



**LUISS GUIDO CARLI UNIVERSITY**

# **Two Problems in Control Theory and Applications to Economics**

by

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# Abstract

The thesis is composed by two different parts, which are not related each other.

The first part is devoted to study a class of optimal control problems, where the state equation is an ordinary differential equation with delay in the control variable. This class of problems arises in economic applications, in particular in optimal advertising problems. The control problem is embedded in a suitable Hilbert space and the associated Hamilton-Jacobi-Bellman (HJB) equation considered in this space. It is proved that the value function is continuous with respect to a weak norm and that it solves in the viscosity sense the associated HJB equation. The main result is the proof of a directional  $C^1$ -regularity result for the value function. This result represents the starting point to define a feedback map in classical sense going towards a verification theorem yielding optimal feedback controls for the problem.

In the second part of the thesis, the techniques of the Malliavin Calculus are applied to a stochastic differential equation whose coefficients depend on a control process, in particular in the special case of Markovian controls. It is calculated the stochastic derivative of the stochastic differential equation and it is proved that the Malliavin matrix is strictly positive, assuring the results of existence and regularity of densities for the controlled process.