UNIVERSITÉ PARIS 13
Paris

et/e

UNIVERSITÀ LUISS GUIDO CARLI
Roma

TESI

pour le grade de **DOCTEUR** de l'**Université de Paris 13**
Discipline :**Mathématiques**

per il titolo di **DOTTORE DI RICERCA** dell' **Università LUISS GUIDO CARLI**
Indirizzo : **Metodi Matematici per l'Economia, l'Azienda, la Finanza e le Assicurazioni.**

par

Cristina DI GIROLAMI

Infinite dimensional stochastic calculus via regularization with financial motivations

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Francesco	RUSSO	Université de Paris 13	Direttore di Tesi
Fausto	G0ZZI	LUISS Guido Carli	Co-Direttore di Tesi

December 21, 2009

Title : Infinite dimensional calculus via regularization with financial motivations.

Abstract : This paper develops some aspects of stochastic calculus via regularization to Banach valued processes. An original concept of χ-quadratic variation is introduced, where χ is a subspace of the dual of a tensor product $B \otimes B$ where B is the value space of the process. Particular interest is devoted to the case when B is the space of real continuous functions defined on $[-\tau, 0]$, $\tau > 0$. Itô formulae and stability of finite χ-quadratic variation processes are established. Attention is devoted to a finite real quadratic variation (for instance Dirichlet, weak Dirichlet) process X. The $C([-\tau, 0])$-valued process $Y(\cdot)$ defined by $Y_t(y) = Y_{t+y}$ where $y \in [-\tau, 0]$ is called *window* process. Let $T > 0$. If Y is a finite quadratic variation process such that $[Y]_t = t$ and $h = F(Y_T(\cdot))$ where F is a $C^2(H)$ Fréchet function with $H = L^2([-T, 0])$, it is possible to represent h as a sum of a real number H_0 plus a forward integral of type $\int_0^T \xi d^- Y$ where ξ will be explicitly given. This decomposition generalizes the Clark-Ocone formula which is true when Y is the standard Brownian motion W. The main motivation comes hedging theory of path dependent options without semimartingales in mathematical finance.