ARTICLE IN PRESS

Economic Modelling xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econmod

Is the survival of the euro area at risk? An economic analysis of exit and contagion possibilities☆

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ARTICLE INFO

JEL codes:
F30
F31
F41
G01

Keywords:
Euro breakup
Currency crisis
Contagion
Nash equilibria

ABSTRACT

Our paper analyzes the possibility that the most fragile countries of a monetary union decide to exit when hit by negative external shocks, and can thus also induce the exit of more robust member states (contagion). The paper reaches two main results: (i) the depreciation of the common currency lowers the exit as well as the contagion probabilities; (ii) the non-price competitiveness factors, here stylized in terms of output gap elasticities to the exchange rate, also play a crucial role since higher elasticities of the weakest countries make their exit from the monetary union more likely, and if these same elasticities for the other member states are low enough contagion will never hapepen; The paper also shows that the previous results are deeply affected by the initial assumptions and a number of exogenous variables and structural parameters.

1. Introduction

The dramatic episodes of the Greek crisis (end of June – beginning of July 2015) led some of the main European countries and intergovernmental institutions to explicitly discuss the exit of a member state from the European Economic and Monetary Union (EMU) as a possible way out of an institutional *impasse*. In the euro area's short life, it was the first time that this possibility was formally put on the table.[1] A persistent significant probability that a fragile member state leaves the monetary union would transform the EMU into a reversible fixed exchange rate *régime*. In fact, the euro area remains at risk due to the lack of convergence between the performances of the EMU's 'core' countries and a few fragile countries (mainly, Greece and Italy). In Germany, there is a growing consensus that Italy's economic fundamentals are becoming incompatible with the EMU's rules. On the other hand, Italy is the third most important EMU's economy by size. Hence, despite the progress made by Greece during the last two years, it remains crucial to analyze (i) the conditions that would make it convenient for a member state in trouble to leave the euro area, and (ii) the possible impact of this exit on the behavior of other member states.

Our paper is unable to fully address questions (i) and (ii). The exit process of a given country from a monetary union cannot be reduced to an "in/out" alternative, since it depends on a number of medium-long term variables such as the sustainability of its public debt and related financial charges, the balance of its capital flows in the area, the weaknesses and interdependencies of its banking sector, its price and non-price competitiveness in international markets, and its expectations about the area's future policy. Moreover, the costs of transitioning from the current *régime* (inside the monetary union) to the new one (outside the monetary union) matters a lot for the actual choices of the country potentially leaving and its possible followers; and the main features of this transition are deeply influenced by the legal and institutional settings of the monetary area and by the structural, economic, and social organization of each of the member states (cf. Boltho and Carlin, 2013). Therefore, to assess the convenience of an exit from the euro area and its possible impact on the strategies adopted by other member states, it would be necessary to combine a

☆ We are grateful to the Editor and three anonymous Referees for very helpful comments and suggestions We would like to thank all the participants to a seminar held at the Luiss School of European Political Economy on October 7, 2015 for their helpful comments and acknowledge the suggestions offered by Pier Paolo Benigno and Franco Bruni. We remain responsible for any mistakes.

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[1] The *Treaty for the Functioning of the European Union* (TFEU) does not conceive of at possible exit from the EMU but just from the EU (cf. Athanassiou, 2009). However, as emphasized by law scholars (see for instance: Tosato, 2015, sec. 10), the Lisbon Treaty does not expressly prohibit a temporary and separate withdrawal from the euro. Moreover, such a prohibition cannot be based on "an inseparable bond between the monetary union and the Union as a whole" as some EU member states do not adopt the euro (at least currently). Hence, it is possible to conceive the exit from the EMU of a given country as a temporary interruption of its participation to the euro area but not of its membership to the EU. Hofmeister (2011) discusses this and other exit options from the euro at length, even if he remains skeptical on their possible implementation.

http://dx.doi.org/10.1016/j.econmod.2017.09.007
Received 20 January 2017; Received in revised form 4 July 2017; Accepted 9 September 2017
0264-9993/

Please cite this article as: Canofari, P., Economic Modelling (2017), http://dx.doi.org/10.1016/j.econmod.2017.09.007

large number of institutional, organizational, financial, and productive components. This is beyond the scope of our model.

Our paper is based on a simplified framework. It focuses on a one-shot game between two stylized economic systems in a monetary union: a member state affected by economic fragilities and lack of competitiveness, and another more robust and competitive member state. This simple setting, which is based on Canofari et al. (2015), can be conceived as a stylized representation of part of the euro area. The first country (country A) can approximate a representative of a set of the most fragile countries (for instance: Greece) with a significant probability of exiting from the monetary union if affected by strong negative shocks; the second country (country B) can approximate either Germany and its satellites (i.e., the 'core' member states in the euro area) or, even better, an intermediate country (typically France), which is exposed to direct contagion due to the possible exit of country A. We adopt the latter view.

Our paper aims to assess the effects of the euro's appreciation/depreciation and other non-price competitiveness factors on the probability that 'peripheral' country A leaves the EMU when hit by a negative and specific demand shock. We also analyze the role played by the euro's depreciation and non-price factors on the contagion probability from country A to country B.[2] Since the two countries' decisions are taken in a one-shot game, they are determined in a point-of-time and hence relate - by definition - to the short-term.[3] However, these decisions can also depend on long-term exogenous expectations.

The dependence of the exit probability of country A on the euro's appreciation/depreciation is determined by its output gap elasticity to its real effective exchange rate—the higher this elasticity, the more likely the country's exit from the euro. However, changes in the euro's relative value cannot be the only variable affecting the exit probability of peripheral member state A. Competitiveness indicators, such as the relative dynamics of labor unit costs and of technical and organizational innovations, matter a lot (see Corsetti, 2015); and the same applies to relative improvements in the institutional setting. Since our theoretical model cannot endogenize either the labor market or institutional and organizational variables, we interpret the elasticity of country A as a proxy for the impact of the euro's appreciation/depreciation with respect to a set of other variables given exogenously. This assumption makes it possible to show that the probability of contagion is affected by the trade balance elasticity of country B to its effective exchange rate relative to the corresponding elasticity of the peripheral country A. In particular, if the elasticity of country B is low enough, contagion never occurs.

Our model could offer a more comprehensive analysis of the workings of the euro area, if it encompassed strategic interactions between three agents: a country E representing the EMU's core member states (i.e., Germany and its satellites), in addition to countries A and B. However, it is well known that referencing a strategic interaction with more than two heterogeneous players severely increases the complexity of the analytical setting (cf. Papadimitriou, 2007; Chen et al., 2009). Hence, in the following analysis, we will assume that Germany and other 'core' EMU member states do not react to measures implemented by countries A and B. The assumption implies that EMU country E plays an even less apparent role than the one played by the extra-EMU world (any currency appreciation/depreciation relative to the euro is taken as a benchmark and, by definition, the euro cannot appreciate/depreciate toward itself).

This simplification is compatible with a well-known result (see Eichengreen and Sachs, 1986), which has been recently refined (see Benigno and Romei, 2014; Fornaro, 2015; Cook and Devereux, 2016)—a unilateral devaluation by a country (such as A) can have weak or even counter-intuitive effects on other countries (such as E and the extra-EMU world), especially if the zero lower bound is binding. On the other hand, our simplified model cannot incorporate two other effects of unilateral devaluations which are encompassed in the literature just quoted: the role of expectations, and the possible micro-founded spillovers which are crucial to design the optimal policy responses (see Corsetti et al., 2000; Corsetti and Pesenti, 2005).

It remains that our model analyzes the strategic interaction between some types of EMU countries by means of a one-shot game: a fragile country, hit by a specific shock which negatively affects its price and non-price competition, can choose to abandon the monetary union; and an intermediate country, not directly hit by any shock, can suffer from contagion. We consider the role of both the currency's depreciation and output gap elasticities in determining possible Nash equilibria. Hence, our paper is based on Canofari et al. (2015) and mainly refers to the literature on the EMU crisis due to exchange rates effects.[4] To be more specific, it belongs to the so-called "second generation models," which started with the contribution by Obstfeld (1986) and later included the role played by economic fundamentals (see Jeanne, 1997). However, we do not follow a recent evolution of these models aimed at stressing the role of credibility, expectations, and policy trade-off to analyze possible self-fulfilling speculative attacks (see Obstfeld, 1994, 1997; De Grauwe and Ji, 2013). We are instead influenced by the models that analyze strategic interactions between countries to explain the collapse of the European exchange rate mechanism (see Buiter et al., 1996; Di Bartolomeo et al., 2006).

The remaining parts of this paper are organized as follows. In Section 2, we define the general framework. Section 3 illustrates the structure of the game and the different in/out *régimes* involved when the peripheral country is hit by an exogenous shock. In the fourth section, we analyze the role played by the currency's depreciation (appreciation) in offering disincentives (incentives) to peripheral country A to leave the monetary union and, hence, in stabilizing (destabilizing) the area. Then, we assume the exit case for country A and consider the probability of a contagion effect from this exit for country B. In particular, Section 5 specifies the peculiar role played by currency depreciation, and Section 6 examines the impact of output gap elasticities. The last section offers some conclusions and proposes avenues for further research.

2. The general model

The basic structure of our model largely reproduces that of Canofari et al. (2015). We consider a monetary union characterized by two countries, A and B, which strategically interact. In a given point-of-time country A may decide to leave the union due to a specific and negative demand shock, and country B can decide to follow the same path due to contagion. The other EMU countries are denoted by E, whereas the rest of the world outside the EMU is denoted by W. Some of the analytical refinements of this basic structure, introduced in the current paper, are quite important since they make the model more general. They can be synthetized in the following five points. First: the common currency exchange rate with respect to the rest of the world

[2] Our definition of contagion is based on the possibility that country A's exit also implies the exit of country B. We concentrate on the exchange rate and other non-price tools without considering policy interventions. However, let us recall that the effectiveness of monetary policy in stabilizing demand shocks has been called into question in the literature (e.g., Chortareas and Mavrodimitrakis, 2016).

[3] Given this framework, our paper cannot address questions such as: does the participation to a monetary union positively or negatively affect the potential growth of a given country? According to Dreyer and Schmid (2016), whereas the participation to the European Union improves the member states' economic growth, the participation to the euro-area does not have further effects but during the financial crises (where the effects become negative).

[4] We are also indebted to other papers analyzing the Greek crisis by means of theoretical tools. Referring to the literature on exchange rate crises, let us quote Arghyrou and Tsoukalas (2011) who argue that the Greek case can be interpreted as the result of a deterioration of Greece's macroeconomic fundamentals between 2001 and 2009. In this view, without a structural convergence, Greece participation to the EMU will be inconsistent in the long term (see also German Council of Economic Experts, 2015).

becomes an instrument to re-adjust the international competitiveness of the monetary union. Second: the euro depreciation also implies costs for both countries A and B. Third: we assume that the output elasticities to the exchange rate have different initial values in the two countries. Fourth: each of these elasticities is partially endogenized in the model, since its initial value would change if the related country decided to leave the monetary union.[5] Fifth: country B would have to face a further cost if it decided to avoid contagion and remain in the monetary union, despite the exit of country A.[6] These refinements confirm that the depreciation of the common currency has a stabilizing effect on the area; however, they also emphasize that the intensity of this effect is affected by the output gap elasticities to the exchange rate and by other institutional factors.

We focus on the possible exit choice of country A from the monetary union and the likely contagion to country B by assuming that the rest of the EMU countries and the world outside the EMU do not react to the initiatives taken by countries A and B, which represent the only strategic agents in our model.[7]

As in Canofari et al. (2015), the output gap of country A (y^A) depends on the exchange rate of this same country (s^A) and on the exchange rate of the other interacting country B (s^B), weighted by the elasticity to the exchange rate of A's output gap (σ_A). In our more general model we assume that y^A is also affected by the average exchange rate of the rest of the world W (s^W). As indicated above, s^z(where $z = A, B,$ and W) are nominal variables, and the world exchange rate s^W represents the channel to assess the role played by the depreciation of the currency of the monetary union under consideration.[8] The exchange rates s^B and s^W are weighted, respectively, by the parameters β and φ, whose values are determined by the importance of different trade partners for country A $(0 < (\beta, \varphi) < 1)$.[9] The same reasoning applies to the output gap of country B (y^B). For the sake of simplicity and without significant loss of generality, the parameters β and φ are the same for countries A and B.[10] Moreover, we assume that the elasticity to the exchange rate of each country's output gap ($\sigma_i > 0$; with $i = A, B$) has different values inside or outside the EMU.

In this setting, the representative peripheral country A can be affected by an exogenous negative demand shock u^A[11]; no exogenous shocks occur for country B.

The output for both countries A and B is:

$$y_N^A = \sigma_{AN}(s^A - \beta s^B - \varphi s^W) - u^A \tag{1a}$$

$$y_E^A = \sigma_{AE}(s^A - \beta s^B) - \varphi\sigma_{AN}s^W - u^A \tag{1b}$$

$$y_N^B = \sigma_{BN}(s^B - \beta s^A - \varphi s^W) \tag{2a}$$

$$y_E^B = \sigma_{BE}(s^B - \beta s^A) - \varphi\sigma_{BN}s^W \tag{2b}$$

where: y_N^A and y_E^A are the output gaps for country A when choosing to remain or leave the monetary union respectively; σ_{AN} and σ_{AE} are the output gap elasticities for country A when choosing to remain or leave the monetary union respectively; y_N^B and y_E^B are the output gaps for country B when it remains or leaves the monetary union respectively; σ_{BN} and σ_{BE} are the output gap elasticities for country B when choosing to remain or leave the monetary union respectively;

Let us momentarily assume that $u^A = 0$, hence we refer just to (1a) and (2a). In equilibrium, outputs and inflation rates are thus at their target levels for both countries. Given the above definition of nominal exchange rates, a positive s^A reflects a real devaluation for country A and a consequent improvement in its price competitiveness.[12] The latter leads to a positive output gap for country A, which also depends on σ_{AN} as well as on the value of the two parameters β and ϕ. On the other hand, positive s^B or s^W determine a real revaluation for country A and a consequent deterioration in its price competitiveness. The latter leads to a negative output gap for country A, which also depends on σ_{AN} as well as on the values of the two parameters β and ϕ. The same reasoning applies to country B.

We can now specify the interactions between countries A and B. As in Canofari et al. (2015), three *régimes* are possible: no country abandons the EMU (*régime N*), only country A exits (*régime* D^A), and both countries A and B leave the EMU (*régime F*). The strategic choice of each country will be determined by the relative losses in each *régime*. The governments of country A and country B aim at minimizing a loss function L^i (where $i = A, B$), depending on the output gap and the possible inflation:

$$L^A = (y^A)^2 + \theta[s^A - \varphi s^W]^2 + \delta_A C \tag{3}$$

$$L^B = (y^B)^2 + \theta[s^B - \varphi s^W]^2 + \delta_B C \ - \delta_A(1 - \delta_B)I_B \tag{4}$$

where: L^A and L^B denote the loss functions to be minimized for country A and B, respectively; θ indicates the coefficients of inflation aversion (with $\theta > 0$), which are equal for the two countries; and δ_i(with $i = A, B$) represents a dummy variable whose value will become zero, if country i remains in the monetary union, and one, if the same country leaves the monetary union.

Note that, differently from Canofari et al. (2015), we also introduce in both the loss functions the weighted depreciation of the whole monetary union s^W. Moreover, consistent with the second generation approach to currency crises (cf. above; and De Grauwe and Ji, 2013), we consider that an opting out has always a positive cost (C; with $C > 0$).[13] Finally, by including δ_A in the loss function of country B, we measure the impact (I_B) of the actual choice of country A on country B: when the former country exits from the monetary union ($\delta_A = 1$) while the latter decides to remain ($\delta_B = 0$), there will be direct and indirect impacts (I_B) on country B's expected losses.

Let us further distinguish I_B from C. I_B is determined by at least two cost components (that is $I_B < 0$): (i) an instantaneous increase in the instability of the monetary union, due to the legal separation procedures and the consequent economic and institutional adjustment

[5] We thank an anonymous referee for suggesting us this improvement.

[6] In fact, we will specify in the following that this cost tends to assume a negative value.

[7] The reference to the extra-EMU world allows us to measure the average effects of the depreciation of the common currency with respect to the world outside the EMU. In particular, we can analyze the exogenous depreciation of the currency of representative country W in terms of the currency of representative country E.

[8] s^W is the average of the exchange rates of the non-EMU world. Hence, since all variables are expressed in logs and thus represent deviations, a negative s^W implies a depreciation of the euro toward any other currency not related to countries A and B (in the event of their exit). At the same way, positive s^A and s^B imply a devaluation for country A and B, respectively.

[9] The parameter β is the weight of the other strategically interacting country, and φ ($\varphi = 1 - \beta - \Upsilon$, *where* Υ takes into account EMU's ‘core’ countries) is the weight of the rest of the extra-EMU world. Obviously, $\beta + \Upsilon$ must be strictly lower than 1; otherwise, the weight of the non-EMU rest of the world would be equal to 0 despite the recent processes of globalization.

[10] We assume that the weights are the same for country A and B to simplify the algebra. As a matter of fact, the two countries tend to have different parameters since the importance of the respective trade partners is specific to each of them. However, we will take into account this specificity by referring to non-price competition.

[11] u^A is the size of an i.i.d. random shock described by a continuous, bell-shaped, and symmetric (around zero) probability density function. Here we are only interested on the possibility of a negative demand shock affecting country A implying an incentive to exit and depreciate. Thus, the size of the shock has a negative sign in Eqs. (1a) and (1b).

[12] As pointed out by an anonymous referee, this result requires that we neglect the exchange rate pass-through so that a nominal devaluation does not affect domestic prices. Our view is that this simplification is justified in our model since it refers to a point-of-time.

[13] The constant C can be interpreted as a non-pecuniary cost due to the loss of reputation that countries A and B would suffer at the international level by breaking the unity of the monetary area. On the other hand, C can also be interpreted as the total monetary cost of exit. This latter cost includes a number of items. For instance: the cost of giving up the commitment to repay the stockpile of old debts in the old currency, the transaction costs involved in the currency change, the reorganization of the national payment system, and the cost of switching to a flexible exchange monetary system. C also represents the strength of the peg. In short, we can state that the constant C summarizes all the costs not explicitly considered in the model, which are connected to abandoning the monetary union as described in the introduction of this paper.

processes ignited by the leaving country; (ii) the strongest attraction to the left-hand side tail of the member states distribution function and the related higher probability to be affected by negative shocks. However, I_B also determines two positive effects on B's situation (that is, $I_B > 0$): (iii) the expected improvements in the medium-term workings and regulation of a more balanced and convergent monetary union, which will not yet include one of its most extreme outliers; (iv) the larger amount of resources in case of difficulty, now available for the remaining countries. In the following, we assume that the impact of points (iii)-(iv) prevail on that of points (i)-(ii). Hence, we have: $I_B > 0$.

We assume, for simplicity, that both countries A and B target their output gaps (y^A, and y^B) and devaluation (s^A and s^B) to zero in log.

3. Exogenous shock: The structure of the game

As in Canofari et al. (2015), let us refer to a game characterized by an exogenous and aggregate demand shock so that the previous assumption of $u^A = 0$ does not apply. The shock (denoted as $u^A > 0$) directly affects only country A, even if it is observed by both A and B and can have an indirect impact on the latter (see Section 2, Eqs. (2) and (4)). This shock implies a deviation in the output and inflation of country A from their respective target levels.[14]

These assumptions do not fit with the 2007-08 banking sector and stock market collapse in the most advanced economies, and with the following great recession. However, at the end of 2009, the United States and EMU followed different trajectories. The recovery phase in the United States was stronger than expected, whereas the EMU's economy flowed into the sovereign debt crisis and/or the banking sector crisis in three peripheral member states (Greece, Ireland, and Portugal), leading to contagion to Italy and Spain. All these EMU's countries (with the partial exception of Italy) inherited severe negative current account imbalances; and, due to the “flight to quality” of the European financial flows, they had to suddenly adjust these external disequilibria. The impossibility of recourse to a depreciation imposed the compression of employment and wages, thus igniting a new recessionary phase.[15] The alternative would have been the exit from the euro area. *A fortiori*, these possible implications should apply to the most recent Greek crisis, which reached its peak between mid-June and mid-July 2015. In addition, according to the most severe critics of the structural flaws characterizing the Italian economy, this could relate to the third biggest euro area country during the next European crisis. Hence, it is useful to evaluate the impact that an exogenous shock such as a recessionary phase or a crisis event can have on the remain/exit choices of countries A and B. Our model and the related game structure performs this duty.

As stated in the previous section, the game structure is characterized by the three *régimes* (N, D^A, and F). Countries A and B will select that or those *régimes* leading to a Nash-equilibrium, that is characterized by absence of incentives to deviate from the acquired position.

As in Canofari et al. (2015), the optimal reaction functions for both countries are determined by the differentiation of (3) and (4) with respect to the nominal exchange rates, subject to conditions (1a-1b) and (2a-2b), respectively.[16] However, in the current model, we included s^W and considered that $\sigma_{iN} \neq \sigma_{iE}$ for both countries. We thus obtain the optimal reaction function for country A:

$$s^A = 0 \quad \text{if} \quad \delta_A = 0$$
$$s^A = \frac{\sigma_{AE}{}^2 \beta s^B}{\sigma_{AE}{}^2 + \theta} + \frac{\varphi(\sigma_{AE}\sigma_{AN} + \theta)s^W}{\sigma_{AE}{}^2 + \theta} + \frac{\sigma_{AE} u^A}{\sigma_{AE}{}^2 + \theta} \quad \text{if} \quad \delta_A = 1 \tag{5A}$$

Eq. (5A) shows how country A reacts to the shock u^A, to the common currency depreciation s^W, and to the other interacting country devaluation s^B.

The optimal reaction function for country B is:

$$s^B = 0 \quad \text{if} \quad \delta_B = 0$$
$$s^B = \frac{\sigma_{BE}{}^2 \beta s^A}{\sigma_{BE}{}^2 + \theta} + \frac{\varphi(\sigma_{BE}\sigma_{BN} + \theta)s^W}{\sigma_{BE}{}^2 + \theta} \quad \text{if} \quad \delta_B = 1 \tag{5B}$$

Eq. (5B) displays the best response that country B can give to the common currency depreciation and the possible depreciation of country A.

Given Eqs. (5A) and (5B), the optimal strategies of the two countries in each *régime* are:

$$RégimeN: \{s^A = 0, s^B = 0\} \tag{6}$$

$$RégimeD^A: \left\{ s^A = \frac{\varphi[\sigma_{AE}\sigma_{AN} + \theta]s^W}{\sigma_{AE}{}^2 + \theta} + \frac{\sigma_{AN} u^A}{\sigma_{AE}{}^2 + \theta},\ s^B = 0 \right\} \tag{7}$$

$$RégimeF: \left\{ \begin{array}{l} s^A = \dfrac{[\varphi\beta s^W(\sigma_{BE}\sigma_{BN} + \theta)\sigma_{AE}{}^2 + (u^A + \sigma_{AN}\varphi s^W)(\sigma_{BE}{}^2 + \theta)\sigma_{AE} + \theta\varphi s^W(\sigma_{BE}{}^2 + \theta)]}{[\theta + (1 - \beta^2)\sigma^2{}_{BE}]\sigma_{AE}{}^2 + \theta(\sigma^2{}_{BE} + \theta)} \\ s^B \dfrac{\beta[(u^A + \sigma_{AN}\varphi s^W)\sigma_{AE} + \theta\varphi s^W]\sigma_{BE}{}^2 + \sigma_{BN}\varphi s^W(\sigma_{AE}{}^2 + \theta)\sigma_{BE} + \theta\varphi s^W(\sigma_{AE}{}^2 + \theta)}{[\theta + (1 - \beta^2)\sigma^2{}_{AE}]\sigma^2{}_{BE} + \theta(\sigma_{AE}{}^2 + \theta)} \end{array} \right\} \tag{8}$$

Let us denote L_h (with $h = N,\ D^A, F$) as the losses associated to each *regime*, so that L_h^i represents the loss for country i (with $i = A, B$) in regime h. Substituting (6)–(8) in (3) subject to (1a) and (1b), the losses for country A in the significant *régimes* become:

$$L_N^A = (u^A + \sigma_{AN}\varphi s^W)^2 + \theta\varphi^2(s^W) \tag{9}$$

$$L_{D^A}^A = \frac{\theta(u^A + \sigma_{AN}\varphi s^W - \sigma_{AE}\varphi s^W)^2}{\sigma_{AE}{}^2 + \theta} + C \tag{10}$$

Analogously, substituting (6)–(8) in (4) subject to (2a) and (2b), the losses for country B in the significant *régimes* become:

$$L_{D^A}^B = \frac{\sigma_{BN}^2\beta^2\sigma_{AE}^2(u^A)^2}{(\sigma_{AE}^2 + \theta)^2} + \frac{2\sigma_{BN}^2\varphi s^W \nabla u^A \beta\sigma_{AE}}{(\sigma_{AE}^2 + \theta)^2} + \frac{\sigma_{BN}^2\varphi^2(s^W)^2(\nabla)^2}{(\sigma_{AE}^2 + \theta)^2} + \varphi^2(s^W)^2\theta - I_B \tag{11}$$

$$L_F^B = \frac{\{[\varphi s^W(\beta^2 - 1)\sigma_{AE}^2 + \beta(u^A + \sigma_{AN}\varphi s^W)\sigma_{AE} + \theta\varphi s^W(\beta - 1)]\sigma_{BE}\theta + \sigma_{BN}\varphi s^W(\sigma_{AE}^2 + \theta)\}^2 (\sigma_{BE}^2 + \theta)}{\{[(1 - \beta^2)\sigma_{AE}^2 + \theta]\sigma_{BE}^2 + \theta(\sigma_{AE}^2 + \theta)\}^2} + C \tag{12}$$

where: $\nabla \equiv \beta\sigma_{AE}\sigma_{AN} + \beta\theta + \sigma^2{}_{AE} + \theta$

4. Opting out, currency depreciation and structural parameters

Our first results allow a re-statement in a more general setting of the conclusions reached in Canofari et al. (2015). We are able to determine a threshold level for the exogenous shock hitting country A (u_A^*), which makes it indifferent for this country to stay in or to opt out from the monetary union. In this respect, by solving the condition $L_{D^A}^A - L_N^A = 0$ for a specific value of u^A, we get:

$$u_A^* = \frac{\sqrt{C(\sigma_{AE}^2 + \theta)} - \varphi s^W(\sigma_{AE}\sigma_{AN} + \theta)}{\sigma_{AE}} \tag{13}$$

Eq. (13) and (1a) and (1b) imply that, if the shock hitting country A

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[14] The economic systems of both countries A and B are in equilibrium in the absence of shocks; and, as mentioned above, output and inflation are assumed at their target levels.

[15] There is a rich collection of literature analyzing the possible impact of adjustment programs and the consequent European policies on the EMU's economic recession. The latter was particularly severe and prolonged in peripheral countries. Let just recall: Barkbu et al. 2015; Baldwin and Giavazzi 2015.

[16] The economic meaning of the constraints (1a-1b) and (2a-2b) in the minimization problem is that the goods market must be in equilibrium in both countries.

u_A --------------------------------|---------------------------------- u_A

N *régime* u_A^* D^A U F *régime*

Fig. 1. .

exceeds u_A^*, this country will choose to leave the monetary union and depreciate its consequent new currency.[17] In this event, *régimes* D^A or F are the Nash-equilibrium solution depending on the reaction of country B. Differently from Canofari et al. (2015), Eq. (13) emphasizes the role played by s^W. In fact, it shows that changes in s^W imply changes in u_A^*. This means that the possible choice of country A to opt out from the monetary union is also indirectly affected by the nominal exchange rate of the rest of the world (s^W), and thus by the possible depreciation of the monetary union currency. A depreciation of the monetary union currency would affect the value of the exogenous shock hitting country A, which makes it indifferent for this country to stay in or opt out of the monetary union.

In order to better assess this new aspect, let us differentiate (13) with respect to s^W, obtaining:

$$\frac{\partial u_A^*}{\partial s^W} = \frac{-\varphi(\sigma_{AE}\sigma_{AN} + \theta)}{\sigma_{AE}} \tag{14}$$

Given that φ, σ_{AE}, and σ_{AN} are positive parameters, Eq. (14) clearly states that $\frac{\partial u_A^*}{\partial s^W}$ is negative. *Ceteris paribus* an appreciation of s^W, improving the price competitiveness of country A, would increase the threshold level u_A^* and, given the probability density function of u_A, would thus make the exit of country A from the monetary union less likely to occur.

This result is significant for the further assessment of the recent evolution of the euro area. The adjustment programs implemented in the EMU led to restrictive fiscal policies and hindered the adoption of expansionary unconventional monetary policies until the end of 2014, except at the peak of the banking and sovereign debt crises (December 2011 – June 2012). Due to more expansionary policies implemented in other main economic areas (the United States, Japan, and the United Kingdom) since mid-2012 to the third quarter of 2014, the euro appreciated with respect to the other most important international currencies. Hence, our previous results state that – other things remaining equal – restrictive European policy measures contributed to increasing the probability that some of the peripheral member states would opt out. However, in March 2015, the European Central Bank (ECB) started the implementation of a strong quantitative easing program, and financial markets anticipated this decision from November 2014, resulting in a depreciation of the euro relative to the US dollar and the other main currencies during the following six months. This trend was strengthened by the gradual shift of European fiscal policies to a neutral stance. Hence, our previous results indicate that the ECB's current monetary policy is reducing the probability of country A's exit.[18]

The more general framework of our new model leads to another interesting result: the positive impact of inflation aversion coefficient (θ) on the robustness and stability of the euro area. Let us reasonably assume that the anti-inflation credibility of a given country is stronger in a monetary union than in a fixed exchange rate regime (Giavazzi and Giovannini, 1991). Then, it becomes interesting to note that any positive variation in θ will increase the threshold level u_A^*, when there is a depreciation in the common currency (a negative s^W). This is clearly stated by the following equation:

$$\frac{\partial u_A^*}{\partial \theta} = \frac{\frac{1}{2}\frac{C}{\sqrt{C(\sigma_{AE}^2 + \theta)}} - \varphi s^W}{\sigma_{AE}} \tag{15}$$

Finally, our more general model offers richer results on the impact of changes in the output gap elasticities to the exchange rate. We have that:

$$\frac{\partial u_A^*}{\partial \sigma_{AE}} = -\frac{\theta(C - \varphi s^W\sqrt{C(\sigma_{AE}^2 + \theta)})}{\sigma_{AN}^2\sqrt{C(\sigma_{AE}^2 + \theta)}} \tag{16}$$

$$\frac{\partial u_A^*}{\partial \sigma_{AN}} = -\varphi s^W \tag{17}$$

Eq. (16) shows that the higher the elasticity outside the monetary union (σ_{AE}), the lower the value of the threshold u_A^*. Hence, despite the currency depreciation, a higher σ_{AE} implies a higher exit probability for country A. On the other hand, according to Eq. (17), the higher the elasticity within the monetary union (σ_{AN}), the higher the value of the threshold u_A^* and - then - the lower the exit probability for country A given the strong impact of the currency depreciation.

We can restate these findings in a more descriptive way. *Régime N,* which corresponds to a stable working of the monetary area, can actually be selected. It is sufficient that the exogenous shock hitting country A satisfies the condition $u_A \le u_A^*$ (see Fig. 1). In this case, as stated by Eq. (13) and (1a) and (1b), country A does not leave the monetary union; and it would be easy to show that also country B stays in. In analytical terms, *régime N* is one of the Nash equilibria of the game. This means that, at least in principle, the EMU can select a stable equilibrium with the capacity to absorb external shock (a case of 'good' equilibrium). However, it is also possible that countries select a spurious *regime* (D^A U F). If the exogenous shock hitting country A implies $u_A > u_A^*$, country A finds it convenient to leave the monetary union (see Fig. 1). In this case, country B can either stay in or opt out due to contagion. Both *régimes*, D^A and F, can be Nash equilibria. This means that, at least in principle, the EMU can select 'bad' equilibria: D^A would transform a monetary union into an unstable fixed exchange rate monetary system,[19] whereas F would lead to a dramatic breakdown of the monetary union.[20]

5. Contagion and currency depreciation

A simple refinement of our new model allows us to specify the selection of the 'bad' equilibria — that is, when country A decides to leave the monetary union due to the condition $u_A > u_A^*$. In this respect, it is possible to derive another threshold level for the shock hitting country A (see also Canofari et al., 2015). This new threshold for u_A, other than satisfying the condition $u_A > u_A^*$ (a necessary condition for the exit of country A), must make it indifferent for country B to stay in or to opt out from the monetary union.

[17] This obviously means that country A will stay in the monetary union if its specific shock is lower than u_A^*. As usual, the former results leave the choice of country A undetermined when the specific shock is strictly equal to u_A^*. Through a bit of algebra, it would be possible to overcome this indeterminacy. However, we assume, for the sake of simplicity, that country A always chooses to stay in when the specific shock is equal to u_A^*.

[18] Our statement could be affected by the fact that the ECB's quantitative easing did not apply to the purchase of Greek bonds. In order to address this issue, we would have to endogenize the impact of the policy tools involved through the European aid program. The structure of our model is too simple to allow this specification.

[19] In the past forty years, European attempts to create a fixed exchange rate system failed, as exemplified by the reaction to the breakdown of the Bretton Wood agreements and the construction of the European Monetary System. A few years after President Nixon's obviation of the gold standard (mid-August 1971), European countries built up a quasi-fixed exchange rate system, which summarily collapsed due to financial speculation. The European Monetary System that started in the European Union in 1978 had a longer life, lasting until 1992-'93. However, this life was characterized by several re-alignments of different currencies (the Italian lira, for instance), and it later collapsed in the fall of 1992, with the exit of the Italian lira and British pound. An empirical justification of the fragilities characterizing pegged regimes is offered by: Bohl et al. (2016).

[20] It is quite clear that it would be impossible to conceive the survival of the EMU with the exit – for example - of Spain and Italy. France could not remain in the euro area. Hence, the new monetary area would be no more than a currency agreement between Germany and its satellites, one centered on the resurgence of the Deutsche mark.

To simplify the analysis of the following analytical results, let drop the previous assumption that the output gap elasticities for country A and B will change when country A abandons the monetary union.[21] Hence, σ_A and σ_B continue to be different but both become invariant with respect to the remain/exit choice.

In principle, our one-shot game is compatible with the two opposite conditions: $\sigma_A < \sigma_B$, and $\sigma_A > \sigma_B$. From an economic point of view, $\sigma_A < \sigma_B$ implies that the potential growth of country A is less dependent on currency depreciation and, hence, on price competition; conversely, $\sigma_A > \sigma_B$, implies that the potential growth of country A is more dependent on currency depreciation and, hence, on price competition. In both cases, the relative dependency of the peripheral country can be justified by its weaker productive organization and more traditional specialization relative to the other EMU intermediate country. In the first case ($\sigma_A < \sigma_B$), the fragile economic structure of country A makes its position in international markets so dependent on low real wages or so marginal that a one-shot depreciation would be insufficient to alter its position towards the rest of the world. Hence the trade balance elasticity of country A will be lower than that of country B, even if the latter does not mainly base its international competitiveness on prices. In the second case ($\sigma_A > \sigma_B$), the economic structure of country A would be too developed to be just focused on low real wages and strong enough to positively react to the depreciation; hence, the traditional specialization of country A can explain why its trade balance elasticity is higher than that of country B, as the latter does not mainly rely on prices for international competitiveness.

In the following analysis, we assume that the EMU's economies are sufficiently involved in the international markets to make the case of $\sigma_A > \sigma_B$ predominant. Therefore, in our model, the assumption $\sigma_A > \sigma_B$ is a stylized representation of two related empirical evidences: with respect to country B, country A faces higher unit labor costs as well as a negative gap in terms of innovative production and organization. This is equivalent to state that country A's economy mostly depends on low productivity and low margin goods or services that have a high price-elasticity, whereas country B's economy depends on high productivity and high margin goods or services that have a lower price-elasticity (in this respect, see also: Wierts et al. (2013).

The threshold value over which contagion displays, has to satisfy the condition $L^B_{D^A} - L^B_F = 0$ for the specific value of u^A. Thus, we get the new threshold level u_A^{**}. The latter implies that country B will remain in the monetary union despite the exit of country A, if the actual exogenous shock affecting the latter country is lower than u_A^{**} ($u_A < u_A^{**}$).[22] On the other hand, country B will choose to opt out of the monetary union (case of contagion), if the actual exogenous shock affecting country A is large enough to determine the condition $u_A > u_A^{**}$.

The expression that determines u_A^{**} is too complex to be explicitly reported and discussed in the text. Its implicit form is:

$$u_A^{**} = f(\beta, \gamma, \theta, \sigma_A, \sigma_B, C, I_B, s^W) \tag{18}$$

Let us emphasize three features of Eq. (18). First, it seems reasonable to assume that $u_A^{**} > u_A^{*}$ since this is equivalent to state that the minimum level of shock generating contagion is greater than the minimum level of shock leading to the exit of country A without contagion.[23] Second, as also shown by Eq. (4) above, the choice of country A to abandon the euro implies an impact cost I_B for the remaining country B. Eq. (18) highlights that I_B affects the level of u_A^{**}: if $I_B > 0$, contagion would be less likely. Finally, u_A^{**} is also affected by the depreciation of the monetary union currency with respect to the rest of the world. In this last respect, we can differentiate u_A^{**} with respect to s^W. Under reasonable value of β, $\frac{\partial u_A^{**}}{\partial s^W}$ is negative as in the case of u_A^{*}.[24] This means that a depreciation of the monetary union currency (decrease in s^W) increases the threshold level u_A^{**} and, hence, reduces the probability of contagion.

[21] This simplification will be applied to all the remaining sections.

[22] In this case as well, we will assume that country B will always choose to remain in if the shock affecting country A is equal to u_A^{**} (see footnote 17, above).

[23] To support the intuition proposed in the text, the Appendix A1 shows that the inequality $u_A^{**} > u_A^{*}$ is satisfied for reasonable values of the parameters and for reasonable intervals in the values of output gap elasticities to the real effective exchange rates. Here, we are interested in the possible effect of euro depreciation on the probability of contagion for country B.

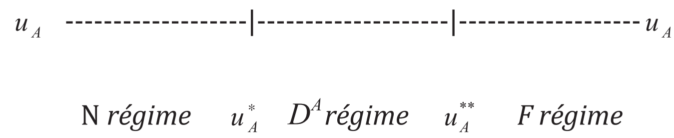

Fig. 2. .

To summarize (see Fig. 2): the exit of country A leads to a Nash-equilibrium without contagion, represented in our game by the *régime* D^A, when $u_A^{*} < u_A \leq u_A^{**}$; this same exit causes instead a Nash-equilibrium with contagion, represented in our game by *régime F*, when $u_A > u_A^{**} > u_A^{*}$.

6. Contagion and output gap elasticities

In this section, we aim to focus our analysis on the output gap elasticities of countries A and B (σ_A and σ_B, respectively) to the effective real exchange rates of these same countries. As we already stated (see Section 5, above), we assume that the elasticity of the representative peripheral country A is higher than that of country B ($\sigma_A > \sigma_B$).[25] This assumption is supported by the fact that peripheral economies have a more traditional specialization, higher unit labor costs and weaker productive organization than other countries of the monetary union. Hence, in our model, country A competes more on the depreciation of its effective real exchange rate (in other words, on prices) than on other factors (such as organizational and technical innovations, product quality, consequent increases in labor productivity, and so on). The opposite holds true for country B.[26] This is the reason why we focus on the elasticity of the two countries under consideration. If we were able to show that the values of σ_A and σ_B have an impact on the decisions of country A and B to stay in or opt out of the monetary union, it would follow that the relative competitiveness of these two countries affects the stability of a monetary union. This possible result is important for the analysis of the EMU. It would confirm that one of the main factors of instability in the euro area is the competitive weakness of peripheral member states towards the rest of the area and the consequent lack of convergence inside the monetary union (cf. Eichengreen, 2010; European Commission, 2012).

In order to investigate the role played by the elasticities to real effective exchange rates in the relationship between country A and B, it is convenient to set the world exchange rate s^W equal to 0. Moreover, in order to avoid any reference to other factors of reciprocal influence exercised by these two countries on each other, let us eliminate the parameter β. Given the previous simplifications, Eq. (1a) becomes:

$$y^A = \sigma_A(s^A - s^B) - u^A. \tag{1 bis}$$

Combining Eq. (5A) and (1 bis), we can solve $L^A_{D^A} - L^A_N = 0$ for u^A. The result is the determination of the threshold level (v^*), which makes it indifferent for country A to stay in or leave the monetary union given the value of σ_A (see also Eq. (13) in Section 4 above and Fig. 3 below):

[24] See Appendix A2.

[25] Formally, we could allow for a weaker condition, that is $\sigma_A \geq \sigma_B$. However, from an economic point of view, the stronger condition adopted in the text is more convincing.

[26] The simplification introduced in our model allows us to compare country B only to country A, keeping in mind that, in a more complex model with three interacting countries (A, B, and E), the representative core country E would be characterized by the lowest elasticity to its real effective exchange rate.

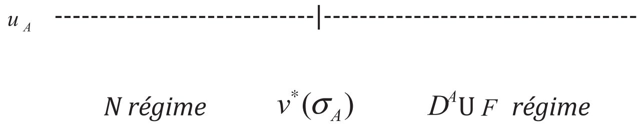

Fig. 3. .

$$v^* = \frac{\sqrt{C({\sigma_A}^2 + \theta)}}{\sigma_A}. \tag{19}$$

Eq. (19) implies that, if country A's specific shock u^A exceeds v^*, this country will leave the monetary union and depreciate its currency such that the possible dominant *régimes* become D^A U F. Instead, if u^A is lower than or equal to v^*, country A will stay in the monetary union making *régime N* a Nash equilibrium. It is then obvious that, if v^* increases, the probability that country A stays in will also increase given the probability density function of u_A.

Eq. (19) stresses that the value of v^* depends on σ_A. Hence, to complete the economic interpretation of this equation and Fig. 3, we have to analyze the effect of the elasticity σ_A on v^*. It is easy to assess that:

$$\frac{\partial v^*}{\partial \sigma_A} = -\frac{C\theta}{\sqrt{C({\sigma_A}^2 + \theta)}\,{\sigma_A}^2} \tag{20}$$

Being C, $\theta > 0$(see Section 2, above), Eq. (20) clearly states that the lower the elasticity of country A the higher v^* becomes. This means that, if country A improved its non-price competitiveness, reducing its gap towards countries B and E, the threshold level v^* would increase; hence, *ceteris paribus*, the exit of country A from the monetary union would be less likely to occur. The opposite would obviously hold true in the case of a further weakening of country's A non-price competitiveness. In this case, the value of v^* would decrease so that, *ceteris paribus*, the exit of country A from the monetary union would be more likely to occur. Applying this result to the actual working of the EMU, we find that the non-price competitiveness of peripheral member states has an impact on the stability of the euro area. The risk of an exit from the EMU would decrease and the euro area would become more stable if peripheral member states improved their organizational and technical innovations and the dynamics of their labor productivity, reducing their gaps vis-à-vis the other member states.

Following the refinements introduced in the previous section, we can now be more precise about the conditions that would lead country B to select the bad equilibrium (D^A) or the worst equilibrium (F) when country A decides to leave the monetary union due to the fact that $u_A > v^*$.[27] In this respect, let us derive another threshold level for the shock hitting country A. This new threshold (v^{**}), other than satisfying the condition $u_A > v^*$ (a necessary condition for the exit of country A), must make it indifferent for country B to stay in or to opt out of the monetary union, satisfying the condition $L^B_{D^A} - L^B_F = 0$ for the specific value of u_A. We thus get the new threshold level v^{**}:

$$v^{**} = \frac{\sqrt{(\theta\,{\sigma_B}^2 + \theta^2 - {\sigma_A}^4)(C + I_B)\theta({\sigma_A}^2 + {\sigma_B}^2 + \theta)({\sigma_A}^2 + \theta)}}{(\theta\,{\sigma_B}^2 + \theta^2 - {\sigma_A}^4){\sigma_B}^2\sigma_A} \tag{21}$$

We can easily show that v^{**} is greater than v^*. From (21) and (19), we derive:

[27] It is obvious that the bad equilibrium D^A is the best possible equilibrium given the exit of country A. This is equivalent to stating that, in the extended form of our game, country B is at a node where the 'bad' and 'worst' equilibria are the only possible outcomes.

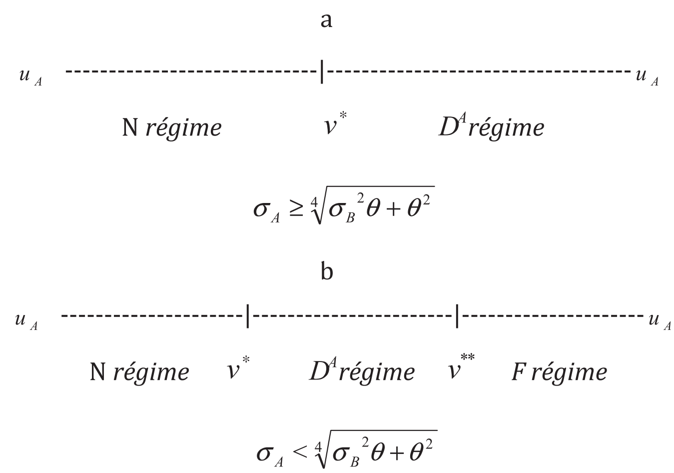

Fig. 4. .

$$\frac{v^{**}}{v^*} = \frac{\sqrt{\theta}({\sigma_A}^2 + {\sigma_B}^2 + \theta)\sqrt{{\sigma_A}^2 + \theta}\sqrt{B_r + C}}{\sqrt{\theta\,{\sigma_B}^2 + \theta^2 - {\sigma_A}^4{\sigma_B}^2}\sqrt{C}} = \frac{\sqrt{{\sigma_A}^2 + \theta}}{\sqrt{{\sigma_B}^2 + \theta - \frac{{\sigma_A}^4}{\theta}}} * \frac{{\sigma_A}^2 + {\sigma_B}^2 + \theta}{{\sigma_B}^2} * \frac{\sqrt{I_B + C}}{\sqrt{C}} > 1 \tag{22}$$

Eqs. (19) and (21) imply that country B will remain in the monetary union despite the exit of country A, if the actual exogenous shock affecting the latter country is lower than or equal to v^{**} ($u_A < v^{**}$). On the other hand, country B will choose to opt out from this monetary union (case of contagion), if the actual exogenous shock affecting country A is large enough to satisfy the condition $u_A > v^{**}$. It follows that *régime* D^A becomes a Nash-equilibrium when $v^* < u_A \le v^{**}$; conversely, *régime F* becomes a Nash-equilibrium when $u_A > v^{**} > v^*$.

Eqs. (19) and (21) also allow the rephrasing of previous results in terms of σ_A and σ_B (see Fig. 4a–b). Whenever $\sigma_A \ge \sqrt[4]{{\sigma_B}^2\theta + \theta^2}$ (or $\sigma_B = 0$), $L^B_{D^A} - L^B_F$ will become negative for each shock level so that F can never be a Nash-equilibrium (see Fig. 4a). On the other hand, whenever $\sigma_A < \sqrt[4]{{\sigma_B}^2\theta + \theta^2}$, $L^B_{D^A} - L^B_F$ will become positive and, hence, F will be the selected *régime* if the shock exceeds the threshold level v^{**} (see Fig. 4b).

In other words, given the elasticity of country A, these new results imply that the selection between the bad and worst equilibrium will depend on the elasticity of country B. If the latter is zero or small enough ($\sigma_A \ge \sqrt[4]{{\sigma_B}^2\theta + \theta^2}$), contagion can never be observed; conversely, if σ_B is high enough ($\sigma_A < \sqrt[4]{{\sigma_B}^2\theta + \theta^2}$) and u_A strong enough, contagion and the direct breakdown of the monetary union can happen. Hence, country B—the representative of countries in between 'central' and 'peripheral' member states—can avoid contagion if it can maintain a low elasticity, achieving competitiveness through innovation and labor productivity dynamics rather than currency depreciation. This conclusion offers a stimulating interpretation of the workings of the euro area. The EMU could better survive the exit of a peripheral member state if there were a convergence between the remaining member states in terms of non-price competition.

7. Conclusion

This paper, which is a significant extension of Canofari et al. (2015), analyzes the effects of common currency depreciation and output gap elasticities to the real effective exchange rate on the stability of a monetary union.

Our model reaches four main findings: (a) the depreciation (appreciation) of the common currency under examination as well as an increase (decrease) in inflation aversion reduce (increase) the probability that the most fragile countries of this monetary union, hit by an idiosyncratic and negative shock, choose to exit; (b) the same effect applies to the probability that this exit implies contagion to more robust member states of the same area; (c) the higher the elasticity of the most fragile countries affected by the shock, the higher the probability of them abandoning the monetary union; (d) given this elasticity, if the elasticities of other more robust countries of the same area are low enough, the contagion effect will never display. These findings lead us to conclude that the risk of an exit from a monetary union does not only depend on price competition. In fact, points (c) and (d) show that this risk would increase if the subset of its most fragile member states relied too much on price competition. Moreover, the risk that this exit results in a breakdown of the monetary union due to widespread contagion effects could be reduced, if other more robust member states mainly based their competitiveness on organizational and technical innovations and, hence, on a positive dynamics of labor productivity.

Leaving a monetary union involves many other economic factors, as well as institutional and organizational factors that the basic version of our theoretical model does not take into account. Hence, we enriched this version with a number of structural parameters in the attempt to account for some of the institutional complexities of an exit/remain choice. We introduced an exogenous exit cost in the loss functions of the several types of countries as well as an impact cost in the loss function of the more robust member states in case of a remain choice despite the contagion. Then, we characterized the output gap equations of the different countries by differentiating the initial values of their output elasticities to the exchange rate and by varying these values in case of exit. Finally, we referred to the relationships between the monetary union and the rest of the world by analyzing the role of the common currency exchange rate with respect to the rest of the world.

These improvements allow important refinements of the basic results. However, our analytical conclusions are still too stylized to be econometrically proven or directly applied to a specific monetary union such as the EMU. The possible exit of a given member state from a monetary union mainly depends on the policy stance and policy reactions implemented in the area; even the enriched version of our model does not endogenize any policy tool. The first analytical improvement for future research is to include a representation of further economic, institutional, and organizational factors to allow the model to analyze different stances in monetary and fiscal policy. The main steps in these directions would have to be the following two: (i) introduction of a centralized monetary policy and of national fiscal policies in the countries analyzed, constrained by the respective legacy in terms of the public balance sheet; (ii) reference to the specific banking sectors of these countries.

These steps are difficult to implement. Hence, our strategy will be to pursue a gradual introduction of new variables, which will probably require a further simplification in the structure of the model.

From an analytical point of view, there are two other obvious improvements for future research. The first is the re-design of the structure of our two-agent game, which analyzes the strategic interactions between a peripheral member state and a more robust, but not core, member state in the monetary union, thus confining the core member states to the background with a passive position. It would be convenient to build a three-agent game containing a strategic interaction between three member states representative of peripheral, intermediate, and core countries. The second analytical improvement consists of transforming our "one shot" analysis into a dynamic one. This would also allow us to build a solid setting for a better incorporation of institutional and organizational cost functions. Let us emphasize that it would be useless to pass through a "repeated game." As shown by our simple "one shot" model, each round of interactions between two countries changes the structure regardless of an exit or non-exit outcome. Hence, it would be necessary to build a dynamic game (see Weibull, 1995; Basar and Olsder 1999; see also: van Aarle at al. 2002).

Appendix A1

In the text, we maintained that it is reasonable to assume $u_A^{**} > u_A^{*}$ since this inequality is equivalent to stating that the minimum shock capable of generating contagion is greater than the minimum shock necessary for triggering just the exit of peripheral country *A*. In the text (see footnote 23), we also pointed that it would have been possible to go beyond this intuition by proving that $u_A^{**} > u_A^{*}$ under reasonable values for the parameters in Eq. (22) and reasonable intervals for the values of output gap elasticities to the real effective exchange rates (σ_A and σ_B). Here, we prove this result by means of Fig. A1.

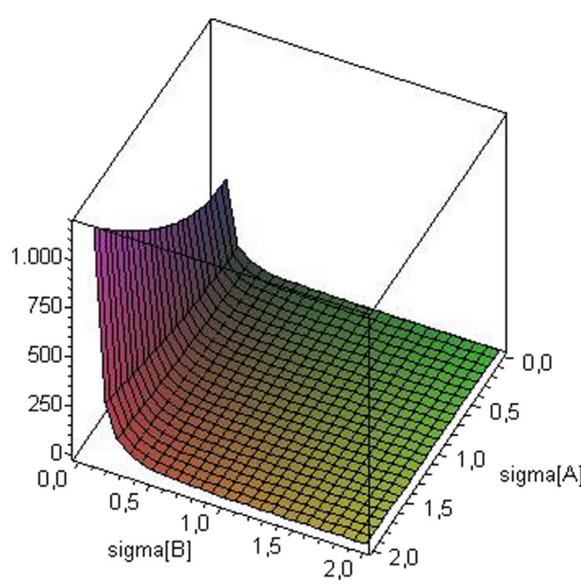

Fig. A1. .

Fig. A1 represents the values assumed by $(u_A^{**}/u_A^{*}) - 1$ for different values of σ_A and σ_B. The positive value of this function obviously satisfies the condition $u_A^{**} > u_A^{*}$. Let us assume that the values of the parameters are:

$$\beta = \frac{1}{3},\ \gamma = \frac{1}{3},\ \theta = 1,\ C = 1,\ I_B = 0,\ s^W = 0.5$$

Given these values, which appear reasonable, it follows that we remain in the positive portion of the figure if the elasticities σ_A and σ_B assume values between 0 and 2, which are also reasonable values.

Appendix A2

$$\frac{\partial u_A^{**}}{\partial s^W} = A^*B - X$$

$$A \equiv \frac{\varphi^2[\theta^2 + (\sigma_A^2 + \sigma_B^2)\theta(1 - \beta^2)\sigma_A^2\sigma_B^2\beta^3]^2\sigma_A^3\theta^2(\sigma_A^2 + \theta)^2 s^W}{\sqrt{\{[(1 - \beta^2)\sigma_A^2 + \theta]\sigma_B^2 + \theta(\sigma_A^2 + \theta)\}^2\{(I_B + C)[(1 - \beta^2)\sigma_A^2 + \theta]^2\sigma_B^2 + \theta\{[\theta\phi^2(s^W)^2\beta^4 - 2(I_B + C)(\beta^2 - \tfrac{1}{2})]\sigma_A^4(\sigma_A^2 + \theta)^2\sigma_B^4 - 2\theta(\beta^2 - 1)(I_B + C)\sigma_A^2 + (I_B + C)\theta^2\}\}}}$$

$$B \equiv \{[(1 - 2\beta^2)\theta + \sigma_B^2(\beta^2 - 1)^2]\sigma_A^4 - 2\theta(\beta^2 - 1)(\sigma_B^2 + \theta)\sigma_A^2 + \theta^2(\sigma_B^2 + \theta)\}$$

$$X \equiv \frac{\varphi(\sigma_A^2 + \theta)\left\{-\left[(\beta - 1)\theta^2 + 2\left(\beta^2 - 1 + \frac{1}{2}\beta\right)\right]\sigma_B^2\theta - \sigma_B^4(\beta^2 - 1)^2\right\}(\beta + 1)\sigma_A^4 - [(\beta^2 - 2)\theta + 2\sigma_B^2(\beta - 1)(\beta + 1)^2]\theta(\sigma_B^2 + \theta)\sigma_A^2 + [\theta + \sigma_B^2(\beta + 1)]\theta^2(\sigma_B^2 + \theta)}{\sigma_B^2\beta\sigma_A\left\{\left[(1 - 2\beta^2)\theta + \sigma_B^2(\beta^2 - 1)^2\right]\sigma_A^4 - 2\theta(\beta^2 - 1)(\sigma_B^2 + \theta)\sigma_A^2 + \theta^2(\sigma_B^2 + \theta)\right\}}$$

Under reasonable value of β is easy to check that $\frac{\partial u_A^{**}}{\partial s^W} < 0$ since $A < 0$, $B > 0$, $X > 0$.

References

Aarle, van B., Engwerda, J., Plasmans, J., 2002. Monetary and fiscal policy interaction in the EMU: a dynamic game approach. Ann. Oper. Res. 109 (1), 229–264.

Arghyrou, M.G., Tsoukalas, J.D., 2011. The greek debt crisis: likely causes, mechanics and outcomes. World Econ. 34 (2), 173–191.

Athanassiou, P., 2009. Withdrawal and Expulsion from the EU and EMU – Some Reflections, ECB Legal Working Paper Series, No. 10.

Baldwin, R., Giavazzi, F. (Eds.), 2015. The Eurozone Crisis: A Consensus View of the Causes and a Few Possible Solutions. CEPR, London.

Barkbu, B., Berkmen, S.P., Lukyantsau, P., Saksonovs, S., Schoelermann, H., 2015. Investment in the Euro Area: Why Has It Been Weak? IMF Working Paper, 15/32.

Basar, T., Olsder, G.J., 1999. Dynamic non Cooperative Game Theory 2d ed. SIAM, Philadelphia.

Benigno, P., Romei, F., 2014. Debt deleveraging and the exchange rate. J. Int. Econ. 93 (1), 1–16.

Bohl, M.T., Michaelis, P., Siklos, P.L., 2016. Austerity and recovery: exchange rate regime choice, economic growth, and financial crises. Econ. Model. 53, 195–207.

Boltho, A., Carlin, W., 2013. EMU's problems: asymmetric shocks or asymmetric behavior? Comp. Econ. Stud. 55 (3), 387–403.

Buiter, W.H., Corsetti, G., Pesenti, P., 1996. Interpreting the ERM Crisis: Country-Specific and Systemic Issue. CEPR Discussion Paper, No. 1466. CEPR, London, (October).

Canofari, P., Di Bartolomeo, G., Piersanti, G., 2015. Strategic interactions and contagion effects under monetary unions. World Econ. 38 (10), 1618–1629.

Chen, X., Deng, X., Teng, S.H., 2009. Settling the complexity of computing two-player nash equilibria. J. ACM 56 (3), 1–57.

Chortareas, G., Mavrodimitrakis, C., 2016. Can monetary policy fully stabilize pure demand shocks in a monetary union with a fiscal leader? Econ. Model. 54, 463–468.

Cook, D.E., Devereux, M.B., 2016. Exchange rate flexibility under the zero lower bound. J. Int. Econ. 101, 52–69.

Corsetti, G., 2015. Roots of the EZ crisis: incomplete development and imperfect credibility of institutions. In: Baldwin, R., Giavazzi, F. (Eds.), The Eurozone Crisis: A Consensus View of the Causes and a Few Possible Solutions. CEPR, London, (VoxEU.org.).

Corsetti, G., Pesenti, P., 2005. International dimension of optimal monetary policy. J. Monet. Econ. 52 (2), 281–305.

Corsetti, G., Pesenti, P., Roubini, N., Tille, C., 2000. Competitive devaluations: toward a welfare-based approach. J. Int. Econ. 51 (1), 217–241.

De Grauwe, P., Ji, Y., 2013. Self-fulfilling crises in the eurozone: an empirical test. J. Int. Money Financ. 34, 15–36.

Di Bartolomeo, G., Engwerda, J., Plasmans, J., van Aarle, B., 2006. Staying together or breaking apart: policy-makers' endogenous coalitions formation in the european economic and monetary union. Comput. Oper. Res. 33 (2), 438–463.

Dreyer, J.K., Schmid, P.A., 2016. Growth effects of EU and EZ memberships: empirical findings from the first 15 years of the Euro. Econ. Model.. http://dx.doi.org/10.1016/j.econmod.2016.09.007.

Eichengreen, B., 2010. Imbalances in the Euro Area. University of California, (November, http:// eml.Berkeley.Edu).

Eichengreen, B., Sachs, Jeffrey, 1986. Competitive devaluation and the Great Depression: a theoretical reassessment. Econ. Lett. 22 (1), 67–71.

European Commission, 2012. Current account surpluses in the EU. Eur. Econ., 2012, (n. 9).

Fornaro, L., 2015. International debt deleveraging, Oesterreichische National Bank, Working Paper No. 182.

German Council of Economic Experts, 2015. Consequences of the Greek Crisis for a More Stable Euro Area, *Special Report*.

Giavazzi, F., Giovannini, Alberto, 1991. Limited Exchange Rate Flexibility. MIT Press, Cambridge MA.

Hofmeister, H., 2011. Goodbye euro: legal aspects of withdrawal from the Eurozone. Columbia J. Eur. Law 18, 111–134.

Jeanne, O., 1997. Are currency crises self-fulfilling? A test. J. Int. Econ. 43 (3-4), 263–286.

Obstfeld, M., 1986. Rational and self-fulfilling balance of payments crises. Am. Econ. Rev. 76 (1), 72–81.

Obstfeld, M., 1994. The logic of currency crises. Cah. Économiques Et. Monétaires 43 (1), 189–213.

Obstfeld, M., 1997. Destabilizing effects of exchange-rate escape clauses. J. Int. Econ. 43 (1–2), 61–77.

Papadimitriou, C.H., 2007. The complexity of finding nash equilibria. In: Nisan, N., Roughgarden, T., Tardos, E., Vezirani, V.V. (Eds.), Algorithmic Game Theory. Cambridge University Press, Cambridge, 29–51.

Tosato, G.L., 2015. The European Monetary Union (EMU): Evolution and Legal Challenges. SEP Policy Brief, 17.

Weibull, J.W., 1995. Evolutionary Game Theory. Cambridge Ma.: MIT Press.

Wierts, P., Kerkhoff, Van, H., De Haan, J., 2013. Composition of exports and export performance of Eurozone countries. J. Common Mark. Stud. 52 (4), 928–941.