaum (1994), and Cooley, Hansen and Prescott (1995).

$$\omega > 1$$

The schooling-education production function is not endogenous, it is determined by the parameter ω; s_t variable which influences the law motion of knowledge H_t.

Finally in respect to the shock, the model considers an investment-specific technological change. In both sectors of production we have Cobb Douglas production functions, but they do not include a TFP variable to measure productivity. (The equations that follow refer to the shock over the schooling-education variable s_t)

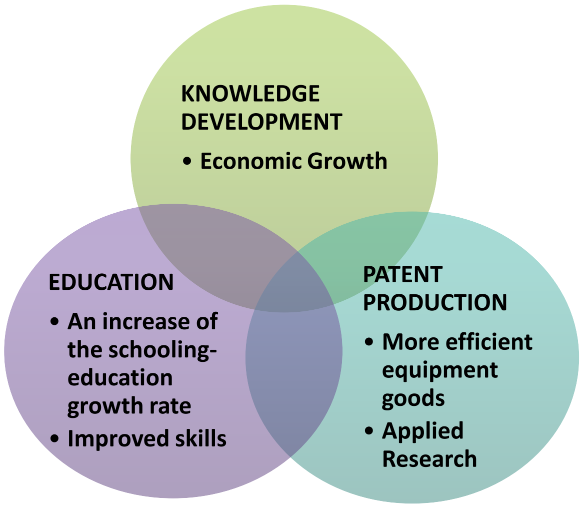
$$H_{t+1} = (1 - \delta_s)H_t + F(s_t)H_t$$

$$F(s_t) = \exp(z_t)s_t^{\omega}$$

$$z_t = \rho z_{t-1} + \varepsilon_t \quad \varepsilon_t \sim (0, \sigma)$$

This is the dynamic problem, the shock.

Figure 3.1: Economic Environment



The above graph represents the functioning of the economy. The representative agents supply two different types of capital ($k_{c,t}$;$k_{e,t}$), and two different types of labor ($l_{c,t}$;$l_{e,t}$) to firms. During each period of time a new stock of knowledge H_t is produced in the economy, and it will be the most important engine for growth. On one hand we have the equipment goods production sector which uses this new knowledge to improve its research and to continually produce more efficient equipment goods $i_{e,t}$. On the other hand we have the representative agent who decides to invest his time in improving his education and training s_t.

3.3 The Social Planner Problem

I assume the existence of a benevolent social planner who makes all the decisions in the economy. The social planner is benevolent because he only cares about the welfare of the consumer. Therefore, instead of having prices to guide the economy to equilibrium, the social planner decides how the consumer's time will be allocated between leisure, labor, and schooling-education. The only constraints, the planner faces are: the consumer's time endowment, the resource constraint, and the state of technology. Obviously, the social planner problem gives us the highest possible utility for the consumer.

$$\max_{\{c_t,k_{c,(t+1)},k_{e,(t+1)},H_t,l_{c,t},l_{e,t},s_t,\lambda,\eta,\mu\}_{t=0}^{\infty}} E_t[\sum_{t=0}^{\infty}\beta^t\{lnc_t+\psi(1-l_{c,t}-l_{e,t}-s_t)\}]$$

subject to the resource constraint:

$$c_t+k_{c,t}=k_{c,(t-1)}^{\alpha}k_{e,(t-1)}^{\gamma}l_{c,(t-1)}^{1-\alpha-\gamma}+(1-\delta)k_{c,(t-1)}$$

the law of motion for equipment capital:

$$k_{e,t}=H_{t-1}^{\theta}l_{e,t}^{1-\theta}+(1-\delta_e)k_{e,(t-1)}$$

and the law of motion for knowledge:

$$H_t=(1-\delta_s)H_{t-1}+s^{\omega}H_{t-1}$$

As I said before the prices are absent. The social planner maximizes the consumer's utility function using the method of the Lagrangian multipliers.[21]

$$\begin{aligned} L = E_t[\sum_{t=0}^{\infty}\beta^t\{[lnc_t + \psi ln(1 - l_{c,t} - l_{e,t} - s_t)] - \\ -\lambda_t(c_t + k_{c,t} - k_{c,(t-1)}^{\alpha}k_{e,(t-1)}^{\gamma}l_{c,t}^{1-\alpha-\gamma} - (1-\delta)k_{c,(t-1)}) \\ -\eta_t(k_{e,t} - H_{t-1}^{\theta}l_{e,t}^{1-\theta} - (1-\delta_e)k_{e,(t-1)}) \\ -\mu_t(H_t - (1-\delta_s)H_{t-1} + s_t^{\omega}H_{t-1})\}] \end{aligned}$$

Let λ_t denotes the Lagrange multiplier on the consumption constraint, η_t denotes the Lagrangian multiplier on the equipment capital constraint, and finally μ_t denotes the Lagrangian multiplier on the knowledge constraint.

First order conditions for the problem:

The marginal utilities of consumption and leisure are:

$$\frac{\partial L}{\partial c_t} = [\beta^t\{\frac{1}{c_t}\}] = 0; \ \lambda_t = \beta\frac{1}{c_t}$$

$$\frac{\partial L}{\partial k_{c,t}} = [\beta^t\{-\lambda_t\} + \beta^{t+1}\{\lambda_{t+1}(\alpha k_{c,t}^{\alpha-1}k_{e,t}^{\gamma}l_{c,t}^{1-\alpha-\gamma} + 1 - \delta)]\} = 0$$

$$\frac{\partial L}{\partial k_{e,t}} = [\beta^t\{-\eta_t\} + \beta^{t+1}\{\lambda_{t+1}(\gamma k_{c,t}^{\alpha}k_{e,t}^{\gamma-1}l_t^{c^{1-\alpha-\gamma}} + 1 - \delta) + \eta_{t+1}(1-\delta_e)]\} = 0$$

$$\frac{\partial L}{\partial H_t} = \beta^t\{-\mu_t\} + \{\beta^{t+1}\{\eta_{t+1}\theta H_t^{\theta-1}l_{e,(t+1)}^{1-\theta} + \eta_{t+1}[(1-\delta_s) + s_t^{\omega}] = 0$$

[21]The household equates the cost from saving one additional unit of today's consumption to the benefit of obtaining more consumption tomorrow (consumption smoothing). Consumption depends upon expected future wealth as opposed to current income.

Here we have the first order conditions that show how to manage labor time:

$$\frac{\partial L}{\partial l_{c,t}} = [\beta^t\{\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t} + \lambda_t k^{\alpha}_{c,(t-1)} k^{\gamma}_{e,(t-1)}(1-\alpha-\gamma) l^{-\alpha-\gamma}_{c,(t-1)}] = 0$$

$$\frac{\partial L}{\partial l_{e,t}} = [\beta^t - \frac{\psi}{1-l_{c,t}-l_{e,t}-s_t} + \eta_t(1-\theta)H^{\theta}_{t-1} l^{-\theta}_{e,t}] = 0$$

$$\frac{\partial L}{\partial s_t} = [\beta^t\{-\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t} + \mu_t \omega s_t^{\omega-1} H_{t-1}\}] = 0$$

And finally $\frac{\partial L}{\partial \lambda_t}$, $\frac{\partial L}{\partial \eta_t}$ and $\frac{\partial L}{\partial \mu_t}$ give us the constraint.

$$\frac{\partial L}{\partial \lambda_t} = [\beta^t(-1)\{c_t + k_{c,t} - k^{\alpha}_{c,(t-1)} k^{\gamma}_{e,(t-1)} l^{1-\alpha-\gamma}_{c,t} - (1-\delta)k_{c,(t-1)}\}] = 0$$

$$\frac{\partial L}{\partial \eta_t} = [-\beta^t\{k_{e,t} - H^{\theta}_{t-1} l^{1-\theta}_{e,t} - (1-\delta_e)k_{e,(t-1)}\}] = 0$$

$$\frac{\partial L}{\partial \mu_t} = [\beta^t + \{H_t - (1-\delta_s)H_{t-1} - s_t^{\omega} H_{t-1}\}] = 0$$

Now from the first order equations we obtain the social planner solution.

The Social Planner Solution

From the previous first order conditions we obtain the social planner solution equations, and then the steady state of the model. Finally from the steady state we obtain the initial values of the main variables: λ_t, η_t, μ_t, c_t, $k_{c,t}$, $k_{e,t}$, $l_{c,t}$, $l_{e,t}$, H_t, and s_t^{ω}.[22]

[22]See appendix for a more detailed explanation

The Lagrangian multipliers are the marginal utility of relaxing constraints; they represent the shadow values of the constraints (constraints marginal cost). They tell us how much better off we would be if the constraints slackened slightly.

Considering a social planner problem there are no differences between investment costs on consumption capital $k_{c,t}$, equipment capital $k_{e,t}$, and knowledge H_t. That means they have the same value in consumption terms.

Looking for the marginal utilities on consumption and leisure, the **Euler Equation** concerning the consumption goods production sector is as follow:

$$\frac{1}{c_t} = \frac{1}{c_{t+1}}[\alpha(\frac{y_{t+1}}{k_{c,(t+1)}} + 1 - \delta)]$$

The **Euler Equation** for the equipment goods production sector is given as:

$$\eta_t = \beta\frac{1}{c_{t+1}}[(\gamma\frac{y_{t+1}}{k_{e,(t-1)}} + 1 - \delta) + \eta_{t+1}(1 - \delta_e)];$$

And finally looking to the **Euler Equation** for knowledge we have that:

$$\mu_t = \beta[\eta_{t+1}\theta H_t^{\theta-1} l_{e,(t+1)}^{1-\theta} + \mu_{t+1}(s_t^{\omega} + 1 - \delta_s)]$$

Utility function describes $l_{c,t}$, $l_{e,t}$, and s_t as perfect substitute goods, that means the representative agent does not have preference in spending time working (in the consumption or in the equipment goods production sector) or spending time in improving his education. Without prices in the model, we have no differences between the wage earned by workers. These first order conditions indicate the marginal benefit of working a little more, but they do not indicate if it is better spending time in working (in both sectors), or in educational advancement. At the optimum level we balance marginal benefits to marginal costs which are the losses in utility due to the losses of a unit of leisure.

Concerning the labor force in the consumption goods production sector:

$$\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}=\frac{1}{c}(1-\alpha-\gamma)\frac{y_t}{l_{c,t}}=0$$

The same concerning the labor force in the equipment goods production sector:

$$\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}=\eta_t(1-\theta)H_{t-1}^{\theta}l_{e,t}^{-\theta}$$

And finally concerning the schooling-education variable:

$$\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}=\mu_t\omega s_t^{\omega-1}H_{t-1}$$

Now we write down the budget constraint for the entire economy:

$$c_t+i_t=y_t$$

$$y_t=k_{c,(t-1)}^{\alpha}k_{e,(t-1)}^{\gamma}l_{c,t}^{1-\alpha-\gamma}$$

$$k_{e,t}=H_{t-1}^{\theta}l_{e,t}^{1-\theta}+(1-\delta_e)k_{c,(t-1)}$$

$$H_t=(1-\delta_s)H_{t-1}+s^{\omega}H_{t-1}$$

The equations in the previous page describe the welfare maximizing bundle. Let's go on to finding the deterministic solution, we look at the steady state of the model which is given by the following equations (if $\sigma = 0$):

$$\frac{\psi}{(1-l_c-l_e-s)} = \frac{(1-\alpha-\gamma)y}{l_c c}$$

$$y = k_c^{\alpha} k_e^{\gamma} l_c^{1-\alpha-\gamma}$$

$$\frac{\psi}{(1-l_c-l_e-s)} = (1-\theta)\eta H^{\theta} l_e^{-\theta}$$

$$\frac{\psi}{(1-l_c-l_e-s)} = \omega\mu s^{\omega-1} H$$

$$y = c + \delta k_c$$

$$s^{\omega} = \delta_s$$

$$1 = \beta[\frac{\alpha y}{k_c} + 1 - \delta]$$

$$\eta[1-\beta(1-\delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e} + 1 - \delta);$$

$$\mu(1-\beta) = \beta\eta\theta H^{(\theta-1)} l_e^{(1-\theta)}$$

$$H^{\theta} l_e^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

3.4 Results

Calibration

Before simulating the model it's necessary to assign values to preference parameters: β, and ψ, and to technology parameters: α, γ, θ, δ, δ_e, δ_s.
The procedure adopted: as many parameters as possible are set in advance based on literature information. Additional parameters are set on model' balanced growth variables, matching with the average values of the European data between 1990-2007. The nominal data used to construct these series are: y as nominal GDP - chain linked values, base year 2000, l as labor unit, k as total gross capital - years $1980 - 2009$ - chain linked values - base year 2000), and $R\&D$ as millions of $R\&D$ expenditure (GERD) over structural capital value.
I end up with the following values: k_c productivity is given by $\alpha = 0.3$; k_e productivity is given by $\gamma = 0.2$; the discount rate is given by $\beta = 0.97$; the utility value of leisure is given by $\psi = 1.5$; the depreciation rate for structures capital is given by $\delta = 0.035$; the depreciation rate for equipment capital is given by $\delta_e = 0.050$; the depreciation rate for knowledge is given by $\delta_s = 0.050$; s productivity is given by $\omega = 1.5$; H productivity is given by $\theta = 0.3$; l_e productivity is given by $1 - \theta$. Finally concerning the shock: $\rho = 0.95$ and $\sigma = 0.010$. See table 1.

Starting from the parameter values, and the set of the first order conditions, I look for the initial values of the main variables in the model (see Table 2). I end up with a system of twelve equations, and consequently twelve unknown variables, that I solved recursively. Then I used Dynare in order to analyze how the system behaved in response to the stochastic temporary shock. The model's statistics are generated by simulating the artificial economy developed above 10000 periods and impulse response functions equal to 100. A table with the resulting steady state values is in the appendix section.

Simulation

Examples of models without investment-specific technological change have been extensively explored in many studies (King and Plosser, 1982; Hansen, 1985, King Plosser and Rebelo, 1988; Villaverde, 2005-2009). The following figure shows the impulse response functions[23] resulting from a sector-neutral productivity shock (TFP shock) on the economy.

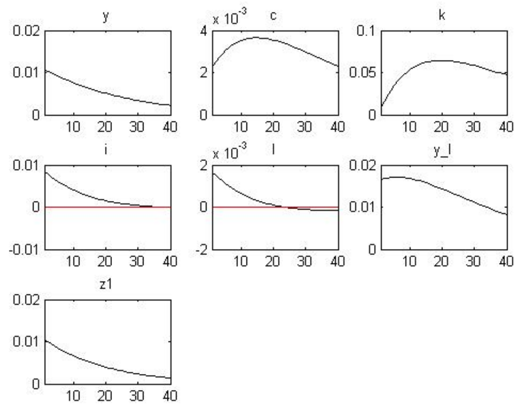


Figure 3.2: Orthogonalized shock to TFP

The figure above refers to a standard RBC framework. In this case we have a decentralized economy with one production sector which uses one type of labor and one type of capital to produce the consumption good. The shock (z_1) is a temporary shock over labor into the production function, so we have that:

$$y_{t+1} = (k_t)^{\alpha}(\exp(z_{1,t})l_t)^{1-\alpha}$$

As we can see from the IRF the shock immediately hits output, consumption, and the investment rate. Anyway the persistence of the shock is not too

[23]The impulse response functions (IRF) refer to the reaction of any dynamic system in response to some external change, they describe the reaction of the system as a function of time.

high. Using the same RBC framework, Greenwood et. al (1998) adds a new type of shock. This new shock does not concern the labor production factor as in the previous example, but instead, it concerns the investment variable. Particularly, the shock represents the investment-specific technological change.[24]. Since the shock is *capital embodied* we have that:

$$k_{t+1} = (1 - \delta)k_t + \exp(z_{2,t})i_t$$

As we can see from the following figure the main difference from the TFP shock, is that the investment-specific technological shock (z_2) does not directly affect consumption in the current period. At the beginning consumption is negative, and even later it does not increase a lot. Hence, in Greenwood et.al (1998) consumption is negatively correlated with the shock in the short-time analysis. Concerning output and investment rates both of them are not increasing in the short-period, on the contrary they soon come back to their steady state. For that reason the persistence of the shock is low.

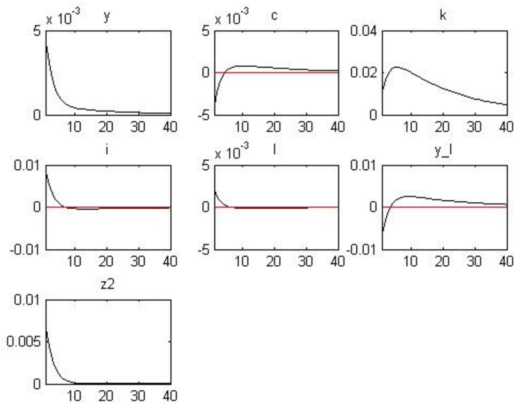


Figure 3.3: Orthogonalized shock to investment-specific technological change

[24]Examples of this type of technology change are: more powerful computers, faster and more efficient telecommunication infrastructure or transportation, etc

New model simulations

Now I present the IRF results obtained from the simulation of the *new model*. A key objective of the present analysis is, on one hand, to quantify the contribution to economic growth resulting from the investment-specific technological shock.[25] On the other hand, taking advantage from a more detailed economic environment (particularly form the endogenous explanation of the technological progress), it is possible to understand how variables are directly affected by the investment-specific technological shock.[26] I simulate the model twice. During the first time, in order to be as close as possible to Greenwood et. al (1998) model, the shock is over the equipment goods production sector's investment variable $i_{e,t}$, and I consider a shorter period in the analysis.[27] Instead during the second time, the shock concerns the schooling-education variable.

The following figures refer to the first simulation exercise. The shock (z_t) is over the equipment goods production sector's investment variable, and it is capital embodied:

$$k_{e,(t+1)} = (1 - \delta_e)k_{e,t} + F(i_{e,t})$$

$$F(i_{e,t}) = [H_t, l_{e,t}] = H_t^{\theta} l_{e,t}^{(1-\theta)} \quad 0 < \theta < 1$$

$$F(i_{e,t}) = \exp(z_t) i_{e,t}$$

[25]The model tries to replicate the results obtained in the Greenwood et al. (1998) framework article.

[26]The new model features a centralized economy with two production sectors which use two different types of capital ($k_{c,t}$, $k_{e,t}$) and two different types of labor ($l_{c,t}$, $l_{e,t}$), hence the simulations will not be strictly the same as Greenwood et. al (1998)

[27]The model presents two different investment variables (i_t, $i_{e,t}$), here I consider the equipment production sector investment variable $i_{e,t}$.

$$z_t = \rho z_{t-1} + \varepsilon_t \quad \varepsilon_t \sim (0, \sigma)$$

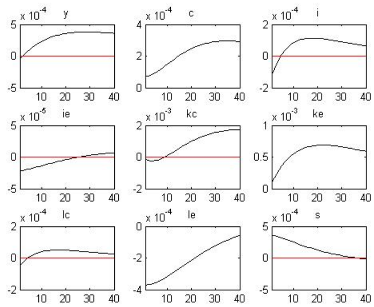


Figure 3.4: Orthogonalized shock to the equipment investment variable in the short period

As in Greenwood et al (1998) the shock does not immediately hit output, consumption, and investment rates. Consumption is negative at the beginning and then tends to increase. It is important to highlight the differences between k_c and k_e. Only k_e is directly affected by the shock. The same happens for l_c and l_e. As we can see the shock shows more persistence than in the Greenwood et. al (1998) model.
Concerning the observations about the negative co-movements between the price and quantity of new equipment goods produced, the model confirms the Greenwood et. al (1998) assumptions.

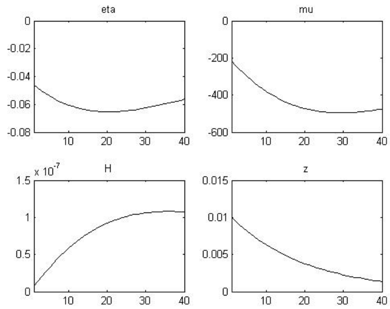


Figure 3.5: Orthogonalized shock to equipment investments in the short period

Where η_t represents the investment cost in knowledge (constraint marginal cost); μ_t represents the investment cost in schooling-education (constraint marginal cost); and where H_t represents the stock of knowledge.

The second simulation exercise considers a longer period of analysis, features the shock (z_t) over the schooling-education variable[28] and it is still capital embodied:

$$H_{t+1} = (1 - \delta_s)H_t + F(s_t)H_t$$

$$F(s_t) = s_t^{\omega} \quad \omega > 1$$

$$F(s_t) = \exp(z_t)s_t^{\omega}$$

$$z_t = \rho z_{t-1} + \varepsilon_t \quad \varepsilon_t \sim (0, \sigma)$$

[28]Obviously the parameter ω has an important role for that shock.

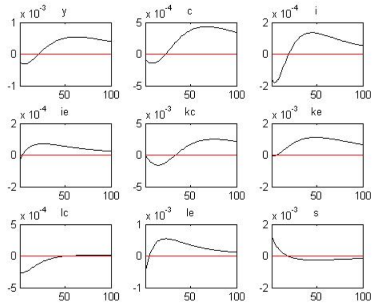


Figure 3.6: Orthogonalized shock to schooling level in the long period

The shock directly hits the knowledge stock H_t[29], increasing equipment investments, and the production of equipment goods. Current output is indirectly affected by the shock, through an increased employment of capital and labor which occurred in response to a change in investment opportunities. Since the model considers two types of labor and divides the capital stock into structures and equipment capital, we can demonstrate that only one kind of labor increases (the equipment goods production sector's labor), and in the same way that there is only one capital extension (the equipment capital).

Concerning the price observation, also in that second simulation exercise, the Greenwood et. al (1998) assumptions are proved. That negative co-movement affects the production of new equipment goods, and it is increases in the long-period. The shock is a temporary shock but its effects hit the economy in the long-period.[30]

[29]The IRF about knowledge stock, H_t, is inclueded in figure 3.7

[30]As I said before, this is a crucial result in contrast with the sectoral-neutral model theory where relative prices are assumed to be fixed.

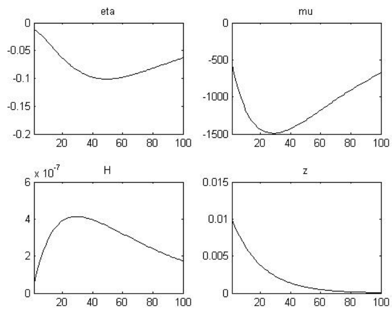


Figure 3.7: Orthogonalized shock to schooling level in the long period

Where η_t represents the investment cost in knowledge (constraint marginal cost); μ_t represents the investment cost in schooling-education (constraint marginal cost), and where H_t represents the stock of knowledge.

Finally we can resume the transmission mechanism as: a positive shock immediately raises the level of schooling-education level, and consequently (in the short-period) the stock of knowledge in the economy. That entices equipment investments, and hence a higher equipment goods stock in the next period. The resulting decline in the equipment capital's value implies a lower marginal utilization cost. This promotes a more intensive utilization of the existing equipment goods, which leads to increased employment of the equipment production sector's labor, and to output expansion (in the long-period). The increases in the rate of return on equipment investments stimulate production, however, at the same time they operate to dissuade consumption.
In the short-period consumption decreases and it comes back to grow only in the long period, the same happens concerning the output variable. Hence, it is a priori uncertainty whether consumption is procyclical in the model, even if it is in the actual data.

3.5 Conclusion

The analysis in this chapter suggests that investment-specific technological changes may trigger innovative investments and be a persistent source of economic fluctuations. The present model considers two types of labor and divides capital stock into structures and equipment capital, hence we can demonstrate that only one kind of labor increases (equipment goods production sector's labor), and in the same way that there is only one capital extension (the equipment capital).
Here I demonstrate that a good way to entice investments in *R&D* can be a positive shock over the schooling-education variable. That shock directly affects the existing stock of knowledge, and indirectly it contributes to improving the efficiency in production of the equipment goods.
Therefore, it is not how many people devote their time to working in the equipment goods production sector which determine the rate of economic growth. It is, rather, an increase in the efficiency of the equipment goods, as demonstrated in the empirical data. Looking at the European data it is easy to see that too little knowledge is really devoted to research and to patent production, and that having a large population is not sufficient in generating growth. Time series about annual growth in neutral productivity have an opposite trend, if compared to the time series about the growth rate in investment-specific technological change.

Future Direction

First of all, the most important future direction for the model is to estimate it.

Greenwood et al. (1997) found that about 60 per cent of U.S. post-war growth can be explained by investment-specific technological change. More recent analysis suggest that investment-specific technological change contributes relatively less to the business cycle, than to long-term growth.
An investment-specific technological shock, as I demonstrate here, is strictly

associated to a knowledge advancement process, so equipment capital and dedicated labor utilization will increase as well as in response to changed investment opportunities. At this point, it becomes interesting interpreting the positive shock over the schooling-education level, as a governmental transfer that affects economic growth.

To this aim we add the public sector in a stylized standard way. The balanced budget-rule is given by:

$$g_t = tax_t$$

where g_t is exogenous government spending subject to shocks:

$$g_t = g_t \exp(\zeta_t)$$

$$\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_{\zeta,t}$$

and $\varepsilon_{\zeta,t} \sim iid.N(0, \sigma_\zeta^0)$

Conclusion

The analysis in this work suggests that investment-specific technological changes may trigger innovative investments and be a persistent source of economic fluctuations. The present model considers two types of labor and divides capital stock into structures and equipment capital. Hence, we can demonstrate that only one kind of labor increases (the equipment production sector's labor), and in the same way that there is only one capital extension (the equipment capital). It is important to highlight that since the model considers an investment-specific technological shock, and not a TFP shock, current output is only indirectly affected by the shock. The model's simulation results demonstrate that a good way to entice investments in $R\&D$ can be a positive shock over the schooling-education variable. Hence, it is not how many people devote their time to working in the equipment goods production sector, which determine the rate of economic growth. But it is, rather, an increase in the efficiency of the equipment goods, as demonstrated in the empirical data.

There are two ways to follow in order to improve the model are:

- On one side the model could be estimated by econometric techniques.
- On the second side, it would be interesting to add a government sector. Looking at how a government grant-strategy could affect investment-

specific technological changes and the relative economic growth.

Possible policies for the government are:

1. To increase savings as a source of capital formation.

2. To encourage investments in knowledge resources, such as education and training to enable an increased output per worker without an increase in capital.

3. To encourage investments in structures capitals, such as factories and infrastructures, to develop the capital deepening process.

4. To provide incentives for research and development to enable a continuous revival of capital deepening.

Appendix A

Appendix

A.1 First Order Conditions

Starting from the Lagrangian we obtain the solution of the model:

$$
\begin{aligned}
L = E_t[\sum_{t=0}^{\infty} \beta^t \{[lnc_t + \psi ln(1 - l_{c,t} - l_{e,t} - s_t)] - \\
-\lambda_t(c_t + k_{c,t} - k_{c,(t-1)}^{\alpha} k_{e,(t-1)}^{\gamma} l_{c,t}^{1-\alpha-\gamma} - (1-\delta)k_{c,(t-1)}) \\
-\eta_t(k_{e,t} - H_{t-1}^{\theta} l_{e,t}^{1-\theta} - (1-\delta_e)k_{e,(t-1)}) \\
-\mu_t(H_t - (1-\delta_s)H_{t-1} + s_t^{\omega} H_{t-1})\}]
\end{aligned}
$$

where λ_t η_t and μ_t are the lagrangian multipliers. First order conditions for this problem are:

$$
\begin{aligned}
\frac{\partial L}{\partial c_t} = [\beta^t\{\frac{1}{c_t}\}] = 0; \\
\lambda_t = \beta \frac{1}{c_t}
\end{aligned}
$$

$$
\frac{\partial L}{\partial l_{c,t}} = [\beta^t\{\frac{\psi}{l_{c,t} - l_{e,t} - s_t} + \lambda_t k_{c,(t-1)}^{\alpha} k_{e,(t-1)}^{\gamma} (1-\alpha-\gamma) l_{c,(t-1)}^{-\alpha-\gamma}] = 0
$$

$$\implies -\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}+\frac{\lambda_t k_{c,(t-1)}^{\alpha}k_{e,(t-1)}^{\gamma}(1-\alpha-\gamma)l_{c,(t-1)}^{-\alpha-\gamma}}{l_{c,t}}=0$$

$$\implies -\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}=-\lambda_t\frac{(1-\alpha-\gamma)y_t}{l_{c,t}}=0$$

$$\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}=\lambda_t\frac{(1-\alpha-\gamma)y_t}{l_{c,t}}=0$$

$$y_t=k_{c,(t-1)}^{\alpha}k_{e,(t-1)}^{\gamma}l_{c,t}^{1-\alpha-\gamma}$$

$$\frac{\partial L}{\partial l_{e,t}}=[\beta^t-\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}+\eta_t(1-\theta)H_{t-1}^{\theta}l_{e,t}^{-\theta}=0]$$

$$\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}=\eta_t(1-\theta)H_{t-1}^{\theta}l_{e,t}^{-\theta}$$

$$\frac{\partial L}{\partial s_t}=[\beta^t\{-\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}+\mu_t\omega s_t^{\omega-1}H_{t-1}\}=0$$

$$\frac{\psi}{1-l_{c,t}-l_{e,t}-s_t}=\mu_t\omega s_t^{\omega-1}H_{t-1}=0$$

$$\frac{\partial L}{\partial \lambda_t}=[\beta^t(-1)\{c_t+k_{c,t}-k_{c,(t-1)}^{\alpha}k_{e,t-1}^{\gamma}l_{c,t}^{1-\alpha-\gamma}-(1-\delta)k_{c,(t-1)}\}]=0$$

$$c_t+k_{c,t}=k_{c,(t-1)}^{\alpha}k_{e,(t-1)}^{\gamma}l_{c,t}^{1-\alpha-\gamma}+(1-\delta)k_{c,(t-1)}$$

$$\frac{\partial L}{\partial \eta_t}=[-\beta^t\{k_{e,t}-H_{t-1}^{\theta}l_{e,t}^{1-\theta}-(1-\delta_e)k_{e,(t-1)}\}]=0$$

$$k_{e,t} = H_{t-1}^{\theta} l_{e,t}^{1-\theta} + (1-\delta_e) k_{e,(t-1)} \tag{A.1}$$

$$\frac{\partial L}{\partial \mu_t} = [\beta^t + \{H_t - (1-\delta_s)H_{t-1} - s_t^{\omega} H_{t-1}\}] = 0$$

$$H_t = s_t^{\omega} H_{t-1} + (1-\delta_s)H_{t-1} = 0$$

$$\frac{\partial L}{\partial k_{c,t}} = [\beta^t\{-\lambda_t\} + \beta^{t+1}\{\lambda_{t+1}(\alpha k_{c,t}^{\alpha-1} k_{e,t}^{\gamma} l_{c,t}^{1-\alpha-\gamma} + 1 - \delta)]\} = 0$$

$$\Longrightarrow \lambda_t = \beta\lambda_{t+1}[\alpha k_{c,t}^{\alpha-1} k_{e,t}^{\gamma} l_{c,t}^{1-\alpha-\gamma} + 1 - \delta)] = 0$$

$$\lambda_t = \beta\lambda_{t+1}(\alpha \frac{y_{t+1}}{k_{c,t}} + 1 - \delta) = 0$$

$$\frac{\partial L}{\partial k_{e,t}} = [\beta^t\{-\eta_t\} + \beta^{t+1}\{\lambda_{t+1}(\gamma k_{c,t}^{\alpha} k_{e,t}^{\gamma-1} l_{c,t}^{1-\alpha-\gamma} + 1 - \delta) + \eta_{t+1}(1-\delta_e)]\} = 0$$

$$\eta_t = \beta^t[\lambda_{t+1}[\gamma k_{c,t}^{\alpha} k_{e,t}^{\gamma-1} l_{c,t}^{1-\alpha-\gamma} + 1 - \delta) + \eta_{t+1}(1-\delta_e)] = 0$$

$$\eta_t = \beta[\lambda_{t+1}(\gamma \frac{y_{t+1}}{k_{e,t}} + 1 - \delta) + \eta_{t+1}(1-\delta_e)] = 0$$

$$\frac{\partial L}{\partial H_t} = \beta^t\{-\mu_t\} + \{\beta^{t+1}\{\eta_{t+1}\theta H_t^{\theta-1} l_{e,(t+1)}^{1-\theta} + \eta_{t+1}[(1-\delta_s) + s_t^{\omega}] = 0$$

$$\mu_t = \beta[\mu_{t+1}\theta H_t^{\theta-1} l_{e,(t+1)}^{1-\theta} + \mu_{t+1}(s_t^{\omega} + (1-\delta_s)]$$

A.2 Steady State Values

Now simplifying the steady state conditions we get the model as follow:

$$\frac{\psi}{(1-l_c-l_e-s)} = \frac{(1-\alpha-\gamma)y}{l_c c}$$

$$y = k_c^{\alpha} k_e^{\gamma} l_c^{1-\alpha-\gamma}$$

$$\frac{\psi}{(1-l_c-l_e-s)} = (1-\theta)\eta H^{\theta} l_e^{-\theta}$$

$$\frac{\psi}{(1-l_c-l_e-s)} = \omega\mu s^{\omega-1} H$$

$$y = c + \delta k_c$$

$$s^{\omega} = \delta_s$$

$$1 = \beta[\frac{\alpha y}{k_c} + 1 - \delta]$$

$$\eta[1-\beta(1-\delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e} + 1 - \delta);$$

$$\mu(1-\beta) = \beta\eta\theta H^{(\theta-1)} l_e^{(1-\theta)}$$

$$H^{\theta} l_e^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

Now equation 6 defines s in the steady state, so we have:

$$\bar{s} = \delta_s^{\frac{1}{\omega}}$$

Now the model is:

$$\frac{\psi}{(1 - l_c - l_e - \bar{s})} = \frac{(1-\alpha-\gamma)y}{l_c c}$$

$$y = k_c^{\alpha} k_e^{\gamma} l_c^{1-\alpha-\gamma}$$

$$\frac{\psi}{(1 - l_c - l_e - \bar{s})} = (1-\theta)\eta H^{\theta} l_e^{-\theta}$$

$$\frac{\psi}{(1 - l_c - l_e - \bar{s})} = \omega\mu\bar{s}^{\omega-1} H$$

$$y = c + \delta k_c$$

$$1 = \beta[\frac{\alpha y}{k_c} + 1 - \delta]$$

$$\eta[1 - \beta(1-\delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e} + 1 - \delta);$$

$$\mu(1-\beta) = \beta\eta\theta H^{(\theta-1)} l_e^{(1-\theta)}$$

$$H^{\theta} l_e^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

Modifying the left side of equation 3 we can write:

$$\frac{\psi}{(1 - l_c - l_e - \bar{s})} = \frac{(1 - \alpha - \gamma)y}{l_c c}$$

$$y = k_c^{\alpha} k_e^{\gamma} l_c^{1-\alpha-\gamma}$$

$$\eta = \frac{\omega}{(1 - \theta)} \bar{s}^{\omega-1} H^{1-\theta} l_e^{\theta} \mu$$

$$\frac{\psi}{(1 - l_c - l_e - \bar{s})} = \omega \mu \bar{s}^{\omega-1} H$$

$$y = c + \delta k_c$$

$$1 = \beta[\frac{\alpha y}{k_c} + 1 - \delta]$$

$$\eta[1 - \beta(1 - \delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e} + 1 - \delta);$$

$$\mu = \frac{\beta}{(1 - \beta)} \eta \theta H^{(\theta-1)} l_e^{(1-\theta)}$$

$$H^{\theta} l_e^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

Now substituting the right side of equation 3 into equation 8 we have:

$$\frac{\psi}{(1 - l_c - l_e - \bar{s})} = \frac{(1 - \alpha - \gamma)y}{l_c c}$$

$$y = k_c^{\alpha} k_e^{\gamma} l_c^{1-\alpha-\gamma}$$

$$\eta = \frac{\omega}{(1-\theta)} \bar{s}^{\omega-1} H^{1-\theta} l_e^{\theta} \mu$$

$$\frac{\psi}{(1 - l_c - l_e - \bar{s})} = \omega \mu \bar{s}^{\omega-1} H$$

$$c + \delta k_c = y$$

$$1 = \beta[\frac{\alpha y}{k_c} + 1 - \delta]$$

$$\eta[1 - \beta(1 - \delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e} + 1 - \delta);$$

$$\bar{l_e} = \frac{(1-\beta)(1-\theta)}{\omega\beta\theta} \bar{s}^{1-\omega}$$

$$H^{\theta} l_e^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

We can now write the model as follow:

$$\frac{c}{y} = \frac{(1-\alpha-\gamma)(1 - l_c - \bar{l_e} - \bar{s})}{\psi l_c}$$

$$y = k_c^{\alpha} k_e^{\gamma} l_c^{1-\alpha-\gamma}$$

$$\eta = \frac{\omega}{(1-\theta)} \bar{s}^{\omega-1} H^{1-\theta} \bar{l_e}^{\theta} \mu$$

$$\frac{\psi}{(1 - l_c - \bar{l_e} - \bar{s})} = \omega\mu\bar{s}^{\omega-1}H$$

$$\frac{c}{y} + \frac{i}{y} = 1$$

$$\frac{i}{y} = \frac{\alpha\beta\delta}{1 - \beta + \beta\delta}$$

$$\eta[1 - \beta(1 - \delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e} + 1 - \delta);$$

$$H^{\theta}\bar{l_e}^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

we continue on substituting equation 1 and 6 into equation 5:

$$\frac{c}{y} = \frac{(1-\alpha-\gamma)(1-l_c-\bar{l}_e-\bar{s})}{\psi l_c}$$

$$y = k_c^{\alpha} k_e^{\gamma} l_c^{1-\alpha-\gamma}$$

$$\eta = \frac{\omega}{(1-\theta)} \bar{s}^{\omega-1} H^{1-\theta} \bar{l}_e^{\,\theta} \mu$$

$$\frac{\psi}{(1-l_c-\bar{l}_e-\bar{s})} = \omega\mu\bar{s}^{\omega-1} H$$

$$\bar{l}_c = \frac{(1-\alpha-\gamma)(1-\beta+\beta\delta)(1-\bar{l}_e-\bar{s})}{[(1-\beta+\beta\delta-\alpha\beta\gamma)\psi+(1-\alpha-\gamma)(1-\beta+\beta\delta)]}$$

$$\frac{i}{y} = \frac{\alpha\beta\delta}{1-\beta+\beta\delta}$$

$$\eta[1-\beta(1-\delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e}+1-\delta);$$

$$H^{\theta} \bar{l}_e^{\,1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

again the system is:

$$\frac{c}{y} = \frac{(1-\alpha-\gamma)(1-\bar{l}_c-\bar{l}_e-\bar{s})}{\psi \bar{l}_c}$$

$$y = k_c^{\alpha} k_e^{\gamma} \bar{l_c}^{1-\alpha-\gamma}$$

$$\eta(1-\theta)H^{1-\theta}\bar{l_e}^{\theta} = \omega\mu\bar{s}^{\omega-1}H$$

$$\frac{\psi}{(1-\bar{l_c}-\bar{l_e}-\bar{s})} = \omega\mu\bar{s}^{\omega-1}H$$

$$\frac{i}{y} = \frac{\alpha\beta\delta}{1-\beta+\beta\delta}$$

$$\eta[1-\beta(1-\delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e}+1-\delta);$$

$$H^{\theta}\bar{l_e}^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

substituting the left side of the equation 4 into the 3 we come out with:

$$\frac{c}{y} = \frac{(1-\alpha-\gamma)(1-\bar{l_c}-\bar{l_e}-\bar{s})}{\psi\bar{l_c}}$$

$$y = k_c^{\alpha} k_e^{\gamma} \bar{l_c}^{1-\alpha-\gamma}$$

$$\eta(1-\theta)H^{1-\theta}\bar{l_e}^{\theta} = \frac{\psi}{(1-\bar{l_c}-\bar{l_e}-\bar{s})}$$

$$\frac{\psi}{(1-\bar{l_c}-\bar{l_e}-\bar{s})} = \omega\mu\bar{s}^{\omega-1}H$$

$$\frac{i}{y} = \frac{\alpha\beta\delta}{1-\beta+\beta\delta}$$

$$\eta[1-\beta(1-\delta_e)] = \frac{\beta}{c}(\frac{\gamma y}{k_e} + 1 - \delta);$$

$$H^{\theta}\bar{l_e}^{1-\theta} = \delta_e k_e$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

and again substituting equation 1 into the right side of equation 6

$$\frac{c}{y} = \frac{(1-\alpha-\gamma)(1-\bar{l_c}-\bar{l_e}-\bar{s})}{\psi\bar{l_c}}$$

$$y = k_c^{\alpha} k_e^{\gamma} \bar{l_c}^{1-\alpha-\gamma}$$

$$\eta = \frac{\psi\bar{l_e}^{\theta} H^{-\theta}}{(1-\bar{l_c}-\bar{l_e}-\bar{s})(1-\theta)}$$

$$\frac{\psi}{(1-\bar{l_c}-\bar{l_e}-\bar{s})} = \omega\mu\bar{s}^{\omega-1}H$$

$$\frac{i}{y} = \frac{\alpha\beta\delta}{1-\beta+\beta\delta}$$

$$\eta[1-\beta(1-\delta_e)] = \frac{\beta\psi\bar{l_c}}{[y(1-\alpha-\gamma)(1-\bar{l_c}-\bar{l_e}-\bar{s})]}(\frac{\gamma y}{H^{\theta}\bar{l_e}^{1-\theta}}\delta_s + 1 - \delta);$$

$$k_e = \frac{H^{\theta}\bar{l_e}^{1-\theta}}{\delta_e}$$

$$i_e = \delta_e k_c$$

$$i = \delta k_c$$

Now substituting equation 3 into equation 6 (η value) we end up with:

$$\frac{\psi \bar{l_e}^{\theta} H^{-\theta}}{(1-\bar{l_c}-\bar{l_e}-\bar{s})(1-\theta)}[1-\beta(1-\delta_e)] = \frac{\beta\psi\bar{l_c}}{[y(1-\alpha-\gamma)(1-\bar{l_c}-\bar{l_e}-\bar{s})]}(\gamma y H^{-\theta}\bar{l_e}^{\theta-1}\delta_s + 1 - \delta)$$

$$\frac{\bar{l_e}^{\theta} H^{-\theta}}{(1-\theta)} y(1-\alpha-\gamma)(1-\beta+\beta\delta_e) = \beta\bar{l_c}\gamma y H^{-\theta}\bar{l_e}^{\theta-1}\delta_e + \beta\bar{l_c}(1-\delta)$$

$$y[\frac{\bar{l_e}^{\theta} H^{-\theta}}{(1-\theta)}(1-\alpha-\gamma)(1-\beta+\beta\delta_e) - \beta\bar{l_c}\gamma y H^{-\theta}\bar{l_e}^{\theta-1}\delta_e] = \beta\bar{l_c}(1-\delta)$$

$$yH^{-\theta}[\frac{\bar{l_e}^{\theta}(1-\alpha-\gamma)(1-\beta+\beta\delta_e)}{(1-\theta)} - \beta\bar{l_c}\gamma\bar{l_e}^{\theta-1}\delta_e] = \beta\bar{l_c}(1-\delta)$$

$$y = H^{-\theta}[\frac{\beta\bar{l_c}(1-\delta)\bar{l_e}^{\theta}(1-\alpha-\gamma)(1-\beta+\beta\delta_e)}{(1-\theta)} - \beta\bar{l_c}\gamma\bar{l_e}^{\theta-1}\delta_e]$$

For simplicity we called the right side of the above equation ξ. So the steady state value of y is:

$$y = H^{-\theta}\xi$$

then we have that:

$$k_e = \frac{H^{\theta}\bar{l_e}^{1-\theta}}{\delta_e}$$

$$\frac{k_e}{y} = \frac{H^{\theta}\bar{l_e}^{1-\theta}}{\delta_e H^{\theta}\xi}$$

$$\frac{k_e}{y} = \frac{\bar{l_e}^{1-\theta}}{\delta_e \xi}$$

and,

$$y = k_c^{\alpha} k_e^{\gamma} \bar{l_c}^{1-\alpha-\gamma}$$

$$1 = \frac{k_c^{\alpha} k_e^{\gamma} \bar{l_c}^{1-\alpha-\gamma}}{y^{\alpha} y^{\gamma} y^{1-\alpha-\gamma}} = (\frac{k_c}{y})^{\alpha} (\frac{k_e}{y})^{\gamma} (\frac{\bar{l_c}}{y})^{1-\alpha-\gamma}$$

Finally we have that:

$$\frac{k_c}{y} = \frac{\alpha\beta}{(1-\beta+\beta\delta)}$$

$$\frac{k_e}{y} = \frac{\bar{l_e}^{1-\theta}}{\delta_e \xi}$$

$$\bar{y}^{1-\alpha-\gamma} = (\frac{\alpha\beta}{1-\beta+\beta\delta})^{\alpha} (\frac{\bar{l_e}^{1-\theta}}{\delta_e \xi})^{\gamma} \bar{l_c}^{1-\alpha-\gamma}$$

$$\bar{y} = [(\frac{\alpha\beta}{(1-\beta+\beta\delta)}^{\alpha})(\frac{\bar{l_e}^{1-\theta}}{\delta_e \xi})^{\gamma} \bar{l_c}^{1-\alpha-\gamma}]^{\frac{1}{1-\alpha-\gamma}}$$

Now writing the model again we have:

$$\frac{c}{y} = \frac{(1-\alpha-\gamma)(1-\bar{l_c}-\bar{l_e}-\bar{s})}{\psi \bar{l_c}}$$

$$y = k_c^{\alpha} k_e^{\gamma} \bar{l_c}^{1-\alpha-\gamma}$$

$$\eta = \frac{\psi \bar{l_e}^{\theta} H^{-\theta}}{(1-\bar{l_c}-\bar{l_e}-\bar{s})(1-\theta)}$$

$$\frac{\psi}{(1-\bar{l}_c-\bar{l}_e-\bar{s})}=\omega\mu\bar{s}^{\omega-1}H$$

$$\frac{i}{y}=\frac{\alpha\beta\delta}{1-\beta+\beta\delta}$$

$$y=H^{\theta}\xi$$

$$k_e=\frac{H^{\theta}\bar{l}_e^{\,1-\theta}}{\delta_e}$$

$$i_e=\delta_e k_c$$

$$i=\delta k_c$$

As I said before, equation 2 can be written as:

$$1=\frac{k_c^{\alpha}k_e^{\gamma}\bar{l}_c^{\,1-\alpha-\gamma}}{y^{\alpha}y^{\gamma}y^{1-\alpha-\gamma}}=(\frac{k_c}{y})^{\alpha}(\frac{k_e}{y})^{\gamma}(\frac{\bar{l}_c}{y})^{1-\alpha-\gamma}$$

and

$$\frac{k_c}{y}=\frac{\alpha\beta}{(1-\beta+\beta\delta)}$$

so we get the steady state of H^{θ}:

$$H^{\theta}=\frac{y}{\xi}$$

from equation 7:

$$k_e=\frac{y}{\xi\delta_e}\bar{l}_e^{\,1-\theta}$$

$$\frac{k_e}{y} = \frac{\bar{l_e}^{1-\theta}}{\delta_e \xi}$$

again as I said before:

$$1 = (\frac{\alpha\beta}{(1-\beta+\beta\delta)})^{\alpha}(\frac{\bar{l_e}^{1-\theta}}{\delta_e \xi})^{\gamma}\frac{\bar{l_c}^{1-\alpha-\gamma}}{y^{1-\alpha-\gamma}}$$

$$\bar{y} = [(\frac{\alpha\beta}{(1-\beta+\beta\delta)}^{\alpha})(\frac{\bar{l_e}^{1-\theta}}{\delta_e \xi})^{\gamma}\bar{l_c}^{1-\alpha-\gamma}]^{\frac{1}{1-\alpha-\gamma}}$$

In order to have a look of all the steady state values together I write down the model one more time:

$$\bar{s} = \delta_s^{\frac{1}{\omega}}$$

$$\bar{c} = \bar{y}\frac{(1-\alpha-\gamma)(1-\bar{l_c}-\bar{l_e}-\bar{s})}{\psi \bar{l_c}}$$

$$\bar{y} = [(\frac{\alpha\beta}{(1-\beta+\beta\delta)}^{\alpha})(\frac{\bar{l_e}^{1-\theta}}{\delta_e \xi})^{\gamma}\bar{l_c}^{1-\alpha-\gamma}]^{\frac{1}{1-\alpha-\gamma}}$$

$$\bar{\eta} = \frac{\psi \bar{l_e}^{\theta}\bar{H}^{-\theta}}{(1-\bar{l_c}-\bar{l_e}-\bar{s})(1-\theta)}$$

$$\bar{\mu} = \frac{\psi}{(1-\bar{l_c}-\bar{l_e}-\bar{s})\omega \bar{s}^{\omega-1}\bar{H}}$$

$$\bar{i} = \bar{y}\frac{\alpha\beta\delta}{1-\beta+\beta\delta}$$

$$\bar{H} = \frac{\bar{y}}{\xi}^{\frac{1}{\theta}}$$

$$\bar{l_e} = \frac{(1-\beta)(1-\theta)}{\omega\beta\theta}\bar{s}^{1-\omega}$$

$$\bar{l_c} = \frac{(1-\alpha-\gamma)(1-\beta+\beta\delta)(1-\bar{l_e}-\bar{s})}{[(1-\beta+\beta\delta-\alpha\beta\gamma)\psi+(1-\alpha-\gamma)(1-\beta+\beta\delta)]}$$

$$\bar{k_e} = \frac{H^{\theta}\bar{l_e}^{1-\theta}}{\delta_e}$$

$$\bar{i_e} = \delta_e k_e$$

$$\bar{k_c} = \frac{i}{\delta}$$

A.3 Decentralized Economy

In this section I analyze the equipment goods production sector. That production sector plays in a monopolistic economy; since it produces something new each time, it has the power to decide the goods' price.

Decentralized economy: equipment goods production sector.

Households maximize utility over consumption, c_t, and leisure, l_t, according to the following utility function:

$$E_t[\sum_{t=0}^{\infty} \beta^t U_t(c_t; l_t)]$$

with

$$U(c_t, l_t) = lnc_t + \psi ln[(1 - l_{e,t} - s_t)]$$

The equipment goods production function is:

$$i_{e,t} = [H_t, l_{e,t}] = H_t^{\theta} l_{e,t}^{(1-\theta)} \quad 0 < \theta < 1$$

the knowledge stock is defined recursively as:

$$H_{t+1} = (1 - \delta_s)H_t + F(s_t)H_t$$

where

$$F(s_t) = s_t^{\omega}$$

and

$$\omega > 1$$

Households maximize utility subject to the following budget constraint:

$$c_t + H_{t+1} = w_t l_{e,t} + r_t H_t + c_s s_t + (1 - \delta_s) H_t + F(s_t) H_t$$

where H_t is knowledge capital stock, w_t real wages, c_s cost of schooling, r_t real interest rates or cost of knowledge capital and δ_s the depreciation rate.

Finally we end up with the following *profit equation* to maximize:

$$\pi = p_t i_{e,t}[H_t, l_{e,t}] - w_t l_{e,t} - r_t H_t - c_s s_t$$

A.4 Tables

Table A.1: Parameters

α	γ	β	ψ	δ	δ_e	δ_s	ω	ϕ	ρ	σ
0.3	0.2	0.97	1.5	0.035	0.05	0.05	1.5	0.3	0.95	0.01

Table A.2: Steady State Values

Variables	*Calibrated Values*
y	0.40524079
c	0.34070009
i	0.06454069
i_e	0.01101906
k_c	1.84401986
k_e	0.22038118
l_c	0.20830378
l_e	0.13059054
s	0.13572088
η	48.33735023
μ	150145.206
h	0.00003441
z	0
e	0

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