

Banking Frictions and Integrated Financial Markets in a Two Country DSGE Model

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Abstract

Financial frictions and integrated financial markets matter by spreading and amplifying country specific shocks. We develop a two country DSGE world with incomplete markets to address these issues. The main reference for the model's framework is a work by Gertler and Kiyotaki¹. In the basic version of the model countries trade in goods but financial markets are closed. Then, we enrich the model by allowing for integrated financial markets but portfolio shares remain exogenously set. We end up with the complete model that also allows for portfolio choice by implementing a method developed by Devereux and Sutherland². We find home bias in international portfolios, that under incomplete markets allows for less volatility than under full portfolio diversification. The model provides a simple two country world framework that may also be used for monetary policy issues.

Key Words: two country DSGE model, incomplete markets, endogenous portfolio choice

*Supervisor Prof. Pierpaolo Benigno. I would like to thank my Supervisor for all the insightful suggestions. All errors are my own responsibility.

¹Gertler, M. and Kiyotaki, N., "Financial Intermediation and Credit Policy in Business Cycle Analysis", *N.Y.U. and Princeton*, March 2010.

²See Bibliography for the many papers of this topic. Among the others, Devereux, M. B. and Sutherland, A., "Solving for Country Portfolios in Open Economy Macro Models", "International Capital Flows", *CEPR Discussion Papers 5966, C.E.P.R. Discussion Papers*, 2006.

1 Introduction

Banking balance sheets and leverage constraints have been crucial in transmitting and amplifying shocks from a country to another one during last financial crises. These two channels can separately act in propagating the contagion, how were pointed out, among the others, by Devereux and Yetman (2010). What we developed here is a DSGE model that aims to keep into account these real-world features in order to understand better what mechanisms are at work during a modern financial crisis.

By trying to analyze how the current Eurozone crisis developed, we see that both banking system's integrated financial portfolios and leverage constraints were crucial factors in transforming a single country's problem in a systemic one. What directly exposes a national banking system to external shocks, indeed, are not just portfolio's linkages (by owning Government bonds subject to speculative attack), but also the presence of constraints on the ratio asset/liability in the balance sheets.

When the banks are obliged to maintain (or reach) precise capital requirements (Basel Accords, or last European Banking Authority's requirements), they may be obliged to sell potentially "toxic" assets, by causing in this way a chain of sells that further exacerbates the crisis.

The basic framework of our model closely follows that contained in Gertler and Kiyotaki(2010). We complicate their model by extending the analysis to a two country world with integrated good and financial markets and endogenous portfolio shares. We simplify, however, the analysis by not allowing for idiosyncratic shock to hit separately banks in the same country.

We know that according to empirical evidence markets in the real world are incomplete. We therefore choose to introduce these feature into our model by considering four independent sources of shocks, two for each country (to the technology and to the quality of capital), while only two assets are available as investment instruments for the banking system: home and foreign equities.

The deposit markets, where depositors and banks meet, are segmented at the national level.

The presence of incomplete markets can cause stationarity problems to

the model. In order to avoid these, we followed Schmitt-Groh and Uribe (2002,2004)³ and the literature related by making the discount factor endogenous. We chose this solution among the others proposed by the literature because it is the same that also Devereux and Yetman (2010) used in their work, similar in many ways to the one proposed here.

In order to understand better how the portfolio integration channel works under leverage constraints and incomplete markets, we consider three different versions of the same model: in the first version, the good markets are integrated but financial markets are closed (Model A). In the second version, both good and financial markets are integrated, but portfolio's shares are exogenously given as parameters (Model BK). In the third and last case, both markets are open and portfolio's shares are endogenous (Model BH/F). In order to determine the equilibrium portfolio's shares we followed the method developed by Devereux and Sutherland (2006-2011).

With this baseline model we would like to build a theoretical framework that might be used to two purposes. First of all, for analyzing the implication of coordinated versus uncoordinated unconventional monetary policies, such those used during the last financial turmoils by the main Central Banks. Secondly, for studying the linkages in balance sheets across Governments and banks, that revealed to be a crucial channel in transmitting the current Eurozone debt's crisis.

We will deepen the first task in the second part of our PhD dissertation, while we leave the second one for a future research.

Our work is organized in the following sequence: 1) the model 2)the steady state 3) a description of the method developed by Devereux and Sutherland (2006-2011) for finding the equilibrium portfolios 4)calibration 5)results. Theoretical models are in the Appendix.

2 The Model

We extend a work by Gertler and Kiyotaky (2010) by analyzing a two country world with international trade and banking frictions. In order to determine endogenous portfolio's shares in an equilibrium with international asset markets we refer to the approach followed by Devereux and Sutherland (2006-2011). Tille and Van Wincoop (2007) also developed a similar method

³See Schmitt-Groh and Uribe for a more detailed discussion

to get the same results. It is worth mentioning also the work by Devereux and Yetman (2010) as a main reference for this model.

2.1 General Hypotheses

We assume a two country model, where each country is specialized in the production of a single traded good. In each country there is a continuum of households. Population size is normalized to 1. Every time a fraction $1 - f$ of the home (foreign) representative household is done by depositors, while the remaining f is done by bankers. This turnover between the two groups is random, keeping the relative proportion of each type fixed. Depositors supply labour and return the wage to the household. Moreover, they deposit funds in a bank different from the one they own. Bankers manage a financial intermediary and transfer earnings back to the household. Within the family, we assume perfect consumption insurance.

We assume that with probability θ a banker will be banker also at the following time. Instead with probability $1 - \theta$ she will become a depositor. Therefore, each period $(1 - \theta)f$ bankers exit and become depositors, θf bankers remain bankers, $(1 - f)\theta$ depositors become bankers and $(1 - f)(1 - \theta)$ depositors remain depositors. There is one risk-free asset (deposit) D_t for the home country (D_t^* abroad) called in composite consumption units. Only household within a country hold the deposit of that country (deposits are not internationally traded). The Cobb-Douglas consumption index for the representative home household will be:

$$C_t \equiv C_{h,t}^\gamma C_{f,t}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

where $C_{h,t}$ is the domestic consumption of the home good and $C_{f,t}$ is the domestic consumption of the foreign good.

Similarly, for the foreign representative agent we have identical preferences:

$$C_t^* \equiv C_{f,t}^{\gamma} C_{h,t}^{*(1-\gamma)} \quad 0 < \gamma < 1 \quad (1^*)$$

where γ is the "economic size" of the home economy (i.e. the share of the home good in the consumption basket, while $1 - \gamma$ is the economic size of the foreign country). We assume $\gamma = \frac{1}{2}$ be the same for both home/foreign agents, meaning that there is no home consumption bias. Consumption-based price indexes will be:

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$$P_t \equiv \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} P_{h,t}^\gamma P_{f,t}^{(1-\gamma)} \quad (2)$$

$$P_t^* \equiv \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}} P_{f,t}^{*\gamma} P_{h,t}^{*(1-\gamma)} \quad (2^*)$$

where $P_{h,t}$ and $P_{f,t}$ are the prices of home/foreign goods in domestic currency, while $P_{h,t}^*$ and $P_{f,t}^*$ are the prices of home and foreign goods in foreign currency. P_t (or P_t^*) represents the minimum expenditure required to buy 1 unit of the composite consumption good. We assume the law of one price for both home and foreign good, that is:

$$P_{h,t} = \epsilon_t P_{h,t}^* \quad (3a)$$

$$P_{f,t} = \epsilon_t P_{f,t}^* \quad (3b)$$

Moreover, being households' preferences identical across borders with no home bias, the consumption baskets will also be identical and then the relative price of consumption (real exchange rate ReR_t) will be equal to one (Purchase Power Parity holds), that is:

$$P_t = \epsilon_t P_t^* \quad (3c)$$

where we define ϵ_t as the nominal exchange rate (the price of the foreign currency in terms of the home one).

Home consumption expenditure will be given by:

$$P_t C_t = P_{h,t} C_{h,t} + P_{f,t} C_{f,t}$$

while the lifetime utility of the home household will be:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{C_\tau^{1-\rho}}{1-\rho} - \frac{\kappa}{2} L_\tau^2 \quad (4)$$

where β is the discount rate, that can be exogenous or endogenous. We set ρ , γ and κ equal to the same value both home and abroad.

Generic home household chooses consumption, labour supply and deposits in order to maximize (4) subject to the following flow of funds constraint:

$$P_t C_t + P_t D_{t+1} = P_t R_t D_t + W_t L_t + \Pi_t \quad (5)$$

where we define:

$W_t(W_t^*)$: nominal wage rate of the generic household.

$R_t(R_t^*)$: real gross return on deposit from period $t - 1$ to period t .

$D_t(D_t^*)$: quantity of risk-less deposit in composite consumption units.

$\Pi_t(\Pi_t^*)$: net distribution from the ownership of both banks and non-financial firms (nominal), net the transfers that the household gives to its members that start a new activity by entering at t in the banking system.

Symmetrically, for the foreign household we have:

$$P_t^* C_t^* + P_t^* D_{t+1}^* = P_t^* R_t^* D_t^* + W_t^* L_t^* + \Pi_t^* \quad (5^*)$$

Consumption based power parity always holds due to the law of one price plus the assumptions on preferences (from the (3)). Within/across countries preferences and constraints are symmetric. Since agents (households and firms) are equal within countries, in what follows all the variables will be considered in per capita (aggregate) terms.

2.2 The Production Side

2.2.1 Final Good Firms

Each firm faces an identical Cobb Douglas, therefore in aggregate we can assume:

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

or, for the foreign country:

$$Y_t^* = A_t^* K_t^{*(1-\alpha)} L_t^{*\alpha} \quad (6^*)$$

Each home/foreign competitive final good firm produces a single (but differentiated among countries) output by using an identical CRTS Cobb Douglas production function with capital and labour as inputs. Labour is provided by the household of the same country. Now we define:

δ : rate of physical depreciation.

$\Psi_t(\Psi_t^*)$: shock to the quality of capital.

$I_t(I_t^*)$: aggregate home (foreign) investment in unit of consumption good.

Every period the law of motion for aggregate capital (in units of consumption good) will be given by:

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$$K_{t+1} = \Psi_{t+1}[I_t + (1 - \delta)K_t] \quad (7)$$

The generic firm's maximization function (in real terms) at time t will then be the following:

$$\begin{aligned} & E_t \beta \Lambda_{t,t+1} \left\{ \frac{P_{h,t}}{P_t} A_t K_t^{(1-\alpha)} L_t^\alpha + Q_{h,t} \frac{K_{t+1}}{\Psi_{t+1}} - R_{hk,t} Q_{h,t-1} \frac{K_t}{\Psi_t} - \frac{W_t}{P_t} L_t - Q_{h,t} I_t \right\} = \\ & = E_t \beta \Lambda_{t,t+1} \left\{ \frac{P_{h,t}}{P_t} A_t K_t^{(1-\alpha)} L_t^\alpha + Q_{h,t} (I_t + (1 - \delta)K_t) - R_{hk,t} Q_{h,t-1} \frac{K_t}{\Psi_t} - \frac{W_t}{P_t} L_t - Q_{h,t} I_t \right\} \end{aligned}$$

where $\beta \Lambda_{t,t+1}$ is the firms' stochastic discount factor (by assumption, firms belong to the households), $R_{hk,t}$ is the real return on home capital required by investing banks and $Q_{hk,t}$ is the real price of one unit of home capital. We are assuming here that the replacement price of capital that has depreciated is equal to 1. Each period the firm issues new state-contingent securities in order to obtain funds from an intermediary at price $Q_{hk,t}$ and to buy new capital good at the same price, and then it has to pay back capital returns on the securities issued at the previous period.

From the FOCs w.r.t. L_t and K_t :

$$W_t/P_{h,t} = \alpha Y_t/L_t \quad (8)$$

$$R_{hk,t+1} = \Psi_{t+1} \frac{(1-\alpha) \frac{P_{h,t+1}}{P_{t+1}} \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) Q_{h,t+1}}{Q_{h,t}} \quad (9)$$

where the real gross profit per unit of capital is:

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left(\frac{L_t}{K_t} \right)^\alpha \quad (10)$$

Therefore, each unit of home equity bought at time t will be a state-contingent claim to the future returns from one unit of investment:

$$\Psi_{t+1} Z_{t+1}, (1 - \delta) \Psi_{t+1} \Psi_{t+2} Z_{t+2}, (1 - \delta)^2 \Psi_{t+1} \Psi_{t+2} \Psi_{t+3} Z_{t+3}$$

2.2.2 Capital Good Firms

Home competitive capital good firms belong to home households and operate in a national market. They produce capital good using national output as input subject to adjustment costs. They choose I_t (in unit of the composite consumption good) in order to solve the following problem:

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,\tau} \left\{ \left[Q_{h,\tau} I_\tau - \left(1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right) \right) \right] I_\tau \right\}$$

where $f(\frac{I_t}{I_{t-1}})$ is the physical adjustment cost (in units of consumption good) with $f(1) = f'(1) = 0$ and $f''(1) > 0$. From the FOC:

$$Q_{h,t} = 1 + f(\frac{I_t}{I_{t-1}}) + f'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}} - E_t \Lambda_{t,t+1} f'(\frac{I_{t+1}}{I_t}) \frac{I_{t+1}}{I_t}^2 \quad (11)$$

where the real price of the capital good has to be equal to the marginal cost of producing the investment good. Profits arise only outside the steady state, and are lump-sum distributed to the home households.

2.2.3 The Households

The home generic household takes the nominal prices as given in maximizing the following function:

$$E_t \left\{ \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{C_{\tau}^{1-\rho}}{1-\rho} - \frac{\kappa}{2} L_{\tau}^2 \right] + \sum_{\tau=t}^{\infty} \beta^{\tau-t} \mu_{\tau} [P_{\tau} R_{\tau} D_{\tau} + W_{\tau} L_{\tau} + \Pi_{\tau} - P_{\tau} C_{\tau} - P_{\tau} D_{\tau+1}] \right\}$$

where the FOCs with respect to C_t, D_{t+1}, L_t are:

$$C_t^{-\rho} = P_{\tau} \mu_t \quad (12)$$

$$E_t \left\{ \frac{P_{\tau+1} \mu_{\tau+1}}{P_{\tau} \mu_{\tau}} R_{\tau+1} \right\} = \frac{1}{\beta} \quad (13)$$

$$W_t = \frac{\kappa L_t}{\mu_t} \quad (14)$$

where μ_t is the Lagrange multiplier. The Euler Equation is therefore given by:

$$1 = E_t \left\{ \beta R_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \right\} \quad (15)$$

while the following is the labour supply equation:

$$\frac{W_t}{P_t} = \kappa L_t C_t^{\rho} \quad (16)$$

To complete the household's problem we now need to find the demands for home and foreign consumption goods. The representative household solves:

$$\min P_{h,t} C_{h,t} + P_{f,t}^* \epsilon_t C_{f,t}$$

s.t.

$$C_t = C_{h,t}^\gamma C_{f,t}^{(1-\gamma)}$$

in order to get from the FOCs:

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$C_{f,t} = (1 - \gamma) C_t \frac{P_t}{P_{f,t}} \quad (18)$$

while for the foreign household we will have:

$$C_{h,t}^* = (1 - \gamma) C_t^* \frac{P_t^*}{P_{h,t}^*} \quad (17^*)$$

$$C_{f,t}^* = \gamma C_t^* \frac{P_t^*}{P_{f,t}^*} \quad (18^*)$$

2.2.4 The Banks

Each home bank borrows d_t (in composite consumption units) in the national deposit market at the real gross deposit rate R_{t+1} , and then purchases $s_{h,t}$ and $s_{f,t}$ units of financial claims on final goods' producing firms at the real prices $Q_{h,t}$ and $Q_{f,t}$. These are the prices of the banking system's claim on the future returns from one unit of present capital of the non-financial firm at the end of period. Firms are able to offer to the banks perfectly state-contingent debt. For an individual bank, the flow of funds constraint (where everything is in units of the composite consumption good) holds (intermediary balance sheet):

$$Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} = n_t + d_t \quad (19)$$

where $s_{h,t}$ and $s_{f,t}$ are held by an individual national bank ($s_{h,t}^*$ and $s_{f,t}^*$ are held abroad) and where $Q_{f,t} = \frac{Q_{f,t}^* P_t}{\epsilon_t P_t^*}$ with ϵ_t as the nominal exchange rate. Home and foreign financial claims respectively pay the real gross returns $R_{hk,t}$ and $R_{fk,t}$. On the LHS of the (19) we have the value of the loans funded within a given period, while and on the RHS there are the equity capital (the net worth of the bank) plus the debt. Net worth evolves according to:

$$n_t = R_{hk,t}Q_{h,t-1}s_{h,t-1} + R_{fk,t}Q_{f,t-1}s_{f,t-1} - R_t d_{t-1} \quad (20)$$

or, given that

$$n_t = R_{hk,t}Q_{h,t-1}s_{h,t-1} + R_{fk,t}Q_{f,t-1}s_{f,t-1} - R_t[Q_{h,t-1}s_{h,t-1} + Q_{f,t-1}s_{f,t-1} - n_{t-1}]$$

we can simplify the net worth as:

$$n_t = [R_{hk,t} - R_t]Q_{h,t-1}s_{h,t-1} + [R_{fk,t} - R_t]Q_{f,t-1}s_{f,t-1} + R_t n_{t-1}$$

The end of period objective of the bank will be:

$$V_t = E_t \sum_{\tau=t+1}^{\infty} (1 - \theta)\theta^{\tau-(t+1)}\beta^{\tau-(t+1)}\Lambda_{t,\tau}n_{\tau}$$

where $\beta\Lambda_{t,t+1}$ is the stochastic discount factor, equal to the marginal rate of substitution of the representative household that owns the bank. In order to solve its maximization's problem, the bank will have to take into account an endogenous constraint. We limit the bank's ability to obtain funds in the deposit market in this way. Suppose that after a bank obtains funds, the banker managing the bank may transfer a fraction ζ of "divertable" assets to her household ($0 < \zeta < 1$). Since creditors recognize this bank's incentive to divert funds, a borrowing constraint must arise:

$$V(s_{h,t}, s_{f,t}, d_t) \geq \zeta(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) \quad (21)$$

where on the LHS we have the maximized value of the bank's objective at the end of t . The value of the bank at the end of $t - 1$ will be then:

$$\begin{aligned} & V(s_{h,t-1}, s_{f,t-1}, d_{t-1}) = \\ & = E_{t-1}\Lambda_{t-1,t} [(1 - \theta)n_t + \theta(\max_{s_{h,t}, s_{f,t}, d_t} V(s_{h,t}, s_{f,t}, d_t))] \end{aligned}$$

where quantities are all in units of composite consumption good. In order to solve it, we guess that the value function is linear (we will verify this guess later):

$$V(s_{h,t}, s_{f,t}, d_t) = v_{h,t}s_{h,t} + v_{f,t}s_{f,t} - v_t d_t$$

where the parameters are defined as:

$v_{h,t}$: the value to the bank at the end of t of an additional unit of home asset

$v_{f,t}$: the value to the bank at the end of t of an additional unit of foreign asset

v_t : the marginal cost of deposit

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Being now λ_t the Lagrange multiplier for the incentive constraint, we have that:

$$\max V(s_{h,t}, s_{f,t}, d_t) = v_{h,t}s_{h,t} + v_{f,t}s_{f,t} - v_t(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} - n_t) - \lambda_t [\zeta(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) - v_{h,t}s_{h,t} - v_{f,t}s_{f,t} + v_t(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} - n_t)]$$

We derive the FOCs with respect to $s_{h,t}$, $s_{f,t}$ and λ_t :

$$\left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right)(1 + \lambda_t) = \zeta\lambda_t \quad (22)$$

$$\left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right)(1 + \lambda_t) = \zeta\lambda_t \quad (23)$$

$$Q_{h,t}s_{h,t} \left[\zeta - \left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right)\right] + Q_{f,t}s_{f,t} \left[\zeta - \left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right)\right] \leq v_t n_t \quad (24)$$

where it must be that

$$\lambda_t \left\{ Q_{h,t}s_{h,t} \left[\zeta - \left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right)\right] + Q_{f,t}s_{f,t} \left[\zeta - \left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right)\right] - v_t n_t \right\} \quad (25)$$

Then, from the above equations, as long as the incentive constraint is binding ($\lambda_t > 0$), the marginal value of asset (in terms of home goods) will be greater than the marginal cost of deposit. Now, by defining:

$$\mu_{h,t} = \frac{v_{h,t}}{Q_{h,t}} - v_t = \mu_{f,t} = \frac{v_{f,t}}{Q_{f,t}} - v_t > 0$$

at an equilibrium where both assets (home and foreign) are held and the incentive constraint is binding, the last ones becomes:

$$Q_{h,t}s_{h,t} [\zeta - \mu_{h,t}] + Q_{f,t}s_{f,t} [\zeta - \mu_{f,t}] = v_t n_t \quad (26)$$

or, since $\mu_{h,t} = \mu_{f,t} = \mu_t$,

$$(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) [\zeta - \mu_t] = v_t n_t$$

$$(Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t}) = \frac{v_t n_t}{[\zeta - \mu_t]}$$

where

$$\phi_t = \frac{v_t}{[\zeta - \mu_t]}$$

is the leverage ratio net of interbank borrowing. Note that the leverage ratio is increasing in μ_t . Note also that the constraint is binding as long as $0 < \mu_t < \zeta$. If, instead, $\mu_t > \zeta$ the constraint is not binding. What we assume is that at an equilibrium, for reasonable values of parameters, the incentive constraint always binds. Now, by combining the conjectured value function with the Bellman equation, and given that, at the equilibrium, FOCs plus binding incentive constraint imply that:

$$Q_{h,t}s_{h,t} + Q_{f,t}s_{f,t} = \phi_t n_t = n_t + d_t$$

we can write:

$$V_{t-1} = E_{t-1}\beta\Lambda_{t-1,t} \{(1 - \theta)n_t + \theta V_t\}$$

Therefore, holding everything else constant, the expected discounted marginal gain to the banker of expanding asset $Q_{h,t}s_{h,t}$ or $Q_{f,t}s_{f,t}$ by one unit will be:

$$\frac{\partial V_t}{\partial Q_{h,t}s_{h,t}} = E_t\beta\Lambda_{t,t+1} [R_{hk,t+1} - R_{t+1}] (1 - \theta + \theta(v_{t+1} + \phi_{t+1}\mu_{t+1}))$$

$$\frac{\partial V_t}{\partial Q_{f,t}s_{f,t}} = E_t\beta\Lambda_{t,t+1} [R_{fk,t+1} - R_{t+1}] (1 - \theta + \theta(v_{t+1} + \phi_{t+1}\mu_{t+1}))$$

while the expected marginal cost of expanding deposit by one unit will be:

$$\frac{\partial V_t}{\partial d_t} = -E_t\beta\Lambda_{t,t+1}R_{t+1} (1 - \theta + \theta(v_{t+1} + \phi_{t+1}\mu_{t+1}))$$

Therefore, if we rewrite the conjectured value function in this way:

$$\begin{aligned} V_t &= \left(\frac{v_{h,t}}{Q_{h,t}} - v_t\right) s_{h,t}Q_{h,t} + \left(\frac{v_{f,t}}{Q_{f,t}} - v_t\right) s_{f,t}Q_{f,t} + v_t n_t = \\ &= \mu_{h,t}Q_{h,t}s_{h,t} + \mu_{f,t}Q_{f,t}s_{f,t} + v_t n_t \end{aligned}$$

we can verify that the value function is linear in $(s_{h,t}, s_{f,t}, d_t)$ if $\mu_{h,t}$, $\mu_{f,t}$ (the marginal value of holding home/foreign assets net marginal costs) and v_t (the marginal cost of deposit) satisfy:

$$\begin{aligned} \frac{\partial V_t}{\partial Q_{h,t}s_{h,t}} &= \mu_{h,t} = \\ = E_t\beta\Lambda_{t,t+1} [R_{hk,t+1} - R_{t+1}] (1 - \theta + \theta(v_{t+1} + \phi_{t+1}\mu_{t+1})) \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{\partial V_t}{\partial Q_{f,t}s_{f,t}} &= \mu_{f,t} = \\ = E_t\beta\Lambda_{t,t+1} [R_{fk,t+1} - R_{t+1}] (1 - \theta + \theta(v_{t+1} + \phi_{t+1}\mu_{t+1})) \end{aligned} \quad (28)$$

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$$= E_t \beta \Lambda_{t,t+1} R_{t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) \quad (29)$$

where using the definition of $\mu_{h,t}, \mu_{f,t}$ we can also rewrite the equations above as:

$$v_{h,t} = E_t \beta \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta) Q_{h,t+1}] \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) \quad (30)$$

$$v_{f,t} = E_t \beta \Lambda_{t,t+1} [Z_{t+1}^* + (1 - \delta) Q_{f,t+1}] \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) \quad (31)$$

$$v_t = E_t \beta \Lambda_{t,t+1} R_{t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) \quad (32)$$

or, for the foreign country,

$$v_{h,t}^* = E_t \beta \Lambda_{t,t+1}^* [Z_{t+1} + (1 - \delta) Q_{h,t+1}] \left(1 - \theta + \theta \left((v_{t+1}^* + \phi_{t+1}^* \mu_{t+1}^*) \right) \right) \quad (30^*)$$

$$= E_t \beta \Lambda_{t,t+1}^* [Z_{t+1}^* + (1 - \delta) Q_{f,t+1}] \left(1 - \theta + \theta \left((v_{t+1}^* + \phi_{t+1}^* \mu_{t+1}^*) \right) \right) \quad (31^*)$$

$$v_t^* = E_t \beta \Lambda_{t,t+1}^* R_{t+1}^* \left(1 - \theta + \theta \left(v_{t+1}^* \right) \right) \quad (32^*)$$

To conclude, at the equilibrium, if the incentive constraint is binding:

$$\begin{aligned} & E_t \beta \Lambda_{t,t+1} R_{h,t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) = \\ & = E_t \beta \Lambda_{t,t+1} R_{f,t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) > \\ & \quad ; E_t \beta \Lambda_{t,t+1} R_{t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) \end{aligned}$$

Otherwise, the banks will increase the amount of home and foreign assets to the point where:

$$\begin{aligned} & E_t \beta \Lambda_{t,t+1} R_{h,t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) = \\ & = E_t \beta \Lambda_{t,t+1} R_{f,t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) = \\ & = E_t \beta \Lambda_{t,t+1} R_{t+1} \left(1 - \theta + \theta \left((v_{t+1} + \phi_{t+1} \mu_{t+1}) \right) \right) \end{aligned}$$

We sum up across individual banks in order to obtain the demand for total bank assets $Q_{h,t} S_{h,t}$ and $Q_{f,t} S_{f,t}$ as a function of total net worth N_t as:

$$Q_{h,t} S_{h,t} + Q_{f,t} S_{f,t} = \frac{v_t}{\zeta - \mu_t} N_t \quad (33)$$

2.2.5 Evolution of Bank Net Worth

The total net worth for the home banks is given by the sum of the total net worth of existing bankers (o for old) and of entering bankers (y for young):

$$N_t = N_{o,t} + N_{y,t} \quad (34)$$

where

$$N_{o,t} = \theta [R_{hk,t}Q_{h,t-1}S_{h,t-1} + R_{fk,t}Q_{f,t-1}S_{f,t-1} - R_t D_{t-1}] \quad (35)$$

$$N_{y,t} = \frac{\xi(1-\theta)}{(1-\theta)} [R_{hk,t}Q_{h,t-1}S_{h,t-1} + R_{fk,t}Q_{f,t-1}S_{f,t-1}] \quad (36)$$

with θ being the fraction of bankers that survive from the previous period, $(1 - \theta)$ the fraction of "new" bankers that were workers the previous period, $\frac{\xi}{(1-\theta)}$ is the fraction of total value assets gained by exiting bankers that is transferred from the family to each new banker, and D_t is the aggregate deposit (in real terms). Therefore, from the whole banking sector's balance sheet, deposits will be given by:

$$D_t = Q_{h,t}S_{h,t} + Q_{f,t}S_{f,t} - N_t \quad (37)$$

2.3 Equilibrium

Since households are symmetric, we can rewrite the previously defined equations in terms of the following equilibrium conditions.

Optimal intertemporal allocation of consumption will be given by :

$$1 = \beta E_t R_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \quad (15)$$

$$1 = \beta E_t R_{t+1}^* \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\rho} \quad (15^*)$$

Equilibrium in the goods market is described by:

$$Y_t = C_{h,t} + C_{h,t}^* + \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \frac{P_t}{P_{h,t}} \quad (38)$$

$$Y_t^* = C_{f,t} + C_{f,t}^* + \left[1 + f \left(\frac{I_t^*}{I_{t-1}^*} \right) \right] I_t^* \frac{P_t}{P_{f,t}} \quad (38^*)$$

For the labour markets we get the following equilibrium conditions:

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$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

$$\alpha \frac{P_{f,t}^*}{P_t^*} \frac{Y_t^*}{L_t^*} = \kappa L_t^* C_t^{*\rho} \quad (39^*)$$

The supply for home securities will be given by:

$$S_{h,t} + S_{h,t}^* = [I_t + (1 - \delta) K_t] \quad (40)$$

$$S_{f,t}^* + S_{f,t} = [I_t^* + (1 - \delta) K_t^*] \quad (40^*)$$

while the demands are obtained by the (33), giving the equilibrium in the security markets.

Lastly, being the household also the owner of both national banks and firms, it follows that in the equation (5):

$$C_t = -D_{t+1} + R_t D_t + \frac{W_t}{P_t} L_t + \frac{\Pi_t}{P_t} \quad (5)$$

the real dividend will be given by the sum of the following terms:

- real profits from both final and capital good sectors
- the difference in a generic bank's wealth
- the transfers from the exiting banker to the household net from the transfer from the household to the new banker

By summing up all these terms, the representative household's budget constraint in the home country will be given by:

$$\begin{aligned} \frac{P_{h,t}}{P_t} Y_t + Q_{h,t} S_{h,t}^* + R_{f,t} Q_{f,t-1} S_{f,t-1} - R_{h,t} Q_{h,t-1} S_{h,t-1}^* - Q_{f,t} S_{f,t} &= \\ &= C_t + \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] I_t \quad (41) \end{aligned}$$

and, symmetrically, for the foreign country:

$$\begin{aligned} \frac{P_{f,t}}{P_t} Y_t^* + Q_{f,t} S_{f,t} + R_{h,t} Q_{h,t-1} S_{h,t-1}^* - R_{f,t} Q_{f,t-1} S_{f,t-1} - Q_{h,t} S_{h,t}^* &= \\ &= C_t^* + \left[1 + f \left(\frac{I_t^*}{I_{t-1}^*} \right) \right] I_t^* \quad (41^*) \end{aligned}$$

Lastly, notice that from the equilibrium conditions in the financial markets, we must have:

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$$\begin{aligned}\frac{v_{h,t}}{Q_{h,t}} - v_t &= \frac{v_{f,t}}{Q_{f,t}} - v_t \\ \frac{v_{h,t}^*}{Q_{h,t}} - v_t^* &= \frac{v_{f,t}^*}{Q_{f,t}} - v_t^*\end{aligned}$$

that can be summarized as:

$$E_t \{R_{h,t+1} - R_{f,t+1}\} = 0 \quad (42)$$

Then, given that A_t, A_t^*, Ψ_t and Ψ_t^* follow exogenous stochastic processes,

we need to find the real prices:

$$\begin{aligned}Q_{h,t}, R_{ht}, R_{t+1}, \frac{P_{h,t}}{P_t} \\ Q_{f,t}, R_{ft}, R_{t+1}^*, \frac{P_{f,t}}{P_t}\end{aligned}$$

the quantities:

$$\begin{aligned}Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, S_{h,t}, S_{f,t}, Yu_t \\ Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\ D_t^*, N_t^*, S_{h,t}^*, S_{f,t}^*, Yu_t^*\end{aligned}$$

the shadow prices:

$$\begin{aligned}v_t, v_{h,t}, v_{f,t}, \lambda_t, \Phi_t \\ v_t^*, v_{h,t}^*, v_{f,t}^*, \lambda_t^*, \Phi_t^*\end{aligned}$$

determined as a function of the state variables:

$$\begin{aligned}K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Yu_{t-1} \\ K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Yu_{t-1}^*\end{aligned}$$

by the sequence of the following 44 equations:

$$\begin{aligned}(1) - (6) - (7) - (9) - (10) - (11) - (15) - (17) \\ (1^*) - (6^*) - (7^*) - (9^*) - (10^*) - (11^*) - (15^*) - (17^*)\end{aligned}$$

that describe the optimization conditions of households and final good firms (home and abroad),

$$\begin{aligned}(22) - (23) - (24) - (30) - (31) - (32) - (34) \\ (22^*) - (23^*) - (24^*) - (30^*) - (31^*) - (32^*) - (34^*)\end{aligned}$$

that describe the optimization conditions for banks,

$$\begin{aligned} & (37) - (38) - (39) - (40) - (41) \\ & (37^*) - (38^*) - (39^*) - (40^*) - (41^*) \end{aligned}$$

that describe the market clearing conditions for goods, securities and labour.

We also include the definitions of Φ_t and of $Y u_t$:

$$\begin{aligned} \Phi_t &= \frac{v_t}{\zeta - \left(\frac{v_{h,t}}{Q_{h,t}} - v_t \right)} \\ Y u_t &= \frac{I_t}{I_{t-1}} \end{aligned}$$

2.4 The Steady State

By assumption, at steady state the real capital prices $Q_{h,t}$ and $Q_{f,t}$ are equal to unity. Then, being at equilibrium $\mu_{h,t} = \mu_{f,t} = \mu$, it must be the case that

$$R_{hk} = R_{fk} = R_k$$

and also from the demands for securities we have:

$$\begin{aligned} S_h + S_f &= \phi N \\ S_h^* + S_f^* &= \phi N^* \end{aligned}$$

where ϕ is defined as:

$$\phi_t = \frac{v_t}{\zeta - \mu_t} \quad (42)$$

while from the supply side we get:

$$\begin{aligned} S_h + S_f &= N + D \\ S_h^* + S_f^* &= N^* + D^* \end{aligned}$$

Lastly, from the definition of bank's wealth we have:

$$\begin{aligned} S_h + S_h^* &= K \\ S_f + S_f^* &= K^* \end{aligned}$$

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Moreover, from Euler Equation (15) and (15*) we know that at steady state:

$$R = R^* = \frac{1}{\beta}$$

Then, from equations (34) – (35) – (36) and (34*) – (35*) – (36*) it follows that at the equilibrium (in the binding case, the only one that we consider):

$$R_k = \frac{\beta + \theta\phi - \theta}{(\theta + \xi)\beta\phi} \quad (43)$$

and therefore from equations (9) – (9*) also follows that:

$$\frac{P_h Y}{P K} = \frac{P_f^* Y^*}{P^* K^*} = \left(\delta - 1 + \frac{1}{\Psi} \frac{\beta + \theta\phi - \theta}{(\theta + \xi)\beta\phi} \right) \frac{1}{1 - \alpha} \quad (44)$$

Then, by considering the definition of ϕ_t and the equations (27) – (28) – (29) and (27*) – (28*) – (29*) we have that:

$$\phi_t = \frac{v_t}{\zeta - \mu_t}$$

where at steady state:

$$v = [1 - \theta + \theta v + \theta\phi\mu]$$

and then:

$$v = \frac{(1 - \theta)(\zeta - \mu)}{(\zeta - \mu - \theta\zeta)} \quad (45)$$

$$\phi = \frac{(1 - \theta)}{(\zeta - \mu - \theta\zeta)} \quad (46)$$

and therefore we find that at the steady state:

$$\phi = \phi^*$$

since at equilibrium

$$v_h = v_f = v$$

and also

$$\mu_h = \mu_f = \mu$$

where μ from the (27) – (28) and (27*) – (28*) is given by:

$$\begin{aligned}\mu &= \beta \left[R_k - \frac{1}{\beta} \right] (1 - \theta + \theta v + \theta \phi \mu) = \\ &= (\beta - \theta - \phi \xi) (1 - \theta + \theta \phi \zeta) \frac{1}{\phi(\theta + \xi)} \quad (47)\end{aligned}$$

Therefore, by substituting the (47) into the (46) we find a second-order equation in ϕ :

$$\begin{aligned}\phi [\zeta \xi + \zeta \theta - \zeta \theta \xi - \zeta \theta^2 + \xi - \xi \theta - \beta \theta \zeta + \zeta \theta^2] + \\ + \phi^2 [\theta \xi \zeta] + [\theta - \beta + \beta \theta - \theta^2 - (1 - \theta) (\xi + \theta)] = 0\end{aligned}$$

Then, since we know that an equilibrium where the constraints are binding we must have $\mu < \zeta$, and since we assume v to be positive (being it a marginal cost), only one root of the above equation will guarantee that $\phi > 0$.

From (46), we can derive the numerical steady state values for R_k , while from (43) – (44) we can obtain $\frac{P_h Y}{P K}$ and $\frac{P_f^* Y^*}{P^* K^*}$.

Now, by considering $\frac{P_h Y}{P K} = \frac{P_f^* Y^*}{P^* K^*}$ into the equilibrium conditions in the Good Markets (38) – (38*) it follows that:

$$\frac{K}{K^*} = \frac{\gamma}{(1-\gamma)} \quad (48)$$

Therefore, from equations (8) – (9) and (8*) – (9*) it follows that:

$$\frac{Y}{Y^*} = \frac{\Psi^* P_f K}{\Psi P_h K^*} + \frac{(1-\delta)(\Psi^* - \Psi) K P}{\Psi(1-\alpha) Y^* P_h}$$

that, since at steady state we set $\Psi = \Psi^* = 1$, can be rewritten as:

$$\frac{Y}{Y^*} = \frac{P_f K}{P_h K^*} \quad (49)$$

and the labour ratio across countries

$$\frac{L}{L^*} = \frac{K}{K^*} \quad (50)$$

while from the production function (6) – (6*) we obtain the relative prices:

$$\frac{P_h}{P_f} = \frac{A^*}{A} \quad (51)$$

Therefore from the equation (49) and (50) we get the output and labour ratios:

$$\frac{Y}{Y^*} = \frac{1}{(1-\gamma)} \frac{A}{A^*}$$

$$\frac{L}{L^*} = \frac{\gamma}{(1-\gamma)}$$

where the above equations can be simplified since we are assuming A, A^* equal to unity at steady state.

Also by considering equation (51) into the definitions of home and foreign price of one unit of the composite consumption good (equations (2) – (2*)) we derive the relative prices $\frac{P_h}{P}$ and $\frac{P_f}{P}$ where:

$$\frac{P_h}{P} = \gamma^\gamma (1 - \gamma)^{(1-\gamma)} \left(\frac{P_h}{P_f} \right)^{1-\gamma}$$

$$\frac{P_f}{P} = \gamma^\gamma (1 - \gamma)^{(1-\gamma)} \left(\frac{P_f}{P_h} \right)^{1-\gamma}$$

From the definitions of Z_t and Z_t^* (equations (10) – (10*)) we get now the labour to capital ratios:

$$\frac{L}{K} = \left(\frac{Z}{1-\alpha} \frac{P}{AP_h} \right)^{\frac{1}{\alpha}} \quad (52)$$

$$\frac{L^*}{K^*} = \left(\frac{Z^*}{1-\alpha} \frac{P^*}{A^*P_f^*} \right)^{\frac{1}{\alpha}} \quad (52^*)$$

where

$$Z = \frac{\beta + \theta\phi - \theta}{(\theta + \xi)\beta\phi} + \delta - 1$$

Now from the equations (9) – (9*) and knowing the capital to labour ratios we find the real wages:

$$\frac{W}{P} = \alpha \frac{PhY}{PL} \quad (53)$$

$$\frac{W^*}{P^*} = \alpha \frac{PfY^*}{P^*L^*} \quad (53^*)$$

Then by considering the above results into the equilibrium conditions in the Good Markets (38) – (38*)) it follows that:

$$\begin{aligned} (C + C^*) &= (\tilde{\alpha} - \delta) (K + K^*) = \\ &= (K + K^*) \quad (54) \end{aligned}$$

where we set

$$\tilde{\alpha} = \frac{P_h Y}{PK} = \frac{P_f^* Y^*}{P^* K^*}$$

$$\tilde{\beta} = \tilde{\alpha} - \delta$$

Therefore, by combining the labour supply equations (16) – (16*) with equations (54) and knowing the capital to labour ratios from equations (52) and the capital ratio from equation (48) we obtain:

$$K^* = \left[\left(\frac{W}{\kappa P \tilde{\gamma}} \right)^{\frac{1}{\rho}} \left(\frac{1 + \delta^{-\frac{1}{\rho}}}{\tilde{\beta} (1 + \delta)} \right) \right]^{\frac{\rho}{1 + \rho}} \quad (54)$$

where we set

$$\tilde{\gamma} = \frac{L}{K} = \frac{L^*}{K^*}$$

and

$$\tilde{\delta} = \frac{K}{K^*}$$

From the above results it is easy to obtain K, Y, Y^*, L, L^* . Then, from the labour supply equations, we also derive C and C^* , while from equations (17) – (18) and (17*) – (18*) we end up with C_h, C_f, C_h^*, C_f^* .

Lastly, from equations (40) – (40*) we obtain $S_h + S_h^*$ and $S_f + S_f^*$. Then, by combining the (40) – (40*) with the home and foreign household's budget constraint (41) – (41*), we find that at steady state:

$$\frac{C - \frac{P_h}{P} Y + \delta K}{(1 - R_k)} = (S_h^* - S_f)$$

$$\frac{C^* - \frac{P_f^*}{P^*} Y^* + \delta K^*}{(1 - R_k)} = (S_f - S_h^*)$$

In a steady state where trade balances are equal to zero, $S_h^* = S_f$. Otherwise (if we not constraint the trade balances to be equal to zero at steady state), we can derive a relationship between S_h^* and S_f . From the last above equations we can also obtain $S_h + S_f$ and $S_h^* + S_f^*$ and therefore we get N and N^* from the demands for securities. Lastly, from equations (37) – (37*) we find D and D^* and then N_y, N_o, N_y^* and N_o^* .

There is, however, no way to distinguish across S_h and S_f (in the home financial portfolio) and across S_h^* and S_f^* (in the foreign financial portfolio). The steady state is therefore not completely determined yet. We need a method in order to overcome this step.

3 The Devereux and Sutherland's approach

Should we try to solve a model like this by using the traditional methods for solving DSGE models (that is, by log-linearizing the equations until the first order), it will remain undetermined.

This happens because the equilibrium conditions (22) – (23) and (22*) – (23*) imply that, at the first-order,

$$E_t [R_{h,t+1}] = E_t [R_{f,t+1}] \quad (42)$$

in order to allow for portfolio's optimization. Since there is no way to distinguish across the two internationally traded assets at a first-order approximation, we can not know the equilibrium asset holdings and we therefore do not have the zero-order components of portfolio's shares around which to linearize the model. Devereux and Sutherland (since 2006 onwards) and Tille-Van Wincoop (2007), separately developed a method for solving for the zero-order components of asset holdings in models such that we developed here. They all based their approach on the idea that in a second-order approximation it would be possible to distinguish across home and foreign assets, since at that order second moments would appear. However, how Devereux and Sutherland pointed out, we do not need to solve all the model at the second-order in order to tie down the equilibrium portfolio shares. It is enough to find the second-order approximation of the portfolio's equilibrium conditions and then to add these to the rest of the first-order approximated model.

In the followings we try to summarize their approach.

From banks' first order conditions (22) – (23) and (22*) – (23*) it follows that, at an equilibrium, the following equations must hold:

$$E_t [C_{t+1}^{-\rho} \Omega_{t+1} R_{hk,t+1}] = E_t [C_{t+1}^{-\rho} \Omega_{t+1} R_{fk,t+1}] \quad (58a)$$

$$E_t [C_{t+1}^{*\rho} \Omega_{t+1}^* R_{hk,t+1}] = E_t [C_{t+1}^{*\rho} \Omega_{t+1}^* R_{fk,t+1}] \quad (58a^*)$$

where, according to our model, Ω_{t+1} is the stochastic marginal value of net worth, such that:

$$\Omega_{t+1} = 1 - \theta + \theta \zeta \phi_{t+1}$$

We can rewrite (58a) – (58a*) as:

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3 THE DEVEREUX AND SUTHERLAND'S APPROACH

$$E_t \left[\tilde{C}_{t+1}^{-\rho} R_{hk,t+1} \right] = E_t \left[\tilde{C}_{t+1}^{-\rho} R_{fk,t+1} \right] \quad (58b)$$

$$E_t \left[\tilde{C}_{t+1}^{*- \rho} R_{hk,t+1} \right] = E_t \left[\tilde{C}_{t+1}^{*- \rho} R_{fk,t+1} \right] \quad (58b^*)$$

where

$$\begin{aligned} \tilde{C}_{t+1}^{-\rho} &= C_{t+1}^{-\rho} \Omega_{t+1} \\ \tilde{C}_{t+1}^{*- \rho} &= C_{t+1}^{*- \rho} \Omega_{t+1}^* \end{aligned}$$

Then, by defining $\hat{R}_{x,t+1}$ as:

$$\hat{R}_{x,t+1} = \hat{R}_{hk,t+1} - \hat{R}_{fk,t+1} = 0$$

we can derive the second-order approximation of the home and foreign banks' optimality conditions of the portfolio's problem (58b)-(58b*) :

$$E_t \left[\hat{R}_{x,t+1} + 0.5 \left(\hat{R}_{hk,t+1}^2 - \hat{R}_{fk,t+1}^2 \right) - \rho \hat{C}_{t+1} \hat{R}_{x,t+1} \right] = O(\epsilon^3) \quad (59)$$

$$E_t \left[\hat{R}_{x,t+1} + 0.5 \left(\hat{R}_{hk,t+1}^2 - \hat{R}_{fk,t+1}^2 \right) - \rho \hat{C}_{t+1}^* \hat{R}_{x,t+1} \right] = O(\epsilon^3) \quad (59^*)$$

where $O(\epsilon^3)$ is a residual that collects all the terms of order higher than two. Note that we use the hat to mean the log-deviation of a variable from its steady state. Now, by aggregating the (59) – (59*) we find that, at the equilibrium, the following equations must hold:

$$E_t \left[\rho \left(\hat{C}_{t+1} - \hat{C}_{t+1}^* \right) \left(\hat{R}_{hk,t+1} - \hat{R}_{fk,t+1} \right) \right] = 0 + O(\epsilon^3) \quad (60)$$

$$\begin{aligned} E_t \left[\hat{R}_{hk,t+1} - \hat{R}_{fk,t+1} \right] &= -0.5 E_t \left[\hat{R}_{hk,t+1}^2 - \hat{R}_{fk,t+1}^2 \right] + \\ &+ 0.5 \rho E_t \left[\left(\hat{C}_{t+1} + \hat{C}_{t+1}^* \right) \hat{R}_{x,t+1} \right] \quad (61) \end{aligned}$$

Equation (60) must be satisfied by equilibrium portfolio's holdings while equation (61) shows the corresponding set of equilibrium expected excess of returns. Devereux and Sutherland (since 2006 onwards) show that the (60) provides a sufficient condition to tie down the zero-order component of portfolio's holdings. This happens thanks to two important properties of the approximated model:

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3 THE DEVEREUX AND SUTHERLAND'S APPROACH

1) *Second order accurate solutions for products may be derived by first order accurate solutions for individual variables. Since the only terms that appear at the left hand side of equation (60) are products, in order to evaluate them it is sufficient to derive expressions for the first order accurate behaviour of the “modified” consumption \hat{C}_{t+1} (\hat{C}_{t+1}^*) and for the excess of returns $\hat{R}_{x,t+1}$.*

2) *The zero order component for*

$$S_h, S_f, S_h^*, S_f^*$$

it is the only aspect of the portfolio's decision that affects the first-order accurate behaviour of the “modified” consumption and of the excess of return $\hat{R}_{x,t+1}$.

To sum up, according to the second property we can evaluate the first-order behaviour of:

$$\hat{R}_{x,t+1}, \hat{C}_{t+1}, \hat{C}_{t+1}^*$$

conditional on a given value of:

$$S_h, S_f, S_h^*, S_f^*$$

while according to the first property the last one will also be enough to evaluate the left hand side of equation (60).

Then, according to equation (60) the steady state portfolio holdings that solve the model are the ones that satisfy it.

If the model is sufficiently small, the Devereux and Sutherland's approach allows to derive analytical formulas for the portfolio's holdings. Larger models need instead to be solved through numerical methods.

Notice that it is not necessary to repeat the whole procedure each time in order to solve for the equilibrium portfolio's holdings. As long as the model satisfies standard properties, it is sufficient to derive the state-space form of a slightly modified version of the model, approximated at the first order, and then to apply the Devereux and Sutherland's formula to tie down S_h, S_f, S_h^*, S_f^* .

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3 THE DEVEREUX AND SUTHERLAND'S APPROACH

This “modified model” (See the Appendix C) is done by adding the budget constraints (41) – (41*) to the other non-portfolio equations of the complete model, possibly modified in order to hide the portfolio's holdings.

Then, by extracting the appropriated rows from the state space form of this modified model, that are :

$$\begin{aligned}\hat{R}_{x,t+1} &= R_1 \hat{\xi}_{t+1} + R_2 \epsilon_{t+1} \\ {}_{t+1} - \hat{C}_{t+1}^* &= D_1 \hat{\xi}_{t+1} + D_2 \epsilon_{t+1}\end{aligned}$$

where $\hat{\xi}_{t+1}$ is built such that:

$$\hat{\xi}_{t+1} = S_{h,t} \hat{R}_{x,t+1}$$

and can then be treated as an i.i.d. exogenous variable and where ϵ_{t+1} it is a (4X1) vector containing all the home/foreign exogenous shocks (to technology and to quality of capital).

We end up by applying the formula derived by Devereux and Sutherland, that allows us to find the steady state equilibrium portfolio's holdings:

$$S_h = [R_2 \Sigma D_2' R_1' - D_1 R_2 \Sigma R_2']^{-1} R_2 \Sigma D_2'$$

Lastly, we use this 0-order components of portfolio holdings as a parameter in the model contained in Appendix B.

“Our method can be applied to any standard open economy model with any number of assets, any number of state variables and complete or incomplete markets. We find a general formula for asset holdings which fits naturally into the standard solution approach for DSGE models. In fact, our solution formula can be applied directly using a standard first-order accurate solution that is generally derived in the analysis of DSGE models. It is not necessary to repeat the derivation of our formula for every model. The technique is simple to implement and can be used to derive either analytical results (for sufficiently small models) or numerical results for larger models.”

From M. Devereux and A. Sutherland, “Solving for Country Portfolios in Open Economy Macro Models”, CEPR, University of British Columbia and University of St. Andrews, 2006

4 How the Model works

4.1 Calibration

We would like to show how frictions in the credit markets together with integrated financial portfolios could act to propagate the effects of a disturbance to asset values or to aggregate production. In our model we have four standard preference/technology parameters, that we get from the traditional literature. These are the inverse of risk aversion $\rho = 0.99$, the utility weight of labour $\kappa = 5.584$, the capital share $\alpha = 0.33$ and the depreciation rate $\delta = 0.025$. Three parameters are instead specific of the Gertler and Kiyotaki's model (2010). These are the fraction of assets that banks may divert $\zeta = 0.383$, the transfer to entering bankers $\xi = 0.003$ and the quarterly survival rate of the bankers $\theta = 0.972$. In this case we maintain the original calibration of Gertler and Kiyotaki where the first two parameters are chosen to hit an economy-wide leverage ratio of 4.

We suppose that the capital quality and the technology shocks obey to a first-order autoregressive process, symmetric across countries. Disturbances are uncorrelated across countries (we have four independent sources of disturbances, therefore markets are incomplete).

We also need to assign a form to the physical adjustment cost $f(\frac{I_t}{I_{t-1}})$ in equation (11). We know that it must be such that $f(1) = f'(1) = 0$ and $f''(1) > 0$ and therefore we allow Matlab to find:

$$f\left(\frac{I_t}{I_{t-1}}\right) = cq\left(\frac{I_t}{I_{t-1}}\right)^2 - 2cq\left(\frac{I_t}{I_{t-1}}\right) + cq$$

where we get $cq = 1.728$ from the work by Gertler and Karadi (2009) that also considers similar investment adjustment cost.

Lastly, for what concerns the parameters that drive the endogenous discount factor, we set $\eta = 0.7$ while ω adjusts in order to maintain the steady state value of the discount factor $\bar{\beta} = \omega \bar{C}_A^\eta = 0.99$ (we follow, here, Schmitt-Groh and Uribe (2002) in order to avoid the presence of unit root in the model's first-order approximation).

4.2 The Results

Following Gertler and Kiyotaki (2010), we try to catch the initiating feature of a financial crisis through the deterioration of the value of banks' financial portfolios.

We consider a five percent exogenous decline in the quality of capital with an autoregressive factor of 0.66. This size of the shock is fixed by Gertler and Kiyotaki (2010) as the one able to produce a downturn similar to the one they observed during the US sub-prime mortgages' crisis (2007). We assume that capital quality shocks are symmetric and uncorrelated across countries in order to better understand the transmission mechanism at work across countries. An additional source of risk in the model is due to the presence also of home and foreign technology shocks (again, symmetric and uncorrelated). Our model is, therefore, incomplete: it contains four independent sources of risk, while each country's leveraged banks have to limit their investment opportunities to only two assets, home and foreign equities.

Impulse response functions of the most important economic variables for the home and foreign countries are collected at the end of this section.

The transmission mechanism behind both shocks in each country is the same described by Gertler and Kiyotaki (2010). We find similar results for what concerns the movements in the variables. An exogenous five percent shock to the quality of capital (at home) causes a drop in the market price $Q_{h,t}$ and in the stock of available capital K_t . This decreases the value of the banks' financial portfolio; since banks are leveraged, banks' net worth N_t fall will be worsen by a factor equal to the leverage ratio. The spread between the expected return on home firm and the risk-free rate expands, symptom of financial stress. The fall in the net worth also contributes to tighten bank's borrowing constraints. What follows is a sale of assets that further depress $Q_{h,t}$. The loss of capital also implies a drop in output and consumption. The relative price $\frac{P_{h,t}}{P_t}$ starts to increase, while $\frac{P_{f,t}}{P_t}$ falls. Then, high returns on capital induce an increase in investment and labour. The recovery is slowed down because banks start a de-leveraging process (buying equities and selling deposits) in order to reduce the spread (the difference between the equity returns and the risk-free rate). At a SINGLE country level, our results appear to be very similar to those reached by Gertler and Kiyotaki (2010).

We then need to expand the analysis in order to understand what our model implies also for what concerns the transmission mechanism ACROSS countries.

In the first four tables of graphs at the end of this section we tried to compare the impulse response functions to the main economic variables provided by three different versions of the same model. Notice that we always assume

perfectly integrated good markets ($\gamma = 0.5$). In the Model A, financial markets are closed. In the Model B, financial markets are open but steady state portfolio shares are exogenously given: in the first case (BK), we set $S_h = K$ - and, being home and foreign countries perfectly symmetric, also equal to S_f^* (meaning that each country's equilibrium portfolio has to be invested in that country's equity: financial markets are closed at steady state), while in the second case (BH) we set $S_h = S_f = K/2$ (full international diversification). Notice that this last portfolio's allocation is the same that we would obtain endogenously under complete markets.

What we observe is a similar behavior in the three versions of the model in case of shock to technology. In response to a 1 percent home technology shock, home consumption drops of -0.1 percent in all the models. Foreign consumption also drops, of almost the same amount. Despite the high and positive correlation across home and foreign consumption (due to the perfectly integrated good markets), the home and foreign outputs are instead negatively related. Therefore, when home output drops by -0.7 , foreign output increases (strongly in the Model B, where it reaches the maximum level $+0.7$ after ten periods). The excess of return between home equity and the risk-free rate appears to be similar in all the three versions of the model. It increases by $+0.6$ percent in both versions of Model B and by $+0.7$ percent in Model A, where financial stress is greater since portfolio's diversification it is not allowed. Home and foreign prices of capital move in the same direction.

Therefore, according to what we observed above, the steady state composition of portfolio holdings has no effect on the first order behaviour of the model when we limit our analysis to a technology shock: the impulse response functions from models BH and BK are exactly the same. How Cole and Obstfeld (1991) and Halova (2011) (among the others) pointed out, when the elasticity of substitution between home and foreign goods equals one, optimal portfolios become indeterminate. This happens because in this case the total term of trade moves exactly one to one with the relative productivity, making the risks perfectly shared across countries even with no equity trade. In this case portfolio diversification provides no benefit.

When we consider instead a 5 percent shock to the quality of capital, Models A and BK behave similarly, with home and foreign consumption levels positively related (both decrease in a similar manner) and home and foreign outputs negatively related (at least initially). After period 15, however, in Model BK foreign output becomes negative reaching a minimum of -1 percent between periods 25 – 30. Therefore, when we allow for finan-

cial market integration (outside steady state), the anomaly of cross-countries negative output correlation disappears. Again, home and foreign prices of capital move in the same direction.

By exogenously imposing full financial market integration (Model BH) things, however, change. In general, we observe a remarkable increase in the volatility of the impulse response functions in the cases where the direction of the responses is the same as those in Models A and BK (for what concerns, for example, the behavior of capital, investment, net worth and price of capital). However, in Model BH a shock to the home quality of capital causes a period 1 inverted correlation in home and foreign consumption levels (now highly negatively related) that is added to the high and negative cross-countries correlation in output levels. Moreover, under full portfolio integration the risk-free rate jumps by 25 percent at period 1. A so deep response in the risk-free rate also causes a reversed spread in the excess of returns (between home and foreign equity returns and the home risk-free rate): it drops by 20 percent (on home equity) and by 25 percent (on foreign equity).

This period 1 jump in the home risk-free and consumption level is an undoubtedly puzzling results of our full integrated model. Under incomplete markets we have no benefit from full international portfolio integration. By exogenously imposing it, we only introduce more volatility and an inverse correlation across home and foreign endogenous stochastic discount factors

$$\beta_t = \omega \bar{C}_t^{-\eta}, \beta_t^* = \omega \bar{C}_t^{*-\eta}$$

that depend on the consumption levels.

Trying to improve our understanding of what mechanisms are at work under financial market integration, in the last part of our analysis we compare the model BH with a model where portfolio shares are endogenous. We obtain endogenous portfolio shares by using the method developed by Devereux and Sutherland (since 2006 onwards). According to our results, home banks hold the 70.5 percent of home equities at steady state (and, symmetrically, foreign banks hold the 70.5 percent of foreign equities).

We therefore find home bias in financial portfolios. Home banks prefer to hold a greater share of home equities instead of holding a perfectly diversified portfolio. French and Poterba (1991) and Tesar and Werner (1995) first discovered this seemingly puzzling phenomenon (according to what portfolio diversification theory would suggest).

Many authors explain the financial portfolio's home bias with the presence of home bias in consumptions or with the presence of different trading

costs for home or foreign equities. According to what they found, the reason of the bias appears to be directly related with the presence of asymmetries across countries. Home and foreign countries are, however, perfectly symmetric in our model. Good markets are perfectly integrated, and elasticity of substitution across home and foreign goods is equal to one. It is therefore the incompleteness in the markets the only possible explanation of the observed home bias in financial portfolios, together with the presence of leverage constraints for the banking system.

In the last four tables of graphs at the end of this section we compare the model with full portfolio integration (called “F” as “full”) and the model with endogenous portfolio shares (called “NF” as “no full” integration).

Under a 1 percent home technology shock we see, again, almost perfectly and positively related home and foreign consumption levels, while home and abroad outputs move in opposite directions. At home the output level immediately drops and then moves gradually to the equilibrium, while in the foreign country it starts an upward gradual movement. Again, the two models behave in an identical manner during a 1 percent home technology shock, for the reason we mentioned before.⁵

Instead, during a 5 percent shock to the quality of capital the model with endogenous portfolio shares deeply reduces the volatility of the responses, especially for what concerns risk-free rates and excess of returns, despite all variables move in the same direction regardless of the presence or the absence of full portfolio integration.

Under incomplete markets, indeed, even perfectly integrated financial markets do not allow the perfect transmission of one country’s idiosyncratic risk to the other country. Therefore, a perfectly equity sharing across home and foreign countries can become sub-optimal. Moreover, our banks are leveraged: after a serious financial shock, the leveraged banks will be obliged to sell equities. If their portfolio both contains home and foreign equities, what will they choose to sell the most, home or foreign equities? They will sell the foreign equity in order to maintain the price of their own asset as higher as possible, I would say. The reason is that home profits depend on the home price of capital, and home profits enter in the home household’s (depositor/banker) budget constraint. Apparently, this seems to be the best explanation of the home bias result we obtained here. People tend to invest more in their own country in order to protect that country’s production sys-

⁵See Cole and Obstfeld (1991) and Halova (2011) for a deeper analysis.

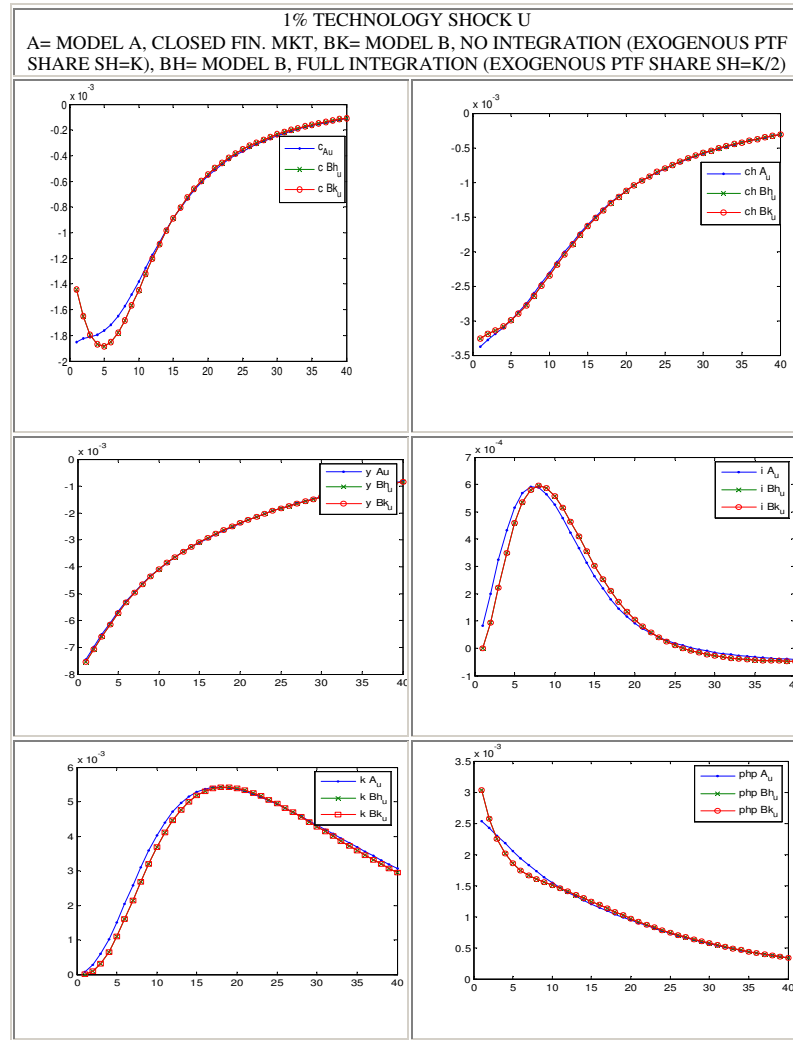
tem, by obtaining in this way more profits (that only can be gained outside steady state). By analyzing the impulse response functions after a 5 percent shock to the home quality of capital, we see that from periods 2 onwards the foreign price of capital Q_f will fall as much as there are foreign equities in the home equilibrium portfolio. This drop in Q_f also causes a steeper decrease in K^* and in Y^* from period 2 onwards.

In the above analysis we tried to better understand what our model implies for what concerns the transmission mechanism across countries. The results we obtain seem, in some cases, counterintuitive. According to empirical evidence, we expected to find, for example, a positive correlation across home and foreign outputs and home and foreign consumption levels. Our results are, however, consistent with what found by a wide literature on two country real business cycle model. The most influent example of such genre is probably represented by the work by Backus, Kehoe and Kydland (1992) that documented a number of discrepancies between the theory and the empirical observations. The most important discrepancy is exactly the negative correlations across home and foreign outputs implied by standard real business cycle models. Again, in the Model A with financial autarky we expected to find a very low (or absent) correlation across net worths. Instead, what we found is that even in the Model A the impact of a serious crisis on a country deeply strikes also the other country financial markets. It is also difficult to explain why the risk-free rate and the consumption levels behave in such a puzzling way during a capital quality shock under full or partial financial integration (Model BH/F, Model NF).

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4.2 The Results

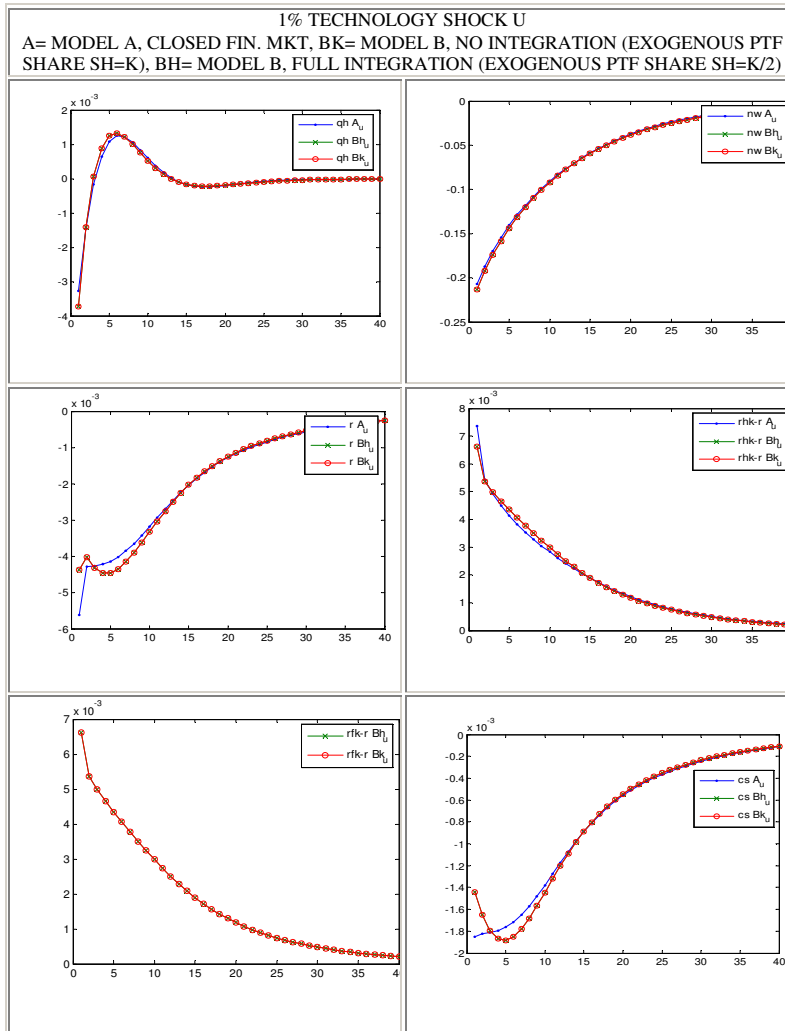
4 HOW THE MODEL WORKS



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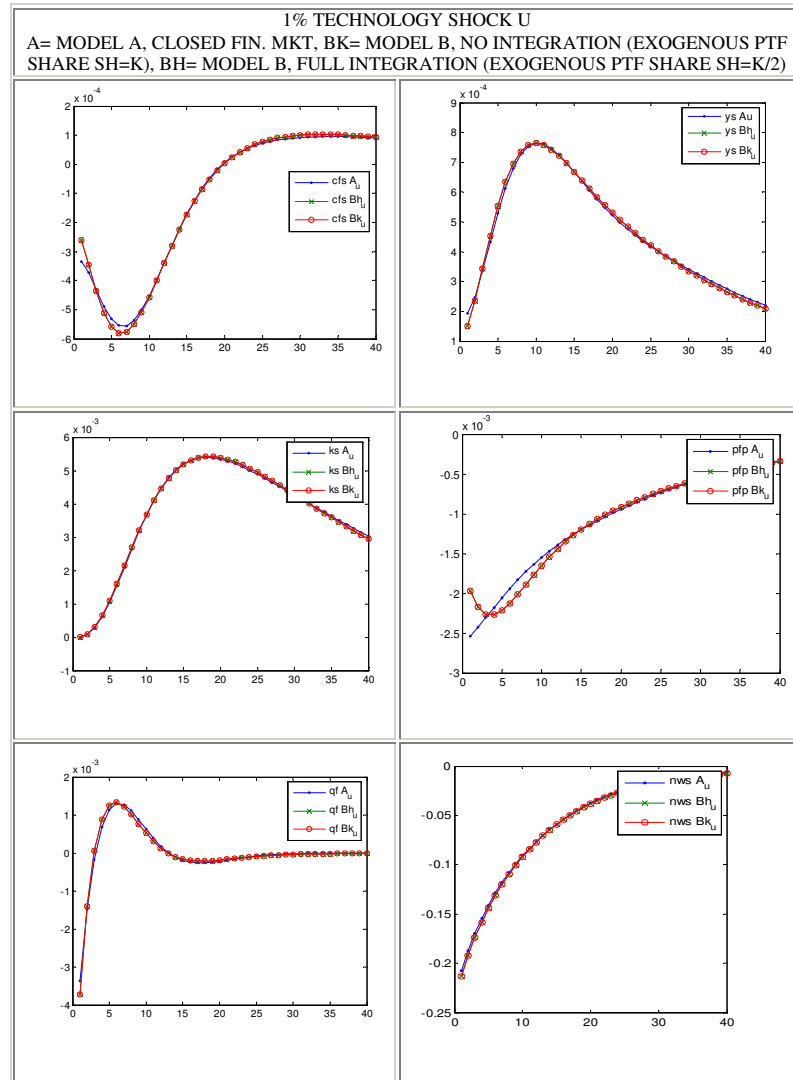
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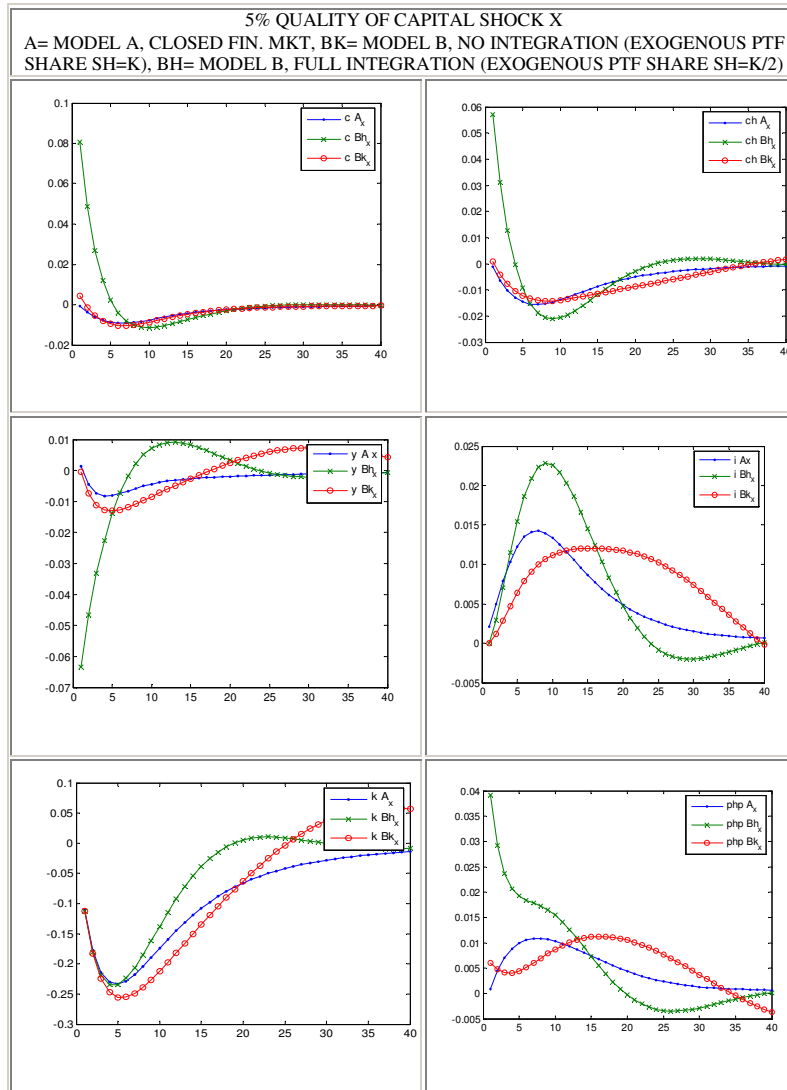
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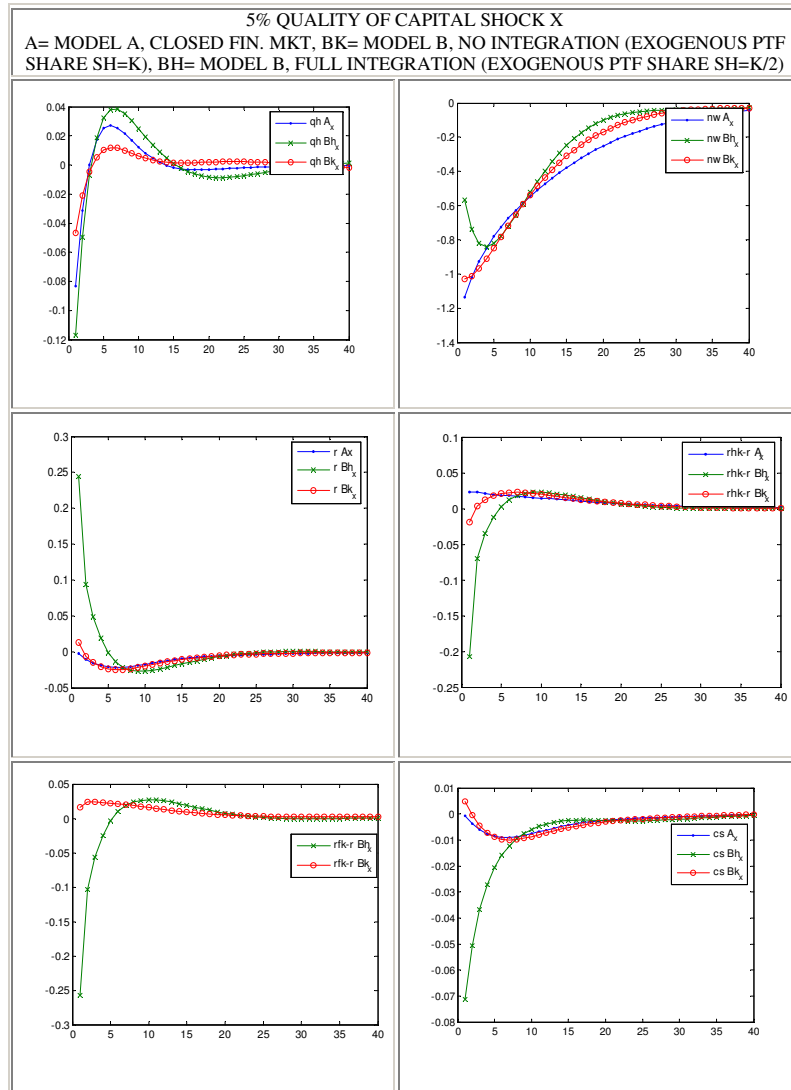
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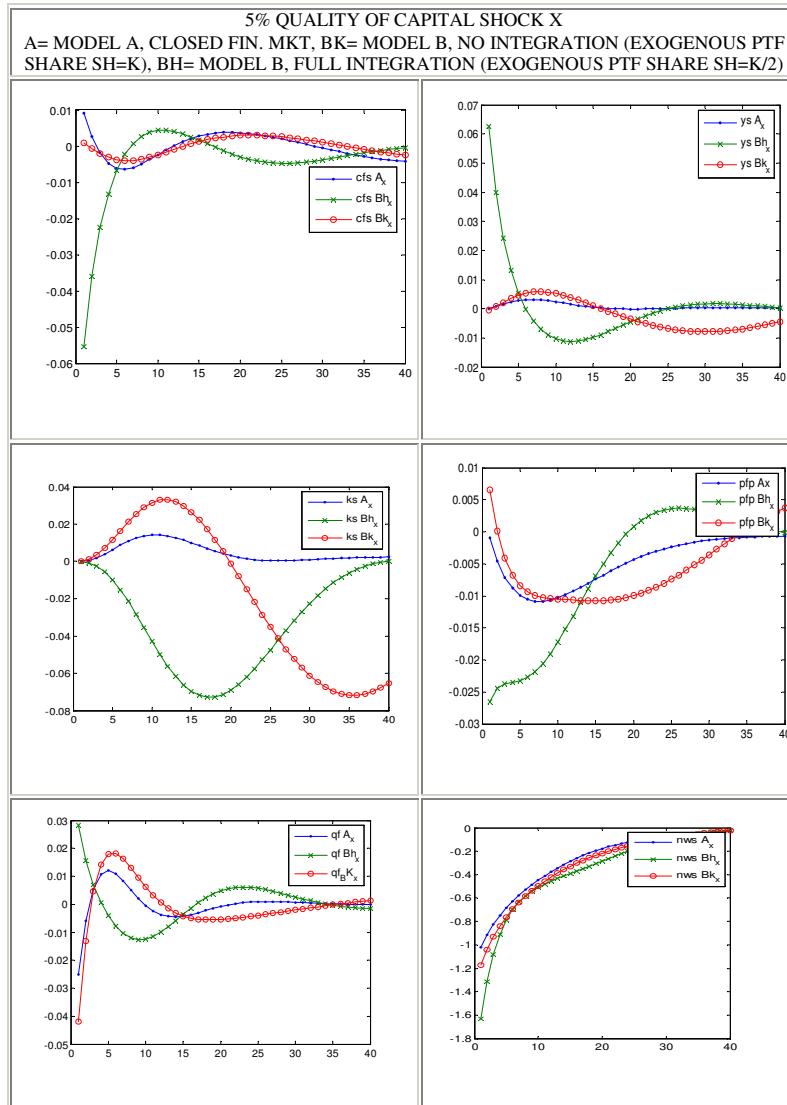
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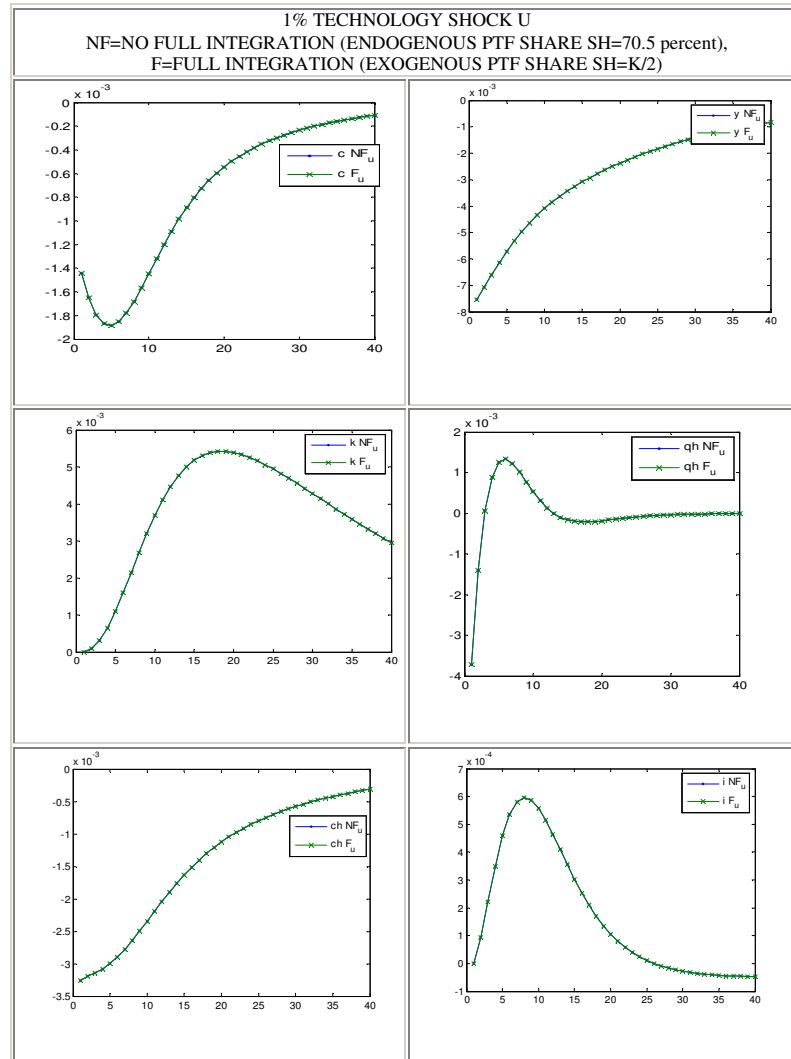
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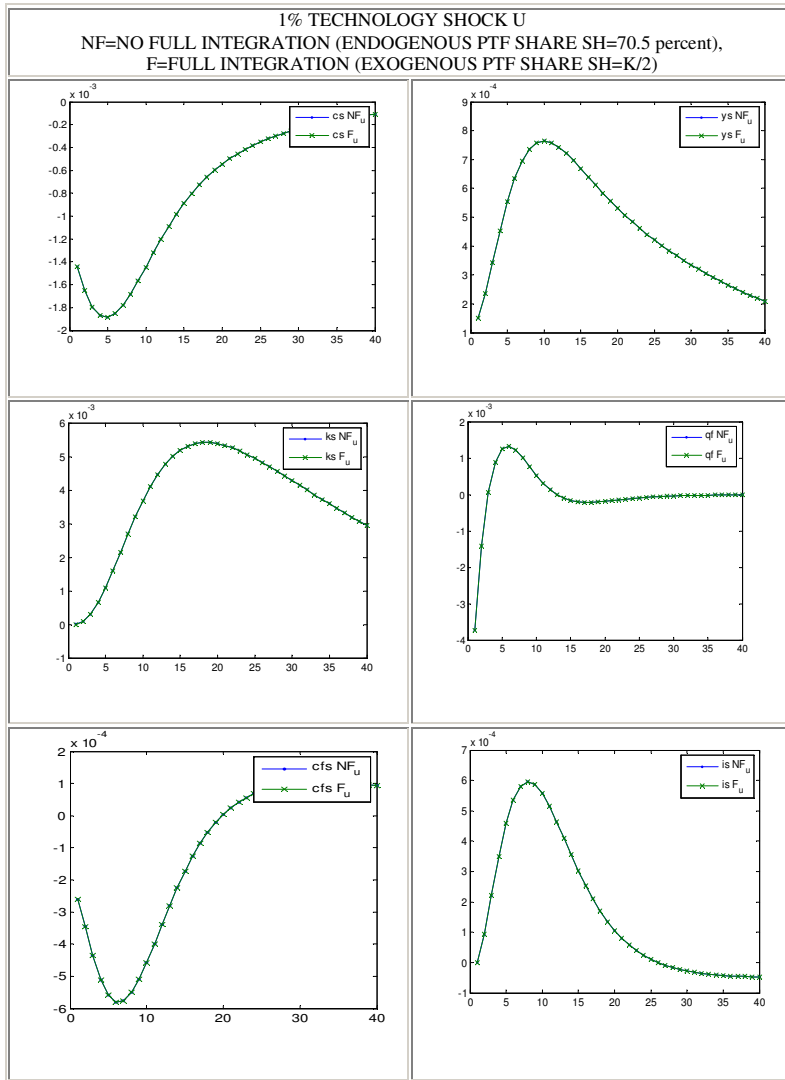
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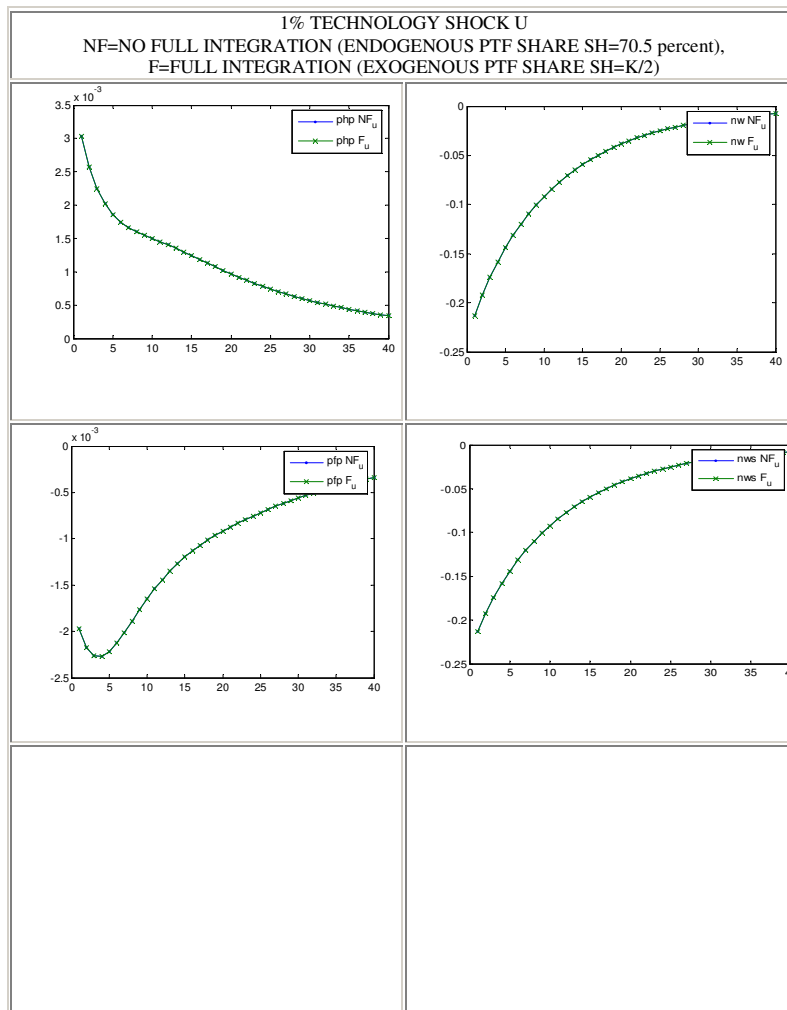
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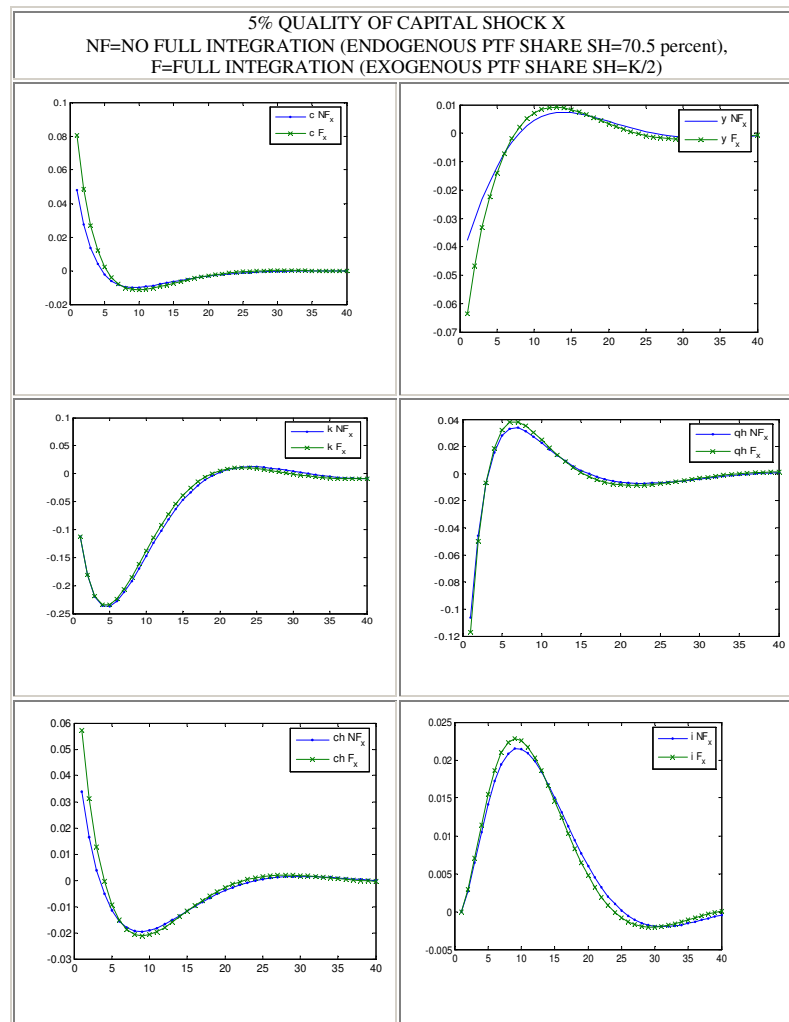
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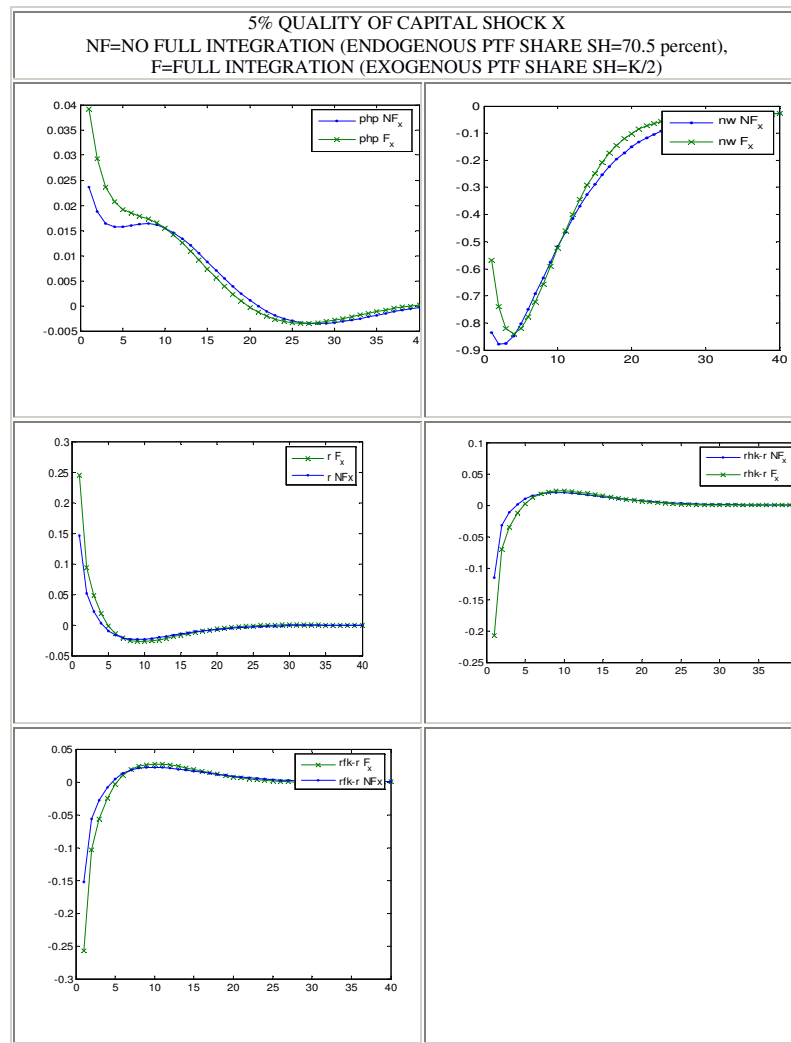
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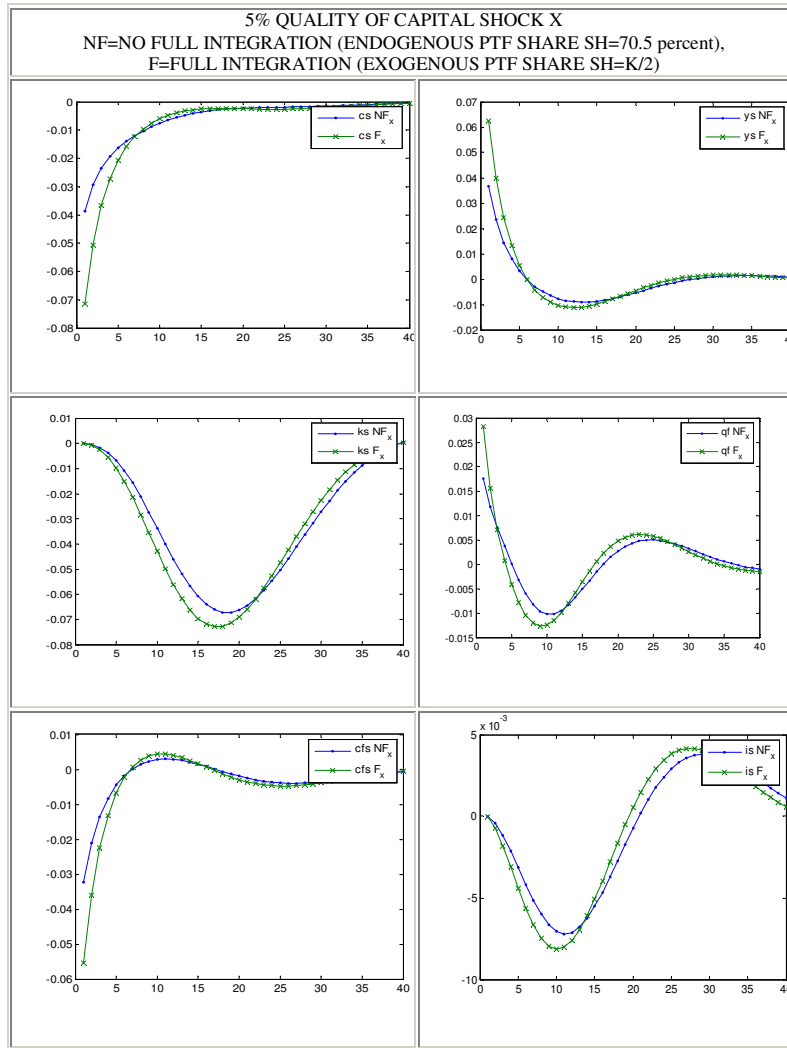
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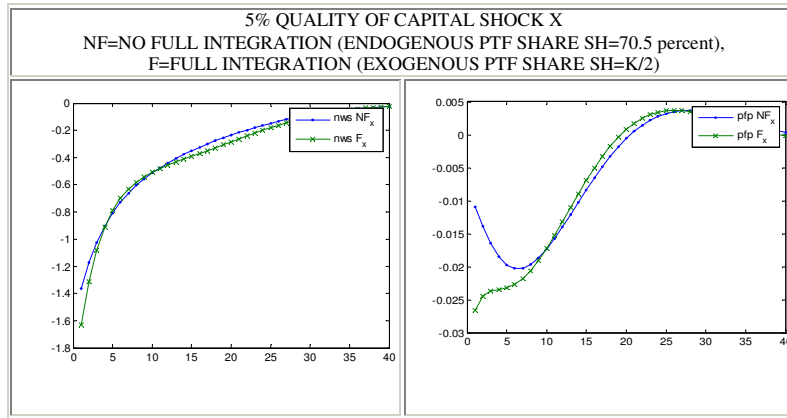
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4.2 The Results

4 HOW THE MODEL WORKS



4.3 Conclusions

What we have done here is to create a simple two country framework with credit frictions and incomplete markets to analyze the transmission mechanism of some shocks from a country to the rest of the world. The main references for our model are the works by Gertler and Kyiotaki (2010), Devereux and Yetman (2010) and Devereux and Sutherland (since 2006 onwards). In order to better analyze how each shock spreads, we start with a simple version of the model, where good markets are perfectly integrated but financial markets are closed. Then, we complicate the model by allowing for integrated financial markets but keeping portfolio shares exogenous. We end up with the complete model that also allow for portfolio choice by implementing a method developed by Devereux and Sutherland (since 2006 onwards).

This two country world framework can also be used for monetary policy issues.

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5 Appendix A

5.1 The model with integrated good markets but closed financial markets

$A_t, A_t^*, \Psi_t, \Psi_t^*$ follow an exogenous stochastic process. We need to find the real prices:

$$\begin{aligned} & Q_{h,t}, R_{ht}, R_{t+1}, \frac{P_{h,t}}{P_t} \\ & Q_{f,t}, R_{ft}, R_{t+1}^*, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities:

$$\begin{aligned} & Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, S_{h,t}, Yu_t \\ & Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\ & D_t^*, N_t^*, S_{f,t}^*, Yu_t^* \end{aligned}$$

the shadow prices:

$$\begin{aligned} & v_t, v_{h,t}, \lambda_t, \Phi_t \\ & v_t^*, v_{f,t}^*, \lambda_t^*, \Phi_t^* \end{aligned}$$

determined as a function of the state variables:

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$$\begin{aligned} &K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Y u_{t-1} \\ &K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Y u_{t-1}^* \end{aligned}$$

by the sequence of the following 40 equations (we only show home equations):

From the household's problem:

$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left(\frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

From the final good production firms' problem:

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

$$K_{t+1} = \Psi_{t+1} [I_t + (1 - \delta) K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta) Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left(\frac{L_t}{K_t} \right)^\alpha \quad (10)$$

From the banks' problem:

$$\left(\frac{v_{h,t}}{Q_{h,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$Q_{h,t} S_{h,t} \left[\zeta - \left(\frac{v_{h,t}}{Q_{h,t}} - v_t \right) \right] - v_t n_t = 0 \quad (24)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta) Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

$$N_{t+1} = (\theta + \xi) [R_{hk,t+1} Q_{h,t} S_{h,t}] - \theta R_{t+1} D_t \quad (34)$$

$$D_t = Q_{h,t} S_{h,t} - N_t \quad (37)$$

$$S_{h,t} = [I_t + (1 - \delta) K_t] \quad (40)$$

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where the last one is the supply in the home financial market and where Φ_t is given by:

$$\phi_t = \Phi_t = \frac{v_t}{\zeta - \left(\frac{v_{h,t}}{Q_{h,t} - v_t} \right)}$$

From the capital good firms' problem:

$$Q_{h,t-1} = 1 + f(Yu_{t-1}) + f'(Yu_{t-1})Yu_{t-1} - E_{t-1}\Lambda_{t-1,t}f'(Yu_t)Yu_t^2 \quad (11)$$

where we define the investment ratio between $t, t - 1$ as:

$$Yu_t = \frac{I_t}{I_{t-1}}$$

Equilibrium condition in the (integrated) good markets:

$$Y_t = C_{h,t} + C_{h,t}^* + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \frac{P_t}{P_{h,t}} \quad (38)$$

Equilibrium condition in the labour market:

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

Lastly, to complete the model we have to add the home household budget constraint:

$$\frac{P_{h,t}}{P_t} Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \quad (41)$$

Notice that in the equations above the discount factor β is endogenous:

$$\beta_t = \frac{\psi_{t+1}}{\psi_t} = \omega C_{A,t}^{-\eta}$$

with initial conditions $\psi_0 = 1$ and $0 \leq \eta < \rho$. Since C_A is the aggregate consumption of home households, it is not internalized in the households' decision ($0 < \omega C_A^{-\eta} < 1$).

We set $\eta = 0.7$ and ω such that $\beta = \omega \bar{C}_A^{-\eta} = 0.99$ at steady state. We followed here Schmitt-Groh and Uribe (2002) in order to avoid the unit root in the first order approximated model (since we have incomplete markets).

6 Appendix B

6.1 The Model with integrated good and financial markets but constant (and arbitrary) portfolio shares

We assume steady state trade balances equal to 0. Therefore it follows that:

$$S_h = S_f^*$$

We then set $S_h = S_f^*$ equal to an arbitrary constant, for example 0 (that means, home households hold only foreign stocks and vice versa) or K (home household only invest in home stock market), or K/2 (perfect portfolio's diversification).

We also show that, by applying the Devereux and Sutherland (since 2006 onwards) method, it is possible to derive the endogenous 0-order component (steady state) of the portfolio shares. We can also set $S_h = S_f^* = 70$, 5 percent.

Then,

$$A_t, A_t^*, \Psi_t, \Psi_t^*$$

follow an exogenous stochastic process. We need to find the real prices:

$$\begin{aligned} & Q_{h,t}, R_{h,t}, R_{t+1}, \frac{P_{h,t}}{P_t} \\ & Q_{f,t}, R_{f,t}, R_{t+1}^*, \frac{P_{f,t}}{P_t} \end{aligned}$$

the quantities

$$\begin{aligned} & Y_t, C_t, C_{h,t}, C_{f,t}, L_t, I_t, K_{t+1}, Z_t, D_t, N_t, Yu_t \\ & Y_t^*, C_t^*, C_{h,t}^*, C_{f,t}^*, L_t^*, I_t^*, K_{t+1}^*, Z_t^*, \\ & D_t^*, N_t^*, Yu_t^* \end{aligned}$$

the shadow prices

$$\begin{aligned} & v_t, v_{h,t}, v_{f,t}, \lambda_t, \Phi_t \\ & v_t^*, v_{h,t}^*, v_{f,t}^*, \lambda_t^*, \Phi_t^* \end{aligned}$$

determined as a function of the state variables

$$\begin{aligned} & K_t, C_{t-1}, I_{t-1}, A_t, \Psi_t, Q_{h,t-1}, Yu_{t-1} \\ & K_t^*, C_{t-1}^*, I_{t-1}^*, A_t^*, \Psi_t^*, Q_{f,t-1}, Yu_{t-1}^* \end{aligned}$$

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by the sequence of the following 40 equations (we just show the the Home equations):

From the household's problem:

$$C_t \equiv C_{ht}^\gamma C_{ft}^{(1-\gamma)} \quad 0 < \gamma < 1 \quad (1)$$

$$C_{h,t} = \gamma C_t \frac{P_t}{P_{h,t}} \quad (17)$$

$$1 = E_t \left\{ \omega C_{A,t-1}^{-\eta} R_t \left(\frac{C_t}{C_{t-1}} \right)^{-\rho} \right\} \quad (15)$$

From the final good production firms' problem:

$$Y_t = A_t K_t^{(1-\alpha)} L_t^\alpha \quad (6)$$

$$K_{t+1} = \Psi_{t+1} [I_t + (1 - \delta)K_t] \quad (7)$$

$$R_{hk,t} = \Psi_t \frac{(1-\alpha) \frac{P_{h,t}}{P_t} \frac{Y_t}{K_t} + (1-\delta)Q_{h,t}}{Q_{h,t-1}} \quad (9)$$

$$Z_t = (1 - \alpha) \frac{P_{h,t}}{P_t} A_t \left(\frac{L_t}{K_t} \right)^\alpha \quad (10)$$

From the banks' problem:

$$\left(\frac{v_{h,t}}{Q_{h,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (22)$$

$$\left(\frac{v_{f,t}}{Q_{f,t}} - v_t \right) (1 + \lambda_t) = \zeta \lambda_t \quad (23)$$

$$Q_{h,t} S_h + Q_{f,t} S_f = \Phi_t N_t \quad (24)$$

$$v_{h,t} = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} [Z_{t+1} + (1 - \delta)Q_{h,t+1}] (1 - \theta + \theta \zeta \phi_{t+1}) \quad (30)$$

$$v_t = E_t \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} R_{t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \quad (32)$$

where the definition of Φ_t is given by:

$$\Phi_t = \frac{v_t}{\zeta - \left(\frac{v_{h,t}}{Q_{h,t} - v_t} \right)}$$

$$N_{t+1} = (\theta + \xi) [R_{hk,t+1} Q_{h,t} S_h + R_{fk,t+1} Q_{f,t} S_f] - \theta R_{t+1} D_t \quad (34)$$

$$D_t = Q_{h,t}S_h + Q_{f,t}S_f - N_t \quad (37)$$

From the capital good firms' problem:

$$Q_{h,t-1} = 1 + f(Yu_{t-1}) + f'(Yu_{t-1})Yu_{t-1} - E_{t-1}\Lambda_{t-1,t}f'(Yu_t)Yu_t^2 \quad (11)$$

where we define the investment ratio between $t, t - 1$ as:

$$Yu_t = \frac{I_t}{I_{t-1}}$$

Equilibrium condition in the (integrated) good markets:

$$Y_t = C_{h,t} + C_{h,t}^* + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \frac{P_t}{P_{h,t}} \quad (38)$$

Equilibrium condition in the labour market:

$$\alpha \frac{P_{h,t}}{P_t} \frac{Y_t}{L_t} = \kappa L_t C_t^\rho \quad (39)$$

Lastly, to complete the model we have to add the home household budget constraint:

$$\begin{aligned} \frac{P_{h,t}}{P_t} Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - \\ - Q_{h,t}S_h^* - R_{fk,t}Q_{f,t-1}S_f + R_{hk,t}Q_{h,t-1}S_h^* + Q_{f,t}S_f \end{aligned} \quad (41)$$

7 Appendix C

7.1 The Modified Baseline Model for the Devereux-Sutherland approach

In what follows we described the non-portfolio part of our model, the so called "modified" final system. We followed here step by step the method described by Devereux and Sutherland (since 2006 onwards).

In order to obtain it, we need to modify some equations in order to drop the portfolio's shares.

We rewrite equation (41) in this way:

$$\begin{aligned} C_t = \frac{P_{h,t}}{P_t} Y_t + Q_{h,t} (I_t + (1 - \delta) K_t) - \Phi_t N_t - R_{h,t} Q_{h,t-1} \frac{K_t}{\psi_t} + \\ - \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + \\ + R_{f,t} \Phi_{t-1} N_{t-1} + \xi_{t-1} Q_{h,t-1} \end{aligned}$$

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where we set:

$$\xi_{t-1} = R_{x,t} S_{h,t-1}$$

and:

$$[R_{h,t} - R_{f,t}] = R_{x,t}$$

Since at steady state we know that:

$$[R_{h,t} - R_{f,t}] = 0$$

we are allowed to consider ξ_{t-1} as an exogenous i.i.d. variable with 0 mean. At a first step, we solve the model by considering it as exogenous.

We modified in the same way all the remaining equations that still contain the portfolio's variables. Equation (37) becomes:

$$D_t = \Phi_t N_t - N_t$$

where we used (33):

$$Q_{h,t-1} S_{h,t-1} + Q_{f,t-1} S_{f,t-1} = \Phi_{t-1} N_{t-1}$$

Equation (34) can be modified as:

$$N_{t+1} = (\theta + \xi) \xi_t Q_{h,t} + R_{f,t+1} \Phi_t N_t - \theta R_{t+1} D_t$$

Lastly, equation (24) becomes:

$$\begin{aligned} v_t n_t &= \Phi_t N_t \left[\zeta + v_t - \frac{v_{f,t}}{Q_{f,t}} \right] - \\ &- \omega C_{A,t}^{-\eta} \Lambda_{t,t+1} (1 - \theta + \theta \zeta \phi_{t+1}) \xi_t Q_{h,t} \end{aligned}$$

We derive the state space form of this modified model (log-linearized at the first order) by setting \hat{x}_t as an exogenous variable.

Then, by extracting the equations for:

$$\hat{R}_{x,t+1}, \left(\hat{C}_{t+1} - \hat{C}_{t+1}^* \right)$$

from the state space system, we derive

$$S_h = [R_2 \Sigma D_2' R_1' - D_1 R_2 \Sigma R_2']^{-1} R_2 \Sigma D_2'$$

We find that:

$$S_h = S_f^* = 0.75K$$

that means that at the steady state we have home equity bias.