

# An analysis of credit risk financial indicators

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The recent financial crisis has highlighted the importance of one of the main components of the financial risk: the credit risk. Of particular interest in finance has become the modeling of credit risk for measuring portfolio risk and for pricing securities exposed to credit risk, as defaultable bonds and credit derivatives.

In this thesis we focus on modeling the aggregated risk of a portfolio, and on studying the relationship between default risk and yield rates for defaultable bonds. In particular we analyze financial indicators of the default risk of a portfolio of credit risky assets and of a bond.

The first chapter aims at contributing to the literature on the pricing of portfolios of credit derivatives (such as CDOs or basket CDS) where the goal is to compute the joint probability of default of a portfolio of risky assets. In this regard, the risk of default of each asset in the portfolio depends on mostly two sources of randomness: an individual risk factor and a common market factor. The latter represents the uncertainty affecting all assets simultaneously. Our scope is to model the aggregate portfolio risk. Such main theme is fundamental in finance both for the valuation of many credit derivatives and for extracting information from market prices that can be relevant from a macro-prudential point of view (such as estimating joint probabilities of default or probabilities of default conditional on other assets being in default). The recent international financial crisis has just highlighted the missing of correct models for valuating credit derivatives as CDO. From the theoretical point of view we develop a dynamic multivariate default model. A recent paper of Mai and Scherer (2008) uses a stochastic time change to introduce dependence in a portfolio of credit risky assets. In that paper the default times are modeled as random variables with possibly different marginal distribution. By restricting the time change to suitable Lévy subordinators the authors can separate the dependence structure and the marginal default probabilities. Using a so-called time normalization they compute the survival copula of all default times. In order to compute the portfolio loss distribution and to apply their model to the pricing of CDO tranches, an homogeneous portfolio is assumed, in which all default times share the same marginal distribution. Our model develops the ideas of Mai and Scherer (2008) by introducing the case of different marginal distributions for the different assets in the portfolio; we aim at introduc-

ing heterogeneity in the model by allowing for an heterogeneous portfolio, as in the implied copula model of Hull and White (2009). In particular we define and model a cumulative dynamic hazard process as a Lévy subordinator, which allows for jumps and induces positive probabilities of joint defaults, and we model the dependence structure by the implied survival copula, which is related to the choice of the subordinator. In our model we allow the main asset classes in the portfolio to have different cumulative default probabilities and corresponding different cumulative hazard processes. We find an analytical closed formula for the distribution of the portfolio-loss process under this heterogeneous assumption, and we prove an approximation formula for the loss distribution that is useful for empirical applications. Once we have specified a suitable Lévy subordinator, our model can be calibrated to portfolio-CDS spreads and CDO tranche spreads, properly choosing the model parameters that determine the dependence structure. Once we have developed the model for the heterogeneous case we may use it to determine the possible testable implications that our theoretical model for the dependence of the joint defaults has for the characteristics of the CDS prices. From an empirical point of view we calibrate the parameters of our model to the tranches of the iTraxx Europe, which is a basket of 125 CDS on the European firms. Once we have estimated the multivariate default distribution of the companies included in the iTraxx we can follow Segoviano and Goodhart (2010) and analyze the distress dependence in the portfolio computing indicators of systemic risk by estimating a set of stability measures that incorporate changes in distress dependence that are consistent with the economic cycle. Examples of these stability measures are the Stability Index (that reflects the expected number of firms becoming distressed given that at least one firm has become distressed), the Distress Dependence Matrix (in which we estimate the set of pairwise conditional probabilities of distress providing some insights into inter-linkages and likelihood of contagion between the firms), and the Probability of Cascade Effects (that characterizes the likelihood that one or more institutions becomes distressed given that a specific firm becomes distressed). These stability measures can be used to verify which firms are more systemically relevant for the index as a whole.

In the second chapter we study the relationship between the risk of default and the yield-to-maturity of a bond. Credit Default Swap (CDS) spreads and bond spreads (i.e. the spreads between the bond yield rate and the risk free rate) have become commonly used as default risk indicators for risk analysis (see for example Bank of England, 2009, and Fitch, 2010a, 2010b). However, both CDS and bond spreads depend not only on variables directly linked to the risk of default but also on the specific structure of the contract. In particular, in this chapter we show that bond spreads can be misleading if used to infer the default probability of the issuers, and consequently the yield-to-maturity must

be cautiously interpreted as an indicator of the bond default risk. In fact, the yield rate, for given default probabilities and recovery rates, can considerably vary as a function of the residual life and the coupon value of the bond. In particular, when there is default risk, bonds with high coupons are more likely to have high yield rates too (and viceversa). The intuition behind this result is that greater coupons are associated with less than proportional price increases, because there is a probability, linked to the default likelihood, that the coupons are not actually payed-off. Bond prices which are relatively low with respect to the nominal payment flows (on which the yield is computed) determine nominal yields which are relatively higher. This implies that bonds with higher default risk can have lower yields (and viceversa), just as a consequence of their coupon structures. Also the slope of the yield curve must be cautiously interpreted as in general it does not convey enough information to establish if the default risk is higher in one period than in another period. We show that a downward sloping yield curve does not necessarily imply that the default probability on shorter maturities is higher than on longer maturities. This result arises from the fact that also the yield curve slope is linked to the coupon rate: taking fixed the other variables (in particular the default probability and the recovery rate) bonds with low coupons determine decreasing yield curves, while bonds with high coupons imply increasing yield curves. The intuition behind this result is that higher coupons determine losses relatively higher for bonds with longer maturity in case of default, and consequently higher yields for these bonds. On the contrary, when the coupons are low, the nominal losses in case of default are similar both for short term and long term bonds; it follows that the prices of longer maturity bonds, for which the losses are likely in far away time horizons, are relatively higher, and the corresponding yields are lower. Most of the literature available on the valuation of fixed income securities (see for example Fabozzi, 2003, 2007), is focused on the interest rate risk (i.e. the bond's price sensitivity to the change in interest rates) and the concept of duration is used to describe the relationship between the bond maturity and coupon rate, and the bond price sensitivity (a longer maturity and a lower coupon rate are linked to a greater price sensitivity to interest rate changes); in this context a higher bond yield is considered as a premium for the higher interest rate risk. In this chapter we study similar financial indicators that could be used in presence of credit (or default) risk to properly evaluate the relationship between the defaultable bond yield and its default probability.