Overconfidence, Subjective Perception and Pricing Behavior*

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Abstract

We study the implications of overconfidence for price setting in a monopolistic competition setup with incomplete information. Our price-setters overestimate their abilities to infer aggregate shocks from private signals. The fraction of uninformed firms is endogenous; firms can obtain information by paying a fixed cost. We find two results: i) overconfident firms are less inclined to acquire information relative to the rational benchmark; ii) prices might exhibit excess volatility driven by non-fundamental noise. We explore the empirical predictions of our model for idiosyncratic price volatility.

Keywords: Overconfidence, overprecision, imperfect common knowledge, information acquisition, inflation volatility.

JEL classification: D4; D8; E3.

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1 Introduction

This paper studies the implications of the well-documented psychological bias of *overconfidence* on the price-setting behavior of firms in a monopolistically competitive market with incomplete information. In our model, firms receive a private signal about an aggregate shock that influences their marginal costs. They can acquire better information by paying a fixed cost. Our price-setters are overconfident in their signals, i.e. they overestimate the precision of their private information.

There are two important conclusions that we draw from this framework: first, overconfidence implies that a large fraction of firms is going to optimally stay uninformed; second, overconfident firms set prices that may be excessively volatile, since they are driven by under-estimated noise. This volatility at the individual level is coupled with an aggregate inflation rate that is smooth and persistent. So we provide a unified "noise" explanation of prices at both the idiosyncratic and the aggregate level.

There is a vast psychological literature on overconfidence. The term has borne different interpretations depending on the particular study but here we focus on the concept of *overprecision*. The overconfidence bias is rather new in macro models but has been influential in the finance literature. For example, Barber and Odean (2001) analyze the implications of overconfident male investors for excessive stock trading whereas Daniel et al. (1998) and Daniel et al. (2001) focus on the effects of investor's overconfidence on market overreaction and asset pricing.

Overconfidence has not been found to be important only in experimental studies or in setups where monetary loss is inconsequential: Oberlechner and Osler (2009) and Oberlechner and Osler (2012) provide empirical evidence that overconfidence, in the sense of underestimation of uncertainty, is ubiquitous in currency markets and that overconfident traders actually survive in the long-run. In a similar vein, Burnside et al. (2011) associate investor's overconfidence to the forward premium puzzle. Moreover, there have been recently various studies that document in detail biases like overconfidence in the forecasts of actual decision-makers like managers or CFOs. A separate section later provides detailed evidence.

The prevalence of overconfidence has led to work that generates endogenously optimistic biases in beliefs, by taking into account the utility benefits of good outcomes. A prominent example is the theory of optimal expectations of Brunnermeier and Parker (2005). Another example is the work of Ortoleva and Snowberg (2015), who generate overconfidence as a consequence of correlational neglect and explore the implications for political ideology. For our own purposes, we take overconfidence as *given* but explore how it interacts with fundamental and *strategic* uncertainty in our monopolistic competition setup. In the baseline version of our economy, overconfidence can deter agents from obtaining information, since they rely more on their subjective perception of the world, but can also increase the incentives to obtain information by increasing the implicit cost of staying uninformed through the mechanism of higher-order beliefs.

In a dynamic extension of our economy, the presence of strategic complementarities and the interaction with higher-order beliefs generates persistent responses of output to nominal shocks, as in the imperfect common knowledge setup of Woodford (2002), where all agents are uninformed. The persistence at the aggregate level, together with the excess volatility of over-confident uninformed price-setters at the idiosyncratic level, leads to interesting empirical predictions. To flesh them out, we explore how much idiosyncratic price volatility can be generated by overconfidence and contrast it to some well known features of micro price data.

More specifically, Klenow and Kryvtsov (2008) have documented that U.S. consumer prices change frequently with an average absolute size of about 10% on a monthly basis and that these frequent movements are equally likely to be positive or negative in sign. In contrast, aggregate inflation averages just at 0.8% over an horizon of 3 months. Depending on how we calibrate fundamental uncertainty, our economy generates an average absolute size of price changes of up to 2% when it is populated with rational price-setters. Instead, for a reasonable degree of overconfidence that is documented in experimental studies and managerial surveys, we find that the average absolute size can increase up to 5%. We think that this result is indicative of the potential quantitative importance of overconfidence for macroeconomic models.

1.1 Related literature

Our setup is based on the imperfect common knowledge framework of Woodford (2002). Lorenzoni (2009) provides a theory of demand shocks in setups with incomplete information, whereas Melosi (2014) devises methods for the structural estimation of these models. Adam (2007) considers optimal monetary policy in such setups.

More generally, the imperfect common knowledge setup shares similarities with the rational inattention theory of Sims (2003) in its emphasis on the costs of processing information.[1] Moscarini (2004) analyzes in a continuous time model the optimal sampling frequency of noisy information under information-processing constraints and shows that it can also generate inertia. Morris and Shin (2006) have emphasized that in models with forward-looking expectations even the existence of a small fraction of uninformed agents about the future path of fundamentals can generate persistence in the price behavior.

The option to obtain information resembles Reis (2006), who builds a model where producers decide when to acquire information but do not receive any private signals. The contribution of Hellwig and Veldkamp (2009) focuses on the strategic complementarities in information acqui-

[1] See Maćkowiak and Wiederholt (2009) for the implications of rational inattention for sticky prices and Steiner et al. (2017) for an elaborate dynamic model of information processing. Sims (2010) provides a thorough survey of the rational inattention literature.

sition and their implications for multiple equilibria in setups without overconfidence.

The literature in psychology is nicely summarized by Moore and Healy (2008). They provide an extensive survey of the studies that document overconfidence and differentiate between *overestimation* (of one's abilities), *overplacement* (relative to others), and *overprecision*, which is the type of overconfidence we focus on. An example of overstimation is Clayson (2005), who shows that students overestimate their performance in exams. A prominent example of overplacement is the study of Svenson (1981): in a simultaneous study of US and Swedish drivers, 88% of the US group and 77% of the Swedish group asked believed that they are safer drivers than the median. Another example is Guthrie et al. (2001): 90% of 168 federal magistrate judges thought that they are above average as far as their reversal rate on appeal is concerned. Regarding overprecision, studies like Soll and Klayman (2004) have shown that subjective confidence intervals are systematically too narrow given the accuracy of one's information. Another classic reference is Lichtenstein et al. (1982).

In finance, other influential studies beyond the ones mentioned in the introduction are Malmendier and Tate (2005), who devise methods of measuring CEO overconfidence based on corporate investment and Scheinkman and Xiong (2003), who explore the potential of overconfidence to generate speculative bubbles in a dynamic setup. Daniel et al. (2002) consider the policy implications of overconfidence. The survey of Daniel and Hirshleifer (2015) provides additional references.

Various studies in experimental economics have explored the interactions of overconfidence and excess entry, see for example Camerer and Lovallo (1999). Furthermore, in micro theory there are models on the effects of overconfidence on performance as in Compte and Postlewaite (2004) and on the optimal menu of wage contracts as in Fan and Moscarini (2005). In macroeconomics, Caliendo and Huang (2008) analyze the implications of overconfidence for consumption and savings problems.

1.2 Organization

Section 2 provides motivating empirical evidence on the overprecision of decision-makers like managers and CFOs that lend credibility to the bias of overconfidence outside the laboratory environment. Section 3 provides the basic static economy of incomplete information, derives the optimal prices, and characterizes the decision to obtain information. Section 4 explores the pricing implications of overconfidence. Section 5 extends the basic analysis to an infinite horizon model of imperfect common knowledge under overconfidence. Section 6 explores the empirical predictions of the model for idiosyncratic price volatility. Section 7 concludes and an Appendix follows.

2 Motivating empirical evidence on overprecision

A recent literature has emerged that investigates the presence of overconfidence in the forecasts of decision makers like managers and CFOs. Progress has been made by designing confidential surveys that elicit subjective managerial beliefs.

Barrero (2018) uses data from a confidential monthly survey led by the Federal Reserve Bank of Atlanta of a panel of about 1,200 U.S. managers. The survey elicits subjective probability distributions regarding growth outcomes at their own firm, in particular sales growth over the four quarters following the survey.[2] Findings are: the average forecast error is not distinguishable from zero, so managers are neither optimistic nor pessimistic on average. Nevertheless, managers are overconfident, that is, they underestimate the volatility of future sales growth and overestimate their forecasts accuracy. While the manager subjective distributions would imply an average absolute forecast error of about 4 percentage points, in reality the mean absolute forecast error is close to 18 percentage points, more than four times as large.

While Barrero (2018) examines in detail biases in the managerial forecasts of the firm's own variables, Ben-David et al. (2013) examine the overprecision in the forecasts of senior finance executives of the S&P 500 returns. They use quarterly survey data of 10 years and document that CFOs of mid-size and large U.S. corporations are severely miscalibrated: realized market returns are within the 80% confidence intervals that CFOs provide only 36% of the time. They further examine the effect of over-precision on corporate investment and financial leverage.

Huang et al. (2016) have distinguished between over-optimism which is a "better than average" effect, while the overprecision bias is "an unwarranted belief in the correctness of one's answers". The former bias can be interpreted as overconfidence regarding the mean, while the latter as overconfidence regarding the precision, which is indeed the focus of our paper. They study earnings forecasts which are presented in the form of a range, allowing the measurement of the over-precision bias. They find that CEOs are overly precise in their earnings forecast. 67% of actual earnings fall outside the forecast range. Overly precise CEOs invest more in real assets.

We will close this section by citing evidence on behavioral biases coming from surveys of professional forecasters. Bordalo et al. (2018) examine the rationality of individual and consensus professional forecasts of macroeconomic and financial variables. They find that forecasters typically over-react to information at the individual level, while consensus forecasts exhibit underreaction. To model over-reaction in processing noisy signals, they assume that agents have diagnostic expectations, a concept closely related to overconfidence. They manage to reconcile over-reaction at the individual level with under-reaction at the consensus level. Similarly, Broer

[2]The survey elicits also managerial beliefs about employment levels, average unit cost growth, and capital investment expenditures. An overview for the public is provided here. Further details can be found in Altig et al. (2018).

Table 1: Summary of studies on overprecision.

Study	Subjects	Overprecision in forecasts of
Barrero (2018), Altig et al. (2018)	CFOs and CEOs	Sales growth
Ben-David et al. (2013)	CFOs	S&P 500 returns
Huang et al. (2016)	CEOs	Earnings
Bordalo et al. (2018)	Professional forecasters	Macroeconomic variables
Broer and Kohlhas (2018)	Professional forecasters	Inflation

and Kohlhas (2018) document how US professional forecasters overreact to information, in the sense that their forecast revisions are too large. They show how a model of overconfidence may explain overreaction to private information.

To sum up, our paper is about overreaction of individual perceptions about an aggregate shock on the side of managers who make pricing decisions. The study of Barrero shows that managers are overconfident in predicting sales growth, providing evidence of overconfidence within the firms and not only among financial markets participants. Moreover, evidence from professional forecasts supports the notion that overconfidence is relevant when forecasting macroeconomic variables. These variables are directly related to the filtering problem that our price-setters face. Motivated by this evidence, our paper is the first to analyze the overprecision bias in a macro context.

3 Static economy

In this section, we present a static partial-equilibrium model of price-setting behavior in which firms have full information on the structure, parameters and variables of interest.[3]

Firms. We consider a continuum of firms indexed by j on the unit interval $[0, 1]$. Each firm produces a good that is differentiated in the preferences of consumers. We do not explicitly model neither consumer preferences nor their optimization problem. We just assume what is needed to characterize the price-setting problem of firms. Firms use labor $L(j)$ to produce goods through the production function $Y(j) = AL(j)$, where A is a productivity shock common to all firms; W is the nominal wage paid for one unit of labor in the labor market. Firms are profit maximizers and set their prices in a monopolistic-competitive market. The problem of a generic

[3] The model is similar to the one used in Ball and Romer (1989), Ball and Romer (1991) and Blanchard and Kiyotaki (1987).

firm j is to choose the price of its product $P(j)$ to maximize real profits given by

$$\frac{P(j)}{P}Y(j) - \frac{W}{P}L(j). \tag{1}$$

$Y(j)$ is the demand of good j given by

$$Y(j) = \left(\frac{P(j)}{P}\right)^{-\varepsilon} Y, \tag{2}$$

that depends on the price of good j relative to the general price index P, given by

$$P = \left[\int_0^1 P(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}, \tag{3}$$

and on aggregate production Y. The parameter $\varepsilon > 1$ denotes the elasticity of substitution across differentiated goods in consumer preferences.

Labor supply. We do not explicitly model the supply of labor. Instead, we assume that the labor-supply schedule implies the following relation between real wage and aggregate production

$$\frac{W}{P} = Y^{\eta} \tag{4}$$

with $\eta > 0$.[4]

Government. We assume the existence of a monetary authority that has perfect control over the level of nominal spending in the economy. It follows that

$$M = PY \tag{5}$$

where M, which may be labeled as money supply, is indeed controlled by the monetary authority.

Profit function. We can substitute (2), (4) and (5) into (1) to define the profit function of firm j as

$$\pi(P(j), P, \theta) \equiv \left[\frac{P(j)}{P} - \frac{1}{A}\left(\frac{M}{P}\right)^{\eta}\right]\left(\frac{P(j)}{P}\right)^{-\varepsilon}\left(\frac{M}{P}\right). \tag{6}$$

[4] This labor-supply schedule can be derived from the optimizing-behavior of households in a general-equilibrium model. In particular, η would be a combination of the risk-aversion coefficient in consumer preferences and of the Frisch elasticity of substitution of labor supply, or in case of local labor market of ε as well. Assuming a more general labor-supply schedule does not change the subsequent analysis.

The money supply enters the profit function by determining (together with the price level) real output Y. Real output Y has a positive demand effect on good j and increases marginal cost through equilibrium wages, as can been seen by (2) and (4) respectively. Profits of firm j become a function of the action of firm j, $P(j)$, the actions of all other firms synthesized by the index P, and the vector of exogenous shocks θ, $\theta \equiv (A, M)$. Firm j is of measure zero with respect to the aggregate, so its pricing decision does not affect the general price index P.

3.1 Full information

In the full information economy firm j chooses price $P(j)$ to maximize profits (6). Let $P^{\dagger}(j)$ denote the profit-maximizing price, given by

$$P^{\dagger}(j) = P^{\dagger} = \frac{\varepsilon}{\varepsilon - 1}\frac{W}{A} = \frac{\varepsilon}{\varepsilon - 1}\frac{P}{A}\left(\frac{M}{P}\right)^{\eta} \tag{7}$$

which is just the familiar markup rule over nominal marginal cost. Since nominal marginal costs are the same across firms, all firms set the same price, $P^{\dagger} = P$. Money is neutral and the full information output equals $Y^{*} = (\frac{A}{\varepsilon/(\varepsilon-1)})^{\frac{1}{\eta}}$. The respective full information price level is a function of the exogenous money supply and equilibrium output,

$$P^{*} = \frac{M}{Y^{*}} = \left(\frac{\varepsilon}{\varepsilon - 1}\frac{1}{A}\right)^{\frac{1}{\eta}} M. \tag{8}$$

P^{*} is function solely of the *exogenous* aggregate shock θ and will be a useful statistic in the next section. For later use, note that since $P^{*\eta} = \frac{\varepsilon}{\varepsilon-1}\frac{M^{\eta}}{A}$, we can rewrite (7) as

$$P^{\dagger} = P^{1-\eta}P^{*\eta}. \tag{9}$$

This rewriting reveals how the optimal price of firm j depends in *equilibrium* on the actions of other firms (P), and on a particular combination of exogenous shocks (P^{*}). The parameter η captures the elasticity of the individual optimal price with respect to the aggregate price level. Therefore, η determines the equilibrium substitutability or complementarity of pricing decisions: price-setting decisions are strategic complements, if $0 < \eta < 1$, and strategic substitutes if $\eta > 1$.[5]

3.2 Incomplete information

We provide here an overview of the incomplete information setup and then we will proceed to the details of the signal-extraction problem.

[5] See Woodford (2003, ch. 3) for a detailed discussion of equilibrium strategic complementarities in price-setting.

Our information structure is based on the "limited attention" setup of Woodford (2002) and the subsequent imperfect common knowledge literature. This amounts effectively to making two assumptions. First, firms do not know the realization of the aggregate shock θ. Second, we assume that *none* of the actions of the other agents, as captured by the general price level P, is in the information set of price-setters.

In particular, each firm receives a *private* signal s^j that is correlated with the aggregate hidden state θ. This signal is the only element in their information set and captures the firm's "subjective perception" about θ. We assume an additive structure for the signals with i.i.d. idiosyncratic noise. The firm uses its private signal to make inferences on productivity and money supply shocks that affect marginal costs and aggregate demand. Furthermore, the signal is used to infer the beliefs, and therefore the pricing decisions, of other firms. An entire hierarchy of beliefs about the price level is formed, so price-setters face *both* fundamental *and* strategic uncertainty. Overconfidence enters the inference problem by making agents put too much weight on their own subjective perception of the aggregate shock. Moreover, we allow the fraction of uninformed firms to be *endogenous*: each firm j can pay a real fixed idiosyncratic cost $\tilde{c}^j$ and obtain full information about the shock θ and the pricing decisions of the rest of the firms. These fixed shocks are i.i.d. across firms and independent of the private signals.

Timing. Each firm j gets a realization of the fixed cost at the beginning of time and receives a private signal. Given the signal, the firm decides to obtain or not information. If the firm becomes "informed", it pays cost $\tilde{c}^j$ and sets price $P^\dagger(j)$. If the firm decides to stay "uninformed," it sets price $\tilde{P}(j)$. We can characterize this problem by working backwards.

Informed firm. The optimal price when the firm has complete information is just the typical markup over nominal marginal cost, as in equation (7). Since nominal marginal costs are the same across firms, informed firms set again the same price, $P^\dagger(j) = P^\dagger$. As before, we can use the definition of the full information price in (8), to rewrite the price of an informed firm as a function of the price of aggregate price level P and the full information price P^*, $P^\dagger = P^{1-\eta}P^{*\eta}$. Note the similarity with equation (9). However, the price level P is obviously *not* the same as in an environment with full information, since it may include prices of firms that chose to stay uninformed.

Uninformed firm. If the firm does not obtain information, it sets a price that maximizes its *subjective* expectation of real profits, given its perception of the world. Let E^j denote the subjective expectation operator of firm j, conditional on its private signal, so $E^jX \equiv E(X|s^j)$ for a generic random variable X.[6] $\tilde{P}(j)$ maximizes $E^j\pi(P(j), P, \theta)$ which implies that the optimal

[6] The operator E always refers to the subjective expectation operator. This coincides with the objective expectation operator except for the case of overconfidence in the private signals. It will be always clear from the

price of an uninformed firm takes the form

$$\tilde{P}(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{E^j\{\frac{W}{A} Z\}}{E^j Z}, \tag{10}$$

where $Z \equiv MP^{\varepsilon-2}$. So the optimal price of the uninformed firm is an "average" of the typical markup over nominal marginal cost.[7] Note again that $\frac{\varepsilon}{\varepsilon-1}\frac{W}{A} = P^{1-\eta}P^{*\eta} = P^{\dagger}$. So we can express the optimal price of the uninformed firm in (10) as an average of the price it would set if it were informed,

$$\tilde{P}(j) = \frac{E^j\{P^{\dagger} Z\}}{E^j\{Z\}}. \tag{11}$$

Obtaining information. A generic firm j chooses to pay the fixed cost and acquire complete information when the expected increase in profits in doing this is higher than the cost $\tilde{c}^j$,

$$E^j\{\pi(P^{\dagger}, P, \theta) - \pi(\tilde{P}(j), P, \theta)\} \geq \tilde{c}^j. \tag{12}$$

Having observed the realization of its own signal, a firm j evaluates the left-hand side (LHS) of (12). The firm acquires information and sets price $P^{\dagger}$, if (12) holds, otherwise it stays uninformed and sets $\tilde{P}(j)$.

3.3 Approximation to the incomplete-information model

We will simplify the problem by proceeding with an approximation around the deterministic steady state where $\theta = \bar{\theta} = (\bar{A}, \bar{M})$.[8]

Proposition 1. *A second-order approximation to the LHS of (12) leads to a criterion of the form*

$$var^j\{p^{\dagger}\} \geq c^j, \tag{13}$$

where $var^j\{\cdot\}$ denotes the variance operator conditional on the subjective information set of firm j, while $p^{\dagger}$ is the log of the price that firm j would set with complete information and c^j is a reparametrization of the idiosyncratic fixed cost $\tilde{c}^j$.

Proof. See Appendix. □

context if we refer to the objective or the subjective distribution.

[7] The average is according to the change of measure $x \equiv Z/E^j Z \geq 0$ with $E^j x = 1$.

[8] This simplification is particularly useful for the derivation of the higher-order hierarchy of beliefs. See proposition 2.

The decision of acquiring or not information depends on whether the subjective variance of the price that a firm sets under complete information is higher than the cost c^j. An important implication of proposition 1 is that (13) can be evaluated using just a log-linear approximation to the equilibrium conditions. In this log-linear approximation, equation (11) implies that the log of the price under incomplete information is the expected value of the log of the price under complete information

$$\tilde{p}(j) = E^j p^\dagger, \tag{14}$$

where lower-case letters denote logarithms of the respective variables.[9] Moreover, $p^\dagger$ satisfies an exact log-linear relationship given by

$$p^\dagger = (1-\eta)p + \eta p^*, \tag{15}$$

where p^* is the log of the full-information equilibrium price level and p is given by

$$p = \int_0^1 p(i)di \tag{16}$$

as a result of a first-order approximation of (3).

Let μ denote the fraction of firms that in equilibrium decide to keep their subjective information set. Assuming without loss of generality that agents $j \in [0, \mu]$ are the ones who remain uninformed, we can write (16) as

$$p = \mu\tilde{p} + (1-\mu)p^\dagger, \tag{17}$$

where $\tilde{p}$ stands for the average price of the subjectively-informed firms,

$$\tilde{p} \equiv \frac{1}{\mu}\int_0^\mu \tilde{p}(i)di. \tag{18}$$

We can then plug (17) into (15) to obtain

$$p^\dagger = \delta p^* + (1-\delta)\tilde{p}, \tag{19}$$

where

$$\delta \equiv \frac{\eta}{\eta + (1-\eta)\mu}. \tag{20}$$

Equation (19) expresses the price of informed firms as a linear combination of the exogenous full information price, which captures the fundamentals, and the price of the uninformed firms. The weight on the full information price is equal to unity, $\delta = 1$, when all firms are informed

[9] Note that the change of measure $Z/E^j Z$ is not relevant anymore in the log-linear approximation. Furthermore, all firms set the same constant price at $\bar{\theta}$.

($\mu = 0$), which would happen if the cost of acquiring information were zero. If costs are large enough so that every firm stays uninformed ($\mu = 1$), we are in the case of Woodford (2002), where every firm is exogenously uninformed, and the weight δ becomes equal to the degree of complementarity, $\delta = \eta$. The weight δ is a decreasing function of μ and $\eta \leq \delta \leq 1$ if $\eta < 1$. Thus, the larger the mass of uninformed firms, the smaller the weight on fundamentals, and the larger the weight on the prices of other firms, if we have strategic complements. In contrast, δ is an increasing function of μ with $1 \leq \delta \leq \eta$ in case of strategic substitutes ($\eta > 1$).

To sum up, the set of equations (14), one for each firm that remains uninformed, together with (18) and (19) determine the equilibrium prices of informed and uninformed firms in a first-order approximation to the equilibrium conditions.

3.4 Signal extraction, overconfidence and optimal pricing

Consider now the signal-extraction problem of the firm. We note that uncertainty about the aggregate shock θ has collapsed to uncertainty about the full-information price p^*, which is a linear combination of the logarithms of the technology and the money supply shock, $p^* = \text{const.} + \ln M - \frac{1}{\eta} \ln A$. Without loss of generality for our purpose of exploring the effects of overconfidence on pricing, we are going to assume that firms receive signals about this particular linear combination p^*, which captures the "fundamental" shocks in the economy.[10] Let p^* be a random variable of the form

$$p^* = \bar{p}^* + u, \tag{21}$$

where $\bar{p}^*$ is a constant and u is a Gaussian white-noise process with variance σ_u^2, $u \sim N(0, \sigma_u^2)$. This distribution is common knowledge and corresponds to the objective probability distribution of p^*.

Overconfidence. Each firm receives a *private* signal s^j that is linearly related to p^* as

$$s^j = p^* + \xi^j, \tag{22}$$

where ξ^j is an idiosyncratic Gaussian noise with mean zero. Moreover, ξ^j is statistically independent of u as well as of ξ^i for each $i \neq j$. Our departure from objective probabilities is in the variance of the noise. Let $\bar{\sigma}_\xi^2$ and σ_ξ^2 denote respectively the *perceived* (subjective) and *true* variance of ξ^j. We introduce overconfidence as in the influential contributions of Daniel et al.

[10] We invite the reader to entertain more elaborate signal structures where each firms get multiple signals about different components of P^*. For example, we could think of each firm getting a private idiosyncratic productivity shock A^j (leading to idiosyncratic marginal costs), that is correlated with the aggregate shock A. This shock could be used as a signal in order to infer prices of other firms. We could also envision private (or even public) signals about the money supply M. We think that these directions, albeit interesting, would hinder our main goal of analyzing sharply the effects of overconfidence on macroeconomic pricing models.

(1998) and Daniel et al. (2001) in the behavioral finance literature: we assume that $\bar{\sigma}_\xi^2 < \sigma_\xi^2$. So price-setters overestimate their ability to infer the hidden state from their private signals. This corresponds to the behavioral bias of *overprecision*, a term which we will use interchangeably with the term overconfidence.[11]

Given this information structure, each firm can form its own expectation of the full-information price p^* as in a standard signal-extraction problem

$$E^j p^* = (1 - \bar{r}^j)\bar{p}^* + \bar{r}^j s^j, \tag{23}$$

where the weight $\bar{r}^j$ is defined as

$$\bar{r}^j = \bar{r} \equiv \frac{1}{1+\bar{\lambda}} \quad \text{with} \quad \bar{\lambda} = \frac{\bar{\sigma}_\xi^2}{\sigma_u^2}. \tag{24}$$

Since $\bar{\lambda}$ is common and common knowledge across the different firms, then $\bar{r}^j$ is independent of j and equal to a common $\bar{r}$. In particular, $\bar{\lambda}$ represents the noise-to-fundamental variance ratio and can be interpreted as an index of confidence in how a firm's private signal is a good representation of the full-information price. Lower values of $\bar{\lambda}$ implies a higher weight to the signal when firms form expectations of the full-information price and, therefore, a high degree of confidence on the subjective information set. We can define a "true" degree of confidence λ as $\lambda \equiv \frac{\sigma_\xi^2}{\sigma_u^2}$ with a respective weight in the filtering problem, $r = 1/(1+\lambda)$. Overconfidence implies that $\bar{\lambda} < \lambda$, resulting into an excessive weight on the private signal, $\bar{r} > r$.

Higher-order beliefs and optimal pricing. To solve for the equilibrium prices of informed and uninformed agents, we first guess that μ is known to each firm j. We then verify that this is indeed the case. Given this guess, we can substitute (19) into (14) to get

$$\tilde{p}(j) = \delta E^j p^* + (1-\delta) E^j \tilde{p} \tag{25}$$

which can be averaged across all uninformed price setters to obtain

$$\tilde{p} = \delta \bar{E} p^* + (1-\delta)\bar{E}\tilde{p}, \tag{26}$$

where we have defined the operator $\bar{E}(\cdot) \equiv \frac{1}{\mu}\int_0^\mu E^i(\cdot) di$, the average expectation across uninformed firms. So the price index of uninformed firms is a linear combination of the average (across uninformed firms) forecast of the fundamentals and the average forecast of the price index. By iterating the above expression, it follows that $\tilde{p}$ is a linear combination of all higher-order

[11] See the survey of Moore and Healy (2008) for a succinct taxonomy of overconfidence studies in terms of overestimation, overplacement and overprecision.

average expectations of the full-information price,

$$\tilde{p} = \delta \sum_{k=0}^{\infty} (1-\delta)^k \bar{E}^{(k+1)} p^*, \tag{27}$$

where the $(k+1)-$order average expectation operator is defined as $\bar{E}^{(k+1)}(\cdot) \equiv \bar{E}(\bar{E}^{(k)}(\cdot)), k \geq 1$, with $\bar{E}^{(1)}(\cdot) \equiv \bar{E}(\cdot)$.[12] As in the literature pioneered by Morris and Shin (2002), higher-order beliefs are an integral part of optimal prices.[13] Using the signal-extraction formula (23) we get the following optimal prices.

Proposition 2. *("Optimal prices in the static economy")*

- *The price of an informed firm is*

$$p^\dagger = \frac{(1-\eta)(1-\bar{r})\mu}{\eta + (1-\eta)(1-\bar{r})\mu} \bar{p}^* + \frac{\eta}{\eta + (1-\eta)(1-\bar{r})\mu} p^*. \tag{28}$$

- *The price index of uninformed firms is*

$$\tilde{p} = \frac{\eta(1-\bar{r}) + (1-\eta)(1-\bar{r})\mu}{\eta + (1-\eta)(1-\bar{r})\mu} \bar{p}^* + \frac{\bar{r}\eta}{\eta + (1-\eta)(1-\bar{r})\mu} p^*. \tag{29}$$

- *The price of uninformed firm $j \in [0, \mu]$ is*

$$\tilde{p}(j) = \frac{\eta(1-\bar{r}) + (1-\eta)(1-\bar{r})\mu}{\eta + (1-\eta)(1-\bar{r})\mu} \bar{p}^* + \frac{\bar{r}\eta}{\eta + (1-\eta)(1-\bar{r})\mu} s^j. \tag{30}$$

Proof. By using (23), averaging across uninformed firms, assuming that a law of large numbers holds on a positive measure, $\frac{1}{\mu}\int_0^\mu \xi^i di = 0$, and iterating to derive the higher-order expectations, we arrive at[14]

$$\bar{E}^{(k)} p^* = (1-\bar{r}^k)\bar{p}^* + \bar{r}^k p^*, k \geq 1. \tag{31}$$

We can substitute (31) into (27) to get $\tilde{p}$, and therefore $p^\dagger$ and $\tilde{p}(j)$ from (19) and (14)

[12] Under the restriction that $\eta(2\mu - 1) < 2\mu$, δ is such that $|1-\delta| < 1$.

[13] See Allen et al. (2006), Amato and Shin (2003) and Amato and Shin (2006) for further examples of problems with iterated expectations.

[14] See Uhlig (1996) for the conditions under which a law of large numbers holds.

respectively,

$$\begin{aligned}
\tilde{p} &= \frac{1-\bar{r}}{1-(1-\delta)\bar{r}}\bar{p}^* + \frac{\bar{r}\delta}{1-(1-\delta)\bar{r}}p^* \\
p^\dagger &= \frac{(1-\delta)(1-\bar{r})}{1-\bar{r}(1-\delta)}\bar{p}^* + \frac{\delta}{1-(1-\delta)\bar{r}}p^* \\
\tilde{p}(j) &= \frac{1-\bar{r}}{1-(1-\delta)\bar{r}}\bar{p}^* + \frac{\bar{r}\delta}{1-(1-\delta)\bar{r}}s^j
\end{aligned}$$

Use the definition of δ in (20) to get the respective expressions in terms of the degree of complementarity η and the fraction μ.

□

3.5 Information acquisition

Corollary. *Given (28) and Proposition 1, it follows that a generic firm j decides to acquire information if the following inequality holds*

$$var^j\{p^\dagger\} = \left[\frac{\eta}{\eta+(1-\eta)(1-\bar{r})\mu}\right]^2 var^j\{p^*\} \geq c^j,$$

where $var^j\{p^*\}$ is the variance of the full-information price level conditional on the subjective information set of firm j. This is given by $var^j\{p^*\} = \sigma_u^2(1-\bar{r}^j)$, so we have

$$\left[\frac{\eta}{\eta+(1-\eta)(1-\bar{r})\mu}\right]^2 \sigma_u^2(1-\bar{r}^j) \geq c^j. \tag{32}$$

Note that in (32) we have kept the distinction –since it matters for the discussion that follows- between the own degree of confidence $\bar{r}^j$ and the others' degree of confidence $\bar{r}$– although we have assumed that they are the same.[15]

According to (32), several parameters of the model drive the incentives for firm j to acquire information.[16] The higher is the variance of the full-information price, σ_u^2, the higher are the incentives to acquire information. Obviously, the lower the cost c^j, the higher those incentives.

[15] Indeed, we could have derived the same equilibrium price $p^\dagger$ and, thus, criterion (32), even if we had heterogeneity in overconfidence. In that case, let $\bar{r} \equiv \left(\int_0^\mu \bar{r}^i di\right)/\mu$ and assume that a law of large numbers holds for $\int_0^\mu \xi^i \bar{r}^i di = 0$. Then (28) follows.

[16] Note that the criterion to obtain information does not depend on the signal realization s^j. This is an outcome of the log-linear approximation and the Gaussian setup we are using, which leads to conditional variances that are independent of the signals. This is the price to pay for being able to calculate high-order beliefs as iterations of higher-order average expectations, a tractability feature which explains the ubiquity of the log-linear approach in the imperfect common knowledge literature. The same feature emerges in the infinite horizon model of section 5, where we employ the Kalman filter: forecast error variances depend only on time and not on signal realizations.

In the case of strategic complementarity in the pricing decision, $0 < \eta < 1$, the higher is the fraction of firms that are acquiring information (i.e. the lower the μ) the higher are the incentives for the individual firm to acquire information. This result is of the same nature as the one found by Ball and Romer (1989) in a similar model but with only imperfect information, in which firms's decisions are on whether to change or not prices.

Each firm's decision is also influenced by the degree of confidence in the informativeness of the signal. If $\bar{r}^j$ is high ($\bar{\lambda}^j$ is low), then the firm will not have incentives to acquire finer information. A high degree of confidence implies that firms are going to be stuck with their perceptions of the world when setting their prices.

Interestingly, if the confidence of others increases ($\bar{\lambda}$ decreases and $\bar{r}$ increases) then the price under complete information has higher subjective variance since the average forecast of uninformed firms is getting closer to the full information price, as shown in (31) for $k = 1$. Then, each individual firm has higher incentives to acquire information and imitate other firms –when pricing decisions are strategic complements.

We move to characterize the equilibrium value of μ, under the assumption $\bar{r}^j = \bar{r}$ for each j. We define

$$c^* \equiv \left[\frac{\eta(1-\bar{r})^{\frac{1}{2}}}{\eta + (1-\eta)(1-\bar{r})\mu} \right]^2 \sigma_u^2 \tag{33}$$

and note that (32) implies that all firms with costs c^j less than the threshold value c^* obtain information. Let the cumulative distribution function of costs be F with respective density f on support $[\underline{c}\ , \bar{c}]$. Therefore, μ has to satisfy

$$1 - F(c^*) = \mu. \tag{34}$$

The value c^* depends on μ, so (34) determines implicitly the equilibrium fraction of uninformed firms. This solution confirms our initial guess that μ is a function of known parameters and then known to each firm j. The properties of $F(\cdot)$ determine the existence and the characteristics of the equilibrium. Indeed, when $c^j = c$ for each j, multiple equilibria are possible for the same reasons as they occur in the imperfect-information model of Ball and Romer (1989). For other $F(\cdot)$ multiple equilibria might disappear. Since this is not the focus of this work, we assume that $F(\cdot)$ and $f(\cdot)$ are such that there exists an equilibrium and its unique. We get the following proposition.

Proposition 3. *("Confidence on private signals and uninformed firms")*

Assume that we are at a stable equilibrium where $1 + f(c^*)\frac{\partial c^*}{\partial \mu} > 0$.

$$\text{If} \quad (1-\eta)(1-\bar{r})\mu < \eta \quad \text{then} \quad \frac{d\mu}{d\bar{r}} > 0. \tag{35}$$

Thus, the fraction of uninformed firms increases when the weight on the private signal increases.

Proof. See the Appendix.

□

The proposition implies that an increased reliance on private signals, which would occur if we had a low subjective variance due to overconfidence, can lead to a larger mass of uninformed firms relative to a rational signal-extraction benchmark.

Discussion. Although intuitive, Proposition 3 is not obvious. It involves the effects of firm's own confidence and the opposing effects of the confidence of other firms through the mechanism of higher-order expectations. An increase in one's confidence decreases the incentives to acquire information. But an increase in the confidence of others makes the price of informed firms more volatile, since there is larger reliance on private signals in equilibrium, *amplifying* the effect of higher-order beliefs. To see that, consider the extreme case where private signals are completely *uninformative* (in the eyes of the price-setters), $\bar{r} = 0$. In that case, the entire mechanism of higher-order beliefs is mute: uninformed firms set a price equal to their prior, $\tilde{p} = \bar{p}^*$, and the price of informed firms becomes $p^\dagger = (1-\delta)\bar{p}^* + \delta p^*$. So the weight on the full information price p^* reaches its minimum, leading to small volatility and reduced incentives to acquire information.

The inequality condition in (35) requires that the effect of the firm's own confidence, which leads to a larger equilibrium fraction μ, is *stronger* than the higher-order beliefs effect, which reduces μ. As expected, the condition always holds in the case of strategic substitutes, $\eta > 1$. In the case of strategic complements, the condition holds if we effectively limit the effect of higher-order beliefs. This would happen in an equilibrium where μ is small (so the complementarities are not strong enough), $\mu < \frac{\eta}{(1-\eta)(1-\bar{r})}$. From another angle, the higher-order beliefs effect would be contained if we bounded η away from a lower bound, by writing the condition as $\eta > \frac{(1-\bar{r})\mu}{1+(1-\bar{r})\mu}$. The lower bound for η is always smaller than $1/2$ and decreases to zero when $\bar{r}$ increases to unity. The conclusion is that if complementarities are not too "large," a high degree of confidence in private signals increases the equilibrium fraction of uninformed firms.

4 Price implications of incomplete information and overconfidence

In this section we study the price implications of the model and in particular the relation between excess volatility of prices and overconfidence.

A first important implication is that the model displays two levels of heterogeneity: at a first stage, there are differences in prices between informed and uninformed firms. At a second stage, within uninformed firms, prices are related to the realization of subjective signals. By using (21), we can rewrite the prices of proposition 2 as

$$p^{\dagger} = \bar{p}^{*} + (1+\bar{\lambda})\hat{k}u \tag{36}$$

$$\tilde{p}(j) = \bar{p}^{*} + \hat{k}u + \hat{k}\xi^{j}, j \in [0, \mu] \tag{37}$$

$$\tilde{p} = \bar{p}^{*} + \hat{k}u, \tag{38}$$

where

$$\hat{k} \equiv \frac{\bar{r}\eta}{\eta + (1-\eta)(1-\bar{r})\mu}. \tag{39}$$

Thus, the prices of informed firms react only to the fundamental shock u, whereas the price of uninformed firm j reacts also to the noise shock ξ^j. In particular, equation (36) shows that prices of informed firms react less than proportionally to fundamental shocks when pricing decisions are strategic complements ($\eta < 1$), since in that case $(1+\bar{\lambda})\hat{k} < 1$, but more than proportionally in the strategic-substitute case ($\eta > 1$), since in that case $(1+\bar{\lambda})\hat{k} > 1$. As shown in (37), the response of uninformed firms is always smaller than that of informed firms, since $\bar{\lambda} > 0$. The dicrepancy is coming from the fact that the informed firms do not have to filter the hidden shock. Moreover, prices of uninformed firms react also to non-fundamental shocks, ξ^j, in the *same* proportion as they do to fundamental shocks.

Overconfidence can affect the volatility of prices. Using equation (37), we obtain that the "true" variance of prices for a generic uninformed agent j is

$$var\{\tilde{p}(j)\} = (1+\lambda)\hat{k}^2\sigma_u^2. \tag{40}$$

Equation (36) implies that the variance of the prices of informed firms is given by

$$var\{p^{\dagger}\} = (1+\bar{\lambda})^2\hat{k}^2\sigma_u^2. \tag{41}$$

It follows that the ratio of the volatilities of prices of uninformed and informed firms is given by

$$\frac{var\{\tilde{p}(j)\}}{var\{p^{\dagger}\}} = \left[\frac{(1+\lambda)^{\frac{1}{2}}}{(1+\bar{\lambda})}\right]^2. \tag{42}$$

When the signal-extraction problem is rational (i.e. $\lambda = \bar{\lambda}$), prices of uninformed firms are always less volatile than the prices of informed firms. With overconfident firms, it is instead possible for the reverse to happen. It is sufficient that $(1+\bar{\lambda}) < (1+\lambda)^{\frac{1}{2}}$, which requires that the true volatility of the idiosyncratic noise σ_ξ^2 is large enough relative to the perceived $\bar{\sigma}_\xi^2$, so sufficient overprecision is needed.

A second important implication of overconfidence is that it is even possible to have excess volatility of the price of an individual uninformed firm with respect to the full-information (fundamental) price. Indeed, we obtain that

$$\frac{var\{\tilde{p}(j)\}}{var\{p^*\}} = \left[\frac{\zeta(1+\lambda)^{\frac{1}{2}}}{(1+\bar{\lambda})}\right]^2 \tag{43}$$

where ζ is a positive parameter given by $\zeta \equiv \frac{\hat{k}}{\bar{r}} = \frac{\eta}{\eta+(1-\eta)(1-\bar{r})\mu}$, such that $\zeta < 1$ ($\zeta > 1$) when pricing decisions are strategic complements (substitutes). To have excess volatility of the prices of uninformed firms with respect to fundamentals, it is required that $(1+\bar{\lambda}) < \zeta(1+\lambda)^{\frac{1}{2}}$ which is then a more (less) stringent condition than before when pricing decisions are strategic complements (substitutes).[17]

Overconfidence has two important roles in this model. First, a higher fraction of firms is going to decide optimally not to acquire information and just pay attention to their own private signals, relative to the rational benchmark. Second, the prices of uninformed firms can be more volatile than fundamental disturbances and this volatility is driven by the noise in the perception of fundamentals.

In a dynamic extension of the above model, the fact that overconfident price setters are less prone to acquire information implies that there can be a high proportion of this kind of subjectively-driven price setters. Woodford (2002) has shown that higher-order expectations matter for determining persistent effects of output and prices following exactly those shocks agents are subjectively informed about. In addition to this persistence result, the existence of subjectively-informed firms with overconfident beliefs can produce excess *volatility* of prices, as section 5 shows.

5 Infinite-horizon model

Consider a simple dynamic extension of our setup. Time is discrete and the horizon is infinite. We assume that each firm does not know the realization of the sequence $\{\theta_t\}_{t=t_0}^{\infty}$. However, each firm has a prior distribution on the sequence $\{\theta_t\}_{t=t_0}^{\infty}$ that coincides with the correct distribution and which is common knowledge. In each period and contingency, each firm can observe a private signal s_t^j that is correlated with the hidden state θ_t. In particular, the sequence of signals $\{s_t^j\}_{t=t_0}^{\infty}$, one for each j, is related to the sequence $\{\theta_t\}_{t=t_0}^{\infty}$ through a likelihood function which is known and common knowledge but, as before, does not necessarily coincide with the correct likelihood function. As in the static economy, incomplete information is modelled by assuming that each firm knows only its own private history of signals and not those of the others, as well as not the

[17] Note that with no overconfidence ($\lambda = \bar{\lambda}$) the ratio is always smaller that unity even in the case of strategic substitutes ($\zeta > 1$). This is clear if we note that $\hat{k} = \delta/(\delta+\lambda)$ and that the ratio is less than unity when $Q(\delta) = \lambda\delta^2 - 2\lambda\delta - \lambda^2 < 0$, which holds for the permissible δ, i.e. such that $|1-\delta| < 1$.

price index and the individual prices. Fixed costs $\tilde{c}^j$ are realized identically and independently across firms at the beginning of time and stay constant thereafter. Each firm has the option to acquire information by paying cost $\tilde{c}^j$. We simplify the information acquisition problem by assuming that once the cost is paid, the firm remains in the "informed" state forever.

We assume that firms choose prices to maximize the expected discounted value of profits given by

$$E^j_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \pi(P_t(j), P_t, \theta_t), \tag{44}$$

where β is such that $0 < \beta < 1$.[18] $E^j_{t_0}$ is the subjective expectation operator conditional on information at time t_0. Prices are set freely in each period. As in the static model, an "informed" firm sets its price as

$$P^\dagger_t = P_t^{1-\eta} P_t^{*\eta} \tag{45}$$

for each period t after having paid the information cost. An "uninformed" firm instead sets its price as

$$\tilde{P}_t(j) = \frac{E^j_t\{P^\dagger_t Z_t\}}{E^j_t\{Z_t\}}, \tag{46}$$

where Z_t has the same definition as in section 3.

To characterize the decision for a generic firm j to acquire or not information, we guess an equilibrium and then verify that prices and information decisions are consistent with that equilibrium. The analysis is simplified by noting that the fraction of firms that remain uninformed each period cannot increase over time, i.e. $\{\mu_t\}_{t=t_0}^{+\infty}$ is a non-increasing sequence. Of the many equilibria that can exist, we are interested in ones in which $\mu_t = \mu$ for each $t \geq t_0$. In particular, in these stationary equilibria, whichever firm decides to be informed does it in the first period. For this to be optimal, the strategy of getting information in the first period should give higher expected discounted profits than the strategy of waiting until a generic time T, given the equilibrium strategies of all other firms. In particular at time t_0 the expected profits to acquire immediately information and pay the cost should be higher than the strategy of remaining with the subjective information until a generic period T and pay the cost in that period. For a generic firm j to become informed at time t_0, the following inequality should hold for each $T > t_0$

$$E^j_{t_0} \sum_{t=t_0}^{T-1} \beta^{t-t_0} \pi(\tilde{P}_t(j), P_t, \theta_t) - \beta^T \tilde{c}^j \leq E^j_{t_0} \sum_{t=t_0}^{T-1} \beta^{t-t_0} \pi(P^\dagger_t, P_t, \theta_t) - \tilde{c}^j,$$

[18] We can generalize the analysis that follows by assuming a stochastic discount factor to evaluate real profits across contingencies and time.

which can be rewritten

$$E_{t_0}^{j}\sum_{t=t_0}^{T-1}\beta^{t-t_0}\left\{E_t^j[\pi(P_t^{\dagger},P_t,\theta_t)-\pi(\tilde{P}_t(j),P_t,\theta_t)]\right\}\geq\tilde{c}^j(1-\beta^T). \tag{47}$$

We take a second-order approximation of the above problem around a stationary point with unitary relative prices to obtain

$$E_{t_0}^{j}\sum_{t=t_0}^{T-1}\beta^{t-t_0}var_t^j\{p_t^{\dagger}\}\geq c^j(1-\beta^T), \tag{48}$$

where c^j is a reparameterization of $\tilde{c}^j$ as in Proposition 1. We guess, and verify later, that in the equilibrium $var_t^j\{p_t^{\dagger}\}$ is a constant that does not depend on j and is also independent of t in a stationary filtering problem. Thus, (48) simplifies to

$$var_t^j\{p_t^{\dagger}\}\geq c^j(1-\beta), \tag{49}$$

which is also independent of T.

We verify now that $var_t^j\{p_t^{\dagger}\}$ is constant and independent of j, and that μ is also a constant and known within the information set of each type of firm at time t_0. As before, we just need to characterize the equilibrium values of prices in a log-linear approximation to the equilibrium. It is still true that the set of equations (14), one for each firm that remains uninformed, together with (18) and (19) determine the equilibrium prices of informed and uninformed firms in a first-order approximation to the equilibrium conditions. We continue to assume that each firm receives a private signal s_t^j that is related linearly to p_t^* as

$$s_t^j=p_t^*+\xi_t^j, \tag{50}$$

where ξ_t^j is an idiosyncratic Gaussian noise with mean zero, perceived variance $\bar{\sigma}_\xi^2$ and true variance σ_ξ^2 for each j, with $\bar{\sigma}_\xi^2<\sigma_\xi^2$. We assume that ξ_t^j, for each j, is statistically independent of the sequence $\{p_t^*\}$ as well as of the sequence $\{\xi_t^i\}$ for each $i\neq j$. We allow now $\{p_t^*\}$ to be a first-order autoregressive stochastic process of the form

$$p_t^*=\bar{p}^*+\rho p_{t-1}^*+u_t \tag{51}$$

with $|\rho|\leq 1$ where u_t is Gaussian noise with mean zero and variance σ_u^2.

The assumption of persistence of the unobservable shock can in principle be a source of complication in the solution of the model, for an infinite dimensional state might be necessary to keep track of the higher-order beliefs of other firms. Woodford (2002) has shown that the dimension of the hidden-state space is finite in a model of imperfect common knowledge where all firms are exogenously uninformed. We modify his approach in order to account for the endogenous fraction of uninformed firms and derive the following proposition.

Proposition 4. *("Optimal prices in the dynamic model")*

- *The general price index evolves according to*

$$p_t = \bar{p}^* + \rho(1-\hat{k})p_{t-1} + \rho\hat{k}p^*_{t-1} + [\delta\,(1-\mu)\,(1-\hat{k}) + \hat{k}]u_t, \tag{52}$$

  where[19]

$$\hat{k} \equiv \frac{1}{2\rho^2}\left\{\rho^2 - 1 - \frac{\delta}{\bar{\lambda}} + \sqrt{\left[1-\rho^2+\frac{\delta}{\bar{\lambda}}\right]^2 + 4\rho^2\frac{\delta}{\bar{\lambda}}}\right\}. \tag{53}$$

  We have $\frac{\partial\hat{k}}{\partial\eta} > 0$ *and* $\frac{\partial\hat{k}}{\partial\bar{\lambda}} < 0$*. If* $\eta < (>)1$*, then* $\frac{\partial\hat{k}}{\partial\mu} < (>)0$*.*

- *The price of informed firms follows*

$$p^\dagger_t = \bar{p}^* + \rho(1-\hat{k})p^\dagger_{t-1} + \rho\hat{k}p^*_{t-1} + [\delta(1-\hat{k}) + \hat{k}]u_t. \tag{54}$$

- *The price of uninformed firms follows*

$$\tilde{p}_t(j) = \bar{p}^* + \rho(1-\hat{k})\tilde{p}_{t-1}(j) + \rho\hat{k}p^*_{t-1} + \hat{k}(u_t + \xi^j_t). \tag{55}$$

- *The contemporaneous variance of* $p^\dagger_t$ *is constant, does not depend on* j*, and is also independent of* t *in a stationary solution,*

$$var^j_t\{p^\dagger_t\} = \frac{1+\bar{\lambda}[1-\rho^2(1-\hat{k})]^2}{1-\rho^2(1-\hat{k})^2}\bar{\lambda}\hat{k}^2\sigma^2_u. \tag{56}$$

  Therefore, (49) implies that the equilibrium fraction of uninformed firms is determined implicitly by

$$\mu = 1 - F(c^*), \quad \textit{with} \quad c^* \equiv \frac{var^j_t\{p^\dagger_t\}}{1-\beta}. \tag{57}$$

[19] The parameter $\hat{k}$ represents a linear combination of the vector of Kalman gains and is a different function of other (exogenous) parameters than the $\hat{k}$ of the static economy in (39). We use the same notation, since when $\rho = 0$ the two expressions coincide.

Proof. See the Appendix for the details of the derivations. □

The analysis of the static economy is retrieved if we set $\rho = 0$. The main qualitative results of sections 3 and 4 hold in the dynamic extension with some qualifications. Indeed, it is still the case that overconfidence is needed for the volatility of prices of uninformed to be higher than that of informed. The ratio of the unconditional variances between informed and uninformed firms is higher than the unitary value when the following criterion holds:

Proposition 5. *("Excess price volatility")*

$$\frac{var\{\tilde{p}_t(i)\}}{var\{p_t^\dagger\}} > 1$$

if and only if

$$\lambda > 2\bar{\lambda} + \bar{\lambda}^2[1 - \rho^2(1 - \hat{k})^2]. \tag{58}$$

Proof. See the Appendix. □

Note that the criterion nests the static case result for $\rho = 0$. In this dynamic model, it does not only matter the difference between the 'true' and the subjective degree of confidence, but also other parameters. Indeed, since $\hat{k} < 1$, the discrepancy between λ and $\bar{\lambda}$ that is needed in order to have excessive volatility of the uninformed prices is smaller than in the static case. The reason is that the persistence of the shock process makes past estimates useful to forecast the future evolution of the state. But this leads to a larger reliance on private signals and therefore, comparatively to the static case, agents are driven more by their subjective perceptions. As a consequence, the amount of overconfidence needed to have excess volatility is less. This is also the case if the mass of uninformed agents (μ) increases since $\hat{k}$ becomes smaller (when $\eta < 1$) and if the degree of strategic complementarity increases, i.e. η becomes smaller.

6 An exploration

A recurring theme in our analysis is that the combination of a model of endogenous imperfect common knowledge and overconfidence has implications for both aggregate dynamics and the cross-section of individual price changes, and especially their idiosyncratic volatility. In particular, it will be made clear in this section that the behavioral bias of overprecision cannot be identified with aggregate data only; additional information on micro data is needed.

Our theoretical model is simple enough to illustrate sharply the implications of overconfidence on price-setting in macroeconomic setups and abstracts from other types of shocks like idiosyncratic productivity and demand shocks. Nevertheless, it is useful to get an idea of the potential quantitative predictions of overprecision in such a setup and to see if this particular mechanism can in some ways complement other explanations of price volatility.

Table 2: Baseline calibration.

Parameter	Value	Source/Justification
ρ	1	Data
σ_u	0.0097	Melosi (2014)
$\sigma_u/\bar{\sigma}_\xi$	0.1	Melosi (2014)
$\bar{p}^*$	0.0098	Klenow and Kryvtsov (2008)
η	0.30	Melosi (2014)
ε	6	Markup of 20%
Costs (% of s.s. profits)	U[0, 2] or U[0,10]	Managerial cost studies

Melosi (2014) has estimated a rich, quantitative imperfect common knowledge model with likelihood methods in post-war U.S. data and showed its relative success in generating persistence and monetary non-neutralities versus sticky-prices alternatives. For our purposes, we are going to inform our calibration with the estimated values of the parameters that he bears to the table. Furthermore, since our model has implications for volatility in the cross-section, we also use information from the study of Klenow and Kryvtsov (2008) (KK henceforth) on micro price data.[20]

6.1 Calibration

A main finding of KK is that prices are sticky with a median duration that can range from 3.7 to 7.2 months (if sales are excluded).[21] But in our model firms change their price in each period, even if they have incomplete information. For that reason, we decide to measure the time period of our model in quarters, during which we might reasonably assume that all the firms had the time to adjust their prices. Thus, KK statistics on monthly price changes will be taken as representing inflation statistics within a quarter.

For calibration purposes and for ease of comparison to Woodford (2002), we abstract first from technology shocks and treat the shock in the full information price p_t^* as a nominal spending shock. Furthermore, we proceed under the assumption that there is a unit root in nominal spending (51), $\rho = 1$, which is in general consistent with the data.[22]

We set the volatility of the fundamentals equal to the unconditional volatility that Melosi

[20]KK have analyzed monthly CPI data from the top 3 metropolitan areas (New York, Los Angeles and Chicago) for the period from 1988 till 2004. See Klenow and Malin (2010) for a detailed survey of price data studies.

[21]See their Table I. Similarly, Bils and Klenow (2004) report a median duration of 4.3 months.

[22]In post-war U.S. data the autocorrelation in the growth rate of quarterly nominal spending per capita is low and about 0.4. If we constrain ourselves to more recent data like the sample period of KK (1988:1-2004:4), the autocorrelation is only 0.12.

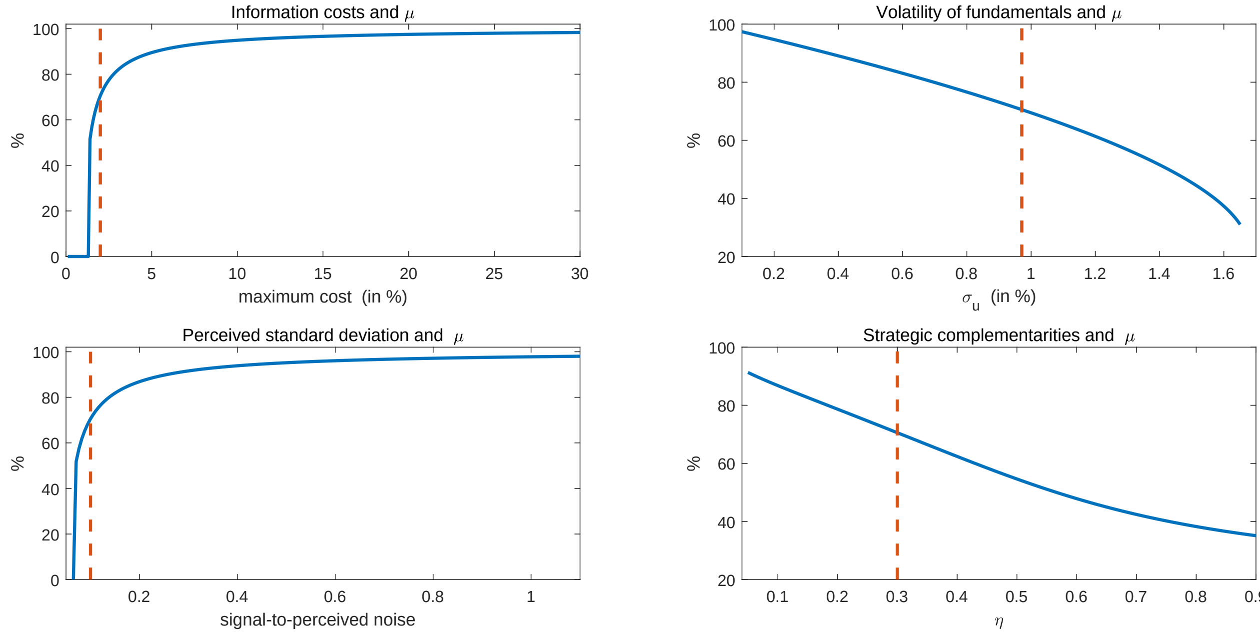

Figure 1: These panels depict the equilibrium fraction μ when we vary the information costs, the volatility of the fundamentals σ_u, the perceived signal noise $\bar{\sigma}_\xi$ (altering therefore the signal-to-noise ratio), and the degree of strategic complementarity. The vertical dotted lines correspond to the baseline calibration. Concerning the information costs, the dotted line refers to the low-cost specification.

(2014) estimates, $\sigma_u = 0.0097$.[23] Furthermore, Melosi's estimate of the signal-to-noise ratio for nominal spending shocks is small and about 0.10. We use his estimate and set $\bar{\sigma}_\xi$ so that $\sigma_u/\bar{\sigma}_\xi = 0.1$. Thus, signals about nominal spending shocks are estimated to be noisy. This should be expected, since this is the way how an imperfect common knowledge model matches the monetary non-neutralities found in the data. We set $\bar{p}^* = 0.0098$, in order to match the mean price change of 0.98% that KK report.[24]

We assume an elasticity of substitution between the differentiated products $\varepsilon = 6$, which corresponds to a markup under full information of 20%. The parameter η is critical for determining whether pricing decisions are strategic complements or substitutes and plays a crucial role in determining the persistence of the response of output to a monetary shock in sticky-price models and the strength of higher-order expectations in imperfect common knowledge models. Woodford (2002) set the degree of strategic complementarity equal to 0.15. We are more conservative and use $\eta = 0.30$ for our baseline calibration, which is the value that Melosi (2014) has estimated, but we explore also other values of η.[25]

[23] Melosi (2014) estimates an AR(1) process for the growth rate in nominal spending with 0.4 persistence and 0.0089 conditional volatility. So we set $\sigma_u = \frac{0.0089}{\sqrt{1-0.4^2}} = 0.0097$.

[24] See their Table VI. The mean price change that we set is also consistent with the mean quarter-to-quarter growth in nominal spending per capita in the period 1988:1-2004:4, which equals about 1%.

[25] Chari et al. (2000) assume that pricing decisions are strategic substitutes by setting $\eta = 2.25$. Since the data seem to favor strategic complementarities, we are going to abstain in the exercise from values of η higher than

Studies on the costs of price adjustment like Zbaracki et al. (2004) have shown that managerial and customer costs of price adjustment constitute a large fraction of firms profits even if actual menu costs are quite small. In fact, managerial costs (which refer to information gathering, decision-making and communicating-to-sales-team costs) are 4.61% of the profits, whereas customer costs (communication and negotiation costs) can reach even 15.01% of profits. The concept of managerial costs is the closest to the notion of information-processing costs in our model, so an average measure of cost that corresponds to 4.61% of profits maybe the most relevant for us. However, since the equilibrium fraction of uninformed firms crucially depends on the calibration of costs, we want to be more agnostic about their value and consider a low and high cost specification. More specifically, we assume a uniform distribution of costs per period as a fraction of steady state profits with a minimum cost of zero and a maximum cost that is either 2% or 10%, so the average cost is 1% and 5% respectively.[26]

6.2 Information acquisition, monetary non-neutrality and inflation

Information acquisition. The decision to obtain information depends on the level of fixed costs, the volatility of fundamentals σ_u, the *perceived* signal-to-noise ratio $\sigma_u/\bar{\sigma}_\xi$ (which is equal to $\bar{\lambda}^{-1/2}$) and the degree of strategic complementarity η. Figure 1 depicts the equilibrium fraction of uninformed firms, that is, the solution to the fixed point problem in (57), when each of these respective parameters change. The top left panel shows that if information becomes more costly, then more firms stay uninformed. The top right panel shows the effect of an increase in fundamental uncertainty σ_u. An increase in σ_u leads to higher volatility of the price of informed firms, as we can see in both the static and the dynamic setup, in equations (32) and (56) respectively. Thus, the firm has more incentives to acquire information. However, an increase in the standard deviation of the hidden state increases the signal-to-noise ratio, or in other words, it increases the confidence of firms on their own signals. This diminishes the incentives to acquire information, mitigating the effect of a rise in σ_u. Overall, the volatility effect dominates, and the equilibrium fraction of firms falls when σ_u increases.

The bottom left panel in Figure 1 increases the degree of confidence on the own signals by decreasing the perceived volatility $\bar{\sigma}_\xi$, but keeping the level of fundamental uncertainty constant. This leads to a higher signal-to-noise ratio, and to stronger incentives to stay uninformed. Thus, the equilibrium fraction of uninformed firms is increasing. This result in the dynamic economy corresponds to the static result of proposition 3. Therefore, higher subjective precision leads to more uninformed firms in equilibrium relative to the rational benchmark. Finally, the bottom right panel shows that an increase in strategic complementarities, captured by a reduction in η,

unity.

[26]See for further details the Appendix. Note that the maximum cost even in the high cost parametrization is still smaller than the measure of customer costs in Zbaracki et al. (2004).

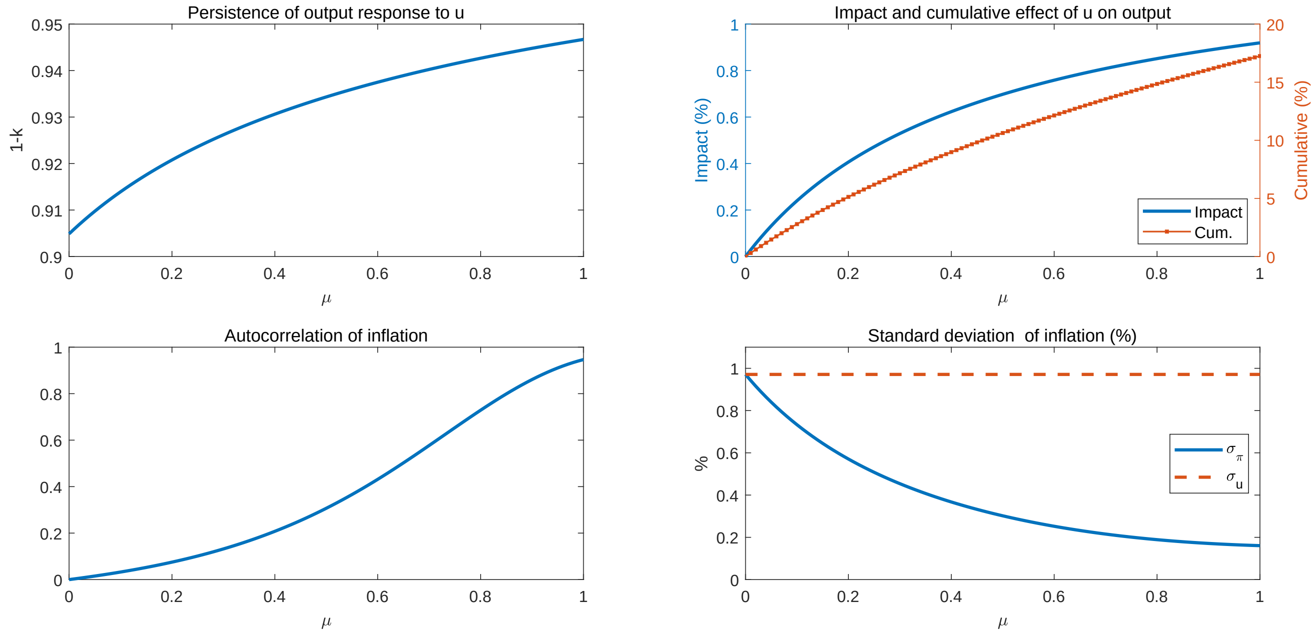

Figure 2: The top left panel depicts the persistence of output $(1-\hat{k})$ to nominal spending shocks as a function of μ. The top right panel depicts the impact and cumulative effect of a nominal spending shock of size σ_u. The bottom left and right panels plot the autocorrelation and standard deviation of inflation respectively. The formulas are provided in the Appendix. All graphs use the baseline calibration.

leads to more uninformed firms.

Persistence and monetary non-neutrality. Since we restrict out attention to nominal spending shocks, equilibrium output in deviation from the steady state (i.e. $y_t \equiv \ln Y_t/\bar{Y}$) satisfies $y_t + p_t = p_t^*$.[27] Using the law of motion for the price index (52) for $\rho = 1$, allows us to express y_t as an autoregressive process of order 1,

$$y_t = (1-\hat{k})y_{t-1} + \frac{1-\hat{k}}{\eta + (1-\eta)\mu}\mu u_t, \tag{59}$$

where $\hat{k}$ satisfies equation (53) for $\rho = 1$.

The degree of monetary non-neutrality is captured by the degree of output persistence to a monetary shock, measured by $1-\hat{k}$, and the impact effect of a monetary shock, $\frac{\partial y_t}{\partial u_t} = \frac{1-\hat{k}}{\eta+(1-\eta)\mu}\mu$. When there is no endogenous margin of information acquisition as in Woodford (2002) ($\mu = 1$), $\hat{k}$ depends only on the signal-to-noise ratio and the degree of strategic complementarity η. With endogenous information acquisition though, $\hat{k}$ changes also with the endogenous changes in μ. Similarly, the impact effect of a monetary shock on output depends on the fraction of uninformed firms, both directly and through $\hat{k}$. For example, if all firms are informed ($\mu = 0$), output does

[27]Recall that $p_t^* = const. + \ln M_t - \frac{1}{\eta}\ln A_t$.

not respond to a monetary shock, $\partial y_{t+i}/\partial u_t = 0, \forall i \geq 0$, and we retrieve monetary neutrality.

Figure 2 makes these points graphically. The top left panel shows that the persistence of the output response increases as the fraction of uninformed firms increases. The top right panel plots the respective impact and cumulative effect of a nominal spending shock of size σ_u on output.[28] Depending on how many firms are uninformed, which will be determined by the size of information costs, the cumulative effect can be from zero to about 17%.

To sum up, the degree of monetary non-neutrality depends in a non-trivial way on the mass of uninformed firms. In contrast to setups where there is no choice to obtain information, as in Woodford (2002) and Melosi (2014), the signal-to-noise ratio and the degree of strategic complementarity are not sufficient anymore to determine the effects of a nominal spending shock on output. Instead, the *level* of uncertainty in fundamentals σ_u matters for information acquisition and therefore, for monetary non-neutrality.

Aggregate inflation dynamics. Under the assumption of $\rho = 1$, the process of the inflation rate is given by

$$\pi_t = \hat{k}\bar{p}^* + (1-\hat{k})\pi_{t-1} + \Big(1 - \frac{(1-\hat{k})\mu}{\eta + (1-\eta)\mu}\Big)u_t - \frac{(1-\hat{k})\eta(1-\mu)}{\eta + (1-\eta)\mu}u_{t-1}. \tag{60}$$

The average rate of inflation therefore corresponds to the average value of nominal spending, $E\pi_t = \bar{p}^*$. When everybody is informed ($\mu = 0$), inflation collapses to the i.i.d. full information process Δp_t^* in (51). In contrast, when some firms stay uninformed, $(0 < \mu < 1)$, inflation acquires an autoregressive and a moving average component, both of order one. If everybody is uninformed as in Woodford (2002), only the autoregressive part of order one survives. The bottom panels of figure 2 display the first-order autocorrelation and standard deviation of inflation as a function of μ. In these panels we see the tension between variance and persistence that arises in models with imperfect information. A larger fraction of uninformed firms increases persistence, but reduces the standard deviation of inflation since firms are solving a filtering problem, so their responses on aggregate are smoother. It is important to note that the smooth (but persistent) responses concern aggregate inflation. The response of idiosyncratic inflation rates differs, since it is affected by the overconfidence of price-setters, a subject we now turn to.

6.3 Overprecision experiments and micro price volatility

Macro and micro data. The processes for output and inflation in (59) and (60) show that aggregate (macro) data are not sufficient to identify the extent of overprecision. The reason is that the idiosyncratic noise washes out in the aggregate due to a law of large numbers.

[28]The cumulative output effect is $\sum_{i=0}^{\infty} \frac{\partial y_{t+i}}{\partial u_t} = \sum_{i=0}^{\infty}(1-\hat{k})^i \frac{\partial y_t}{\partial u_t} = \frac{\partial y_t}{\partial u_t}/\hat{k}$.

Table 3: Overprecision and price changes.

	Baseline			$\sigma_u = 1.5 \times$ **Baseline**		
	Endogenous μ		*Exogenous* μ	*Endogenous* μ		*Exogenous* μ
	cost 2%	**cost 10%**		**cost** 2%	**cost** 10%	
μ (%)	70.55	94.90	100	48.25	92.29	100
$1 - \hat{k}$	0.94	0.946	0.947	0.902	0.919	0.921
Impact eff. on y (%)	0.81	0.90	0.92	0.99	1.31	1.34
Cum. eff. on y (%)	13.61	16.66	17.25	10.17	16.13	17.02
Std(dp)	0.19	0.13	0.12	0.49	0.26	0.24
$\gamma = 1$						
$\|dp\|$	1.00	1.00	0.99	1.11	1.08	1.07
Std(dp_i)	0.54	0.54	0.54	0.89	0.82	0.82
$\gamma = 2$						
$\|dp\|$	1.17	1.18	1.18	1.40	1.49	1.50
Std(dp_i)	0.95	1.04	1.06	1.35	1.55	1.59
$\gamma = 3$						
$\|dp\|$	1.44	1.49	1.50	1.74	2.01	2.04
Std(dp_i)	1.37	1.54	1.58	1.81	2.28	2.36
$\gamma = 4$						
$\|dp\|$	1.73	1.84	1.86	2.10	2.56	2.63
Std(dp_i)	1.78	2.05	2.11	2.28	3.02	3.14
$\gamma = 6$						
$\|dp\|$	2.35	2.59	2.63	2.83	3.70	3.83
Std(dp_i)	2.61	3.07	3.16	3.22	4.50	4.70
$\gamma = 8$						
$\|dp\|$	3.00	3.37	3.44	3.57	4.87	5.05
Std(dp_i)	3.44	4.08	4.21	4.16	5.99	6.27

All price changes are reported in %. Average inflation is given by $\bar{p}^*$, so it is 0.98% for all parameterizations. The impact and cumulative effects are multiplied by σ_u. The left panel of the table reports statistics for the baseline calibration (for low and high c) and for the case of Woodford (2002), where $\mu = 1$. The right panel of the table increases σ_u by 50% ($\sigma_u = 0.0146$), so the signal-to-noise ratio increases to 0.15. *Std*(dp_i) corresponds to the average standard deviation of each firm's prices. *Std*(dp) corresponds to the standard deviation of inflation. We use 2,000 simulations for each parametrization and report averages across simulations. For each simulation we used 20,000 firms for 68 quarters.

Therefore, any aggregate data on inflation and output will *not* reflect the true variance of the noise shocks σ_ξ, but *only* the perceived variance of the noise shocks, $\bar{\sigma}_\xi$, through the filtering and the information acquisition problem. As a result, unless we use information from micro price studies (as the study of KK), we *cannot* tell from aggregate data only if the responses of inflation and output reflect overconfident price-setters.[29]

The above point warrants further discussion. One would think that the overconfidence bias, i.e. the fact that $\bar{\sigma}_\xi < \sigma_\xi$, should show up in the aggregate through the *filtering* problem of the firm, which is captured by the $\hat{k}$, a linear combination of the vector of Kalman gains. It does not, due to the linearity of Kalman filtering, a necessary feature of our setup that makes manageable the calculation of higher-order beliefs.[30] It all boils down to the fact that we have followed a conservative route in introducing the behavioral bias of overconfidence: it only affects the "subjective perceptions" of price-setters about the world. If we had instead assumed that agents are overconfident in their estimates of the variance of the fundamentals, $\bar{\sigma}_u < \sigma_u$, then both perceived and true variances, $\bar{\sigma}_u$ and σ_u, would affect aggregate dynamics.

Fixing macro implications: increase true noise. We proceed now to the implications of overprecision on idiosyncratic price volatility. Of particular interest is the finding of KK in their monthly sample from years 1988 to 2004 that the absolute price change for goods that compose the CPI index averages about 10%.[31] So idiosyncratic prices change substantially in absolute value, with changes that are both positive and negative in sign.[32] How does overprecision affect price volatility, even in a stripped down environment that lacks idiosyncratic shocks?

In the current exercise we will introduce overprecision by keeping the perceived standard deviation *constant* and by increasing the true standard deviation, σ_ξ. This choice allows us to investigate the effects of overprecision on idiosyncratic price volatility without altering the aggregate implications of imperfect information, since both the decision to obtain information and the filtering problem depend on the perceived volatility $\bar{\sigma}_\xi$ and not the true one. We choose as an index of overconfidence the parameter $\gamma \equiv \sigma_\xi / \bar{\sigma}_\xi$.

Table 3 reports both macro and micro moments from a simulation of 20,000 firms for 68 quarters (so that we have the same sample length as KK). We use both the small and the high cost specification and consider also the case of Woodford (2002) by setting exogenously $\mu = 1$. Consider first the left panel of the table that refers to the baseline calibration of table 2. If costs are small, the fraction of uninformed firms is about 70%. The persistence of the output response to a nominal spending shock is 0.94 with an impact effect of 0.8% and a cumulative effect of

[29]This fact allows us to use the estimate of Melosi (2014), who uses aggregate data only, in our calibration of the perceived variance of noise. Melosi treats it as the true noise variance, since there is no such distinction in his setup. Through the lens of our model though, using only aggregate data provides information only about the perceived noise variance.

[30]See footnote 16 for the same point in the static economy.

[31]This is how KK summarize the findings that they report in Table III.

[32]Recall that for the same sample the average price change is just 0.98%.

13.6%. The respective half-life is about 11 quarters, which is consistent with the estimates that Melosi reports.[33] Furthermore, we see that the volatility of the inflation rate is small and about 0.19%, whereas in a full information setup it would be equal to σ_u, so it would amount to 0.97%.

Turning to the micro price statistics, consider first the case of $\gamma = 1$, which corresponds to a rational signal-extraction problem. The mean absolute change $|dp|$ is 1% and the average (across firms) standard deviation of the idiosyncratic inflation rate is about 0.5%. By increasing overprecision up to $\gamma = 8$ we see that the average absolute change becomes 3% and the standard deviation of idiosyncratic inflation is raised to 3.44%. Similar figures are obtained when we consider a higher cost specification that leads to 95% of firms staying uninformed. The persistence increases slightly and the cumulative effect of nominal spending shocks is about 17%. Overprecision increases again the average absolute change of prices and the respective standard deviation. Note that for γ larger or equal than 2, the standard deviation of idiosyncratic inflation rates exceeds also the volatility of the fundamentals, σ_u.

What are we learning from this exercise? It all depends on what amounts of overconfidence we consider as "reasonable". Indeed, experimental studies, like Soll and Klayman (2004), have shown that on a series of questions where individuals are asked to form an 80% confidence interval the actual hit rate is around 40%, which can be translated in γ being approximately equal to 2.5.[34] In more complicated tasks, as forecasting the level of the exchange rate with a confidence interval of 90% (see Oberlechner and Osler (2009) and Oberlechner and Osler (2012)), the hit rate ranges from 5% to 70% with an average of 40%, rationalizing values of γ higher than 3. Similarly, CFOs in the study of Ben-David et al. (2013) have a hit rate of only 36.3% when they are asked to form 80% confidence intervals on market-wide stock returns. Barrero (2018) uses managerial survey data to structurally estimate a model of firm dynamics and finds a $\gamma = 2$. In general values from 2 to 4 can be considered as reasonable.

Thus, for γ up to 4 the average absolute price change doubles and the standard deviation of idiosyncratic inflation rates quadruples. Similar results obtain if we increase conservatively the baseline volatility of the nominal spending shock by 50%, as the right part of table 3 shows. Note that, as expected from the analysis of figure 1, an increase in volatility reduces the mass of uninformed firms to 50% and the persistence to 0.9, implying now a half-life of 6.7 quarters. Increasing the costs increases obviously the fraction of uninformed firms. The cumulative effect of a monetary shock can vary from 10% to 17% in the case of $\mu = 1$. Regarding the micro

[33] The half-life of the output response to a nominal spending shock is given by $\tau = -\ln 2/\ln(1-\hat{k})$. In Melosi (2014) the half-life is about 12 quarters (see Figure 3, page 19). Christiano et al. (2005), who consider shocks to interest rates, show that the effect of a monetary shock on output dies after 12 quarters, so the half-life is smaller. Note that in VAR studies the output and inflation responses are hump-shaped; this is not true in our simple setup, but it would be true in a larger imperfect common knowledge model with persistence in the growth rate of nominal spending. Such a setup would require additional state variables. See Melosi (2014) for further details.

[34] This value can be obtained by rough computation on confidence intervals for normal distributions.

Table 4: Overprecision with σ_u reflecting both nominal spending and technology shocks.

		$\sigma_u = 0.0306$				
		$\frac{\sigma_u}{\bar{\sigma_\xi}} = 0.4$			$\frac{\sigma_u}{\bar{\sigma_\xi}} = 0.66$	
		μ	$1-\hat{k}$		μ	$1-\hat{k}$
		87.39 %	0.795		92.84 %	0.692
γ	Std(dp)	$\lvert dp \rvert$	Std(dp_i)	Std(dp)	$\lvert dp \rvert$	Std(dp_i)
1	1	1.71	1.87	1.27	1.79	1.98
2		2.69	3.20		2.72	3.24
3		3.78	4.61		3.77	4.62
4		4.90	6.04		4.88	6.03
6		7.16	8.91		7.13	8.88
8		9.45	11.80		9.40	11.76

The high information cost specification was used for this exercise. The perceived (or else subjective) standard deviation of the noise is adjusted so that the signal-to-noise ratio is either 0.4 or 0.66. The standard deviation of inflation, Std(dp), does not depend on the true variance of the shocks, so it is reported only once for each calibration.

price moments, $|dp|$ ranges from 1.11% to 3.57% and the standard deviation from 0.89% to 4.16% when we raise γ from unity to $\gamma = 8$ for the low-cost specification. If everybody were uninformed, then $|dp|$ can increase up to 5% and the standard deviation up to 6.3%. For a more conservative specification of γ from 2 to 4, we get again that the absolute price change doubles and the standard deviation quadruples.[35]

More variable fundamentals? By focusing on nominal spending shocks, we are constrained in how we calibrate the volatility of fundamentals. This is reflected in the low variance of aggregate inflation, which in Table 3 ranges from 0.12 to 0.5% for either the baseline or the higher volatility scenario. This outcome is related to the tension between volatility and persistence in filtering problems that we highlighted in figure 2. Note that the respective standard deviation of price changes in the KK data is 1.19%. We could get more volatility of both the aggregate inflation rate and the idiosyncratic inflation rates (raising therefore $|dp|$) if we also considered the volatility of aggregate technology shocks. Remember that our full information price is a linear

[35] In the Appendix we perform two additional overprecision exercises. First, we increase the signal-to-noise ratio to $\sigma_u/\bar{\sigma}_\xi = 0.4$ by decreasing the perceived variance (keeping σ_u to its baseline value). This is the value that Woodford (2002) uses. Second, we consider the case where strategic complementarities are stronger, $\eta = 0.15$. Table C.1 provides the results. Similar conclusions about $|dp|$ and idiosyncratic inflation volatility can be drawn.

combination of both nominal spending and technology shocks. Taking that into consideration, we could set $\sigma_u = 0.0306$, that is, a volatility of about 3%.[36]

Table 4 reports the results of the same overprecision exercise as the one we performed in table 3, using now a σ_u that reflects both sources of fundamental uncertainty. Before we comment on the table, we want to highlight two potential issues when we consider technology shocks in the calibration of our one-signal setup. First, it is not clear what the volatility of the idiosyncratic noise should be. Melosi (2014), who uses an environment with two signals, showed that the signal-to-noise ratio for nominal spending shocks is low, of the order of 0.1, implying noisy private signals about nominal spending, whereas the signal-to-noise ratio for technology shocks is high, of the order of 0.66, implying informative signals about technology shocks. For our case, we are going to set a signal-to-noise ratio that is 0.4. This is about the average of the two extremes of Melosi (2014), and corresponds also to the baseline value of Woodford (2002). Moreover, we are also going to derive results for informative signals with a signal-to-noise ratio of 0.66.[37]

The second issue that emerges is that when u_t is a linear combination of the two shocks, we cannot derive anymore the output response as in (59). Thus, we have to be more careful about the interpretation of results about persistence and the degree of monetary non-neutrality. The inflation response in (60) is valid though, so inflation persistence comments are legitimate.

Turning now to table 4, we note that aggregate persistence ranges from 0.8 to 0.7 for the more informative signal specification. More importantly, for both signal-to-noise ratios, the absolute value of price changes increases a lot with overconfidence. For values of γ between 2 and 4, $|dp|$ ranges from 3% to 5%, with a respective idiosyncratic price volatility that ranges from about 3% to 6%. Aggregate inflation has also larger volatility, of the order of 1-1.27 % which is similar to the value that KK find in the data.[38]

To sum up, when we calibrate the fundamental uncertainty more realistically, "reasonable" amounts of overconfidence do not only double or triple the average absolute price change relative to the rational benchmark, but are also able to explain up to 50% of the 10% absolute change that KK found in the data. We think that these results suggest that our proposed mechanism can have some value in explaining price volatility –although we acknowledge that there can be other important mechanisms from which we have abstracted in this analysis.

[36] Recall that $\Delta p_t^* = \Delta \ln M_t - \frac{1}{\eta}\Delta \ln A_t$. Thus, assuming independence, we have $Var(\Delta p_t^*) = Var(\Delta \ln M_t) + \frac{1}{\eta^2}Var(\Delta \ln A_t)$. Use as before the value of 0.0097 for the standard deviation of the nominal spending shock, and let $\eta = 0.30$. Melosi's estimate of the volatility of the growth rate in technology is 0.0087. Thus, $\sigma_u = \sqrt{0.0097^2 + \frac{0.0087^2}{0.30^2}} = 0.0306$.

[37] Note also that if we set a signal-to-noise ratio equal to 0.1, firm's confidence on its unreliable signals would be low, which, in combination with a high volatility of the fundamentals, would lead to everybody becoming informed, $\mu = 0$. The same outcome would emerge if we used the low-cost specification, which is why we use the high-cost specification in table 4.

[38] KK do not report idiosyncratic inflation volatilities.

Table 5: Overprecision: fixing the true variance of noise shocks.

				$\sigma_\xi = 0.0971$				
$\bar{\sigma}_\xi$	γ	μ (%)	$1-\hat{k}$	Impact (%)	Cum. (%)	Std(dp)	$\|dp\|$	Std(dp_i)
0.0486	2	86.87	0.891	0.83	7.63	0.22	1.18	1.01
0.0324	3	91.58	0.844	0.80	5.13	0.26	1.47	1.50
0.0243	4	93.85	0.800	0.76	3.80	0.31	1.78	1.98
0.0194	5	95.18	0.758	0.72	2.99	0.34	2.12	2.44
0.0162	6	96.06	0.718	0.69	2.44	0.38	2.45	2.89
0.0139	7	96.68	0.680	0.65	2.04	0.41	2.78	3.33
0.0121	8	97.14	0.645	0.62	1.75	0.44	3.10	3.75

In this table we fix the true variance of the noise to the baseline perceived noise ($\sigma_\xi = 0.0971$) and decrease the perceived variance. The rest of the baseline calibration is the same. The exercise is performed for the low maximum cost case of 2%. The signal-to-noise ratio increases from 0.1 ($\gamma = 1$) to 0.8 ($\gamma = 8$). The $\gamma = 1$ case is reported in the left part of table 3. Both aggregate and idiosyncratic statistics vary in this experiment.

Altering both macro and micro implications: decrease perceived noise. We will finish our analysis by illustrating what would happen if we fixed the true noise variance and decreased the perceived variance. Table 5 reports the results of this experiment for the baseline calibration. On the one hand, for values of γ up to 4, we again obtain the result that $|dp|$ doubles and idiosyncratic inflation volatility quadruples. On the other hand, such a way of introducing overconfidence is *not* consistent with having considerable monetary non-neutralities. To see that, note that aggregate statistics will vary, since we reduce the perceived variance of noise. More reliance on the private signal leads to a higher fraction of uninformed firms. However, since the signal-to-noise ratio increases, signals become subjectively more informative, resulting into a decrease in the persistence of the output response to a monetary shock. This can be seen sharply in the cumulative output effect of a nominal spending shock that falls from 13%, when $\gamma = 1$, to just 3.8%, when $\gamma = 4$, or even less if we decrease further $\bar{\sigma}_\xi$.

7 Concluding remarks

In this paper, we study the behavior of individual and aggregate prices in an economy with monopolistic-competitive firms that is driven by aggregate shocks observed with noise. Each firm receives a private signal about the hidden state. We assume that firms are overconfident in their signals, that is, their overestimate the precision of their own perception of the aggregate state.

This model can rationalize a persistent response of output to the aggregate hidden state and be consistent at the same time with excess volatility of individual prices, providing therefore a unified "noise" interpretation of aggregate and idiosyncratic prices. We see our approach as complementary to setups where idiosyncratic fundamental shocks are the main driver of price volatility.

More generally, we believe that behavioral biases, especially in the processing and analysis of information, are not easy to dismiss. There has been substantial progress in recent years in collecting data about actual decision makers in real market conditions. For example, Bachmann and Elstner (2015) use confidential German manufacturing survey data and document non-negligible instances of systematic positive and negative expectation errors. Studies like Ben-David et al. (2013) and Barrero (2018) document substantial second-moment biases in the forecasts of CFOs and managers. Such evidence opens the avenue for a thorough quantitative evaluation of overconfidence in more elaborate models.

A Static model

A.1 Proof of proposition 1

Derivation of condition (13) from (12). We first note that by using condition (7) we can rewrite marginal costs in terms of $P^{\dagger}$ as $W/A = \frac{\varepsilon-1}{\varepsilon}P^{\dagger}$. Use also condition (11) and rewrite the expected profits as functions of $P^{\dagger}$, $\tilde{P}(j)$ and $Z \equiv MP^{\varepsilon-2}$,

$$E^j\{\pi(P^{\dagger}, P, \theta)\} = \frac{1}{\varepsilon}E^j\{\left(P^{\dagger}\right)^{1-\varepsilon} Z\}, \tag{A.1}$$

$$E^j\{\pi(\tilde{P}(j), P, \theta)\} = \frac{1}{\varepsilon}\tilde{P}^{1-\varepsilon}(j)E^jZ. \tag{A.2}$$

We take a second order approximation of expected profits around a deterministic steady state where $\theta = \bar{\theta}$ and as a result $P = P^{\dagger} = \tilde{P} \equiv \bar{P}$. Let lowercase variables denote log-deviations from the steady state and let $\|p\|$ and $\|z\|$ denote a bound on the size of fluctuations for the price of each differentiated good and for the variable Z respectively. We can obtain by approximating equation (A.1) that

$$\begin{aligned} E^j\left\{\pi(P^{\dagger}, P, \theta)\right\} &= \frac{1}{\varepsilon}\bar{P}^{1-\varepsilon}\bar{Z}\Big[1 + (1-\varepsilon)E^jp^{\dagger} + \frac{1}{2}(1-\varepsilon)^2E^j\left(p^{\dagger}\right)^2 + \\ &\quad + E^jz + \frac{1}{2}E^jz^2 + (1-\varepsilon)E^jp^{\dagger}z\Big] + \mathcal{O}(\|p,z\|^3). \end{aligned} \tag{A.3}$$

Similarly by approximating equation (A.2) we obtain that

$$\begin{aligned} E^j\{\pi(\tilde{P}(j), P, \theta)\} &= \frac{1}{\varepsilon}\bar{P}^{1-\varepsilon}\bar{Z}\Big[1 + (1-\varepsilon)\tilde{p}(j) + \frac{1}{2}(1-\varepsilon)^2\tilde{p}(j)^2 \\ &\quad + E^jz + \frac{1}{2}E^jz^2 + (1-\varepsilon)\tilde{p}(j)E^jz\Big] + \mathcal{O}(\|p,z\|^3). \end{aligned} \tag{A.4}$$

Note that $\bar{P}^{1-\varepsilon}\bar{Z} = \frac{\bar{M}}{\bar{P}} = \bar{Y}$ and let

$$W(j) \equiv \pi(P^{\dagger}, P, \theta) - \pi(\tilde{P}(j), P, \theta)$$

denote the difference in profits. Then using (A.3) and (A.4) we have

$$\begin{aligned} E^j\{W(j)\} &= \frac{\varepsilon-1}{\varepsilon}\bar{Y}\Big\{\tilde{p}(j) - E^jp^{\dagger} + \frac{1}{2}(\varepsilon-1)\left[E^j\left(p^{\dagger}\right)^2 - (\tilde{p}(j))^2\right] \\ &\quad - E^j[\left(p^{\dagger} - \tilde{p}(j)\right)z]\Big\} + \mathcal{O}(\|p,z\|^3). \end{aligned} \tag{A.5}$$

Note that the price of the uninformed agents (1) at a first order approximation is

$$\tilde{p}(j) = E^jp^{\dagger} + \mathcal{O}(\|p,z\|^2). \tag{A.6}$$

Furthermore, if we take a second-order approximation of it we obtain

$$\tilde{p}\,(j) - E^j p^\dagger = \frac{1}{2} Var^j\left(p^\dagger\right) + E^j\left[p^\dagger - E^j p^\dagger\right] z + \mathcal{O}(\|p, z\|^3), \tag{A.7}$$

where $Var^j(p^\dagger) \equiv E^j(p^\dagger)^2 - (E^j p^\dagger)^2$. Using (A.6) and (A.7) into (A.5) we observe that the terms involving z cancel out and that $E^j(p^{\dagger 2} - \tilde{p}(j))^2 = Var^j(p^\dagger)$, so ignoring third order terms we obtain

$$\begin{aligned} E^j\{W(j)\} &= \frac{1}{2}\frac{\varepsilon - 1}{\varepsilon}\bar{Y}\{Var^j(p^\dagger) + (\varepsilon - 1)\,Var^j(p^\dagger)\} \\ &= \frac{\bar{Y}}{2}\,(\varepsilon - 1)\,Var^j(p^\dagger). \end{aligned}$$

Thus firms acquire information if and only if

$$Var^j\left(p^\dagger\right) \geq c^j$$

where $c^j \equiv \frac{2}{\bar{Y}(\varepsilon - 1)}\tilde{c}^j$ which is expression (13) in the text.

The same calculations apply in the dynamic case, where each variable is indexed with t. In this case the expected difference of profits at time t, in (47), is equal to

$$E_t^j\{\pi(P_t^\dagger, P_t, \theta_t) - \pi(\tilde{P}_t(j), P_t, \theta_t)\} = \frac{\bar{Y}}{2}\,(\varepsilon - 1)\,Var_t^j(p_t^\dagger)$$

in a second-order approximation, where the expectation operator is conditional on the private history of signals including time t. So $Var_t^j(p_t^\dagger)$ corresponds to the subjective contemporaneous variance of $p_t^\dagger$.

A.2 Proof of proposition 3

Equation (34) defines implicitly the equilibrium fraction μ as a function of $\bar{r}$, $\mu(\bar{r})$. We have $c^* = c^*\left(\bar{r}, \mu\left(\bar{r}\right)\right)$. Differentiate implicitly (34) to get

$$\frac{d\mu}{d\bar{r}} = -\frac{f(c^*)\frac{\partial c^*}{\partial \bar{r}}}{1 + f(c^*)\frac{\partial c^*}{\partial \mu}}. \tag{A.8}$$

Note that

$$\frac{\partial c^*}{\partial \mu} = \frac{-2\eta^2(1-\bar{r})^2\sigma_u^2(1-\eta)}{\left(\eta + (1-\eta)(1-\bar{r})\mu\right)^3} \tag{A.9}$$

$$\frac{\partial c^*}{\partial \bar{r}} = \frac{\eta^2\sigma_u^2}{\left(\eta + (1-\eta)(1-\bar{r})\mu\right)^3}\left[(1-\eta)(1-\bar{r})\mu - \eta\right] \tag{A.10}$$

As we noted in the text, when $\eta < 1$, we have strategic complementarities in information acquisition, $\frac{\partial c^*}{\partial \mu} < 0$. The opposite happens when $\eta > 1$, $\frac{\partial c^*}{\partial \mu} > 0$.

The denominator of (A.8) is always positive when we have strategic substitutes. In the case of strategic complements, we are going to assume that the complementarities are not large enough to make the denominator negative, so we assume that $-f(c^*)\frac{\partial c^*}{\partial \mu} < 1$. This makes the LHS of (34) an increasing function of μ with slope less than unity. This is what we call a "stable" equilibrium.

The above discussion implies that the sign of $dc^*/d\bar{r}$ is determined by the sign of $\partial c^*/\partial \bar{r}$. The result follows from (A.10).

B Infinite-horizon model

B.1 Proof of proposition 4

Derivations of (54) and (55). We proceed using the method developed in Woodford (2002). We claim that the relevant hidden state is

$$X_t = \begin{bmatrix} p_t^* \\ p_t \end{bmatrix}$$

and guess that it evolves according to a linear law of motion

$$X_t = f + MX_{t-1} + mu_t, \tag{B.1}$$

where

$$f \equiv \begin{bmatrix} \bar{p}^* \\ \bar{p} \end{bmatrix}, \qquad M \equiv \begin{bmatrix} \rho & 0 \\ a & b \end{bmatrix}, \qquad m \equiv \begin{bmatrix} 1 \\ c \end{bmatrix},$$

are vectors and matrices to be determined. Note that our variables of interest are the prices of the informed firms which can be written as $p^\dagger = \bar{\eta}' X_t$ and that of the uninformed which can be written as $\tilde{p}_t(i) = \bar{\eta}' E_t^i X_t$, where $\bar{\eta}' = (\eta, 1-\eta)$.

Let $e_1 = (1,0)'$. We can write the following system

$$\begin{aligned} X_t &= f + MX_{t-1} + mu_t \\ s_t^i &= e_1' X_t + \xi_t^j \end{aligned}$$

where the second line corresponds to the observational equation. We proceed assuming a stationary filtering problem. The filtering equation of a generic uninformed firm j is given by

$$E_t^j X_t = E_{t-1}^j X_t + K(s_t^j - E_{t-1}^j p_t^*), \tag{B.2}$$

where K is the vector of Kalman gains pre-multiplied with M^{-1}. Using (B.1) we obtain that $E_{t-1}^j X_t = f + ME_{t-1}^j X_{t-1}$ and $E_{t-1}^j p_t^* = \bar{p}^* + \rho E_{t-1}^j p_{t-1}^*$; we can then write (B.2) as

$$E_t^j X_t = f + ME_{t-1}^j X_{t-1} + K(s_t^j - \bar{p}^* - \rho E_{t-1}^j p_{t-1}^*).$$

Aggregating among all agents j that are uninformed and guessing that in equilibrium μ will be non-random (as in the static case) we obtain

$$\begin{aligned} \bar{E}_t X_t &= f + M\bar{E}_{t-1} X_{t-1} + K\left(p_t^* - \bar{p}^* - \rho \bar{E}_{t-1} p_{t-1}^*\right), \\ &= f + M\bar{E}_{t-1} X_{t-1} + \rho K\left(p_{t-1}^* - \bar{E}_{t-1} p_{t-1}^*\right) + Ku_t, \end{aligned} \tag{B.3}$$

which is the law of motion of the average estimate, where we have used the law of large numbers.

Our target is to express the price level p_t in terms of X_{t-1}. The general price index can be expressed as a function of the full information price and average expectations as

$$\begin{aligned} p_t &= \delta\left[(1-\mu)\, p_t^* + \mu \bar{E}_t p_t^*\right] + (1-\delta)\, \bar{E}_t p_t \\ &= (\delta\mu, 1-\delta)\, \bar{E}_t X_t + \delta\,(1-\mu)\, p_t^* \\ &= (\delta\mu, 1-\delta)\, \bar{E}_t X_t + \delta\,(1-\mu)\left(\bar{p}^* + \rho p_{t-1}^* + u_t\right). \end{aligned}$$

Using the law of motion (B.3) to substitute for $\bar{E}_t X_t$ and collecting terms we have

$$\begin{aligned} p_t = {} & \left[(\delta\mu, 1-\delta)\, f + \delta\,(1-\mu)\, \bar{p}^*\right] + \rho\left[\delta\,(1-\mu) + \bar{K}\right] p_{t-1}^* + (\delta\mu, 1-\delta)\, M \bar{E}_{t-1} X_{t-1} \\ & -\rho \bar{K} \bar{E}_{t-1} p_{t-1}^* + \left[\delta\,(1-\mu) + \bar{K}\right] u_t, \end{aligned}$$

where $\bar{K} \equiv (\delta\mu, 1-\delta)\, K$. Finally, using the definition of M and $\bar{E}_{t-1} X_{t-1}$ and noting that

$$(\delta\mu, 1-\delta)\, f + \delta\,(1-\mu)\, \bar{p}^* = \delta \bar{p}^* + (1-\delta)\, \bar{p}$$

we obtain that

$$\begin{aligned} p_t = {} & \delta \bar{p}^* + (1-\delta)\, \bar{p} + \rho\left[\delta\,(1-\mu) + \bar{K}\right] p_{t-1}^* + \left[\delta\mu\rho + (1-\delta)\, a - \rho \bar{K}\right] \bar{E}_{t-1} p_{t-1}^* \\ & + (1-\delta)\, b \bar{E}_{t-1} p_{t-1} + \left[\delta\,(1-\mu) + \bar{K}\right] u_t. \end{aligned} \tag{B.4}$$

Since

$$p_{t-1} = \delta\mu \bar{E}_{t-1} p_{t-1}^* + (1-\delta)\, \bar{E}_{t-1} p_{t-1} + \delta\,(1-\mu)\, p_{t-1}^*,$$

we can use this expression to substitute for $\bar{E}_{t-1} p_{t-1}$ in (B.4) and arrive at

$$\begin{aligned} p_t = {} & \delta \bar{p}^* + (1-\delta)\, \bar{p} + \left[\delta\,(1-\mu)\,(\rho - b) + \rho \bar{K}\right] p_{t-1}^* \\ & + \left[\delta\mu\,(\rho - b) + (1-\delta)\, a - \rho \bar{K}\right] \bar{E}_{t-1} p_{t-1}^* \\ & + b p_{t-1} + \left[\delta\,(1-\mu) + \bar{K}\right] u_t. \end{aligned} \tag{B.5}$$

We note that (B.1) implies

$$p_t = \bar{p} + a p_{t-1}^* + b p_{t-1} + c u_t. \tag{B.6}$$

We can then match the coefficients between (B.5) and (B.6) and obtain

$$\begin{aligned} \delta \bar{p}^* + (1-\delta)\, \bar{p} &= \bar{p} \\ \delta\,(1-\mu)\,(\rho - b) + \rho \bar{K} &= a \\ \delta\mu\,(\rho - b) + (1-\delta)\, a - \rho \bar{K} &= 0 \\ \delta\,(1-\mu) + \bar{K} &= c. \end{aligned}$$

Solving this system and using the definition of $\bar{K} = (\delta\mu, 1-\delta)\, K$, we get

$$\begin{aligned} \bar{p} &= \bar{p}^* \\ a &= \frac{\rho}{\delta\mu + 1 - \delta} \bar{K} = \rho \bar{\eta}' K = \rho \hat{k} \end{aligned}$$

since $\eta = \delta\mu/(\delta\mu + 1 - \delta)$ and $\hat{k} \equiv \bar{\eta}' K$. Moreover

$$b = \rho - a = \rho\left(1 - \hat{k}\right)$$

and

$$c = (\delta\mu + 1 - \delta)\,\hat{k} + \delta\,(1 - \mu)\,.$$

The vector of (pre-multiplied) Kalman gains satisfies the equation

$$K = \Sigma e_1 \left(e_1' \Sigma e_1 + \bar{\sigma}_\xi^2\right)^{-1}, \tag{B.7}$$

where Σ is the variance of the one step ahead forecast error which satisfies the following stationary version of the Riccatti equation

$$\Sigma = M\Sigma M' + mm'\sigma_u^2 - \left(e_1'\Sigma e_1 + \bar{\sigma}_\xi^2\right)^{-1} M\Sigma e_1 e_1' \Sigma M'. \tag{B.8}$$

Thus in our guess-and-verify approach we expressed M and m as a function of $\hat{k}$ which depends on the vector of Kalman gains K which in turn depends on Σ. But Σ depends on M and m by (B.8). So it remains to solve for this fixed point. Let

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}.$$

Solving the upper left block of the Riccatti equation (B.8) we find that σ_{11} satisfies the quadratic

$$\sigma_{11}^2 + \left[\left(1 - \rho^2\right)\bar{\sigma}_\xi^2 - \sigma_u^2\right]\sigma_{11} - \bar{\sigma}_\xi^2 \sigma_u^2 = 0.$$

The positive root (since σ_{11} is a variance) of this quadratic is

$$\sigma_{11} = \frac{1}{2}\sigma_u^2 \left\{1 - \left(1 - \rho^2\right)\bar{\lambda} + \sqrt{\left(1 - \left(1 - \rho^2\right)\bar{\lambda}\right)^2 + 4\bar{\lambda}}\right\}$$

where $\bar{\lambda} \equiv \frac{\bar{\sigma}_\xi^2}{\sigma_u^2}$. From the lower left block of the Riccatti we derive

$$\sigma_{12} = \frac{\rho^2 \hat{k} \sigma_{11} \bar{\sigma}_\xi^2 + \left(\sigma_{11} + \bar{\sigma}_\xi^2\right)\left[(\delta\mu + 1 - \delta)\,\hat{k}\sigma_u^2 + \delta\,(1 - \mu)\,\sigma_u^2\right]}{\left[1 - \rho^2(1 - \hat{k})\right]\bar{\sigma}_\xi^2 + \sigma_{11}}$$

and -using (B.7)- we obtain

$$\hat{k} = \bar{\eta}' K = \eta \frac{\sigma_{11}}{\sigma_{11} + \bar{\sigma}_\xi^2} + (1 - \eta)\,\frac{\sigma_{12}}{\sigma_{11} + \bar{\sigma}_\xi^2}.$$

This is a system of two equations in the two unknowns $(\sigma_{12}, \hat{k})$. Solving the system and using our solution for σ_{11} we finally arrive at the quadratic expression for $\hat{k}$

$$Q(\hat{k}) = \rho^2 \bar{\lambda} \hat{k}^2 + \left[\bar{\lambda}\left(1 - \rho^2\right) + \delta\right]\hat{k} - \delta = 0. \tag{B.9}$$

At first, note that the $\hat{k}$ that solves the quadratic for $\rho = 0$ agrees -as expected- with the static case. Proceeding with the case of $\rho \neq 0$, we note that the discriminant of (B.9) is positive, so there are two real roots. Furthermore, since $Q(0) < 0$ and $Q(1) = \bar{\lambda} > 0$, one is negative and the other positive and less than unity. Note that subtracting p_t from p_t^* we get an expression for the output deviation

$$y_t = \rho(1 - \hat{k})y_{t-1} + (1 - c)\, u_t,$$

since $p_t^* + \ln \bar{Y} = \ln M_t = p_t + \ln Y_t$. In order to have a stationary solution for output we need $\left|1 - \hat{k}\right| < |\rho|^{-1}$. It follows that $\hat{k}$ should satisfy the restriction $1 - \rho^{-1} < \hat{k} < 1 + \rho^{-1}$. Since $Q\left(1 - \rho^{-1}\right) = -\rho^{-1}\left((\rho - 1)^2 \lambda + \delta\right) < 0$, only the positive root of the quadratic satisfies the restriction. Thus

$$\hat{k} = \frac{1}{2\rho^2}\left\{\rho^2 - 1 - \frac{\delta}{\bar{\lambda}} + \sqrt{\left[1 - \rho^2 + \frac{\delta}{\bar{\lambda}}\right]^2 + 4\rho^2\frac{\delta}{\bar{\lambda}}}\right\}.$$

Having solved for the laws of motion of X_t and $E_t^j X_t$, we can derive the laws of motion of the prices of interest $p^\dagger = \bar{\eta}' X_t$ and $\tilde{p}_t(j) = \bar{\eta}' E_t^j X_t$ and obtain expressions (54) and (55) in the main text, respectively.

Derivatives. Recall that $\hat{k}$ depends on μ and η through the weight on the full information price, $\delta(\mu, \eta) \equiv \frac{\eta}{\eta + (1-\eta)\mu}$. The derivatives of $\hat{k}$ are as follows:

$$\begin{aligned}
\frac{\partial \hat{k}}{\partial \mu} &= \frac{1}{2\rho^2}\frac{1}{\bar{\lambda}}\frac{\partial \delta}{\partial \mu} \cdot \Lambda \\
\frac{\partial \hat{k}}{\partial \eta} &= \frac{1}{2\rho^2}\frac{1}{\bar{\lambda}}\frac{\partial \delta}{\partial \eta} \cdot \Lambda \\
\frac{\partial \hat{k}}{\partial \bar{\lambda}} &= -\frac{1}{2\rho^2}\frac{1}{\bar{\lambda}^2}\delta \cdot \Lambda,
\end{aligned}$$

where

$$\Lambda \equiv -1 + \frac{1 + \rho^2 + \frac{\delta}{\bar{\lambda}}}{\left[(1 - \rho^2 + \frac{\delta}{\bar{\lambda}})^2 + 4\rho^2\frac{\delta}{\bar{\lambda}}\right]^{1/2}}.$$

It is easy to see that $\Lambda > 0$. Thus, $\partial\hat{k}/\partial\bar{\lambda} < 0$. Note also that $\frac{\partial \delta}{\partial \mu} = -\frac{\eta(1-\eta)}{(\eta + (1-\eta)\mu)^2} < (>) 0$ if $\eta < (>) 1$. Furthermore, $\frac{\partial \delta}{\partial \eta} = \frac{\mu}{(\eta + (1-\eta)\mu)^2} > 0$. The result follows.

Derivation of (56). Defining $q_t^j \equiv p_t^\dagger - \tilde{p}_t(j) = p_t^\dagger - E_t^j p_t^\dagger$ we obtain

$$q_t^j = \rho(1 - \hat{k})q_{t-1}^j + \delta(1 - \hat{k})u_t - \hat{k}\xi_t^j. \tag{B.10}$$

At first notice that $E_t^j q_t^j = E_t^j(p_t^\dagger - \tilde{p}_t(j)) = 0$ and $E_{t-1}^j q_t^j = E_{t-1}^j(p_t^\dagger - \tilde{p}_t(j)) = E_{t-1}^j p_t^\dagger - E_{t-1}^j E_t^j p_t^\dagger = 0$. Calculating variances conditional on the private history until last period we get

$$Var_{t-1}^j(q_t^j) = \rho^2(1-\hat{k})^2 Var_{t-1}^j(q_{t-1}^j) + \delta^2(1-\hat{k})^2\sigma_u^2 + \hat{k}^2\bar{\sigma}_\xi^2.$$

Note that $Var_{t-1}^j(q_{t-1}^j) = E_{t-1}^j(q_{t-1}^j)^2 = Var_{t-1}^j(p_{t-1}^\dagger)$. Moreover, $Var_{t-1}^j(q_t^j) = E_{t-1}^j(q_t^j)^2 = E_{t-1}^j\{E_t^j(q_t^j)^2\} = E_{t-1}^j Var_t^j(p_t^\dagger) = Var_t^j(p_t^\dagger)$, where the last step follows from the non-randomness of the variances of the filter. The expression in the text for the contemporaneous variance follows by using the stationarity of the filter and the fact that (B.9) implies $\delta(1-\hat{k}) = \bar{\lambda}\hat{k}[1-\rho^2(1-\hat{k})]$.

B.2 Proof of proposition 5

We will now proceed to derive the condition for excess volatility of the prices of the uninformed firms. Taking unconditional variances in (B.10) we obtain that

$$var(q_t^j) = \frac{1}{1-\rho^2(1-\hat{k})^2}[\delta^2(1-\hat{k})^2\sigma_u^2 + \hat{k}^2\sigma_\xi^2].$$

Furthermore note that, since

$$var(q_t^j) = var(p_t^\dagger) + var(\tilde{p}_t(j)) - 2cov(p_t^\dagger, \tilde{p}_t(j))$$

and

$$cov(p_t^\dagger, \tilde{p}_t(j)) = cov(q_t^j, \tilde{p}_t(j)) + var(\tilde{p}_t(j)),$$

we have

$$var(p_t^\dagger) = var(q_t^j) + 2cov(q_t^j, \tilde{p}_t(j)) + var(\tilde{p}_t(j)).$$

Dividing over $var(p_t^\dagger)$ we obtain

$$\frac{var(\tilde{p}_t(j))}{var(p_t^\dagger)} = 1 - \frac{var(q_t^j) + 2cov(q_t^j, \tilde{p}_t(j))}{var(p_t^\dagger)}.$$

So the ratio can exceed unity only if $I \equiv var(q_t^j) + 2cov(q_t^j, \tilde{p}_t(j)) < 0$. Note that

$$cov(q_t^j, \tilde{p}_t(j)) = \frac{1}{1-\rho^2(1-\hat{k})^2}[\rho^2(1-\hat{k})\hat{k}\cdot cov(p_{t-1}^*, q_{t-1}^j) + \delta(1-\hat{k})\hat{k}\sigma_u^2 - \hat{k}^2\sigma_\xi^2].$$

Using the law of motion for the full information price and q_t^j we derive that

$$cov(p_t^*, q_t^j) = \frac{\delta(1-\hat{k})\sigma_u^2}{1-\rho^2(1-\hat{k})}$$

and plugging it in the previous expression we finally obtain

$$cov(q_t^j, \tilde{p}_t(j)) = \frac{1}{1-\rho^2(1-\hat{k})^2}\left[\frac{\delta(1-\hat{k})\hat{k}}{1-\rho^2(1-\hat{k})}\sigma_u^2 - \hat{k}^2\sigma_\xi^2\right].$$

Therefore

$$I = \frac{\hat{k}^2}{1-\rho^2(1-\hat{k})^2}\left[\frac{\delta(1-\hat{k})}{\hat{k}}\left(\frac{\delta(1-\hat{k})}{\hat{k}} + \frac{2}{1-\rho^2(1-\hat{k})}\right) - \lambda\right]\sigma_u^2.$$

In order to have $I < 0$, we need

$$\frac{\delta(1-\hat{k})}{\hat{k}}\left[\frac{\delta(1-\hat{k})}{\hat{k}} + \frac{2}{1-\rho^2(1-\hat{k})}\right] - \lambda < 0.$$

Using as before the fact that $\delta(1-\hat{k}) = \bar{\lambda}\hat{k}[1-\rho^2(1-\hat{k})]$ we derive condition (58).

C Pricing exercise

C.1 Information costs

We convert the lifetime costs to costs per period by multiplying it by the factor $(1-\beta)$. The steady state real profits are $\bar{\pi} = \bar{Y}/\varepsilon$. Recall that

$$c^j = \frac{2}{(\varepsilon-1)}\frac{\tilde{c}^j}{\bar{Y}}.$$

Then the equilibrium fraction of uninformed agents is given by

$$\mu = 1 - Pr(c^j \le c^*).$$

where, Pr refers to the probability measure of costs. In particular

$$Pr(c^j \le c^*) = Pr(c^j\,(1-\beta) \le c^*\,(1-\beta) = Var_t^j(p_t^\dagger))$$

and

$$Pr\left(\frac{\tilde{c}^j\,(1-\beta)}{\bar{Y}} \le \frac{(\varepsilon-1)}{2}Var_t^j(p_t^\dagger)\right) = Pr\left(\frac{\tilde{c}^j\,(1-\beta)}{\bar{\pi}} \le \frac{\varepsilon\,(\varepsilon-1)}{2}Var_t^j(p_t^\dagger)\right).$$

Given our assumption of a uniform distribution for the costs as a fraction of steady state real profits, it follows that

$$\mu = 1 - U\left(\frac{\varepsilon\,(\varepsilon-1)}{2}Var_t^j(p_t^\dagger)\right),$$

where $U(\cdot)$ is the corresponding c.d.f.

C.2 Variance and persistence of inflation

For completeness, we list the formulas for the variance and the first-order autocorrelation of inflation in (60). Rewrite (60) as

$$\pi_t = \hat{k}\bar{p}^* + (1-\hat{k})\pi_{t-1} + cu_t - (c-\hat{k})u_{t-1}$$

where

$$
\begin{aligned}
c &\equiv (1-\hat{k})\delta(1-\mu)+\hat{k} = 1-(1-\hat{k})\big[1-\delta(1-\mu)\big] = 1-\frac{(1-\hat{k})\mu}{\eta+(1-\eta)\mu} \\
c-\hat{k} &= (1-\hat{k})\delta(1-\mu) = \frac{(1-\hat{k})\eta(1-\mu)}{\eta+(1-\eta)\mu}.
\end{aligned}
$$

We have

$$
\begin{aligned}
Var(\pi_t) &= \frac{c^2+(c-\hat{k})^2-2(1-\hat{k})(c-\hat{k})c}{1-(1-\hat{k})^2}\sigma_u^2 \qquad \text{(C.1)}\\
Corr(\pi_t,\pi_{t-1}) &= (1-\hat{k})-\frac{(c-\hat{k})c}{Var(\pi_t)}\sigma_u^2. \qquad \text{(C.2)}
\end{aligned}
$$

Note that when everybody is informed, $\mu=0$, we have $c=1$ and therefore we get $Var(\pi_t)=\sigma_u^2$ and $Corr(\pi_t,\pi_{t-1})=0$. This is expected since the process collapses to the full information price Δp_t^*, which is i.i.d. At the other extreme of $\mu=1$, we get that $c=\hat{k}$ with $Var(\pi_t)=\frac{\hat{k}^2}{1-(1-\hat{k})^2}\sigma_u^2$ and $Corr(\pi_t,\pi_{t-1})=1-\hat{k}$. Formulas (C.1) and (C.2) were used for the derivation of the bottom panels of figure 2.

C.3 Another overprecision experiment

In table C.1 we provide two additional experiments: we either reduce the perceived volatility of noise $\bar{\sigma}_\xi$, or increase the degree of strategic complementarity, and perform the same overprecision exercise as in table 3.

Table C.1: Signal-to-noise ratio, strategic complementarity and price changes.

	$\sigma_u/\bar{\sigma}_\xi = 0.4$			$\eta = 0.15$		
	Endogenous μ		*Exogenous* μ	*Endogenous* μ		*Exogenous* μ
	cost 2%	**cost 10%**		**cost** 2%	**cost** 10%	
μ (%)	93.85	98.80	100	82.67	96.81	100
$1-\hat{k}$	0.80	0.803	0.804	0.959	0.961	0.962
Impact eff. on y (%)	0.76	0.78	0.78	0.90	0.93	0.93
Cum. eff. on y (%)	3.80	3.94	3.97	21.98	24.13	24.59
Std(dp)	0.31	0.30	0.30	0.11	0.10	0.09
$\gamma = 1$						
$\|dp\|$	1.01	1.01	1.00	0.98	0.98	0.98
Std(dp_i)	0.59	0.59	0.59	0.38	0.38	0.38
$\gamma = 2$						
$\|dp\|$	1.18	1.18	1.18	1.06	1.05	1.05
Std(dp_i)	1.03	1.05	1.05	0.71	0.74	0.75
$\gamma = 3$						
$\|dp\|$	1.46	1.47	1.47	1.22	1.22	1.22
Std(dp_i)	1.50	1.53	1.54	1.04	1.11	1.12
$\gamma = 4$						
$\|dp\|$	1.79	1.80	1.81	1.43	1.44	1.44
Std(dp_i)	1.98	2.03	2.04	1.37	1.47	1.49
$\gamma = 6$						
$\|dp\|$	2.49	2.53	2.54	1.90	1.95	1.95
Std(dp_i)	2.94	3.02	3.04	2.04	2.20	2.24
$\gamma = 8$						
$\|dp\|$	3.23	3.29	3.31	2.40	2.49	2.51
Std(dp_i)	3.90	4.01	4.04	2.71	2.93	2.98

In the left panel we decrease the perceived volatility to $\bar{\sigma}_\xi = 0.0243$ (so that the signal-to-noise ratio becomes 0.4) and perform the same simulations as in table 3. The rest of the baseline calibration is the same. The right panel uses the baseline calibration of table 2 except for η; the degree of strategic complementarity is increased by setting $\eta = 0.15$.

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