Phd Thesis

Essays on tail risk in international finance

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Abstract

The unexpected movements of financial assets' returns far from the mean, the so-called *tail risk*, has regained much attention after the Global Financial Crisis. In these essays we have been focused on the implications of such extreme events with respect to the relationship between different asset classes (commodities and exchange rates) and in terms of systemic risk. The first chapter introduces the topic and offers a review of the literature.

The second chapter, done jointly with Massimiliano Bondatti[1] studies the downside tail-risk between currencies and commodities. We use the novel MCoVaR with Elastic-Net of [Bonaccolto et al., 2021] to simultaneously account for the potential ties among a large set of commodities. We show that exchange rates are significantly exposed to downside tail-risk with respect to several commodities. Additionally, we find that exchange rates are vulnerable to tail-risk in different commodities. Lastly, while most of the currencies are significantly exposed to conditional tail-risks in the commodity markets, the results with respect to gold indicate that the Japanese yen and the Swiss franc can be considered safe haven assets.

The third chapter studies the effects of a policy provision aimed at reducing the sovereign - bank nexus. We first implement the methodology by [Frazzini et al., 2013] to reproduce banks' characteristics and risk profiles so as to reshuffle banks' balance sheet composition and estimate changes in systemic risk using the ΔCoVaR by [Adrian and Brunnermeier, 2016a]. Results show that the simulated effects of this policy are mixed: indeed it could reduce systemic risk in peripheral countries, while on the other side could have a negative impact for banks in countries like UK, Swiss and Sweden and, finally, ambiguous effects on core countries' banks.

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1 Introduction and Literature Review

Tail risk is defined as a form of portfolio risk that arises when the possibility that an investment will move more than three standard deviations from the mean is greater than what is shown by a normal distribution. Such risk, while being a well-known and intensively studied topic, has re-gained much attention after the Global Financial Crisis. Such interest has been both within the financial services industry and the academic world. As said, however, practitioners and scholars realized in the early 1960s that market returns may violate normality assumptions, with fundamental implications in terms of *asset pricing* and portfolio optimization.

Empirical evidences reported a series of stylized facts related to the returns of fianncial assets. As showed by [Cont, 2001], among the others, the data generating process associated to financial assets tend to display heavy tails. That is, the (unconditional) distribution of returns seems to display a power-law or Pareto-like tail, with a tail index which is finite, higher than two and less than five. In particular, by focusing on the kurtosis of the increments of asset prices, it has been found that it is far from the Gaussian associated value. The result could be summarized saying that "the distribution fit tends to be non-Gaussian, sharp peaked and heavy tailed, these properties being more pronounced for intraday values". [2]

The aim of this essay is to further develop evidences of the implication of fat-tailed distribution in terms of international finance and systemic risk. Before delving deeper into the structure of the two main chapters devoted to this topic, in this introduction we provide a brief review of the current state of the literature, highlighting the progressive step that have been made on the tractability of tail risk in the finance literature and on the implication of such architecture.

[2] The other facts reported by Cont are: the absence of autocorrlation; the gainasymmetry; the aggregational Gaussianity; the intermittency; the volatility clustering; the conditionally heavy tails; the slow decay in autocorrelation in absolute terms; the leverage effect; the volumecorrelation and the asymmetry in time scales.

The first work related to tail risk and its measurement dates back to 1963 [Mandelbrot, 1963] where the author challenged the usual assumption of Gaussian return distributions by applying the power law to describe the unconditional tail distributions of financial returns. Following, still in the early 1960s [Fama, 1963] provided evidence that, in certain markets, prices show large, abrupt movements that one would not expect under a model of Gaussian distributed returns. With respect to the methodology to manage such kind of distribution [Blattberg and Gonedes, 1974] proposed to use the Student distribution in order to account for the fat tails of returns.

Starting from the late 1980s, a series of articles demonstrate that for those assets that displays fat tails, the behavior of returns in the tail of the distribution are significantly different from those in the center. The first work in this sense is that of [Akgiray and Booth, 1988].

Financial economists then realized that the different behavior of returns in the tails should be treated as a source of risk for financial asset. [Sortino and Price, 1994] moved from the classic risk measures based on the assumption of normal distribution, such as standard deviation and beta, and advocated the use of downside deviation as measure of risk, the so-called *Sortino ratio*.

Despite the innovative fundamentals of this new risk measure, the fact that the Sortino ratio was only based on the lower part of the distribution, led to the fact that it has always been disregarded with respect to other measures, such as the popular Value at Risk (VaR). Indeed, as it is well known, the VaR has been treated for a long time as the most efficient risk measure. Despite VaR's conceptual simplicity, its measurement raised challenging statistical problem. Because VaR is simply a particular quantile of future portfolio values, conditional on current information, and because the distribution of portfolio returns typically changes over time, the challenge has been to find a suitable model for time-varying conditional quantiles. In their seminal work, [Engle and Manganelli, 2004] provide a formula that, at the same time allowed to compute the VaR_t as a function of variables known at time $t-1$ and a set of parameters that need to be estimated; provide a procedure to estimate the set of unknown parameters and, finally, provide a test to establish the quality of the estimate. Such risk measure, known as *conditional autoregressive VaR* (CAViaR), has been fundamental to capture the time varying tail behavior of asset returns and made use of an approach that does not model the entire distribution, but just focuses on the regression quantile, which does not entail strict assumptions.

However, researchers soon demonstrated how the Value at Risk has several significant drawbacks. For example, [Beder, 1995] showed that VaR is extremely sensitive to parameter choice and that it does not possess desirable properties of a risk measure, such as subadditivity, under certain market circumstances.

After the bankrupt of the Long Term Capital Management, where the VaR poorly performed in accounting for the effective risk borne by the fund, researchers began to examine new measures to better estimate the extreme tail. [Li, 1999] proposed a new approach to estimate VaR based on skewness and kurtosis in addition to volatility. Similarly, [Favre and Galeano, 2002] developed a new method called Modified Value at Risk in which they use a Corner-Fisher expansion in computing VaR.

In terms of empirical evidences, researchers have used Extreme Value Theory (EVT) to model tail-behavior, based only on the extreme values. In this sense, [Bali, 2003], in analysing the U.S. Treasury market, claimed that standard VaR approaches can be significantly improved by utilizing EVT, while [Marimoutou et al., 2009] found that the use of the EVT models in the energy market provides significant improvements in estimating tail risk when compared to other traditional techniques such as GARCH, historical simulation and filtered historical simulation.

As previously stated, researchers devoted much attention to the topic after the Global Financial Crisis, where some financial assets displayed returns far above the three standard deviation from the mean. In this sense, one of the most relevant work is

CoVaR (Conditional Value at Risk), provided by [Adrian and Brunnermeier, 2016a]. The risk measure they proposed, ΔCoVaR has been generally adopted as a proxy for systemic risk, that is the risk of a collapse of the entire financial system, typically triggered by the default of one, or more, large and interconnected financial institutions. In this sense, systemic risk measures the increase in tail comovement that can arise due to the spreading of distress across different financial markets. Indeed, given two variables Y and X, ΔCoVaR is defined as the contemporaneous change in Value at Risk (VaR) of Y conditional on X being at its VaR relative to its median state, and it measures the conditional tail-dependency in a non-causal sense. Hence, ΔCoVaR is a measure of risk conditional upon an adverse shock, where risk is the standard Value at Risk (VaR). From a dynamic point of view, [White et al., 2015] study a multivariate extension of CAViaR, which can be used to generate a dynamic version of CoVaR. Lastly, [Bonaccolto et al., 2021] proposed an extension of this work with respect to two dimensions. First, they allow for a multiple-regression version of CoVaR to account for additional relevant variables, and their interactions with the independent variables. Second, they deal with the curse of dimensionality using machine learning regularization techniques. The combination of a multiple-regression version of CoVaR with the elastic net enables to identify the systemic risk contributors out of a large sample of candidate factors

This essay extensively builds on the CoVaR risk measure, both the original one provided by [Adrian and Brunnermeier, 2016a] and the extended version of [Bonaccolto et al., 2021]. In the first chapter we studied the downside tail-risk relationship between currencies and commodities by using MCoVaR (Multiple Conditional Value at Risk) and, hence, simultaneously account for the potential ties among a large set of commodities. We show that exchange rates are significantly exposed to downside tail-risk with respect to several commodities. Lastly, while most of the currencies are significantly exposed to conditional tail-risks in the commodity markets, the results with respect to gold indicate that the Japanese yen and the Swiss franc can be considered safe-haven assets. These findings can be of interest to global investors, financial institutions and firms, as they can serve as a useful tool for risk management and portfolio decisions. To this end, results are relevant to correctly measure the risk and diversification of financial portfolios, as well as to implement strategies to hedge forex exposures from commodity shocks. Additionally, these findings have interesting implications for policymakers in terms of designing policies and provisions to curb the volatility of portfolio allocations of financial investors, such as controls on international trades and production of commodities, or fiscal and monetary policies, aimed at stabilizing commodity prices and, consequently, fluctuations in exchange rates. Eventually, these conditional tail-risk relationships uncovered have implications for the choice of commodity producers over the currency invoicing in international trade of commodities (i.e. local currencies vs U.S. dollar as invoicing currency), which is currently a relevant policy debate

In the second chapter we estimate the effects, in terms of systemic risk, of a policy provision aimed at reducing the sovereign - bank nexus. We first reproduce banks' characteristics and risk profiles by means of a multi-factor model. Following, we are able to artificially reshuffle banks' balance sheet composition to estimate changes in systemic risk using the ΔCoVaR. Preliminary results show that the simulated effects of this policy are mixed: indeed it could reduce systemic risk in peripheral countries, while on the other side could have a negative impact for banks in countries like UK, Swiss and Sweden and, finally, ambiguous effects on core countries' banks. These findings enlarge the recent debate with respect to the possible beneficial effect that the banking system could experience with the introduction of a European-safe asset. At the same time, financial investors could make use of the multi-factor model based on sovereign debt in order to price banks' stocks.

2 Commodity Tail Risk and Exchange Rate[3]

2.1 Introduction

In many countries, especially among emerging economies, commodities are an important driver of output and prospective inflows, and as such are potentially potentially correlated with exchange rates. Moreover, emerging countries tend to experience more macroeconomic volatility than rich countries, a fact that has negative consequences on their growth and can be traced back, among other explanations, to commodity price volatility ([Jacks et al., 2011]). Commodity prices and futures can indeed be extremely volatile at times ([Fong and See, 2001] and [Pindyck, 2004]). In this paper, we study the relation between commodities and currencies from the perspective of (downside) commodity tail-risks in exchange rates[4]. In particular, we consider the exchange rates of emerging economies as well as those of advanced countries, which we use as a control group, and analyse their exposures to tail-risks in a broad set of different individual commodities. At the same time, we account for the ties among all the commodities and their potential simultaneous effects on each exchange rate. In order to do so, we provide a novel application of the new MCoVaR (Multiple Conditional Value at Risk) with Elastic-Net of [Bonaccolto et al., 2021], which in turn is based on the popular CoVaR of [Adrian and Brunnermeier, 2016b]. Consequently, our work also addresses the interest in understanding how financial asset returns co-move. In this direction, an increasing number of papers in recent years have been documenting that returns of financial assets exhibit a stronger degree of co-movement at the lower part of their distributions. Additionally, our question directly speaks to the open puzzle of the apparent disconnection of exchange rates from economic fundamentals. While this puzzle first

[3]This chapter is written jointly with Massimiliano Bondatti, PhD candidate in Economics and Finance, Nova School of Business and Economics, R. Holanda 1, Carcavelos, Portugal.

[4]The link between exchange rates and commodities has been framed, for example, from an international trade perspective ([Kohlscheen et al., 2017]) and within sticky-price open economy models with non-traded goods, a portfolio-balance model, and the terms-of-trade hypothesis ([Chen and Rogoff, 2003] and [Chen, 2003]).

emerged from the perspective of currencies' predictability ([Meese and Rogoff, 1983]), recent studies have also been debating the difficulty of uncovering macro-economic and financial variables that co-move with currencies, with the latter appearing a fruitful avenue of research ([Lilley et al., 2020]).

We show that exchange rates are significantly exposed to downside tail-risk with respect to several commodities. Moreover, we find heterogeneous exposure across currencies, as different exchange rates are sometimes vulnerable to tail-risks in different commodities. The vulnerability of exchange rates to commodities is estimated by means of the ΔMCoVaR, which is an extension of the measure proposed by [Adrian and Brunnermeier, 2016b]. In particular, given two variables Y and X, the ΔCoVaR is defined as the contemporaneous change in Value at Risk (VaR) of Y conditional on X being at its VaR relative to its median state, and it measures the conditional tail-dependency in a non-causal sense. The Multiple CoVaR method extends this concept to the multivariate case. Hence, looking at the estimated values for the ΔMCoVaR, we see that, for example, especially agricultural commodities expose the Brazilian real to its largest downside (commodity) tail-risks. When instead energy commodities are hit by a negative shock equal to their Value-at-Risk, currencies such as the Canadian dollar, Norwegian krone and the Russian ruble experience relevant losses, while the Chilean peso and the Australian dollar exhibit their strongest vulnerabilities against copper. Interestingly, while several of the most economically significant downside tail-risk vulnerabilities seem to be related to the share of that commodity in the total export of commodities for that country (e.g. [Kohlscheen et al., 2017]), not all the estimates appear directly connected to the country's reliance on the export of commodities. Additionally, our results point out that tail-risks are not only relevant with respect to oil or gold but also statistically and economically significant with respect to several different commodities. Lastly, we find that the Swiss franc and, especially, the Japanese yen exhibit safe-haven behaviours. Interestingly, all these patterns are more clear in an MCoVaR with Elastic-Net framework than through the lens of a standard CoVaR. The MCoVaR, indeed, estimates all the commodities simultaneously, and as such it takes into account their possible ties and interactions when posing tail-risk threats to each currency. Moreover, the MCoVaR shrinks to zero less relevant vulnerabilities that might incorrectly show as non-null in the CoVaR.

Overall, the paper contributes to two main streams of the literature. First, our work contributes to the literature analysing (conditional) tail-risk within and across asset classes. Several papers use the CoVaR of [Adrian and Brunnermeier, 2016b], in different specifications, to analyse different assets and markets: sovereign bonds/CDSs ([Fong and Wong, 2012], [Reboredo and Ugolini, 2015] and [Borri, 2019b]), cryptocurrencies ([Borri, 2019a] and [Xu et al., 2021]), currencies and stocks ([Reboredo et al., 2016]), stocks and oil ([Mensi et al., 2017]), sovereign CDS spreads and oil ([Wang et al., 2020]) and Chinese and Asian stock markets ([Jin, 2018]). Recently, [Bonaccolto et al., 2021], in estimating breakup and default risks in the Eurozone, extended the CoVaR model in two simultaneous directions. Specifically, they work at the intersection between two recent methodologies: multivariate, multi-quantile regression (e.g. [White et al., 2015]) and high-dimensional networks (e.g. [Fan et al., 2018]). Their new estimator is the so-called multiple-regression-CoVaR (MCoVaR) with Elastic-Net, on which our analysis builds.

Second, our paper contributes to the literature investigating the relation between commodities and exchange rates. Starting from the seminal paper of [Chen and Rogoff, 2003], several works have analysed this link either from the perspective of time-series predictability ([Chen et al., 2010] and [Kohlscheen et al., 2017]) or of commodity-currencies cross-sectional risk premium ([Ready et al., 2017] and [Byrne et al., 2019]) or tried to reconcile both the dimensions ([Passari, 2019]). Other papers, instead, explore Granger causality (e.g. [Zhang et al., 2016]). Moreover, some existing works investigate the tail dependence between exchange rates and (only) oil or gold ([Aloui et al., 2013], [Chen et al., 2013], [Reboredo, 2013] and [Bedoui et al., 2018]). Nevertheless, by providing a novel application of the MCoVaR with Elastic-Net, we take a different stance from these studies in several dimensions. First, our focus is on measuring and analysing non-causal *conditional tail-risk*. Second, we do not limit our attention exclusively to oil and gold: rather, we assess vulnerabilities of exchange rates to tail-risk in *several* different individual commodities ($j = 1, \dots, J$). Hence, we are also able to assess whether the exchange rates of different countries are related to different commodities. Third, the MCoVaR allows us to study the tail-risk relationship between the exchange rate-i and a commodity-j while, at the same time, accounting for the potential ties among the commodity-j and all the remaining $J - 1$ commodities. The coefficients attached to the different regressors are now jointly estimated (differently, for example, from a simple CoVaR approach, which would estimate J-separate regressions). In doing so, we consider the possibility that one commodity is in a state of distress because of the effects of another (otherwise omitted) commodity; i.e. we address a potential omitted variable bias problem. The previous literature has shown that commodity prices exhibit idiosyncrasies as well as commonalities (see [Byrne et al., 2013], [Alquist et al., 2020] and [Delle Chiaie et al., 2021]).

The paper is organized as follows: in subsection 2 we describe the data. Subsection 3 explains the MCoVaR with Elastic-Net (and the CoVaR) methodology we use to derive the results presented in subsection 4. Eventually, subsection 5 concludes the paper.

2.2 Data

We collect data for exchange rates from Reuters and for commodity prices from the S&P Goldman Sachs Commodity Index (henceforth, S&P GSCI), both via Datastream. The data are daily and cover the period from 30/1/2004 to 29/06/2021. The choice of this time frame is consistent with the common practice of avoiding periods of high inflation and the introduction of the euro and allows us to have all the commodity prices from the beginning of the sample available for the joint estimation of the parameters.

We express spot exchange rates as the number of U.S. dollars per unit of foreign currency. Thus, an increase in the spot rate represents a depreciation of the U.S. dollar. Our sample covers eigth advanced economies: Australia, Canada, Japan, New Zealand, Norway, Sweden, Switzerland and United Kingdom; and eigth emerging economies: Brazil, Chile, India, Mexico, Russia, Singapore, South Africa and South Korea.

Data for commodity prices – measured in U.S. dollar per unit – refer to commodities that are traded on active, liquid future markets. We collect 19 S&P GSCI individual sub-indexes[5]: Cocoa, Coffee, Cotton, Corn, Soybeans, Sugar, Wheat, Brent, Crude Oil, Natural Gas, Gold, Silver, Platinum, Copper, Lead, Nickel, Zinc, Lean Hogs and Feeder Cattle. As standard, commodities and exchange rates returns are then computed as the change in the respective log spot prices (S_t):

$$r_{t+1} = log(S_{t+1}) - log(S_t)$$

2.2.1 Descriptive Statistics

Table 1 shows the descriptive statistics for the returns series. Average daily returns are close to zero, mainly left skewed and exhibit excess kurtosis. These characteristics of the data point at non-normality of the returns' distributions, consistent with the analysis of the previous literature. For many return series, there is a greater probability of extremely negative realizations than the one of a normal distribution. Moreover, commodity returns are overall more volatile than exchange rate returns, both in terms of standard deviations and the spreads between maximum and minimum values observed over the sample, although often with slightly lower (in absolute terms) values for

[5]Belonging to Agriculture (Cocoa, Coffee, Cotton, Corn, Soybeans, Sugar, Wheat), Energy (Brent, Crude Oil, Natural Gas), Precious Metals (Gold, Silver, Platinum), Industrial Metals (Copper, Lead, Nickel, Zinc) and Livestock (Lean Hogs, Feeder Cattle).

skeweness and kurtosis. Lastly, unconditional Value at Risk at the $\tau = 0.01$ quantile (i.e. 0.99 confidence level) for exchange rate returns ranges from -0.91% to, at most, -2.96%; for commodities the range is from -3.01% to -7.52%.

Table 1: Descriptive Statistics

Country	$Mean\%$	$Std\%$	Skw	Krt	$Max\%$	$Min\%$	VaR 1%	Acf
				Exchange Rates				
Australia	-0.00	0.82	-0.78	16.91	6.94	-8.83	-2.20	-0.04
Brazil	-0.01	1.00	-0.33	10.04	7.45	-8.12	-2.77	-0.03
Chile	-0.01	0.69	-0.39	8.09	4.92	-5.46	-1.86	0.04
Canada	0.00	0.60	0.08	8.06	5.05	-4.34	-1.57	-0.01
India	-0.01	0.42	-0.44	9.80	3.06	-3.97	-1.27	0.02
Japan	-0.00	0.63	0.28	7.61	4.61	-3.71	-1.67	-0.02
Mexico	-0.01	0.75	-0.71	12.85	5.33	-7.55	-2.17	0.02
New Zealand	0.00	0.85	-0.40	9.00	5.88	-6.65	-2.35	-0.01
Norway	-0.00	0.80	-0.20	8.35	6.46	-6.05	-2.28	-0.00
Russia	-0.02	0.88	-0.52	50.67	15.52	-14.27	-2.53	0.03
Singapore	0.01	0.34	-0.00	7.82	2.34	-2.14	-0.91	-0.03
South Africa	-0.02	1.06	-0.37	6.78	6.39	-9.81	-2.96	0.02
South Korea	0.00	0.69	0.79	56.23	13.26	-10.35	-1.88	-0.01
Sweden	-0.00	0.75	0.08	7.06	5.55	-5.09	-1.97	-0.02
Switzerland	0.01	0.66	0.89	35.77	11.42	-9.00	-1.52	0.02
United Kingdom	-0.01	0.63	-0.74	15.14	4.47	-8.31	-1.59	0.03
				Commodities				
Cocoa	0.01	1.77	-0.17	5.23	8.99	-9.78	-4.74	-0.02
Coffee	0.02	1.97	0.11	4.72	12.06	-11.25	-4.97	0.05
Cotton	0.00	1.66	-0.16	4.48	6.94	-7.58	-4.65	0.04
Corn	0.02	1.76	0.01	5.16	9.17	-8.12	-4.83	0.02
Soybeans	0.01	1.51	-0.23	5.43	6.78	-7.34	-4.58	-0.01
Sugar	0.03	1.99	-0.45	8.24	8.56	-22.46	-5.34	0.01
Wheat	0.01	1.83	0.11	4.44	8.10	-8.99	-4.77	0.01
Brent	0.02	2.23	-0.54	15.05	19.08	-26.83	-6.35	-0.06
Crude Oil	0.02	2.73	-1.61	72.44	43.79	-56.86	-6.82	-0.04
Natural Gas	-0.01	2.90	0.18	5.47	17.13	-19.18	-7.52	-0.02
Copper	0.03	1.73	-0.13	7.36	11.90	-10.38	-4.76	-0.08
Lead	0.03	2.07	-0.17	6.33	12.84	-13.03	-5.79	-0.04
Nickel	0.00	2.28	-0.13	6.51	13.16	-18.22	-6.26	0.00
Zinc	0.02	1.94	-0.15	5.44	9.93	-11.13	-5.15	-0.02
Feeder Cattle	0.01	1.07	-0.07	5.27	7.48	-5.87	-3.01	0.1
Lean Hogs	0.01	1.84	-0.02	5.08	9.81	-12.53	-4.49	0.12
Gold	0.03	1.15	-0.37	8.44	8.59	-9.81	-3.29	0.01
Silver	0.03	2.12	-0.91	9.80	12.47	-19.49	-7.11	-0.02
Platinum	0.01	1.54	-0.52	8.20	11.19	-12.22	-4.54	0.04

Notes: This table reports mean, standard deviation, kurtosis, skeweness, maximum value, minimum value, value-at-risk (VaR) and (lag-1) auto-correlation for the log daily returns on exchange rates (top panel) and on commodities (bottom panel). Mean, standard deviation, maximum, minimum and VaR are in percentages. For the Value at Risk, the confidence level is $\tau = 1\%$. Data are daily, for the period 01/2004 to 06/2021, and retrieved from Datastream.

2.3 Methodology

[Bonaccolto et al., 2021] propose a multiple-regression CoVaR (MCoVaR) with shrinkage estimator that extends the popular CoVaR measure of systemic risk of [Adrian and Brunnermeier, 2016b]. This new model, by *jointly* (instead of separately) estimating all the regressors, captures their potential simultaneous effects and accounts for all of their ties. Hence, the MCoVaR limits the potential omitted variable bias problem that arises when estimating *non-causal* conditional tail-risk through models, such as the CoVaR, that estimate the covariates (here, the commodities) *independently* from each other.

This section follows Section 3 of [Bonaccolto et al., 2021], to which we refer the interested reader for further details.

2.3.1 CoVaR

We follow [Adrian and Brunnermeier, 2016b] in estimating CoVaR with quantile regressions (introduced by [Koenker and Bassett Jr, 1978]). In particular, to estimate conditional risk, we run the following quantile regression[6]:

$$Q_\theta(y_t^i) = \delta_\theta + \lambda_\theta x_t^j + \gamma_\theta M'_{t-1} \tag{1}$$

In Equation (1), y_t^i represents the returns on exchange rate-i, and x_t^j denotes the residuals of an OLS regression of the returns on commodity-j (r^j) on a basket of equally weighted currency returns excluding currency-i ($base_t^i$):

$$r_t^j = \alpha^{j,i} + \psi^{j,i} base_t^i + x_t^j \tag{2}$$

with

$$base_t^i = \frac{1}{I-1} \sum_{n=1, n \neq i}^{I} y_t^n$$

[6]For a more comprehensive discussion relative to quantile regression, look at Section 2.6.1

Using the residuals from Equation (2), we consider the orthogonalised commodity returns with respect to all the other currencies. We do that to exclude the possibility of capturing the interaction between the currency component of the commodity returns on the right-hand side (commodity prices are expressed in U.S. dollars) and the U.S. dollar component of the bilateral exchange rate in the left-hand side of the regression. Hence, to avoid this problem, we use as our right-hand side variable in (1) the residuals (x^j) of the regression in (2), which contain only the part of the commodity returns that is not related to their USD-component ($base^i$), rather than using the plain commodity returns themselves. Lastly, M_{t-1} are the lagged returns on a set (a $1 \times K$ vector) of common factors employed as conditioning variables. Specifically, we employ controls that are standard in the CoVaR literature (see [Borri, 2019a]): the returns on the S&P500, on the CBOE VIX volatility index and on a U.S. corporate bond index. Overall, this set of lagged state variables captures time variations (in the conditional quantiles) in exchange rate returns that are not directly related to shocks in commodity returns.

$CoVaR^{y_t^i|x_t^j}$ is then defined as the Value at Risk (VaR) of currency-i conditional upon commodity-j being in a state of distress, and it is obtained as the fitted values of the quantile regression in (1) *when* commodity-j is at its VaR ($\hat{q}_\tau(x_t^j)$):

$$CoVaR_{t,\theta,\tau}^{y_t^i|x_t^j=\hat{q}_\tau(x_t^j)} = \hat{\delta}_\theta + \hat{\lambda}_\theta \hat{q}_\tau(x_t^j) + \hat{\gamma}_\theta M'_{t-1} \tag{3}$$

The vulnerability of currency-i to tail-risk in commodity-j is then measured with the ΔCoVaR, i.e. as the difference between the CoVaR of exchange rate-i conditional on a state of distress in commodity-j and conditional on its median state:

$$\Delta CoVaR_{\theta,\tau}^{y^i|x^j} = \hat{\lambda}_\theta [\hat{q}_\tau(x_t^j) - \hat{q}_{\frac{1}{2}}(x_t^j)] \tag{4}$$

So, ΔCoVaR quantifies the marginal impact of commodity-j (x_t^j) on the VaR of the exchange rate-i (y_t^i), i.e. when x_t^j moves from its normal state to its state of distress (its VaR).

2.3.2 MCoVaR with Elastic-Net

The MCoVaR with Elastic-Net of [Bonaccolto et al., 2021] consists in a multiple-regression CoVaR. In our context, the underlying idea is to extend the baseline CoVaR to be able to capture the potential effects of other $(J-1)$ commodities, and their ties, when estimating the relationship between one commodity-j $(j \in J)$ and one exchange rate-i in their quantile. This procedure also limits the potential omitted variable bias problem arising when estimating the *non-causal* tail-risk relationships but omitting the other $J-1$ potentially relevant commodities in the regression (as it would be the case in the CoVaR). Hence, we start estimating a specification similar to (1):

$$Q_\theta(y_t^i) = \delta_\theta + \lambda_\theta X'_{t,J} + \psi_\theta y_{t-1}^i + \gamma_\theta M'_{t-1} \tag{5}$$

where, now, $X_{t,J}$ is a $1\times J$ vector $(X_{t,J} = [x_t^1 \ x_t^2 \ \ldots x_t^J])$, which contains all the commodities-returns (again estimated as in (2)); and $\lambda_\theta = [\lambda_\theta^1 \ \lambda_\theta^2 \ \ldots \lambda_\theta^J]$ are the corresponding parameters that quantify the impact of a (downside) shock to commodities on exchange rate-i. The lagged values of the dependent variable, instead, address the possible problem of serial correlation (not captured in CoVaR). Additionally, the quantile regression in equation (5) is simultaneously extended to account for the Elastic-Net estimation functional:

$$L(\delta_\theta, \beta_\theta) = \frac{1}{T-1}\sum_{t=2}^{T} \rho_\theta(y_t^i - \delta_\theta - \beta_\theta Z_t) + \nu\left[\alpha\|\beta_\theta\|_1 + \frac{1-\alpha}{2}\|\beta_\theta\|_2^2\right] \tag{6}$$

where Z_t is the vector containing the regressors in (5), $\beta_\theta = [\lambda_\theta \ \psi_\theta \ \gamma_\theta]$ and

$$\|\beta_\theta\|_p = \left[\sum_{j=1}^{J} |\lambda_{\theta,j}|^p + |\psi_\theta|^p + \sum_{k=1}^{K} |\gamma_{\theta,j}|^p\right]^{\frac{1}{p}}$$

for $p \in \{1, 2\}$. The parameters ν>0 and $0 \leq \alpha \leq 1$, respectively, control the magnitude of the penalization and assign the weights to the penalty functions $\|\beta_\theta\|_1$ and $\|\beta_\theta\|_2^2$. The optimal values of ν and α are jointly estimated through a 10-fold cross-validation[7] (see, among others, [Gross and Siklos, 2020])[8] The goal of the machine learning extension (6) of the quantile regression in (5) is twofold. First, the quantile regression has now to deal with a potentially high number of regressors (1+J+K parameters to be simultaneously estimated), and an high correlation among the covariates raises the problem of estimation errors. Second, it addresses the issue that it is not known ex-ante which regressors are relevant to explain the dependent variable. Hence, the inclusion of the Elastic-Net of [Zou and Hastie, 2005] makes it possible to balance the bias and variance[9] of the estimates and to perform variable selection thanks to the l_1 component. Practically, it can be hard to know ex ante which regressors are the most relevant to explain the dependent variable, especially in high-dimensional and noisy variables environments. So, the Elastic-Net in MCoVaR provides a data-driven methodology to select the most relevant regressors from a large set of candidates. As [Bonaccolto et al., 2021] explain, since now only the parameters attached to the regressors that have a high effect on the VaR of the dependent variable take non-zero values, their methodology improves the baseline CoVaR, which always returns non-zero ΔCoVaRs. Hence, the use of the Elastic Net as regularization techniques is due to the ability of the EN to combine the properties of the Least Absolute Shrinkage and Selection Operator (LASSO) and Ridge Regression, that is to select variables and reduce over-fitting, respectively.

Then, in parallel to eq. (3), the MCoVaR of the exchange rate-i conditional on, respectively, the state of distress and the median state of commodity-j is now computed as follows:

[7] For a more comprehensive discussion over cross-validation techniques see section 2.6.3

[8] The function in (6) is minimized through the R package "hqreg" of [Yi and Huang, 2017], which recurs to a Semi-Smooth Newton Coordinate Descent algorithm.

[9] For a theoretical discussion on the bias variance look at section 2.6.2

$$MCoVaR_{t,\theta,\tau}^{y_t^i|x_t^j=\hat{q}_\tau(x_t^j)} = \hat{\delta}_\theta + \hat{\lambda}_{j,\theta}\hat{q_\tau}(x_t^j) + \sum_{l=1,l\neq j}^{J} \hat{\lambda}_{l,\theta}x_t^l + \hat{\psi}_\theta y_{t-1}^i + \hat{\gamma}_\theta M_{t-1}^{'} \tag{7}$$

$$MCoVaR_{t,\theta,\frac{1}{2}}^{y_t^i|x_t^j=\hat{q}_{\frac{1}{2}}(x_t^j)} = \hat{\delta}_\theta + \hat{\lambda}_{j,\theta}\hat{q_{\frac{1}{2}}}(x_t^j) + \sum_{l=1,l\neq j}^{J} \hat{\lambda}_{l,\theta}x_t^l + \hat{\psi}_\theta y_{t-1}^i + \hat{\gamma}_\theta M_{t-1}^{'} \tag{8}$$

ΔMCoVaR is then calculated, similarly to before, by subtracting (8) from (7):

$$\Delta MCoVaR_\theta^{y^i|x^j} = \hat{\lambda}_{j,\theta}[\hat{q_\tau}(x_t^j) - \hat{q}_{\frac{1}{2}}(x_t^j)] \tag{9}$$

where, however, now the estimated coefficients $\hat{\lambda}_\theta$ are all simultaneously estimated in (5), and as such, $\hat{\lambda}_{j,\theta}$ is affected by the possible interactions with all the other commodities.

Lastly, standard errors are computed by the "wild-bootstrap[10]" method ([Wang et al., 2018]).This choice is justified since this approach outperforms other resampling techniques when estimating quantile regression models including ([Wang et al., 2018]), or not including ([Feng et al., 2011a]), penalty functions.

[10]For a more comprehensive discussion on this topic, see Section 2.6.4

2.4 Results

This section discusses the estimation results presented in Table 2. We select the quantile to be $\theta = \tau = 1\%$. The results can be summarized as follows.

First, we find that the exchange rates are vulnerable to tail-risks in the commodity markets. As we can see also from the heatmap in Figure 1, around 34% of all the estimated exposures are statistically significant and several of them are also economically meaningful[11]. This highlights that currencies tend to experience relevant fluctuations on a day when commodities are hit by a shock equal to their VaR. Since the estimated ΔMCoVaR values tend to be negative, exchange rates tend to depreciate against the U.S. dollar when commodities are in a state of distress.

Second, these downside tail-risk relationships are heterogeneous across currencies. Different exchange rates are vulnerable to tail-risks with respect to different commodities. Several of the results point at a relation between currencies and commodities in terms of the relevance of that commodity with respect to the overall commodity exports for that country[12]. However, interestingly, we find that this commodity–currency link is not necessarily subject to the dependence of a country on the export of a commodity[13]. We summarize our results by describing in details the findings for one emerging economy commonly identified as a commodity-currency associated to oil, namely the Russian

[11] [Ranaldo and Söderlind, 2010], for example, show that on 2% of the days the returns to the stock market (respectively, treasury notes) are associated with at least a 0.26% (respectively, 0.19%) return on the Swiss franc against the U.S. dollar, and denote these effects as economically significant. Our results indicate that states of distress in individual commodities are associated to fluctuations in foreign currencies (mainly depreciations against the U.S. dollar) often comparable, in terms of magnitudes, to their definition of economically significant forex fluctuations.

[12] Table 6 in the Appendix reports, in the spirit of [Kohlscheen et al., 2017], the share of a commodity export revenues in total commodity export revenues for each country.

[13] One potential economic reason underlying this finding is that global factors drive some of these relations between exchange rates and tail-risk in commodities. However, an exhaustive explanation in this direction would require a theoretical macro-finance model which is beyond the scope of this paper and, as such, we leave this interesting discussion open for future research. Partly related, in a recent work [Ayres et al., 2020] develop a general equilibrium model of trade in primary commodities with productivity shocks and shocks to the supply of commodities, and show a comovement between real exchange rates of three developed economies (against the U.S. dollar) and four commodity prices due to common factors.

Table 2: ΔMCoVaR with Elastic-Net

Country	Coc$^{\|}$	Cof$^{\|}$	Cot$^{\|}$	Cor$^{\|}$	Soy$^{\|}$	Sug$^{\|}$	Wht$^{\|}$	Bre$^{'}$	CrO$^{'}$	NaG$^{'}$	Cop§	Led§	Nic§	Znc§	FeC$^{\#}$	LeH$^{\#}$	Gld†	Slv†	Pla†
Emerging Countries																			
Brazil	-0.08	-0.13***	-0.16***	-0.13***	-0.17***	-0.22***	-0.03	-0.13***	-0.00	-	-0.15***	-	-	-	-	-	0.07**	-	-0.10***
Chile	-	-	0.02	-0.00	-0.02	-0.01	-0.19***	-	-	-0.14**	-0.32***	-	0.08**	-	-0.16***	-	0.29***	-0.20***	-
India	-	-	-0.01	-	-	-	-	-	-	-	-0.30***	-	-	-	-	-	0.00	-	-
Mexico	-0.04	-	-0.04	-0.25**	-	-	-	-0.18***	-	-	-0.24**	-	-	0.17*	-0.25***	-0.00	0.76***	-	-0.72***
Russia	-	-	-	-	-	-	-	-0.26*	-0.45***	-	-	-	-	-	-0.35***	-	-	-	-
Singapore	-	-	-	-0.04**	-	-	-0.06***	-0.03**	-0.01	-0.05**	-	-	-	0.04***	-0.05**	-0.02*	-	-	-
South Africa	-0.05	-0.05	-	-	-	-	-0.07	-0.07*	-0.06*	-	-0.17***	-	-0.05	-	-0.01	-	0.03	-	-
South Korea	-0.03	-	0.01	-	-	-	-	-0.29***	-	-	-0.21**	-0.02	-0.00	-	-0.25***	-	0.43***	-	-0.21***
Developed Countries																			
Australia	-0.08	0.05	-	-0.25**	-	-	-	-	-	-0.19***	-0.72***	-	0.09***	0.24***	-0.04	-	-	-0.23***	0.07***
Canada	-0.10**	0.07	-	-0.46***	0.13**	-	-	-	-0.20***	-0.14***	-0.25***	-0.06*	-	0.43***	-0.04	0.19***	-	-0.07*	-
Japan	-	-0.08*	0.10**	0.00	0.21***	0.25***	-0.12**	-	-	0.14**	0.03	0.18***	0.11*	-	-	-	-0.45***	-	-0.11**
New Zealand	-	-	-	-0.06***	-	-	-	-	-	-0.30***	-0.05*	-	-	-	-0.13**	-	-	-0.12*	-
Norway	-0.15**	0.06*	-	-	-0.30***	-	-	-0.24***	-	-0.35***	-	-	0.01	0.05	-0.21**	-0.04	-	-0.07	-
Sweden	-	0.05**	-	-0.13***	-0.09*	-	-0.01	-0.00	-	-0.12***	-	-	0.03	0.09***	-0.14***	-	0.09**	-	-
Switzerland	-0.11***	-	-	0.00	-	0.10***	-	0.11***	0.08***	-0.10***	0.03	0.02	0.01	0.07***	0.07**	-0.01	-0.10***	-0.10***	0.02
United Kingdom	-0.17***	-	-	-0.11***	-0.00	-	-	-0.14***	-0.05**	-0.06**	-	-	0.02	0.21***	-0.07***	-	0.10***	0.00	-

Notes: This table reports the estimated values for the ΔMCoVaR with Elastic-Net in (9), with confidence level τ=θ=1%. In all the estimates, the left-hand side variables of the regressions in (5) (i.e. the exchange rate-i) are on the rows of the table, and the right-hand conditioning variable (i.e. commodity-j) on the columns of the table. The other independent variables in each regressions in (5) are the remaining j = $1, \dots, J-1$ commodities, the $t-1$ values of the dependent variable, and the set of common factors which include: the returns on the S&P500, on the CBOE VIX volatility index and on the US Corporate Bond Total Return Index. We divide exchange rates of developed and emerging countries in two different panels. Standard errors are computed by wild-bootstrap as in [Bonaccolto et al., 2021]. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level. Statistical significance refers to the coefficients $\hat{\lambda}_{j,\theta}$ from equation (9). We report with - values that the MCoVaR with Elastic-Net method shrinks below $10^{-3}. Data are daily, for the period 01/2004-06/2021, and obtained from Datastream. The commodities are: Cocoa(Coc), Coffee(Cof), Cotton(Cot), Corn(Cor), Soybeans(Soy), Sugar(Sug), Wheat(Wht), Brent(Bre), CrudeOil(CrO), NaturalGas(NaG), Copper(Cop), Lead(Led), Nickel(Nic), Zinc(Znc), FeederCattle(FeC), LeanHogs(LeH), Gold(Gld), Silver(Slv), Platinum$ refers to commodities belonging to the agricultural-category, $'$ to the energy-category, † to the precious metals-category, § to the industrial metals-category and $^{\#}$ to livestock-category.

Table 3: ΔCoVaR

Country	Coc$^{\|}$	Cof$^{\|}$	Cot$^{\|}$	Cor$^{\|}$	Soy$^{\|}$	Sug$^{\|}$	Wht$^{\|}$	Bre$^{\prime}$	CrO$^{\prime}$	NaG$^{\prime}$	Cop§	Led§	Nic§	Znc§	FeC$^{\#}$	LeH$^{\#}$	Gld†	Slv†	Pla†
Emerging Countries																			
Brazil	-0.73***	-0.84**	-0.90***	-0.80	-1.08***	-1.29***	-0.41	-0.78**	-0.53	0.91**	-0.82**	-0.43**	-0.47*	-0.47	-0.31	0.47	-0.13	-1.41***	-0.45**
Chile	0.71***	-0.12	-0.11	-0.47***	-0.58***	-0.41***	0.18	-0.43***	-0.24	-0.41***	-0.81***	-0.36*	-0.21	-0.44	-0.27***	-0.20	0.18	-0.49***	-0.13
India	0.12	-0.03	-0.34***	-0.02	-0.08	-0.08	-0.01	0.34***	-0.04	0.18	-0.42***	-0.30***	-0.23*	-0.29***	-0.05	0.11	0.07	-0.01	0.03
Mexico	-0.01	-0.09	-0.53***	-0.57***	-0.56***	-0.25	-0.28	-0.47***	-0.42**	0.62**	-0.44***	-0.04	-0.28*	0.05	-0.65***	-0.33**	0.38**	-0.08	-0.47***
Russia	0.27	-0.56**	-0.61*	-0.84***	-0.13	-0.39	-0.77**	-1.09***	-0.63***	-0.46*	0.09	0.46	0.44**	0.43	-0.59**	-0.03	-0.17	-0.27	-0.32
Singapore	-0.09*	0.81***	-0.17***	-0.21***	-0.10**	0.75***	-0.37***	-0.12***	-0.10**	-0.17***	-0.10**	-0.05	-0.07	0.03**	0.83***	-0.10*	-0.00	-0.01	-0.01
South Africa	0.60**	-0.57***	-0.18*	0.21**	-0.26	1.14***	-0.19	-0.38**	-0.40***	-0.47***	-0.38***	-0.29*	-0.37*	-0.22***	-0.29**	0.76***	0.04	-0.24	-0.33***
South Korea	0.13	-0.34***	0.06	-0.22	-0.57***	-0.14	0.71***	0.11	-0.35***	0.41*	-1.30***	-0.45***	-0.46***	-0.01	-0.63***	-0.59***	0.24	0.04	-0.44***
Developed Countries																			
Australia	-0.45**	-0.33**	-0.48***	-0.64**	-0.39***	-0.29	0.03	-0.35***	-0.40***	-0.47***	-0.62**	-0.41***	-0.30*	-0.29**	-0.42**	-0.12	-0.01	-0.28	-0.08
Canada	-0.15	0.16***	-0.45***	-0.48***	-0.33***	-0.15	-0.03	-0.38**	-0.25***	-0.20	-0.29***	-0.09	-0.06	-0.01	-0.19	-0.00	-0.12	-0.29	-0.02
Japan	-0.16	-0.15	0.20**	0.10	0.37***	0.46***	0.10**	0.38	0.21	0.39***	0.32***	0.53**	0.48***	0.26***	0.17	-0.01	-0.63***	-0.62***	-0.53*
New Zealand	-0.05	-0.19	-0.14	-0.76***	-0.35**	-0.33	0.60***	-0.34***	-1.33***	-0.59***	-0.37***	-0.31***	0.01	0.30*	-0.03	-0.76***	-0.07	-0.53***	0.06
Norway	-0.43***	-0.65***	-0.28**	-0.52***	-0.57***	-0.00	-0.17	-0.54	-0.40***	-0.06	-0.39	-0.14***	0.00	0.71*	-0.72***	-0.41***	-0.14	-0.33***	-0.23**
Sweden	-0.14**	0.52***	-0.08	-0.42***	-0.34*	-1.31***	-0.26	-0.08	-1.03***	0.39***	0.02	0.05	0.58***	0.43***	-0.60***	-0.19	0.21***	0.01	0.02
Switzerland	-0.22**	0.01	0.04	0.06	0.06	0.25*	0.74**	0.38***	0.17**	-0.07	0.29**	0.18***	0.11	0.21	0.40*	-0.27***	-0.18***	-0.21***	-0.10*
United Kingdom	-0.41***	-0.03	-0.15*	-0.28***	-0.36***	-0.17**	-0.04	-0.34***	0.51**	-0.32***	0.01	0.39***	0.08**	0.23***	-0.32***	-0.05	0.11	-0.03	-0.15***

Notes: This table reports the estimated values for the ΔCoVaR in (4), with confidence level τ=θ=1%. In all the estimates, the left-hand side variables of the regressions in (1) (i.e. the exchange rate-i) are on the rows of the table, and the right-hand conditioning variable (i.e. commodity-j) on the columns of the table. The other independent variables in each regressions in (1) are the set of common factors which include: the returns on the S&P500, on the CBOE VIX volatility index and on the US Corporate Bond Total Return Index. We divide exchange rates of developed and emerging countries in two different panels. Standard errors are computed by wild-bootstrap as in [Bonaccolto et al., 2021]. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level. Statistical significance refers to the coefficients $\hat{\lambda}_\theta$ from equation (3). Data are daily, for the period 01/2004-06/2021, and obtained from Datastream. The commodities are: Cocoa (Coc), Coffee (Cof), Cotton (Cot), Corn (Cor), Soybeans (Soy), Sugar (Sug), Wheat (Wht), Brent (Bre), Crude Oil (CrO), Natural Gas (NaG), Copper (Cop), Lead (Led), Nickel (Nic), Zinc (Znc), Feeder Cattle (FeC), Lean Hogs (LeH), Gold (Gld), Silver (Slv), Platinum (Pla). $^{\|}$ refers to commodities belonging to the agricultural-category, $^{\prime}$ to the energy-category, † to the precious metals-category, § to the industrial metals-category and $^{\#}$ to livestock-category.

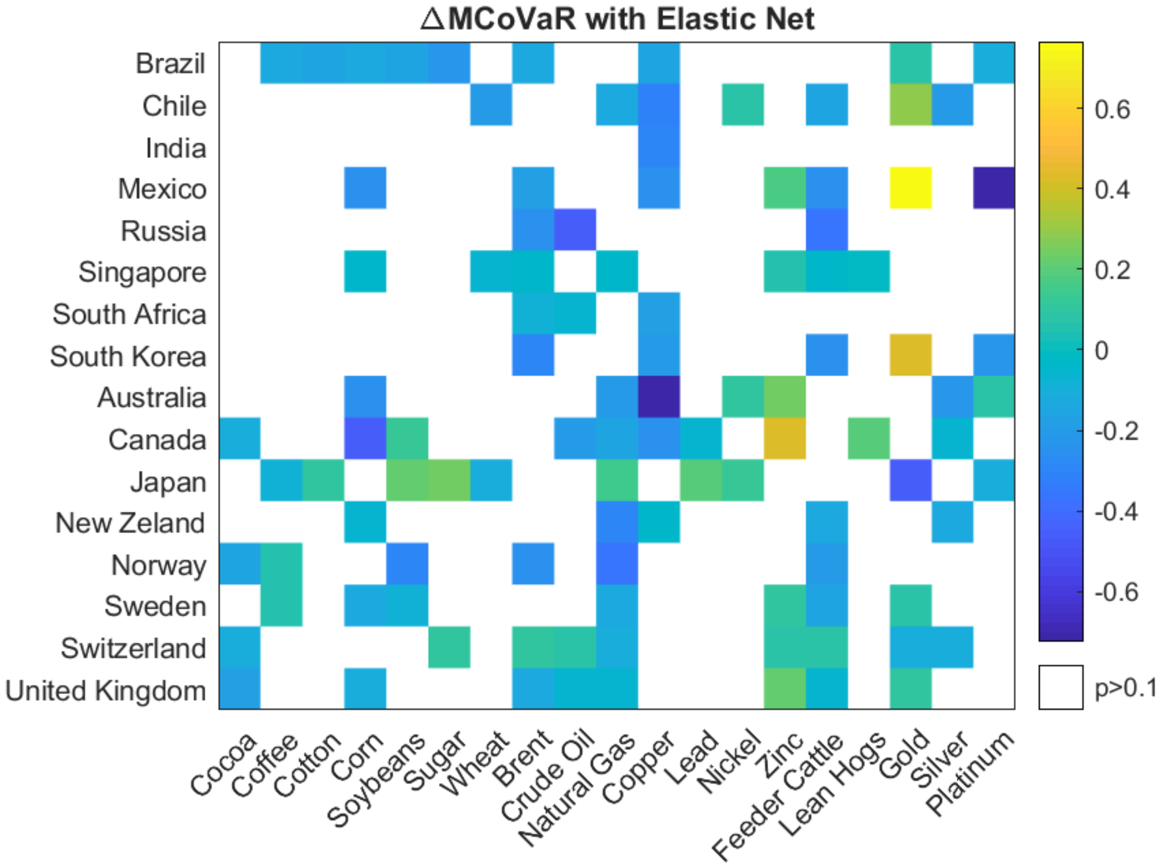


Figure 1: Heatmap: ΔMCoVaR with Elastic-Net
This figure shows the heatmap for the results of the estimated values for the ΔMCoVaR with Elastic-Net reported in Table 2. In white, we display values that are either not selected by the ΔMCoVaR with Elastic-Net or statistically not significant at the 10% level. In parallel to Table 2, we report in the top half of the figure the exchange rates belonging to emerging economies, and in the bottom half the exchange rates belonging to developed economies.

ruble. We find that the ruble has strong estimated vulnerabilities against brent and crude oil, with ΔMCoVaRs equal to -0.26% and -0.45%. This means that on a day when, for example, crude oil is in a state of distress, the ruble experiences an additional -0.45% drop with respect to the case where the commodity is in its median state. Nevertheless, the ruble exhibits an economically significant vulnerability against feeder cattle as well.

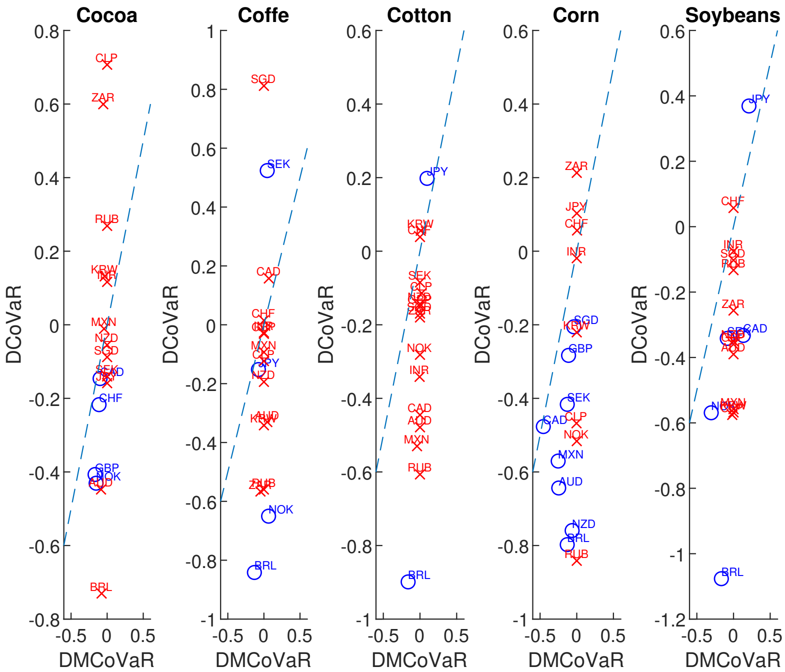
Third, while our paper studies the vulnerabilities to *conditional tail-risk* in a *broad* set of individual commodities (which are also *jointly* estimated), the previous literature on tail-dependence focused on the relationship between exchange rates and only oil or gold. With respect to these two commodities, the negative signs of our oil estimates are consistent with [Aloui et al., 2013] and [Bedoui et al., 2018]. However, the significant tail-risk vulnerabilities to oil we find for the Canadian dollar and the British pound are different from the results in [Aloui et al., 2013], who find little tail dependence for these currencies. Thus, together with the significant estimates we get also for the Norwegian krone and Russian ruble, our results seem partly at odds with their claim that higher oil reserves and production positions of a country reduce its exchange rates' tail dependence on oil price fluctuations[14]. Additionally, the vulnerabilities to tail-risk in gold present several positive values, except for the Swiss franc and the Japanese yen, which are negative. Overall, we believe our results for gold can be interpreted as follows. Gold returns tend to be high in periods of distress and low in good times. At the same time, the U.S. dollar has flight-to-safety properties ([Lilley et al., 2020]); meanwhile, the Swiss franc and the Japanese yen are the two leading safe-haven currencies ([Ranaldo and Söderlind, 2010]). Hence, when gold is in a state of distress (good times) the U.S. dollar depreciates against all exchange rates but the Swiss franc and the Japanese yen. Gold and, in this order, the Japanese yen, Swiss franc and U.S. dollar behave as safe assets for international investors.

Finally, in Figure 2 we compare the results between the MCoVaR and the basic CoVaR methodology (reported in Table 3). Overall, estimating the *conditional* tail-risk relationships with the CoVaR would have still allowed us to draw one of our main conclusions, namely that exchange rates are vulnerable to tail-risk in several commodities. However, as we see in the graph, downside tail-risks measured by the ΔMCoVaR tend to be often different from the ones measured by the ΔCoVaR[15]. Hence, if we overlooked a potential omitted variable bias problem, we would have erroneously attributed to exchange rates several vulnerabilities which are actually not significant.

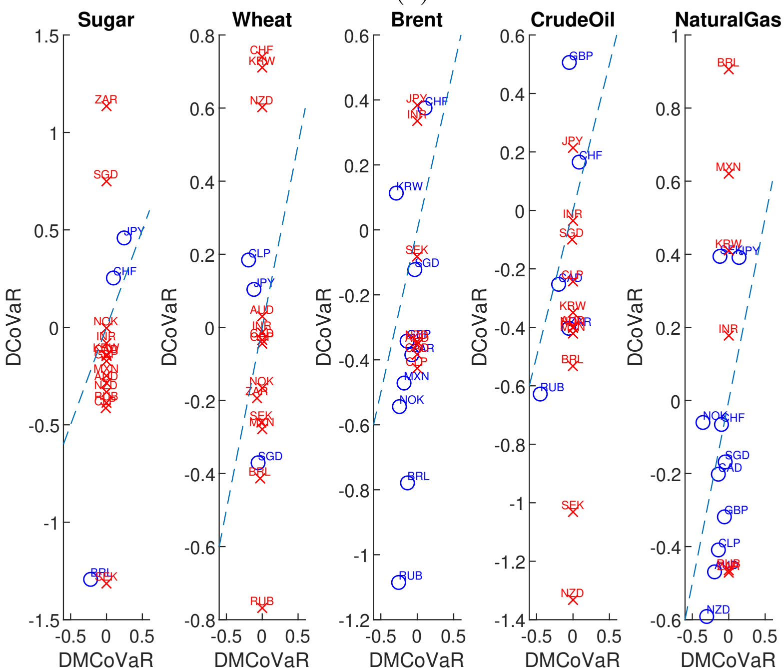
[14] This interpretation appears potentially in contrast with the results we find also across other types of commodities and with the (partial) connection of these relationships with the export channel.

[15] Similarly, as we show in Figure 3 in the Appendix, for exchange rates *conditional* tail-risk values (ΔMCoVaRs) are significantly different from uncondtional risk values (VaR).

Additionally, in a few circumstances, considering the ties among the commodities on the right-hand side of the quantile regression makes some vulnerabilities emerge or reinforce. Eventually, the magnitudes arising from the MCoVaR estimation are in general smaller.



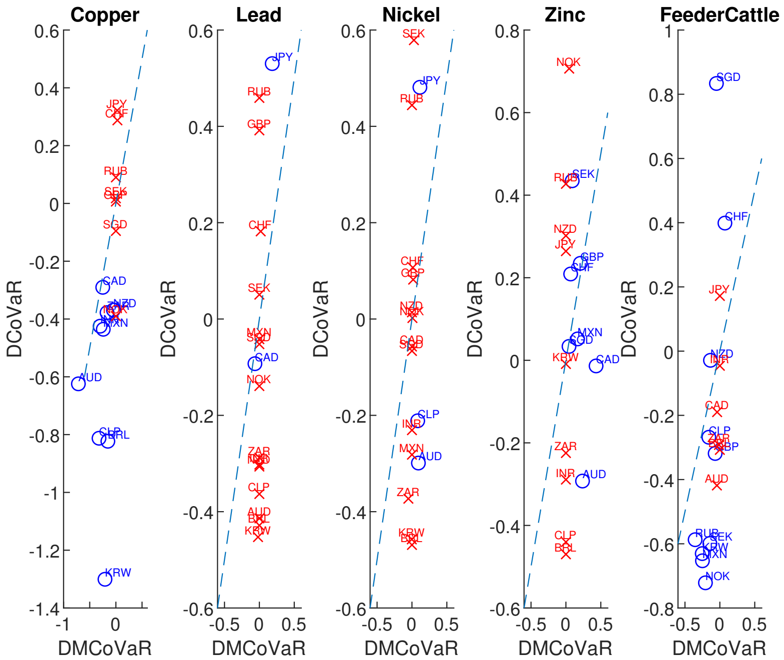
(a)



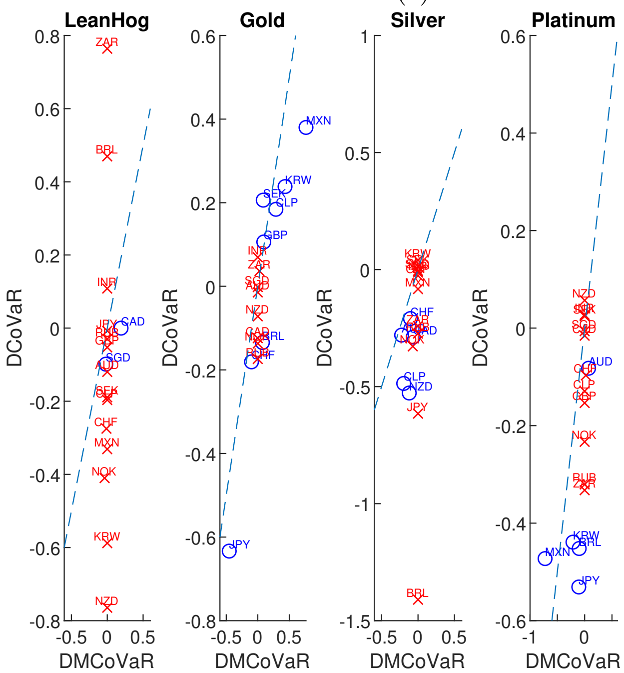
(b)

Figure 2: ΔCoVaR vs ΔMCoVaR

The scatter plot shows the weak correlation between exchange rates' tail-risk to extreme commodity returns when the effects of the commodities are considered independently from each other, measured by ΔCoVaR (y-axis), and exchange rates tail-risk to extreme commodity returns when the effects of the commodities are considered simultaneously, measured by the ΔMCoVaR (x-axis). The ΔCoVaR and ΔMCoVaR are conditional 99% measures and are reported in daily percent returns. ΔCoVaR refers to equation (4), while ΔMCoVaR to equation (9). The exchange rates (our y variables) and commodities (our x variables) names are listed in Section 2. Red crosses represent values of the ΔMCoVaR that are not statistically different from zero and/or are not selected by the MCoVaR with Elastic-Net method.



(c)



(d)

Figure 2: ΔCoVaR vs ΔMCoVaR

The scatter plot shows the weak correlation between exchange rates' tail-risks to extreme commodity returns when the effects of the commodities are considered independently from each other, measured by ΔCoVaR (y-axis), and exchange rates tail-risk to extreme commodity returns when the effects of the commodities are considered simultaneously, measured by the ΔMCoVaR (x-axis). The ΔCoVaR and ΔMCoVaR are conditional 99% measures and are reported in daily percent returns. ΔCoVaR refers to equation (4), while ΔMCoVaR to equation (9). The exchange rates (our y variables) and commodities (our x variables) names are listed in Section 2. Red crosses represent values of the ΔMCoVaR that are not statistically different from zero and/or are not selected by the MCoVaR with Elastic-Net method.

2.4.1 Upside Tail-Risk

Our main goal was to assess whether exchange rates are vulnerable to downside tail-risk in the commodity markets. Moreover, we showed in the descriptive statistics that the data are mainly left skewed and exhibit excess kurtosis. However, for completeness, we analyse upside tail-risk spillovers as well; i.e. we assess exchange rates' vulnerabilities to commodity returns being in the right tail of their distribution (now, considering the upper 1% quantile). The results for the MCoVaR with Elastic-Net in Table 4 might be of interest (especially) to investors with open short positions. As expected, the estimated vulnerabilities are now often economically smaller than in the case of downside tail-risk. Hence, exchange rates tend to be more correlated to negative shocks in commodities fluctuations than to positive shocks. Nevertheless, we still find some economically and statistically significant results. Moreover, the results again show some degree of heterogeneity across currencies. In some circumstances, they are also symmetric to the findings for the downside MCoVaR. Both developed and emerging currencies, such as the Australian dollar and Brazilian real among others, present relevant vulnerabilities to upside commodity tail-risks. Meanwhile, exchange rates such as the Swedish krone and the Singapore dollar show again few and weak estimates of ΔMCoVaR. As an example, we see that in a day when sugar (coffee) is experiencing high returns, the Brazilian real experiences an additional +0.49% (+0.32%) gain compared to when the commodity is in its median state; i.e. the BRL now appreciates against the U.S. dollar. Interestingly, we also see that the Japanese yen and the Swiss franc present strong vulnerabilities to gold, but now with a positive sign. This means that in a situation when gold returns spike up, i.e. in a bad state of the world (a period of financial market turmoil), the U.S. dollar depreciates against these two currencies. Hence, the Japanese yen and the Swiss franc once again confirm their safe-haven properties.

Table 4: ΔMCoVaR with Elastic-Net (Upside Tail-Risk)

Country	Coc$^{\parallel}$	Cof$^{\parallel}$	Cot$^{\parallel}$	Cor$^{\parallel}$	Soy$^{\parallel}$	Sug$^{\parallel}$	Wht$^{\parallel}$	Bre$^{'}$	CrO$^{'}$	NaG$^{'}$	Cop§	Led§	Nic§	Znc§	FeC$^{\#}$	LeH$^{\#}$	Gld†	Slv†	Pla†
Emerging Countries																			
Brazil	-0.22***	0.32***	-	0.01	0.23**	0.49***	-0.13*	0.20***	0.11***	-0.05	0.32***	0.16*	0.01	-	-	-0.28***	-0.36***	0.11**	0.07
Chile	-0.10**	-	-	-	-	0.16***	-0.15**	-	0.00	-	-	-	0.05	-	-	-0.01	-0.12**	-0.06	-
India	0.03	-0.00	-	-	-0.06*	0.27***	-	-0.16***	-	0.13***	-	-	-	0.13***	-	-	0.16**	-0.26***	-
Mexico	-0.42***	-	-	0.07	-0.05	0.16**	-	0.06	0.06	0.01	0.00	0.19**	0.21**	-0.42***	0.22***	-0.15**	-0.10*	0.03	-0.22***
Russia	-	-0.06**	-	-	-0.05*	0.01	-0.09***	0.15***	0.09***	-	-	-	-	0.02	0.06**	-	-0.04	-0.03	-0.02
Singapore	-	-0.00	-	-0.02	-	0.12***	-0.11**	-0.00	-0.07**	-	-	-	-0.06*	-	0.04**	-	-	-	-
South Africa	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
South Korea	-0.03	0.01	-0.03	-0.02	0.12**	0.13*	-	-	-0.07*	0.11*	-0.09**	0.24***	-0.03	-0.08*	0.22***	-0.05	-0.08	-0.20***	0.07**
Developed Countries																			
Australia	-0.09*	-	-	0.21***	-	0.13**	-0.04	-0.00	-0.13***	-	0.24***	0.03	-	-	0.22***	-0.05*	-0.08**	-	-
Canada	-	-	-	-	-	-	-	0.07***	-	-	0.08***	0.03	-	-	-	-	-	-	-
Japan	0.00	-0.02	-0.08*	-0.30***	-	-0.07	0.12**	-0.24***	-	-	-0.46***	0.13**	-0.07	0.02	-0.55***	0.15***	0.65***	-	-0.08*
New Zealand	0.13**	-	-	0.15***	-0.24***	0.13***	-	-0.05	-0.11***	-	0.11**	0.05	0.06	0.02	0.12**	-0.00	-	0.11**	-0.03
Norway	-	-	-	-	-	-	-	-	-	-	-	-	-	-0.01	-	-	0.20***	-	-
Sweden	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Switzerland	0.07*	-0.07**	0.14***	-	-0.02	0.02	0.03	-0.12**	-0.17***	0.01	0.08**	-	-0.14***	-	-0.14***	0.03	0.44***	-	-0.15***
United Kingdom	0.42***	-0.10*	-	0.08**	-0.06	0.20***	-0.15**	-	-0.23***	0.09*	0.24***	-	-0.11**	-0.11*	0.04	-	-	0.07	-

Notes: This table reports the estimated values for the ΔMCoVaR with Elastic-Net in (9), but now with confidence level τ=θ=99%. In all the estimates, the left-hand side variables of the regressions in (5) (i.e. the exchange rate-i) are on the rows of the table, and the right-hand conditioning variable (i.e. commodity-j) on the columns of the table. The other independent variables in each regressions in (5) are the remaining j = $1, \ldots, J-1$ commodities, the $t-1$ values of the dependent variable, and the set of common factors which include: the returns on the S&P500, on the CBOE VIX volatility index and on the US Corporate Bond Total Return Index. We divide exchange rates of developed and emerging countries in two different panels. Standard errors are computed by wild-bootstrap. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level. Statistical significance refers to the coefficients $\hat{\lambda}_{j,\theta}$ from equation (9). We report with - values that the MCoVaR with Elastic-Net method shrinks below $10^{-3}. Data are daily, for the period 01/2004-06/2021, and obtained from Datastream. The commodities are:$ $Cocoa(Coc), Coffee(Cof), Cotton(Cot), Corn(Cor), Soybeans(Soy), Sugar(Sug), Wheat(Wht), Brent(Bre), CrudeOil(CrO), NaturalGas(NaG), Copper(Cop), Lead(Led), Nickel(Nic), Zinc(Znc), FeederCattle(FeC), LeanHogs(LeH), Gold(Gld), Silver(Slv), Platinu$ refers to commodities belonging to the agricultural-category, $'$ to the energy-category, $\dagger$ to the precious metals-category, $\S$ to the industrial metals-category and # to livestock-category.

2.5 Conclusions

Connecting exchange rates to other financial and macroeconomic variables has always been a challenging task for the macro-finance literature. However, commodity prices are an important driver of countries' output and are volatile. In this paper, we take advantage of these features and study the relation between exchange rates and commodity prices through the lens of a conditional tail-risk model, namely the novel MCoVaR with Elastic-Net of [Bonaccolto et al., 2021]. This methodology allows us to overcome the potential omitted variable bias problem present in more basic models, such as the CoVaR of [Adrian and Brunnermeier, 2016b], in which we would estimate the effects of each commodity independently from the others (rather than simultaneously) and, as such, ignore their ties. Overall, we show that different exchange rates are vulnerable to a state of distress of different individual commodities, with these tail-risk threats coming from several different (types of) individual commodities, even beyond oil and gold.

The findings of this paper can be of interest to policymakers in regulating forex and commodity markets and international trades as well as in monitoring changes in commodity prices to protect exchange rates from the shocks of extreme commodity returns. Additionally, the results are important to international investors who want to hedge and diversify their portfolios, (forex) exposures and trade activities against extreme movements in these financial markets. Although beyond the scope of this work, interesting avenues for future research might include exploring how a portfolio containing both currencies and commodities can perform in reducing overall risk or estimating a forward-looking MCoVaR to determine which specific and macro variables can forecast these conditional tail-risk correlations. Our results suggest that policymakers and international investors have to take into account that each exchange rate is exposed to downside tail-risk threats that come from different individual commodities. This finding has important consequences in the decisions on how to hedge and diversify investors' portfolios, as well as on how to regulate international trade and potentially orient monetary policy decisions.

Our main next steps will be to extend the vector of control variables, to show that the results are not specific to the base currency used (namely the USD), and to provide investors with a forward-looking systemic risk measure that evaluates which variables can be used to predict these vulnerabilities and, as such, that can serve as a useful risk-management and portfolio decision tool. Further, future research may also consider to extend the current framework by considering the approach suggested by [Taylor, 2019], where VaR is computed with *expectile* models.

2.6 Appendix

This section is dedicated to a more technical discussion with respect to several aspects encountered in the Chapter. Namely, the econometric aspects related to quantile regression; the bias variance trade-off and, hence, the regularization techniques; cross-validation techniques; "wild-bootrstrap". Finally, table 6, in the spirit of [Kohlscheen et al., 2017], reports the most important commodities for each country according to their share in total export revenues while Figure 3 plot the estimated ΔMCoVaR against simple VaR.

2.6.1 Quantile regression

The conditional Value at Risk, also known as CoVaR [Adrian and Brunnermeier, 2016a] is the main risk measure employed in this work. In this appendix we will review the generic model that has to be estimated in order to get this risk measure and the general features of the quantile regression (as proposed by [Koenker and Bassett, 1978]) by means of which it is estimated.

For illustrative purposes, [Adrian and Brunnermeier, 2016a] made the case of a simple financial system that can be split into two groups, institutions of type i and of type j, subject to two latent independent risk factors, ΔZ^i and ΔZ^j. The authors conjecture that institutions of type i are directly exposed to the sector specific shock ΔZ^i, and indirectly exposed to ΔZ^j via spillover effects. The assumed data generating process is

$$-X^i_{t+1} = \bar{\mu^i}(.) + \bar{\sigma^{ii}}(.)\Delta Z^i_{t+1}\bar{\sigma^{ij}}(.)\Delta Z^j_{t+1} \tag{A2.10}$$

where the notation (.) means that the geometric drift and volatility loadings are functions of the state variables. Following similar reasoning, the data generating process for j-type institutions is:

$$-X^j_{t+1} = \bar{\mu^j}(.) + \bar{\sigma^{jj}}(.)\Delta Z^j_{t+1}\bar{\sigma^{ji}}(.)\Delta Z^i_{t+1} \tag{A2.11}$$

Hence, the model to be estimated is given by:

$$X^j_{t+1} = \phi_0 + M_t\phi_1 + X^i_{t+1}\phi_2 + (\phi_3 + M_t\phi_4)\Delta Z^j_{t+1} \tag{A2.12}$$

where M_t is a vector of state variables. The error term ΔZ^j_{t+1} is assumed to be i.i.d. with zero mean and unit variance, and $E[\Delta Z^j_{t+1}|M_t, X^i_{t+1}] = 0$. The conditional expected return $\mu^j[X^j_{t+1}|M_t, X^i_{t+1}] = \phi_0 + M_t\phi_1 + X^i_{t+1}\phi_2$ depends on the set of state variables M_t and on X^i_{t+1} and the conditional volatility $\sigma^{ij}_t[X^j_{t+1}|M_t, X^i_{t+1}] = (\phi_3 + M_t\phi_4)$ is a direct function of the state variables M_t. The coefficients ϕ_0, ϕ_1, and ϕ_2 could be estimated consistently via OLS of X^j_{t+1} on M_t and X^i_{t+1}. The predicted value of such an OLS regression would be the mean of X^j_{t+1} conditional on M_t and X^i_{t+1}. In order to compute the VaR and CoVaR from OLS regressions, one would have to also estimate ϕ_3, ϕ_4, and ϕ_5 and then make distributional assumptions about ΔZ^j_{t+1}. The quantile regressions incorporate estimates of the conditional mean and the conditional assumptions that would be needed for estimation via OLS.

The cumulative distribution function of ΔZ^j is denoted by $F_{\Delta Z^j}(.)$ and its inverse cumulative distribution function by $F^{-1}_{\Delta Z^j}(q)$ for the q%-quantile. Then the inverse cumulative distribution function of X^j_{t+1} is:

$$F^{-1}_{X^j_{t+1}}(q|M_t, X^i_{t+1}) = \alpha_1 + M_t\gamma_q + X^i_{t+1}\beta_q \tag{A2.13}$$

where $\alpha_q = \phi_0 + \phi_3 F^{-1}_{\Delta Z^j}(q), \gamma_q = \phi_1 + \phi_4 F^{-1}_{\Delta Z^j}(q)$, and $\beta_q = \phi_2$ for quantiles $q \in (0, 100)$. We then call $F^{-1}_{X^j{t+1}}(q|M_t, X^i_{t+1})$ the conditional quantile function.

By the definition of VaR we then obtain:

$$VaR^j_{q,t+1} = \inf_{VaR^j_{q,t+1}} \{Pr(X^j_{t+1}|\{M_t, X^i_{t+1}\} \leq VaR^j_{q,t+1}) \geq q\%\} = F^{-1}_{X^j_{t+1}}(q|M_t, X^i_{t+1}) \tag{A2.14}$$

Indeed, since the conditional quantile function $F^{-1}_{X^j_{t+1}}(q|M_t, X^i_{t+1})$ is the $VaR^j_{q,t+1}$ conditional on M_t and X^i_{t+1}, by conditioning on $X^i_{t+1} = VaR^i_{q,t+1}$ we obtain the $CoVaR^{j|i}_{q,t+1}$

$$CoVaR^{j|i}_{q,t+1} = \inf_{VaR^j_{q,t+1}} \{Pr(X^j_{t+1}|\{M_t, X^i_{t+1} = VaR^i_{q,t+1}\} \leq VaR^j_{q,t+1}) \geq q\%\} =$$

$$=\mathrm{F}^{-1}_{X^j_{t+1}}(q|M_t, X^i_{t+1} = VaR^i_{q,t+1})\text{(A2.15)}$$

Finally, the quantile function are estimated ad the predicted value of the q%-quantile regression of X^i on M_t and X^j_{t+1} by solving:

$$\min_{\alpha_q,\beta_q,\gamma_q} \sum_t \begin{cases} q\%|X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q| & \text{if} \quad (X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q) \geq 0 \\ (1 - q\%)|X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q| & \text{if} \quad (X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q) < 0 \end{cases} \tag{A2.16}$$

With respect to the properties of the estimator, [Koenker and Bassett Jr, 1978] has provided formal proof of them. Under a set of conditions, it can be showed that the quantile regression estimator ($\hat{\beta}(\theta)$) is asymptotically distributed as:

$$\sqrt{n}(\hat{\beta}(\theta) - \beta(\theta)) \qquad \underset{\rightarrow}{d} \qquad N(\mathbf{0}, \omega^2(\theta)\mathbf{D}^{-1})$$

with scale parameter $\omega^2(\theta) = \frac{\theta(1-\theta)}{f(F^{-1}(\theta))^2}$ being a function of $s = \frac{1}{f(F^{-1}(\theta))}$ which is known as the sparsity function and $\mathbf{D} = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1} x_i^T x_i$ being a positive definite matrix, and x_i the (1,p) row vector comprising the i-th observation of the p explanatory variables.

2.6.2 Regularization techniques

One of the main contribution of this work is that of enlarging the univariate CoVaR analysis by taking into consideration possible "ties" among different covariates (commodities). As the inclusion of a large set of different covariates may undermine the reliability of the final estimator, the model is extended so as to account for the Elastic-Net estimation form. This model allows to select only relevant regressors from a large set of candidates. In particular, the inclusion of the Elastic-Net makes it possible to balance the bias and variance of the estimates. In this section we will review the main theoretical basis behind the bias-variance trade-off.

Let's consider the standard case of an Ordinary Least Squares (OLS) approach to estimate the true parameter β

$$Y = X\beta + \epsilon \tag{A2.17}$$

$$\epsilon \sim N(0, \sigma^2)$$

which is obtained by minimizing the sum of squares residuals

$$L_{OLS}(\hat{\beta}) = \sum_{i=1}^{n}(y_i - x_i'\hat{\beta})^2 \tag{A2.18}$$

which leads to obtain the OLS parameter estimate $\hat{\beta}_{OLS} = (X'X)^{-1}(X'Y)$. The reliability of this parameter will depend on two characteristics of the estimator: the bias and the variance defined, respectively, as:

$$Bias(\hat{\beta}_{OLS}) = E(\hat{\beta}_{OLS} - \beta) \tag{A2.19}$$

$$Var(\hat{\beta}_{OLS}) = \sigma^2(X'X)^{-1} \tag{A2.20}$$

where the error variance σ^2 can be estimated from the residuals as:

$$\hat{\sigma^2} = \frac{e^{'}e}{n-m}^{16} \tag{A2.21}$$

$$e = y - X\hat{\beta} \tag{A2.22}$$

An estimator is desired to have both bias and variance as low as possible. In general, estimator are unbiased but could have a high variance. The possible solution to this problem is to reduce the variance at the cost of introducing some bias. That is, we want to reduce the *complexity* of the model (the number of regressors) in order to reduce the variance. The goal of the analysis is, then, focused on how to select the optimal number of regressors. The so-called *regularization techniques*(to whom the Elastic Net method belongs) allow to take rid of the non-relevant regressors. In what it follows, we will go through the three main techniques (Ridge Regression, LASSO and EN), analysing how these method behave in terms of the bias variance trade-off.

Ridge Regression

This technique works by augmenting the loss function that will, then, not only minimize the sum of squared residuals but also penalize the size of the parameter estimates in order to shrink them towards zero:

$$L_{ridge}(\hat{\beta}) = \sum_{i=1}^{n}(y_i - x_i^{'}\hat{\beta})^2 + \lambda\sum_{j=1}^{m}\hat{\beta}_j^{\;2} \tag{A2.23}$$

The $\hat{\beta}$ that solves the equation is then equal to

$$\hat{\beta}_{ridge} = (X^{'}X + \lambda I)^{-1}(X^{'}Y)$$

where I is the identity matrix. The λ is known as the regularization penalty and:

- as $\lambda \rightarrow 0,_{ridge} \rightarrow \hat{\beta}_{OLS}$;
- as $\lambda \rightarrow \infty$, $\hat{\beta}_{ridge} \rightarrow 0$.

[16] where n is the number of observation and m is the number of predictors

In other words, as the value of *lambda* increases, the stronger the *penalization* imposed on the coefficients' size. [17]

LASSO

The second regularization technique is known as Least Absolute Shrinkage and Selection Operator (LASSO), and is conceptually similar to the above Ridge method. As in the previous model, with LASSO there is a *penalty* for non-zero coefficients. Unlike the Ridge model, where the penalization is for the sum of squared coefficients (L2 penalty), with LASSO the penalty is for the sum of the absolute value (L1 penalty). The estimator, $\hat{\beta}_{LASSO}$ is then the β that solves:

$$L_{LASSO}(\hat{\beta}) = \sum_{i=1}^{n}(y_i - x_i^{'}\hat{\beta})^2 + \lambda \sum_{j=1}^{m} |\hat{\beta}_j| \quad \text{(A2.24)}$$

Unlike in Ridge Regression, this minimization problem cannot be solved analytically. Fortunately, there are numerical algorithms able to deal with it.
With respect to the properties of the estimator, a well known result[18] is that Lasso estimates are consistent. Essentially, they are biased with finite sample size (n $<\infty$) but their bias approaches zero as n approaches infinity.

Elastic Net

The LASSO method allows to perform variable selection among the set of possible regressors. For this reason, LASSO model has been intensively used by researchers in data-driven application. That being said, the method has received some critiques due to the selection process that could be too much dependent on the dataset used and, hence, unstable. A solution proposed for this problem emerged as the Elastic Net, which combines the penalties of Ridge regression and LASSO method. In particular,

[17] In terms of the bias-variance trade off, with the ridge model they become:

$$Bias(\hat{\beta}_{ridge}) = -\lambda(X^{'}X + \lambda I)^{-1}\beta$$

$$Var(\hat{\beta}_{Ridge}) = \sigma^2(X^{'}X + \lambda I)^{-1}(X^{'}X)^{-1}(X^{'}X + \lambda I)^{-1}$$

such that as the penalty increases (λ increases) the variance shrinks while the bias increases.

[18] The interested reader could find a formal proof of this result in [Zhao and Yu, 2006]

the Elastic Net minimizes the following loss function:

$$L_{EN}(\hat{\beta}) = \frac{\sum_{i=1}^{n}(y_i - x^{'}\hat{\beta})^2}{2n} + \lambda(\frac{1-\alpha}{2}\sum_{j=1}^{m}\hat{\beta}_j^2 + \alpha\sum_{j=1}^{m}|\hat{\beta}_j|) \tag{A2.25}$$

where α is the mixing parameter between Ridge ($\alpha = 0$) and LASSO ($\alpha = 1$). In this model, differently from Ridge and LASSO, the parameters to be found are then two: α and λ.

2.6.3 Cross-validation techniques

For all of the three regularization methods exposed above, one of the key passage is that of choosing the correct value of the parameters of the model, also known as *tuning parameter*([Tibshirani, 1996]. That is, λ for Ridge Regression and LASSO and λ and α for the Elastic Net. The tuning parameter controls the amount of regularization, so choosing a good value of the tuning parameter is crucial. Because each tuning parameter value corresponds to a fitted model, we also refer to this task as model selection.
The objective of this process is defined as choosing the parameter in such a way that the accuracy of our prediction is maximized (that is, prediction error is minimized).

From [Tibshirani, 1996] consider the case in which we observe

$$y_i = f(x_i) + \epsilon_i, \quad i = 1, ..., n \tag{A2.26}$$

where x_i are predictor measurements, f is the true function we are trying to predict and ϵ_i are random errors. We call $(x_i, y_i), i = 1, ..., n$ the *training data* and, based on such set, we consider the estimator $\hat{f}$. Then, the average prediction error is:

$$PE(\hat{f}) = E[\frac{1}{n}\sum_{i=1}^{n}(y_i^{'} - \hat{f}(x_i))^2] \tag{A2.27}$$

where $y_i^{'} = f(x_i) + \epsilon_i^{'}, \quad i = 1, ..., n$ are another set of observation, independent of $y_1, ..., y_n$.

The goal of model selection is choosing the parameter of the model on which the estimator depends (θ) so that we can minimize the PE. The problem emerges as, usually, we don't have the so-called *test data* $(y_1^{'}, ..., y_n^{'})$. If that would have been the case, we could have computed the *test error* as an estimate for $PE(\hat{f}_\theta)$:

$$TestErr(\hat{f}_\theta) = \frac{1}{n}\sum_{1=1}^{n}(y_i^{'} - \hat{f}_\theta(x_i))^2 \tag{A2.28}$$

So, when we could not observe the test data, we have to compute the *training error*, which is similar to the test error, but it is computed using the data which lead to the estimation of $\hat{f}$

$$TrainErr(\hat{f}_\theta) = \frac{1}{n}\sum_{1=1}^{n}(y_i - \hat{f}_\theta(x_i))^2 \tag{A2.29}$$

The training error rate often is quite different from the test error rate, and in particular the former can underestimate the latter.

In the absence of a very large designated test set that can be used to directly estimate the test error rate, a number of techniques can be used to estimate this quantity using the available training data.

The Validation Set approach

The Validation Set approach belongs to the method in which the available set of observation is split in two parts, the *training set* and a *validation set.* In this case, the two are formed at random and then the model is fit on the training set, and the fitted model is used to predict the responses for the observations in the validation set. The resulting validation set error rate provides an estimate of the test error rate.
This method has two potential drawbacks: *i)* the validation estimate of the test error rate can be highly variable, depending on which observations are included in the training set and which observations are included in the validation set; *ii)* only a subset of the observations is used to fit the model and the validation set error rate may tend to overestimate the test error rate for the model fit on the entire data set, since estimation models tend to perform worse when run on a reduced number of observations.

Leave-One-Out Cross-Validation

This approach builds on the Validation Set one, but it attempts to address its drawbacks. Also in this case, the set of observations is split into two parts but, instead of creating two subsets of comparable size, a single observation (x_1, y_1) is used for the validation set, and the remaining observations $\{(x_2, y_2), ..., (x_n, y_n)\}$ make up the training set. Hence, the model is fit on the $(n-1)$ training observations, and a prediction $\hat{y}_1$ is made for the excluded observation. Then the Mean Squared Error $(MSE_1) = (y_1 - \hat{y})^2$ provides an approximately unbiased estimate for the test error. Even if unbiased, the MSE is still a poor estimate because it is highly variable, since it is based upon a single observation. For this reason, we can repeat the procedure by selecting a different couple of observations (x_2, y_2) for the validation data, fit the model on the remaining $(n-1)$ observations $\{(x_1, y_1), (x_3, y_3)..., (x_n, y_n)\}$ and compute again the $(MSE_2) = (y_2 - \hat{y})^2$. Repeating this procedure n times produces n different MSE, such that the final estimate for the test MSE is the average of these n test error estimates

$$CV_{(n)} = \frac{1}{n}\sum_{i=1}^{n} MSE_i \tag{A2.30}$$

As compared to the Validation Set method, this approach has less bias but also less volatility of the results that in the former model was due to the random split of the observation data set.

K-Fold Cross-validation

An alternative to the Leave-One-Out Cross-Validation is the K-fold cross-validation [19]. Under this approach, the training set is randomly split in K different "folds" of roughly equal size $(F_1, ..., F_K)$. Then, for $k = 1, ..., K$ we consider (x_i, y_i) $i \notin F_k$ as the observation in the training set, and (y_i, x_i), $i \in F_k$ as those belonging to the validation set. Then, given the spectrum of possible values for θ, we compute the estimate $\hat{f}_\theta^{-k}$ on the training set, and then we calculate the error on the validation set

$$e_k(\theta) = \sum_{i \in F_k} (y_i - \hat{f}_\theta^{-k}(x_i))^2 \tag{A2.31}$$

[19] The interested reader could find the other possible approaches to cross validation in [Hastie et al., 2009]

Finally, for each θ we compute the average error over all fold

$$CV(\theta) = \frac{1}{n}\sum_{k=1}^{K} e_k(\theta) = \frac{1}{n}\sum_{k=1}^{K}\sum_{i\in F_k}(y_i - \hat{f}_\theta^{-k}(x_i))^2 \tag{A2.32}$$

which leads to the *cross validation error curve*, that is a function of the parameter, which reports the CV error for all the values of θ. We will choose the value of the parameter that minimizes this curve

$$\hat{\theta} = \underset{\theta\{\theta_1,\ldots,\theta_k\}}{argmin} CV(\theta) \tag{A2.33}$$

2.6.4 Wild-Bootstrap vs. Standard Bootstrap

In this paper, following [Bonaccolto et al., 2021] we make use of the "wild-bootstrap" method to estimate the standard errors of the model. This choice is justified since this approach outperforms other resampling techniques when estimating quantile regression models including ([Wang et al., 2018]), or not including ([Feng et al., 2011a]), penalty functions. An alternative approach would have been that of using the "standard bootstrap" method ([Davino et al., 2014]). The choice of the former has been due to the better performance when estimating a penalized quantile regression [Feng et al., 2011b].

In this section we recall the main properties of the estimators estimated by means of the two different methods, and the operational steps to compute them.

Wild-bootstrap[20]

[Wang et al., 2018] showed that wild residual bootstrap procedure for unpenalized quantile regression is asymptotically valid for approximating the distribution of the quantile regression estimator with adaptive L_1 penalty. In particular, the quantile regression estimator with the adaptive L_1 penalty performs simultaneous estimation and variable selection by minimizing a penalized quantile loss function:

$$\tilde{\beta} = \underset{\beta}{argmin} = \left\{ \sum_{i=1}^{n} \rho_\tau(Y_i - x_i^T \beta) + \lambda_n \sum_{j=1}^{p} w_j |\beta_j| \right. \tag{A2.34}$$

where $\lambda_n > 0$ is a tuning parameter, and $w_j = |\bar{\beta}|^{-\gamma}$ are the adaptive weights. Defining as $\tilde{\beta} = (\tilde{\beta}_0, ..., \tilde{\beta}_p)^T$ the subvector that contains the first (q+1) elements of $\tilde{\beta}$ and $\tilde{A} = 1 \leq j \leq p : \beta_j \neq 0$. Let $D_0 = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} x_{iA} x_{iA}^T$ and $D_1 = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} f_i(0) x_{iA} x_{iA}^T$, where $f_i(0)$ is the density function of the error term evaluated at zero. If these conditions are satisfied and $n^{-1/2}\lambda_n \to 0$ and $n^{(\gamma-1)/2}\lambda_n \to \infty$, then the adaptive L_1-penalised quantile regression estimator $\tilde{\beta}$ has:

- pr$(\tilde{A} = A) \to 1$ as n $\to \infty$;

[20] This part follows [Wang et al., 2018]

- $n^{1/2}(\tilde{\beta}_1 - \beta_{01}) \rightarrow N\{O_{q+1}, \tau(1-\tau)D_1^{-1}D_0D_1^{-1}\}$ in distribution as n $\rightarrow \infty$

Finally, the asymptotic distribution of $\tilde{\beta}$ can be obtained by means of a wild residual bootstrap procedure ([Wang et al., 2018]). That is, to obtain the wild bootstrap sample, the following steps have to be followed:

- Compute the residuals from the adaptively penalized quantile regression: $\hat{\epsilon_i} = Y_i - x_i^T\tilde{\beta}(i = 1, ..., n)$ and obtain $\tilde{\beta}$;
- Let $\epsilon_i^* = r_i|\hat{\epsilon_i}|$, where $r_i(i = 1, ..., n)$ are generated as a random sample from a distribution with a cumulative distribution function G with specific characteristics listed below;
- Generate the bootstrap sample as $Y_i^* = x_i^T\tilde{\beta} + \epsilon_i^*(i = 1, ..., n)$.

Then by using the bootstrap sample it should be recalculate the adaptively penalized quantile regression estimator as

$$\tilde{\beta^*} = \underset{\beta}{argmin} = \begin{cases} \sum_{i=1}^n \rho_\tau(Y_i^* - x_i^T\beta) + \lambda_n \sum_{j=1}^p w_j^*|\beta_j| \end{cases} \tag{A2.35}$$

where $w_j^* = |\bar{\beta_j^*}|^{-\gamma}, \bar{\beta^*} = (\bar{\beta_0^*}, ..., \bar{\beta_p^*})^T$ is the ordinary quantile regression estimator recomputed on the bootstrap sample. For $j = 1, ..., p$ and $0 < \alpha < 1$, let $d_j^{*(\alpha/2)}$ and $d_j^{*(1-\alpha/2)}$ be the $(\alpha/2)$-th and $(1-\alpha/2)$-th quantiles of the bootstrap distribution of $n^{1/2}(\tilde{\beta_j^*} - \tilde{\beta}_j)$, respectively.
The asymptotic $100(1-\alpha)\%$ bootstrap confidence interval for $\beta_{0j}, j = 1, ..., p$, is given by $[\tilde{\beta}_j - n^{-1/2}d_j^{*(1-\alpha/2)}, \tilde{\beta}_j - n^{-1/2}d_j^{*(\alpha/2)}]$.

Finally, assuming that:

- the true value of β_0 is an interior point of a compact set in $\mathbf{R}^p$ and the density of the error term is Lipschitz continuous and bounded away from 0 and ∞ in a neighborhood around 0;

- $\lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} x_i x_i^T \to B_0$ and $\lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} f_i(0) x_i x_i^T{}_1$ for some positive definite matrices B_0 and B_1. Furthermore, $\sum_{i=1}^{n} ||x_i||^3 = O(n)$ and $max_{1\leq i\leq n}||x_i|| = O(n^{1/4})$, where $||.||$ is the Euclidean norm;

- for some strictly positive constants c_1 and c_2, $sup\{r \in \mathbf{G} : r \leq\} = -c_1$ and $inf\{r \in \mathbf{G} : r > 0\} = -c_2$, where $\mathbf{G}$ is the support of the weight distribution of G;

- the weight distribution G satisfies $\int_0^{+\infty} r^{-1} dG(r) = -\int_{-\infty}^{0} r^{-1} dG(r) = 1/2$ and $\mathbf{E}_G(|r|) < \infty$, where the expectation is taken under G;

- the quantile of the distribution of G is zero.

than the conditional distribution of $n^{1/2}(\tilde{\beta}^* - \tilde{\beta})$ provides an asymptotically valid approximation of that of $n^{1/2}(\tilde{\beta}^* - \beta)$. Let $\tilde{A}^* = \{j = 1, ..., p : \tilde{\beta}_j^* \neq' 0\}$, and let $\tilde{\beta}_1^*$ be the subvector that contains the first (q + 1) elements of $\tilde{\beta}^*$. Let $r = \{r_1, ..., r_n\}$ be the random bootstrap weights and $z = \{z_1, ..., z_n\}$ be the random sample. By the wild bootstrap mechanism, the distribution of r is independent of that of z. Let pr_z denote the probability under the joint distribution of z, and let $pr_{r|z}$ denote the probability of r conditional on z. Then:

- $pr_{r|z}(\tilde{A}^* = A) = 1 + o_{pr_z}(1)$. Furthermore,

- $$\sup_t |pr_{r|z}\{n^{1/2}(\tilde{\beta}_1^* - \tilde{\beta}_1) \leq t\} - pr_z\{n^{1/2}(\tilde{\beta}_1^* - \beta_{01}) \leq t\}| = o_{pr_z}(1)$$

Standard-bootstrap (xy-pair)[21]

An alternative to the "wild-bootstrap" method can be found in the "standard bootstrap" (or the xy-pair) method. Let's consider the simple quantile regression (QR) framework:

$$Q_\theta(\mathbf{y}|x) = \hat{\beta}_0 + \hat{\beta}_1(\theta)x \tag{A2.36}$$

[21] This part is from [Davino et al., 2014]

Given the above model, the method consists in constructing a given number of samples (B) where each sample is obtained by a random sampling procedure with replacement from the original dataset. The resampling procedure is simultaneously applied to the x and y vectors. Then, B quantile regressions are performed on the bootstrap samples, and a vector of the parameter estimates is retained for each quantile of interest. The standard error of the vector of parameter bootstrap estimates represents an estimate of the QR standard error useful in confidence intervals and hypothesis tests. If k quantiles are considered, the bootstrap procedure produces a matrix of parameter estimates from which a variance-covariance matrix can be derived.

In cases of a multiple QR performed with p explanatory variables, the following bootstrap variance can be considered as an estimator of the asymptotic variance for each explanatory variable j and for each quantile q:

$$\hat{V_{q,j}} = \frac{1}{B}\sum_{b=1}^{B}(\hat{\beta_{b,j}}(\theta_q) - \bar{\hat{\beta}}_j(\theta_q))(\hat{\beta_{b,j}}(\theta_q) - \bar{\hat{\beta}}_j(\theta_q))^T \tag{A2.37}$$

where $j = 1, ..., p; q = 1, ..., k$ and $\bar{\hat{\beta}}_j(\theta_q)) = \frac{1}{B}\sum_{b=1}^{B}\hat{\beta_{b,j}}(\theta_q)$. Confidence interval can be constructed using the above equation for the bootstrap variance, or by using the percentile method. In the first case, exploiting the asymptotic normal limit, a confidence interval for the generic jth parameter and the qth quantile, is:

$$\bar{\hat{\beta}}_j(\theta_q) \pm z_{\alpha/2} SD(\hat{\beta}_j(\theta_q)) \tag{A2.38}$$

where $\bar{\hat{\beta}}_j(\theta_q)$ is the average value of the B bootstrap estimates, and $SD(\hat{\beta}_j(\theta_q))$ is the squared root of the variance in the equation for the bootstrap variance.

The percentile method is based on the αth$((\hat{\beta}_j(\theta_q))$ and $(1-\alpha)$th$((\hat{\beta}_j(\theta_q))$ percentiles of the cumulative distribution function of the bootstrap vector of parameter estimates:

$$[\hat{\beta}_j(\theta_q)_{lo}, \hat{\beta}_j(\theta_q)_{up}] = [\hat{F}(\alpha), \hat{F}(1-\alpha)] \tag{A2.39}$$

where *lo* and *up* stand for, respectively, the lower and upper extreme of the confidence interval.

2.6.5 ΔCoVaR without wild-bootstrap

In Table 5, we estimate the ΔCoVaR by computing standard errors by simple bootstrap, as is common in the previous CoVaR literature (see [Borri, 2019b] and [Borri, 2019a], among others), rather than by wild-bootstrap as in [Bonaccolto et al., 2021].

Overall, we see that the takeaways when moving from the basic ΔCoVaR (in Table 3) to the ΔMCoVaR with Elastic-Net (Table 1) remain the same as described at the end of Section 4, where the standard errors for both the measures were computed by wild-bootstrap.

Table 5: ΔCoVaR (without wild-bootstrap)

Country	Coc$^{\Vert}$	Cof$^{\Vert}$	Cot$^{\Vert}$	Cor$^{\Vert}$	Soy$^{\Vert}$	Sug$^{\Vert}$	Wht$^{\Vert}$	Bre$^{'}$	CrO$^{'}$	NaG$^{'}$	Cop§	Led§	Nic§	Znc§	FeC$^{\#}$	LeH$^{\#}$	Gld†	Slv†	Pla†
Emerging Countries																			
Brazil	-0.73**	-0.84***	-0.90***	-0.80***	-1.08***	-1.29***	-0.41	-0.78***	-0.53**	0.91***	-0.82**	-0.43	-0.47	-0.47	-0.31	0.47	-0.13	-1.41***	-0.45
Chile	0.71***	-0.12	-0.11	-0.47**	-0.58***	-0.41**	0.18	-0.43**	-0.24	-0.41**	-0.81***	-0.36*	-0.21	-0.44**	-0.27	-0.20	0.18	-0.49**	-0.13
India	0.12	-0.03	-0.34***	-0.02	-0.08	-0.08	-0.01	0.34**	-0.04	0.18	-0.42***	-0.30**	-0.23*	-0.29**	-0.05	0.11	0.07	-0.01	0.03
Mexico	-0.01	-0.09	-0.53**	-0.57**	-0.56**	-0.25	-0.28	-0.47**	-0.42**	0.62**	-0.44*	-0.04	-0.28	0.05	-0.65**	-0.33	0.38	-0.08	-0.47*
Russia	0.27	-0.56	-0.61	-0.84**	-0.13	-0.39	-0.77**	-1.09***	-0.63***	-0.46	0.09	0.46	0.44	0.43	-0.59	-0.03	-0.17	-0.27	-0.32
Singapore	-0.09	0.81***	-0.17**	-0.21***	-0.10	0.75***	-0.37***	-0.12	-0.10	-0.17**	-0.10	-0.05	-0.07	0.03	0.83***	-0.10	-0.00	-0.01	-0.01
South Africa	0.60**	-0.57**	-0.18	0.21	-0.26	1.14***	-0.19	-0.38	-0.40**	-0.47*	-0.38	-0.29	-0.37	-0.22	-0.29	0.76***	0.04	-0.24	-0.33
South Korea	0.13	-0.34	0.06	-0.22	-0.57**	-0.14	0.71**	0.11	-0.35**	0.41	-1.30***	-0.45	-0.46*	-0.01	-0.63**	-0.59**	0.24	0.04	-0.44
Developed Countries																			
Australia	-0.45**	-0.33	-0.48**	-0.64***	-0.39	-0.29	0.03	-0.35	-0.40**	-0.47**	-0.62***	-0.41*	-0.30	-0.29	-0.42	-0.12	-0.01	-0.28	-0.08
Canada	-0.15	0.16	-0.45**	-0.48***	-0.33**	-0.15	-0.03	-0.38***	-0.25*	-0.20	-0.29*	-0.09	-0.06	-0.01	-0.19	-0.00	-0.12	-0.29*	-0.02
Japan	-0.16	-0.15	0.20	0.10	0.37**	0.46***	0.10	0.38*	0.21	0.39**	0.32*	0.53***	0.48***	0.26	0.17	-0.01	-0.63***	-0.62***	-0.53***
New Zealand	-0.05	-0.19	-0.14	-0.76***	-0.35	-0.33	0.60**	-0.34	-1.33***	-0.59***	-0.37	-0.31	0.01	0.30	-0.03	-0.76***	-0.07	-0.53*	0.06
Norway	-0.43**	-0.65***	-0.28	-0.52**	-0.57***	-0.00	-0.17	-0.54***	-0.40**	-0.06	-0.39**	-0.14	0.00	0.71***	-0.72***	-0.41**	-0.14	-0.33	-0.23
Sweden	-0.14	0.52***	-0.08	-0.42**	-0.34*	-1.31***	-0.26	-0.08	-1.03***	0.39*	0.02	0.05	0.58***	0.43***	-0.60***	-0.19	0.21	0.01	0.02
Switzerland	-0.22	0.01	0.04	0.06	0.06	0.25**	0.74***	0.38***	0.17	-0.07	0.29**	0.18	0.11	0.21	0.40***	-0.27**	-0.18	-0.21	-0.10
United Kingdom	-0.41***	-0.03	-0.15	-0.28**	-0.36***	-0.17	-0.04	-0.34**	0.51***	-0.32**	0.01	0.39**	0.08	0.23*	-0.32**	-0.05	0.11	-0.03	-0.15

Notes: This table reports the estimated values for the ΔCoVaR in (4), with confidence level τ=θ=1%. In all the estimates, the left-hand side variables of the regressions in (1) (i.e. the exchange rate-i) are on the rows of the table, and the right-hand conditioning variable (i.e. commodity-j) on the columns of the table. The other independent variables in each regressions in (1) are the set of common factors which include: the returns on the S&P500, on the CBOE VIX volatility index and on the US Corporate Bond Total Return Index. We divide exchange rates of developed and emerging countries in two different panels. Standard errors are computed by bootstrap as in [Borri, 2019b] and [Borri, 2019a]. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level. Statistical significance refers to the coefficients $\hat{\lambda}_\theta$ from equation (3). Data are daily, for the period 01/2004-06/2021, and obtained from Datastream. The commodities are: Cocoa (Coc), Coffee (Cof), Cotton (Cot), Corn (Cor), Soybeans (Soy), Sugar (Sug), Wheat (Wht), Brent (Bre), Crude Oil (CrO), Natural Gas (NaG), Copper (Cop), Lead (Led), Nickel (Nic), Zinc (Znc), Feeder Cattle (FeC), Lean Hogs (LeH), Gold (Gld), Silver (Slv), Platinum (Pla). $^{\Vert}$ refers to commodities belonging to the agricultural-category, $^{'}$ to the energy-category, † to the precious metals-category, § to the industrial metals-category and $^{\#}$ to livestock-category.

2.6.6 Shares of Commodity in Total Exports

In Table 6, we report the most important commodities for each country according to their share in total export revenues, in a similar vein to Table 9 in [Kohlscheen et al., 2017]. Due to space constraints and for exposition clarity, we include for each country only the commodity groups that account for more than 0.3% of the exports. The data are retrieved for the period 2007–2017 from UN Comtrade. We use classification based on the three-digit level. However, since some commodities are not available or impossible to disentangle at this level of granularity, we recur to the four-digit SIC for certain commodities (e.g. silver and platinum).

Table 6: Shares of Commodity Groups in Exports

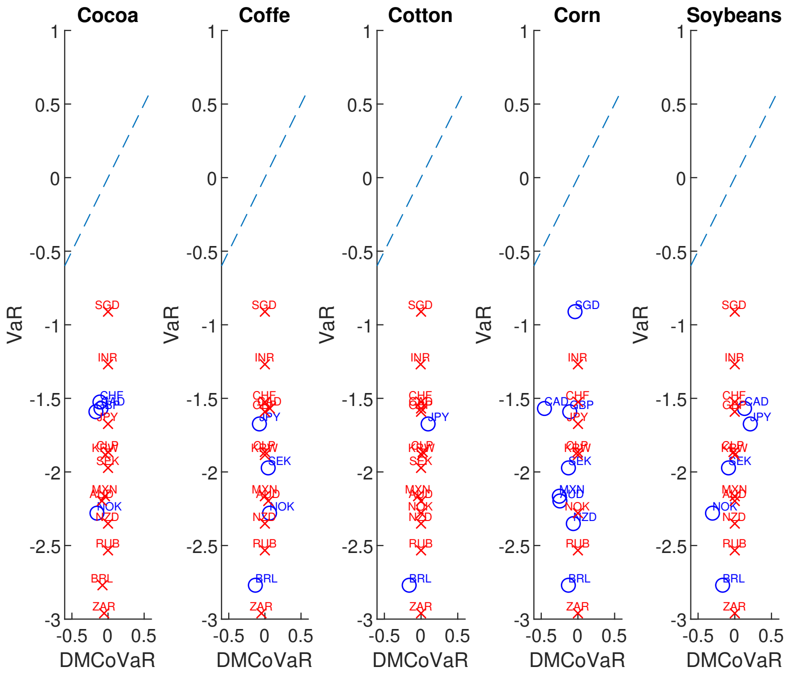
	Description of Group	Export Over Total Exports (%)
Australia	Gold	6,24
	Natural Gas	4,63
	Brent/Crude Oil	3,60
	Wheat	2,08
	Copper	1,38
	Cotton	0,61
	Zinc	0,47
	Lead	0,41
	Feeder Cattle	0,37
Brazil	Soybeans	8,26
	Brent/Crude Oil	6,95
	Sugar	4,92
	Coffee	2,80
	Corn	1,69
	Gold	1,02
	Cotton	0,56
	Copper	0,39
Canada	Brent/Crude Oil	13,76
	Natural Gas	3,38
	Gold	3,04
	Wheat	1,35
	Copper	0,63
	Nickel	0,62
	Soybeans	0,36
Chile	Copper	31,65
	Gold	1,42
	Brent/Crude Oil	1,11
	Silver	0,48
India	Copper	0,99
	Cotton	0,96
	Sugar	0,48
Japan	Copper	0,96
	Gold	0,91
Mexico	Brent/Crude Oil	9,90
	Gold	1,40
	Silver	0,73
	Copper	0,39
New Zealand	Brent/Crude Oil	3,38
	Gold	1,11
	Sugar	0,56
	Silver	0,30
Norway	Brent/Crude Oil	32,71
	Natural Gas	23,08
	Nickel	1,22
Russia	Brent/Crude Oil	30,43
	Natural Gas	10,89
	Copper	1,10
	Nickel	1,00
	Wheat	0,95
	Gold	0,67
Republic of Korea	Copper	0,77
	Gold	0,37
Singapore	Gold	2,42
South Africa	Platinum	10,04
	Gold	4,83
	Corn	0,57
	Brent/Crude Oil	0,51
	Sugar	0,40
	Nickel	0,39
	Copper	0,38
Sweden	Copper	0,90
	Gold	0,44
Switzerland	Gold	15,02
	Platinum	1,27
	Coffee	0,68
United Kingdom	Brent/Crude Oil	5,23
	Gold	3,83
	Platinum	0,87
	Natural Gas	0,66
	Silver	0,37

Notes: This table reports the share in total export revenues for commodity groups in each country. For space constraints, for each country we report only the commodities whose share is above a 0.30% threshold. The share is compiled based on UN Comtrade data between 2007 and 2017, at three-digit level (four-digit, when a three-digit code is not available for a certain commodity).

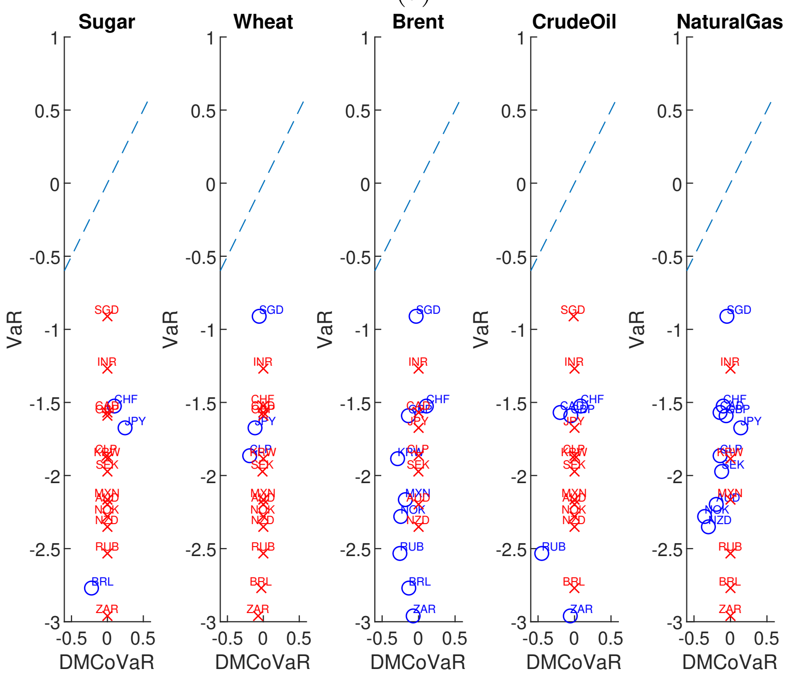
2.6.7 VaR vs ΔMCoVaR

In Figure 2, which is inspired by Figure IV.1 of [Adrian and Brunnermeier, 2016b], we show that exchange rate-i's tail-risk to commodity-j ($\Delta\text{MCoVaR}^{y^i|x^j}$-$i$) is different from the exchange rate's own risk measure (VaR-i). As we see, since the dots do not lie on the 45° degree line, downside ΔMCoVaR values are significantly different from the VaR values among all the exchange rates for each commodity.

Therefore, it is important to regulate the forex market and to hedge FX exposure based not only on exchange rate' risk in isolation but also take into account the *conditional* tail-risks coming from the commodities market.



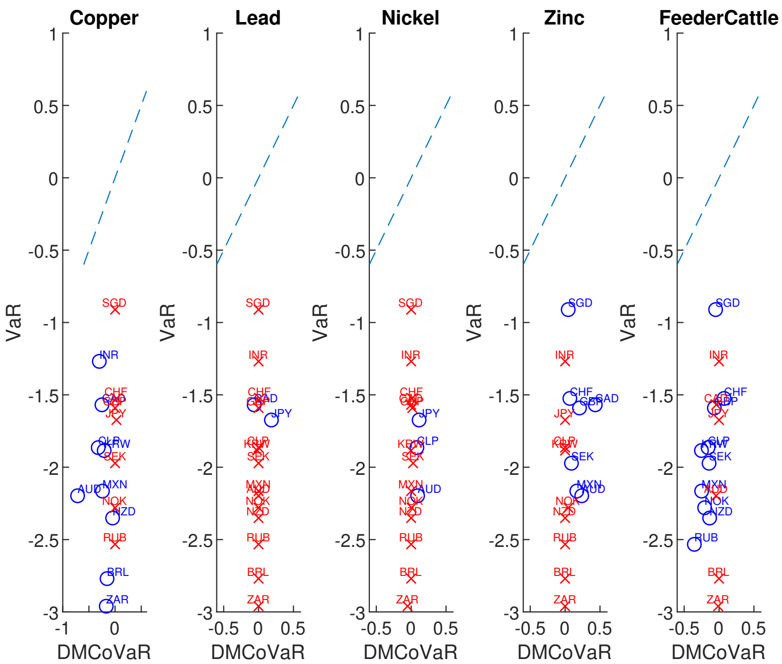
(a)



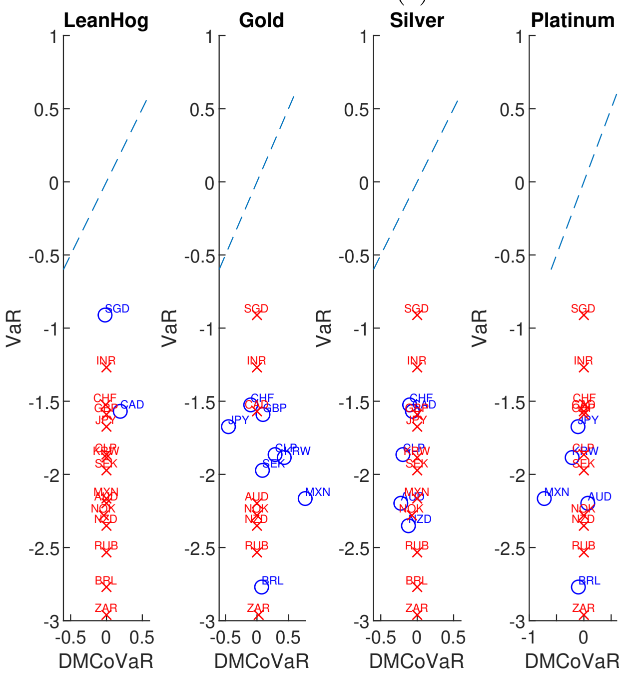
(b)

Figure 3: VaR vs ΔMCoVaR

The scatter plot, inspired to Figure IV.1 of [Adrian and Brunnermeier, 2016b], shows the weak correlation between exchange rates' risk in isolation, measured by VaR (y-axis), and exchange rates' tail-risks to extreme commodity returns, measured by the ΔMCoVaR (x-axis). The VaR and ΔMCoVaR are unconditional 99% measures and are reported in daily percent returns. ΔMCoVaR is the difference between the exchange rates-i's VaR conditional on commodity-j's distress and the exchange rates-i's VaR conditional on commodity-j's median state. The exchange rates (our y variables) and commodities (our x variables) names are listed Section 2. Red crosses represent values of the ΔMCoVaR that are not statistically different from zero and/or are not selected by the MCoVaR with Elastic-Net method.



(c)



(d)

Figure 3: VaR vs ΔMCoVaR
The scatter plot, inspired to Figure IV.1 of [Adrian and Brunnermeier, 2016b], shows the weak correlation between exchange rates' risk in isolation, measured by VaR (y-axis), and exchange rates' tail-risks to extreme commodity returns, measured by the ΔMCoVaR (x-axis). The VaR and ΔMCoVaR are unconditional 99% measures and are reported in daily percent returns. ΔMCoVaR is the difference between the exchange rates-i's VaR conditional on commodity-j's distress and the exchange rates-i's VaR conditional on commodity-j's median state. The exchange rates (our y variables) and commodities (our x variables) names are listed Section 2. Red crosses represent values of the ΔMCoVaR that are not statistically different from zero and/or are not selected by the MCoVaR with Elastic-Net method.

3 Banks, Sovereign Holdings and Systemic Risk

3.1 Introduction

Financial institutions are the main holders of sovereign debt. The origin of this nexus between the banking sector and the asset class of public debt is long standing. Indeed, public debt has always been regarded as a safe asset and, as such, banks tend to hold it to fulfill capital requirements.

The European regulation regime has further contributed to encourage financial institutions in holding sovereign debt. The prudential framework, known as the "large exposure regime", forces banks to limit their exposure to a single issuer or creditor. Such provision is aimed at preventing an institution to incur disproportionately losses as a result of the failure of an individual counter-party. This regime, however, does not apply to sovereign debt. Moreover, all the public debt issued by a European government are considered safe in prudential terms. This means that banks do not have to set aside any capital buffer to cover possible failure of the issuer with respect to European government debt.

The provision was motivated by the construction of the Euro Area: a binding agreement between European countries to join a single currency area. The adoption of a single currency implied a single monetary policy regime that, in turn, prevented national Authority to devalue national currency. This constraint, together with the fiscal provisions set out in the European Treaties, made theoretically safe the public debt issued by European countries.

The aforementioned combination of fiscal and monetary regime allowed, at the beginning of the years 2000, to bring down national spreads computed as the difference between the interest rate borne by national debt and a benchmark, safe, asset. Thanks to the closing gap in the European spreads, and due to the increasing financial integration, the banking sector began to reshuffle their sovereign holdings composition. With respect to Italy, indeed, the banking sector, that before the introduction of the Euro held the 80% of the Italian debt, ended up with less than 50% at the onset of the Great Financial Crisis.

The eruption of the subprime mortgage bubble led, soon, to the European Debt Crisis. Investors started to doubt about the ability of some countries to repay their debt, and the spread widened. The European banking sector has, then, recorded a phenomenon known as flight home: financial institutions divest their holdings of foreign public debt in exchange of national one. Still focusing on the Italian sector it is possible to see how the holding of national debt, after the minimum of 50% in 2008, reached soon pre-euro level. Figure 4 displays the evolution of the Italian banking holdings of domestic debt.

From an empirical point of view, after the European Debt Crisis and the subsequent increase in holdings of domestic debt by the national banking sector, the dependence of the two sectors started to increase. Indeed, the correlation between the returns of national banks' stocks and the returns on sovereign debt increased. In Italy, as documented by Figure 5, the correlation between the returns of the 7 highest capitalized banks stocks and the returns of government debt spiked from an average of almost zero before the crisis to an average of 0.4. The link between the two sectors becomes relevant in the case of an adverse shock hitting the public sector. In this sense, once investors start to fear the the Government would not be able to repay its debt, the value of this asset declines. As long as banks hold significant amount of sovereign debt, the balance sheet value of the asset declines too, eroding the capital buffer of the financial institutions. This erosion could eventually limit the ability of the bank to provide credit to the private sector, hence reducing the ability of the public sector to increase tax revenues and consolidate the financial position.

This transmission mechanism, named the sovereign - bank nexus, has became one of the main concerns for regulators and policymakers. Having realized this risk,

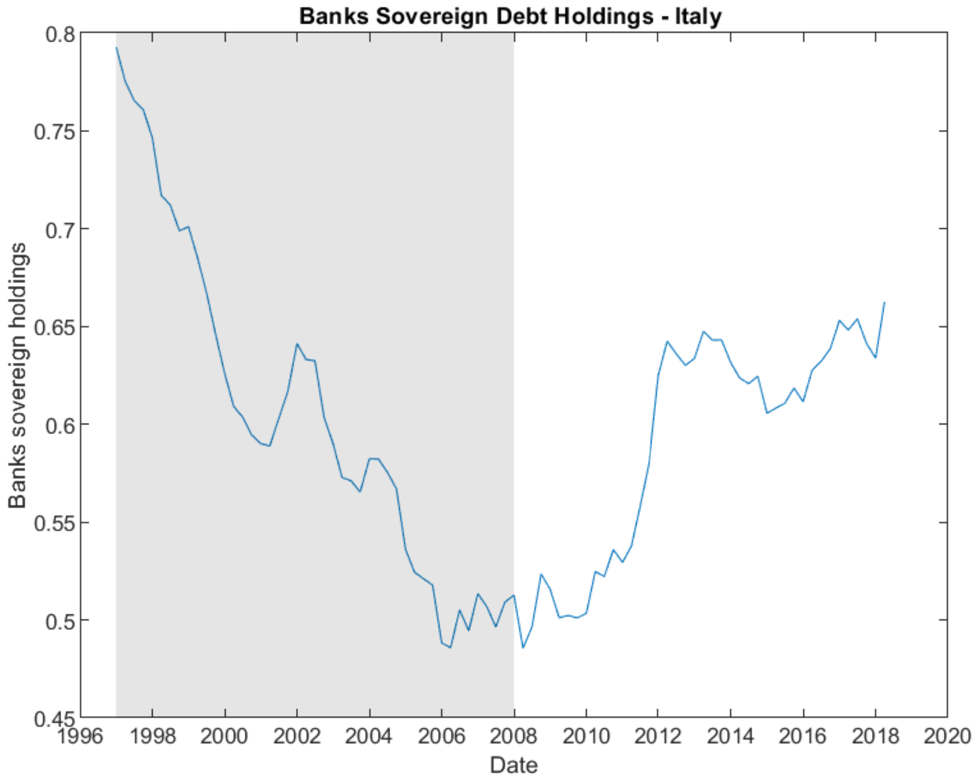


Figure 4: Bank Holdings of Italian Debt
The figure reports, as balance sheet percentage, Italian debt held by Italian banks. Data are monthly and come from the Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012), covering the years between 1997 and 2019.

different proposals have been made to break the nexus. These proposals rely on the benefits of a diversified balance sheet, which would reduce the dependence of one sector on the other.

It is then crucial to understand how do banks' risk profile will change following a re-organization of their balance sheets, towards different sovereign debt assets. This paper represents one of the first attempt to estimate the effects of a policy provision aimed at reducing the sovereign - bank nexus. Breaking the nexus should be beneficial in terms of the contribution that a financial institution provides to the overall risk of the market. To this end, throughout the paper we rely on the systemic risk measure proposed by [Adrian and Brunnermeier, 2016a].

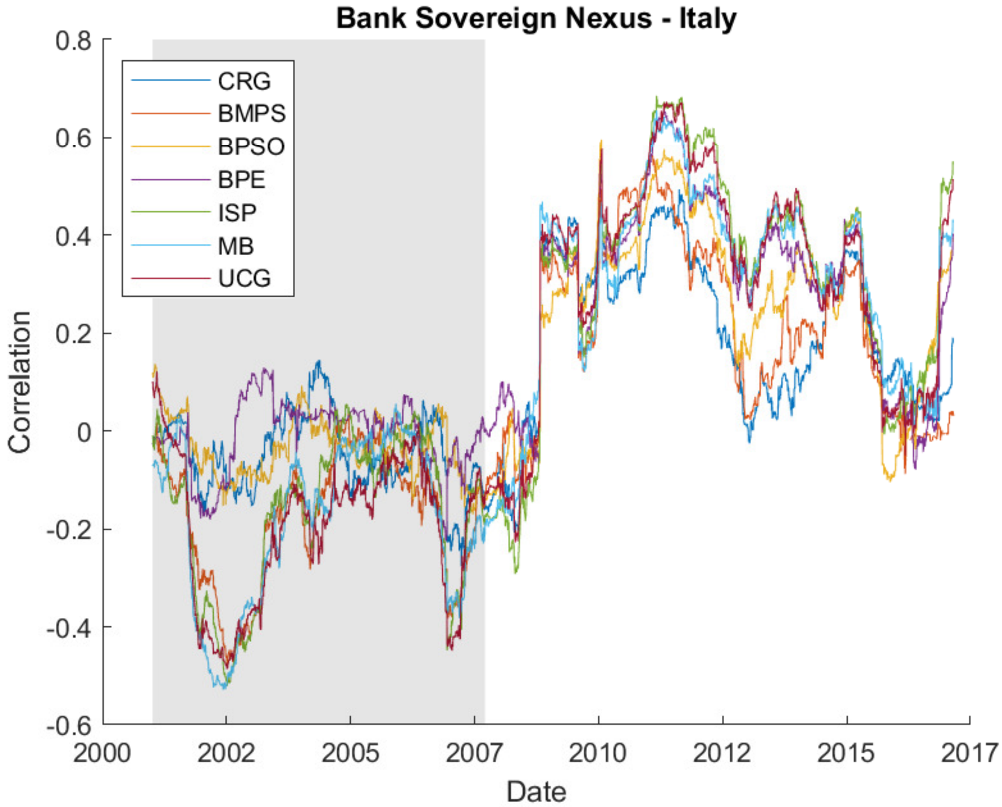


Figure 5: Banks' and Government Debt Correlation
The figure reports the correlation between Italian banks' stock and Italian government debt returns. The Italian banks are the seven highest capitalized bank, and the Italian government debt is the 10 years treasury bill. Data are daily and come from Thomson Reuters Datastream, for the time span 2000/2017.

To quantify the effect of a policy measure aimed at reducing the dependence between domestic debt and the banking sector we need a model that allows to simulate bank's performances in an alternative scenario. In this work we rely on the methodology proposed by [Frazzini et al., 2013] to reproduce banks' characteristics and risk profiles. Eventually, in the spirit of this model, we are able to change banks' balance sheet composition so as to estimate changes in the new risk profile.

In this sense, preliminary conclusions evidence that a factor model based on sovereign debt is able to replicate banks' performances and risk profiles. Further, it is possible to estimate potential benefits of a policy aimed at reducing banks' exposure towards domestic debt. The simulated effects of this policy are mixed: indeed it could reduce systemic risk in peripheral countries, while on the other side could have a negative impact for banks in countries like UK, Swiss and Sweden and, finally, ambiguous effects on core countries' banks.

3.2 Literature Review

The main theoretical contribution to the topic has been given by [Brunnermeier et al., 2016] and [Farhi and Tirole, 2018] where both formalize how a debt crisis may be transmitted to the banking system and, consequently, impact the real economy, eventually generating a loop, due to a worsening in the current account. The central point of these models is that shocks of bad state of nature which hit the government are automatically transmitted to financial institutions that make use of these assets as reserves of capital. As reserves shrink, banks are less able to provide credit, further weakening the health of public finances and, consequently, further eroding the value of banks' reserves. Both works contain policy prescriptions aimed at reducing the effect and the surge of this adverse loop and, not surprisingly, both suggest the introduction of more risk sharing across European countries. In this way the initial shock will not instantaneously get transmitted to the banking sector, hence providing a shield for the sustainability of the availability of credit. Our contribution to this strand of the literature consists in showing how the bank-sovereign nexus could be used to reproduce banks' performances. In particular, by means of a factors model, it is possible to construct synthetic portfolios composed by sovereign debts that replicate banks' stocks returns.

From an empirical point of view [Dell'Ariccia et al., 2018] have worked on the causes of the bank-sovereign nexus. They found that there are three channels through which they are connected: firstly banks have incentives to hold government debt because of their need for safe asset to be used as reserve; secondly domestic banks enjoys state guarantee on deposits, hence government will be forced to bail-out deposits in case of bank failure, accumulating more debt; lastly, both banks and government are affected by the business cycle fluctuations. All these three channels seem to have an impact on determining the aforementioned nexus. It has been documented by [Giannetti and Laeven, 2012] that in the awake of international turmoil, domestic banks are less prone to lend abroad. In particular, in response to an adverse shock to the business cycle, banks respond by lowering their credit abroad and increasing their position in the domestic market. This phenomenon has been called by the authors as *flight home* and it is the forerunner of the home bias. Further analysis of the dynamic link between debt and bank is that of [Buch et al., 2016], that monitor the conduct of German banks in the aftermath of the debt crisis. Their work shows how, in Germany, only a bunch of banks used to hold debt assets and, furthermore, they reallocate such holdings towards assets with low risk and lower yields. This works has two fundamental results: first it shows how in a country with good economics fundamentals, banks prefer to invest in safer assets, hence suggesting that different behavior should be observed between banks in countries with good or bad fundamentals and, second, from a systemic point of view, the entire industry becomes less risky in response to adverse shock. In this work we suggest that a measure of risk sharing based on a diversified banks' portfolios may be beneficial for some countries but also detrimental for some others.

The last part of this review is related to the so-called systemic risk, defined as the risk that a distress of a single financial institute transmits to the entire system, impairing the correct functioning of the latter. The literature has proposed different measures of such risk, namely the CoVaR [Adrian and Brunnermeier, 2016a] and the SRISK [Brownlees and Engle, 2017]. CoVaR is defined as the change in the Value at Risk of the financial system conditional on an institution being under distress relative to its median state. Authors' estimates show that characteristics such as leverage, size, maturity mismatch, and asset price booms significantly predict CoVaR. They also provide out-of-sample forecasts of a countercyclical, forward-looking measure of systemic risk, and show that the 2006 value of this measure would have predicted more than one-third of realized CoVaR during the 2007-2009 financial crisis. SRISK is a measure for the systemic risk contribution of a financial firm, and it is computed as the capital shortfall of a firm conditional on a severe market decline, and is a function of its size, leverage and risk. Aggregate SRISK provides early warning signals of distress in indicators of real activity. The literature on CoVaR at European level includes [Borri et al., 2012], which uses this risk measure to compute systemic risk contribution in a large panel of European banks. An important result they found is that banks with headquarters in countries with a more concentrated banking system tend to contribute more to European wide systemic risk, even after controlling for their size. In this paper we make use of CoVaR in the spirit of [Adrian and Brunnermeier, 2016a] to compute the systemic risk contribution of the single financial institute.

3.3 Data and Methodology

One of the main concern with respect to the banking system is systemic risk, defined as the risk that following the bankrupt of one financial institution, the whole market may experience a generalized collapse. As previously mentioned, the excessive holdings of domestic debt may expose the single bank to insolvency risk, due to the reduced value of these assets. Eventually, the crisis of the single bank may cause a generalized crisis for the sector. In this sense, excessive holdings of domestic debt may represent one of the most important fragility in the context of systemic risk. The aim of this paper is to quantify the effects of a policy implementation that modifies banks' exposure to sovereign debt. In this section we will then introduce: i) the risk measure that will be used as benchmark (CoVaR) and ii) the empirical methodology used to conduct the policy experiment. Several scholars have already provided evidence of possible benefits that would reduce after some changes in the public debt market. To this end the most known example is probably that of the ESBies by [Brunnermeier et al., 2016] that suggest how the creation of a synthetic European debt instrument, composed by different tranches of single European debt, would produce beneficial effects in breaking the sovereign - bank nexus. This works aims at extending further this field, providing first quantitative estimates of the beneficial effect of a different bank exposure to sovereign debt. Namely, the beneficial effects would come from a reduction in the systemic risk. In this section we will first provide the methodology to estimate the systemic risk measure (CoVaR), which is the variable through which the effect of the policy measure would be quantified. Following we will review the methodology that allows to simulate the banking characteristics in the alternative scenario, that is when the exposure to sovereign debt has been modified. To do this we will make use of a highly-dimension factor model, and to select only relevant ones, using regularization technique (LASSO) that allows to select only significant factors.

3.3.1 CoVaR

The financial literature has proposed a variety of possible risk measures in the context of systemic risk that is, the risk that the intermediation capacity of the entire financial system is impaired due to the spreading of distress from one institution to the whole system. One of the most famous has been proposed by [Adrian and Brunnermeier, 2016a], known as *CoVaR*. They build on one of the most famous and widely used measure of risk for financial institutions that is, the Value at Risk *(VaR)*, that focuses on the risk of an individual institutions. Specifically the *q% - VaR* is defined as the maximum loss within the *q%*-confidence interval.

$$Pr(X^i \leq VaR_q^i) = q \tag{40}$$

where X^i is the (return) loss of institution i for which the VaR_q^i is defined. In the context of systemic risk the limit of this measure is that does not include the spillovers from other banks. Tail *co*movement have to be considered in order to add the systemic component to the risk measure. Hence, the *CoVaR* of institution *i* relative to the system is then defined as the *VaR* of the whole financial sector *conditional* on institution *i* being in distress.

$$Pr(X^j \leq CoVaR_q^{j|\mathbf{C}(X^i)}|\mathbf{C}(X^i)) = q \tag{41}$$

Where, as in the notation used by [Adrian and Brunnermeier, 2016a] $\mathbf{C}(X^i)$ represents an event occurred to institution i. In particular,$\mathbf{C}(X^i)$ describes an event that makes institution i's loss being at, or above, its VaR_q^i level which, by definition, occurs with likelihood $(1-q)\%$.

If we consider two different quantiles, $\hat{Q}_{0.01}$ and $\hat{Q}_{0.50}$, estimated with a given level of probability, respectively, 1 percent and 50 percent, then we can compute the Δ*CoVaR* as the difference between the two quantiles, which captures the marginal contribution of a specific bank to the entire systemic risk.

$$\Delta CoVar_q^{system|i} = CoVar_q^{system|\hat{Q}^i_{0.01}} - CoVar_q^{system|\hat{Q}^i_{0.50}} \tag{42}$$

There are many advantages that led the *CoVaR* to be one of the most used systemic risk measure, above all the generality of its definition, that allows to study the spillovers from one bank to another in the entire financial network. With respect to the estimation strategy, recall that, by definition the VaR is a quantile of the the returns' distribution. The CoVar is then estimated through means of *quantile* regression of the financial sector returns on a specific bank's returns.

$$\hat{R}_q^{System} = \hat{\alpha}_q^i + \hat{\beta}_q^i R^i \tag{43}$$

By construction $\hat{R}_q^{System}$ is equal to the VaR of the financial system conditional on R^i.

$$CoVar_q^{system|\hat{Q}^i_{0.01}} = \hat{\alpha}_q^i + \hat{\beta}_q^i Q^i_{0.01} \tag{44}$$

$$\Delta CoVar_q^{system|i} = \hat{\beta}_q^i (Q^i_{0.01} - Q^i_{0.50}) \tag{45}$$

3.3.2 Portfolio Replication

The CoVaR provides the systemic risk measure we will use to estimate the change in risk due to a change in banks' balance sheet composition. To compute this measure is however needed a model that allows to simulate banks' returns in a counterfactual scenario. The precise solution would imply decomposing banks' balance sheet, and modifying their composition on a daily basis. It is easy to see that this kind of approach has some limitations, as long as it is tough to reconstruct a daily time series for banks' balance sheet. In this context we then rely on an asset pricing style of analysis.

The intuition goes as follows: we can consider a bank as a portfolio composed by different assets, among them, real investment (i.e. to firms and households) and financial activities (i.e. sovereign debt and other securities). We can replicate such portfolio by means of different factors (Fama-French, Quality Minus Junk. etc.), in particular making use of European Sovereign debt. The replicated portfolio will then have the same characteristics of bank's *portfolio*, and we could then treat the loadings of each factor as the exposure to that security. This approach builds on [Frazzini et al., 2013] that allows to disentangle the factors that determine the returns of a portfolio of assets. This kind of approach has several advantages: firstly it is intuitive and quite simple to implement. Moreover this approach allows to study banks without needs to use balance sheet's micro-data.

The construction of this *synthetic portfolio* implies different phases: First we have to understand which are the relevant *factors* for each bank i, which implies running the following regression

$$r_{it}^{bank} - r_t^f = \alpha_i + \beta_{i1}MKT + \beta_{i2}SMB + \beta_{i3}HML + \beta_{i4}ITGDB10 + \beta_{i5}DEGDB10 + ... + \epsilon_{it} \tag{46}$$

where ϵ represents the error term, distributed as i.i.d. with zero mean and unit variance.

The loadings of this regression represents the exposure of bank i to each factor. Next we want to replicate bank returns, hence we construct a portfolio with the same market exposure which behave in a similar fashion with respect the *i-th* bank.

We then capture the bank market exposure, β^{bank}, by means of a univariate regression of excess return on the market portfolio and we next consider beta-adjusted returns and run a regression on the factors that explain bank performance

$$r_{it}^{bank} - r_t^f - \beta_i^{bank}MKT = \alpha_i + \beta_{i1}SMB + \beta_{i2}HML + \beta_{i3}ITGDB10 + \beta_{i4}DEGDB10 + ... + \epsilon_{it} \tag{47}$$

where ϵ represents the error term, distributed as i.i.d. with zero mean and unit variance.

The returns of the *synthetic portfolio* are then computed as

$$r_{it}^{a} = \hat{\beta_{i1}} MKT + \hat{\beta_{i2}} SMB + \hat{\beta_{i3}} HML + \hat{\beta_{i4}} ITGDB10 + \hat{\beta_{i5}} DEGDB10 + ... \quad (48)$$

We then rescale the return, r_{it}^{a}, as to match the bank idiosyncratic volatility to approximate banks' leverage, where in (49) σ_I represents bank's i idiosyncratic volatility, and $\sigma_{r_t^a}$ is the volatility of the bank's i *replicated* returns (r_t^a). Finally we add back the market exposure and the risk free rate to construct the mimicking style portfolio

$$r_{it}^{adjusted} = r_{it}^{a} * \frac{\sigma_I}{\sigma_{r_{it}^{a}}} \quad (49)$$

$$r_{it}^{bank} = r_t^f + \hat{\beta}_i^{bank} MKT + r_{it}^{adjusted} \quad (50)$$

3.3.3 LASSO

In the context of Factors Model one well-known problem is represented by the selection of significant factors among a pool of plausible candidates. In other words, asset returns may be explained by a lot of *factors*, leading to a *Factors Zoo* [Guanhao et al., 2019]. The shrinkage of the high-dimensional set of possible factors to a feasible number of relevant risk connections is a Model selection problem that has been tackled down by means of Least Absolute Shrinkage and Selection Operator (LASSO) model. LASSO methods are standard for high-dimensional conditional mean regression problems [Tibshirani, 1996].

This method works through the selection of relevant regressors according to the absolute value of their respective estimated marginal effect. LASSO minimizes the residual sum of squares subject to the sum of the absolute value of the coefficients being less than a constant.

The LASSO procedure entails a *penalty* for non-zero coefficients, imposed on the sum of the absolute value (L1 penalty). The estimator, $\hat{\beta}_{LASSO}$ is then the β that solves[22]:

$$\underset{\beta}{minimize} \left\{ \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 \right\} \quad \text{subject to} \quad \sum_{j=1}^{p} |\beta_j| \leq s \tag{51}$$

which could be rearranged, such that the estimator is the β that solves

$$L_{LASSO}(\hat{\beta}_{LASSO}) = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{p} |\hat{\beta}_j| \tag{52}$$

The solution to the minimization problem is a quadratic programming problem with linear inequality constraints. LASSO de-selects those regressors that contribute only little variation, and once the regressors are selected, the unrestricted model is re-estimated. It is important to note that factors selection via LASSO crucially depends on the choice of the company-specific penalty parameter(λ). The larger the penalty the more regressors are eliminated. The process of selecting the proper value of λ, also stated as "model selection", is based on the cross-validation approach[23]. The goal of model selection is choosing the parameter of the model on which the estimator depends (in this case, λ) so that we can minimize the Prediction Error(PE). The problem emerges as, usually, we don't have the so-called *test data* $(y_1^i, ..., y_n^{'})$. If that would have been the case, we could have computed the *test error* as an estimate for PE. Then, when we could not observe the test data, we have to compute the *training error*, which is similar to the test error, but it is computed using the data which lead to the computation of the estimator. The training error rate often is quite different from the test error rate, and in particular the former can dramatically underestimate the latter.

In the absence of a very large designated test set that can be used to directly estimate the test error rate, a number of techniques can be used to estimate this quantity using the available training data. *Cross validation* is one of the aforementioned techniques

[22]Note that n is the number of observation and p is the number of predictors

[23]For more on the cross-validation approach see section 2.6.2

and is based on estimating the test error rate by holding out a subset of the training observations from the fitting process, and then applying the statistical learning method to those held out observations.

In this *family* of method, the K-fold cross validation is the most used. Under this approach, the training set is split in K different "folds" of roughly equal size $(F_1, ..., F_K)$. Then, for $k = 1, ..., K$ we consider (x_i, y_i) such that $i \notin F_k$ as the observation in the training set, and (y_i, x_i), $i \in F_k$ as those belonging to the validation set. Then, given the spectrum of possible values for λ, we compute the estimator on the training set, and then we calculate the error on the validation set. Finally, for each λ we compute the average error over all folds which leads to the *cross validation error curve*, that is a function of the parameter, which reports the CV error for all the values of λ. We will choose the value of the parameter that minimizes this curve

3.3.4 Data

The dataset that we used is composed by daily data. We collect data from Thomson Reuters Datastream and the Fama French online library. We have data for 7 European countries, specifically Germany, France, the Netherlands, Italy, Spain, Portugal and Belgium.

Table 7: Descriptive Statistics - Banks

Banks					
Country	N	Mean	Std	Max	Min
Greece	4	-0.0	0.02	0.13	-0.17
Ireland	1	-0.0	0.02	0.17	-0.34
Spain	2	-0.0	0.01	0.09	-0.09
United Kingdom	4	-0.0	0.01	0.17	0.17
Italy	7	-0.0	0.01	0.10	-0.18
Portugal	1	-0.0	0.01	0.10	-0.07
France	3	0.0	0.01	0.11	-0.09
Germany	2	-0.0	0.01	0.09	-0.10
Switzerland	3	-0.0	0.01	0.08	-0.07
Denmark	3	0.0	0.01	0.06	-0.07
Belgium	2	-0.0	0.02	0.24	-0.16
Austria	1	0.0	0.01	0.07	-0.09
Sweden	4	0.0	0.01	0.08	-0.07

Notes: This table reports mean, standard deviation, maximum value and minimum value for the log daily returns on banks' stocks. Mean, standard deviation, maximum and minimum are in percentages.Data are daily, for the period 01/2004 to 06/2020, and retrieved from Datastream.

For each country we have the price of three different government debt indexes, of different maturity. Specifically we have 10, 5 and 2 years government debt assets. We then have daily bank's stock price for different national banks. For those assets, returns are computed taking the difference of the price in log scale. As a risk free rate we took daily returns of the Euribor. Descriptive statistics for the sample of European banks are reported in Table 7,

Finally we have daily returns of the Fama French factors, *SMB, HML and MOM*.

Table 8: Descriptive Statistics - Factors

Factors					
Factor	Mean	Std	Min	Max	Obs
MKT	0.0	0.012	-0.080	0.118	4612
SMB	0.0	0.006	-0.042	0.043	4612
HML	0.0	0.004	-0.031	0.048	4612
MOM	0.0	0.008	-0.050	0.055	4612
ITGDB10	0.0	0.002	-0.016	0.025	4612
PTGDB10	0.0	0.003	-0.049	0.049	4612
ESGDB10	0.0	0.002	-0.012	0.028	4612
BEGDB10	0.0	0.002	-0.011	0.014	4612
NLGDB10	0.0	0.001	-0.007	0.008	4612
DEGDB10	0.0	0.001	-0.008	0.010	4612
FRGDB10	0.0	0.001	-0.009	0.010	4612
ITGDB2	0.0	0.001	-0.013	0.007	4612
PTGDB2	0.0	0.001	-0.024	0.023	4612
ESGDB2	0.0	0.001	-0.006	0.007	4612
BEGDB2	0.0	0.001	-0.006	0.007	4612
NLGDB2	0.0	0.000	-0.003	0.002	4612
DEGDB2	0.0	0.000	-0.002	0.002	4612
FRGDB2	0.0	0.000	-0.002	0.002	4612

Notes: This table reports mean, standard deviation, maximum value and minimum value for the log daily returns of factors. Mean, standard deviation, maximum and minimum are in percentages.Data are daily, for the period 01/2004 to 06/2020, and retrieved from Datastream.

Those returns are the returns of different portfolio constructed by sorting stocks according to a criteria, and then going long on the first stocks and short on the last ones. In the case of SMB stocks are sorted according to their capitalization. For HML are sorted according to their book-to-market value, while for MOM it is considered past performances. Table 8 displays summary statistics for the factors.

3.4 Results

In this section we report the result of the empirical analysis. As laid down in the previous section, the analysis entails different steps: we start by showing how does the model by [Frazzini et al., 2013] works in replicating banks' portfolios. In doing so we will report: *i)* the loading of each factors for different banks[24] in the dataset and *ii)* how these synthetic portfolios replicate banks' performances. Following we will focus on *CoVaR* estimation. In doing so we will first estimate the *CoVaR* for the true banks and then for the *replicated* ones. We will finally present the estimate for the counterfactual policy experiment. To do so we will assume a different and fixed banks' balance sheet composition, and then re-estimate our systemic risk measure, as to understand the effects of the policy.

3.4.1 Portfolio Replication and LASSO Selection

The first step of the portfolio replication is aimed at selecting the relevant factors. This has been done by means of the LASSO selection procedure. Results display some level of heterogeneity, indeed the Market factor, as well as the Italian, Portuguese, German and Belgium 10 years government debt factors, are significant for all the banks. By contrast the Small Minus Big factor (SMB), the Italian and Spanish 2 years government debt factors, are relevant only for a fraction of banks. This evidence is consistent with the hypothesis that banks' balance sheets are composed in a different way.

Once we have obtained the significant factors, we estimate the exposure of each bank to the market by means of a univariate regression of excessive returns on market returns. All the returns have then been *de-marketed* that is, to each bank it has been subtracted the market component ($\hat{\beta}^{bank} * mkt$). This newly computed returns has then been regressed against relevant factors computed with LASSO The loadings of this

[24] In this section we will report results for five representative banks, leaving to the appendix the entire dataset. We pick one bank for five different countries: Italy, Germany, France, UK and Spain.

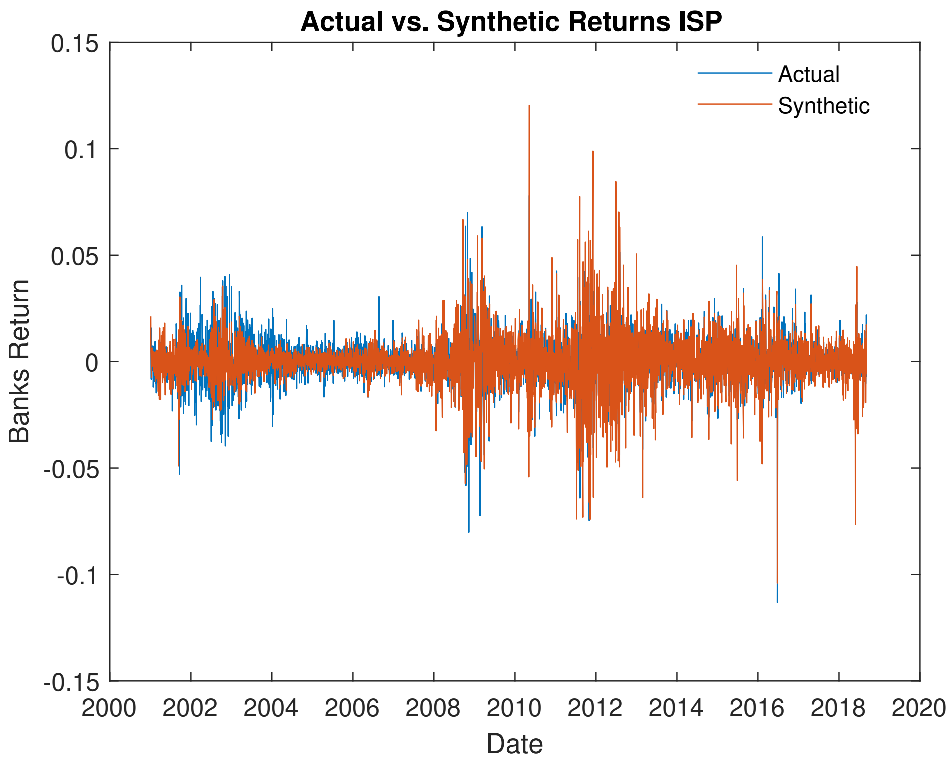
Table 9: Factor Loadings

Factor Loadings					
Factor Loading	ISP	DBKX	BBVA	BARC	SGE
Const					
MKT	0.02	0.01	0.02	0.01	0.02
SMB	-0.03	0.02			
HML					
MOM	0.04		0.03	0.04	0.02
ITGDB10	1.40	0.74	0.59	0.70	0.84
PTGDB10	0.21	0.11	0.27	0.34	0.24
ESGDB10	0.25		0.34		0.18
BEGDB10	1.20	0.61	0.94	1.24	1.15
NLGDB10	-0.26				
DEGDB10	-2.90	-2.51	-2.88	-3.05	-3.61
FRGDB10	-0.72				
ITGDB2	0.14			-0.49	
PTGDB2	-0.33		-0.33	-0.20	
ESGDB2	0.20		0.64	-0.93	0.19
BEGDB2					
NLGDB2	-0.92	-2.53	-0.46	-2.49	-2.04
DEGDB2	-2.38	-2.78	-2.66	-4.63	-4.12
FRGDB2	-1.71	-1.55	-3.25		

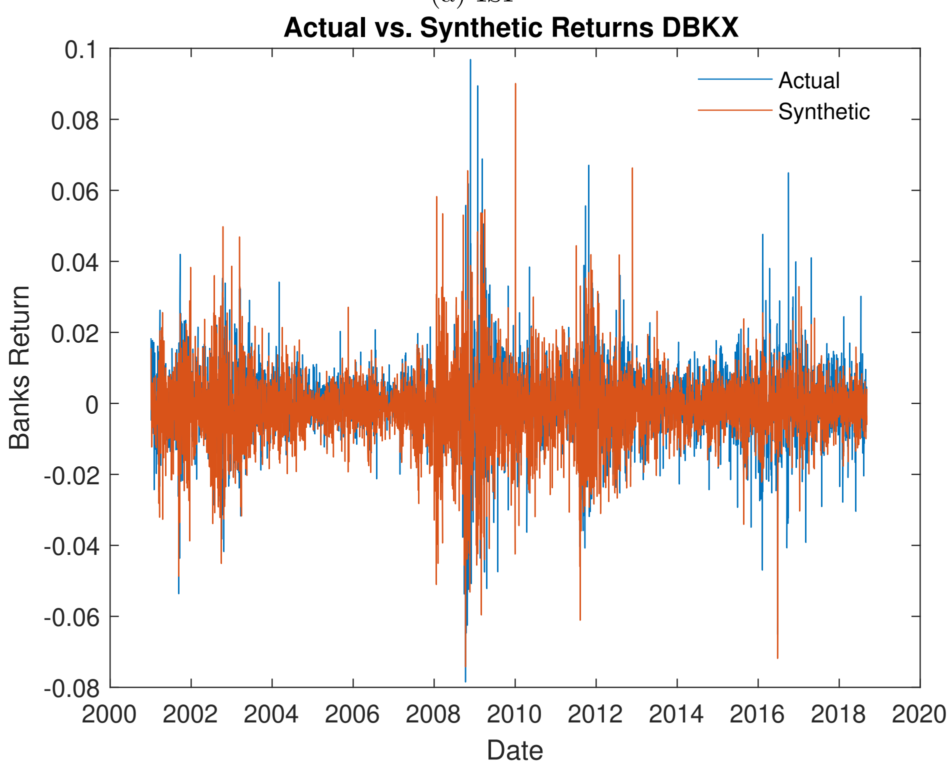
Notes: This table reports the coefficients of the multivariate regression of bank returns on relevant factors returns for the 5 banks used as benchmark.

regression represent the exposure of the bank to each factor. Table 9 reports the results of this estimation. What we can observe is that there is a strong degree of correlation among banks with respect to the sign of their exposure to the same factor, while there is huge discrepancy with respect to the magnitude of this exposure. We have finally computed synthetic returns by fitting data, matching volatility and adding back the risk-free rate of return and the market component. In Figure 6 below we can see how synthetic returns behave compared to true ones.

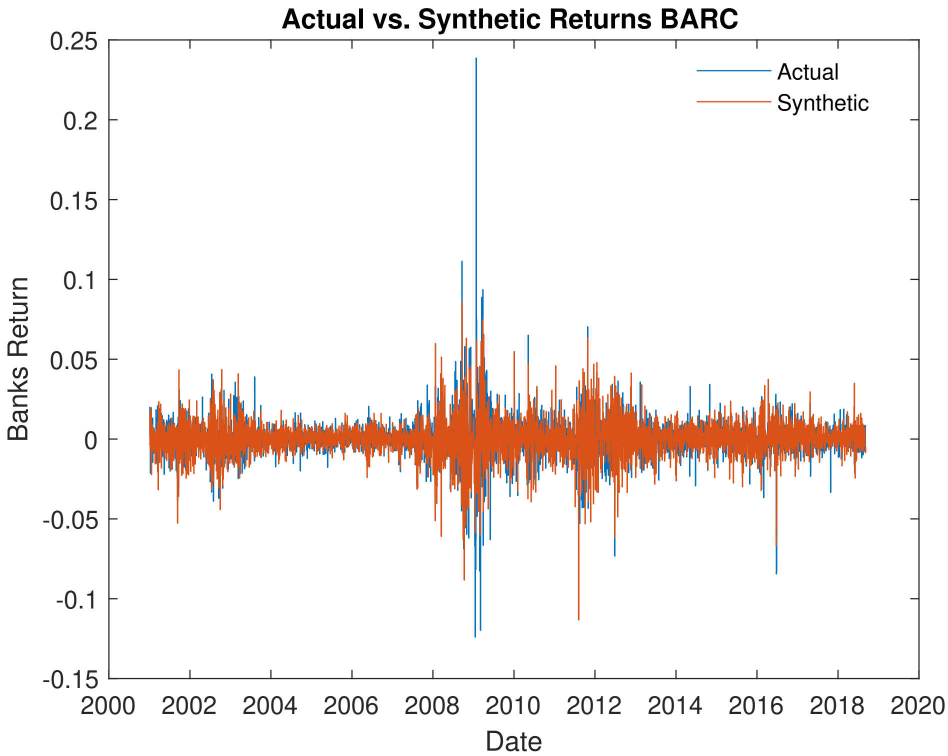
True vs. Synthetic Banks' returns



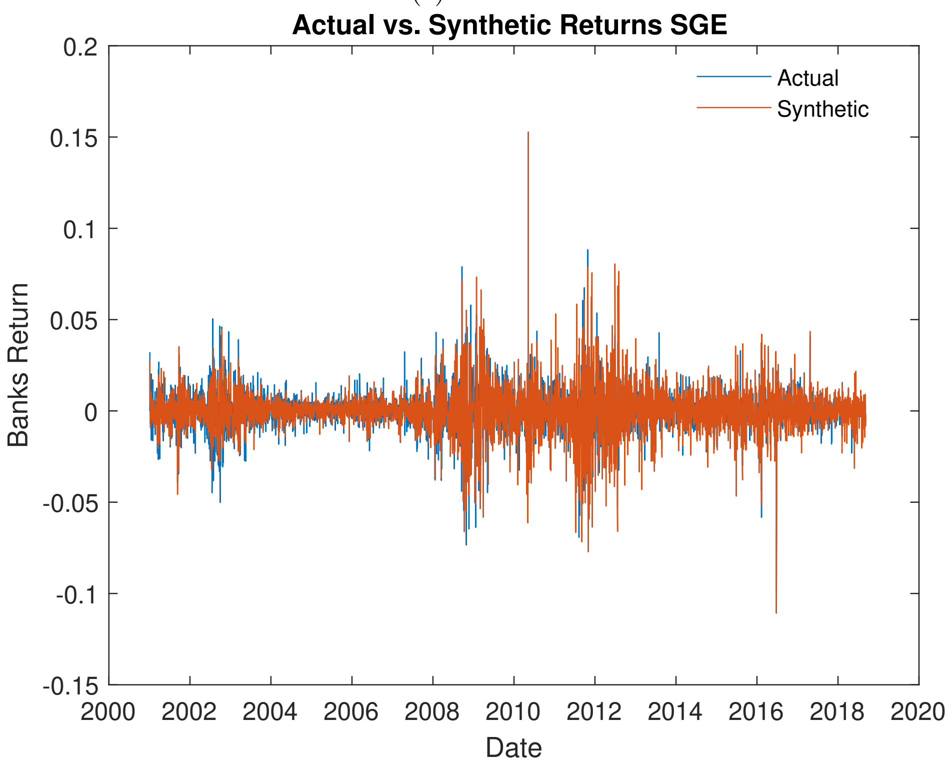
(a) ISP



(b) DBKX



(c) BARC



(d) SGE

Figure 6: **Actual vs. Replicated Banks Returns** The figure reports banks returns, both true ones that the replicated ones. The correlation coefficients are equal to: a) 0.71 for ISP; b) 0.63 for DBKX; c) 0.72 for BARC; d) 0.68 for SGE.

3.4.2 True Banks vs. Synthetic - CoVaR and Δ CoVaR

In the previous subsection we report how do replicated returns performed in comparison with true data.

Table 10: CoVaR and ΔCoVaR (Sub Sample)

CoVaR and ΔCoVaR						
Bank	CoVaR	Synthetic CoVaR	Error	ΔCoVaR	Synthetic ΔCoVaR	Error
ISP	-3.05*	-3.12**	0.07	-1.79*	-1.95*	0.16
DBKX	-3.09*	-2.95*	-0.14	-1.82*	-1.66*	-0.16
BBVA	-2.99*	-2.93**	-0.06	-1.88*	-1.92*	0.05
BARC	-2.68*	-2.93**	0.25	-1.44*	-1.46	0.02
SGE	-2.95*	-2.98***	0.03	-1.83**	-1.96**	0.13

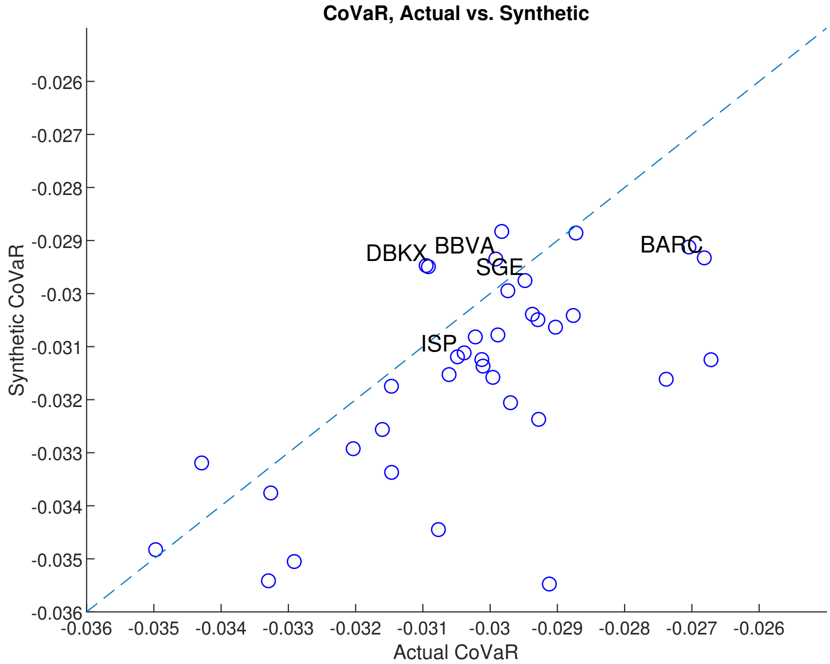
Notes: This table reports the estimates for CoVaR and ΔCoVaR, both real and synthetic ones. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level.

In this subsection we will turn to the estimation of the systemic risk measure, the aforementioned *CoVaR*. We have then estimated *CoVaR* twice, one with true data and one with the synthetic portfolios. This has been done by means of quantile regression. In the figure below we have then plotted the true measure against its synthetic counterpart.

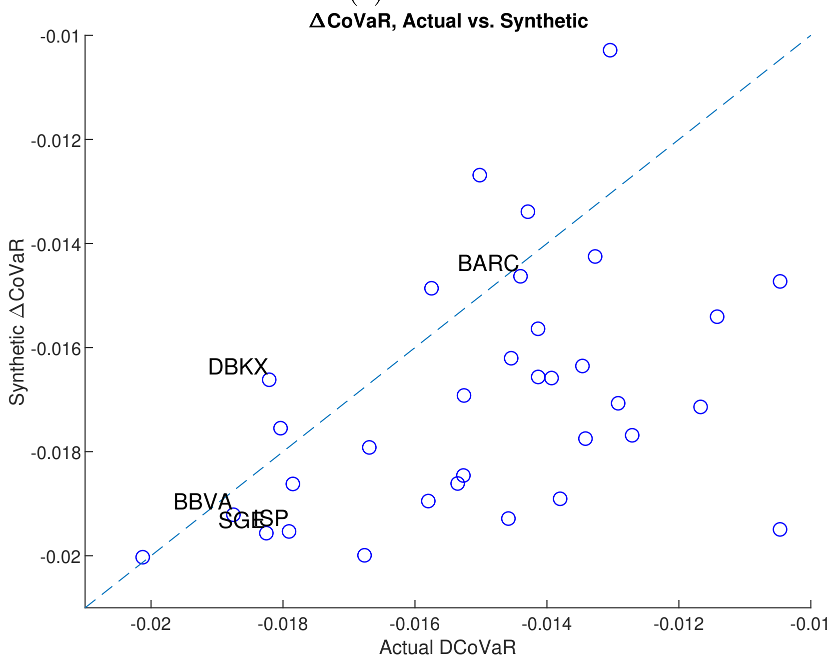
A perfect match would imply all the points to lie on the 45 degrees line. As we can see from the figure the two estimates are quite similar. On average, the difference between the true measure and the synthetic one is 0.0013 for the CoVaR and 0.0024 for the ΔCovar.

The bottom line of these estimates is that our model is able to replicate banks' behaviour and banks' risk profile. We can then build on this model a counterfactual exercise.

Real vs. Synthetic Banks' CoVaR and ΔCoVaR



(a) CoVaR



(b) ΔCoVaR

Figure 7: **Actual vs. Replicated ΔCoVaR** The figure reports banks' CoVaR and ΔCoVaR , both the true ones and the replicated ones.

3.5 Policy Experiment

The two previous subsections allowed us to understand whether our synthetic portfolio replication of banks' characteristics was able to match the returns time series, as well as the true estimate of systemic risk.

As we saw, both synthetic returns and CoVaR have a good level of fit with respect to true data. At this stage we then have a factor model, mainly based on European sovereign debt, that is able to replicate banks' behaviour. The intuition is that each factor loading could be interpreted as the exposure of the single bank towards that factor. We could exploit this interpretation by arbitrarily change the factor loading so as to reshuffling banks' portfolios.

Our policy experiment is aimed at quantifying the potential effect of a regulatory reform to limit excessive exposure towards sovereign debt. In this sense, different kind of hypothesis and simulation could be performed[25].

Table 11: Balance Sheet Composition

Balance Sheet Composition	
Balance Sheet Exposure Towards	Percentage
Belgium	3
Germany	21
Spain	8
France	15
Italy	11
Netherlands	5
Portugal	1

Notes: This table reports the balance sheet composition, with respect to sovereign exposure, for banks in case of the policy experiment.

[25] As an example we could also implement a time-varying coefficient. For example, taking into account the potential negative effect of reducing debt holdings during recession, banks could be allowed to increase their exposure over a given threshold during crisis, but then counterbalancing this over-exposure in the following periods.

In this work, we decided to focus on the aforementioned ESBies[26] kind of proposal by [Brunnermeier et al., 2016], that, pulling together all the European debt assets, would result in a *safe European sovereign debt.* In this spirit, the resulting asset could be composed by the different sovereign debt in proportion to their contribution to the European GDP. Table 11 represents the balance sheet of a bank whether such a policy would be implemented.

Table 12: CoVaR and ΔCoVaR - Experiment (Sub Sample)

CoVaR and ΔCoVaR Experiment						
Bank	CoVaR	Synthetic CoVaR	Difference	ΔCoVaR	Synthetic ΔCoVaR	Difference
ISP	-3.12**	-3.15*	0.03	-1.95*	-1.82*	-0.14
DBKX	-2.95*	-3.14*	0.19	-1.66*	-1.86	0.20
BBVA	-2.93**	-2.99*	0.06	-1.92*	-1.92*	0.00
BARC	-2.93**	-3.16*	0.22	-1.46	-1.82	0.36
SGE	-2.98*	-3.01*	0.03	-1.96**	-1.92*	-0.04

Notes: This table reports the estimates for CoVaR and ΔCoVaR, both experimental and synthetic ones. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level.

With this new banks' balance sheet composition, we can simulate what would have been the returns on their stocks and, then, estimate our systemic risk measure. Table 12 reports the results for the 5 banks that we used as benchmark, while figure 8 provides a graphical intuition of the results for all the banks in the sample, divided by the effect of the reform. Interestingly, we could observe three different groups: *i)* Beneficiaries of this reform, that is banks located in peripheral countries, such as Italy, Greece and Portugal, will see their systemic risk contribution reduced after the reform. On the other side we have the *ii)* Hurt by the reforms, that is banks located in EU countries but not belonging to the Eurozone, hence having a sovereign national currency

[26]From [Brunnermeier et al., 2016] "*A public or private entity purchases a diversified portfolio of euro area sovereign bonds, weighted according to a strict, well-defined rule, such as euro area countries' relative GDPs or contributions to ECB capita*"

different from the Euro. For these banks, the reform would imply a negative effect in terms of systemic risk. Finally we have the *iii)* Ambiguous banks, those banks located in core-Euro countries, where the effect is mixed, for some national banks is positive (e.g. Commerzbank in Germany, Jyske Bank in Denmark) while for others is negative (e.g. Deutsche Bank in Germany, Danske Bank in Denmark).

These results are consistent with the presence of idiosyncratic risk, based on national characteristics, related to the real economy that could not be differentiated solely with a balance sheet reform.

3.6 Conclusion

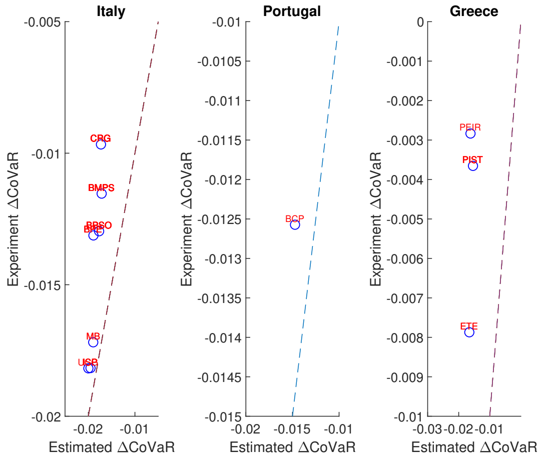
In the awake of the European Sovereign Debt crisis many policy discussion have been focused on the potential benefits of different rules aimed at reducing sovereign debt concentration. An optimal approach would imply disentangling banks' balance sheet and ri-organizing them following the new hypothetical regulation. This kind of approach has several limitations, above all the impossibility to reconstruct daily changes in balance sheet composition.

This paper represents an attempt to quantify potential benefits of a re-modulation of banks' balance sheet in terms of Systemic Risk, by making use of a Factor-Based model à la Fama and French. The advantage of using this approach is that it allows to perform a day-by-day analysis, does not require particular assumptions on banks' behavior and uses public available data relying on Efficient Market Hypothesis.

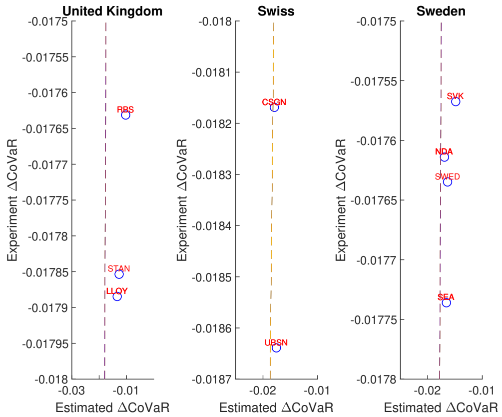
Preliminary conclusions evidences that: *i)* a factor model based on sovereign debt is able to replicate banks' performances and risk profiles. Further, *ii)* by making use of this model it is possible to estimate potential benefits of a policy aimed at reducing banks' exposure towards domestic debt. Finally, *iii)* the estimated effects of this policy are mixed: indeed it could reduce Systemic Risk in peripheral countries, while on the other side could have a negative impact for banks in countries like UK, Swiss and

Sweden and, finally, ambiguous effects on core countries' banks .

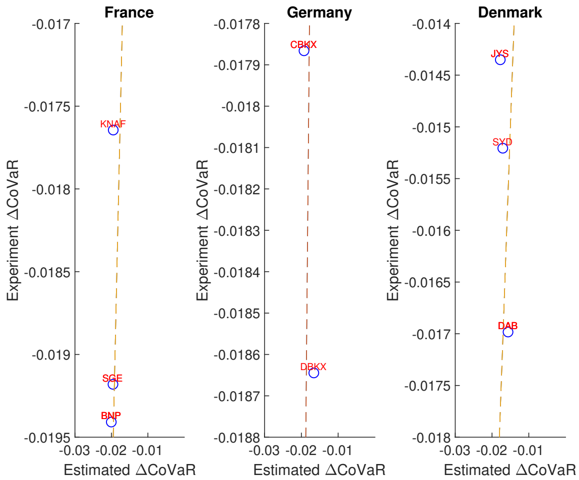
Synthetic vs. Experiment Banks' ΔCoVaR (per Group)



(a) Beneficiaries



(b) Hurt



(c) Ambiguous

Figure 8: **Actual vs. Replicated CoVaR and ΔCoVaR** The figure reports banks' ΔCoVaR , both the replicated ones and the experiment ones. The upper panel reports results for the *Beneficiaries* group; the central one the *Hurt* group and the lower panel the *Ambiguous* group.

3.7 Appendix

In this section we have reported three tables. Table 13 reports banks' names and the associated tickers for the entire dataset. Table 14 and 15 report the CoVaR and ΔDCoVaR before and after the policy experiment for all the banks in the dataset, respectively. Finally, Figure 9 reports the graphic estimate of the real vs. synthtetic banks' ΔDCoVaR divided per peripheries, core and own currency issuer countries, while Table 16 reports the banks' balance sheet exposure in the second policy experiment, and Figure 10 the graphical results.

Table 13: Bank Ticker

	Bank Ticker
Ticker	Bank Name
PIST	ALPHA BANK
BKIR	BANK OF IRELAND
BKT	BANKINTER
BARC	BARCLAYS
CRG	BANCA CARIGE
BMPS	BANCA MONTE DEI PASCHI
BPSO	BANCA PPO.DI SONDRIO
BPE	BANCA PPO.EMILIA ROMAGNA
BBVA	BBV.ARGENTARIA
BCP	BANCO COMR.PORTUGUES
SCH	BANCO SANTANDER
BNP	BNP PARIBAS
CBKX	COMMERZBANK (XET)
CSGN	CREDIT SUISSE GROUP
DAB	DANSKE BANK
DBKX	DEUTSCHE BANK (XET)
DEX	DEXIA
EFG	EFG EUROBANK ERGASIAS
ERS	ERSTE GROUP BANK
SGE	SOCIETE GENERALE
ISP	INTESA SANPAOLO
JYS	JYSKE BANK
KB	KBC GROUP
LLOY	LLOYDS BANKING GROUP
MB	MEDIOBANCA
ETE	NATIONAL BK.OF GREECE
KNAF	NATIXIS
NDA	NORDEA BANK
PEIR	BANK OF PIRAEUS
RBS	ROYAL BANK OF SCTL
SEA	SEB
STAN	STANDARD CHARTERED
SVK	SVENSKA HANDBKN
SWED	SWEDBANK
SYD	SYDBANK
UBSN	UBS
UCG	UNICREDIT
VATN	VALIANT

Notes: This table reports the tickers of each single bank in the dataset.

Table 14: CoVaR and ΔCoVaR (Full Sample)

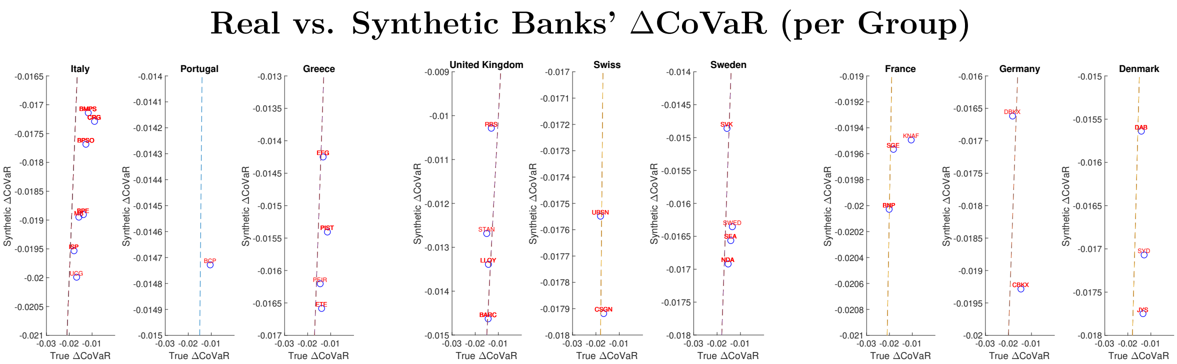
CoVaR and ΔCoVaR						
Bank	CoVaR	Synthetic CoVaR	Error	ΔCoVaR	Synthetic ΔCoVaR	Error
PIST	-3.15*	-3.34***	0.19	-1.14*	-1.54**	0.40
BKIR	-2.74	-3.16*	0.42	-0.87	-1.24*	0.37
BARC	-2.68*	-2.93**	0.25	-1.44*	-1.46	0.02
CRG	-2.91	-3.55*	0.64	-0.89	-1.73**	0.84
BMPS	-3.08**	-3.44**	0.37	-1.17*	-1.71***	0.55
BPSO	-3.33	-3.38*	0.05	-1.27	-1.77*	0.50
BPE	-3.29	-3.51**	0.21	-1.38	-1.89	0.51
BBVA	-2.99*	-2.94**	-0.06	-1.88*	-1.92*	0.05
BCP	-2.97	-3.21	0.24	-1.05	-1.47	0.43
SCH	-2.87	-2.89*	0.01	-1.79*	-1.86	0.08
BNP	-3.09*	-2.95**	-0.15	-2.01*	-2.00*	-0.01
CBKX	-3.01*	-3.14*	0.13	-1.46*	-1.93	0.47
CSGN	-3.06	-3.15**	0.09	-1.67	-1.79*	0.12
DAB	-2.98*	-2.88	-0.10	-1.41	-1.56	0.15
DBKX	-3.09*	-2.95*	-0.14	-1.82*	-1.66*	-0.16
DEX	-2.97*	-2.99***	0.02	-0.90*	-0.99**	0.09
EFG	-3.43*	-3.32***	-0.11	-1.33	-1.42**	0.10
ERS	-3.15	-3.18*	0.03	-1.53	-1.85	0.32
SGE	-2.95*	-2.98***	0.03	-1.83**	-1.96**	0.13
ISP	-3.05*	-3.12**	0.07	-1.79*	-1.95*	0.16
JYS	-3.16	-3.26	0.10	-1.34	-1.77	0.43
KB	-2.93*	-3.24	0.31	-1.54	-1.86*	0.33
LLOY	-2.94*	-3.04	0.10	-1.43*	-1.34	-0.09
MB	-3.20	-3.29*	0.09	-1.58	-1.89*	0.32
ETE	-3.33*	-3.54***	0.21	-1.39**	-1.66**	0.27
KNAF	-2.67	-3.12	0.45	-1.05	-1.95	0.90
NDA	-2.90**	-3.06	0.16	-1.53	-1.69	0.17
PEIR	-3.49	-3.48*	-0.01	-1.45	-1.62**	0.17
RBS	-2.70*	-2.91	0.21	-1.30*	-1.03	-0.28
SEA	-2.88	-3.04	0.16	-1.41	-1.66	0.24
STAN	-2.99*	-3.08*	0.09	-1.50	-1.27	-0.23
SVK	-3.04*	-3.11*	0.07	-1.57	-1.49*	-0.09
SWED	-2.93*	-3.05	0.12	-1.35*	-1.64**	0.29
SYD	-2.99	-3.16*	0.16	-1.29	-1.71	0.41
UBSN	-3.01*	-3.12**	0.11	-1.80	-1.75	-0.05
UCG	-3.02*	-3.08*	0.06	-1.68*	-2.00***	0.32
VATN	-2.34**	-2.66***	0.32	-0.08	-0.61	0.52

Notes: This table reports the estimates for CoVaR and ΔCoVaR, both real and synthetic ones. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level.

Table 15: CoVaR and ΔCoVaR - Experiment (Full Sample)

CoVaR and ΔCoVaR - Experiment						
Bank	CoVaR	Synthetic CoVaR	Diff	ΔCoVaR	Synthetic ΔCoVaR	Diff
PIST	-3.34***	-2.62**	-0.72	-1.54**	-0.37*	-1.18
BKIR	-3.16*	-3.25*	0.09	-1.24*	-1.30*	0.05
BARC	-2.93**	-3.16*	0.22	-1.46	-1.82	0.36
CRG	-3.55*	-3.10	-0.44	-1.73**	-0.97**	-0.76
BMPS	-3.44**	-3.20*	-0.24	-1.71***	-1.15**	-0.56
BPSO	-3.38*	-3.25	-0.13	-1.77*	-1.30	-0.47
BPE	-3.51**	-3.20**	-0.31	-1.89	-1.31	-0.58
BBVA	-2.94**	-2.99*	0.06	-1.92*	-1.92*	0.00
BCP	-3.21	-3.28*	0.07	-1.47	-1.26**	-0.22
SCH	-2.89*	-3.01	0.12	-1.86	-1.92	0.06
BNP	-2.95**	-2.98	0.03	-2.00*	-1.94*	-0.06
CBKX	-3.14*	-3.33	0.19	-1.93	-1.79	-0.14
CSGN	-3.15**	-3.15*	-0.00	-1.79*	-1.82	0.03
DAB	-2.88	-3.39	0.51	-1.56	-1.70	0.13
DBKX	-2.95*	-3.14*	0.19	-1.66*	-1.86	0.20
DEX	-2.99***	-1.89**	-1.10	-0.99**	0.28*	-1.27
EFG	-3.32***	-2.01*	-1.31	-1.42**	0.19**	-1.61
ERS	-3.18*	-3.39	0.21	-1.85	-1.72	-0.12
SGE	-2.98*	-3.01*	0.03	-1.96**	-1.92*	-0.04
ISP	-3.12**	-3.15*	0.03	-1.95*	-1.82*	-0.14
JYS	-3.26	-3.30	0.05	-1.77	-1.44	-0.34
KB	-3.24	-3.37	0.13	-1.86*	-1.77	-0.09
LLOY	-3.04	-3.33	0.29	-1.34	-1.79*	0.45
MB	-3.29*	-3.34	0.04	-1.89*	-1.72	-0.18
ETE	-3.54***	-2.91**	-0.63	-1.66**	-0.79*	-0.87
KNAF	-3.12	-3.40	0.28	-1.95	-1.76	-0.18
NDA	-3.06	-3.25	0.19	-1.69	-1.76	0.07
PEIR	-3.48*	-2.55*	-0.93	-1.62**	-0.28**	-1.34
RBS	-2.91	-3.40	0.48	-1.03	-1.76	0.73
SEA	-3.04	-3.29	0.25	-1.66	-1.77	0.12
STAN	-3.08**	-3.34	0.26	-1.27	-1.79	0.52
SVK	-3.11*	-3.35	0.24	-1.49*	-1.76	0.27
SWED	-3.05	-3.40	0.35	-1.64**	-1.76*	0.13
SYD	-3.16*	-3.31	0.16	-1.71	-1.52	-0.19
UBSN	-3.12**	-3.13	0.01	-1.75	-1.86	0.11
UCG	-3.08*	-3.16	0.08	-2.00***	-1.82*	-0.18
VATN	-2.66***	-1.35	-1.30	-0.61	0.79	-1.39

Notes: This table reports the estimates for CoVaR and ΔCoVaR, both the experimental and synthetic ones. We denote with ***, **, * estimates significant at the, respectively, 1%, 5% and 10% level.



(a) Peripheries (b) Sovereign Currency Issuer (c) Core

Figure 9: **Actual vs. Replicated CoVaR and ΔCoVaR** The figure reports banks' ΔCoVaR , both the true ones and the synthetic ones. The left panel reports results for the *Peripheral* group; the central one the *Sovereign Currency Issuer* group and the right panel the *Core* group.

3.7.1 Policy experiment - B

In this section we report the results of a different policy experiment. Contrary to what have been tested in the previous exercise, now the banks' balance sheets have been reallocated so as to weight more debts from *core* countries. In this case, it has been hypothesized that the 80% of the exposure was towards German, Dutch and French debt, with equal weights. The remaining 20% was towards Belgian, Italian, Spanish and Portuguese debt, with equal weights. The new balance sheet composition is displayed in Table 16.

Table 16: Balance Sheet Composition

Balance Sheet Composition	
Balance Sheet Exposure Towards	Percentage
Belgium	5
Germany	26
Spain	5
France	26
Italy	5
Netherlands	26
Portugal	5

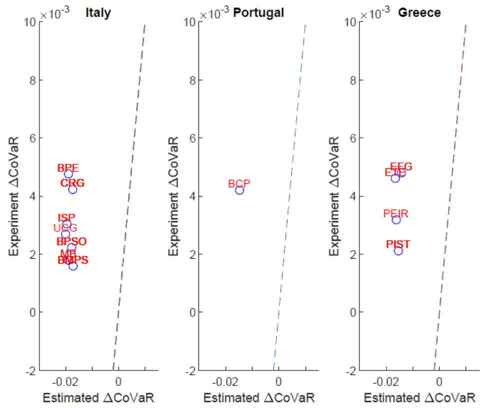
Notes: This table reports the balance sheet composition, with respect to sovereign exposure, for banks in case of the second policy experiment.

Our expectation is that, in this new scenario, the banking system will globally benefit from the new allocation, since now the single institutions are more exposed towards safer debt.

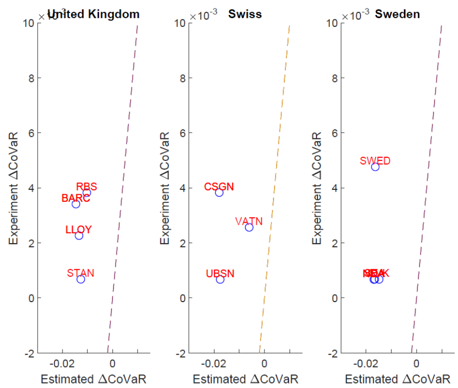
As it is possible to see in Figure 10, this is, indeed, the result we obtain.

Comparing Figure 10 to Figure 8 it is possible to see that now all the new risk measure, ΔCoVaR are above the 45° degrees line, hence implying a *lower* systemic risk contribution as opposed to the current one.

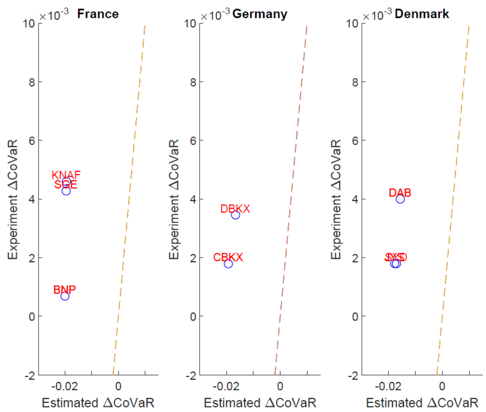
Synthetic vs. Experiment Banks' ΔCoVaR (per Group)



(a) Beneficiaries



(b) Hurt



(c) Ambiguous

Figure 10: **Actual vs. Replicated CoVaR and ΔCoVaR** The figure reports banks' ΔCoVaR , both the replicated ones and the experiment ones, for the second policy experiment (safer). The upper panel reports results for the (old) *Beneficiaries* group; the central one the (old) *Hurt* group and the lower panel the (old) *Ambiguous* group. In this new scenario, all the European banks benefit from the safer exposure, as we would expect.

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