

# Unconventional Monetary Policy Coordination in a Two Country World with Banking Frictions

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## **Abstract**

During the last financial crises, the main Central Banks in the world implemented different examples of unconventional monetary policies in order to soften the tightening in the credit market conditions. In the United States the Federal Reserve (Fed) applied the “quantitative easing” approach in a very massive way, allowing the country to avoid a deep recession from the sub-prime crisis to this day. Similar attempts (even if differently named) were followed during the same period by the Bank of England and the Bank of Switzerland. Even in the Euro Area, despite the more severe stance implied by the influence of the Bundesbank’s heritage, during the current Eurozone crisis the European Central Bank (ECB) was involved in massive shopping of Treasury bonds of the countries under speculative attack in order to restore the transmission mechanism of its monetary policy. Moreover, in the attempt to avoid a credit crunch for the real economy and to mitigate the urgent liquidity needs by the European banking

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system, the ECB also launched last December its unlimited three-years liquidity-refinancing operations at the 1 percent in order to provide longer-term lending facilities to the financial institutions. Unconventional monetary policies seem therefore to get an increased importance during the last few years as alternative instrument of monetary policy when the traditional target rates are close to zero and when the credit market conditions worsen with serious risk for the real economy, justifying a growing interest on this issue. We would like to analyze how different unconventional interventions by the Central Banks might mitigate a country-specific crisis in a two country DSGE model with credit frictions. To this task we use a baseline model we previously developed, whose framework is mainly based on a work by Gertler and Kiyotaki (2010). We specifically focus on the effects of coordinated versus uncoordinated policies.

*Key Words:* two country DSGE model, incomplete markets, endogenous portfolio choice, unconventional monetary policy

## 1 Introduction

Nowadays we are unfortunately experiencing a serious financial distress having its epicenter in the Eurozone. Differently from the previous crisis of some years ago, originated in the 2007 US sub-prime mortgage market and then spread around the world through the deep linkages across financial activities, the current crisis seems to be less related to the innovations in financial markets and their deficient regulatory system, although the first seeds of the Eurozone current turmoil reside exactly in the US-2007 one. It is, instead, the lack of control of the budget imbalances by some member states during the last decades that is responsible for the serious problems that is now facing the Euro Area as a whole. In a monetary union, balance sheets of banks and countries are deeply interconnected; the default risk for a specific country, however small with respect to the rest of the union, can end up by infecting the whole network of relationships that holds up European Governments and financial intermediaries. A lacking regulatory system together with a weak control mechanism completed the work, underestimating the real risks in banking balance sheets. Paradoxically, even countries with a "safe" balance assets/liabilities and a restrained debt to GDP ratio can be seriously involved in the problem. What should a Central Bank do in a situation such the one we are describing now it is not clear at all. How large have to be its interventions, when it is clear that the problem was caused by the national segmented fiscal policies? We all get used in the past with the idea of a Central Bank totally devoted in supporting and stabilizing domestic financial conditions in full independence from the political Government. This is what all we learned at university: there must be total independence between the Government (that decides taxes and public expenditure and has a short-term, political horizon) and the Central Bank's actions. Under normal times, monetary policy acts by setting a target for the overnight interest rate

and by adjusting the supply of money in order to meet that target (through open market operations). The framework used by the Central Bank in providing liquidity to the system is built in a way that minimizes the Central Bank's exposure to private sector risk (all the liquidity operations are done against eligible collateral). In the last few years, however, many structural changes happened in the financial world that made things act differently. First of all, if the institutional framework of all the main modern Central Banks around the world were deeply influenced by the 1980s' lessons of high and worrying inflation levels and high interest rates, today's inflation levels are much lower, and interest rates are in some cases so low that the Central Bank is bound in its usual ability to expand money supply by lowering the overnight rate. This is what happened in Japan in the 1990's (when the country was in liquidity trap). Today the Fed is experiencing a not so different situation, being the Federal Fund rate very close to zero. The Central Bank's response was, however, very different in the two situations. The Japanese experience can probably be a first explanation of why the Central Banks around the world were encouraged to explore other ways to foster real economy when interest rates are very low. A second explanation is the possibility that disruptions in the transmission mechanism can prevent the use of the standard monetary policy even if the interest rates are not close to zero. This happens, for example, when the interbank market suffers malfunctions by the lack of confidence across the counterparts. The financial engineering exploded during the last few years highlighted this problem during the sub-prime mortgage crisis, when extremely complex financial assets were created and then traded in highly integrated financial markets. As a result, the banks' balance sheets around the world ended up full of assets by hardly computable riskiness. It was the lack of liquidity originated in a very specific economic sector (the sub-prime mortgage market) that caused a global crisis, making clear that the task of ensure liquidity to the interbank market and

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## 1 INTRODUCTION

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through this financial stability it is not less important than inflation control as monetary policy task. With the most western economies suffering from the low growth, the main risk for the real economy is the one related to a credit crunch. Not surprisingly, therefore, during the sub-prime mortgage crisis both the Fed and the ECB started to implement same “unconventional monetary policies”, oriented in expanding central intermediation as opposed as to the traditional policies which aim is instead to expand money supply. By implementing these unconventional policies, however, the Central Bank takes upon itself private sector risk (they all imply, even if in different ways, to make risky loans to the private sector). For this reason the use of these policies has to be carefully restrained (for what concerns the Fed’s mandate, for example, these policies can be implemented only under “unusual and exigent circumstances”) . From the last 2007 crisis until now the Fed never stopped to implement its “quantitative easing” policies (a recent example is the so called “twist” operation, oriented to reduce the long term interest rates related to the mortgage market) in order to foster the US economic growth exposed to downside risks. During the same period, other main Central Banks around the world also implemented similar policies (first of all, the Bank of England and the Bank of Switzerland). In all that cases the interest of a country and that of its Central Bank perfectly overlap. Things are, however, different for the Central Bank of a monetary union such as the Eurozone, where fiscal policies remain segmented at a national level. The unconventional measures implemented by the ECB are necessarily much more limited by the binding and in some cases divergent interests of some member states. During the last months the ECB was involved in a growing shopping of Treasury bonds of the countries under speculative attack. However, the Central Bank justified its actions with the urgency to restore the transmission mechanism for its monetary policy, emphasizing that its target it is not the monetary financing of some member states (prohibited by its mandate).

Moreover, in the attempt to avoid a credit crunch for the real economy and to mitigate the urgent liquidity needs by the European banking system, the ECB also launched last December its unlimited three-years liquidity-refinancing operations at the 1 percent in order to provide longer-term lending facilities to the financial institutions.

Nevertheless, many charges are still moved against the ECB's lack of action, as if it was contributing to exacerbate the recession faced by the whole Euro Area. These same voices outline the different approaches adopted in the same period by the other main Central Banks, and accuse the ECB of moving wrongly against the trend. At the opposite corner, the German bankers, strongly fighter against the even limited unconventional policies adopted until now by the ECB. Being this crisis the result of Governments' budget imbalances, they question the opportunity of implementing that kinds of policies. Did not learn all of us that Governments and Central Banks must be independent? The "vicious" Governments could potentially learn a "bad lessons": that the Central Bank will try to avoid their default in all the possible way.

The debate heats. By facing today the empirical evidence of so different guidelines in implementing this kind of monetary policies, it origins the interest in analyzing them in the context of a theoretical framework. Also the issue related to the possible benefits from monetary policy coordination under unusual circumstances deserves study.

We deal here with three different kinds of unconventional monetary policies by using as reference the framework built by Gertler and Kiyotaki (2010), that we adapted to our two country model with banking frictions and portfolio integration: lending and liquidity facilities and equity injections. Again, as we did in the previous section of our PhD Dissertation, we consider different versions of the same model: in Model A good markets are fully integrated, but financial markets are closed. In Model B we instead derive the equilib-

rium portfolio shares endogenously, by applying the method developed by Devereux and Sutherland (since 2006 onwards).

We organized our analysis in the following sections: 1) the general model and its equilibrium 2) the description of the unconventional monetary policies (directly derived by the Gertler and Kiyotaki's work and applied to our model) 3) calibration 4) results. We collected all the equations we used for each model in the final Appendix.

## 2 The Baseline Model

To address monetary policy issues and to analyze how unconventional monetary policies work to mitigate country specific crises, we use the baseline model developed in the first part of our PhD Dissertation. We extend a work by Gertler and Kiyotaky (2010) by analyzing a two country world with international trade and banking frictions. In order to determine endogenous portfolio's shares in an equilibrium with international asset markets we refer to the approach followed by Devereux and Sutherland (since 2006 onwards). Tille and Van Wincoop (2007) also developed a similar method to get the same results. This part is better explained in the first part of the PhD Dissertation.

### 2.1 General Hypotheses

We assume a two country model, where each country is specialized in the production of a single traded good. In each country there is a continuum of households. Population size is normalized to 1. Every time a fraction  $1 - f$  of the home (foreign) representative household is done by depositors, while the remaining  $f$  is done by bankers. This turnover between the two groups is random, keeping the relative proportion of each type fixed. Depositors supply labour and return the wage to the household. Moreover, they deposit

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funds in a bank different from the one they own. Bankers manage a financial intermediary and transfer earnings back to the household. Within the family, we assume perfect consumption insurance.

We assume that with probability  $\theta$  a banker will be banker also at the following time. Instead with probability  $1 - \theta$  she will become a depositor. Therefore, each period  $(1 - \theta)f$  bankers exit and become depositors,  $\theta f$  bankers remain bankers,  $(1 - f)\theta$  depositors become bankers and  $(1 - f)(1 - \theta)$  depositors remain depositors. There is one risk-free asset (deposit)  $D_t$  for the home country ( $D_t^*$  abroad) called in composite consumption units. Only household within a country hold the deposit of that country (deposits are not internationally traded). The Cobb-Douglas consumption index for the representative home household will be:

$$C_t \equiv C_{h,t}^\gamma C_{f,t}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

where  $C_{h,t}$  is the domestic consumption of the home good and  $C_{f,t}$  is the domestic consumption of the foreign good.

Similarly, for the foreign representative agent we have identical preferences:

$$C_t^* \equiv C_{f,t}^{*\gamma} C_{h,t}^{*(1-\gamma)} \quad 0 < \gamma < 1 \quad (1^*)$$

where  $\gamma$  is the "economic size" of the home economy (i.e. the share of the home good in the consumption basket, while  $1 - \gamma$  is the economic size of the foreign country). We assume  $\gamma = \frac{1}{2}$  be the same for both home/foreign agents, meaning that there is no home consumption bias. Consumption-based price indexes will be:

$$P_t \equiv \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} P_{h,t}^\gamma P_{f,t}^{(1-\gamma)} \quad (2)$$

$$P_t^* \equiv \frac{1}{\gamma^\gamma (1-\gamma)^{1-\gamma}} P_{f,t}^{*\gamma} P_{h,t}^{*(1-\gamma)} \quad (2^*)$$



where  $P_{h,t}$  and  $P_{f,t}$  are the prices of home/foreign goods in domestic currency, while  $P_{h,t}^*$  and  $P_{f,t}^*$  are the prices of home and foreign goods in foreign currency.  $P_t$  (or  $P_t^*$ ) represents the minimum expenditure required to buy 1 unit of the composite consumption good. We assume the law of one price for both home and foreign good, that is:

$$P_{h,t} = \epsilon_t P_{h,t}^* \quad (3a)$$

$$P_{f,t} = \epsilon_t P_{f,t}^* \quad (3b)$$

Moreover, being households' preferences identical across borders with no home bias, the consumption baskets will also be identical and then the relative price of consumption (real exchange rate  $ReR_t$ ) will be equal to one (Purchase Power Parity holds), that is:

$$P_t = \epsilon_t P_t^* \quad (3c)$$

where we define  $\epsilon_t$  as the nominal exchange rate (the price of the foreign currency in terms of the home one).

Home consumption expenditure will be given by:

$$P_t C_t = P_{h,t} C_{h,t} + P_{f,t} C_{f,t}$$

while the lifetime utility of the home household will be:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{C_{\tau}^{1-\rho}}{1-\rho} - \frac{\kappa}{2} L_{\tau}^2 \quad (4)$$

where  $\beta$  is the discount rate, that can be exogenous or endogenous. We set  $\rho$ ,  $\gamma$  and  $\kappa$  equal to the same value both home and abroad.

Generic home household chooses consumption, labour supply and deposits in order to maximize (4) subject to the following flow of funds constraint:

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$$P_t C_t + P_t D_{t+1} = P_t R_t D_t + W_t L_t + \Pi_t \quad (5)$$

where we define:

$W_t(W_t^*)$ : nominal wage rate of the generic household.

$R_t(R_t^*)$ : real gross return on deposit from period  $t - 1$  to period  $t$ .

$D_t(D_t^*)$ : quantity of risk-less deposit in composite consumption units.

$\Pi_t(\Pi_t^*)$ : net distribution from the ownership of both banks and non-financial firms (nominal), net the transfers that the household gives to its members that start a new activity by entering at  $t$  in the banking system.

Symmetrically, for the foreign household we have:

$$P_t^* C_t^* + P_t^* D_{t+1}^* = P_t^* R_t^* D_t^* + W_t^* L_t^* + \Pi_t^* \quad (5^*)$$

Consumption based power parity always holds due to the law of one price plus the assumptions on preferences (from the (3)). Within/across countries preferences and constraints are symmetric. Since agents (households and firms) are equal within countries, in what follows all the variables will be considered in per capita (aggregate) terms.

## 2.2 The Production Side

### 2.2.1 Final Good Firms

Each firm faces an identical Cobb Douglas, therefore in aggregate we can assume:

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

or, for the foreign country:

$$Y_t^* = A_t^* K_t^{*(1-\alpha)} L_t^{*\alpha} \quad (6^*)$$

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Each home/foreign competitive final good firm produces a single (but differentiated among countries) output by using an identical CRTS Cobb Douglas production function with capital and labour as inputs. Labour is provided by the household of the same country. Now we define:

$\delta$ : rate of physical depreciation.

$\Psi_t(\Psi_t^*)$ : shock to the quality of capital.

$I_t(I_t^*)$ : aggregate home (foreign) investment in unit of consumption good.

Every period the law of motion for aggregate capital (in units of consumption good) will be given by:

$$K_{t+1} = \Psi_{t+1}[I_t + (1 - \delta)K_t] \quad (7)$$

The generic firm's maximization function (in real terms) at time  $t$  will then be the following:

$$\begin{aligned} E_t \beta \Lambda_{t,t+1} \left\{ \frac{P_{h,t}}{P_t} A_t K_t^{(1-\alpha)} L_t^\alpha + Q_{h,t} \frac{K_{t+1}}{\Psi_{t+1}} - R_{hk,t} Q_{h,t-1} \frac{K_t}{\Psi_t} - \frac{W_t}{P_t} L_t - Q_{h,t} I_t \right\} = \\ = E_t \beta \Lambda_{t,t+1} \left\{ \frac{P_{h,t}}{P_t} A_t K_t^{(1-\alpha)} L_t^\alpha + Q_{h,t} (I_t + (1 - \delta)K_t) - R_{hk,t} Q_{h,t-1} \frac{K_t}{\Psi_t} - \frac{W_t}{P_t} L_t - Q_{h,t} I_t \right\} \end{aligned}$$

where  $\beta \Lambda_{t,t+1}$  is the firms' stochastic discount factor (by assumption, firms belong to the households),  $R_{hk,t}$  is the real return on home capital required by investing banks and  $Q_{hk,t}$  is the real price of one unit of home capital. We are assuming here that the replacement price of capital that has depreciated is equal to 1. Each period the firm issues new state-contingent securities in order to obtain funds from an intermediary at price  $Q_{hk,t}$  and to buy new capital good at the same price, and then it has to pay back capital returns on the securities issued at the previous period.

From the FOCs w.r.t.  $L_t$  and  $K_t$ :

$$W_t/P_{h,t} = \alpha Y_t/L_t \quad (8)$$

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$$R_{hk,t+1} = \Psi_{t+1} \frac{(1-\alpha) \frac{P_{h,t+1}}{P_{t+1}} \frac{Y_{t+1}}{K_{t+1}} + (1-\delta)Q_{h,t+1}}{Q_{h,t}} \quad (9)$$

where the real gross profit per unit of capital is:

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left( \frac{L_t}{K_t} \right)^\alpha \quad (10)$$

Therefore, each unit of home equity bought at time  $t$  will be a state-contingent claim to the future returns from one unit of investment:

$$\Psi_{t+1} Z_{t+1}, (1 - \delta) \Psi_{t+1} \Psi_{t+2} Z_{t+2}, (1 - \delta)^2 \Psi_{t+1} \Psi_{t+2} \Psi_{t+3} Z_{t+3}$$

### 2.2.2 Capital Good Firms

Home competitive capital good firms belong to home households and operate in a national market. They produce capital good using national output as input subject to adjustment costs. They choose  $I_t$  (in unit of the composite consumption good) in order to solve the following problem:

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,\tau} \left\{ \left[ Q_{h,\tau} I_\tau - \left( 1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right) \right) I_\tau \right] \right\}$$

where  $f\left(\frac{I_t}{I_{t-1}}\right)$  is the physical adjustment cost (in units of consumption good) with  $f(1) = f'(1) = 0$  and  $f''(1) > 0$ . From the FOC:

$$Q_{h,t} = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + f'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} - E_t \Lambda_{t,t+1} f'\left(\frac{I_{t+1}}{I_t}\right) \frac{I_{t+1}}{I_t}^2 \quad (11)$$

where the real price of the capital good has to be equal to the marginal cost of producing the investment good. Profits arise only outside the steady state, and are lump-sum distributed to the home households.

### 2.2.3 The Households

The home generic household takes the nominal prices as given in maximizing the following function:

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$$E_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \frac{C_{\tau}^{1-\rho}}{1-\rho} - \frac{\kappa}{2} L_{\tau}^2 \right] + \sum_{\tau=t}^{\infty} \beta^{\tau-t} \mu_{\tau} [P_{\tau} R_{\tau} D_{\tau} + W_{\tau} L_{\tau} + \Pi_{\tau} - P_{\tau} C_{\tau} - P_{\tau} D_{\tau+1}] \right\}$$

where the FOCs with respect to  $C_t, D_{t+1}, L_t$  are:

$$C_t^{-\rho} = P_t \mu_t \quad (12)$$

$$E_t \left\{ \frac{P_{t+1} \mu_{t+1}}{P_t \mu_t} R_{t+1} \right\} = \frac{1}{\beta} \quad (13)$$

$$W_t = \frac{\kappa L_t}{\mu_t} \quad (14)$$

where  $\mu_t$  is the Lagrange multiplier. The Euler Equation is therefore given by:

$$1 = E_t \left\{ \beta R_{t+1} \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \right\} \quad (15)$$

while the following is the labour supply equation:

$$\frac{W_t}{P_t} = \kappa L_t C_t^{\rho} \quad (16)$$

To complete the household's problem we now need to find the demands for home and foreign consumption goods. The representative household solves:

$$\min P_{h,t} C_{h,t} + P_{f,t}^* C_{f,t}$$

s.t.

$$C_t = C_{h,t}^{\gamma} C_{f,t}^{(1-\gamma)}$$

in order to get from the FOCs:

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$C_{f,t} = (1 - \gamma) C_t \frac{P_t}{P_{f,t}} \quad (18)$$

while for the foreign household we will have:

$$C_{h,t}^* = (1 - \gamma) C_t^* \frac{P_t^*}{P_{h,t}^*} \quad (17^*)$$

$$C_{f,t}^* = \gamma C_t^* \frac{P_t^*}{P_{f,t}^*} \quad (18^*)$$

### 2.2.4 The Banks

Each home bank borrows  $d_t$  (in composite consumption units) in the national deposit market at the real gross deposit rate  $R_{t+1}$ , and then purchases  $s_{h,t}$  and  $s_{f,t}$  units of financial claims on final goods' producing firms at the real prices  $Q_{h,t}$  and  $Q_{f,t}$ . These are the prices of the banking system's claim on the future returns from one unit of present capital of the non-financial firm at the end of period. Firms are able to offer to the banks perfectly state-contingent debt. For an individual bank, the flow of funds constraint (where everything is in units of the composite consumption good) holds (intermediary balance sheet):

$$Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} = n_t + d_t \quad (19)$$

where  $s_{h,t}$  and  $s_{f,t}$  are held by an individual national bank ( $s_{h,t}^*$  and  $s_{f,t}^*$  are held abroad) and where  $Q_{f,t} = \frac{Q_{f,t}^*P_t}{\epsilon_t P_t^*}$  with  $\epsilon_t$  as the nominal exchange rate. Home and foreign financial claims respectively pay the real gross returns  $R_{hk,t}$  and  $R_{fk,t}$ . On the LHS of the (19) we have the value of the loans funded within a given period, while and on the RHS there are the equity capital (the net worth of the bank) plus the debt. Net worth evolves according to:

$$n_t = R_{hk,t}Q_{h,t-1}s_{h,t-1} + R_{fk,t}Q_{f,t-1}s_{f,t-1} - R_t d_{t-1} \quad (20)$$

or, given that

$$n_t = R_{hk,t}Q_{h,t-1}s_{h,t-1} + R_{fk,t}Q_{f,t-1}s_{f,t-1} - R_t[Q_{h,t-1}s_{h,t-1} + Q_{f,t-1}s_{f,t-1} - n_{t-1}]$$

we can simplify the net worth as:

$$n_t = [R_{hk,t} - R_t] Q_{h,t-1}s_{h,t-1} + [R_{fk,t} - R_t] Q_{f,t-1}s_{f,t-1} + R_t n_{t-1}$$

The end of period objective of the bank will be:

$$V_t = E_t \sum_{\tau=t+1}^{\infty} (1 - \theta) \theta^{\tau-(t+1)} \beta^{\tau-(t+1)} \Lambda_{t,\tau} n_{\tau}$$

where  $\beta\Lambda_{t,t+1}$  is the stochastic discount factor, equal to the marginal rate of substitution of the representative household that owns the bank. In order to solve its maximization's problem, the bank will have to take into account an endogenous constraint. We limit the bank's ability to obtain funds in the deposit market in this way. Suppose that after a bank obtains funds, the banker managing the bank may transfer a fraction  $\zeta$  of "divertable" assets to her household ( $0 < \zeta < 1$ ). Since creditors recognize this bank's incentive to divert funds, a borrowing constraint must arise:

$$V(s_{h,t}, s_{f,t}, d_t) \geq \zeta(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) \quad (21)$$

where on the LHS we have the maximized value of the bank's objective at the end of  $t$ . The value of the bank at the end of  $t - 1$  will be then:

$$\begin{aligned} & V(s_{h,t-1}, s_{f,t-1}, d_{t-1}) = \\ & = E_{t-1}\Lambda_{t-1,t} [(1 - \theta) n_t + \theta (\max_{s_{h,t}, s_{f,t}, d_t} V(s_{h,t}, s_{f,t}, d_t))] \end{aligned}$$

where quantities are all in units of composite consumption good. In order to solve it, we guess that the value function is linear (we will verify this guess later):

$$V(s_{h,t}, s_{f,t}, d_t) = v_{h,t}s_{h,t} + v_{f,t}s_{f,t} - v_t d_t$$

where the parameters are defined as:

$v_{h,t}$ : the value to the bank at the end of  $t$  of an additional unit of home asset

$v_{f,t}$ : the value to the bank at the end of  $t$  of an additional unit of foreign asset

$v_t$ : the marginal cost of deposit

Being now  $\lambda_t$  the Lagrange multiplier for the incentive constraint, we have that:

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$$\begin{aligned} \max V(s_{h,t}, s_{f,t}, d_t) &= v_{h,t}s_{h,t} + v_{f,t}s_{f,t} - v_t(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} - n_t) - \\ - \lambda_t [\zeta(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) - v_{h,t}s_{h,t} - v_{f,t}s_{f,t} + v_t(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} - n_t)] \end{aligned}$$

We derive the FOCs with respect to  $s_{h,t}$ ,  $s_{f,t}$  and  $\lambda_t$ :

$$\left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right)(1 + \lambda_t) = \zeta\lambda_t \quad (22)$$

$$\left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right)(1 + \lambda_t) = \zeta\lambda_t \quad (23)$$

$$Q_{h,t}s_{h,t} \left[\zeta - \left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right)\right] + Q_{f,t}s_{f,t} \left[\zeta - \left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right)\right] \leq v_t n_t \quad (24)$$

where it must be that

$$\lambda_t \left\{ Q_{h,t}s_{h,t} \left[\zeta - \left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right)\right] + Q_{f,t}s_{f,t} \left[\zeta - \left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right)\right] - v_t n_t \right\} \quad (25)$$

Then, from the above equations, as long as the incentive constraint is binding ( $\lambda_t > 0$ ), the marginal value of asset (in terms of home goods) will be greater than the marginal cost of deposit. Now, by defining:

$$\mu_{h,t} = \frac{v_{h,t}}{Q_{h,t}} - v_t = \mu_{f,t} = \frac{v_{f,t}}{Q_{f,t}} - v_t > 0$$

at an equilibrium where both assets (home and foreign) are held and the incentive constraint is binding, the last ones becomes:

$$Q_{h,t}s_{h,t} [\zeta - \mu_{h,t}] + Q_{f,t}s_{f,t} [\zeta - \mu_{f,t}] = v_t n_t \quad (26)$$

or, since  $\mu_{h,t} = \mu_{f,t} = \mu_t$ ,

$$(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) [\zeta - \mu_t] = v_t n_t$$

$$(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) = \frac{v_t n_t}{[\zeta - \mu_t]}$$

where

$$\phi_t = \frac{v_t}{[\zeta - \mu_t]}$$



is the leverage ratio net of interbank borrowing. Note that the leverage ratio is increasing in  $\mu_t$ . Note also that the constraint is binding as long as  $0 < \mu_t < \zeta$ . If, instead,  $\mu_t > \zeta$  the constraint is not binding. What we assume is that at an equilibrium, for reasonable values of parameters, the incentive constraint always binds. Now, by combining the conjectured value function with the Bellman equation, and given that, at the equilibrium, FOCs plus binding incentive constraint imply that:

$$Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} = \phi_t n_t = n_t + d_t$$

we can write:

$$V_{t-1} = E_{t-1}\beta\Lambda_{t-1,t} \{(1 - \theta)n_t + \theta V_t\}$$

Therefore, holding everything else constant, the expected discounted marginal gain to the banker of expanding asset  $Q_{h,t}s_{h,t}$  or  $Q_{f,t}s_{f,t}$  by one unit will be:

$$\frac{\partial V_t}{\partial Q_{h,t}s_{h,t}} = E_t\beta\Lambda_{t,t+1} [R_{hk,t+1} - R_{t+1}] (1 - \theta + \theta(v_{t+1} + \phi_{t+1}\mu_{t+1}))$$

$$\frac{\partial V_t}{\partial Q_{f,t}s_{f,t}} = E_t\beta\Lambda_{t,t+1} [R_{fk,t+1} - R_{t+1}] (1 - \theta + \theta(v_{t+1} + \phi_{t+1}\mu_{t+1}))$$

while the expected marginal cost of expanding deposit by one unit will be:

$$\frac{\partial V_t}{\partial d_t} = -E_t\beta\Lambda_{t,t+1}R_{t+1} (1 - \theta + \theta((v_{t+1} + \phi_{t+1}\mu_{t+1})))$$

Therefore, if we rewrite the conjectured value function in this way:

$$\begin{aligned} V_t &= \left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right) s_{h,t}Q_{h,t} + \left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right) s_{f,t}Q_{f,t} + v_t n_t = \\ &= \mu_{h,t}Q_{h,t}s_{h,t} + \mu_{f,t}Q_{f,t}s_{f,t} + v_t n_t \end{aligned}$$

we can verify that the value function is linear in  $(s_{h,t}, s_{f,t}, d_t)$  if  $\mu_{h,t}$ ,  $\mu_{f,t}$  (the marginal value of holding home/foreign assets net marginal costs) and  $v_t$  (the marginal cost of deposit) satisfy:

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$$\begin{aligned} & \frac{\partial V_t}{\partial Q_{h,t} s_{h,t}} = \mu_{h,t} = \\ & = E_t \beta \Lambda_{t,t+1} [R_{hk,t+1} - R_{t+1}] (1 - \theta + \theta (v_{t+1} + \phi_{t+1} \mu_{t+1})) \quad (27) \end{aligned}$$

$$\begin{aligned} & \frac{\partial V_t}{\partial Q_{f,t} s_{f,t}} = \mu_{f,t} = \\ & = E_t \beta \Lambda_{t,t+1} [R_{fk,t+1} - R_{t+1}] (1 - \theta + \theta (v_{t+1} + \phi_{t+1} \mu_{t+1})) \quad (28) \end{aligned}$$

$$\begin{aligned} & \frac{\partial V_t}{\partial n_t} = - \frac{\partial V_t}{\partial d_t} = \\ & = E_t \beta \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) \quad (29) \end{aligned}$$

where using the definition of  $\mu_{h,t}, \mu_{f,t}$  we can also rewrite the equations above as:

$$v_{h,t} = E_t \beta \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta) Q_{h,t+1}] (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) \quad (30)$$

$$v_{f,t} = E_t \beta \Lambda_{t,t+1} [Z_{t+1}^* + (1 - \delta) Q_{f,t+1}] (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) \quad (31)$$

$$v_t = E_t \beta \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) \quad (32)$$

or, for the foreign country,

$$v_{h,t}^* = E_t \beta \Lambda_{t,t+1}^* [Z_{t+1} + (1 - \delta) Q_{h,t+1}] (1 - \theta + \theta ((v_{t+1}^* + \phi_{t+1}^* \mu_{t+1}^*))) \quad (30^*)$$

$$\begin{aligned} & v_{f,t}^* = \\ & = E_t \beta \Lambda_{t,t+1}^* [Z_{t+1}^* + (1 - \delta) Q_{f,t+1}] (1 - \theta + \theta ((v_{t+1}^* + \phi_{t+1}^* \mu_{t+1}^*))) \quad (31^*) \end{aligned}$$

$$v_t^* = E_t \beta \Lambda_{t,t+1}^* R_{t+1}^* (1 - \theta + \theta (v_{t+1}^*)) \quad (32^*)$$

To conclude, at the equilibrium, if the incentive constraint is binding:

$$\begin{aligned} & E_t \beta \Lambda_{t,t+1} R_{h,t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) = \\ & = E_t \beta \Lambda_{t,t+1} R_{f,t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) > \\ & \quad \wr E_t \beta \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) \end{aligned}$$

Otherwise, the banks will increase the amount of home and foreign assets to the point where:

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$$\begin{aligned}
& E_t \beta \Lambda_{t,t+1} R_{h,t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) = \\
& = E_t \beta \Lambda_{t,t+1} R_{f,t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1}))) = \\
& = E_t \beta \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta ((v_{t+1} + \phi_{t+1} \mu_{t+1})))
\end{aligned}$$

We sum up across individual banks in order to obtain the demand for total bank assets  $Q_{h,t} S_{h,t}$  and  $Q_{f,t} S_{f,t}$  as a function of total net worth  $N_t$  as:

$$Q_{h,t} S_{h,t} + Q_{f,t} S_{f,t} = \frac{v_t}{\zeta - \mu_t} N_t \quad (33)$$

### 2.2.5 Evolution of Bank Net Worth

The total net worth for the home banks is given by the sum of the total net worth of existing bankers (*o* for old) and of entering bankers (*y* for young):

$$N_t = N_{o,t} + N_{y,t} \quad (34)$$

where

$$N_{o,t} = \theta [R_{hk,t} Q_{h,t-1} S_{h,t-1} + R_{fk,t} Q_{f,t-1} S_{f,t-1} - R_t D_{t-1}] \quad (35)$$

$$N_{y,t} = \frac{\xi(1-\theta)}{(1-\theta)} [R_{hk,t} Q_{h,t-1} S_{h,t-1} + R_{fk,t} Q_{f,t-1} S_{f,t-1}] \quad (36)$$

with  $\theta$  being the fraction of bankers that survive from the previous period,  $(1 - \theta)$  the fraction of "new" bankers that were workers the previous period,  $\frac{\xi}{(1-\theta)}$  is the fraction of total value assets gained by exiting bankers that is transferred from the family to each new banker, and  $D_t$  is the aggregate deposit (in real terms). Therefore, from the whole banking sector's balance sheet, deposits will be given by:

$$D_t = Q_{h,t} S_{h,t} + Q_{f,t} S_{f,t} - N_t \quad (37)$$

## 2.3 Equilibrium

Since households are symmetric, we can rewrite the previously defined equations in terms of the following equilibrium conditions.

Optimal intertemporal allocation of consumption will be given by :

$$1 = \beta E_t R_{t+1} \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \quad (15)$$

$$1 = \beta E_t R_{t+1}^* \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\rho} \quad (15^*)$$

Equilibrium in the goods market is described by:

$$Y_t = C_{h,t} + C_{h,t}^* + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \frac{P_t}{P_{h,t}} \quad (38)$$

$$Y_t^* = C_{f,t} + C_{f,t}^* + \left[ 1 + f \left( \frac{I_t^*}{I_{t-1}^*} \right) \right] I_t^* \frac{P_t}{P_{f,t}} \quad (38^*)$$

For the labour markets we get the following equilibrium conditions:

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

$$\alpha \frac{P_{f,t}^*}{P_t^*} \frac{Y_t^*}{L_t^*} = \kappa L_t^* C_t^{*\rho} \quad (39^*)$$

The supply for home securities will be given by:

$$S_{h,t} + S_{h,t}^* = [I_t + (1 - \delta) K_t] \quad (40)$$

$$S_{f,t}^* + S_{f,t} = [I_t^* + (1 - \delta) K_t^*] \quad (40^*)$$

while the demands are obtained by the (33), giving the equilibrium in the security markets.

Lastly, being the household also the owner of both national banks and firms, it follows that in the equation (5):

$$C_t = -D_{t+1} + R_t D_t + \frac{W_t}{P_t} L_t + \frac{\Pi_t}{P_t} \quad (5)$$

the real dividend will be given by the sum of the following terms:

- real profits from both final and capital good sectors
- the difference in a generic bank's wealth
- the transfers from the exiting banker to the household net from the transfer from the household to the new banker

By summing up all these terms, the representative household's budget constraint in the home country will be given by:

$$\begin{aligned} \frac{P_{h,t}}{P_t} Y_t + Q_{h,t} S_{h,t}^* + R_{f,t} Q_{f,t-1} S_{f,t-1} - R_{h,t} Q_{h,t-1} S_{h,t-1}^* - Q_{f,t} S_{f,t} &= \\ &= C_t + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \quad (41) \end{aligned}$$

and, symmetrically, for the foreign country:

$$\begin{aligned} \frac{P_{f,t}}{P_t} Y_t^* + Q_{f,t} S_{f,t} + R_{h,t} Q_{h,t-1} S_{h,t-1}^* - R_{f,t} Q_{f,t-1} S_{f,t-1} - Q_{h,t} S_{h,t}^* &= \\ &= C_t^* + \left[ 1 + f \left( \frac{I_t^*}{I_{t-1}^*} \right) \right] I_t^* \quad (41^*) \end{aligned}$$

Lastly, notice that from the equilibrium conditions in the financial markets, we must have:

$$\begin{aligned} \frac{v_{h,t}}{Q_{h,t}} - v_t &= \frac{v_{f,t}}{Q_{f,t}} - v_t \\ \frac{v_{h,t}^*}{Q_{h,t}} - v_t^* &= \frac{v_{f,t}^*}{Q_{f,t}} - v_t^* \end{aligned}$$

that can be summarized as:

$$E_t \{ R_{h,t+1} - R_{f,t+1} \} = 0 \quad (42)$$

Then, given that  $A_t, A_t^*, \Psi_t$  and  $\Psi_t^*$  follow exogenous stochastic processes,

we need to find the real prices:

$$\begin{aligned} Q_{h,t}, R_{ht}, R_{t+1}, \frac{P_{h,t}}{P_t} \\ Q_{f,t}, R_{ft}, R_{t+1}^*, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities:

$$\begin{aligned}
 & Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, S_{h,t}, S_{f,t}, Yu_t \\
 & Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\
 & D_t^*, N_t^*, S_{h,t}^*, S_{f,t}^*, Yu_t^*
 \end{aligned}$$

the shadow prices:

$$\begin{aligned}
 & v_t, v_{h,t}, v_{f,t}, \lambda_t, \Phi_t \\
 & v_t^*, v_{h,t}^*, v_{f,t}^*, \lambda_t^*, \Phi_t^*
 \end{aligned}$$

determined as a function of the state variables:

$$\begin{aligned}
 & K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Yu_{t-1} \\
 & K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Yu_{t-1}^*
 \end{aligned}$$

by the sequence of the following 44 equations:

$$\begin{aligned}
 & (1) - (6) - (7) - (9) - (10) - (11) - (15) - (17) \\
 & (1^*) - (6^*) - (7^*) - (9^*) - (10^*) - (11^*) - (15^*) - (17^*)
 \end{aligned}$$

that describe the optimization conditions of households and final good firms (home and abroad),

$$\begin{aligned}
 & (22) - (23) - (24) - (30) - (31) - (32) - (34) \\
 & (22^*) - (23^*) - (24^*) - (30^*) - (31^*) - (32^*) - (34^*)
 \end{aligned}$$

that describe the optimization conditions for banks,

$$\begin{aligned}
 & (37) - (38) - (39) - (40) - (41) \\
 & (37^*) - (38^*) - (39^*) - (40^*) - (41^*)
 \end{aligned}$$

that describe the market clearing conditions for goods, securities and labour.

We also include the definitions of  $\Phi_t$  and of  $Yu_t$  :

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### 3 UNCONVENTIONAL MONETARY POLICIES

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$$\Phi_t = \frac{v_t}{\zeta - \left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right)}$$

$$Y u_t = \frac{I_t}{I_{t-1}}$$

## 3 Unconventional Monetary Policies

In what follows we summarize the unconventional monetary policies as they are described in the Gertler and Kiyotaki model (2010). We apply them to our two country world context.

### 3.1 Lending Facilities: Direct Lending

With ‘Lending Facilities’ we mean the direct acquisition of high quality private securities by the Central Bank (Gertler and Kiyotaki, 2010). During a crisis, when the financially constrained banks are unable to obtain additional funds, the Central Bank can obtain these funds and then lend them to the markets where excess of returns are abnormally high. By expanding the supply of funds available in the market the Central Bank will reduce the equilibrium lending rates.

Following the Gertler and Kiyotaki’s framework, we assume that the Central Bank has both an advantage and a disadvantage with respect to the private financial intermediaries. From the one side, the Central Bank it is not balance sheet constrained. From the other side, however, it is necessarily less efficient than the private sector in evaluating and monitoring borrowers (it is, in a way, too ”far away” from the final good market). It then has to pay an efficiency cost  $\tau$  per unit.

The Central Bank issues government bonds (borrowed from the Treasury) that then sells to the household sector at the rate  $R_{t+1}$  (deposits and government bonds are therefore perfect substitutes in the households’ budget

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### 3.2 Liquidity Facilities:

#### *Discount Window Lending* 3 *UNCONVENTIONAL MONETARY POLICIES*

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constraints). The funds collected in its own country by each Central Bank are therefore used to make loans to domestic borrowers (final good firms) at the private rate  $R_{hk,t+1}$  ( $R_{fk,t+1}$  abroad).

Let

$$Q_{h,t}S_{h,t} + Q_{h,t}S_{h,t}^*$$

be the total intermediation of type home security, where we assume that

$$Q_{h,t}S_{h,t} = Q_{h,t}S_{hp,t} + Q_{h,t}S_{hg,t} \quad (62)$$

where  $S_{hp,t}$  are the total type home securities intermediated by home private banks,  $S_{hg,t}$  are the total type home securities intermediated by the domestic Central Bank, while  $S_{h,t}^*$  are the total home securities intermediated by foreign banks (when financial markets are opened).

We assume that each country's Central Bank chooses to intermediate a fraction  $\varphi_t$  of the total credit in its own country, that means (for the home market):

$$S_{hg,t} = \varphi_t [S_{h,t} + S_{h,t}^*] \quad (63)$$

and then from the home banks' problem the demand for assets by home banks (24) becomes:

$$Q_{h,t} [(1 - \varphi_t) S_{h,t} - \varphi_t S_{h,t}^*] + Q_{f,t} S_{f,t} = \Phi_t N_t \quad (64)$$

### **3.2 Liquidity Facilities: Discount Window Lending**

The Central Bank uses the discount window to lend funds (at a penalty rate) to banks that in turn lend them to the real economy.

It should be considered as a way to avoid financial disruptions in the interbank market (that we did not explicitly model here). In a liquidity



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### 3.2 *Liquidity Facilities:*

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crisis borrowers only have limited access to credit because of a breakdown of the market for short term funds. The private sector is available to borrow even at a penalty rate because of the abnormally high excess of returns. The existence of a penalty rate, however, limits the inefficient use of the funds by the private sector.

Again, the Central Banks finance their activities as discount window lenders by issuing government bonds at the deposit/bond rate  $R_{t+1}$  (bonds and deposits are, as in the previous case of direct lending, perfect substitutes). These funds are then offered to the banks at the non contingent interest rate  $R_{m,t+1}$ . In order to allow the liquidity facility to expand the supply of funds available for the banking sector, we must assume that the Central Bank has an advantage in providing funds to the borrowers with respect to the private sector. In this model the advantage consists in the fact that banks may divert only a fraction  $\zeta(1 - \omega_g) < \zeta$  of assets borrowed by the Central Bank, with  $0 < \omega_g \leq 1$ .

As Gertler and Kiyotaki (2010) explained in their analysis, the Government has many means at its disposal to retrieve funds (legal punishments, etc). However, they assumed that there exists a sort of capacity constraint on the Central Bank's ability to monitoring private banks. After a certain threshold, this Central Bank's ability disappears. In this way they are able to explain why in their model the Central Banks do not simply expand the discount window lending to drive excess returns to zero. Since a capacity constraint limits the Central Bank's ability in monitoring private banks, during financial crises it becomes necessary to introduce also other tools (such as, lending facilities or equity injections) in order to allow the abnormally high excess of returns to fall.

Now, by considering the banks' problem, we assume  $m_t$  to be the discount window borrowing for a generic bank. Then, the flow of funds (19) of the generic home bank becomes:

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### 3.2 Liquidity Facilities:

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$$Q_{h,t}s_{p,t} + Q_{f,t}s_{f,t} = n_t + m_t + d_t \quad (65)$$

where  $m_t > 0$ . Then, the incentive constraint (21) for the generic bank becomes:

$$V_t(s_{h,t}, s_{f,t}, d_t, m_t) \geq \zeta(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} - \omega_g m_t)$$

In addition to the baseline model's first order conditions with respect to  $s_{h,t}$  and  $s_{f,t}$  (22) – (23), we need now to consider also a first order condition w.r.t.  $m_t$ :

$$(v_{m,t} - v_t)(1 + \lambda_t) \geq \omega_g \zeta \lambda_t \quad (66)$$

where  $v_{m,t}$  is the marginal cost of an additional unit of discount window credit. Lastly, equation (24) changes now in:

$$Q_{h,t}s_{h,t} \left[ \zeta - \left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right) \right] + Q_{f,t}s_{f,t} \left[ \zeta - \left( \frac{v_{f,t}}{Q_{f,t}} - v_t \right) \right] + (v_{m,t} - v_t - \zeta \omega_g) m_t \leq v_t n_t$$

that at an equilibrium (where both home and foreign assets are held) becomes:

$$Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} = \frac{v_t}{\zeta - \left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right)} n_t - \frac{(v_{m,t} - v_t - \zeta \omega_g)}{\zeta - \left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right)} m_t \quad (67)$$

Let us define now  $\mu_{m,t}$  to be the excess cost for a bank that borrows from the Central Bank:

$$\mu_{m,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} \Omega_{t+1} [R_{m,t+1} - R_{t+1}] \quad (68)$$

Then, from the FOCs, both private and discount window lending will be used when:

$$\mu_{m,t} = \omega_g \left( \frac{v_{k,t}}{Q_{k,t} - v_t} \right) \quad (69)$$

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### 3.3 Equity Injections 3 UNCONVENTIONAL MONETARY POLICIES

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where  $k = h, f$ . Borrowers have to be indifferent (at the margin) between discount window lending and private credit. In order to allow for that the Central Bank will then set the rate  $R_{m,t+1}$  in order to equal the extra cost implied by the liquidity facility to the fraction  $\omega_g$  of the excess value of home/foreign assets.

Now, by considering the banking sector as a whole, we end up with the following demand for assets by domestic banks (instead of equation (24)):

$$Q_{h,t}S_{h,t} + Q_{f,t}S_{f,t} = \Phi_t N_t + \omega_g M_t \quad (70)$$

As long as  $\omega_g > 0$ , the liquidity facility will expand the total level of funds borrowed by the banking sector.

### 3.3 Equity Injections

For “Equity Injections” is intended the direct acquisition of ownership positions by the Central Bank in private intermediaries (in coordination with the Government). As in Gertler and Kyiotaki (2010), we assume that the injection of shares happens at the beginning of each period, while direct and discount window lending happens instead at the end of the same period. In this way we catch the fact that the equity injections are slower than others unconventional monetary policies.

There are efficiency costs related to equity injections, that we assume to be  $\tau_e$  per unit of equity acquired. However, benefits from this credit policy are greater than costs during financial crises. By assumption, we divide the securities held by each country’s banking system between privately owned securities,  $S_{hp,t}$  (or  $S_{fp,t}^*$  for the foreign country), and publicly owned securities,  $S_{ge,t}$ . Therefore:

$$S_{h,t} = S_{ge,t} + S_{hp,t} \quad (71)$$

(for the domestic country), while abroad we have:

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### 3.3 Equity Injections      3 UNCONVENTIONAL MONETARY POLICIES

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$$S_{f,t}^* = S_{ge,t}^* + S_{fp,t}^* (71^*)$$

Notice that we are assuming here that the Central Bank of each country is only interested in doing equity injections to support its own banking system.

Then, by defining  $n_{g,t}$  as the market value of government equity, the balance sheet of a generic home bank becomes:

$$Q_{h,t}s_{hp,t} + Q_{f,t}s_{f,t} = n_t + d_t + n_{g,t} (72)$$

where (from the baseline model)  $s_{hp,t}$  is the investment of the home bank in domestic final good firms, while  $s_{f,t}$  is the investment of the same bank in the foreign final good firms.

By assumption, the securities held by the home Central Bank are evaluated at the market price  $Q_{h,t}$  ( $Q_{f,t}$  for the foreign country), that means:

$$n_{g,t} = Q_{h,t}s_{ge,t} (73)$$

Instead,  $Q_{hg,t}$  is the price paid by the Central Bank in order to acquire home equities, with  $Q_{hg,t} > Q_{h,t}$ . We assume that  $Q_{hg,t}$  is chosen by the Central Bank in order to make the excess return on government equity  $\mu_{g,t}$  equal to zero, or:

$$\mu_{g,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} \Omega_{t+1} [R_{hg,t+1} - R_{t+1}] = 0 (74)$$

where  $R_{gh,t+1}$  is the gross return on a unit of home Central Bank equity:

$$R_{hg,t+1} = \psi_{t+1} \frac{Z_{t+1} + (1-\delta)Q_{h,t+1}}{Q_{hg,t}} (75)$$

Then, by considering the net worth of a generic bank, we have that:

$$\begin{aligned} n_t = & R_{h,t}Q_{h,t-1}s_{hp,t-1} + R_{f,t}Q_{f,t-1}s_{f,t-1} - R_t d_{t-1} + \\ & + (Q_{hg,t} - Q_{h,t}) [s_{ge,t} - (1-\delta)\psi_t s_{ge,t-1}] \end{aligned} (76)$$

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### 3.4 The Policy Maker 3 UNCONVENTIONAL MONETARY POLICIES

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where the last term is the transfer to the generic bank by the Government (the “gift” from equity injections).

By assumption, banks can not divert assets financed by equity injections. Lastly, by aggregating the whole banking sector we end up with the market demand for assets (in the baseline model, equation (24)), that in our case becomes:

$$Q_{h,t}S_{h,t} + Q_{f,t}S_{f,t} = \Phi_t N_t + N_{g,t} \quad (77)$$

while the evolution of net worth will be:

$$N_t = (\theta + \xi) [R_{h,t}Q_{h,t-1}S_{hp,t-1} + R_{f,t}Q_{f,t-1}S_{f,t-1}] - R_t D_{t-1} + (Q_{hg,t} - Q_{h,t}) [S_{ge,t} - (1 - \delta) \Psi_t S_{ge,t-1}] \quad (78)$$

### 3.4 The Policy Maker

We do not distinguish in this work between Central Bank and Government. We assume instead that they act together, in the attempt to protect the financial stability of the country which they belong to. We define Government's consumption  $G_t$  in the home country as:

$$G_t = \bar{G} + \tau_e S_{ge,t} + \tau S_{g,t} \quad (79)$$

where  $\bar{G}$  is the Government's expenditure under normal times, that we assume equal to 0 (in our model, the Policy Maker do not exist during normal times).

The budget constraint for the Policy Maker is then given by:

$$\begin{aligned} G_t + Q_{h,t}S_{g,t} + Q_{hg,t}S_{ge,t} = \\ = T_t + R_{h,t}Q_{h,t-1}S_{g,t-1} + R_{hg,t}Q_{hg,t-1}S_{ge,t-1} + \\ + R_{m,t}M_{t-1} - M_t + D_{g,t} - R_t D_{g,t-1} \end{aligned} \quad (80)$$

where  $T_t$  are lump-sum taxes that endogenously adjust to finance the losses and  $D_{g,t}$  is the Government's bond.

## 4 How the Model works

### 4.1 Calibration

In our model we have four standard preference/technology parameters, that we get from the traditional literature. These are the inverse of risk aversion  $\rho = 0.99$ , the utility weight of labour  $\kappa = 5.584$ , the capital share  $\alpha = 0.33$  and the depreciation rate  $\delta = 0.025$ . Three parameters are instead specific of the Gertler and Kiyotaki's model (2010). These are the fraction of assets that banks may divert  $\zeta = 0.383$ , the transfer to entering bankers  $\xi = 0.003$  and the quarterly survival rate of the bankers  $\theta = 0.972$ . In this case we maintain the original calibration of Gertler and Kiyotaki where the first two parameters are chosen to hit an economy-wide leverage ratio of 4.

We suppose that the capital quality and the technology shocks obey to a first-order autoregressive process, symmetric across countries. Disturbances are uncorrelated across countries (we have four independent sources of disturbances, therefore markets are incomplete).

We also need to assign a form to the physical adjustment cost  $f(\frac{I_t}{I_{t-1}})$  in equation (11). We know that it must be such that  $f(1) = f'(1) = 0$  and  $f''(1) > 0$  and therefore we allow Matlab to find:

$$f\left(\frac{I_t}{I_{t-1}}\right) = cq\left(\frac{I_t}{I_{t-1}}\right)^2 - 2cq\left(\frac{I_t}{I_{t-1}}\right) + cq$$

where we get  $cq = 1.728$  from the work by Gertler and Karadi (2009) that also considers similar investment adjustment cost.

Lastly, for what concerns the parameters that drive the endogenous discount factor, we set  $\eta = 0.7$  while  $\omega$  adjusts in order to maintain the steady state value of the discount factor  $\bar{\beta} = \omega\bar{C}_A^\eta = 0.99$  (we follow, here, Schmitt-Groh and Uribe (2002) in order to avoid the presence of unit root in the model's first-order approximation).

Lastly, we set the steady state Government's expenditure  $\bar{G} = 0$ .

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Following Gertler and Kiyotaki (2010), we try to catch the initiating feature of a financial crisis through the deterioration of the value of banks' financial portfolios.

We consider a five percent exogenous decline in the quality of capital with an autoregressive factor of 0.66. This size of the shock is fixed by Gertler and Kiyotaki (2010) as the one able to produce a downturn similar to the one they observed during the US sub-prime mortgages' crisis (2007). We assume that capital quality shocks are symmetric and uncorrelated across countries in order to better understand the transmission mechanism at work across countries. An additional source of risk in the model is due to the presence also of home and foreign technology shocks (again, symmetric and uncorrelated). Our model is, therefore, incomplete: it contains four independent sources of risk, while each country's leveraged banks have to limit their investment opportunities to only two assets, home and foreign equities.

In such an environment we introduce a Policy Maker and its unconventional monetary policy. We start with the simpler Model A (perfectly integrated good markets, closed financial markets) and then we move to Model B, where both good and financial markets are open and equilibrium portfolio's shares are endogenously determined (by following the method developed by Devereux and Sutherland, since 2006 onwards). As we explained in the previous chapter of our PhD Dissertation, according to our analysis the banking sector approximately holds the 70,5 percent of domestic equities. We justify this "home bias" result with the presence of incomplete markets that prevent a full risk-sharing across countries.

The three kinds of unconventional monetary measures are not considered all at the same time, in order to make clearer the impact from different policies. We separately implement the first two policies (liquidity and lending facilities) and the third one (equity injection).

## 4.2 The Results

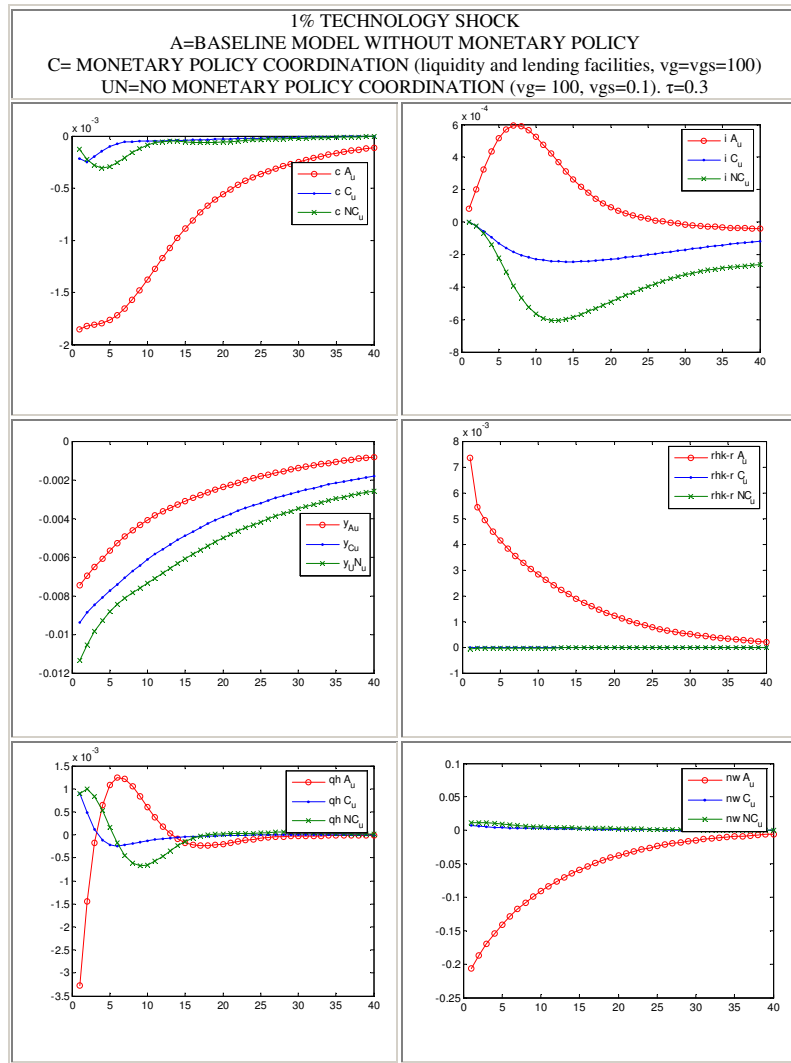
Impulse response functions (for the main economic variables) to a 1 percent technology shock in the home country are collected at the end of this section. We separately described what happens in Model A (closed financial markets) and in Model B with endogenous portfolio shares (partial portfolio integration). Under coordinated monetary system, both countries implement the same policy in response to the same, symmetric, shocks. Under uncoordinated monetary system, the policies adopted by the two countries have instead different strength. One country (home) chooses to implement stronger unconventional monetary policies, while abroad softer monetary measures are adopted. According to our analysis, when the coordinated monetary policies implemented are strong enough, they have a meaningful positive effect on the macroeconomic variables (output, consumption and spread) home and abroad. This is evident in the first graphs, where we analyze the IRFs from a 1 percent shock in the home technology in the context of Model A (with perfectly integrated good markets and closed financial markets) under liquidity and lending facilities. In this case we set the parameter  $v_g = 100$  for both countries in the coordinated case, while in the uncoordinated case the parameter differs home and abroad ( $v_g = 100$  at home, while it is close to zero in the foreign country). We see, however, that in all the other cases, where the parameters chosen to determine the strength of the monetary answer are lower (we set, for example,  $v_g = 0.1$  for both countries in the coordinated case and  $v_g = 0.1$  at home but  $v_g^* = 0$  abroad in the case without coordination), the benefits from unconventional monetary coordination disappear. If the monetary answer to the shock it is too weak, the cost implied in the unconventional monetary policy becomes greater than the benefits.



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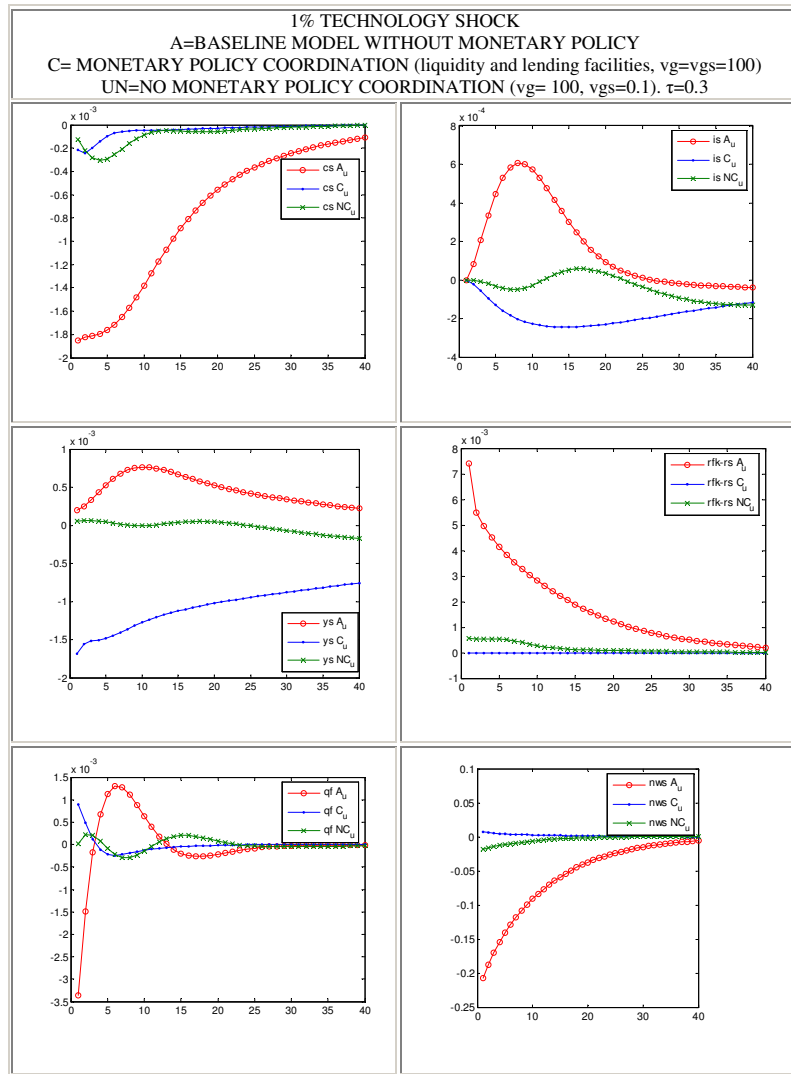
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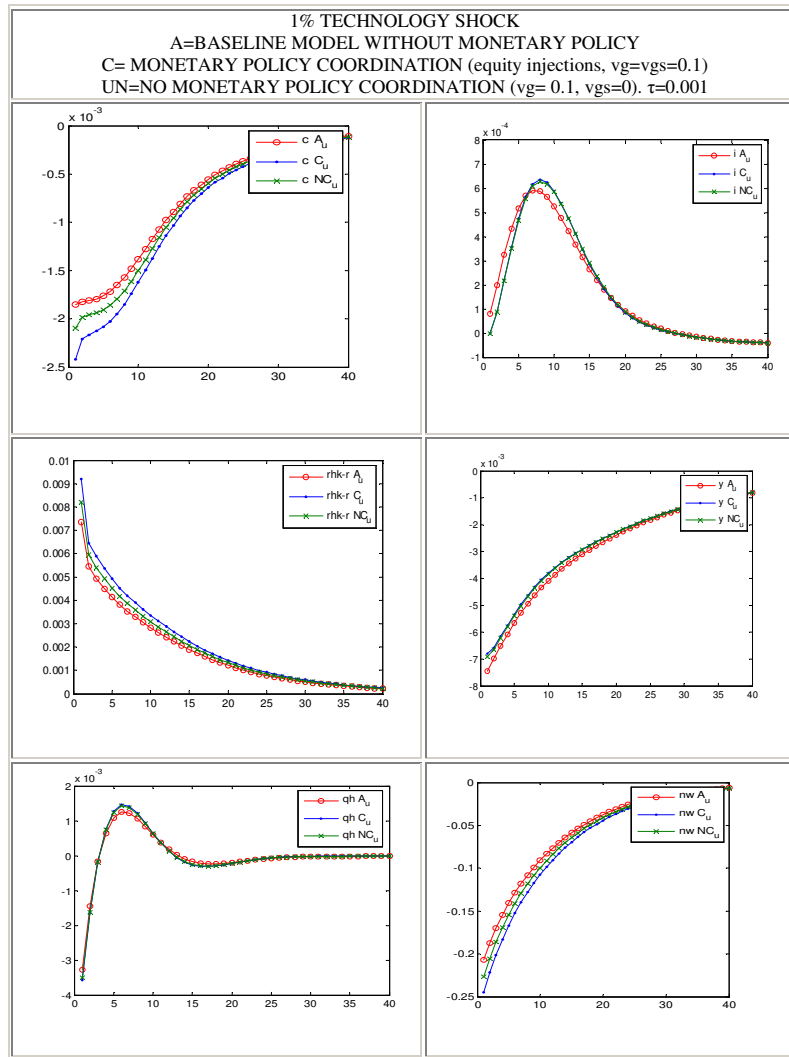
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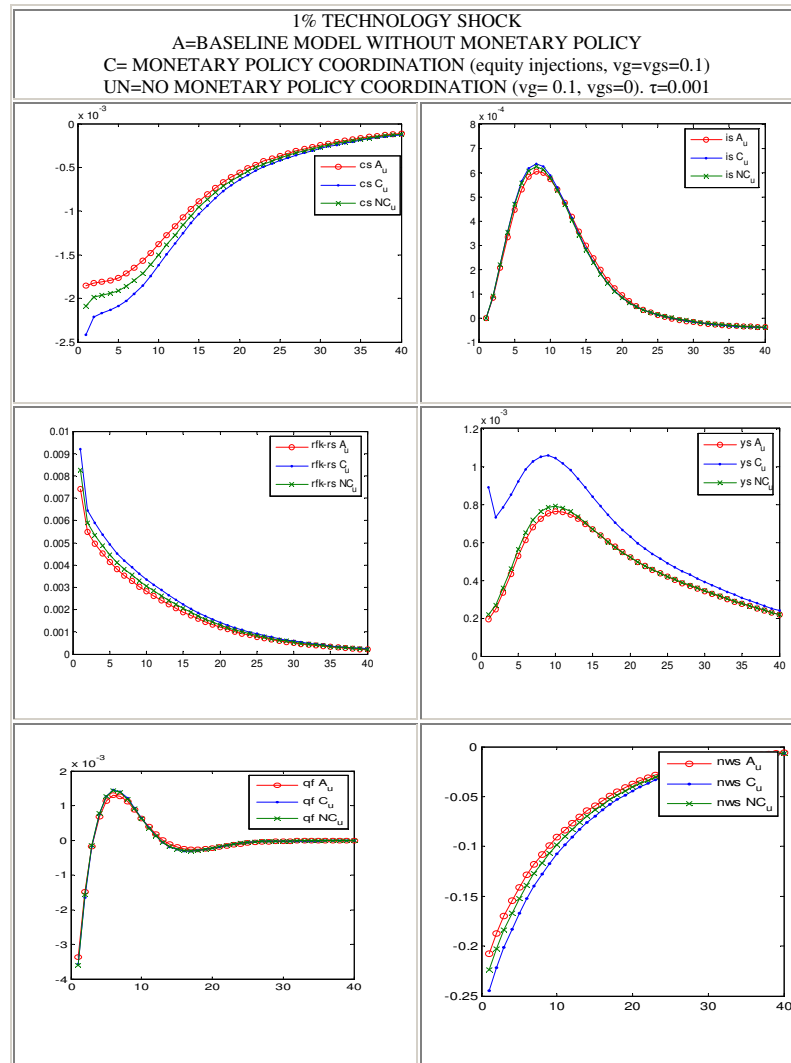
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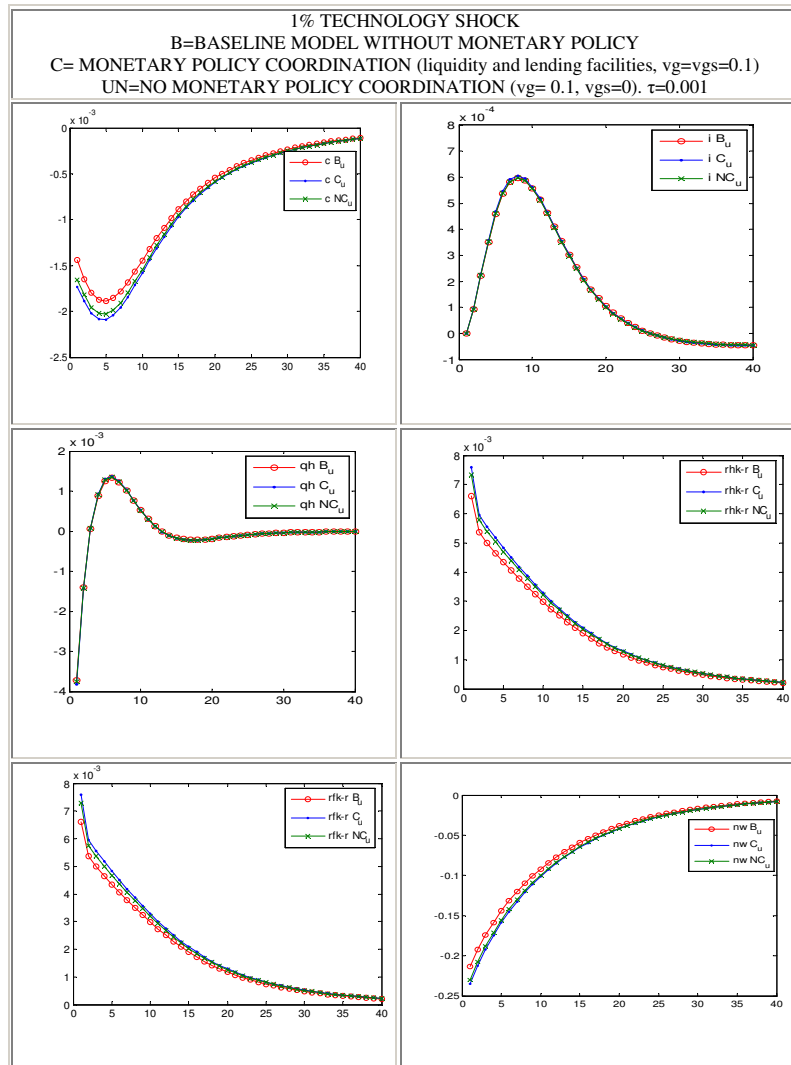
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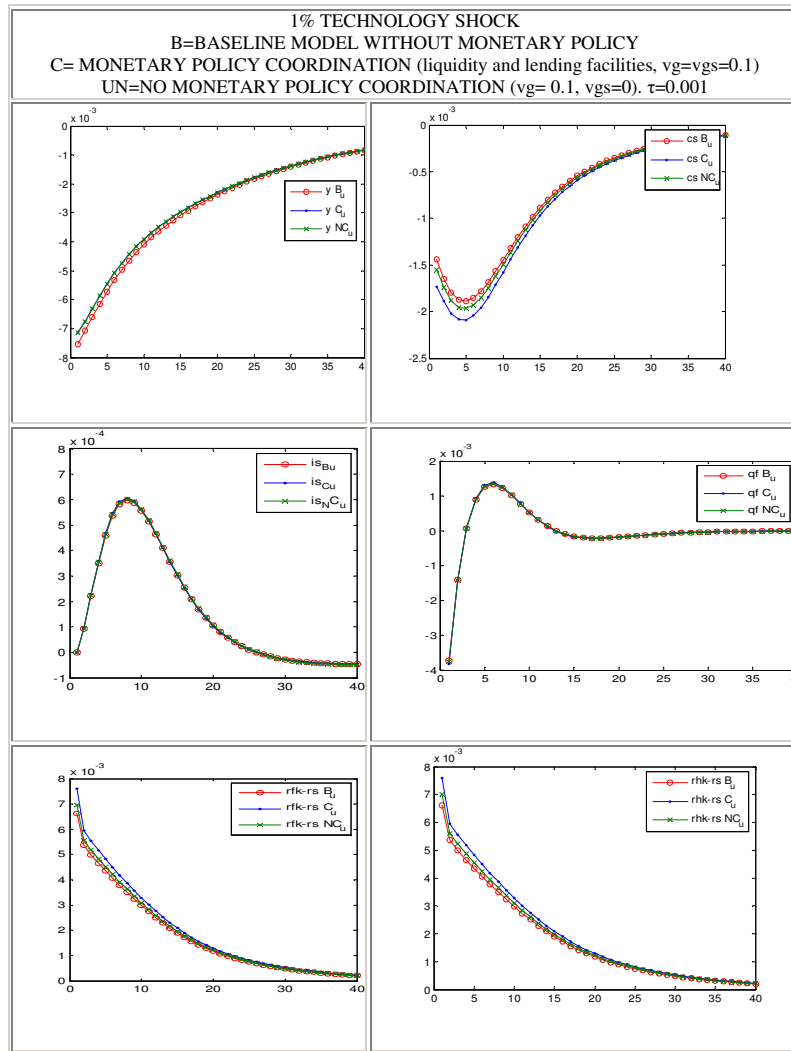
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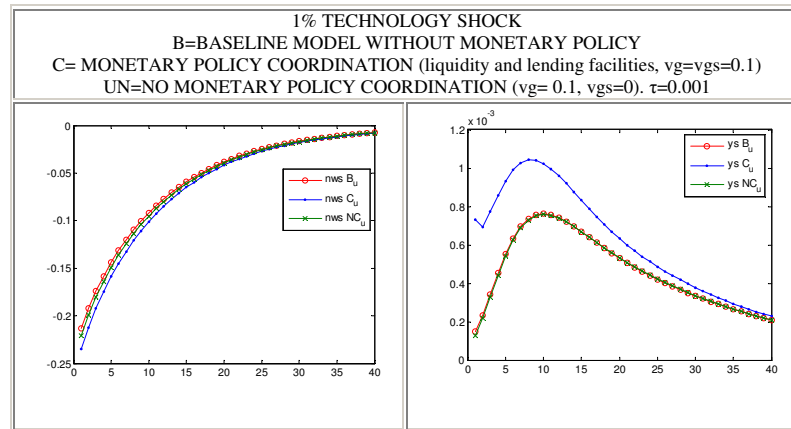
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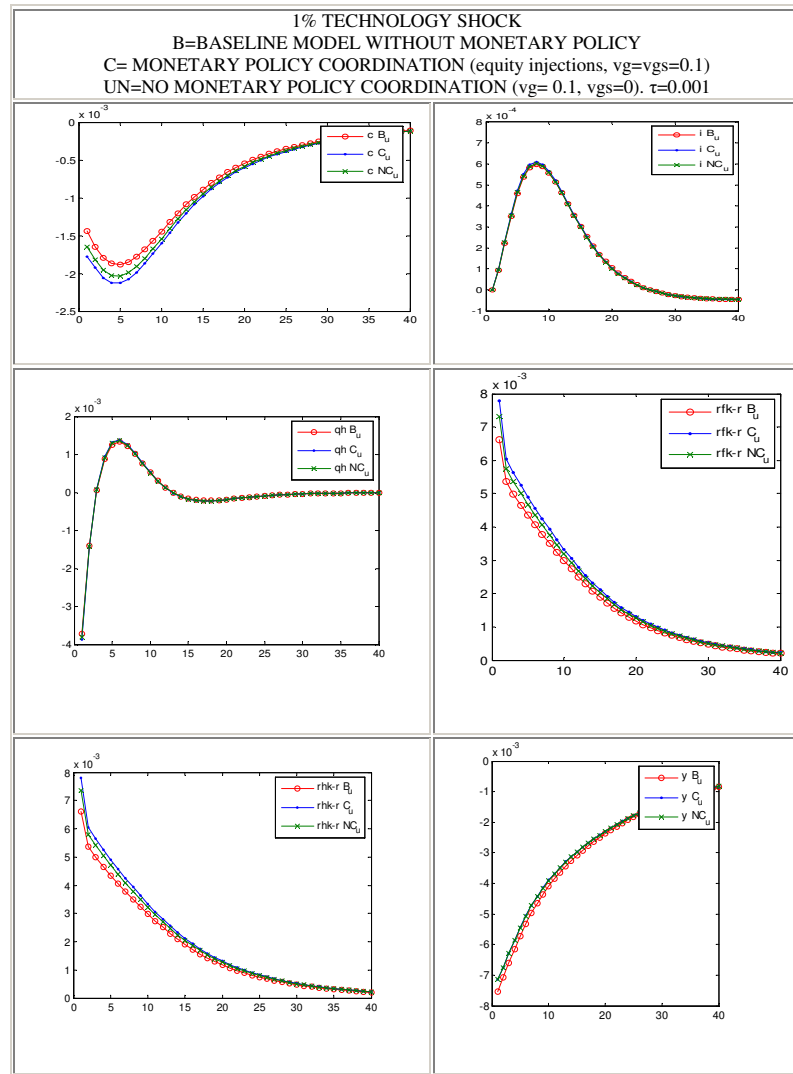
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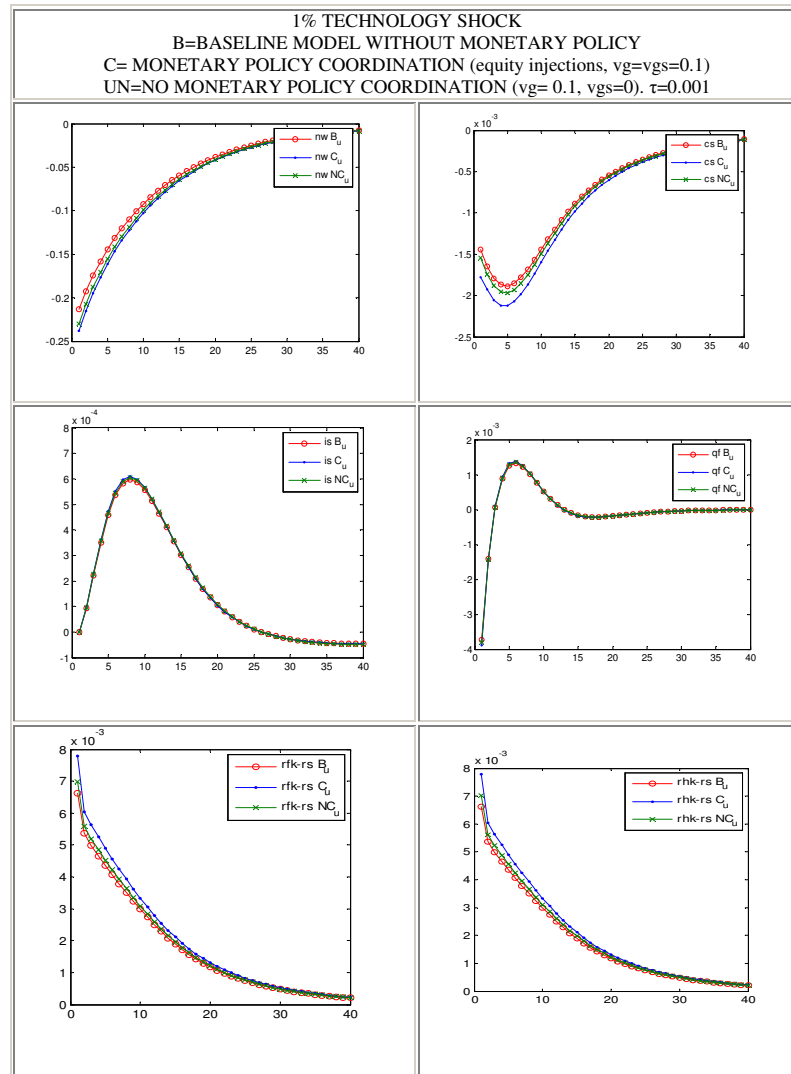




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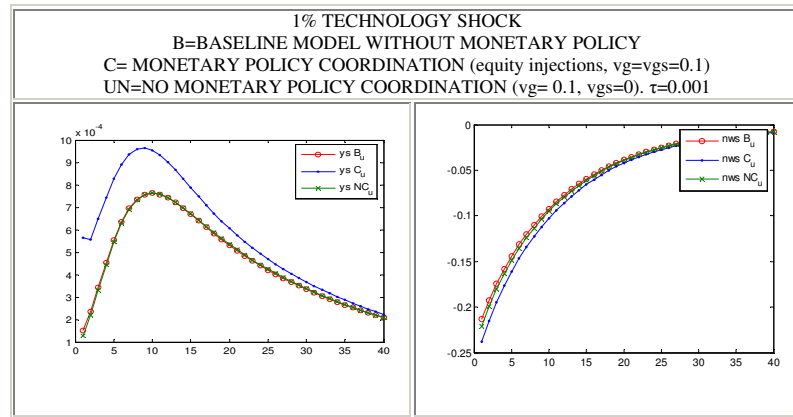
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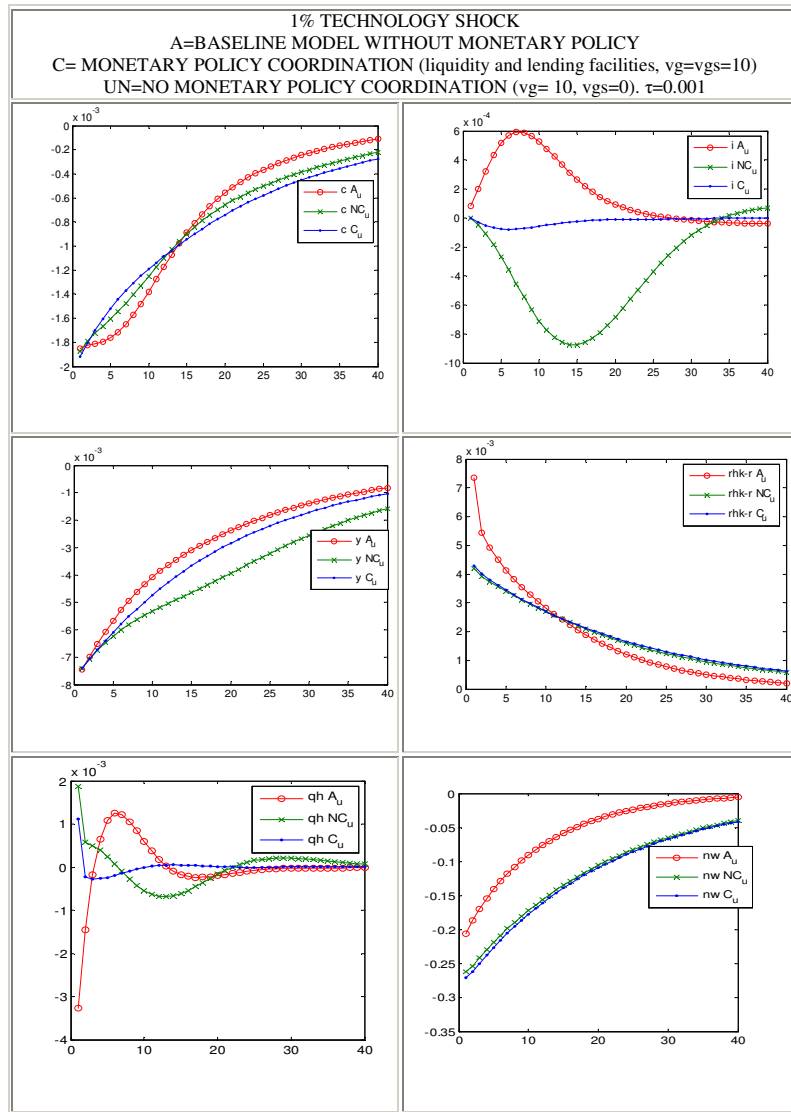
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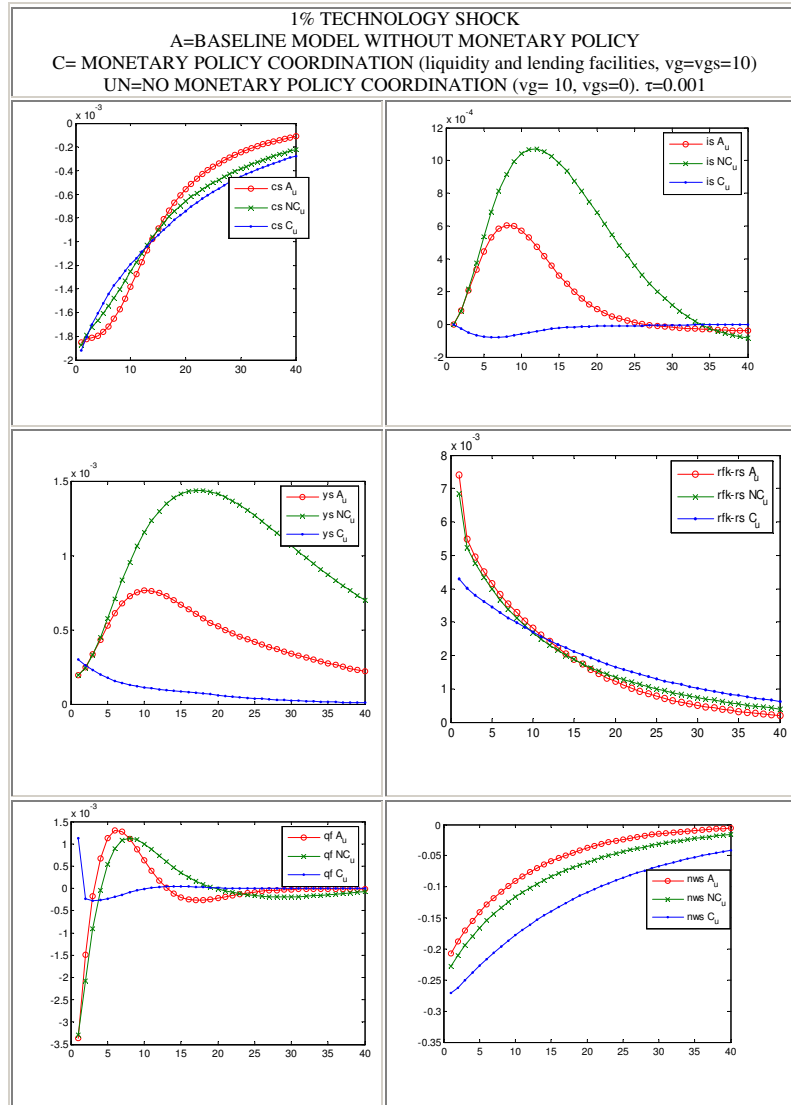
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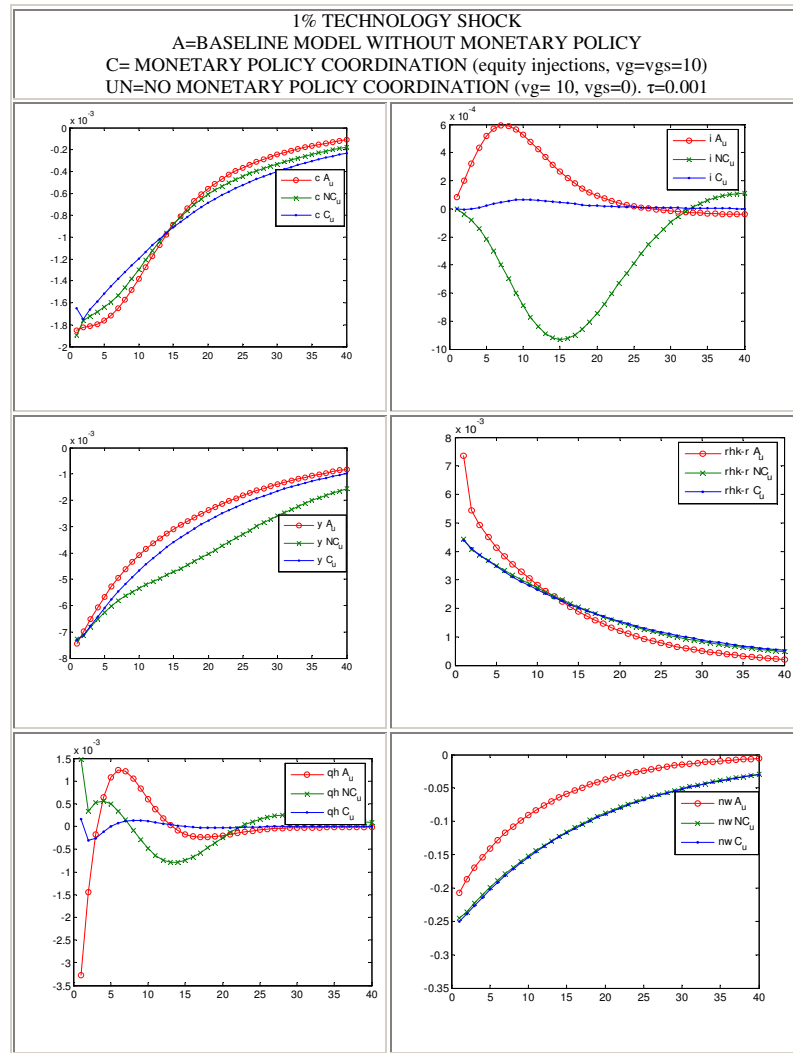
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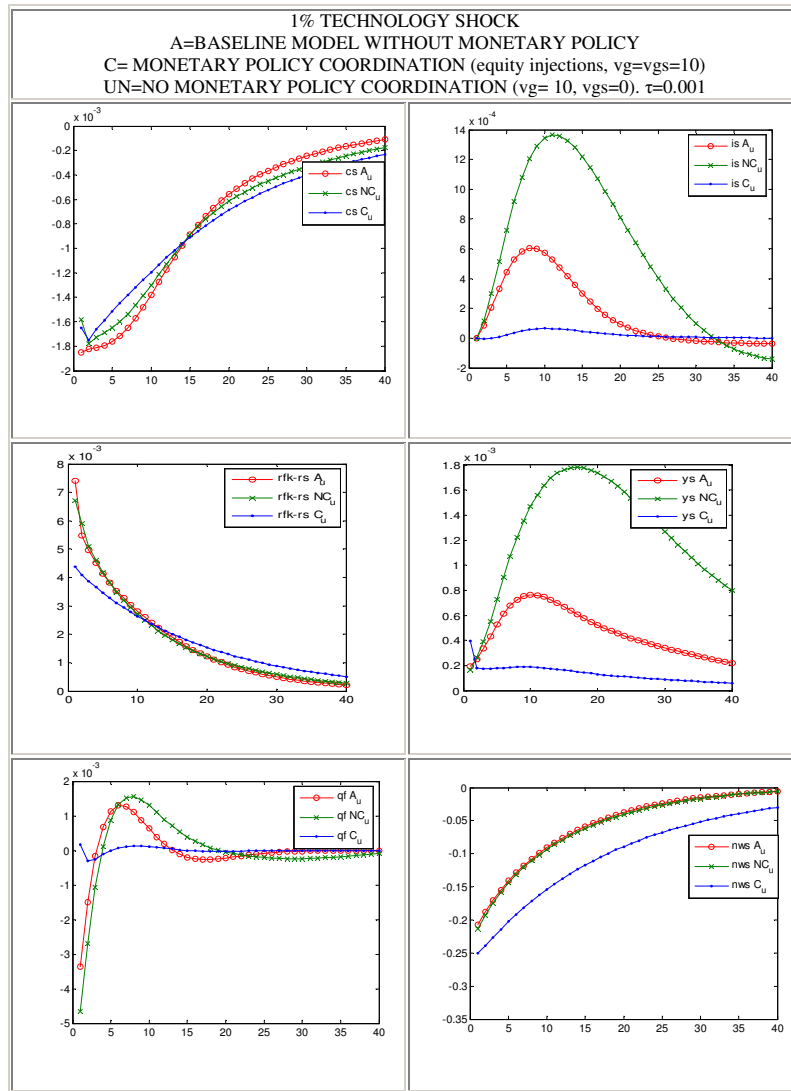
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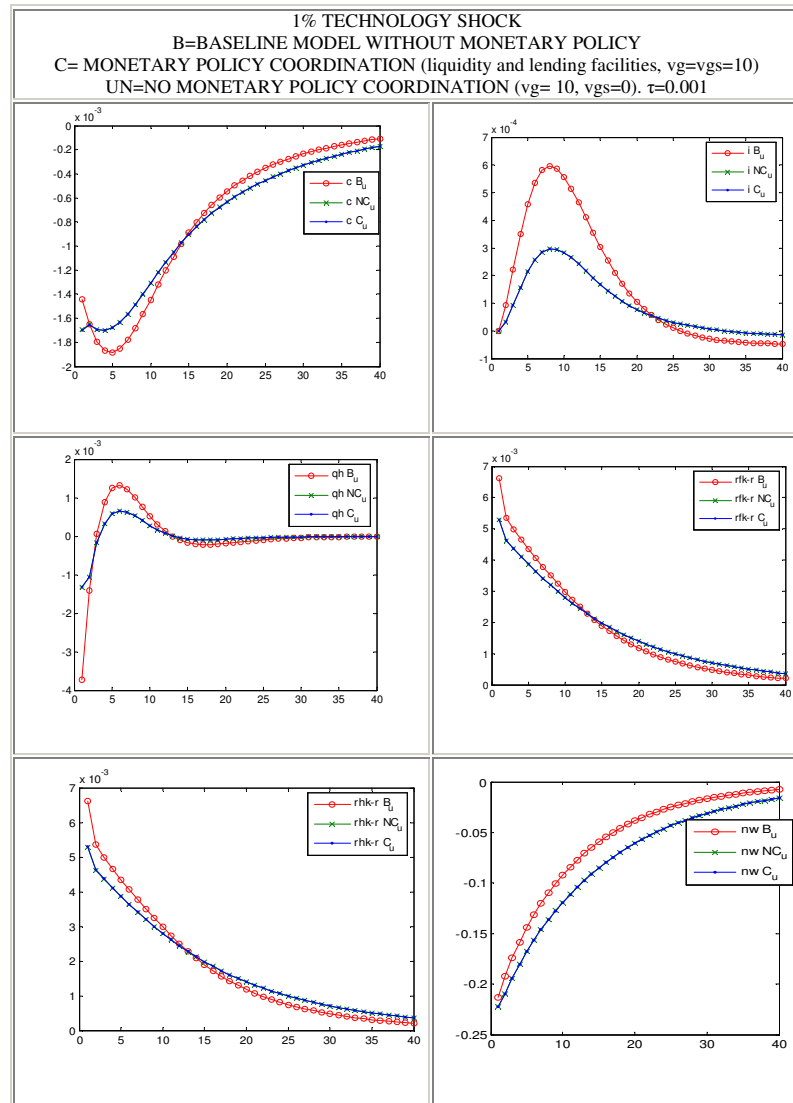
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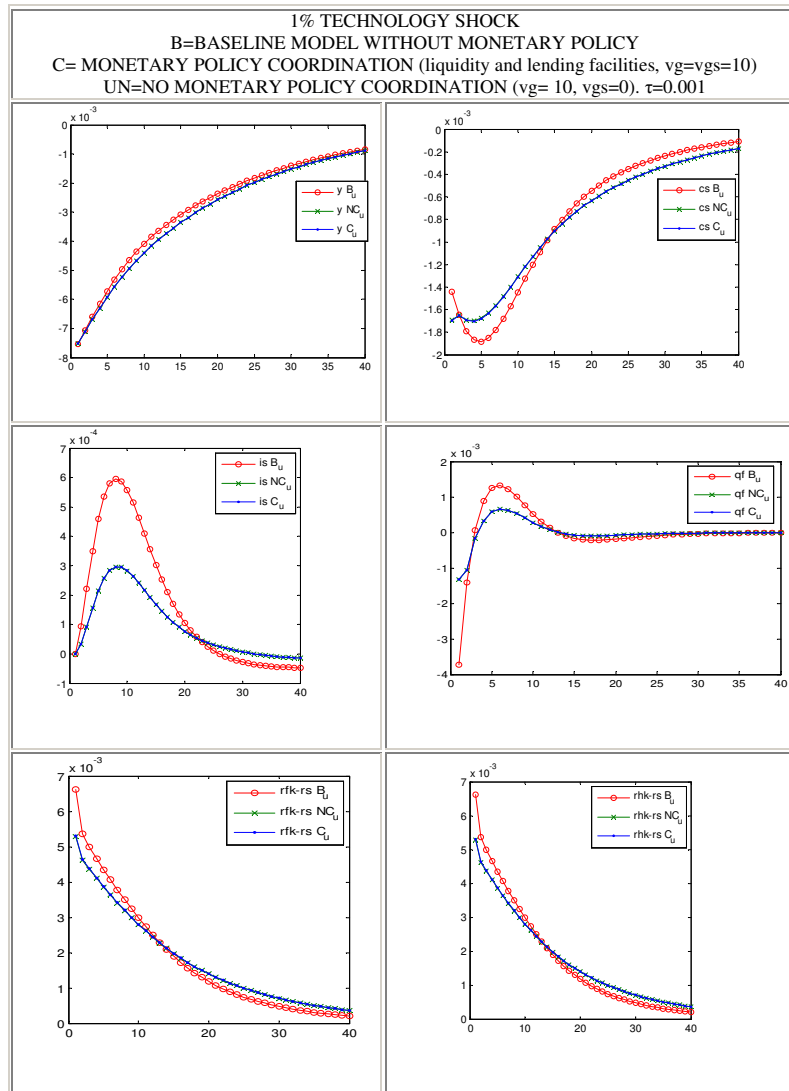
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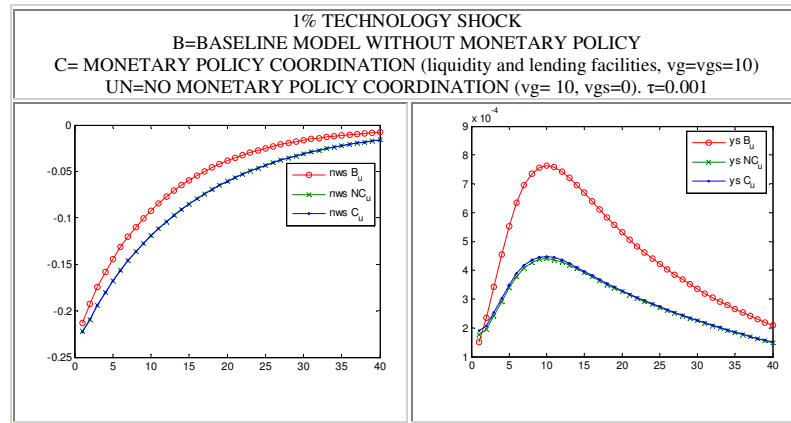




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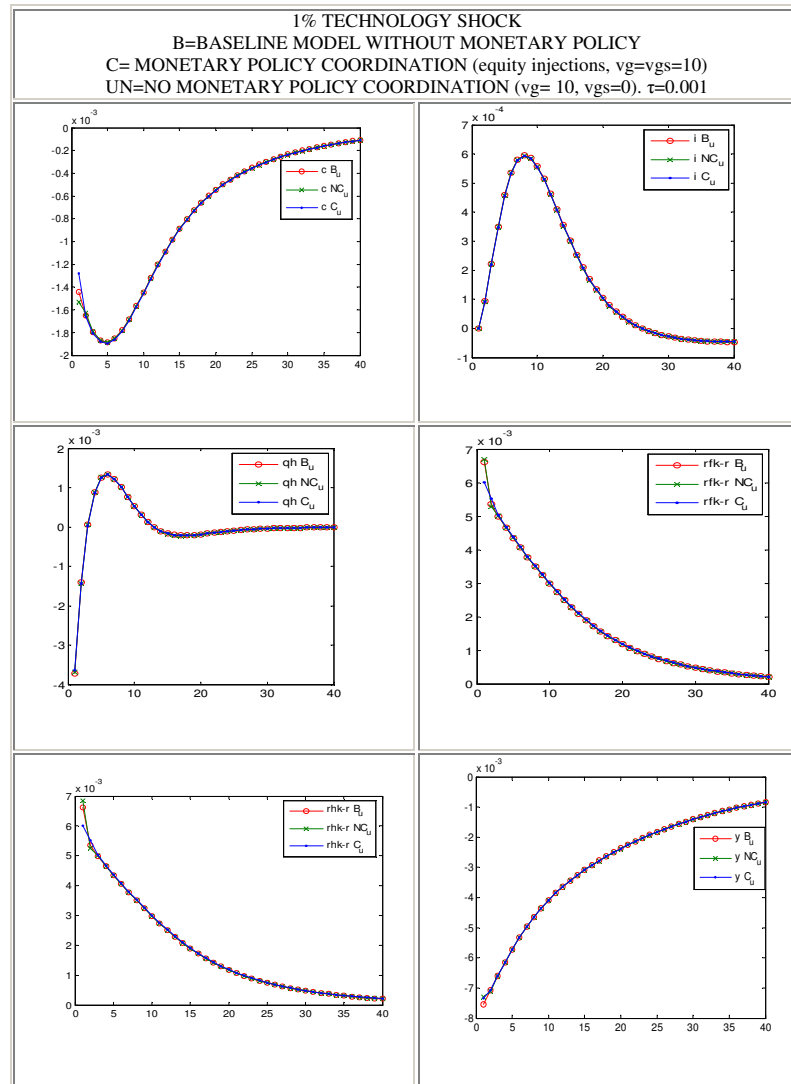
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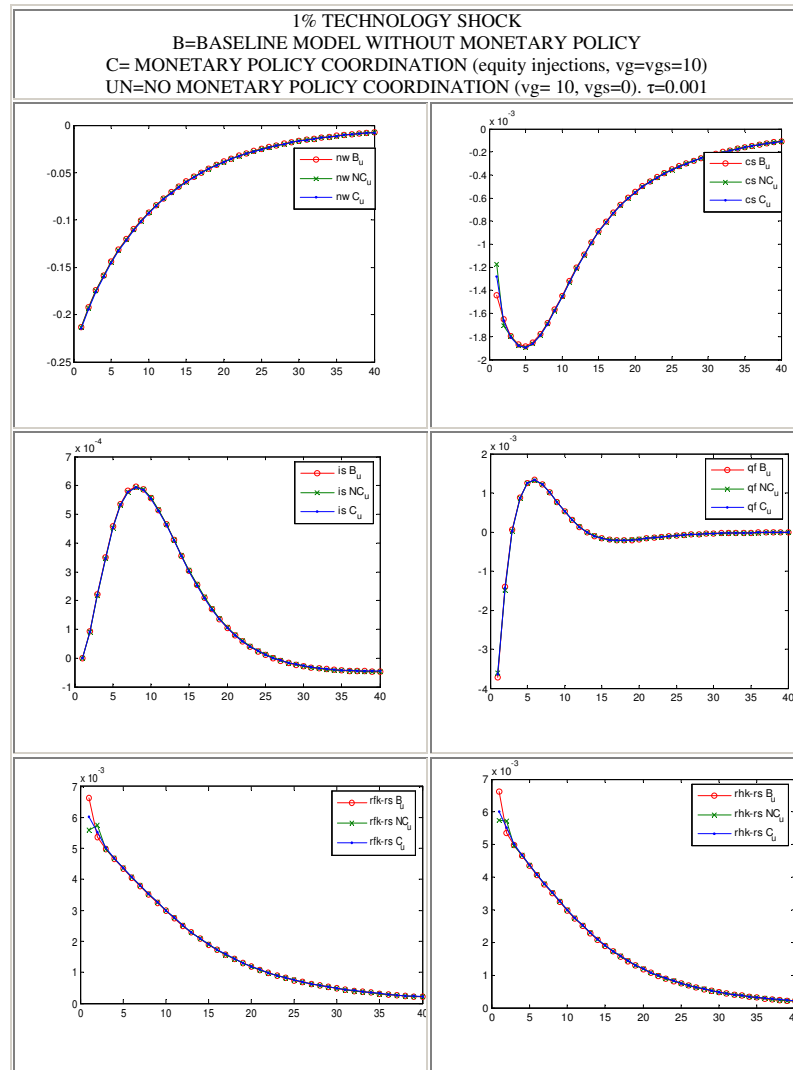
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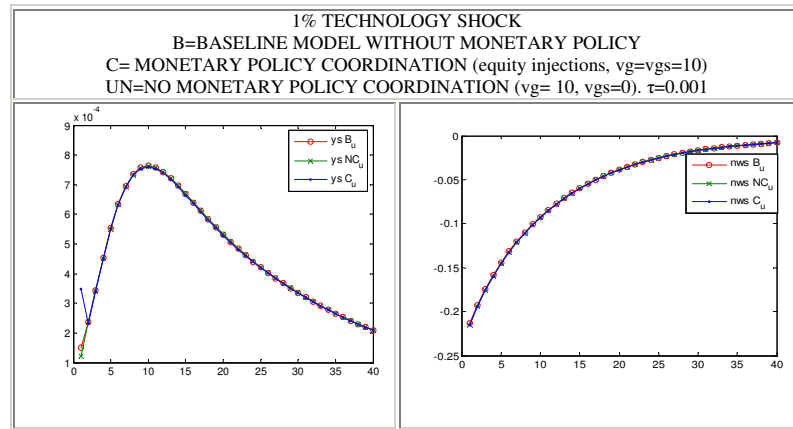
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4.2 *The Results*

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## 5 Appendix A

### 5.1 The model with integrated good markets but closed financial markets

### 5.2 The Baseline Model without credit policies

$A_t, A_t^*, \Psi_t, \Psi_t^*$  follows exogenous stochastic process. We need to find the real prices:

$$\begin{aligned} Q_{h,t}, R_{ht}, R_{t+1}, \frac{P_{h,t}}{P_t} \\ Q_{f,t}, R_{ft}, R_{t+1}^*, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities

$$\begin{aligned} Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, S_{h,t}, Yu_t \\ Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\ D_t^*, N_t^*, S_{f,t}^*, Yu_t^* \end{aligned}$$

the shadow prices

$$\begin{aligned} v_t, v_{h,t}, \lambda_t, \Phi_t \\ v_t^*, v_{f,t}^*, \lambda_t^*, \Phi_t^* \end{aligned}$$

determined as a function of the state variables

$$\begin{aligned} K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Yu_{t-1} \\ K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Yu_{t-1}^* \end{aligned}$$

by the sequence of the following 40 equations (we just show the the Home equations):

From the Household's problem:

$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$



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$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left( \frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

From the Final Good Production Firms' problem:

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

$$K_{t+1} = \Psi_{t+1} [I_t + (1 - \delta) K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta) Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left( \frac{L_t}{K_t} \right)^\alpha \quad (10)$$

From the Banks' problem:

$$\left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$Q_{h,t} S_{h,t} \left[ \zeta - \left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right) \right] - v_t n_t = 0 \quad (24)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta) Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

where the definition of  $\Phi_t$  is given by:

$$\Phi_t = \frac{v_t}{\zeta - \left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right)}$$

$$N_{t+1} = (\theta + \xi) [R_{hk,t+1} Q_{h,t} S_{h,t}] - \theta R_{t+1} D_t \quad (34)$$

$$D_t = Q_{h,t} S_{h,t} - N_t \quad (37)$$

$$S_{h,t} = [I_t + (1 - \delta) K_t] \quad (40)$$

where the last one is the Supply in the Home financial market.

From the Capital Good Firms' problem:

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$$Q_{h,t-1} = 1 + f(Yu_{t-1}) + f'(Yu_{t-1})Yu_{t-1} - E_{t-1}\Lambda_{t-1,t}f'(Yu_t)Yu_t^2 \quad (11)$$

where we define the investment ratio between  $t, t - 1$  as:

$$Yu_t = \frac{I_t}{I_{t-1}}$$

Equilibrium condition in the (integrated) good markets:

$$Y_t = C_{h,t} + C_{h,t}^* + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \frac{P_t}{P_{h,t}} \quad (38)$$

Equilibrium condition in the labour market:

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

Lastly, to complete the model we have to add the home household budget constraint:

$$\frac{P_{h,t}}{P_t} Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \quad (41)$$

Notice that in the equations above we consider the discount factor  $\beta$  as endogenous, or  $\beta_t = \frac{\psi_{t+1}}{\psi_t} = \omega C_{A,t}^{-\eta}$  with initial conditions  $\psi_0 = 1$  and  $0 \leq \eta < \rho$ , and where  $C_A$  is the aggregate consumption of home households and therefore it is not internalized in the households' decision. ( $0 < \omega C_A^{-\eta} < 1$ ).

We set  $\eta = 0.7$  and  $\omega$  such that  $\beta = \omega \bar{C}_A^{-\eta} = 0.99$  at steady state. We followed here Sutherland and Devereux, Schmitt-Groh and Uribe in order to avoid the possible presence of unit root in the first order approximated model implied by possible incomplete markets.

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5.3 *The Model with liquidity and lending facilities*

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### 5.3 The Model with liquidity and lending facilities

$A_t, A_t^*, \Psi_t, \Psi_t^*$  follows exogenous stochastic process. We need to find the real prices:

$$\begin{aligned} & Q_{h,t}, R_{h,t}, R_{t+1}, R_{m,t+1}, \frac{P_{h,t}}{P_t} \\ & Q_{f,t}, R_{f,t}, R_{t+1}^*, R_{m,t+1}^*, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities

$$\begin{aligned} & Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, S_{h,t}, S_{hp,t}, Y u_t, G_t, S_{g,t}, \varphi_t, M_t \\ & Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\ & D_t^*, N_t^*, S_{f,t}^*, S_{fp,t}^*, Y u_t^*, G_t^*, S_{g,t}^*, \varphi_t^*, M_t^* \end{aligned}$$

the shadow prices

$$\begin{aligned} & v_t, v_{h,t}, v_{m,t}, \lambda_t, \Phi_t \\ & v_t^*, v_{f,t}^*, v_{m,t}^*, \lambda_t^*, \Phi_t^* \end{aligned}$$

determined as a function of the state variables

$$\begin{aligned} & K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Y u_{t-1} \\ & K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Y u_{t-1}^* \end{aligned}$$

by the sequence of the following 54 equations (we just show the the Home equations):

$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left( \frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

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5.3 *The Model with liquidity and lending facilities*

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$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

$$K_{t+1} = \Psi_{t+1} [I_t + (1 - \delta) K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta) Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left( \frac{L_t}{K_t} \right)^\alpha \quad (10)$$

$$\left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$Q_{h,t} S_{hp,t} = \Phi_t N_t + \omega_g M_t \quad (70)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta) Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

$$\Phi_t = \frac{v_t}{\zeta - \left( \frac{v_{h,t}}{Q_{h,t} - v_t} \right)}$$

$$N_{t+1} = (\theta + \xi) [R_{hk,t+1} Q_{h,t} S_{hp,t}] - \theta R_{t+1} D_t - \theta R_{m,t+1} M_t \quad (34)$$

$$D_t = Q_{h,t} S_{hp,t} - N_t - M_t \quad (65)$$

$$S_{h,t} = [I_t + (1 - \delta) K_t] \quad (40)$$

$$Q_{h,t-1} = 1 + f(Y u_{t-1}) + f'(Y u_{t-1}) Y u_{t-1} - E_{t-1} \Lambda_{t-1,t} f'(Y u_t) Y u_t^2 \quad (11)$$

$$Y u_t = \frac{I_t}{I_{t-1}}$$

$$Y_t = C_{h,t} + C_{h,t}^* + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \frac{P_t}{P_{h,t}} + G_t \frac{P_t}{P_{h,t}} \quad (38)$$

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

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5.3 *The Model with liquidity and lending facilities*

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We need to add now seven equations for the new variables related to the introduction of the Policy Maker and its unconventional monetary policy. The definition of Government's consumption expenditure:

$$G_t = \bar{G} + \tau S_{g,t} \quad (79)$$

where we set  $\bar{G} = 0$  during normal times. The distinction between privately and publicly held securities in the home securities intermediated by home agents:

$$S_{h,t} = S_{hp,t} + S_{g,t} \quad (71)$$

Then, from the bank's problem and by modifying equation (68) we find:

$$\lambda_t = (v_{m,t} - v_t) \frac{(1+\lambda_t)}{\omega_g \zeta} \quad (66)$$

$$v_{m,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} \Omega_{t+1} R_{m,t+1} \quad (81)$$

The "rule" used by the Central Bank for implementing the credit policy under unusual times of crisis:

$$\varphi_t = v_g [(R_{hk,t+1} - R_{t+1}) - (\bar{R}_{hk} - \bar{R})] \quad (82)$$

where  $\bar{R}_{hk}$  and  $\bar{R}$  are steady state values of home equity and deposit returns and  $v_g$  is a parameter. We assume that the Central Bank adjusts the fraction of private credit/amount of funds it lends to the home banks when the excess of returns on home assets increases over its steady state value:

$$S_{g,t} = \varphi_t S_{h,t} \quad (63)$$

$$M_t = \varphi_t D_t \quad (83)$$

Lastly, to complete the model we have to add the home household budget constraint:

$$\begin{aligned} & \frac{P_{h,t}}{P_t} Y_t = \\ & = C_t + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t + G_t \quad (41) \end{aligned}$$

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5.4 *The Model with equity injections*

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## 5.4 The Model with equity injections

$A_t, A_t^*, \Psi_t, \Psi_t^*$  follows exogenous stochastic process. We need to find the real prices:

$$\begin{aligned} & Q_{h,t}, Q_{hg,t}, R_{h,t}, R_{t+1}, R_{hg,t}, \frac{P_{h,t}}{P_t} \\ & Q_{f,t}, Q_{fg,t}, R_{f,t}, R_{t+1}^*, R_{fg,t}, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities

$$\begin{aligned} & Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, S_{hp,t}, S_{h,t}, Y u_t, G_t, S_{ge,t}, \varphi_t, N_{g,t} \\ & Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\ & D_t^*, N_t^*, S_{fp,t}^*, S_{f,t}^*, Y u_t^*, G_t^*, S_{ge,t}^*, \varphi_t^*, N_{g,t}^* \end{aligned}$$

the shadow prices

$$\begin{aligned} & v_t, v_{h,t}, \lambda_t, \Phi_t \\ & v_t^*, v_{f,t}^*, \lambda_t^*, \Phi_t^* \end{aligned}$$

determined as a function of the state variables

$$\begin{aligned} & K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Y u_{t-1} \\ & K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Y u_{t-1}^* \end{aligned}$$

by the sequence of the following 54 equations (we just show the the Home equations):

$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left( \frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

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$$K_{t+1} = \Psi_{t+1}[I_t + (1 - \delta)K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta)Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left(\frac{L_t}{K_t}\right)^\alpha \quad (10)$$

$$\left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right) (1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$Q_{h,t} S_{h,t} = \Phi_t N_t + N_{g,t} \quad (77)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta)Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

$$\Phi_t = \frac{v_t}{\zeta - \left(\frac{v_{h,t}}{Q_{h,t} - v_t}\right)}$$

$$N_{t+1} = (\theta + \xi) [R_{hk,t+1} Q_{h,t} S_{hp,t}] - \theta R_{t+1} D_t + (Q_{hg,t+1} - Q_{t+1}) (S_{ge,t+1} - (1 - \delta) S_{ge,t}) \quad (76)$$

$$D_t = Q_{h,t} S_{h,t} - N_t - N_{g,t} \quad (72)$$

$$S_{h,t} = [I_t + (1 - \delta) K_t] \quad (40)$$

$$Q_{h,t-1} = 1 + f(Y u_{t-1}) + f'(Y u_{t-1}) Y u_{t-1} - E_{t-1} \Lambda_{t-1,t} f'(Y u_t) Y u_t^2 \quad (11)$$

$$Y u_t = \frac{I_t}{I_{t-1}}$$

$$Y_t = C_{h,t} + C_{h,t}^* + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \frac{P_t}{P_{h,t}} + G_t \frac{P_t}{P_{h,t}} \quad (38)$$

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

We now need to add seven equations for the new variables related to the introduction of the policy maker and its unconventional monetary policy. The definition of Government's consumption expenditure:

$$G_t = \bar{G} + \tau_e S_{ge,t} \quad (79)$$

where we set  $\bar{G} = 0$  during normal times. The distinction between privately and publicly held securities in the home securities intermediated by home agents:

$$S_{h,t} = S_{hp,t} + S_{ge,t} \quad (71)$$

The market value of Government's equity:

$$N_{g,t} = Q_{h,t} + S_{ge,t} \quad (73)$$

The "rule" used by the Central Bank for implementing the credit policy under unusual times of crisis:

$$\varphi_t = v_g [(R_{hk,t+1} - R_{t+1}) - (\bar{R}_{hk} - \bar{R})] \quad (82)$$

where  $\bar{R}_{hk}$  and  $\bar{R}$  are steady state values of home equity and deposit returns and  $v_g$  is a parameter. We assume that the Central Bank adjusts the amount of its equity injections to the spread between excess of returns on home assets and their steady state value:

$$S_{ge,t} = \varphi_t S_{h,t} \quad (84)$$

The equations that define the gross return on one unit of Central Bank equity and  $Q_{hg,t}$ :

$$E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [R_{hg,t+1} - R_{t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) = 0 \quad (74)$$

$$R_{hg,t} = \psi_{t+1} \frac{Z_{t+1} + (1-\delta)Q_{h,t+1}}{Q_{hg,t}} \quad (75)$$



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Lastly, to complete the model we have to add the home household budget constraint:

$$\frac{P_{h,t}}{P_t} Y_t = C_t + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t + G_t + (Q_{hg,t+1} - Q_{t+1}) (S_{ge,t+1} - (1 - \delta) S_{ge,t}) \quad (41)$$

## 6 Appendix B

### 6.1 The Model with integrated good and financial markets

In Model B steady state portfolio shares are endogenously determined. As we described in the previous chapter, by following the method developed by Devereux and Sutherland (since 2006 onwards) we found  $\bar{S}_h = \bar{S}_f^* \cong 0.705 * \bar{K}$ .

### 6.2 The Baseline Model (without credit policies)

In our perfectly symmetric world, it follows that:

$$S_h = S_f^*$$

We then set  $S_h = S_f^*$  equal to an arbitrary constant (endogenously determined). Then,

$A_t, A_t^*, \Psi_t, \Psi_t^*$  follows exogenous stochastic process. We need to find the real prices:

$$\begin{aligned} & Q_{h,t}, R_{h,t}, R_{t+1}, \frac{P_{h,t}}{P_t} \\ & Q_{f,t}, R_{f,t}, R_{t+1}^*, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities

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6.2 *The Baseline Model*  
(without credit policies)

6 APPENDIX B

$$Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, Yu_t$$

$$Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*,$$

$$D_t^*, N_t^*, Yu_t^*$$

the shadow prices

$$v_t, v_{h,t}, v_{f,t}, \lambda_t, \Phi_t$$

$$v_t^*, v_{h,t}^*, v_{f,t}^*, \lambda_t^*, \Phi_t^*$$

determined as a function of the state variables

$$K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Yu_{t-1}$$

$$K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Yu_{t-1}^*$$

by the sequence of the following 40 equations (we just show the the Home equations):

From the Household's problem:

$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left( \frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

From the Final Good Production Firms' problem:

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

$$K_{t+1} = \Psi_{t+1} [I_t + (1 - \delta)K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta)Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left( \frac{L_t}{K_t} \right)^\alpha \quad (10)$$

From the Banks' problem:

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6.2 *The Baseline Model*  
(without credit policies)

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$$\left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right)(1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$\left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right)(1 + \lambda_t) = \zeta \lambda_t \quad (23)$$

$$Q_{h,t}S_h + Q_{f,t}S_f = \Phi_t N_t \quad (24)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta)Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

where the definition of  $\Phi_t$  is given by:

$$\Phi_t = \frac{v_t}{\zeta - \left(\frac{v_{h,t}}{Q_{h,t} - v_t}\right)}$$

$$N_{t+1} = (\theta + \xi) [R_{hk,t+1}Q_{h,t}S_h + R_{fk,t+1}Q_{f,t}S_f] - \theta R_{t+1}D_t \quad (34)$$

$$D_t = Q_{h,t}S_h + Q_{f,t}S_f - N_t \quad (37)$$

From the Capital Good Firms' problem:

$$Q_{h,t-1} = 1 + f(Yu_{t-1}) + f'(Yu_{t-1})Yu_{t-1} - E_{t-1}\Lambda_{t-1,t}f'(Yu_t)Yu_t^2 \quad (11)$$

where we define the investment ratio between  $t, t - 1$  as:

$$Yu_t = \frac{I_t}{I_{t-1}}$$

Equilibrium condition in the (integrated) good markets:

$$Y_t = C_{h,t} + C_{h,t}^* + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \frac{P_t}{P_{h,t}} \quad (38)$$

Equilibrium condition in the labour market:

$$\alpha \frac{P_{h,t} Y_t}{P_t L_t} = \kappa L_t C_t^\rho \quad (39)$$

Lastly, to complete the model we have to add the home household budget constraint:

$$\frac{P_{h,t} Y_t}{P_t} = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - Q_{h,t}S_h^* - R_{fk,t}Q_{f,t-1}S_f + R_{hk,t}Q_{h,t-1}S_h^* + Q_{f,t}S_f \quad (41)$$

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6.3 *The Model with liquidity and lending facilities*

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### 6.3 The Model with liquidity and lending facilities

In this case we have to fix in advance how much the home banking system will invest in home equities  $S_{hp}$  and how much it will invest in foreign equities  $S_f$ . Let us set  $S_f = S_h^*$  equal to an arbitrary constant (again, endogenously determined), that in this perfectly symmetric two country world also implies  $S_{hp} = S_{fp}^*$ .

Then,

$A_t, A_t^*, \Psi_t, \Psi_t^*$  follows exogenous stochastic process. We need to find the real prices:

$$Q_{h,t}, R_{ht}, R_{m,t+1}, R_{t+1}, \frac{P_{h,t}}{P_t}$$

$$Q_{f,t}, R_{ft}, R_{m,t+1}^*, R_{t+1}^*, \frac{P_{f,t}}{P_t}$$

the quantities

$$Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, Yu_t, G_t, S_{g,t}, S_{h,t}, \varphi_t, M_t$$

$$Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*,$$

$$D_t^*, N_t^*, Yu_t^*, G_t^*, S_{g,t}^*, S_{f,t}^*, \varphi_t^*, M_t^*$$

the shadow prices

$$v_t, v_{m,t}, v_{h,t}, v_{f,t}, \lambda_t, \Phi_t$$

$$v_t^*, v_{m,t}^*, v_{h,t}^*, v_{f,t}^*, \lambda_t^*, \Phi_t^*$$

determined as a function of the state variables

$$K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Yu_{t-1}$$

$$K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Yu_{t-1}^*$$

by the sequence of the following 54 equations (we just show the the Home equations):

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6.3 *The Model with liquidity and lending facilities*

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$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left( \frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

$$K_{t+1} = \Psi_{t+1} [I_t + (1 - \delta)K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta)Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left( \frac{L_t}{K_t} \right)^\alpha \quad (10)$$

$$\left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$\left( \frac{v_{f,t}}{Q_{f,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (23)$$

$$Q_{h,t} S_{h,t} + Q_{f,t} S_f = \Phi_t N_t + \omega_g M_t \quad (70)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta)Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

$$\Phi_t = \frac{v_t}{\zeta - \left( \frac{v_{h,t}}{Q_{h,t} - v_t} \right)}$$

$$N_{t+1} = (\theta + \xi) [R_{hk,t+1} Q_{h,t} S_{hp} + R_{fk,t+1} Q_{f,t} S_f] - \theta R_{t+1} D_t - \theta R_{m,t+1} M_t \quad (34)$$

$$D_t = Q_{h,t} S_{hp} + Q_{f,t} S_f - N_t - M_t \quad (65)$$

$$Q_{h,t-1} = 1 + f(Y u_{t-1}) + f'(Y u_{t-1}) Y u_{t-1} - E_{t-1} \Lambda_{t-1,t} f'(Y u_t) Y u_t^2 \quad (11)$$

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6.3 *The Model with liquidity and lending facilities*

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$$Y u_t = \frac{I_t}{I_{t-1}}$$

$$Y_t = C_{h,t} + C_{h,t}^* + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \frac{P_t}{P_{h,t}} + G_t \frac{P_t}{P_{h,t}} \quad (38)$$

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

We need, again, to add seven new equations in order to take into account the presence of the Policy Maker with its credit policy's variables. From the first order conditions of banks we now need to add the following equations:

$$\lambda_t = (v_{m,t} - v_t) \frac{(1+\lambda_t)}{\omega_g \zeta} \quad (66)$$

$$v_{m,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} (1 - \theta + \theta \zeta \phi_{t+1}) R_{m,t+1} \quad (81)$$

where the last one comes out by modifying equation (68).

The definition of Government's consumption expenditure:

$$G_t = \bar{G} + \tau S_{g,t} \quad (79)$$

where we set  $\bar{G} = 0$  during normal times. The distinction between privately and publicly held securities in the home securities intermediated by home agents:

$$S_{h,t} = S_{hp} + S_{g,t} \quad (71)$$

The "rule" used by the Central Bank for implementing the credit policy under unusual times of crisis:

$$\varphi_t = v_g \left[ (R_{hk,t+1} - R_{t+1}) - (\bar{R}_{hk} - \bar{R}) \right] \quad (82)$$

where  $\bar{R}_{hk}$  and  $\bar{R}$  are steady state values of home equity and deposit returns and  $v_g$  is a parameter. We assume that the Central Bank adjusts the fraction of private credit/amount of funds it lends to the home banks when the excess of returns on home assets increases over its steady state value:

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$$S_{g,t} = \varphi_t S_{h,t} \quad (63)$$

$$M_t = \varphi_t D_t \quad (83)$$

Lastly, to complete the model we have to add the home household budget constraint:

$$\begin{aligned} & \frac{P_{h,t}}{P_t} Y_t = \\ = & C_t + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t - Q_{h,t} S_h^* - R_{fk,t} Q_{f,t-1} S_f + R_{hk,t} Q_{h,t-1} S_h^* + Q_{f,t} S_f + \\ & G_t \quad (41) \end{aligned}$$

## 6.4 The Model with equity injections

We have to fix in advance how much the home banking system will invest in home equity  $S_{hp}$  and how much it will invest in foreign equity  $S_f$ . Let us set  $S_f = S_h^*$  equal to an arbitrary constant (endogenously determined); in this perfectly symmetric two country world this also implies  $S_{hp} = S_{fp}^*$ .

Then,

$A_t, A_t^*, \Psi_t, \Psi_t^*$  follows exogenous stochastic process. We need to find the real prices:

$$\begin{aligned} & Q_{h,t}, Q_{hg,t}, R_{h,t}, R_{hg,t}, R_{t+1}, \frac{P_{h,t}}{P_t} \\ & Q_{f,t}, Q_{fg,t}, R_{f,t}, R_{fg,t}, R_{t+1}^*, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities

$$\begin{aligned} & Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, Y u_t, G_t, S_{ge,t}, S_{h,t}, \varphi_t, N_{g,t} \\ & Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\ & D_t^*, N_t^*, Y u_t^*, G_t^*, S_{ge,t}^*, S_{f,t}^*, \varphi_t^*, N_{g,t}^* \end{aligned}$$

the shadow prices

$$\begin{aligned} & v_t, v_{h,t}, v_{f,t}, \lambda_t, \Phi_t \\ & v_t^*, v_{h,t}^*, v_{f,t}^*, \lambda_t^*, \Phi_t^* \end{aligned}$$

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determined as a function of the state variables

$$K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Y u_{t-1}$$

$$K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Y u_{t-1}^*$$

by the sequence of the following 54 equations (we just show the the Home equations):

$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left( \frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

$$K_{t+1} = \Psi_{t+1} [I_t + (1 - \delta) K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta) Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left( \frac{L_t}{K_t} \right)^\alpha \quad (10)$$

$$\left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$\left( \frac{v_{f,t}}{Q_{f,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (23)$$

$$Q_{h,t} S_{h,t} + Q_{f,t} S_{f,t} = \Phi_t N_t + N_{g,t} \quad (77)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta) Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

$$\Phi_t = \frac{v_t}{\zeta - \left( \frac{v_{h,t}}{Q_{h,t}} - v_t \right)}$$



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$$N_{t+1} = (\theta + \xi) [R_{hk,t+1}Q_{h,t}S_{hp} + R_{fk,t+1}Q_{f,t}S_f] - \theta R_{t+1}D_t + (Q_{hg,t+1} - Q_{h,t+1})(S_{ge,t+1} - (1 - \delta)S_{ge,t}) \quad (78)$$

$$D_t = Q_{h,t}S_{hp} + Q_{f,t}S_f - N_t - N_{g,t} \quad (72)$$

$$Q_{h,t-1} = 1 + f(Yu_{t-1}) + f'(Yu_{t-1})Yu_{t-1} - E_{t-1}\Lambda_{t-1,t}f'(Yu_t)Yu_t^2 \quad (11)$$

$$Yu_t = \frac{I_t}{I_{t-1}}$$

$$Y_t = C_{h,t} + C_{h,t}^* + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \frac{P_t}{P_{h,t}} + G_t \frac{P_t}{P_{h,t}} \quad (38)$$

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{I_t} = \kappa L_t C_t^\rho \quad (39)$$

We need to add seven equations for the new variables related to the introduction of the policy maker and its unconventional monetary policy. The definition of Government's consumption expenditure:

$$G_t = \bar{G} + \tau_e S_{ge,t} \quad (79)$$

where we set  $\bar{G} = 0$  during normal times. The distinction between privately and publicly held securities in the home securities intermediated by home agents:

$$S_{h,t} = S_{hp} + S_{ge,t} \quad (71)$$

The market value of Government's equity:

$$N_{g,t} = Q_{h,t} + S_{ge,t} \quad (73)$$

The "rule" used by the Central Bank for implementing the credit policy under unusual times of crisis:

$$\varphi_t = v_g [(R_{hk,t+1} - R_{t+1}) - (\bar{R}_{hk} - \bar{R})] \quad (82)$$

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where  $\bar{R}_{hk}$  and  $\bar{R}$  are steady state values of home equity and deposit returns and  $v_g$  is a parameter. We assume that the Central Bank adjusts the amount of equity injections to the spread between excess of returns on home assets and its steady state value:

$$S_{ge,t} = \varphi_t S_{h,t} \quad (84)$$

The equations that define the gross return on one unit of Central Bank equity and  $Q_{hg,t}$ :

$$E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [R_{hg,t+1} - R_{t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) = 0 \quad (74)$$

$$R_{hg,t} = \psi_{t+1} \frac{Z_{t+1} + (1-\delta)Q_{h,t+1}}{Q_{hg,t}} \quad (75)$$

Lastly, to complete the model we have to add the home household budget constraint:

$$\begin{aligned} & \frac{P_{h,t}}{P_t} Y_t = \\ = & C_t + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t - Q_{h,t} S_h^* - R_{fk,t} Q_{f,t-1} S_f + R_{hk,t} Q_{h,t-1} S_h^* + Q_{f,t} S_f + \\ & G_t \quad (41) \end{aligned}$$