Exclusionary Pricing and Rebates When Scale Matters*

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Abstract

We consider an incumbent firm and a more efficient entrant, both offering a network good to several asymmetric buyers, and both being able to price discriminate. The incumbent disposes of an installed base, while the entrant has a network of size zero, and needs to attract a critical mass of buyers to operate. We analyze different price schemes (uniform pricing, implicit price discrimination - or rebates, explicit price discrimination) and show that the schemes which - for given market structure - induce lower equilibrium prices are also those under which the incumbent is more likely to exclude the rival.

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1 Introduction

This paper deals with exclusionary pricing practices. One such practice which has recently received renewed attention is *rebates*, i.e. discounts applicable where a customer exceeds a specified target for sales in a defined period.

There are different types of rebates. They can be made contingent on the buyer making most or all of its purchases from the same supplier ("fidelity" or "loyalty" rebates), on increasing its purchases relative to previous years, or on purchasing certain quantity thresholds specified in absolute terms. It is on this last category of rebates that we focus here.

Under US case law, rebates are generally said to promote competition on the merits, and the (high) burden of demonstrating their anticompetitive effect is on the plaintiff.[1] In the EU, instead, the European Commission and the Community Courts have systematically imposed large fines on dominant firms applying different forms of rebates.[2] The recent *Michelin II* judgment has established that even standardized quantity discounts (that is, standardized rebates given to any buyer whose purchases exceed a predetermined number of units) are anticompetitive if used by a dominant firm.[3]

One of the objectives of this paper is to study whether rebates, in the form of pure quantity discounts, can have anticompetitive effects. In an industry exhibiting network effects, we find that if rebates are allowed, an incumbent firm having a critical customer base is more likely to exclude a more efficient entrant that can use the same rebate schemes but does not have a customer base yet. Rebates are a form of implicit discrimination, and the incumbent can use them to make more attractive offers to some crucial group of consumers, thereby depriving the entrant of the critical mass of consumers it needs (in our model, network externalities imply that consumers will want to consume a network product only if demand has reached a critical threshold). Only very efficient entrants will be able to overcome the entry barriers that incumbents can raise in this manner.

To give an example of the type of industry that we have in mind, consider the *Microsoft Licensing Case* of 1994-95 (Civil Action No. 94-1564). Microsoft markets its PC operating systems (Windows and MS-DOS) primarily through original equipment manufacturers ("OEMs"), which manufacture PCs. When discussing the substantial barriers to entry for potential rivals of Microsoft, the Complaint explicitly mentions "the difficulty in convincing OEMs to offer and promote a non-Microsoft PC operating system, particularly one with a small installed base".

[1] See Kobayashi (2005) for a review of the US case law on rebates.

[2] For a review of the EU case law on rebates, see e.g. Gyselen (2003).

[3] The (almost) per se illegality of exclusive contracts, rebates and discriminatory prices by dominant firms has led to a hot debate on the EU policy towards abuse of dominance. See Gual et al. (2006) for a contribution to the debate.

The US Department of Justice alleged that Microsoft designed its pricing policy to deter OEMs from entering into licensing agreements with competing operating system providers, thereby reinforcing the entry barriers raised by the network effects that are inherent in this industry.

Although rebates may have exclusionary effects, it is far from clear that they should be presumed to be welfare-detrimental, even if used by a dominant firm. As John Vickers, then Chairman of the UK Office of Fair Trading, put it:

> "These cases about discounts and rebates, on both sides of the Atlantic, illustrate sharply a fundamental dilemma for the competition law treatment of abuse of market power. A firm with market power that offers discount or rebate schemes to dealers is likely to sell more, and its rivals less, than in the absence of the incentives. But that is equally true of low pricing generally." (Vickers, 2005: F252)

Discriminatory pricing has similar contrasting effects. Consider for instance an oligopolistic industry. On the procompetitive side, it allows firms to decrease prices to particular customers, thereby intensifying competition: each firm can be more aggressive in the rival's customer segments while maintaining higher prices with the own customer base, but since each firm will do the same, discriminatory pricing will result in fiercer competition than uniform pricing, and consumers will benefit from it.[4] On the anticompetitive side, though, in asymmetric situations discriminatory pricing may allow a dominant firm to achieve cheaper exclusion of a weaker rival: prices do not need to be decreased for all customers but only for the marginal customers.[5]

This fundamental dilemma between the efficiency effects created by discriminatory pricing and their potential exclusionary effects is one of the main themes of the paper. We show that explicit price discrimination is the pricing scheme with the highest exclusionary potential (and hence the worst welfare outcome if exclusion does occur), followed by implicit price discrimination (i.e., rebates, or pure quantity discounts) and then uniform pricing. However, for given market structure (i.e., when we look at equilibria where entry does occur), the welfare ranking is exactly reversed: the more aggressive the pricing scheme the lower the prices (and thus the higher the surplus) at equilibrium. This trade-off between maximizing the entrant's chances to enter and maximizing consumer welfare for given market structure, illustrates the difficulties that antitrust agencies and courts find in practice: a tough stance against discounts and other aggressive pricing strategies may well increase the likelihood that monopolies or dominant positions are successfully contested, but may also deprive consumers of the possibility to enjoy lower prices, if entry did occur.

Although it deals with pricing schemes rather than contracts, our paper is closely related to the literature on anticompetitive exclusive dealing. Segal and

[4] See Thisse and Vives (1988). For a survey on discriminatory pricing, see e.g. Stole (2005).
[5] See e.g. Armstrong and Vickers (1993).

Whinston (2000) is probably the closest work to ours.[6] Building on Rasmusen et al. (1991), they show the exclusionary potential of exclusive contracts when the incumbent can discriminate on the compensatory offers it makes to buyers. Our study differs from theirs in several respects: (i) in their game the incumbent has a (first-mover) strategic advantage in that it is allowed to contract with buyers before entry occurs; (ii) if buyers accept the exclusivity offer of the incumbent, they commit to it and cannot renegotiate it even if entry occurs; (iii) buyers are symmetric and only linear pricing is considered. In our paper, instead, (i) the incumbent and the entrant choose price schedules simultaneously, (ii) buyers simply observe prices and decide which firm to buy from (therefore avoiding any problems related to assumptions on commitment and renegotiation); (iii) we explore the role of rebates and quantity discounts in a world where buyers differ in size. Yet, the mechanisms which lead to exclusion in the two papers are very similar: both papers present issues of buyers' miscoordination, and scale economies which are created by fixed costs in their model are created instead by network effects in ours.

Our paper is also related to the literature on divide-and-conquer strategies, in particular to Innes and Sexton (1993, 1994) and Segal (2003). A major innovation of our work relative to theirs is that we allow the entrant to use the same discriminatory tools available to the incumbent. Also, contrary to Innes and Sexton's (1994) finding, in our case a ban on discrimination cannot prevent inefficient outcomes: in our setting, exclusion can arise also under uniform linear pricing.[7]

Finally, our paper is related to the literature on incompatible entry in network industries. The very nature of network effects provides a strong incumbency advantage, shielding dominant firms against competitors even in the absence of any anticompetitive conduct (Farrell and Klemperer, 2006). Crémer et al. (2000) show that an incumbent can strategically use compatibility decisions so as to deter entry.[8] More closely related to our paper, Jullien (2001) studies how an entrant can use divide-and-conquer strategies to induce buyers to coordinate on the entrant instead of the incumbent. This insight reappears in an extension (Section 5.2), where we show that negative prices (i.e. usage subsidies) may indeed break miscoordination equilibria, thus making successful

[6] Bernheim and Whinston (1998) analyze the possible exclusionary effects of exclusive dealing when firms make simultaneous offers (as in our paper), but in *non-coincident markets*: first, exclusivity is offered to a buyer in a first market; afterwards, offers are made to a buyer in a second market. In their terminology, our paper is looking at *coincident market* effects, which makes our analysis closer to Aghion and Bolton (1985), Rasmusen et al. (1991), Segal and Whinston (2000) and Fumagalli and Motta (2006). All these papers, however, study only exclusive dealing arrangements and assume that the entrant can enter the market (if at all) only *after* the incumbent and the buyers have negotiated an exclusive contract.

[7] Innes and Sexton (1993, 1994) consider a very different contracting environment, strategic variables, and timing of the game. In particular, after the incumbent made its offers, they allow the buyers to contract with the entrant (or to enter themselves), so as to create countervailing power to the incumbent's.

[8] Where incompatibility could be overcome through multi-homing, Shapiro (1999) argues that incumbents can use exclusive dealing contracts to block multi-homing, thus excluding a technologically superior firm.

entry more likely.

The paper continues in the following way. Section 2 describes the model, Section 3 solves the model under the assumption that prices have to be non-negative. Three cases are analyzed: uniform pricing, explicit price discrimination and implicit price discrimination (that is, rebates). Section 4 studies the effects of the different pricing schemes on consumer surplus. Section 5 shows how our results are affected when relaxing the assumptions of the basic model. Section 6 concludes the paper.

2 The setup

Consider an industry composed of two firms, the incumbent I, and an entrant E. The incumbent supplies a network good, and has an installed consumer base of size $\beta_I > 0$. (The network good is durable: "old" buyers will continue to consume it but no longer need to buy it.) I incurs constant marginal cost $c_I \in (0, 1)$ for each unit it produces of the network good.

The entrant can supply a competing network good at marginal cost $c_E < c_I$, i.e. it is more cost-efficient than the incumbent. Since it has not been active in the market so far, it has installed base $\beta_E = 0$. To focus on the role of network externalities, we assume away any fixed costs of entry.

The good can be sold to $m + 1$ different "new" buyers, indexed by $j = 1, \ldots, m + 1$. There are $m \geq 1$ identical small buyers, and 1 large buyer.[9] Goods acquired by one buyer cannot be resold to another buyer, but they can be disposed of at no cost by the buyer who bought them (in case the latter cannot consume them). Side payments of any kind between buyers are ruled out. Define firm i's network size s_i (where $i = I, E$) as $s_i = \beta_i + q_i^1 + \ldots + q_i^{m+1}$, i.e. the firm's installed base plus its total sales to all "new" buyers.

To simplify the analysis, we assume that demands are inelastic. A buyer will either buy from the incumbent, or from the entrant (but not from both). The large buyer can consume at most $Q^l = 1 - k$ units, while any small buyer can consume at most $Q^s = \frac{k}{m}$ units. Total market size is normalized to 1: $m(k/m) + (1 - k) = 1$.[10] The parameter $k \in (0, 1)$ measures the market share of the group of small buyers, $1 - k$ measures the large buyer's share. Assume that $1 - k > k/m$, so that the large buyer's demand is always larger than a small buyer's demand (provided they both demand strictly positive quantities). This implies that:

$$k < \frac{m}{m+1} \in \left[\frac{1}{2}, 1\right). \tag{1}$$

[9] We assume $m \geq 1$ so as to allow for the large buyer to be smaller than the set of all small buyers (which in turn allows for the large buyer to receive better price offers) and to show that prices under rebates depend on the degree of fragmentation of small buyers (and converge to prices under explicit discrimination as $m \to \infty$).

[10] These quantities apply for general (positive or negative) prices. In the base model we restrict prices to be non-negative. Section 5 considers the case where prices can be negative.

Buyers exert positive consumption externalities on each other: If firm i's network size s_i is below the threshold level $\bar{s}$, consumption of i's good gives zero surplus to its buyer. The goods produced by the two firms are incompatible, so that buyers of firm i do not exert network externalities on buyers of firm j. For a network good of sufficient size, large and small buyers have the same maximum willingness to pay of $\bar{p} = 1$.

The assumption that a buyer's utility from consuming is positive only if the network in question reaches the threshold size $\bar{s}$ is designed to capture in an admittedly simple way the presence of network effects.[11] We will deal with the case of continuous utility functions in Section 5.3.

We assume that $\beta_I \geq \bar{s}$: the incumbent has already reached the critical size, while the entrant will have to attract enough buyers to reach $\bar{s}$.[12]

Let the unit prices offered by the two firms to a buyer of type $j = l, s$ be $p_I^j \leq 1$ and $p_E^j \leq 1$. A buyer's net consumer surplus is given by gross consumer surplus minus total expenditure, $CS^j\left(q_i^j, p_i^j, s_i\right) = grossCS^j\left(q_i^j, s_i\right) - p_i^j q_i^j$, where gross consumer surplus is defined as:

$$\begin{aligned} grossCS^l\left(q_i^l, s_i\right) &= \begin{cases} \min\left\{q_i^l, 1-k\right\} & \text{if } s_i \geq \bar{s} \\ 0 & \text{otherwise} \end{cases} \\ grossCS^s\left(q_i^s, s_i\right) &= \begin{cases} \min\left\{q_i^s, k/m\right\} & \text{if } s_i \geq \bar{s} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{2}$$

Since both types of buyers have the same prohibitive price $\bar{p} = 1$, a monopolist who could price discriminate would set a uniform unit price $p_i^m = 1$. Thus, discriminatory pricing can arise only as a result of strategic interaction.

We assume that neither demand of the large buyer alone, nor demand of all small buyers taken together, is sufficient for the entrant to reach the minimum size:

$$\bar{s} > \max\ \{1-k, k\}. \tag{3}$$

In other words, in order to reach the minimum size, the entrant has to serve the large buyer plus at least one (and possibly more than one) small buyer.[13] We relax this assumption in Section 5.1.

We also assume that the threshold level $\bar{s}$ is such that if the entrant sells to all $m+1$ new buyers, then it will reach the minimum size: $\bar{s} \leq 1$. This, together with the assumption $c_E < c_I$, implies that the social planner would want the entrant (and not the incumbent) to serve all buyers.

[11] It also has the advantage that the old generation of buyers can be ignored when studying welfare effects: since we assume that they have already attained the highest level of utility, new buyers' decisions will never affect old buyers' utility. Of course, this means that we cannot formalise here the possibility that entry may hurt the old generation of buyers, but this is a well-known effect which does not need to be emphasised again.

[12] Note that if the entrant manages to reach the minimum size $\bar{s}$, then consumers will consider I's and E's networks as being of homogenous quality, even if $s_I \neq s_E$.

[13] Note that only units which are actually consumed count towards firm i's network size.

The game. Play occurs in the following sequence: At time $t = 0$, the incumbent and the entrant simultaneously announce their prices, which will be binding at $t = 1$. At time $t = 1$, each of the $m + 1$ buyers decides whether to patronize the incumbent or the entrant. We also assume that offers are observable to everyone.

As for the prices that firms can offer in $t = 0$, in the base model (Section 3) we consider three different possibilities: uniform prices (Section 3.2.1); explicit (or third-degree) price discrimination (Section 3.2.2); and implicit (or second-degree) price discrimination, i.e. the case of standardized quantity discounts or "rebates" (Section 3.2.3).

3 Equilibrium solutions

In this Section, we assume that firms set non-negative prices, and we find the equilibria under the three different price regimes. In line with Segal and Whinston (2000), we find that in each regime our game has two types of pure-strategy Nash equilibria: an exclusionary (miscoordination) equilibrium where all buyers buy from the incumbent, and an entry equilibrium where all buyers buy from the entrant. For each type of equilibria, we shall focus on the highest prices that can be sustained.

Since miscoordination equilibria are the same independently of the pricing regime, we first state a general miscoordination result which holds for any regime (Section 3.1), and we then analyze entry equilibria under the different regimes (Section 3.2).

3.1 Miscoordination equilibria

Proposition 1 *(Miscoordination equilibrium under all price regimes) If firms can only set non-negative prices, the following pure-strategy Nash equilibrium exists: I sets* $p_I^s = p_I^l = p_I^m = 1$*, and in all continuation equilibria where* $p_E^j \leq p_I^j$*, with* $j = s, l$*, all buyers buy from I. The prices identified are the highest that can be sustained in a miscoordination equilibrium.*

Proof: see Appendix A

To understand this Proposition, note that when $p_E^j \leq p_I^j$ (with $j = s, l$) there is a miscoordination equilibrium where all buyers buy from the incumbent: despite the higher price p_I, no buyer has a unilateral incentive to deviate, since - given that all other buyers buy from I - the entrant's network would be below the critical size, and buying from the entrant would then give zero (gross) utility.

Continuation equilibria play a role for the equilibrium at the firms' decision stage. Consider the candidate miscoordination equilibrium where $p_E = p_I = 1$ and all buyers buy from the incumbent. This equilibrium is sustained by having that when $p_E \leq p_I$ the chosen continuation equilibria are those where all buyers will buy from the incumbent.[14] (As we shall see later, when $p_E \leq p_I$ there are also continuation equilibria where all buyers buy from the entrant). Otherwise, a deviation by the entrant could attract all the buyers, undermining the candidate equilibrium.

The equilibrium characterized in this Proposition represents an extreme case, in the sense that the underlying continuation equilibrium is the most favorable one for firm I. *This equilibrium is by no means the only miscoordination equilibrium that can arise in our game.* For instance, there are other equilibria where all buyers miscoordinate on the incumbent, but the latter can at most charge some price $\tilde{p}_I^j < p_I^m = 1$. Such an equilibrium is sustained by continuation equilibria where buyers buy from I as long as $p_E^j \leq p_I^j \leq \tilde{p}_I^j$, but would switch to E if p_I^j exceeded $\tilde{p}_I^j$. *For the rest of the paper, for both exclusionary and entry equilibria we will focus on those continuation equilibria which are the most profitable ones for the firm that eventually serves the buyers.* The motivation for this choice is two-fold: First, these equilibria are the Pareto-dominant ones from the point of view of the firms. Second, from a policy point of view, the equilibria with the highest profits are those which cause most concern.

Finally, note that the miscoordination equilibrium identified here does not depend on the price regime, as long as prices are non-negative. The driving force behind this equilibrium is just that a unilateral deviation by a buyer - given that all others buy from the incumbent - is not sufficient to give the entrant the threshold size it needs.[15]

3.2 Entry equilibria

In this Section, we look for the entry equilibria of the game. The conditions for their existence depend on the price regimes assumed, as we show below.

3.2.1 Uniform pricing

Assume that firms can only use uniform linear prices, p_i with $i = I, E$. Recall that any buyer's demand for E's good, $q_E^j(\ldots, s_E)$, depends on the size of E's network, s_E, which in turn depends on E's sales to the buyers, $\left\{q_E^1, \ldots, q_E^{m+1}\right\}$. Thus, the following can be proved.

Proposition 2 *(Entry equilibria under uniform prices) If firms can only use uniform flat prices, the following pure-strategy Nash equilibrium exists:*

E sets $p_E = c_I$, I sets $p_I = c_I$, and in all continuation equilibria where $p_E \leq p_I$, all buyers buy from E.

[14] In this situation, the entrant is indifferent among all prices $p_E \geq 0$ it could charge, and might as well offer the monopoly price, which weakly dominates all other possible equilibrium prices.

[15] This also implies that the equilibria identified in Proposition 1 are not Perfectly Coalition Proof (PCP): In the second stage of the game, a collective deviation by a *group* of buyers sufficiently large to generate critical mass for the entrant could always disrupt a miscoordination equilibrium. However, we show in Section 5 that there are other ways to break miscoordination equilibria: for instance, allowing the entrant to offer negative prices and/or to discriminate perfectly.

Proof: see Appendix A

We have seen in Section 3.1 that when $p_E \leq p_I$, there is a miscoordination equilibrium where all buyers buy from the incumbent. However, there is also an equilibrium where all buyers buy from the entrant: no buyer has an incentive to deviate given that all others buy from the entrant, since he would pay a (weakly) higher price p_I for a product which is as good as the entrant's (if all buy, the entrant reaches critical size).[16]

Continuation equilibria are chosen to prevent deviations in the firms' stage of the game. Consider the candidate entry equilibrium where $p_E = c_I = p_I$ and all buyers buy from the entrant. Because of the multiplicity of equilibria at the buyers' stage, when the incumbent deviates by increasing its price, there might also be a continuation equilibrium where $p_E < p_I$ and all buyers buy from the incumbent. To eliminate such counter-intuitive deviations, it is required that in all continuation equilibria where $p_E \leq p_I$ all buyers buy from the entrant.

As in the case of miscoordination equilibria, there is a continuum of entry equilibria, and the particular equilibrium chosen in Proposition 2 is the most favorable for firm E.[17]

3.2.2 Explicit (3rd degree) discrimination

Under explicit price discrimination, each firm can set one price for the large buyer, and a different price for the small buyers (all buyers of the same type will be charged the same price). When firms can price discriminate, entry equilibria do not necessarily exist, unlike the uniform pricing case. To fix ideas, start with the candidate entry equilibrium where both firms charge c_I and all buyers buy from the entrant (a natural candidate, as this was an entry equilibrium under uniform pricing). This equilibrium can be disrupted by the incumbent setting a price $c_I - \epsilon$ to one category of buyers and the monopoly price to the other category: the loss made on the former would be outweighed by the profits made on the latter. Indeed, under this deviation the former category strictly prefers to buy from I, thus preventing the entrant from reaching the minimum size. Anticipating that for the former buyers it is a dominant strategy to buy from the incumbent, the latter category of buyers would also prefer to buy from I rather than from the entrant, since they would derive zero utility from buying from E.

Therefore, an entry equilibrium can exist only if it is immune to the deviations outlined above, i.e. if the entrant's prices to *both* large and small buyers are so low that the incumbent cannot profitably undercut either of the two prices while charging the monopoly price to the other group. This implies that the highest prices that the entrant can charge in any entry equilibrium will be

[16] To be precise, if $p_I = p_E$, there is also a buyers' equilibrium where some buyers (in a sufficient number for the entrant to reach the critical size) buy from the entrant and the remaining buyers buy from the incumbent.

[17] Under different continuation equilibria, there are also entry equilibria where the entrant must charge a strictly lower price than c_I to induce buyers to coordinate on E.

strictly below c_I to *both* sets of buyers. Thus, for an entry equilibrium to exist, the efficiency gap between entrant and incumbent must be large enough.

Proposition 3 *(Entry equilibria under explicit discrimination) Under explicit price discrimination, entry equilibria only exist if* $c_I \geq \min\left\{\frac{1+c_E}{2}, k+c_E, 1-k+c_E\right\}$. *The highest prices that the entrant can charge in any such entry equilibrium are* $p_E^s = \max\left\{\frac{c_I-(1-k)}{k}, 0\right\} < c_I$ *and* $p_E^l = \max\left\{\frac{c_I-k}{1-k}, 0\right\} < c_I$.

Proof: see Appendix A

Figure 1 illustrates the results of Proposition 3 (recall that miscoordination equilibria exist for all parameter values). The figure shows that, for given k, the

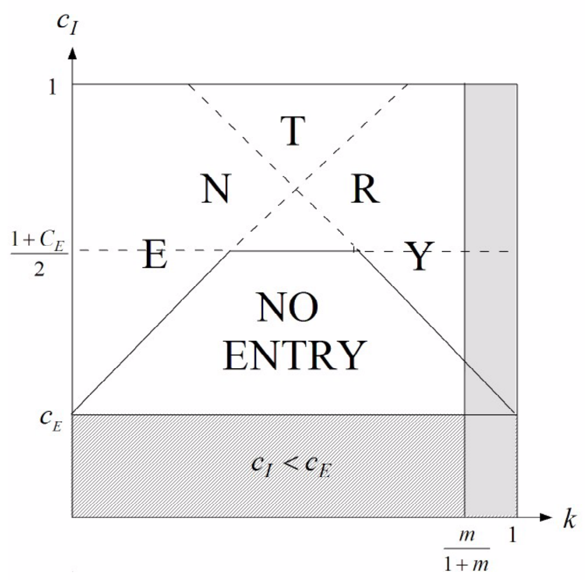

Figure 1: Existence of entry equilibria under explicit price discrimination (the grey areas are outside of the parameter space)

larger c_I with respect to c_E (that is, the larger the efficiency gap) the more likely for entry to be an equilibrium of the game. The intuition is straightforward: if the incumbent is less efficient, it will find it more difficult to profitably make low (discriminatory) price offers, which in turn makes it possible for the entrant to sustain higher (more profitable) prices which are immune to incumbent's deviations. The effect of k on equilibrium outcomes is slightly more complex.

Corollary 4 *The more asymmetry there is between the large buyer and the group of small buyers, the more likely are entry equilibria to exist.*

Proof:

Follows immediately from our existence condition $c_I \geq \min\left\{\frac{1+c_E}{2}, k + c_E, 1 - k + c_E\right\}$. Minimizing the expression in brackets with respect to k, we find that it converges to its global minimum c_E both as k goes to zero and and as k goes to 1. In other words, entry equilibria will exist even for arbitrarily small differences between c_I and c_E, provided the two buyer groups are sufficiently asymmetric.□

To understand why entry is more likely at very low levels and very high levels of k, consider for instance a candidate entry equilibrium (p_E^s, p_E^l) when k is very small. In order to disrupt this equilibrium, the incumbent could discriminate across buyers, by offering the large buyer a very low price and recovering losses by setting a high price to the small buyers, and vice versa. However, since k is very small, the incumbent cannot offer the large buyer a price (much) below c_I, since the profits it could make on the small buyers are very small (they account for a tiny part of the total market). In contrast, it could use the profits it makes on the (very) large buyer to decrease the price offered to the small buyers. But since prices are restricted to be non-negative here, the incumbent's best offer to the small buyers will be $p_I^s = 0$. In order to avoid deviations, the entrant will therefore have to set $p_E^s = 0$ and p_E^l slightly lower than c_I. As small buyers account for a small proportion of demand (k is very small), the entrant will make positive profits at these prices, and the entry equilibrium will exist. The same argument can be used symmetrically to explain why entry equilibria are more likely to exist if k is sufficiently large. Of course, what drives this result is that prices cannot go below zero. We shall see in Section 5.2 that when prices may be negative, k will affect results monotonically.

To summarize this Section, note that relative to uniform pricing, explicit price discrimination: (a) on the one hand, makes entry equilibria less likely to exist (they always exist under uniform pricing, but under discriminatory pricing they only exist only if c_I is high enough relative to c_E); (b) on the other hand, for given market structure, it results in (weakly) lower prices.[18]

3.2.3 Implicit (2nd degree) discrimination (or rebates)

Explicit discrimination may not always be feasible, for instance because of informational constraints (firms cannot observe buyer types), or because of policy constraints. Let us then consider the case where firms cannot condition their offers directly on the type of buyer (large or small), but have to make uniform

[18] For parameter values such that entry equilibria exist under both regimes, prices are strictly lower than c_I under discrimination, while they equal c_I under uniform pricing. The exclusionary equilibrium always exists - and the highest sustainable prices are the same - under both regimes.

offers to both types which may only depend on the quantity bought by buyer $j = 1, \ldots, m+1$:

$$T_i(q_i^j) = \begin{cases} p_{i,1} q_i^j & \text{if } q_i^j \leq \bar{q}_i \\ p_{i,2} q_i^j & \text{if } q_i^j \geq \bar{q}_i \end{cases} \tag{4}$$

(If the buyer buys exactly the threshold quantity, $q_i^j = \bar{q}_i$, the firm may either charge $p_{i,1}$ or $p_{i,2}$.) Each buyer can now choose his tariff from this price menu by buying either below the sales target $\bar{q}_i$ or above it.[19]

If this menu is designed appropriately, buyers will self-select into different tariffs: small buyers will buy below the threshold, while the large buyer will buy above the threshold, thus paying a different price than the small buyers. We say that firm i's offer satisfies the "self-selection conditions" if neither of the two buyer types wants to masquerade as the other, i.e. if

$$\begin{aligned} CS^l\left(p_{i,2}, 1-k\right) &\geq CS^l\left(p_{i,1}, \bar{q}_i\right) \\ \text{and } CS^s\left(p_{i,1}, k/m\right) &\geq CS^s\left(p_{i,2}, \bar{q}_i\right) \end{aligned} \tag{5}$$

For any offer that satisfies the self-selection condition, denote $(p_{i,1})$ by (p_i^s), and $(p_{i,2})$ by $\left(p_i^l\right)$, for $i = I, E$.

We now look for the equilibria that arise in this game when both firms can use quantity discounts.

Entry equilibria under implicit discrimination The implicitly discriminatory effect of rebates gives rise to an exclusionary mechanism similar to the one under explicit discrimination. Since buyers are asymmetric, they can be induced to self-select either into the high-quantity or the low-quantity bracket of the price menu, thus allowing the incumbent to de facto price-discriminate between them. This in turn enables the incumbent to offer a below-cost price to one group, thus winning their orders, while making up for the resulting losses by charging a high price (possibly the monopoly price) to the other group.

The major difference between explicit and implicit discrimination lies in the self-sorting conditions, which reduce the range of prices that the incumbent can offer. Consider for instance the case where, under explicit discrimination, the incumbent charges the monopoly price $p_I^s = 1$ to the small buyers, and $p_I^l = 0$ to the large buyer. Clearly, this offer does not satisfy the small buyers' self-sorting condition: At a zero price, the small buyers would always prefer to "buy" above the quantity threshold (i.e. receive a large quantity for free, and dispose of the

[19] Each buyer is allowed only one transaction. This rules out the possibility that a large buyer makes "multiple small purchases" so as to buy a large amount of units at the lower price. Important transaction costs may be invoked to justify this assumption, which in a way is the counterpart of the assumption that a small buyer cannot buy a large quantity and then resell it to others. In both cases, it is arbitrage which is prevented. Recall also that we exclude reselling of units between buyers (while allowing for free disposal), so the only thing a small buyer can do with units he cannot consume is to throw them away.

units they cannot consume) rather than paying $p_I^s = 1$ (or any other positive price) for a small quantity.

Likewise, an offer where $p_I^s < c_I$ and $p_I^l = 1$ cannot be replicated through a rebate tariff: in this case, it is the large buyer who would prefer to buy below the threshold and enjoy a positive surplus on the (few) units he consumes, rather than buying above the threshold and being left with zero surplus.[20]

Thus, while rebates still have exclusionary potential, the incumbent's deviation offers will be less aggressive under rebates than under explicit discrimination, allowing for entry equilibria to be sustained in some regions where they do not exist if firms can explicitly price discriminate.

Proposition 5 *(entry equilibria under implicit discrimination) Under rebates as defined in (4), entry equilibria only exist if*

(i) $c_E < \frac{1}{2(m+1)}$ *and* $c_I \geq \min\left\{c_E(1+m), k + c_E, \frac{m}{1+m} + c_E - k\right\}$

(ii) or if $c_E \geq \frac{1}{2(m+1)}$ *and* $c_I \geq \min\left\{\frac{m+(1+m)c_E}{1+2m}, k + c_E, \frac{m}{1+m} + c_E - k\right\}$

The highest prices that the entrant can charge in any such entry equilibrium are

$$p_E^s = \begin{cases} 1 - \frac{m(1-c_I)}{k(m+1)} & \text{if } c_I \geq 1 - k - k/m \\ 0 & \text{if } c_I < 1 - k - k/m \end{cases} \quad p_E^l = \begin{cases} \frac{c_I - k}{1-k} & \text{if } c_I \geq \frac{k(1+m)}{m} \\ \frac{c_I}{(1-k)(m+1)} & \text{if } c_I < \frac{k(1+m)}{m} \end{cases}$$

These prices will satisfy the self-sorting conditions if the quantity threshold is $\bar{q}_E = k/m$ *if* $p_E^s \leq p_E^l$*;* $\bar{q}_E = 1 - k$ *if* $p_E^s > p_E^l$*.*

Proof: see Appendix A

Corollary 6 *The parameter space for which entry equilibria exist under explicit discrimination is a proper subset of the parameter space for which entry equilibria exist under rebates.*

Proof: see Appendix A

Figure 2 illustrates the results of the analysis of entry equilibria under rebates and non-negative prices for the case where $c_E \geq \frac{1}{2(m+1)}$ (recall that miscoordination equilibria exist for all parameter values). We see that the region where entry equilibria do not exist is smaller under rebates than under explicit discrimination. While nothing changes for low values of k (rebates exactly replicate the outcome under explicit discrimination), exclusion becomes more difficult for intermediate and high values of k. Intuitively, given m, the large buyer becomes smaller and smaller the higher k is, and so he becomes more and more similar

[20] Such a rebate scheme may appear as somewhat unorthodox, since buyers are "rewarded" for buying little and "penalized" for buying a lot. However, this is a deviation offer which will never be made in equilibrium.

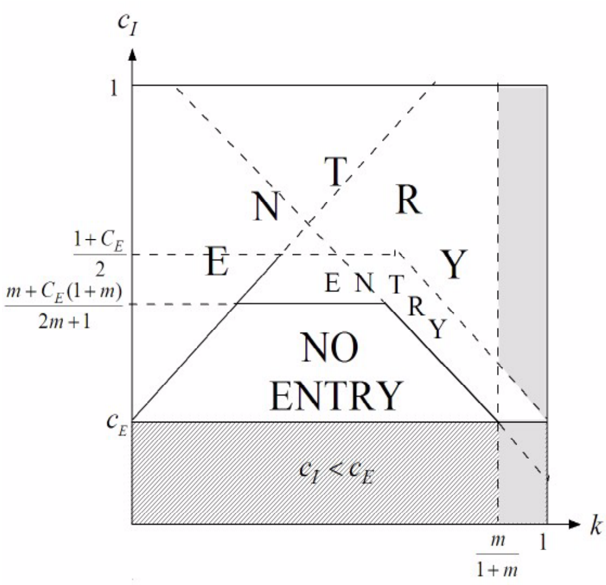

Figure 2: Regions where entry equilibria exist and do not exist under rebates (i.e. implicit price discrimination), compared to explicit discrimination

to the small buyers, making it difficult to discriminate between them through rebates without violating any of the self-sorting conditions.

Note that as m grows (that is, a single small buyer becomes smaller), both the efficiency thresholds and prices under rebates converge to the values under explicit discrimination. In the limit case where $m \to \infty$, the self-selection constraints play no role: the large buyer will never want to behave like a small buyer whose demand is infinitely small, and vice versa for the small buyer, and so the implicit and explicit discrimination cases coincide.

Let us take stock of the results obtained in this section. One of the motivations for this paper was to investigate whether rebates, in the particular form of quantity discounts, can be exclusionary. Our analysis shows that indeed an incumbent firm could use rebates to exclude a more efficient rival, even if the latter can also make use of rebates. The main intuition is that by relying on quantity discounts the incumbent can (implicitly) discriminate across buyers by making attractive offers to some of them, thus subtracting to the rival firm buyers that it critically needs in order to reach the minimum viable size. Therefore, rebates reduce the likelihood that successful entry takes place.

Nevertheless, precisely because they imply competing aggressively for each group of buyers, rebates might also have a procompetitive function: for given market structures (that is, if one compares regions where entry occurs), prices are lower when rebates are allowed than when prices are uniform. It is to explore more formally this basic trade-off between exclusion and lower prices that we now turn to the analysis of consumer welfare under the different price schemes.

4 Consumer welfare

In our model, entry is always socially efficient, because the entrant produces at a lower marginal cost than the incumbent. Thus, all miscoordination equilibria entail a productive inefficiency, which is the only source of inefficiency due to the simplifying assumption of inelastic demands.

Yet, prices do matter, firstly because they determine consumer surplus, which is usually considered the objective function of antitrust agencies; secondly, because if we used an elastic demand function, exclusion would also cause an allocative inefficiency. Comparing equilibrium prices across different price regimes is not straightforward because each price regime gives rise to multiple equilibria, both entry and miscoordination equilibria, and each of these can be sustained by a broad range of prices. The approach we take here is to compare the "worst case scenarios" *given* market structure, i.e. the highest sustainable prices under each price regime given that either the incumbent or the entrant serves the buyers.

Proposition 7 *(Consumer surplus)*

(i) Miscoordination equilibria: Under all three price regimes the highest equilibrium price is the monopoly price, and so consumer surplus is always zero.

(ii) Entry equilibria: At the highest sustainable prices under each regime, consumer surplus is maximal under explicit discrimination, intermediate under rebates, and minimal under uniform pricing.

Proof: see Appendix A

By combining the results about the conditions under which entry exists and about the price comparisons, we identify the fundamental dilemma we mentioned in the introduction. The more aggressive the price regime the less likely entry will take place (entry equilibria always exist under uniform pricing, and they exist under rebates for a larger region of the parameter space than under explicit discrimination). But when entry equilibria exist - that is, for given market structure - the more aggressive the price regime the higher consumer welfare (in regions where entry exists, prices are the highest under uniform pricing, followed by rebates and then by explicit discrimination).

This explains the difficult task faced by competition policy: by banning price discrimination - in its possible forms - one would reduce the risk of anticompetitive exclusion, but at the risk of chilling competition, and ending up with higher prices. By allowing it, one would foster competition but at the risk of exclusionary outcomes.

5 Extensions

In this Section we deal with a number of extensions of the basic model in order to explore the role of each of the key elements of our base model in generating the results obtained in the previous section. In Section 5.1 we modify the threshold level $\bar{s}$. In Section 5.2 we consider the possibility that firms subsidize consumption, i.e. can charge negative prices. In Section 5.3, we study the case where buyers' utility increases continuously in the network size. Section 5.4 discusses the case of perfect price discrimination, where firms are allowed to discriminate even across units sold to the same buyer. As it turns out, each of these modifications allows us to generate cases where discrimination also plays *against* the incumbent (not only in its favor), because they open the possibility for the entrant to use price discrimination in order to disrupt miscoordination equilibria (recall that this was not possible so far).

5.1 The role of the threshold (comparative statics)

In the base model, we assume that $\overline{s} > \max\{k, 1-k\}$: neither by serving all the small buyers nor by serving the large buyer alone would the entrant be able to reach the critical threshold base $\overline{s}$. This assumption is at the heart of the mechanism of exclusion highlighted by this paper, and the following Proposition studies the case where alternative assumptions on $\overline{s}$ are made.[21]

Proposition 8 *(Varying levels of critical thresholds)*

(a) If $k/m < (1-k) < \overline{s} \leq k$ then the equilibrium outcomes are exactly as in the base model.

(b) If $k/m < \overline{s} \leq 1-k$, then under uniform pricing there exists no exclusionary equilibrium, but only an entry equilibrium where all buyers buy from the entrant at $p_E = c_I$. Under discriminatory pricing, if $c_I \leq k$ there is an exclusionary equilibrium where $(p_I^l = 0, p_I^s = 1)$ and all buyers buy from the incumbent. If $c_I > k$, the exclusionary equilibrium does not exist. If $c_I \geq \min\{(1+c_E)/2, c_E + k\}$ there exists an entry equilibrium where $p_E^l = \max\{(c_I - k)/(1-k), 0\}$. Otherwise, the entry equilibrium does not exist.

(c) If $\overline{s} < k/m < 1-k$, there exists a unique entry equilibrium where $p_E^l = p_E^s = c_I$.

Proof. See Appendix A

[21] For shortness we focus on the cases of uniform prices and of explicit discrimination. The case of implicit discrimination - being 'intermediate' among these two - would not give rise to any additional insight.

Comments. This Proposition stresses the importance of network externalities, in the sense that - if consumers value the good only if a network has reached a certain minimum size - the higher that minimum threshold the more difficult for entrants to challenge an incumbent firm.

It also highlights the role of buyers' concentration. For given minimum size $\overline{s}$, part (b) of Proposition 8 tells us that the existence of a very large buyer is sufficient to avoid miscoordination equilibria, and part (c) that if each buyer commands a large enough demand, then network effects become irrelevant. In other words, we would expect industries with fragmented buyers to be more prone to the type of exclusionary mechanism we have highlighted here. Buyer power would increase the size of the orders an individual buyer would bring, and make it less likely that a dominant incumbent firm may exclude a more efficient but new rival.[22]

5.2 Allowing for usage subsidies

In this section, we relax the assumption that prices must be non-negative. Recall that we assume free disposal of the good. Thus, a buyer could exploit negative prices by buying an infinite amount of the good. Therefore, we have to assume that firms can monitor consumption, and that the subsidy is only paid for units that are actually consumed, thus limiting sales to a maximum of $1-k$ for the large buyer, and k/m for any small buyer.

5.2.1 Uniform prices

Under uniform price offers, the results are the same as in the base model. The *miscoordination equilibrium* cannot be disrupted by negative price offers, because the entrant cannot profitably offer negative prices to all buyers. For the same reason, the *entry equilibrium* will also exist for all parameter values. Therefore, *Propositions 1 and 2 still hold good.*

5.2.2 Explicit price discrimination

We consider first miscoordination equilibria and then entry equilibria.

Miscoordination equilibria The possibility to offer negative prices changes dramatically the analysis of miscoordination equilibria. Consider for instance a natural candidate equilibrium, that is the miscoordination equilibrium prevailing under uniform (non-negative) prices: $(p_I^s = 1, p_I^l = 1)$ and all buyers buy from the incumbent. If firm E sets $p_E^l = p_I^l - \varepsilon = 1 - \varepsilon$ and $p_E^s < 0$, then all buyers will buy from the entrant. Indeed, by buying from the entrant each small buyer would receive a strictly positive surplus $(k/m)(-p_E^s) > 0$ even if nobody else consumed the product. Therefore, they will want to consume

[22]The role of buyer power in preventing exclusion is also stressed by Fumagalli and Motta (2008), in a model where - however - scale economies are on the supply-side and discriminatory prices are not considered.

in order to receive the payment. But since it is a dominant strategy for the small buyers to consume the product, the large buyer will now prefer to buy from the entrant as well, since the critical network size will be met, and since $CS^l(p_E^l) = (1-k)(1-p_E^l) > CS^l(p_I^l) = 0$.

More generally, a miscoordination equilibrium with prices (p_I^s, p_I^l) will not exist if the entrant can offer a negative price $p_E^s < 0$ to the small buyers such that $CS^s(p_E^s, s_E < \bar{s}) > CS^s(p_I^s, s_I \geq \bar{s})$ while slightly undercutting the incumbent's offer to the large buyer, $p_E^l = p_I^l - \varepsilon$.[23]

Proposition 9 *(Exclusionary equilibria under negative prices) Let* $\bar{s} > (1-k) + \frac{k}{m}$*. Then, if both firms charge negative prices, a miscoordination equilibrium will only exist if* $c_I \leq k + c_E$*.*

(i) If $c_E \leq 1-k$*, the equilibrium is characterized by* $\left(p_I^l = 1, p_I^s = 1 - \frac{1}{k}\left[1-k-c_E\right]\right)$, $p_E^l \in [0,1], p_E^s = -\frac{1-k-c_E}{k}$.

(ii) If instead $c_E > 1-k$*, the equilibrium is characterized by* $p_I^l = p_I^s = 1$*, and* $p_E^l = p_E^s = 1$ *and it exists for all* c_I*.*

Proof: See Appendix A

Figure 3 illustrates in the space (k, c_I) the region where the miscoordination equilibrium arises, for the case $c_E < 1/2$. It shows that this equilibrium exists only if c_I is sufficiently close to c_E.

The main conclusions from the analysis are as follows. Firstly, when negative prices are possible, allowing for explicit discrimination disrupts miscoordination equilibria when c_I is sufficiently high. Secondly, when a miscoordination equilibrium exists under explicit discrimination (with linear prices which can be negative), the incumbent will not be able to enjoy the monopoly outcome $(p_I^s = 1, p_I^l = 1)$, unless $c_E > 1-k$; the incumbent needs to lower its prices to prevent the entrant from stealing its buyers.

Relative to uniform pricing regimes, where a miscoordination equilibrium which reproduces the monopolistic outcome is always possible, allowing for negative prices has the effect of both rendering miscoordination equilibria less likely, and, where such equilibria survive, of reducing the equilibrium prices. Note that in this case, p_I^s may even be below-cost, i.e. $p_I^s < c_I$!

Entry equilibria The analysis of entry equilibria when we allow for negative prices requires just a small modification of the problem already analyzed in

[23] In the case where $\bar{s} \leq (1-k) + \frac{k}{m}$, the entrant might as well charge a negative price to the *large* buyer, while matching I's offer to the small buyers. In this case, as soon as E attracted the large buyer, E needs just one more buyer to reach the minimum size. Thus, any small buyer will find it optimal to buy from E as well, and the miscoordination equilibrium is broken. This is not the case if $\bar{s} > (1-k) + \frac{k}{m}$, where the entrant needs more than one small buyer to reach the minimum size, so that attracting the large buyer is not sufficient to solve the coordination problem among the small buyers. For shortness, we focus on this case.

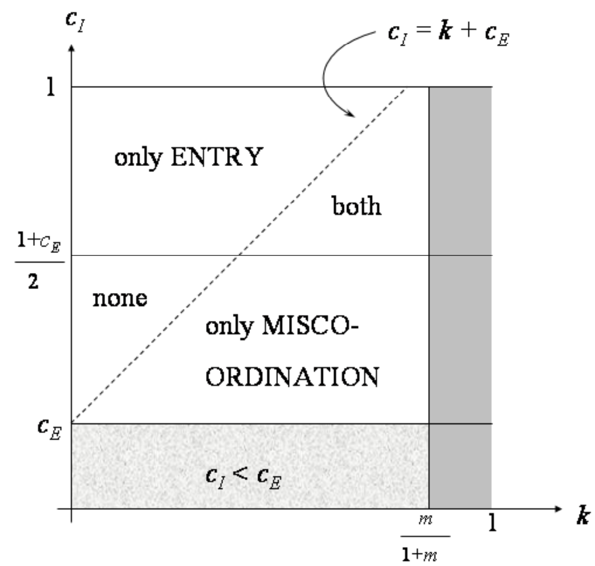

Figure 3: Regions where miscoordination equilibria and/or entry equilibria (or none) exist under negative prices, for $c_E < 1/2$

Section 3.2.2 above, i.e. allowing for p_I^s and p_I^l to take negative values, which was not possible before.

Proposition 10 *(Entry equilibria under negative prices) If both firms can use explicit price discrimination and charge negative prices, entry equilibria only exist if* $c_I \geq \frac{1+c_E}{2}$. *The highest prices that the entrant can charge in any such entry equilibrium are* $p_E^s = \frac{c_I-(1-k)}{k}$ *and* $p_E^l = \frac{c_I-k}{1-k}$.

Proof: see Appendix A

Figure 3 illustrates entry equilibria. Note that under negative pricing, the incumbent can prevent entry for a larger region of parameter values than under non-negative prices: for values such as $c_I < (1 + c_E)/2$, entry may occur under non-negative prices, but not under negative ones.

The figure also shows that under explicit discrimination, there might be a situation where, for given c_E and k, for c_I sufficiently close to c_E a miscoordination equilibrium exists, for intermediate values of c_I no equilibrium in pure strategies exists, and for high values of c_I only the entry equilibrium will exist. (To be precise, such a situation exists if $c_E < 1/3$). For high values of k, there exists an area of parameter values where both miscoordination and entry equilibria will coexist.

To compare results, recall that under uniform pricing both entry and miscoordination equilibria exist under all parameter values. This multiplicity of equilibria in the base case makes it difficult to identify precise policy implications. However incomplete (depending on the values of c_E, there may also exist other regions where no equilibria exist under explicit discrimination, or where multiple equilibria exist also under explicit discrimination), the following Table allows to fix ideas. It shows that for relatively high efficiency gaps between incumbent and entrant, if explicit discrimination schemes are allowed consumer welfare will always be (weakly) higher than under uniform pricing (miscoordination equilibria never exist, and entry equilibria are characterized by (weakly) lower prices). For relatively low efficiency gaps between incumbent and entrant, though, the impact on consumer welfare is not unambiguous: at equilibrium, the incumbent will always serve, and the desirability of explicit discrimination schemes depends on which equilibrium would prevail under uniform pricing: if under uniform pricing a miscoordination equilibrium is played, then explicit discrimination will increase consumer welfare, but if under uniform pricing an entry equilibrium is played, then explicit discrimination leads to exclusion and higher prices. We would then find again the same tension between exclusion and low prices that we have stressed in the main Section above, although it is to be noticed that - apart from very specific cases ($c_E > 1 - k$) - exclusion can be achieved by the incumbent only by decreasing equilibrium prices.

	Uniform pricing	Explicit discrim. (neg. prices)
$c_I > \max\left\{\frac{1+c_E}{2}, k + c_E\right\}$	I serves: $p_I^l = p_I^s = 1$ $\Longrightarrow CS = 0$ E serves: $p_E^l = p_E^s = c_I$ $\Longrightarrow CS = 1 - c_I$	E serves: $\widehat{p}_E^l \leq c_I; \widehat{p}_E^s \leq c_I$ $\Longrightarrow CS \geq 1 - c_I$
$c_I < \min\left\{\frac{1+c_E}{2}, k + c_E\right\}$	I serves: $p_I^l = p_I^s = 1$ $\Longrightarrow CS = 0$ E serves: $p_E^l = p_E^s = c_I$ $\Longrightarrow CS = 1 - c_I$	I serves: $\widehat{p}_I^l = 1; \widehat{p}_I^s \lesseqgtr c_I$ $\Longrightarrow CS \leq 1 - c_I$

5.2.3 Implicit price discrimination (rebates)

It would be tedious to characterize all the equilibrium solutions for the case of rebates as well. Like for the case of explicit discrimination, the possibility to set negative prices allows the incumbent to make more aggressive offers, eliminating entry equilibria which would have existed under uniform prices; also, and again like for explicit discrimination, it allows the entrant to subsidize a group of buyers and induce them to use the product independently of what other buyers do, thus leading to the disruption of miscoordination equilibria. The fact that the self-selection constraint needs to be satisfied does not therefore eliminate the possibility to disrupt some of the equilibria;[24] however, it does imply that competition is softer under rebates than under explicit discrimination. Even in this case, therefore, we find the result that rebates are less exclusionary than explicit discrimination, but lead to higher prices when similar equilibrium market structures are compared.

5.3 Continuous utility function

In this Section, we study the case where consumers' gross utility increases continuously in network size, and neither a lower threshold $\overline{s}$ is required to have a positive utility from consumption, nor is there an upper bound on the network externality.

Assume that a buyer of type $j = l, s$ buying from firm $i = I, E$ has a net surplus $CS_i^j = (v(s_i) - p_i) Q^j$, where $v(s_i)$ is continuous, monotone increasing and concave, and where $v(0) = 0$. For simplicity, we restrict attention to the case where there is only one large and one small buyer. Hence, $Q^l = 1 - k$ and $Q^s = k$, with $k < 1/2$. The incumbent has already an established base $\beta_I > 1 - k$.[25] Also assume that: $c_I < v(\beta_I)$, which guarantees that the market was viable when the incumbent served 'old' consumers; and that:

$$v(\beta_I + 1)(1 + \beta_I) - c_I < \beta_I v(\beta_I) + v(1) - c_E, \tag{6}$$

which implies that social efficiency is the highest when the entrant serves the new cohort of buyers.[26]

We shall look for both exclusionary and entry equilibria, under uniform pricing and price discrimination. As a preliminary remark, note that the miscoordination equilibrium characterized in Proposition 1 does not apply here. When network effects are continuous and there is no minimum threshold size for consumers to reach positive utility, it is no longer true that the entrant necessarily needs both large and small buyers. Accordingly, miscoordination results do not arise, even though an exclusionary equilibrium may still arise as an effect of the established base advantage of the incumbent.

5.3.1 Uniform pricing

Proposition 11 *(Continuous network effects, uniform prices) If firms set uniform prices and there is no minimum threshold base:*

[24] At first sight, one may wonder why a buyer may want to buy at positive prices when it could mimic a buyer who is offered a negative price. But recall that a large buyer may get more surplus from buying $1 - k$ units at a positive price than a smaller number of units k/m at a negative price. However, we have seen in Section 3.2.3 that small buyers will never be willing to buy at positive price if they have the chance to buy more units than they need at zero price. A fortiori, this is true when the price offered for a large number of units is negative.

[25] This is consistent with the assumptions made in the base model, where $\beta_I \geq \overline{s} > \max\{1 - k, k\}$.

[26] Note that not only does the entrant have to offer higher net surplus to the new cohort, but this extra surplus must outweigh the loss in surplus that the "stranded" old cohort experiences if the new cohort is served by the entrant rather than the incumbent.

(a) If $c_I \leq c_E + [v(\beta_I + 1) - v(1-k)]$*, exclusion arises with the Incumbent selling to buyers at* $p_I = c_E + [v(\beta_I + 1) - v(1-k)]$.

Otherwise, the exclusionary equilibrium does not exist.

(b) The entry equilibrium always exists, with the entrant selling to buyers at $p_E = c_I - [v(\beta_I + 1 - k) - v(1)]$.

Proof. See Appendix A

Contrary to the results in the base model, where both equilibria always exist, here the exclusionary equilibrium exists only when the efficiency gap is small enough or when - for given efficiency gap - the incumbency advantage is sufficiently important (i.e., if the established base β_I of the incumbent is sufficiently important and the network externality is not 'too flat').

Clearly, the fact that there was a minimum threshold that the entrant had to reach for customers to derive utility from consumption was an important element to the advantage of the incumbent. The existence of a smoother function makes it less likely that exclusion will arise.

5.3.2 Explicit price discrimination

Although not conceptually more difficult, the case of continuous network effects does make it more lengthy to find the equilibrium solutions. For this reason, rather than fully characterising the equilibrium solutions, we limit ourselves to stating the following result.

Proposition 12 *(Continuous network effects, explicit discrimination) If firms set discriminatory prices and there is no minimum threshold base, relative to uniform pricing:*

(a) Exclusionary equilibria exist under a narrower region of parameter values.

(b) Entry equilibria exist under a narrower region of parameter values.

Proof. See Appendix A

Modelling network effects as continuous we obtain similar results as when allowing for usage subsidies: for certain parameters, the entrant can overcome the coordination problem by targeting individual buyers, making it a dominant strategy for them to buy from the entrant, thus inducing other buyers to switch as well. This means that price discrimination also allows the entrant to break some exclusionary equilibria, a result in contrast to our benchmark model with minimum threshold and non-negative prices.

However, price discrimination also reduces the scope for entry equilibria (again similar to the model with usage subsidies), because it also allows the incumbent to play a "divide-and-conquer" strategy, making very favorable (below-cost) offers to one group of buyers while recouping the losses on the other group.

As usual, the welfare effects are complex because of multiplicity of equilibria, but it remains true that price discrimination has an ambiguous effect: it makes entry (as well as exclusionary) equilibria less likely, but - if comparing regions where entry equilibria exist both under uniform and under discriminatory prices - it lowers the prices that consumers would have to pay for the good.

5.4 Perfect Price Discrimination

Suppose that the firms can set a different price on each unit sold, i.e. they can discriminate even across units, and restrict attention to non-negative prices.

Assume, without loss of generality, that $1-k>k$. For simplicity, also assume that to sell the number of units necessary to prevent the entrant from reaching critical size, the incumbent does not have to split orders among buyers:

$$1-\bar{s}+\epsilon=\overline{m}\frac{k}{m}, \tag{7}$$

where $n\leq m$, and $n,m,\overline{m}\in N$. This assumption will be discussed below.

Proposition 13 *(Perfect discrimination) If the firms can discriminate by units, the following describe existence of entry equilibria:*

(i) If $c_I\geq c_E+\bar{s}\left(1-c_E\right)$, then $p_E^*=\frac{c_I-\bar{s}}{1-\bar{s}}$ and all buyers buy from E.[27]

(ii) If $c_I\geq c_E/\left(1-\bar{s}\right)$, then E sells $\bar{s}$ units at $p_E^{\bar{s}}=0$ and $\left(1-\bar{s}\right)$ units at $p_E^{1-\bar{s}}=c_I$ and all buyers buy from E.

(iii) For lower values of c_I, no entry equilibria exist.

Proof: see Appendix A

Lemma 14 *(Explicit vs. perfect discrimination) Relative to the case of explicit price discrimination, perfect discrimination may either reduce or increase the parameter space where entry equilibria exist.*

Proof: see Appendix A

To understand the logic behind these results, note that there are two effects at play here: On the one hand, perfect price discrimination allows the incumbent to make more aggressive offers, because it can concentrate more rent on fewer buyers; on the other hand, if the entrant can discriminate even among buyers of the same type, making targeted zero-price offers may allow the entrant to secure enough small buyers so that - added to the large buyer - they give the entrant sufficient size $\bar{s}$. Since prices must be non-negative, a zero price cannot be undercut by the incumbent. Once the entrant reached minimum size, it can

[27] For simplicity, we focus on equilibria where the entrant sets the same price p_E^* for all units, but there also exist other equilibria where the entrant charges different prices on different units, but the average price it receives equals p_E^*.

then engage in a Bertrand-style competition for the remaining small buyers, thus recovering losses made on the other buyers. This strategy was not available under explicit discrimination (i.e. discrimination by buyer type), since the entrant could not give lower prices to some small buyers but not to others.

Discussion of assumption 7. In general, the size of orders needed to prevent E from reaching critical size will not correspond precisely to an integer multiple of k/m. Define instead $\overline{m} \leq m$ as the lowest integer number of buyers that firm I needs to secure to prevent E from reaching critical size; that is, $\overline{m}$ will satisfy:

$$(\overline{m} - 1)\frac{k}{m} \leq 1 - \bar{s} < \overline{m}\frac{k}{m}. \tag{8}$$

In words, to implement the deviation identified above, the incumbent would have to offer to one buyer some units at a price $p_E - \epsilon$ and other units at a price 1. Obviously, this buyer would not buy from the incumbent as this would deliver him a lower utility than the entrant's offer (which guarantees a price p_E on all units). Hence, to induce this pivotal buyer to buy from it, the incumbent will have to offer the price $p_E - \epsilon$ for *all* units demanded by him.

As a result, the incumbent's profitability condition becomes $\pi_I = \overline{m} p_E k/m + (1 - \overline{m}k/m) - c_I \geq 0$. For the entry equilibrium to be immune from this deviation, it must therefore be: $p_E^* = \max\left\{0, \frac{c_I - 1 + \overline{m}k/m}{\overline{m}k/m}\right\}$. From $\overline{m}k/m > 1 - \bar{s}$, it follows that the entry equilibrium will be more likely to exist, and that the price which can be sustained at such equilibrium are higher.

Random coupons One possible instrument of price discrimination is to use discount coupons which are randomly sent by firms. One may think that such coupons are another efficient discriminatory strategy to achieve exclusion, because - similar to the case of perfect price discrimination - they allow for different units to sell at different prices (depending on whether the buyer received a coupon for his purchase or not). It turns out, however, that random coupons cannot reproduce the exclusionary results of perfect discrimination. Suppose for instance the incumbent sent $\overline{m}$ coupons entitling their recipients to buy k/m units at the price $p_E^* - \epsilon$. If all the coupons reached small buyers, this strategy would replicate the optimal deviation under perfect discrimination. However, there is a positive probability that one or more coupons will end up in the hands of the large buyer. Understanding he is pivotal, he will not use such coupons, resulting in the incumbent making losses (he will sell some units at a price $p_E^* - \epsilon < c_I$) without preventing E from reaching critical size. In essence, the randomness of these coupons prevents the incumbent from carefully targeting price cuts to the pivotal buyers, which makes random coupons a less effective tool of discrimination than perfect price discrimination (which is itself, as Lemma

14 shows, does not dominate third-degree discrimination as an exclusionary device).

6 Conclusions, and a policy discussion

Our paper demonstrates the exclusionary potential of price discrimination and rebates in a model where - relative to the literature on exclusionary practices - the entrant is in a fairly good initial position: it is more efficient than the incumbent, it does not have to pay any set-up cost, it can approach buyers at the same time as the incumbent, and it can use the same pricing schemes. However, the incumbent does enjoy an incumbency advantage (when the game starts, its network has already reached the minimum threshold size to be viable, whereas the entrant's has not), and this turns out to be crucial.

We show that - if buyers are sufficiently fragmented and/or the threshold size is sufficiently high and prices are non–negative - both exclusionary equilibria and entry equilibria exist under uniform pricing, and that both explicit and implicit discrimination (that is, rebates) increase the likelihood of an exclusionary outcome: while discrimination does not prevent miscoordination, it makes it easier for the incumbent to disrupt entry equilibria. This is done by a "divide and rule" strategy where some buyers are offered a below-cost price, thereby depriving the entrant of the critical mass it needs, and allowing the incumbent to recover losses from the remaining buyers, who become captive to it.

On the other hand, if we look at regions where entry equilibria exist under all pricing regimes, we find that consumers would be better off when discrimination is allowed: to counter aggressive price cuts from the incumbent, the entrant has to reduce prices, resulting in lower prices for consumers.

These results emphasize a fundamental dilemma that is at the origin of the difficulties of dealing with price abuses in competition law. If antitrust agencies and courts pursued a policy of forbidding discriminatory pricing they might avoid exclusion of efficient entry, but at the cost of having higher prices whenever entry-deterrence is not an issue.

One might be tempted to think that an *asymmetric policy which prohibits below-cost or discriminatory prices by dominant incumbent firms*, while letting the entrant free to choose its pricing policy, might be an appropriate policy option. In fact, such a policy would have two limits. First, as showed by Proposition 1, price discrimination does not enable the entrant to break miscoordination equilibria (unless subsidies could be used, which is not always feasible). Second, it is true that if the incumbent cannot engage in below-cost (or discriminatory) prices, then entry equilibria will exist for the entire parameter space. However, in such entry equilibria consumers would pay the entrant a price equal to the incumbent's marginal cost, which is higher than the price they would pay under explicit discrimination (when the incumbent cannot price below cost, it suffices to set a price slightly below c_I for the entrant to get all buyers). Thus, an asymmetric anti-discrimination policy would have the same effects as a symmetric imposition of uniform pricing, making exclusion less likely, but raising prices to consumers in the case of entry.

If anything, an asymmetric rule preventing dominant firms from giving usage subsidies may be a more promising road: on the one hand, the incumbent can still discriminate (as long as all prices it offers are non-negative), which preserves some (if not all) of the beneficial effects of discrimination on equilibrium prices in the entry equilibrium; on the other hand, an entrant who can offer usage subsidies is able to break some miscoordination equilibria that could not be broken otherwise, thus facilitating coordination on the socially desirable entry equilibrium.

We have also identified the conditions under which the exclusionary issues studied here may arise at all. In particular, if buyers were sufficiently concentrated, or critical threshold size was sufficiently small, the game would resemble the standard Bertrand model with asymmetric cost, and only entry equilibria would emerge. Also, we have seen that if network effects are modelled in a continuous way, exclusionary outcomes are somewhat less likely, but it would still be true that discrimination - by making competition fiercer - makes it more likely for the incumbent to prevent entry equilibria, while at the same time resulting in lower prices in case an entry equilibrium does emerge.

Allowing for subsidies (i.e. negative prices) does not fundamentally change this insight: while subsidies might allow the entrant to disrupt miscoordination equilibria, they also allow the incumbent to prevent entry equilibria for an even wider region of parameter values. Furthermore, they reduce further the maximal prices that can be sustained in any entry equilibrium. Overall, usage subsidies (i) make exclusion most likely, but (ii) given market structure, result in the lowest prices. Therefore, the trade-off between exclusionary potential and (for given market structure) lower equilibrium prices reappears even when negative prices are allowed.

The possibility of exclusionary outcomes is intimately linked with the assumption that the incumbent has already reached the minimum threshold size (in the base model), or that it has in any case a strong initial customer base (in the model with continuous network effects). For this reason, the mechanism identified in our paper seems well suited to industries (such as those of recent liberalisation or those where a firm's dominant position is built upon intellectual property rights whose protection is about to expire), where entrants can challenge an incumbent firm only after the latter has developed a strong customer base.

Given the trade-off between exclusion and consumer welfare, and given the fairly specific conditions under which the exclusionary mechanism identified here would take place, it would be difficult to advise a policy *prohibiting* price discrimination (even if such a policy was limited to dominant firms, as discussed above). This is not to say that our analysis favours a laissez-faire policy. Indeed, we have offered here a possible anti-competitive rationale for price discrimination and rebates. In any abuse of dominance (or, in the US, monopolisation) case, Antitrust Agencies and Courts have to formulate a theory of harm. What our paper suggests is that - if network or more generally scale effects are at work, the dominant firm has a strong incumbency advantage, and buyers are sufficiently fragmented - the incumbent might use rebates and discriminatory prices in order to exclude as- or more efficient new rivals. Hence, when facing a case with such features, there would be a strong rationale for agencies and courts to argue the anti-competitive effects of discriminatory practices.

In this paper, we have chosen to model scale effects as a demand-side variable, by using network effects and by considering a network's installed base as the incumbency advantage. However, our results would be identical if we assumed there are scale economies on the supply side, and that there is a firm which has already paid its sunk costs, as the incumbency advantage.

Consider the following game. At time 1, firms I and E simultaneously set prices (according to the different price regimes, prices can be uniform or differentiated); at time 2, all buyers decide which firm they want to buy from and make firm orders; at time 3, firm E decides on entry (if it does enter, it has to pay sunk cost $f > 0$); at time 4, payoffs are realized. Like in Section 2, continue to assume that there are m small buyers and 1 large buyer, and let the sunk cost f be large enough so that entry is profitable only if firm E serves the large buyer plus at least one small buyer. With these modifications, results will be of the same nature as those obtained in this paper, and even the calculations will be to a large extent the same.[28]

Finally, one may wonder how the existence of switching costs (which play an important role in shaping entry in the real world) would change our model. First of all, consider our basic model with network effects. One simple way to take switching costs into account would be to assume that all buyers repeat their purchases, but there are some buyers who (equivalently to the 'old' buyers in our basic model) have arbitrarily large switching costs and therefore would never buy from the entrant, and others who (equivalently to our 'new' buyers) have switching costs σ which are small enough, so that the entrant's effective marginal cost, $c_E + \sigma \equiv \widetilde{c}_E$, is still lower than the incumbent's: $c_E + \sigma < c_I$. Provided that there are both large and small buyers among the latter category of buyers, and after replacing c_E with the effective marginal cost $\widetilde{c}_E$, the analysis would be the same as in our model, and the comparative statics on the switching costs would be straightforward. An increase in switching costs σ would be equivalent to an increase in the marginal cost of the entrant, $\widetilde{c}_E$, and would thus lead to more likely exclusionary equilibria. At the other extreme, if all buyers repeat their purchases but switching costs are very small for all of them, then exclusion would be unlikely.

Of course, one could find more sophisticated and interesting ways to incorporate switching costs in the analysis, but it is clear that the basic mechanisms illustrated in this paper would still be at work and would be exacerbated by the existence of switching costs. Both under consumption externalities and un-

[28] Fumagalli and Motta (2001) study a similar model with economies of scale in production. However, they focus on the role of buyer power and downstream competition, and do not consider price discrimination and rebates (buyers are identical in their model).

der economies of scale, switching costs would add to the incumbency advantage provided by the installed base and the sunk cost, respectively. Note, however, that in our framework, switching costs alone (i.e. without installed base or sunk cost) would not be sufficient to obtain the results.

Similarly, one could think of a model where products have a life of, say, two periods, so that old buyers would not buy today but would buy again tomorrow. Anticipating that the market will include them in the future (absent switching costs and with buyers attaching enough weight to future consumption), only entry equilibria will arise with the current (and the future) cohort of buyers sponsoring the entrant firm. This suggests that the frequency of the renewal of the purchases might have an effect on the structure of the market. Of course, one would need a dynamic model to deal properly with such a situation.

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7 Appendix A - Proofs

Proof of Proposition 1 *(Miscoordination equilibrium, all price regimes)***:**

Consider the candidate equilibrium where $p_I^s = p_I^l = 1$ and all buyers buy from I. Recall that $\bar{s} > \max\ \{1-k, k\}$: none of the individual buyers alone is sufficient for E to reach the minimum size. Thus, no buyer $j = s, k$ will want to deviate and buy from E, even if $p_E^j = 0$, as E's product would have zero value for the deviating buyer. Firm I has no incentive to increase or decrease its price as it is getting the monopoly profits. Since in all continuation equilibria buyers will not switch to E no matter how low p_E^j is, E has no incentive to decrease its price either.

More generally, there exists a continuum of miscoordination equilibria with any price $p_I^j \in [c_I, 1]$ and buyers $j = s, l$ all buying from I, sustained by the appropriate continuation equilibria. The proof is analogous. First, no buyer has an incentive to deviate and buy from the entrant as the latter would not reach size $\bar{s}$. Firm I would not have an incentive to increase its price to $p_I^{j\prime}$ if in the continuation equilibrium where $p_E^j < p_I^{j\prime}$ buyers would all buy from the entrant (recall that for any pair $p_E^j < p_I^j$ there exist two types of equilibria); firm E would have no incentive to change its prices provided in all continuation equilibria where $p_E^j < p_I^j$ all buyers buy from the incumbent.□

Proof of Proposition 2 *(Entry equilibria, uniform prices)*

With all buyers buying from E at $p_E = c_I$, total demand is $1 \geq \bar{s}$: E will reach the minimum size. Since E's product has the same value to the buyers as I's, and the price is the same, no buyer has an incentive to deviate and buy from I. Firm I will not want to deviate either: To attract buyers, it would have to set $p_I < c_I$, i.e. sell at a loss; and increasing p_I above c_I will not attract any buyers under the appropriate continuation equilibria. Firm E has no incentive to change its price either: increasing p_E would imply losing the buyers to I, and decreasing p_E will just reduce profits.

Note also that, following the same logic, there exists a continuum of entry equilibria with any price $p_E^j \in [c_E, c_I]$ and buyers $j = s, l$ all buying from E, sustained by the appropriate continuation equilibria.

Finally, note that there can be no equilibrium where E serves all buyers at a price $p_E > c_I$: In this case, I could profitably undercut E, and all buyers would switch to I.□

Proof of Proposition 3 *(Entry equilibria, explicit discrimination)*

Consider a candidate equilibrium where (p_E^s, p_E^l) and all buyers buy from E. For this to be an equilibrium, it must be immune from deviations by the incumbent, which could set $p_I^j < p_E^j$ to buyers of type j to deprive the entrant of the critical scale, and then charge monopoly price $p_I^{-j} = 1$ to the other group of buyers $-j$ $(j = s, l)$.

The offer $(p_I^s, 1)$ to attract the small buyers is feasible as long as $\pi_I(p_I^s, 1) = m\frac{k}{m}(-c_I + p_I^s) + (1-k)(1-c_I) \geq 0$. Likewise, the offer $(1, p_I^l)$ to attract the large buyer is feasible as long as $\pi_I(1, p_I^l) = (1-k)(-c_I + p_I^l) + m\frac{k}{m}(1-c_I) \geq 0$.

Call $\widehat{p}_I^s$ and $\widehat{p}_I^l$ the prices that solve the equations associated with the two profitability conditions above:

$$\widehat{p}_I^s = \frac{c_I - (1-k)}{k} < c_I; \quad \widehat{p}_I^l = \frac{c_I - k}{1-k} < c_I.$$

The lowest possible deviation prices of the incumbent are identified by $p_I^s = \max(\widehat{p}_I^s, 0)$ and $p_I^l = \max(\widehat{p}_I^l, 0)$, since prices are non-negative.

The entrant can avoid the incumbent's deviations if it can set prices (p_E^s, p_E^l) such that the incumbent will not find it profitable to undercut either the small or the large buyers: $p_E^s = \max(\widehat{p}_I^s, 0)$ and $p_E^l = \max(\widehat{p}_I^l, 0)$, while making positive profits: $\pi_E(p_E^s, p_E^l) \geq 0$. By substitution, the entry equilibrium exists if:

$$k\left(\max(\frac{c_I-(1-k)}{k}, 0) - c_E\right) + (1-k)\left(\max(\frac{c_I-k}{1-k}, 0) - c_E\right) \geq 0.$$

This identifies four regions, according to values of k and c_I:

$$\begin{array}{c} (2) \text{ if } c_I \in [k, 1-k] \text{ and } k < 1/2: \quad \pi_E(0, \widehat{p}_I^l) \geq 0 \\ (3) \text{ if } c_I \in [1-k, k] \text{ and } k \geq 1/2: \quad \pi_E(\widehat{p}_I^s, 0) \geq 0 \\ (4) \text{ if } c_I < \min\{k, 1-k\}: \pi_E(0,0) \geq 0 \\ (1)\ else: \pi_E(\widehat{p}_I^s, \widehat{p}_I^l) \geq 0 \end{array} \tag{9}$$

After replacing, we can then find that:

(1) $\pi_E(\widehat{p}_I^s, \widehat{p}_I^l) = k\left(\frac{c_I-(1-k)}{k} - c_E\right) + (1-k)\left(\frac{c_I-k}{1-k} - c_E\right) \geq 0$, satisfied for $c_I \geq (1+c_E)/2 \equiv \overline{c}_{I1}$.

(2) $\pi_E(0, \widehat{p}_I^l) = -c_E k + (1-k)\left(\frac{c_I-k}{1-k} - c_E\right) \geq 0$, which holds for $c_I \geq k + c_E \equiv \overline{c}_{I2}$.

(3) $\pi_E(\widehat{p}_I^s, 0) = k\left(\frac{c_I-(1-k)}{k} - c_E\right) - c_E(1-k) \geq 0$, which holds for $c_I \geq 1 + c_E - k \equiv \overline{c}_{I3}$.

(4) $\pi_E(0,0) = -c_E \geq 0$, which never holds, apart from the knife-edge case where $c_E = 0$.[29]

Finally, straightforward algebra shows that if $c_I \geq \max\{k, 1-k\}$, so that threshold $\overline{c}_{I1} = \frac{1+c_E}{2}$ applies, we have that $\overline{c}_{I1} = \min\{\overline{c}_{I1}, \overline{c}_{I2}, \overline{c}_{I3}\}$, and the analogous relation holds for the other two threshold values of c_I: in the parameter region where $\overline{c}_{Ii}$ applies, $\overline{c}_{Ii} = \min\{\overline{c}_{I1}, \overline{c}_{I2}, \overline{c}_{I3}\}$.□

Proof of Proposition 5 *(Entry equilibria, implicit discrimination)*

Any equilibrium where the entrant serves the buyers must satisfy two conditions:

[29] Since prices cannot go below zero in this basic model, the best that the incumbent can offer to buyers is to give them the good for free; but when $c_E = 0$, the entrant could match that offer without making losses, and entry equilibria would always exist. Clearly, though, this is a very special case.

(i) the entrant's prices (p_E^s,p_E^l) must be immune to all profitable deviations by the incumbent. The important difference to the case of explicit discrimination is that the incumbent's offers now have to satisfy the self-sorting constraints, either (SSlarge) or (SSsmall), in addition to the break-even constraint (BE).

(ii) the entrant's prices (p_E^s,p_E^l) must themselves satisfy the self-sorting constraints. We will show below that this is implied by the price pairs that are constructed to satisfy condition (i).

Ad (i): When stealing the small buyers at the expense of the large buyer, the incumbent can no longer charge the large buyer $p_I^l = 1$ (as under explicit discrimination): at this price the large buyer is left with zero surplus, and so the large buyer's self-sorting constraint is bound to be violated. Thus, the price pair giving maximum surplus to the small buyers is fully determined by the following two constraints:

$$(p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \tag{BE}$$

$$(1-p_I^l)(1-k) \geq \frac{k}{m}(1-p_I^s) \tag{SSlarge}$$

Note that (SSlarge) will always be binding, and that p_I^s must be non-negative. Call the solution to this problem $\left(\widetilde{p}_I^s, \bar{p}_I^l\right)$.

Likewise, if the incumbent wants to steal the large buyer at the expense of the small buyers, the price offered to the small buyers must satisfy their self-sorting constraint. The price pair that gives maximum surplus to the large buyer solves:

$$(p_I^s - c_I)k + (p_I^l - c_I)(1-k) \geq 0 \tag{BE}$$

$$\frac{k}{m}(1-p_I^s) \geq \frac{k}{m} - p_I^l(1-k) \tag{SSsmall}$$

If the small buyers are sufficiently fragmented, i.e. if m is high enough, then (SSsmall) may not be binding, i.e. the incumbent can charge price $p_I^s = 1$ as under explicit price discrimination (and price $p_I^l = \frac{c_I - k}{1-k}$ to the large buyer). Note that the non-negativity constraint on p_I^l will never be binding (a price of zero is incompatible with self-sorting by small buyers). Call the solution to this problem $\left(\bar{p}_I^s, \widetilde{p}_I^l\right)$.

Then, the incumbent's optimal (deviation) offers to both small and large buyers can be summarized as follows:

$$\widetilde{p}_I^s = \begin{cases} 1 - \frac{m(1-c_I)}{k(m+1)} & \text{if } c_I \geq 1-k-k/m \\ 0 & \text{if } c_I < 1-k-k/m \end{cases} \quad \widetilde{p}_I^l = \begin{cases} \frac{c_I - k}{1-k} & \text{if } c_I \geq \frac{k(1+m)}{m} \\ \frac{c_I}{(1-k)(m+1)} & \text{if } c_I < \frac{k(1+m)}{m} \end{cases}$$

Again, these prices represent the upper bound on the prices that the entrant can charge in any entry equilibrium. For entry to be feasible, $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ must be high enough to allow the entrant to break even. The functions $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ identify four regions, and for each of them we have to verify whether the entrant's break-even condition holds or not:

(i) if $c_I \in \left[1-k-k/m, \frac{k(1+m)}{m}\right]$ and $k \geq \frac{m}{2(1+m)}$: $\pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$

(ii) if $c_I \in \left[\frac{k(1+m)}{m}, 1-k-k/m\right]$ and $k < \frac{m}{2(1+m)}$: $\pi_E(0, \frac{c_I-k}{1-k}) \geq 0$

(iii) if $c_I < \min\left\{\frac{k(1+m)}{m}, 1-k-k/m\right\}$: $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$

(iv) else: $\pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I-k}{1-k}) \geq 0$

After replacing, we can then find that:

(i) $\pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for $c_I \geq \frac{m}{1+m} + c_E - k$

(ii) $\pi_E(0, \frac{c_I-k}{1-k}) \geq 0$ holds for $c_I \geq k + c_E$

(iii) $\pi_E(0, \frac{c_I}{(1-k)(m+1)}) \geq 0$ holds for $c_I \geq c_E(1+m)$

(iv) $\pi_E(1-\frac{m(1-c_I)}{k(m+1)}, \frac{c_I-k}{1-k}) \geq 0$ is satisfied for $c_I \geq \frac{m+(1+m)c_E}{1+2m}$

If $c_E < \frac{1}{2(m+1)}$, then we have that $c_E(1+m) < \frac{m+(1+m)c_E}{1+2m} < \frac{1}{2}$. Tedious algebra shows that in this case, $c_I \geq \frac{m+(1+m)c_E}{1+2m}$ is redundant, and that each of the remaining thresholds is the minimum of all thresholds in the parameter region where it applies. Conversely, if $c_E \geq \frac{1}{2(m+1)}$, then we have that $\frac{m+(1+m)c_E}{1+2m} < c_E(1+m)$ and $c_E(1+m) \geq \frac{1}{2}$. In this case, $c_I \geq c_E(1+m)$ is redundant, and each of the remaining thresholds is the minimum of all thresholds in the parameter region where it applies.

Ad (ii): If firms practice implicit rather than explicit discrimination, then the entrant's equilibrium offers must satisfy the self–sorting constraints. As it turns out, the latter are always satisfied whenever the entrant's offers are constructed to be immune against the incumbent's deviations, i.e. if $(p_E^s, p_E^l) = \left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$: If $\widetilde{p}_I^s < \widetilde{p}_I^l$, then only the large buyer's self-selection constraint could be violated (but not the small buyers'). But recall that $\widetilde{p}_I^s$ satisfies the large buyer's self-selection constraint (SSlarge) by construction. Now, the price that the large buyer is charged in the entry equilibrium is of course lower than the one it would be charged if the incumbent were to steal the small buyers and to recover the losses on the large buyer, i.e. we have that $\overline{p}_I^l > \widetilde{p}_I^l = p_E^l$. But that implies that $\left(\widetilde{p}_I^s, \widetilde{p}_I^l\right)$ must also satisfy the large buyer's self-selection condition. The reasoning is exactly analogous for the case where $\widetilde{p}_I^s > \widetilde{p}_I^l$.□

Proof of Corollary 6

Under explicit discrimination, the lower bound on c_I for entry equilibria to exist is $\min\left\{\frac{1+c_E}{2}, k+c_E, 1-k+c_E\right\}$. Now, if $c_E < \frac{1}{2(m+1)}$, the corresponding condition under rebates reads $c_I \geq \min\left\{c_E(1+m), k+c_E, \frac{m}{1+m} + c_E - k\right\}$. Comparing the components of the two sets, we see that the second component is the same, $k+c_E = k+c_E$. The third component is lower under rebates, $\frac{m}{1+m} +$

$c_E - k < 1 - k + c_E$. Finally, $c_E < \frac{1}{2(m+1)}$ implies that $c_E(1+m) < \frac{1+c_E}{2}$, i.e. the first component is lower under rebates as well. If instead $c_E \geq \frac{1}{2(m+1)}$, the first component under rebates is $\frac{m+(1+m)c_E}{1+2m}$, which is always smaller than $\frac{1+c_E}{2}$. Thus, we can conclude that the parameter space for which entry equilibria exist under rebates fully includes the corresponding parameter space under explicit discrimination.□

Proof of Proposition 7 *(Consumer surplus)*
Under all three price regimes, buyers consume the same quantities. Thus, their consumer surplus is solely determined by the price they pay: the higher the price, the lower is consumer surplus.

(i) It follows immediately from Proposition 1.

(ii) The following table shows the prices buyers pay under each of the three price regimes. The inequalities follow from simple algebra.

Table 1: Highest Sustainable Prices in Entry Equilibria

	Uniform		Implicit		Explicit
Large Buyer:					
$c_I < k$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I}{(1-k)(m+1)}$	$>$	$p_E^l = 0$
$c_I \in \left[k, \frac{k(1+m)}{m}\right)$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I}{(1-k)(m+1)}$	$>$	$p_E^l = \frac{c_I - k}{1-k}$
$c_I \geq \frac{k(1+m)}{m}$	$p_E^l = c_I$	$>$	$p_E^l = \frac{c_I - k}{1-k}$	$=$	$p_E^l = \frac{c_I - k}{1-k}$
Small Buyers:					
$c_I < 1 - k - k/m$	$p_E^s = c_I$	$>$	$p_E^s = 0$	$=$	$p_E^s = 0$
$c_I \in [1-k-k/m, 1-k)$	$p_E^s = c_I$	$>$	$p_E^s = 1 - \frac{m(1-c_I)}{k(m+1)}$	$>$	$p_E^s = 0$
$c_I \geq 1-k$	$p_E^s = c_I$	$>$	$p_E^s = 1 - \frac{m(1-c_I)}{k(m+1)}$	$>$	$p_E^s = \frac{c_I - (1-k)}{k}$

The ranking of consumer surplus is the reverse of the ranking of prices, and can be summarized thus:

- $CS^l_{\text{expl}} \geq CS^l_{\text{impl}} > CS^l_{\text{unif}} > 0$ with strict inequality if $c_I < \frac{k(1+m)}{m}$;
- $CS^s_{\text{expl}} \geq CS^s_{\text{impl}} > CS^s_{\text{unif}} > 0$ with strict inequality if $c_I \geq 1 - k - k/m$. □

Proof of Proposition 8 *(Varying levels of critical threshold)*
(a) Suppose $k > 1-k$, and $k/m < (1-k) < \overline{s} \leq k$, (this implies that $m \geq 2$). In this case, miscoordination would still arise, given that no single buyer would bring enough size to the entrant. As usual, there will also exist the entry equilibrium. Qualitatively, the results of Propositions 1 to 3 still hold good. The only difference is that now, the region where entry equilibria exist is larger: If $(1-k) < \overline{s} \leq k$, the incumbent can lock in the large buyer after stealing the small buyers, but not the other way round: the group of small buyers is sufficient to generate critical size, even if the large buyer does not join E's

network. This implies that the large buyer will never be offered a price below c_I: $p_I^l \geq c_I$. To prevent the incumbent from stealing the small buyers, the entrant must set p_E^s such that $\pi_I = (p_E^s - c_I)k + (1 - c_I)(1 - k) \leq 0$. For the entrant to break even at candidate equilibrium prices $\left(p_E^l = c_I, p_E^s = (c_I - 1 + k)/k\right)$, we must have $\pi_E = p_E^s k + c_I(1 - k) - c_E \geq 0$, which can be rearranged to read $c_I \geq (c_E + 1 - k)/(2 - k)$.

(b) Suppose $k < \overline{s} \leq 1 - k$. Consider first *uniform pricing.* A *miscoordination equilibrium* with $p_I \geq c_I$ and all buyers buying from I cannot exist. If E sets $c_I - \epsilon$, the large buyer would buy from it and get positive utility. Knowing that, small buyers would buy from E as well. It is easy to see that the *entry equilibrium* always exists, with $p_E = p_I = c_I$ and all buyers buying from E.

Consider next *discriminatory* (non-negative) pricing. Consider a *miscoordination equilibrium* where I sets $(p_I^l < c_I, p_I^s = 1)$ and all buyers buy from the incumbent. The entrant could break this equilibrium by setting $(p_E^l = p_I^l - \epsilon, p_E^s = 1 - \epsilon)$, thus making it a dominant strategy for the large buyer to buy from E, and in turn making the small buyers buy from E as well. The incumbent could prevent this deviation only by setting $(p_I^l = 0, p_I^s = 1)$. Under the assumption that prices are non-negative, the entrant cannot attract the large buyer, and the miscoordination equilibrium cannot be broken. The equilibrium is feasible as long as $\pi_I \geq -c_I(1 - k) + (1 - c_I)k \geq 0$, or $c_I \leq k$.

As for the *entry equilibrium*, the only deviation which could threaten it is the one where the incumbent attracts the large buyer, thus preventing the entrant from reaching its minimum base. Therefore, the candidate equilibrium must be of the type $(p_E^l < c_I, p_E^s = c_I)$. To avoid the incumbent's deviation, it must be: $\pi_I = (p_E^l - c_I)(1 - k) + (1 - c_I)k \leq 0$. Hence, $p_E^l = \max\{(c_I - k)/(1 - k), 0\}$. This is profitable for E as long as $c_I \geq \min\{(1 + c_E)/2, c_E + k\}$. (Unlike the base model, here there is no need to lower the price for small buyers, as they are not needed to reach the minimum customer base.)

(c) If $\overline{s} < k/m < 1 - k$. In this case, any buyer would guarantee enough scale to the entrant, and everything will be as in the standard Bertrand game with asymmetric firms. Suppose there is an exclusionary equilibrium with $p_I^j \geq c_I$, $(j = l, s)$ and all buyers buy from I. Clearly, the entrant could undercut the incumbent and profitably get buyer of type j, and this deviation cannot be prevented. It is also straightforward to check that the entry equilibrium with $p_E^j = c_I$ and all buyers buying from E cannot be disrupted. If the incumbent undercuts the entrant on the type-j buyer and set $p_I^j = c_I - \epsilon$ it would just get that buyer; clearly, it would get negative profits and the deviation would not be profitable. In order to obtain enough buyers to prevent entry, it should get all the buyers (recall that discrimination within the same group of buyers is not possible), which is not profitable.□

Proof of Proposition 9 *(Exclusionary equilibria under negative prices)*

To make it a dominant strategy for a small buyer to buy from E, E must offer a price p_E^s that yields a (weakly) higher net surplus as I's offer to the small buyers: $-p_E^s \frac{k}{m} \geq \frac{k}{m}(1 - p_I^s)$, whence $p_E^s \leq -(1 - p_I^s) < 0$. If the small buyers consume E's product for sure, then the large buyer will switch to E whenever $p^l_E \leq p^l_I$.

To check whether E will find it profitable to carry out this deviation, insert $p^s_E = -(1-p^s_I)$ and $p^l_E = p^l_I$ into $\pi_E(p^s_E, p^l_E) \geq 0$, to obtain $\pi_E = -k(1-p^s_I) - c_E + p^l_I(1-k) \geq 0$.

This implies that, for a candidate exclusionary equilibrium to be immune from the deviation of the entrant, given p^l_I, the incumbent should solve the following problem:

$$\begin{array}{l} \max_{p^s_I, p^l_I} \pi_I = (p^s_I - c_I)\,k + \left(p^l_I - c_I\right)(1-k) \\ \text{s.t. } (1)\ p^l_I \leq 1;\ \ (2)\ p^s_I \leq \min\left\{1 - \frac{1}{k}\left[p^l_I(1-k) - c_E\right], 1\right\} \end{array}$$

and obtain positive profits. It is easy to see that:

(i) If $c_E \leq 1-k$, the programme is solved by $p^l_I = 1$ and $p^s_I = \frac{1}{k}[1-k-c_E]$. By substitution, $\pi_I = k + c_E - c_I$, which entails that the equilibrium exists only if $c_I \leq k + c_E$.

(ii) If $c_E > 1-k$, the programme is solved by $p^s_I = p^l_I = 1$, and π_I will always be positive. Therefore, the equilibrium exists for all values of c_I.□

Proof of Proposition 10 *(Entry equilibria under negative prices)*

By following the same steps as in the proof of Proposition 3, one can check that the lowest deviation prices that the incumbent can profitably set are: $\widehat{p}^s_I = \frac{c_I-(1-k)}{k}$ and $\widehat{p}^l_I = \frac{c_I-k}{1-k}$. An entry equilibrium will exist only if the entrant is able to set $p^s_E = \widehat{p}^s_I$, and $p^l_E = \widehat{p}^l_I$ so as to prevent deviations on both large and small buyers. Therefore, such an equilibrium exists if and only if:

$$\pi_E(\widehat{p}^s_I, \widehat{p}^l_I) = k\left(\frac{c_I-(1-k)}{k} - c_E\right) + (1-k)\left(\frac{c_I-k}{1-k} - c_E\right) \geq 0,$$

which is satisfied for $c_I \geq (1+c_E)/2$.□

Proof of Proposition 11 *(Continuous network effects, uniform pricing)*

a) Consider an exclusionary equilibrium where firm I sets p_I and both buyers buy from it. For the entrant to break this equilibrium, it should set a price p_E at which either the small or the large buyer (or both) get higher surplus than at the candidate equilibrium. To win the small buyer, the entrant should set $p_E < p_I - [v(\beta_I + 1) - v(k)] \equiv p^s_E$ and to win the large buyer, it should set $p_E < p_I - [v(\beta_I+1) - v(1-k)] \equiv p^l_E$.[30] Since $p^l_E > p^s_E$ the latter deviation is more profitable. For the exclusionary equilibrium to be immune from deviations, the entrant should find it unprofitable to charge p^l_E, that is,

[30] The key difference relative to the threshold case is as follows. When a single buyer of type $j = s, l$ unilaterally deviates to the entrant, his surplus will be proportional to $v(j) - p_E$. In this Section, $v(1-k) > v(k) > 0$, whereas in the base model, $v(k) = v(1-k) = 0$, so there was no non-negative price firm E could charge that would induce a buyer to unilaterally deviate from the exclusionary equilibrium.

it must be $p_E^l < c_E$, or $p_I = c_E + [v(\beta_I + 1) - v(1-k)]$. Of course, this price can be an equilibrium only if $\pi_I = c_E + [v(\beta_I + 1) - v(1-k)] - c_I \geq 0$, or $c_I \leq c_E + [v(\beta_I + 1) - v(1-k)]$.

(b) Consider an entry equilibrium where firm E sets p_E and both buyers buy from it. Similarly to the analysis above, the incumbent's most profitable deviation would be to attract the large buyer, that is, to set $p_I < p_E + v(\beta_I + 1 - k) - v(1) \equiv p_I^l$. The deviation is profitable as long as $p_I^l > c_I$. Hence, if firm E is able to set the price $p_E \leq c_I - [v(\beta_I + 1 - k) - v(1)]$ the deviation will be avoided. This amounts to requiring that $p_E = c_I - [v(\beta_I + 1 - k) - v(1)] \geq c_E$, which is always verified under the assumption of efficient entry.□

Proof of Proposition 12 *(Continuous network effects, explicit discrimination)*

a) At the candidate equilibrium, buyers' surplus from buying from I is respectively $CS_I^l = \left[v(\beta_I + 1) - p_I^l\right](1-k)$ and $CS_I^s = [v(\beta_I + 1) - p_I^s]\,k$. In order to induce a unilateral deviation from the large buyer, the entrant should set a price p_E^l such that $CS_E^l = \left[v(1-k) - p_E^l\right](1-k) > \left[v(\beta_I + 1) - p_I^l\right](1-k)$, i.e. it should offer a price $p_E^l < p_I^l - [v(\beta_I + 1) - v(1-k)]$. Given such a price p_E^l, the large buyer will buy from E no matter who the small buyer buys from. But then, the small buyer will anticipate that E's network will have at least size $1-k$. If the small buyer decides to switch as well, E's network will have a size of 1. If the small buyer instead stays with I, then I's network will have size $\beta_I + k$. Therefore, to induce the small buyer to switch, E must offer a price p_E^S such that $CS_E^s = [v(1) - p_E^s]\,k > [v(\beta_I + k) - p_I^s]\,k$, i.e. E's offer must satisfy $p_E^s < p_I^s - [v(\beta_I + k) - v(1)]$. This deviation is profitable if $\pi_E = (1-k)p_E^l + kp_E^s - c_E \geq 0$ which after substituting becomes: $(1-k)\left(p_I^l - [v(\beta_I + 1) - v(1-k)]\right) + k\left(p_I^s - [v(\beta_I + k) - v(1)]\right) - c_E \geq 0$. Therefore, for the pair of prices set by the incumbent to be immune from deviations, it must be:

$$(1-k)p_I^l + kp_I^s \leq c_E + (1-k)\left([v(\beta_I + 1) - v(1-k)]\right) + k\left[v(\beta_I + k) - v(1)\right].$$

It will be profitable for I to set this pair of prices if $\pi_I' = (1-k)p_I^l + kp_I^s - c_I \geq 0$, or:

$$c_I \leq c_E + (1-k)\left([v(\beta_I + 1) - v(1-k)]\right) + k\left[v(\beta_I + k) - v(1)\right] \equiv c_I^{pdl}.$$

This is a *necessary* condition for an exclusionary equilibrium to exist under discriminatory pricing. The condition for the equilibrium under uniform pricing was $c_I \leq c_E + v(\beta_I + 1) - v(1-k) \equiv c_I^{up}$. It is easy to check that $c_I^{pdl} < c_I^{up}$, implying that price discrimination makes this equilibrium less likely to exist.

For a miscoordination equilibrium to exist at all, it must also be immune to another deviation, whereby the entrant first tries to induce a unilateral deviation by the small buyer. In that case, E would offer $p_E^s < p_I^s - [v(\beta_I + 1) - v(k)]$. Since for the *small* buyer it would be a dominant strategy to buy from E, the large buyer will anticipate that E's network will have at least size k. Then, to induce the large buyer to switch, it would be sufficient for E to offer a price p_E^l

such that $CS_E^l = \left[v(1) - p_E^l\right](1-k) > \left[v(\beta_I + 1 - k) - p_I^l\right](1-k)$, i.e. E's offer must satisfy $p_E^l < p_I^l - \left[v(\beta_I + 1 - k) - v(1)\right]$.

For a miscoordination equilibrium (p_I^l, p_I^s) to be immune from this deviation as well, it must be:

$$(1-k)p_I^l + kp_I^s - c_I \leq c_E + (1-k)\left(\left[v(\beta_I + 1 - k) - v(1)\right]\right) + k\left[v(\beta_I + 1) - v(k)\right] - c_I,$$

which is profitable for the incumbent if:

$$c_I \leq c_E + (1-k)\left(\left[v(\beta_I + 1 - k) - v(1)\right]\right) + k\left[v(\beta_I + 1) - v(k)\right] \equiv c_I^{pds}.$$

Therefore, the necessary and sufficient condition for a miscoordination equilibrium to exist are as follows: $c_I \leq \min\left\{c_I^{pds}, c_I^{pdl}\right\}$. Simple (albeit tedious) algebra shows that indeed there exist parameter values where $c_E + (1+\beta_I)\,v(\beta_I + 1) - \beta_I v(\beta_I) - v(1) - c_E \leq c_I \leq \min\left\{c_I^{pds}, c_I^{pdl}\right\}$, the first inequality being the assumption of efficient entry.

b) Proceed analogously to prove the conditions for the entry equilibrium. To induce a unilateral deviation from an entry equilibrium by the large buyer, the incumbent should set a price such that $CS_I^l = \left[v(\beta_I + 1 - k) - p_I^l\right](1-k) > CS_E^l = \left[v(1) - p_E^l\right](1-k)$; then, to "steal" the small buyer as well, the offer to the small buyer must satisfy $CS_I^s = \left[v(\beta_I + 1) - p_I^s\right]k > CS_E^s = \left[v(k) - p_E^s\right]k$. The deviation is profitable if $\pi_I = (1-k)p_I^l + kp_I^s - c_I \geq 0$, where $p_I^l = p_E^l + v(\beta_I + 1 - k) - v(1)$, and $p_I^s = p_E^s + v(\beta_I + 1) - v(k)$. Hence, an entry equilibrium would be immune to such a deviation if $(1-k)p_I^l + kp_I^s - c_I = 0$, and satisfies the break-even condition if $(1-k)p_I^l + kp_I^s - c_I \geq c_E$. Equivalently, we can write:

$$c_I \leq c_E + (1-k)\left[v(\beta_I + 1 - k) - v(1)\right] + k\left[v(\beta_I + 1) - v(k)\right] \equiv c_1.$$

The other possible deviation is to first "target" the small buyer, and then the large buyer, i.e. offer $p_I^s < p_E^s + (v(\beta_I + k) - v(1))$ and $p_I^l < p_E^l + (v(\beta_I + 1) - v(1-k))$. Proceeding as above, one obtains that this amounts to requiring:

$$c_I \leq c_E + (1-k)\left(v(\beta_I + 1) - v(1-k)\right) + k\left(v(\beta_I + k) - v(1)\right) \equiv c_2.$$

For an entry equilibrium to exist, it must be that $c_I \leq \min\{c_1, c_2\}$. Simple algebra shows that there exist values which satisfy this condition (while simultaneously satisfying the condition for efficient entry), and under which then an entry equilibrium would exist. However, under uniform pricing the entry equilibrium always existed, whereas here it exists only for some values of the parameter space. □

Proof of Proposition 13 *(Perfect discrimination)*

(i) Consider a candidate equilibrium where E sells all units at a price p_E. The incumbent may deviate by selling $1 - \bar{s} + \epsilon$ units at the price $p_E - \epsilon$ (thereby securing enough units to prevent E from reaching critical size), and the remaining $\bar{s}-\epsilon$ units at the monopoly price 1.[31] This deviation is profitable if $\pi_I=(1-\bar{s})p_E+\bar{s}-c_I\geq 0$. For the entry equilibrium to be immune from this deviation, it must therefore be: $p_E^*=\max\left\{0,\frac{c_I-\bar{s}}{1-\bar{s}}\right\}$. This equilibrium exists if $\pi_E(p_E^*)=\max\left\{0,\frac{c_I-\bar{s}}{1-\bar{s}}\right\}-c_E\geq 0$. Clearly, a necessary condition for the profitability condition to hold must be that $p_E^*\geq 0$, i.e. that $c_I\geq\bar{s}$. Further, it must be that $c_I\geq c_E+\bar{s}\,(1-c_E)$.

(ii) Another natural candidate equilibrium is one where the entrant sells the $\bar{s}$ units it needs to secure at a price $p_E^{\bar{s}}$, and the remaining units at a higher price $p_E^{1-\bar{s}}$, with all units sold by it.

Consider first the case where $0<p_E^{\bar{s}}\leq p_E^{1-\bar{s}}$. In this case, the incumbent's optimal deviation would be to set the price $p_E^{1-\bar{s}}-\epsilon$ for $1-\bar{s}+\epsilon$ units, thereby securing enough units to make sure the entrant does not reach critical size, and set the price 1 for the remaining $\bar{s}-\epsilon$ units. This deviation is profitable if $\pi_I=(1-\bar{s})p_E^{1-\bar{s}}+\bar{s}-c_I\geq 0$, which is the same condition as above. It follows that the entrant should set the price for all units at $p_E^{\bar{s}}=p_E^{1-\bar{s}}=p_E^*$: we fall back to the case analysed under (i).[32]

But consider now the case where $0=p_E^{\bar{s}}\leq p_E^{1-\bar{s}}=c_I$. In this case, due to the assumption that prices are non-negative, the incumbent cannot subtract any of the units sold by the entrant at the zero price. Since the entrant has secured the $\bar{s}$ units it needs, the equilibrium cannot be broken by an incumbent's deviation. The pair $(0,c_I)$ must guarantee positive profits to the entrant: $\pi_E(0,c_I)=-c_E\bar{s}+(c_I-c_E)(1-\bar{s})\geq 0$. The equilibrium then exists if $c_I\geq c_E/(1-\bar{s})$.□

Proof of Lemma 14 *(Explicit vs. perfect discrimination)*

By combining the existence conditions obtained so far, we conclude that an entry equilibrium exists if

$$c_I\geq\min\left\{c_E/\left(1-\bar{s}\right),c_E+\bar{s}\left(1-c_E\right)\right\}.$$

Recall that the analogous condition for entry under explicit discrimination reads:

$$c_I\geq\min\left\{\frac{1+c_E}{2},k+c_E,1-k+c_E\right\}.$$

It is possible to show that $c_E+\bar{s}\left(1-c_E\right)>\min\left\{\frac{1+c_E}{2},k+c_E,1-k+c_E\right\}$, but also that there are values for which $c_E/\left(1-\bar{s}\right)<\min\left\{\frac{1+c_E}{2},k+c_E,1-k+c_E\right\}$.

This implies that perfect discrimination, may either reduce or increase the parameter space for which pure strategy entry equilibria exist. □

[31] It may be useful to recall that by assumption $\bar{s}>\max(k,1-k)$, from which it follows that $\bar{s}>1/2$.

[32] The same result would occur if $0<p_E^{1-\bar{s}}\leq p_E^{\bar{s}}$. The incumbent would set $p_E^{\bar{s}}-\epsilon$ for $1-\bar{s}+\epsilon$ units, and 1 for the remaining $\bar{s}-\epsilon$ units. This deviation is profitable if $\pi_I=(1-\bar{s})p_E^{\bar{s}}+\bar{s}-c_I\geq 0$. Hence, the entrant should sell all units at $p_E^{\bar{s}}=p_E^{1-\bar{s}}=p_E^*$.

8 Appendix B (for referees only)

In this Appendix we present extensions that for shortness we would not include in the final version of the paper, but that referees may want to look at, and that could possibly be published in a web page as supplementary material.

This Appendix is composed of the following material. Section 8.1 examines the case of sequential purchases. Section 8.2 explains in more detail why random coupons are not equivalent to perfect price discrimination. Section 8.3 deals with the case where - in the base model - demand functions are elastic.

8.1 Sequential buyers

The existence of multiple equilibria in our base model raises some difficulties as to its possible implications. In what follows, we show that - similarly to Segal and Whinston (2000) - when buyers move sequentially rather than simultaneously, unique equilibria emerge for any given set of parameter values.

For simplicity, and without losing any insights, we restrict attention to the case where there are only one large and one small buyer, and the entrant needs both buyers to reach minimum size $\overline{s}$ (in other words, we are back to the basic model with discontinuous utility). Play occurs in the following sequence: Stage 1: I and E choose p_I^l and p_E^l. Stage 2: the large buyer decides which firm to buy from. Stage 3: I and E choose p_I^s and p_E^s. Stage 4: The small buyer decides which firm to buy from.[33] We assume that prices can also be negative.[34]

We consider first the case of explicit price discrimination. (Later, we consider the case of sequentiality under uniform pricing and the case of price commitment.) The following can be proved.

Proposition B1 *(sequential buyers, explicit price discrimination)*
(a) If suppliers can set negative prices, then:

- *if* $c_I \leq \frac{c_E+k}{1+k}$*, then there is an exclusionary equilibrium where* $p_E^l = p_I^l = \frac{c_E - kc_I}{1-k}$*,* $p_E^s \leq p_I^l = 1$*, and all buyers buy from* I*;*
- *if* $c_I > \frac{c_E+k}{1+k}$*, then there is an entry equilibrium where* $p_E^l = p_I^l = \frac{c_I - k}{1-k}$*,* $p_E^s = p_I^l = c_I$*, and all buyers buy from* E*.*

(b) If prices are restricted to be non-negative, then multiple equilibria may arise.

Proof. We solve the game by backward induction.

Stage 4 (small buyer's move): Depending on how play evolved in the previous stages, we distinguish two cases: *Case 1*: If the large buyer already bought from

[33] Results would not change if the small buyer moved first. We have analysed the case where there are m buyers and they buy sequentially, and results are the same. Details are available from the authors.

[34] Uniqueness arises because we assume that firms can set negative prices. If prices were non-negative, multiplicity would arise as an artefact of the non-negativity assumption.

E, the small buyer can be sure that E will reach the minimum size, and will therefore buy from E if $p_E^s \leq p_I^s$, and from I otherwise. *Case 2*: If the large buyer bought from I, firm E cannot attract a sufficient number of units. The small buyer will either buy from I (if $p_I^s \leq 1$) or not buy at all.[35]

Stage 3 (offers to the small buyer): If the large buyer chose I in the first stage, the incumbent is the monopolist over the small buyer, and will set $p_I^s = 1$. Firm E would have to charge a negative price to attract the small buyer, but would have no interest in doing so, as there is no other buyer on whom it could recover the resulting losses. If instead the large buyer bought from E, the two suppliers compete for the remaining small buyer. I will want to serve the latter as long as $p_I^s \geq c_I$. A Bertrand-like reasoning leads to equilibrium prices $p_E^s = p_I^s = c_I$.

Stage 2 (large buyer's move): If the large buyer buys from E, he can be sure the small buyer will buy from E as well. Thus, the large buyer will buy from E whenever $p_E^l < p_I^l$. If instead $p_E^l > p_I^l$, he buys from I. In case of a price tie, the large buyer may either buy from E or from I.

Stage 1 (offers to the large buyer): If I wins the large buyer, it will set $p_I^s = 1$ to the small buyer and gain $\pi_I = (p_I^l - c_I)(1-k) + (1-c_I)k$. If instead E wins the large buyer, it will also serve the small buyer, though at price $p_E^s = c_I$. Thus, E's profits are $\pi_E = (p_E^l - c_E)(1-k) + (c_I - c_E)k$. Thus, everything boils down to who can offer the lower price to the large buyer.

An *exclusionary equilibrium* exists if $\pi_E = (p_E^l - c_E)(1-k) + (c_I - c_E)k = 0$ and $\pi_I = (p_E^l - c_I)(1-k) + (1-c_I)k \geq 0$.

If we impose non-negativity on p_E^l, we obtain two possible solutions:

- if $c_I < c_E/k$, $p_E^l = p_I^l = \frac{c_E - kc_I}{1-k}$ and $\pi_I \geq 0$ if $c_I < \frac{c_E+k}{1+k}$;
- if $c_I > c_E/k$, $p_E^l = p_I^l = 0$ and $\pi_I \geq 0$ if $c_I < k$.

An *entry equilibrium* exists if $\pi_I = (p_I^l - c_I)(1-k) + (1-c_I)k = 0$ and $\pi_E = (p_I^l - c_E)(1-k) + (c_I - c_E)k \geq 0$.

Again, restricting prices to be non-negative, we get:

- if $c_I > k$, $p_E^l = p_I^l = \frac{c_I - k}{1-k}$ and $\pi_E \geq 0$ if $c_I > \frac{c_E+k}{1+k}$;
- if $c_I < k$, $p_E^l = p_I^l = 0$ and $\pi_E \geq 0$ if $c_I > c_E/k$.

We see that whenever $c_E/k < c_I < k$, both firms offer price zero to the large buyer. In this case, the large buyer is indifferent between buying from I or from E, and so both the exclusionary and the entry equilibrium exist, as stated in part (b) of the Proposition.

If we allow instead for *negative prices*, we obtain a unique equilibrium, which is either an exclusionary or an entry equilibrium, as indicated in part (a) of Proposition 11:

If $c_I < (c_E + k)/(1+k)$ there is an exclusionary equilibrium; if $c_I > (c_E + k)/(1+k)$ there is an entry equilibrium.□

[35] Even if the entrant could set negative prices, it would not do so with the last buyer.

Note that - as in Segal and Whinston (2000) - the exclusionary equilibrium does not arise from simple miscoordination among buyers. Rather, it exists because the incumbent can exploit the externalities that the large buyer exerts on all other buyers: Similar to Bernheim and Whinston's (1998) setup of non-coincident markets, the first-moving buyer and the incumbent share the rent that can be extracted from future buyers.

Note, however, that two modifications of the sequential game give rise to a unique *entry* equilibrium, as we now show.

Sequential buyers, uniform pricing. Assume prices to be uniform across all buyers, i.e. $p_I^l = p_I^s = p_I$ and $p_E^l = p_E^s = p_E$; then, a straightforward extension of the argument above shows that the entry equilibrium would be the unique equilibrium.

Proposition B2 *(sequential buyers, uniform pricing)*
There is no exclusionary equilibrium. There exists a unique entry equilibrium with $p_E = p_I = c_I$, and all buyers buying from E.

Proof. First of all, note that prices chosen by suppliers at stage 1 of the game apply to both buyers: therefore no new decisions are taken at stage 3. Also keep in mind that under uniform pricing the incumbent cannot sell below marginal cost to only a subset of buyers. Solve the game by backward induction. At stage 4 the proof is identical to the proof of the previous proposition, and at stage 3 prices are determined by stage 1 decisions. At stage 2, if $p_E < p_I$, the large buyer knows that if it buys from E, the small buyer will buy from E as well. Therefore,it will buy from E. If $p_E > p_I$, it will buy from I. If $p_E = p_I$, it will be indifferent. At stage 1, the game played by the two suppliers will have the same features as the usual Bertrand game. The entrant will set the price $p_E = c_I$ (or a shade below it) and all buyers will buy from it.□

In the case of uniform pricing, entry could never be deterred. Thus, if c_I is small enough relative to c_E, we would have entry under uniform pricing but exclusion under price discrimination. For high enough values of c_I, we would have entry in both cases, but under price discrimination the equilibrium prices (to the large buyer) will be lower. In other words, banning discriminatory pricing would increase welfare for small efficiency gaps, but would decrease it for large enough efficiency gaps.

Sequential buyers, discrimination and price commitment Suppose now that buyers will buy sequentially, but that firms commit to (discriminatory) price offers at the beginning of the game. The rest of the game is as above. Play occurs in the following sequence: Stage 1: I and E choose p_I^l, p_I^s and p_E^l, p_E^s. Stage 2: the large buyer decides which firm to buy from. Stage 3: The small buyer decides which firm to buy from. We assume that prices might also be negative.

Proposition B3 *(sequential buyers, discriminatory pricing, price commitment)*

(a1) If negative prices are allowed: There exists no exclusionary equilibrium.

(a2) If prices are non-negative: If $c_I \leq 1-k$, there exists an exclusionary equilibrium. Otherwise, there exists no exclusionary equilibrium.

(b1) If negative prices are allowed: if $c_I \geq (1+c_E)/2$ the entry equilibrium exists, with ($p^l_E = \frac{c_I-k}{1-k}$,$p^s_E = \frac{c_I-(1-k)}{1k}$), and all buyers buy from E. Otherwise, no entry equilibrium exists.

(b2) If prices are non-negative: if $c_I \geq \min\left\{\frac{1+c_E}{2}, k+c_E, 1-k+c_E\right\}$ the entry equilibrium exists with $p^l_E = \max(0, \frac{c_I-k}{1-k})$ and $p^s_E = \max(0, \frac{c_I-(1-k)}{1k})$. Otherwise, no entry equilibrium exists.

Proof. (a1) Consider a candidate equilibrium where firm I sets (p^l_I, p^s_I) and both buyers buy from it. For this to be an equilibrium, it must be $\pi_I(p^l_I, p^s_I) = (p^l_I - c_I)(1-k) + (p^s_I - c_I)k \geq 0$. But if firm E sets $(p^l_I - \epsilon, p^s_I - \epsilon)$ it will get both buyers. Indeed, the large buyer correctly anticipates that if she buys from E, the small buyer will also buy from E, thus allowing E to get the threshold scale. But since $c_E < c_I$, there always exists a value of ϵ such that $\pi_E(p^l_I - \epsilon, p^s_I - \epsilon) > 0$ whenever $\pi_I(p^l_I, p^s_I) \geq 0$.

(a2) If prices are constrained to be non-negative, firm I may avoid a deviation by setting to zero the price to one buyer. Optimally, it would set $(1,0)$, and the large buyer will buy from I because it knows that E cannot induce the small buyer to deviate (simply because prices cannot be negative). this equilibrium exists if $\pi_I(p^l_I, p^s_I) = (1-c_I)(1-k) - c_I k \geq 0$, or $c_I \leq 1-k$.

(b1) Consider a candidate equilibrium where firm E sets (p^l_E, p^s_E) and both buyers buy from it. The incumbent may deviate by undercutting E on *one* of the two buyers, and getting monopoly profits on the other. For the candidate pair of prices to be immune from deviations it must therefore be that: $\pi_I(p^l_E - \epsilon, 1) = (p^l_E - \epsilon - c_I)(1-k) + (1-c_I)k < 0$ and $\pi_I(1, p^s_E - \epsilon) = (1-c_I)(1-k) + (p^s_E - \epsilon - c_I)k < 0$. This amounts to requiring that: $p^l_E = \frac{c_I-k}{1-k}$ and $p^s_E = \frac{c_I-(1-k)}{1k}$. These prices can be an equilibrium only if $\pi_E(p^l_E, p^s_E) \geq 0$, or $c_I \geq (1+c_E)/2$.

(b2) If prices are non-negative, firm E may avoid the incumbent's deviation by setting $p^l_E = \max(0, \frac{c_I-k}{1-k})$ and $p^s_E = \max(0, \frac{c_I-(1-k)}{1k})$. By substitution in the condition $\pi_E(p^l_E, p^s_E) \geq 0$, one finds the conditions indicated in the Proposition. ■

Note that the conditions under which the entry equilibrium exists are precisely the same as under simultaneous (discriminatory) offers, so price discrimination still affects the conditions under which the entry equilibrium exists even when there is a price commitment and sequential moves.

However, introducing price commitment in the sequential game makes exclusionary equilibria less likely. To see the role of price commitment consider for simplicity the case where prices can be negative. When there is *no* price commitment if the incumbent gets the first buyer it can safely set the monopoly price

($p_I^s = 1$) to the second buyer, whereas if the entrant gets the first buyer, it will have to set the lower duopoly price ($p_E^s = c_I$) to the second buyer. This implies that the incumbent's expected gains from obtaining the first buyer is $(1 - c_I)k$, while the entrant's expected profits would be $(c_I - c_E)k$. This makes it possible for the incumbent to be more aggressive than the entrant when competing for the first buyer.

In case of price commitment, instead, the incumbent has to commit to a price for the second buyer: either it sets $p_I^s = 1$, but in that case the entrant will be able to get a high revenue from the small buyer and would not be deterred from undercutting it on the first buyer, or it sets $p_I^s = c_I$, but in that case it would not be able to recover the losses from selling below cost to the first buyer. Therefore, having to commit to prices will favour the entrant.

8.2 Random coupons

One possible instrument of price discrimination are discounting coupons which are randomly sent out by firms. One may think that coupons could be used as an efficient discriminatory strategy to achieve exclusion. In what follows, we will argue that random coupons cannot reproduce the exclusionary effects of perfect discrimination. This result complements our earlier findings that even perfect discrimination does not necessarily lead to more exclusionary outcomes than third-degree discrimination.

Coupons as inefficient exclusionary tool, I Suppose firms set a list price but can send random coupons entitling the holder to buy one unit of the good at a discounted price. Along the lines of Section 5.4, consider a candidate entry equilibrium where the entrant sets a price $p_E \leq c_I$ to all buyers and sends no coupons. (The fact that it could use coupons does not mean it will use them at equilibrium.)

The incumbent would like to replicate the deviation identified in Section 5.4 by using coupons. This deviation would entail setting a list price of 1 and sending $1-\overline{s}+\epsilon$ coupons entitling to buy one unit at the discounted price $p_E - \epsilon$. On average, the coupons will be distributed evenly across all buyers, so that - if he chose to buy from I - each buyer would buy some units at the price $p_E^* - \epsilon$ and the remaining units at the price 1. However, this would give him a lower utility than buying all units from E at the price p_E. The deviation will therefore not be successful. To disrupt the candidate entry equilibrium, the incumbent would need to find a way to target coupons better, i.e. to concentrate the discounts on fewer buyers.

Suppose for instance the incumbent sent out $\overline{m}$ coupons (where $\overline{m}$ is defined by expression (8)) entitling their recipients to buy k/m units at the price $p_E^* - \epsilon$. If all the coupons reached small buyers, this strategy would replicate the optimal deviation under perfect discrimination. However, there is a positive probability that one or more coupons will end up in the hands of the large buyer. Understanding he is pivotal, he will not use such coupons, resulting in the incumbent making losses (he will sell some units at a price $p_E^* - \epsilon < c_I$) without preventing E from reaching critical size.[36]

Coupons as inefficient exclusionary tool, II Consider now the other candidate entry equilibrium, where firm E sets a price $p_E = c_I$ and sends around $\overline{s}$ coupons giving the right to buy one unit at a price $p_E^c < c_I$. One reason why the incumbent is not able to break this equilibrium is that - like above - when a buyer holds the right to buy some units at $p_I = c_I - \epsilon$, and other units at price 1, he will prefer to buy from E.

But to uncover another reason, suppose for the sake of the argument that buyers do not have mass. Under perfect discrimination, the incumbent would optimally deviate by undercutting the buyers who are offered the high price c_I by the entrant, and selling at the monopoly price to those who are offered the low price. However, random coupons would not necessarily allow it to replicate this deviation strategy: if firm I sent around $s + \epsilon$ coupons giving the right to buy at the price $c_I - \epsilon$, some of these coupons will end up in the hands of buyers who are offered the lower price $p_E^c < c_I$, and who therefore would not be enticed to deviate to I. As a result, firm I would either have to send around many more coupons so that on average a sufficient number of discounting coupons end up in the right hands (even so, it cannot be SURE that the coupons will be appropriated by the right persons), or to send around coupons giving right to a higher discount (e.g., the right to buy at price $p_E^c - \epsilon$). Either way, the deviation strategy will be more costly for the incumbent, and its deviation profits lower - which in turn means that the entry equilibrium will exist for a wider range of parameter values.

8.3 Elastic demands

In this Appendix, we analyse the base model under the assumption that demands are elastic.

8.3.1 The Setup

The basic model is analogous to the inelastic-demand case. The incumbent has an installed base of size $\beta_I \geq \bar{s}$, and incurs constant marginal cost $c_I \in \left(0, \frac{1}{2}\right)$ for each unit it produces of the network good. The entrant has marginal cost $c_E = 0$, and has installed base $\beta_E = 0$.

Let the unit prices offered by the two firms to a buyer of type $j = l, s$ be $p_I^j \in [0, 1]$ and $p_E^j \in [0, 1]$. Then, if buyer j buys from firm i, the quantity

[36] One may think that, since the large buyer is not going to use coupons, the incumbent might send around enough coupons to be sure that $\overline{m}$ of them reach the small buyers. But if it anticipated that the incumbent was able to sell more than $1 - \overline{s}$ units, the large buyer would use the coupons that reach him, making the deviation less profitable than under perfect discrimination. (As a matter of fact, to be sure that $\overline{m}$ coupons reach the small buyers, the incumbent should send coupons in sufficient number to cover the whole market - implying that all units would be sold below cost.)

demanded is:

$$q_i^j\left(p_i^j\right)=\begin{cases}(1-K)\left(1-p_i^j\right) & \text{if } j=l\\ \frac{K}{m}\left(1-p_i^j\right) & \text{if } j=s\end{cases} \tag{10}$$

As in the base model, $K\in(0,1)$ is an indicator of the relative weight of the small buyers in total market size, and $\bar{s}>\max\ \{1-K,K\}$.

Our demand functions are identical across buyers up to the size factor ($1-K$ or $\frac{K}{m}$), so that a monopolist who could charge discriminatory linear prices would set a uniform unit price $p_i^*=\frac{1}{2}(1+c_i)$. Recall that $c_E=0$, so that $p_E^*=\frac{1}{2}$; then, our assumption that $c_I<\frac{1}{2}$ implies that the entrant is never radically more efficient than the incumbent.

8.3.2 Uniform Linear Pricing

Proposition B4 *(equilibria under uniform flat prices) If firms can only use uniform flat prices, the following two pure-strategy Nash equilibria exist under the continuation equilibria as specified (after eliminating all equilibria where firms play weakly dominated strategies):*

(i) Entry equilibrium:

- if $\bar{s}\leq 1-c_I$, E sets $p_E=c_I$, I sets $p_I=c_I$, and all buyers, after observing $p_E\leq\min\{p_I,1-\bar{s}\}$, buy from E.

- if $\bar{s}>1-c_I$, E sets $p_E=1-\bar{s}$, I sets $p_I=p_I^*$ (where p_I^* is firm I's monopoly price), and all buyers, after observing $p_E\leq\min\{p_I,1-\bar{s}\}$, buy from E.

(ii) Miscoordination equilibrium: I sets $p_I=p_I^*$, E sets $p_E=p_E^*$ (where p_E^* is firm E's monopoly price), and all buyers, after observing $p_I-p_E\leq p_I^*$, end up buying from I.

Proof:

(i) Let $\bar{s}\leq 1-c_I$. Then, with all buyers buying from E at $p_E=c_I$, total demand is $mq_E^s(p_E)+q_E^l(p_E)=1-c_I\geq\bar{s}$, and so E will reach the minimum size. Thus, E's product has the exact same value to the buyers as I's, and it sells at the same price, so that buyers are indifferent between I's and E's offer. I will not want to deviate either: To attract the buyers, I would have to set a price $p_I<c_I$, i.e. sell at a loss; and increasing p_I above c_I will not attract any buyers. E has no incentive to change anything about its price either: increasing p_E would imply losing the buyers to I, and decreasing p_E will just reduce profits (recall that E is not radically more efficient than I, i.e. $p_E=c_I<p_E^*$, so that E cannot gain from reducing its price below c_I).

Let $\bar{s}>1-c_I$. Then, total demand at $p_E=1-\bar{s}$ is $mq_E^s(p_E)+q_E^l(p_E)=1-p_E=\bar{s}$, and so E will just reach the minimum size, while still breaking even ($\bar{s}\leq 1$ and $c_E=0$ imply $p_E-c_E\geq 0$). E has no incentive to increase its price, as that would imply falling short of the minimum size (followed by a break-down of coordination, i.e. all buyers would buy from I), while charging a price below $1-\bar{s}$ would only reduce profits . The buyers have no incentive to deviate and buy from I, because $p_I > p_E$. If I decreases its price to a value $p_I \in (1-\bar{s}, p_I^*)$ or increases it to some $p_I > p_I^*$, I will not be able to attract any buyers, and selling at a price at or below $1-\bar{s}$ (which is strictly smaller than c_I if $\bar{s} > 1-c_I$) would imply losses.

Note that we eliminate all equilibria in weakly dominated strategies, where
- if $\bar{s} \leq 1-c_I$, I sets $p_I \in [0, c_I)$ instead of $p_I = c_I$, and E sets $p_E = p_I$, and
- if $\bar{s} > 1-c_I$, I sets $p_I \in [1-\bar{s}, p_I^*)$ or $p_I > p_I^*$, and E sets $p_E = 1-\bar{s}$.

(ii) Suppose that all buyers buy from I. Then, recall that $\bar{s} > \max\{1-K, K\}$, implying that none of the individual buyers alone is sufficient for E to reach the minimum size. Thus, E's product has zero value for any single buyer, and so no buyer will want to deviate and buy from E, even though $p_I > p_E$. I sets $p_I = p_I^*$, which is the most profitable among all prices $p_I \leq p_I^* + p_E^*$ under which buyers will miscoordinate on the incumbent. Thus, I has no incentive to increase or decrease its price. Since buyers will not switch to E even if the price difference between the two firms is maximal, i.e. even if E charges $p_E = 0$ (so that $p_I = p_I^* + p_E$), E has no incentive to decrease its price.

We eliminate all equilibria in weakly dominated strategies, where I sets $p_I = p_I^*$, and E sets $p_E \neq p_E^*$.□

Let firms $i = I, E$ use tariffs of the following form:

$$T_i = p_i q_i^j - R_i$$

i.e. firms cannot discriminate among buyers, neither by type nor by the quantity they buy, but they can charge two-part tariffs.

8.3.3 Explicit Price Discrimination

Now, suppose that a supplier $i = I, E$ can offer contracts of the type

$$T_i^j = p_i^j q_i - R_i^j, \text{ with } j = s, l$$

where both the fixed component R_i^j and the variable component p_i^j may vary across buyer types (though not within one group of buyers). If $R_i^j < 0$, it is a franchise fee, i.e. a payment from the buyer to the firm i; if $R_i^j > 0$, it is a slotting allowance, i.e. a payment from the firm i to the buyer), and where p_i^j is the variable component of the tariff.

Buyers seek to maximize total surplus, which is the sum of net consumer surplus and possible lump-sum payments they receive from or have to pay to the firms. Define net consumer surplus as follows:

$$CS_i^j\left(p_i^j\right) = \frac{1}{2}\left(1-p_i^j\right) q_i^j\left(p_i^j\right) \tag{11}$$

$$= \begin{cases} \frac{1}{2}(1-K)\left(1-p_i^j\right)^2 & \text{if } s_i \geq \bar{s} \text{ and } p_i^j \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } j = l \tag{12}$$

$$= \begin{cases} \frac{1}{2}\frac{K}{m}\left(1-p_i^j\right)^2 & \text{if } s_i \geq \bar{s} \text{ and } p_i^j \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } j = s \tag{13}$$

Lemma B5 *(miscoordination under third-degree price discrimination) For all parameter values, there is an equilibrium where I sets*

$$p_I^s = p_I^l = c_I; R_I^s = -\frac{1}{2}\frac{K}{m}(1-c_I)^2, R_I^l = -\frac{1}{2}(1-K)(1-c_I)^2$$

E makes the analogous offer, $p_E^s = p_E^l = c_E = 0; R_E^s = -\frac{1}{2}\frac{K}{m}, R_E^l = -\frac{1}{2}(1-K)$, and all buyers, after observing that I offers non-negative total surplus, buy from I.

Proof:

Buyer j is indifferent between buying from I, and not buying at all: buying from I yields total surplus $CS_I^j(c_I) + R_I^j = 0$, and not buying at all yields zero surplus as well. Buying from E, given that all other buyers buy from I (so that E would not reach the minimum size), would yield $0 + R_E^j < 0$, so buyers strictly prefer to buy from I. Given that buyers buy from I as long as $CS_I^j\left(p_I^j\right) + R_I^j \geq 0$, the incumbent will optimally set price equal to marginal cost to generate maximum consumer surplus, and then use the fixed component of the tariff to fully extract this surplus, thus making maximal profits $CS_I^l(c_I) + mCS_I^s(c_I)$.

Now, suppose the entrant deviates by offering a strictly positive payment to the small buyers: $p_E^s = p_E^l = c_E = 0$ (wlog); $R_E^s = \varepsilon > 0$, and $R_E^l \leq -mR_E^s < 0$. (The analogous reasoning applies to an offer where instead the large buyer would receive $R_E^l > 0$ while $R_E^s \leq -\frac{1}{m}R_E^l < 0$.) Under this offer, the small buyers will no longer buy from I, as they will rather accept $\frac{K}{m}$ units for free from the entrant to qualify for payment R_E^s. Then, even if the large buyer continues to buy from I, so that the entrant will not reach the minimum size (and hence the $\frac{K}{m}$ units remain without value to the small buyers), they will obtain a strictly positive surplus: $CS_E^s(c_E \mid s_E < \bar{s}) + R_E^s = 0 + \varepsilon > 0$.

However, they will be unable to consume the units they received from E, and so they will dispose of their units: Recall that $q_E^j\left(p_E^j\right) = 0$ if $s_E < \bar{s}$, and that $K/m < \bar{s}$, so that no buyer will be able to consume the good if he is the only one to do so. Hence, E's network still has size zero even after E gave $\frac{K}{m}$ units to each of the small buyers, because only units which are actually consumed count towards a firm's network size. But then, the large buyer will not want to switch to E, as he would obtain strictly negative surplus from E, $CS_E^l(c_E \mid s_E < \bar{s}) + R_E^l \leq 0 - m\varepsilon < 0$.

Thus, the large buyer will continue to buy from I, confirming the small buyers' expectation that E's network will remain below the minimum size (so that disposing of their units is the only option left to them). But then, E will not break even: if E sells at marginal cost to the small buyers, E cannot make strictly positive payments to them, unless E makes positive profits on the large buyer.

We can conclude that if buyers are miscoordinated, any feasible alternative offer E can make must satisfy $R_E^s \leq 0$ and $R_E^l \leq 0$. But such an offer will not induce the small buyers to leave I unless the large buyer does so as well, which will never happen precisely because buyers are miscoordinated. But then, E

might as well offer $p_E^s = p_E^l = c_E = 0; R_E^s = -\frac{1}{2}\frac{K}{m}, R_E^l = -\frac{1}{2}(1-K)$, which completes our proof.□

Proposition B6 *(entry under third-degree price discrimination) If firms can use two-part tariffs and discriminate between large and small buyers (but not among small buyers), then the entry equilibrium can only arise if*

$$c_I \geq 1 - \sqrt{\frac{1}{2}} \simeq 0.2929$$

and is characterized by

$$\begin{aligned} p_E^s &= p_E^l = c_E = 0; R_E^s = -\frac{K-(1-c_I)^2}{2m}; R_E^l = -\frac{(1-K)-(1-c_I)^2}{2} \\ p_I^s &= p_I^l = c_I; R_I^l = K\frac{(1-c_I)^2}{2}; R_I^s = \frac{1-K}{m}\frac{(1-c_I)^2}{2} \end{aligned}$$

with all buyers buying from the entrant after observing that the entrant's offer is at least as good as the incumbent's.

Proof:

Suppose there is a candidate equilibrium at which the entrant charges:

$$p_E^s = p_E^l = c_E = 0; R_E^s, R_E^l,$$

where the entrant optimally chooses to set the variable price equal to its marginal cost so as to maximize the rents that arise from the relationships with the buyers (otherwise, surplus would be inefficiently lost).

Since the entrant needs both the large buyer and at least one small buyer, this candidate equilibrium would not survive if the incumbent could make an offer that makes either the large buyer or the small buyers better off, so that it becomes a dominant strategy for this type of buyer to buy from I, no matter what the other buyers do. Let us look at each possibility in turn.

The first question is whether the incumbent can profitably induce the large buyer to switch. In this case, the small buyers would be forced to buy from I as well, even if I's offer to them is much less attractive than E's offer. The best offer I can make to the large buyer is to extract all the surplus from the small buyers and offer it to the large buyer. Such a deviation would take the form:

$$p_I^s = p_I^l = c_I; R_I^s = -CS_I^s(c_I), R_I^l = mCS_I^s(c_I).$$

Therefore, the candidate equilibrium can survive this deviation only if the entrant leaves the large buyer with a larger payoff than the one offered by the incumbent, that is only if:

$$CS_E^l(c_E) + R_E^l \geq CS_I^l(c_I) + mCS_I^s(c_I),$$

which can be rewritten as:

$$-R_E^l \leq CS_E^l(c_E) - CS_I^l(c_I) - mCS_I^s(c_I). \qquad \text{(cond 1)}$$

Second, the incumbent may also induce a deviation of the small buyers. To this end, it could extract all the surplus from the large buyer and offer it to the small buyers. Such a deviation would consist of the offer:

$$p_I^s = p_I^l = c_I; R_I^l = -CS_I^l(c_I), R_I^s = CS_I^l(c_I)/m.$$

In order for the candidate equilibrium to survive this deviation, the entrant must therefore make an offer such that:

$$CS_E^s(c_E) + R_E^s \geq CS_I^s(c_I) + CS_I^l(c_I)/m,$$

or:

$$-R_E^s \leq CS_E^s(c_E) - CS_I^s(c_I) - \frac{CS_I^l(c_I)}{m}. \qquad \text{(cond 2)}$$

Note that conditions (cond 1) and (cond 2) must hold simultaneously. Also note that the entrant's profits must be non-negative: Since, at the candidate equilibrium, the entrant is selling at marginal cost, the following break-even condition must hold:

$$-R_E^l - mR_E^s \geq 0. \qquad \text{(cond 3)}$$

An entry equilibrium where $p_E^s = p_E^l = c_E = 0; R_E^s, R_E^l$ can therefore survive only if conditions (cond 1), (cond 2) and (cond 3) will simultaneously hold. Optimality requires firm E to charge the highest possible fee (or to leave the lowest possible allowance) to the buyers. Therefore, at the optimum conditions (cond 1) and (cond 2) will be binding. By writing (cond 1) and (cond 2) with equality and inserting them in (cond 3) we obtain:

$$CS_E^l(c_E) + mCS_E^s(c_E) \geq 2\left(CS_I^l(c_I) + mCS_I^s(c_I)\right).$$

In words, entry equilibria can only arise if the total rent generated by the entrant is at least twice as high as the total rent generated by the incumbent. We can make use of the demand functions of the buyers and insert the actual consumer surpluses into the previous inequality, which can then be simplified to

$$c_I \geq 1 - \sqrt{\frac{1}{2}}.$$

In other words, an entry equilibrium can exist only if the entrant is sufficiently more efficient than the incumbent.

This contrasts sharply with the case of uniform linear tariffs of Proposition B4, where entry equilibria could arise even if the efficiency gap between the entrant and the incumbent was very small (i.e. even if $c_I \to c_E = 0$). But discriminatory two-part tariffs allow the incumbent to strategically redistribute rent across different types of buyers in order to exclude the entrant from the market.

Finally, the incumbent's offer in such an entry equilibrium (where the incumbent does not sell anything) will depend on how the buyers coordinate on the entrant: If, for instance, buyers buy from E whenever E's offer is at least as good as I's (analogously to the continuation equilibrium of Proposition B4 (i)), then the incumbent's equilibrium offer must exactly match the entrant's, e.g. as follows:

$$p_I^s = p_I^l = c_I; R_I^l = mCS_I^s(c_I), R_I^s = CS_I^l(c_I)/m$$

Note that this offer is not actually feasible (if I sold at marginal cost, it could not afford to make strictly positive payments to all the buyers). But if I offered less than that, the entrant would want to follow suit and reduce its offers as well, thus violating conditions (cond 1) and/or (cond 2), and such offers could not be sustained as an equilibrium.

Finally, the expressions for R_E^s, R_E^l, R_I^s, and R_I^l given in the Proposition were obtained by inserting from definition (11).□

It is worth studying whether the entrant offers a positive or a negative fixed component to the buyers at equilibrium. It is straightforward to note that:

$$R_E^s \leq 0 \quad \text{iff} \quad K \geq (1-c_I)^2; \text{ and } R_E^l \leq 0 \quad \text{iff} \quad K \leq 1-(1-c_I)^2.$$

We can now illustrate the result in the plane $\left((1-c_I)^2, K\right)$, as in Figure 4. The Figure shows the regions where the entry equilibrium exists and characterizes it by showing whether at the equilibrium firm E has to pay or not a fixed fee to the buyers. First of all, note that for any given admissible level of K the more efficient is firm E relative to firm I (that is, as we move horizontally to the left of the plane) the more likely that we find an equilibrium in which firm E is able to extract surplus from both the large and the small buyers.

Let us now look at the comparative statics on K. For any given value of $(1-c_I)^2$, an increase in K makes it more likely that firm E charges a positive fee to the small buyers and a negative fee to the large buyer. In particular, note that:

$$mR_E^s \leq R_E^l \quad \text{iff} \quad K \geq \frac{1}{2}.$$

In other words, when $K \geq \frac{1}{2}$, that is when the small buyers account for most of the market, the entrant will extract more surplus from them than from the large buyer at equilibrium.

To understand these results, note that according to our assumptions the entrant needs to have purchases from both the small and the large buyer. When K is small, what the incumbent will want to do is to extract as much as possible from the large buyer, whose surplus is larger than the aggregate surplus of the small buyers, to induce the small buyers to buy from it; when K is large, the opposite will occur: the share of the small buyers is the largest, and the

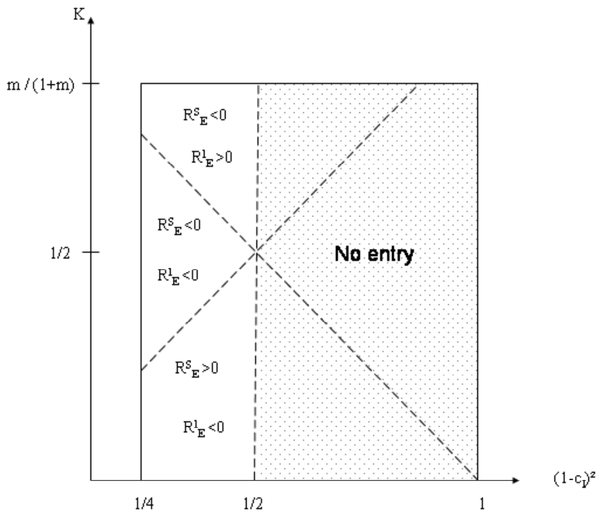

Figure 4: Region where the entry equilibrium exists (plain) and does not exist (dotted)

incumbent will try to extract as much as possible from them to offer it to the large buyer. Hence, when K is small, the entrant will need to make its best offer to the small buyers, whereas when K is large, it is the small buyers' market share which is largest, and therefore it is the large buyer who needs to be induced to buy away from the incumbent.

8.3.4 Implicit Price Discrimination

Let us now consider the case where firms cannot make their offers depend directly on the type of buyer (large or small), but have to make uniform offers to both types which may only depend on the quantity bought by buyer $j = 1, \ldots, m+1$:

$$T_i(q_i^j) = \begin{cases} p_{i,1}q_i^j - R_{i,1} & \text{if } q_i^j \leq \bar{q}_i \\ p_{i,2}q_i^j - R_{i,2} & \text{if } q_i^j \geq \bar{q}_i \end{cases}$$

The fixed component, $R_{i,1}$ or $R_{i,2}$, can again be either positive or negative.[37] The difference is that each buyer can now choose his tariff from this price menu by buying either below the sales target $\bar{q}_i$, or above.

[37] Results are qualitatively similar if we restrict fixed payments to go from firms to buyers only, while ruling out franchise fees paid by buyers to firms.

It is well-known that such quantity discounts or rebates, when applied to buyers who differ in size, will be a tool of (de facto) discrimination, even if the schemes as such are uniform. But to achieve discrimination, the tariffs have to be set in a way that induces buyers to self-select into the right category, with small buyers voluntarily buying below target, and the large buyer choosing to buy above.

First, consider the large buyer $j = l$, and suppose his demand at price $p_{i,2}$ is above the threshold, i.e. $q_i^l(p_{i,2}) \geq \bar{q}_i$ (this will be the only relevant case). Then, the large buyer can either buy $q_i^l(p_{i,2})$, which yields total surplus $CS_i^l(p_{i,2})+R_{i,2}$, or he can buy below the threshold $\bar{q}_i$, i.e. $q_i^l = \min\left\{q_i^l(p_{i,1}), \bar{q}_i\right\}$ at price $p_{i,1}$, in which case his net consumer surplus can be expressed as

$$CS^{l,net}(p_{i,1}, \bar{q}_i) = \begin{cases} CS_i^l(p_{i,1}) & \text{if } s_i \geq \bar{s} \text{ and } q_i^l(p_{i,1}) < \bar{q}_i \\ \bar{q}_i\left(1 - p_{i,1} - \bar{q}_i \frac{1}{2}\frac{1}{1-K}\right) & \text{if } s_i \geq \bar{s} \text{ and } q_i^l(p_{i,1}) \geq \bar{q}_i \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Next, consider a typical small buyer $j = s$, whose demand at price $p_{i,2}$ may be below the threshold, i.e. $q_i^s(p_{i,2}) < \bar{q}_i$. Then, a small buyer may either buy $q_i^s(p_{i,1})$, which yields total surplus $CS_i^s(p_{i,1}) + R_{i,1}$, or he can buy the sales target $\bar{q}_i$ at price $p_{i,2}$ (i.e. a quantity which exceeds his actual demand at this price).

If $\bar{q}_i > \frac{K}{m}$, i.e. if the sales target is above the largest quantity he can consume, $q_i^s(p_i = 0) = \frac{K}{m}$, then the excess units, $\bar{q}_i - \frac{K}{m}$, can be disposed of at no cost.[38] Define the small buyer's net consumer surplus of buying $\bar{q}_i$ units as

$$CS^{s,net}(p_{i,2}, \bar{q}_i) = \begin{cases} CS_i^s(p_{i,2}) & \text{if } s_i \geq \bar{s} \text{ and } q_i^s(p_{i,2}) > \bar{q}_i \\ \bar{q}_i\left(1 - p_{i,2} - \bar{q}_i \frac{1}{2}\frac{m}{K}\right) & \text{if } s_i \geq \bar{s} \text{ and } \bar{q}_i \in \left(q_i^s(p_{i,2}), \frac{K}{m}\right) \\ \frac{1}{2}\frac{K}{m} - p_{i,2}\bar{q}_i & \text{if } s_i \geq \bar{s} \text{ and } \bar{q}_i \geq \frac{K}{m} \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

We say that firm i's offer satisfies the *"self-selection condition"* if the large buyer prefers to buy above the threshold, and the small buyers prefer to buy below the threshold, i.e. if

$$\begin{aligned} CS_i^l(p_{i,2}) + R_{i,2} &\geq CS^{l,net}(p_{i,1}, \bar{q}_i) + R_{i,1} \\ \text{and } CS_i^s(p_{i,1}) + R_{i,1} &\geq CS^{s,net}(p_{i,2}, \bar{q}_i) + R_{i,2} \end{aligned} \tag{16}$$

For any offer that satisfies the self-selection condition, denote $(p_{i,1}, R_{i,1})$ by (p_i^s, R_i^s), and $(p_{i,2}, R_{i,2})$ by $\left(p_i^l, R_i^l\right)$, for $i = I, E$.

Lemma B7 *(miscoordination under rebates) For all parameter values, there is an equilibrium where I sets*

$$p_I^{s,mis} = \frac{mc_I + 1 - \frac{1}{1-K}\frac{K}{m}}{m + 1 - \frac{1}{1-K}\frac{K}{m}} > c_I, p_I^{l,mis} = c_I, \bar{q}_I = q_I^s\left(p_I^{s,mis}\right)$$

[38] Recall that we excluded reselling of units between buyers (while allowing for free disposal), so the only thing a small buyer can do with units he cannot consume is to throw them away.

and fully extracts consumer surplus from the small buyers, while leaving some rent to the large buyer:

$$R_I^s = -CS_I^s\left(p_I^{s,mis}\right), R_I^l = -CS_I^l\left(c_I\right) + \left(CS^{l,net}\left(p_I^{s,mis}, \bar{q}_I\right) + R_I^{s,mis}\right) < 0$$

E makes the analogous offer with $0 < p_E^{s,mis} < p_I^{s,mis}$, $p_E^{l,mis} = c_E = 0$, and all buyers, after observing that I offers non-negative total surplus, buy from I.

Proof:

We will first show that I's equilibrium offer coincides with the solution to the following profit maximization problem:

$$\max_{\{p_I^s, p_I^l, R_I^s, R_I^l\}} m\left(p_I^s - c_I\right) q_I^s\left(p_I^s\right) + m\left(-R_I^s\right) + \left(p_I^l - c_I\right) q_I^l\left(p_I^l\right) + \left(-R_I^l\right)$$

subject to the large and small buyers' self-selection constraints, the large and small buyers' participation constraints, and the break-even constraint:

(i) $CS_I^l\left(p_I^l\right) + R_I^l \geq CS^{l,net}\left(p_I^s, q_I^s\left(p_I^s\right)\right) + R_I^s$

(ii) $CS_I^s\left(p_I^s\right) + R_I^s \geq CS^{s,net}\left(p_I^l\right) + R_I^l$

(iii) $CS_I^l\left(p_I^l\right) + R_I^l \geq 0$

(iv) $CS_I^s\left(p_I^s\right) + R_I^s \geq 0$

(v) $m\left(p_I^s - c_I\right) q_I^s\left(p_I^s\right) + \left(p_I^l - c_I\right) q_I^l\left(p_I^l\right) \geq mR_I^s + R_I^l$

Now, at the optimum, constraints (i) and (iv) will be binding, thus determining R_I^l and R_I^s. This, in turn, implies that $p_I^{l,mis} = c_I$ (which, given p_I^s, maximizes the rent that I can extract from the large buyer).

Therefore, the incumbent's problem reduces to choosing the right p_I^s. The resulting maximization problem is convex in p_I^s, and solves for

$$p_I^{s,mis} = \frac{mc_I + 1 - \frac{1}{1-K}\frac{K}{m}}{m + 1 - \frac{1}{1-K}\frac{K}{m}}$$

Our assumption that $c_I < 1$ implies $p_I^{s,mis} \in (c_I, 1)$.

The large buyer's participation constraint (iii) will be oversatisfied under this solution: Given that constraints (i) and (iv) hold with equality, we have

$$CS_I^l\left(c_I\right) + R_I^{l,mis} = CS^{l,net}\left(p_I^{s,mis}, q_I^s\left(p_I^{s,mis}\right)\right) - CS_I^s\left(p_I^{s,mis}\right)$$

But the right-hand side of this equality is strictly positive because

$$CS^{l,net}\left(p_I^s, q_I^s\left(p_I^s\right)\right) > CS_I^s\left(p_I^s\right) \text{ for all } p_I^s$$

which implies that $CS_I^l\left(c_I\right) + R_I^{l,mis} > 0$.

The small buyers' self-selection condition (ii) holds as well: Suppose a small buyer considers buying $q_I^s\left(c_I\right)$ units at price c_I rather than the sales target $\bar{q}_I = q_I^s\left(p_I^{s,mis}\right) < q_I^s\left(c_I\right)$ at price $p_I^{s,mis} > c_I$. Then, this buyer would enjoy net consumer surplus $CS_I^s\left(c_I\right)$. However, the small buyer will have to pay $R_I^{l,mis}$

(instead of $R_I^{s,mis} = -CS_I^s\left(p_I^{s,mis}\right)$), where $R_I^{l,mis} < -CS_I^s(c_I)$. Thus, a small buyer buying above the threshold would end up with a strictly negative net total surplus, and so he will prefer to buy $\bar{q}_I = q_I^s\left(p_I^{s,mis}\right)$.

Finally, I's break-even constraint (v) reduces to

$$m\left(p_I^{s,mis} - c_I\right) q_I^s\left(p_I^{s,mis}\right) + 0 \geq mR_I^{s,mis} + R_I^{l,mis}$$

which holds because both large and small buyers will pay strictly positive fees to the incumbent ($R_I^{s,mis} < 0$ and $R_I^{l,mis} < 0$).

Hence, the incumbent's equilibrium offer

$$\begin{aligned} p_I^{s,mis} &> c_I,\ p_I^{l,mis} = c_I,\ \bar{q}_I = q_I^s\left(p_I^{s,mis}\right),\ R_I^{s,mis} = -CS_I^s\left(p_I^{s,mis}\right) \\ R_I^l &= CS^{l,net}\left(p_I^{s,mis}, q_I^s\left(p_I^{s,mis}\right)\right) + R_I^{s,mis} - CS_I^l(c_I) \end{aligned}$$

is the most profitable among all feasible offers, and so I will not have any incentive to deviate.

Given that buyers miscoordinate on the incumbent no matter which offer the entrant makes, the entrant is indifferent between all feasible offers (the reasoning is analogous to the Proof of Lemma B5). Eliminating all equilibria where E plays weakly dominated strategies, E will just solve the analogous optimization problem analyzed above, which yields

$$\begin{aligned} p_E^{s,mis} &= \frac{1 - \frac{1}{1-K}\frac{K}{m}}{m + 1 - \frac{1}{1-K}\frac{K}{m}} < p_I^{s,mis} \\ p_E^{l,mis} &= c_E = 0 < p_I^l,\ \bar{q}_E = q_E^s\left(p_E^{s,mis}\right),\ R_E^{s,mis} = -CS_E^s\left(p_E^{s,mis}\right) \\ R_E^l &= CS^{l,net}\left(p_E^{s,mis}, q_E^s\left(p_E^{s,mis}\right)\right) + R_E^{s,mis} - CS_E^l(c_E) \end{aligned}$$

But given that all buyers buy from I, no individual buyer will want to deviate and buy from E, and so all buyers will end up buying from I.□

This is the most profitable miscoordination equilibrium under rebates: The incumbent will receive positive fixed payments from both types of buyers (recall that we explicitly allowed for fixed payments to go from buyers to firms), and on top of that, I will earn a positive mark-up on the sales to the small buyers. Again, there are alternative miscoordination equilibria (under different continuation equilibria), where the incumbent would make lower profits, and may even have to make positive payments to one or both types of buyers.

Note that the incumbent's profits will be lower under rebates than under third-degree price discrimination: I sells above marginal cost to the small buyers, which reduces their consumer surplus (and hence their franchise fee), and I does not extract the full consumer surplus $CS_I^l(c_I)$ from the large buyer. Both features of I's equilibrium offer follow directly from the introduction of personal arbitrage.

Suppose that I wanted to replicate the more profitable offer under third-degree price discrimination, setting a uniform unit price of $p_{I,1} = p_{I,2} = c_I$, and franchise fees

$$\begin{array}{ll} R_{I,1} = -CS_I^s(c_I) & \text{if } q_I^j \leq \bar{q}_I = q_I^s(c_I) \\ R_{I,2} = -CS_I^l(c_I) & \text{if } q_I^j > \bar{q}_I = q_I^s(c_I) \end{array}$$

This "first-best" offer does not satisfy the self-selection condition, because the large buyer would then prefer the small-buyer tariff, i.e. he would buy $\bar{q}_I = q_I^s(c_I)$ and enjoy strictly positive net surplus $CS^{l,net}(c_I, q_I^s(c_I)) - CS_I^s(c_I) > 0$ (although this means the large buyer is quantity-constrained, because $q_I^s(c_I) < q_I^l(c_I)$). Thus, arbitrage implies that I's offer to the large buyer must leave the latter with at least as much rent as buying below the threshold would yield.

This requirement also explains why the incumbent incurs a usage price distortion on the small buyers, rather than charging $p_I^s = p_I^l = c_I$ to all buyers to maximize the consumer surplus of the small buyers. The intuition is as follows. Suppose that I charges c_I to both large and small buyers. It can then extract $CS_I^s(c_I)$ from the small buyers, and must leave at least $CS^{l,net}(c_I, q_I^s(c_I)) - CS_I^s(c_I)$ to the large buyer.

Now, suppose instead that I raises the price p_I^s by an ε, while leaving $p_I^l = c_I$ unchanged: of course, this means efficiency losses, i.e. the additional profits I makes on sales to the m small buyers are lower than the losses in rent that I can extract from them through the fixed fee. On the other hand, I can now extract more rent from the large buyer: under the higher p_I^s (and accordingly lower sales target), it is much less attractive for the large buyer to buy below the threshold.

Thus, no matter how large K or m, there will always be an ε such that the losses of raising p_I^s (on the small buyers) are more than outweighed by the gains (on the large buyer), and so $p_I^s = c_I$ cannot be optimal.

Proposition B8 *(entry under rebates)*

(i) If

- E's joint net surplus is sufficient to match both I's best offer to the large buyer and I's best offer to the small buyers simultaneously, and

- E's offer below satisfies the large buyer's "self-selection condition" of equation (16),

then our game has a pure-strategy equilibrium where all buyers buy from E after observing that E's offers to each of them are at least as good as I's offers. ("first-best entry equilibrium"). Such an equilibrium is characterized by

$$\begin{aligned} p_E^{s*} &= p_E^{l*} = c_E = 0, \bar{q}_E = q_E^s(c_E) \\ R_E^{l*} &= CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} - CS_E^l(c_E) \\ R_E^{s*} &= CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best} - CS_E^s(c_E) \end{aligned}$$

and can arise even when $c_I < 1 - \sqrt{1/2}$, i.e. when there would not be an entry equilibrium under third-degree price discrimination.

(ii) If the above offer violates the large buyer's "self-selection condition" of equation (16), but the offer

$$\begin{aligned} p_E^{l*} &= c_E = 0 \\ p_E^{s*} &= 1 - \sqrt{2\frac{m}{K}\frac{1-K}{1-K-\frac{K}{m}}\left(CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} - \left(CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}\right)\right)} \\ R_E^{s*} &= CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best} - CS_E^s\left(p_E^{s*}\right) \\ R_E^{l*} &= CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} - CS_E^l\left(c_E\right) \end{aligned}$$

satisfies the entrant's break-even constraint and allows E to reach the minimum threshold, $q_E^l(c_E) + mq_E^s(p_E^{s*}) \geq \bar{s}$, then the offer above characterizes a pure-strategy equilibrium where all buyers buy from E after observing that E's offers to each of them are at least as good as I's offers. ("second-best entry equilibrium").

(iii) If the "first-best" offer in (i) violates the large buyer's "self-selection condition" of equation (16), and the "second-best" offer in (ii) violates the entrant's break-even constraint or does not allow E to reach the minimum threshold, no entry equilibrium exists. This situation can arise even when $c_I \geq 1 - \sqrt{1/2}$, i.e. when there would be an entry equilibrium under third-degree price discrimination.

Proof:

(i) We argued in Lemma B5 that miscoordination equilibria exist for all parameter values under the appropriate continuation equilibria. We will now show that entry equilibria will only exist for certain parameter values, even under those continuation equilibria that are "favorable" to the entrant.

Analogously to Proposition B6, a necessary condition for existence of an entry equilibrium is that I can neither match E's offer to the large buyer nor E's offer to the small buyers.

The best offer that I can make to the small buyers solves

$$\max_{\left\{p_I^s, p_I^l, R_I^s, R_I^l\right\}} CS_I^s\left(p_I^s\right) + R_I^s$$

subject to the large and small buyers' self-selection constraints, the large and small buyers' participation constraints, and the break-even constraint:

(i) $CS_I^l\left(p_I^l\right) + R_I^l \geq CS^{l,net}\left(p_I^s, q_I^s\left(p_I^s\right)\right) + R_I^s$

(ii) $CS_I^s\left(p_I^s\right) + R_I^s \geq CS^{s,net}\left(p_I^l\right) + R_I^l$

(iii) $CS_I^l\left(p_I^l\right) + R_I^l \geq 0$

(iv) $CS_I^s\left(p_I^s\right) + R_I^s \geq 0$

(v) $m\left(p_I^s - c_I\right)q_I^s\left(p_I^s\right) + \left(p_I^l - c_I\right)q_I^l\left(p_I^l\right) \geq mR_I^s + R_I^l$

Now, at the optimum, constraints (i) and (v) will be binding, thus determining R_I^l and R_I^s, which implies $p_I^{l,worst} = c_I$ (which, given p_I^s, maximizes the rent that I can extract from the large buyer).

Thus, I's reduced problem is convex in p_I^s, and solves for

$$p_I^{s,best} = \frac{mc_I + 1 - \frac{1}{1-K}\frac{K}{m}}{m + 1 - \frac{1}{1-K}\frac{K}{m}}$$

where $p_I^{s,best} \in (c_I, 1)$. Note that $p_I^{s,best}$ is equal to the incumbent's unit price for small buyers under the miscoordination equilibrium, $p_I^{s,mis}$. But now, all the profits earned on the small buyers as well as the rent extracted from the large buyer will be redistributed to the small buyers through the fixed payment $R_I^{s,best}$:

$$\begin{aligned}
p_I^{s,best} &> c_I, p_I^{l,worst} = c_I,\ \bar{q}_I = q_I^s\left(p_I^{s,best}\right) \\
R_I^{s,best} &= \frac{1}{m+1}\left(m\left(p_I^{s,best} - c_I\right) q_I^s\left(p_I^{s,best}\right) + CS_I^l\left(c_I\right) - CS^{l,net}\left(p_I^{s,best}, \bar{q}_I\right)\right) \\
R_I^{l,worst} &= CS^{l,net}\left(p_I^{s,best}, \bar{q}_I\right) + R_I^{s,best} - CS_I^l\left(c_I\right)
\end{aligned}$$

The small buyers' self-selection condition (ii) reduces to

$$CS_I^l\left(c_I\right) - CS^{l,net}\left(p_I^{s,best}, q_I^s\left(p_I^{s,best}\right)\right) \geq CS^{s,net}\left(c_I\right) - CS_I^s\left(p_I^{s,best}\right)$$

which holds for all $p_I^s \geq c_I$. The participation constraints (iii) and (iv) are over-satisfied at the solution:

$$CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best} > 0 \text{ and } CS_I^l\left(c_I\right) + R_I^{l,worst} > 0.$$

Note that this solution will necessarily yield lower total surplus for the small buyers than the corresponding offer under third-degree price discrimination, where each buyer receives $CS_I^s\left(c_I\right) + CS_I^l\left(c_I\right)/m$.

The incumbent's best offer to the large buyer solves the analogous program:

$$\max_{\left\{p_I^s, p_I^l, R_I^s, R_I^l\right\}} CS_I^l\left(p_I^l\right) + R_I^l$$

subject to the large and small buyers' self-selection constraints, the large and small buyers' participation constraints, and the break-even constraint:

(i) $CS_I^l\left(p_I^l\right) + R_I^l \geq CS^{l,net}\left(p_I^s\right) + R_I^s$
(ii) $CS_I^s\left(p_I^s\right) + R_I^s \geq CS^{s,net}\left(p_I^l, q_I^l\left(p_I^l\right)\right) + R_I^l$
(iii) $CS_I^l\left(p_I^l\right) + R_I^l \geq 0$
(iv) $CS_I^s\left(p_I^s\right) + R_I^s \geq 0$
(v) $m\left(p_I^s - c_I\right) q_I^s\left(p_I^s\right) + \left(p_I^l - c_I\right) q_I^l\left(p_I^l\right) \geq mR_I^s + R_I^l$

This problem has four different solutions:

Case 1: $0 < K \leq \frac{c_I(1-c_I)}{\frac{1}{2}\frac{1}{m} + c_I(1-c_I) + \frac{1}{2}(1-c_I)^2} = K^*\left(c_I, m\right)$

Only constraints (iv) and (v) are binding, all other constraints are oversatisfied. I can extract the full net consumer surplus from the small buyers and transfer it to the large buyer without violating the small buyers' self-selection constraint (ii):

$$\begin{aligned} p_I^{s,worst} &= p_I^{l,best} = c_I,\, \bar{q}_I = q_I^l(c_I) \\ R_I^{s,worst} &= -CS_I^s(c_I),\, R_I^{l,best} = mCS_I^s(c_I) \end{aligned}$$

Case 2: $K^*(c_I,m) < K \leq \frac{c_I\left(1-\frac{1}{m+1}c_I\right)}{\frac{1}{2}\frac{1}{m}+c_I\left(1-\frac{1}{m+1}c_I\right)+\frac{1}{2}(1-c_I)^2} = K^{**}(c_I,m)$

Constraints (ii), (iv), and (v) are binding, while all other constraints are over-satisfied. The solution reads

$$\begin{aligned} p_I^{s,worst} &= c_I,\, p_I^{l,best} = 1 - \frac{1}{2}\frac{1}{c_I}\frac{K}{m}\frac{1}{1-K}\left(1+m(1-c_I)^2\right),\, \bar{q}_I = q_I^l\left(p_I^{l,best}\right) \\ R_I^{s,worst} &= -CS_I^s(c_I),\, R_I^{l,best} = -\frac{1}{2}\frac{K}{m} + p_I^{l,best}\left(1-p_I^{l,best}\right)(1-K) \end{aligned}$$

where $K > K^*(c_I,m)$ implies that $p_I^{l,best} < c_I$, and $K \leq K^{**}(c_I,m)$ implies $p_I^{l,best} > 0$.

Case 3: $K^{**}(c_I,m) < K \leq \frac{m+1-c_I}{m+2+\frac{1}{m}-c_I} = K^{***}(c_I,m)$

Constraints (ii) and (v) are binding, while all other constraints are over-satisfied. The solution reads

$$\begin{aligned} p_I^{s,worst} &= c_I,\, p_I^{l,best} = \frac{1}{m+1}c_I,\, \bar{q}_I = q_I^l\left(p_I^{l,best}\right) \\ R_I^{s,worst} &= \frac{1}{2}\frac{1}{m+1}\frac{K}{m}c_I(2-c_I) - \frac{c_I(1-K)}{m+1}\left(1-\frac{1}{m+1}c_I\right) \\ R_I^{l,best} &= -\frac{1}{m+1}\frac{K}{2}c_I(2-c_I) \end{aligned}$$

Case 4: $K^{***}(c_I,m) < K < \frac{m}{m+1}$

This case is analogous to Case 3, i.e. constraints (ii) and (v) are binding, but now the small buyers are fairly large $q_I^s\left(p_I^{l,best}\right) \leq \frac{K}{m}$, and so the solution reads:

$$\begin{aligned} p_I^{s,worst} &= c_I,\, p_I^{l,best} = \frac{\frac{m}{K}(1-K)-1-\frac{c_I}{m}}{\frac{m}{K}(1-K)-1-\frac{1}{m}},\, \bar{q}_I = q_I^l\left(p_I^{l,best}\right) \\ R_I^{s,worst} &= \frac{1}{m+1}\left(\frac{(1-c_I)^2(1-K)\left(-\frac{1}{m}\right)\left(\frac{m}{K}(1-K)-1-\frac{1}{m}+\frac{1}{2}\frac{1-K}{K}\right)}{\left(\frac{m}{K}(1-K)-1-\frac{1}{m}\right)^2} - \frac{1}{2}\frac{K}{m}(1-c_I)^2\right) \\ R_I^{l,best} &= \frac{1}{2}\frac{1}{m+1}(1-c_I)^2\left[K - \frac{(1-K)^2}{K\left(\frac{m}{K}(1-K)-1-\frac{1}{m}\right)^2}\right] \end{aligned}$$

This problem is not convex in p_I^l; the second-order condition requires $\frac{m}{K}(1-K) - 1 - \frac{1}{m} < 0$; however, $K > K^{***}(c_I,m)$ implies that the second-order condition is satisfied and that $p_I^{l,best} > 0$.

(There is another solution where $p_I^{s,worst} = c_I$, $p_I^{l,best} = 0$, $R_I^{l,best} = -\frac{1}{2}\frac{K}{m}$, $R_I^{s,worst} = -CS_I^s(c_I)$. This offer is feasible iff $K \geq c_I / \left(\frac{1}{2}\frac{1}{m} + c_I + \frac{1}{2}(1-c_I)^2\right)$. But this solution is just a local maximum; it yields less total surplus for the large buyer than the solutions of case 3 and 4, which are the alternative solutions for the relevant range of K).

Note that in cases 2, 3, and 4, we have $p_I^{l,best} \in (0, c_I)$, i.e. I sells to the large buyer below marginal cost, which allows it to raise the quantity threshold, thereby discouraging the small buyers from buying above that threshold. However, the incumbent will incur losses which exceed the extra consumer surplus that the low price generates for the large buyer, and which have to be covered by the buyer's fixed payments. Thus, the large buyer's total net surplus must be lower than under the "first best" offer, where it is $CS_I^l(c_I) + mCS_I^s(c_I)$.

Now, if the total net surplus that E can generate, $CS_E^l(c_E) + mCS_E^s(c_E)$, is smaller than the sum of I's best offer to the large buyer and I's best offer to the small buyers, then I can always match either E's offer to the large buyer, or E's offer to the small buyers, implying that there is no entry equilibrium in pure strategies.

The corresponding "minimum efficiency" condition reads:

$$CS_E^l(c_E) + mCS_E^s(c_E) \geq CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} + m\left(CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}\right)$$

as stated in the proposition.

We argued in Lemma B7 that our game always has an equilibrium where all buyers buy from I under the appropriate coordination equilibria.

For an entry equilibrium to exist, E's offer must satisfy the following conditions:

- it matches I's best offer to both the large and the small buyers;
- it satisfies the large and small buyers' self-selection condition;
- E reaches the minimum size; and
- E breaks even.

More formally, E has to solve the following program:

$$\max_{\{p_E^s, p_E^l, R_E^s, R_E^l\}} m(p_E^s - c_E)q_E^s(p_E^s) + \left(p_E^l - c_E\right)q_E^l\left(p_E^l\right) - mR_E^s - R_E^l$$

subject to:

(i) $CS_E^l\left(p_E^l\right) + R_E^l \geq CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best}$

(ii) $CS_E^s(p_E^s) + R_E^s \geq CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}$

(iii) $CS_E^l\left(p_E^l\right) + R_E^l \geq CS^{l,net}(p_E^s, q_E^s(p_E^s)) + R_E^s$

(iv) $CS_E^s(p_E^s) + R_E^s \geq CS^{s,net}\left(p_E^l\right) + R_E^l$

(v) $q_E^l\left(p_E^l\right) + mq_E^s(p_E^s) \geq \bar{s}$

(vi) $m(p_E^s - c_E)q_E^s(p_E^s) + \left(p_E^l - c_E\right)q_E^l\left(p_E^l\right) \geq mR_E^s + R_E^l$

Case 1: Let the total surplus generated by E be larger than the sum of I's best offer to the large and small buyers:

$$CS_E^l(c_E) + mCS_E^s(c_E) \geq CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} + m\left(CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}\right)$$

and let the large buyer's self-selection condition be satisfied under the "first-best" solution (defined below):

$$CS_I^l\left(p_I^{l,best}\right)+R_I^{l,best}-\left(CS_I^s\left(p_I^{s,best}\right)+R_I^{s,best}\right)\geq CS^{l,net}\left(c_E,q_E^s\left(c_E\right)\right)-CS_E^s\left(c_E\right)$$

Then, the solution to E's problem is ("first-best" solution):

$$\begin{aligned}p_E^{s*} &= p_E^{l*}=c_E=0,\bar{q}_E=q_E^s\left(c_E\right)\\ R_E^{l*} &= CS_I^l\left(p_I^{l,best}\right)+R_I^{l,best}-CS_E^l\left(c_E\right)\\ R_E^{s*} &= CS_I^s\left(p_I^{s,best}\right)+R_I^{s,best}-CS_E^s\left(c_E\right)\end{aligned}$$

Under this solution, constraints (i) and (ii) are satisfied (they hold with equality) by construction of R_E^{l*} and R_E^{s*}. Constraint (iii) holds by assumption. Constraint (v) holds because the critical size is bounded above by 1:

$$mq_E^s\left(c_E\right)+q_E^l\left(c_E\right)=1\geq\bar{s}$$

Constraint (vi) reduces to

$$CS_E^l\left(c_E\right)+mCS_E^s\left(c_E\right)\geq CS_I^l\left(p_I^{l,best}\right)+R_I^{l,best}+m\left(CS_I^s\left(p_I^{s,best}\right)+R_I^{s,best}\right)$$

which is satisfied by assumption.

We will now argue that if constraints (i) and (ii) are satisfied with strict equality and $p_E^l=c_E=0$, then constraint (iv) is satisfied as well, i.e. the small buyers' self-selection constraint is redundant: First of all,

$$CS_E^s\left(c_E\right)+R_E^{s*}=CS_I^s\left(p_I^{s,best}\right)+R_I^{s,best}$$

by strict equality of (ii), and

$$CS_I^s\left(p_I^{s,best}\right)+R_I^{s,best}\geq CS_I^s\left(c_I\right)+R_I^{s,worst}$$

by optimality of $p_I^{s,best}\in(c_I,1)$ among all other feasible pairs (p_I^s,R_I^s), and in particular the pair $\left(p_I^{s,worst}c_I,R_I^{s,worst}\right)$, where

$$R_I^{s,worst}=\max\left\{-CS_I^s\left(c_I\right),CS^{s,net}\left(p_I^{l,best},q_I^l\left(p_I^{l,best}\right)\right)+R_I^{l,best}-CS_I^s\left(c_I\right)\right\}$$

is the fixed payment that the small buyers have to make to I under I's best offer to the large buyer. Then, by definition of $R_I^{s,worst}$, we have

$$CS_I^s\left(c_I\right)+R_I^{s,worst}\geq CS^{s,net}\left(p_I^{l,best},q_I^l\left(p_I^{l,best}\right)\right)+R_I^{l,best}$$

i.e. I's best offer to the large buyer satisfies the small buyers' self-selection condition. Finally, we need to show that

$$CS^{s,net}\left(p_I^{l,best}, q_I^l\left(p_I^{l,best}\right)\right) + R_I^{l,best} \geq CS^{s,net}\left(c_E\right) + R_E^{l*}$$

Since constraint (i) holds with equality, we have that $R_I^{l,best} - R_E^{l*} = CS_E^l\left(c_E\right) - CS_I^l\left(p_I^{l,best}\right)$, so that the above inequality can be rearranged to read

$$CS^{s,net}\left(p_I^{l,best}, q_I^l\left(p_I^{l,best}\right)\right) - CS^{s,net}\left(c_E\right) + \left(CS_E^l\left(c_E\right) - CS_I^l\left(p_I^{l,best}\right)\right) \geq 0$$

If $q_I^l\left(p_I^{l,best}\right) \geq \frac{K}{m}$, we can insert for the consumer surplus terms from equations (12) and (15) and reduce the inequality to $p_I^{l,best} \geq c_E = 0$, which is always true. If instead $q_I^l\left(p_I^{l,best}\right) < \frac{K}{m}$, inserting the appropriate terms yields

$$1 - K \geq \frac{K}{m} + (1-K)\left(1 - p_I^{l,best}\right)^2\left((1-K)\frac{K}{m} - 1\right)$$

which holds as well, because the LHS is strictly larger than $\frac{K}{m}$, while the RHS is strictly smaller than $\frac{K}{m}$. Thus, we can conclude that the small buyers' self-selection condition is implied to hold under our candidate solution:

$$CS_E^s\left(c_E\right) + R_E^{s*} \geq CS^{s,net}\left(c_E, q_E^l\left(c_E\right)\right) + R_E^{l*}$$

The candidate solution satisfies all constraints (i) to (vi), and $p_E^s = p_E^l = c_E = 0$ maximizes consumer surplus, and hence the total surplus that E can appropriate.

Finally, recall that we argued above that $CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} \leq CS_I^l(c_I) + mCS_I^s(c_I)$, and that $m\left(CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}\right) < CS_I^l(c_I) + mCS_I^s(c_I)$. Thus, the candidate entry equilibrium will arise whenever

$$CS_E^l\left(c_E\right) + mCS_E^s\left(c_E\right) \geq CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} + m\left(CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}\right)$$

Comparing this inequality to the corresponding condition under third degree price discrimination:

$$CS_E^l(c_E) + mCS_E^s(c_E) \geq 2\left(CS_I^l(c_I) + mCS_I^s(c_I)\right)$$

we see that

$$CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} + m\left(CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}\right) < 2\left(CS_I^l(c_I) + mCS_I^s(c_I)\right)$$

and so entry is feasible even for values of $c_I < 1 - \sqrt{1/2}$, where it would not have been possible under third-degree price discrimination (see Proposition B6).

(ii) **Case 2:** The "first-best" solution is not feasible because it violates the large buyer's self-sorting condition (iii). Then, constraints (i), (ii), and (iii) determine the "second-best" solution:

$$\begin{aligned}
p_E^{l*} &= c_E = 0 \\
p_E^{s*} &= 1 - \sqrt{2\frac{m}{K}\frac{1-K}{1-K-\frac{K}{m}}\left(CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} - \left(CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best}\right)\right)} \\
R_E^{s*} &= CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best} - CS_E^s\left(p_E^{s*}\right) \\
R_E^{l*} &= CS_I^l\left(p_I^{l,best}\right) + R_I^{l,best} - CS_E^l\left(c_E\right)
\end{aligned}$$

provided that constraints (v) and (vi) are satisfied under this solution. Since constraint (i) holds with strict equality, we can apply the same reasoning developed for the first-best solution to argue that the small buyers' self selection constraint (iv) is implied to be satisfied under the "second-best" solution as well.

(iii) Now, if the "second-best" solution violates either constraint (vi) or (v), i.e. the entrant cannot break even or it cannot generate enough demand ($p_E^{s*} > c_E$ means that $q_E^l\left(p_E^l\right) + mq_E^s\left(p_E^s\right) \geq \bar{s}$ can be violated for $\bar{s}$ large enough), then entry is not feasible. Note that this can even happen for values of $c_I \geq 1 - \sqrt{1/2}$, where entry would have been possible under third-degree price discrimination.

Note also that the entrant's analogous offer to the incumbent's optimal offer under the miscoordination equilibrium,

$$\begin{aligned}
p_E^{l*} &= c_E = 0 \\
p_E^{s*} &= \frac{mc_E + 1 - \frac{K}{m}\frac{1}{1-K}}{m + 1 - \frac{K}{m}\frac{1}{1-K}} > c_E = 0 \\
R_E^{s*} &= CS_I^s\left(p_I^{s,best}\right) + R_I^{s,best} - CS_E^s\left(p_E^{s*}\right) \\
R_E^{l*} &= CS^{l,net}\left(p_E^{s*}, q_E^s\left(p_E^{s*}\right)\right) + R_E^{s*} - CS_E^l\left(c_E\right)
\end{aligned}$$

is not feasible because it violates constraint (i): the large buyer's rent,

$$CS_E^l\left(c_E\right) + R_E^{l*} = CS^{l,net}\left(p_E^{s*}, q_E^s\left(p_E^{s*}\right)\right) + R_E^{s*}$$

does satisfy his self-sorting condition, but it falls short of the best offer that I can make to this buyer. Now, if E was to increase R_E^{l*} up to the point where constraint (i) is satisfied, it would no longer be optimal to charge

$$p_E^{s*} = \frac{mc_E + 1 - \frac{K}{m}\frac{1}{1-K}}{m + 1 - \frac{K}{m}\frac{1}{1-K}}$$

so that only the second-best solution from above remains.□

We see that even uniform rebate schemes will allow the incumbent to strategically redistribute rents between different types of buyers so as to prevent the more efficient entrant from serving the buyers. Thus, if parameters are such that no entry equilibrium exists even under the continuation equilibria specified in Proposition B8 (which are in favor of the entrant), then the miscoordination equilibrium characterized in Lemma B7 is the only pure-strategy equilibrium of our game. Whenever the miscoordination equilibrium is unique, we refer to it as "exclusionary equilibrium".

If instead parameters are such that the entry equilibrium exists, then our game has two pure-strategy equilibria: one where all buyers buy from the entrant, and one where they all buy from the incumbent (miscoordination equilibrium). Then, it depends on the continuation equilibria which of these two types of equilibria will be played.

Note that the entrant's first-best offer under rebates is characterized by marginal cost pricing to *both* groups of buyers; in particular, there will be no usage price distortion for the small buyers if the entrant prevails. Of course, if the entrant was a monopolist, it would charge the analogous unit prices as the incumbent does under the miscoordination equilibrium. But in the presence of the competitor, the entrant cannot replicate this scheme, because the rent left to the large buyer would fall short of the incumbent's best offer to this buyer. Once the entrant has to leave more rent to the large buyer than is necessary for the large buyer's self-selection to hold, it is no longer worthwhile to charge the small buyers above marginal cost, and so the usage price distortion will be removed.

8.3.5 Welfare Comparison: Uniform Linear Prices, Explicit, and Implicit Price Discrimination

We have shown that under uniform linear prices, the incumbent cannot prevent entry when buyers coordinate on the entrant (no matter how small the efficiency gap between I and E), while rebate schemes can indeed be designed so as to break entry even if the entrant is significantly more efficient than the incumbent. The exclusionary potential of two-part tariffs is even greater if these tariffs are allowed to depend on the type of buyer (not just the quantity they buy).

But the price regime does not only affect the likelihood of entry, it also determines the distribution of rents among firms and buyers and the size of possible efficiency losses under those equilibria that exist for a particular type of price regime.

Table 1 shows the total surplus of large and small buyers, the profits of incumbent and entrant, as well as the allocative and productive efficiency loss under each price regime, depending on whether the equilibrium has the incumbent or the entrant serve the buyers. Note that the first best allocation has E serve all buyers at marginal cost $c_E = 0$, which generates total surplus $1/2$, so that the welfare loss under any alternative allocation is the total surplus generated by the allocation under consideration minus $1/2$.

Table B1: Buyers' Surplus, Profits, and Welfare Loss

(a) Buyers' Net Total Surplus			
Price Regime	Small Buyers	Large Buyer	Total
Linear Prices			
I serves	$\frac{1}{8}\frac{K}{m}(1-c_I)^2$	$\frac{1}{8}(1-K)(1-c_I)^2$	$\frac{1}{8}(1-c_I)^2$
E serves	$\frac{K}{2m}\max\left\{(1-c_I)^2,\bar{s}^2\right\}$	$\frac{1-K}{2}\max\left\{(1-c_I)^2,\bar{s}^2\right\}$	$\frac{1}{2}\max\left\{(1-c_I)^2,\bar{s}^2\right\}$
explicit PD			
I serves	0	0	0
E serves	$\frac{1}{m}\frac{1}{2}(1-c_I)^2$	$\frac{1}{2}(1-c_I)^2$	$(1-c_I)^2$
implicit PD			
I serves	0	$\frac{K(1-p_I^s(c_I))^2}{2m}\left(1-\frac{K}{m}\frac{1}{1-K}\right)$	$\frac{K(1-p_I^s(c_I))^2}{2m}\left(1-\frac{K}{m}\frac{1}{1-K}\right)$
E serves	$<\frac{1}{m}\frac{1}{2}(1-c_I)^2$	$\leq\frac{1}{2}(1-c_I)^2$	$<(1-c_I)^2$
(b) Firms' Profits			
Price Regime	Incumbent	Entrant	Total
Linear Prices			
I serves	$\frac{1}{4}(1-c_I)^2$	0	$\frac{1}{4}(1-c_I)^2$
E serves	0	$c_I(1-c_I)$ or $(1-\bar{s})\bar{s}$	$c_I(1-c_I)$ or $(1-\bar{s})\bar{s}$
explicit PD			
I serves	$\frac{1}{2}(1-c_I)^2$	0	$\frac{1}{2}(1-c_I)^2$
E serves	0	$\frac{1}{2}-(1-c_I)^2$	$\frac{1}{2}-(1-c_I)^2$
implicit PD			
I serves	$\in\left(\frac{(1-c_I)^2}{4},\frac{(1-c_I)^2}{2}\right)$	0	$\in\left(\frac{(1-c_I)^2}{4},\frac{(1-c_I)^2}{2}\right)$
E serves	0	$>\frac{1}{2}-(1-c_I)^2$	$>\frac{1}{2}-(1-c_I)^2$
(c) Welfare Loss			
Price Regime	Allocative	Productive	Total
Linear Prices			
I serves	$-\frac{1}{2}\left(\frac{1+c_I}{2}\right)^2$	$-c_I\frac{1}{2}(1-c_I)$	$-\frac{1}{2}+\frac{1}{8}(1-c_I)^2$
E serves	$-\frac{1}{2}+\max\left\{\frac{(1-c_I)^2}{2},\frac{\bar{s}^2}{2}\right\}$	0	$-\frac{1}{2}+\max\left\{\frac{(1-c_I)^2}{2},\frac{\bar{s}^2}{2}\right\}$
3^{rd}-degree PD			
I serves	$-\frac{c_I^2}{2}$	$-c_I(1-c_I)$	$-\frac{1}{2}+\frac{1}{2}(1-c_I)^2$
E serves	0	0	0
2^{nd}-degree PD			
I serves	$-\frac{(1-K)c_I^2}{2}-\frac{K(p_I^s(c_I))^2}{2}$	$-c_I(1-K)(1-c_I)-$ $-c_IK(1-p_I^s(c_I))$	$\frac{(1-c_I)^2}{2}-\frac{1}{2}-$ $-K(p_I^s(c_I)-c_I)^2$
E serves	0	0	0

Given that the first-best outcome has the entrant serve the buyers, it is obvious that whenever the incumbent serves, the equilibrium will be inefficient, because the incumbent produces at a higher marginal cost than the entrant. The resulting efficiency losses will be smallest under third-degree price discrimination, and largest under uniform linear prices, while they are in-between under uniform rebates.[39]

Full efficiency can only arise if the entrant serves, and if the entrant can use two-part tariffs[40], where it is irrelevant if these tariffs are discriminatory or uniform. The smallest possible welfare loss when I serves (namely $-\frac{1}{2} + \frac{1}{2}(1-c_I)^2$) is exactly equal to the largest possible welfare loss when E serves. Of course, when E serves, there is no productive inefficiency, so all remaining welfare losses must be allocative.[41]

We can conclude that there is a trade-off between maximizing E's chances to enter, and minimizing welfare losses (no matter which firm eventually serves the buyers). Discriminatory two-part tariffs raise the highest barriers to E's entry, but yield the most efficient outcome (full efficiency if E serves, lowest-possible inefficiency if I serves). The opposite is true for linear tariffs (lowest entry barriers, but least efficient outcomes).

Uniform rebates are somewhere between these two extremes. Note, however, that they are sufficient to achieve full efficiency. In other words, if E is sufficiently efficient to enter under uniform rebates, then allowing discriminatory two-part tariffs will not yield any efficiency gains, but may jeopardize E's entry.[42]

On the other hand, banning two-part tariffs altogether will only make sense if the efficiency gap between I and E is small, so that even uniform rebates represent a serious barrier to E's entry. But in this case, the welfare gains that can be expected from E's entry are low anyhow. Moreover, having E serve the buyers under linear prices will yield (almost or exactly) the same surplus as having I serve the buyers under discriminatory two-part tariffs.

The conclusions are somewhat different if the welfare criterion is buyers' surplus, not social efficiency. We see that if the incumbent serves, both large and small buyers will prefer linear prices over any type of two-part tariffs. If instead the entrant serves, the small buyers will always prefer discriminatory two-part tariffs over uniform rebates, while the large buyer is indifferent between the two.

If $1-c_I > \bar{s}$, then both types of buyers strictly prefer two-part tariffs over linear prices. If instead $1-c_I < \bar{s}$, then the ranking is ambiguous. The small buyers will prefer discriminatory two-part tariffs over linear prices iff $K < (1-c_I/\bar{s})^2$, and the same holds for the large buyer iff $1-K < (1-c_I/\bar{s})^2$.

[39] Note that the efficiency losses under third-degree price discrimination also represent the lower bound on efficiency losses under any alternative equilibrium (no matter which price regime) where the incumbent cannot fully exploit the buyers because the continuation equilibria are less favorable to the incumbent (e.g. buyers will only miscoordinate on the incumbent if the latter charges at most $p_I = c_I$ to all buyers).

[40] unless $\bar{s} = 1$, in which case even linear prices would yield full efficiency (because the entrant would have to set price equal to marginal cost: $p_E = 1 - \bar{s} = 0$)

[41] Note that the efficiency losses under uniform linear prices also represent the upper bound on efficiency losses under any alternative equilibrium (no matter which price regime) where the continuation equilibria are less favorable to the entrant (e.g. buyers will only coordinate on the entrant if p_E is strictly less than $\min\{c_I, 1-\bar{s}\}$).

[42] Discriminatory two-part tariffs only make sense if one can take for granted that buyers will miscoordinate on the incumbent.