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Habits and demand changes after COVID-19

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ABSTRACT

In this paper, we investigate how a transitory lockdown of a sector of the economy may have changed our habits and, therefore, altered the goods' demand permanently. In a two-sector infinite horizon economy, we show that the demand of the goods produced by the sector closed during the lockdown could shrink or expand with respect to their pre-pandemic level depending on the lockdown's duration and the habits' strength. We also show that the end of a lockdown may be characterized by a price surge due to a combination of strong demand of both goods and rigidities in production.

1. Introduction

Habits have been largely recognized by the psychology and economics literature to influence significantly our consumption behavior. The way habits form and change over time depend among other things on the environment. For example, it is well-documented that people addicted to alcohol or other substances receive cues that trigger further abuse of these substances from the location where they consumed them in the past. Therefore, a change in the environment or context may alter significantly the habits either reinforcing or weakening them (e.g. Danner et al. (2010)).

The COVID-19 epidemics and, specifically, the social distancing and lockdowns have represented a drastic change of context for everybody. Being forced for long periods of time to stay at home and limit the physical interactions with other people have often been accompanied by changes in our consumption behavior. A large literature has already emerged about the effect of these restrictive measures on specific habits. For example, there are contributions on changes in eating/dietary habits and lifestyle during the lockdown (Dixit et al., 2020, Renzo et al., 2020 and Sidor and Rzymski, 2020, among others). More generally, there is evidence that some habits have been reinforced, for example online shopping or using streaming services, while others weakened. Therefore, an open question is whether consumers will go back to their old habits such as shopping in the store or going to a cinema or the new habits will somehow replace the old ones (Sheth, 2020).

From an economic perspective this would mean that a lockdown of a sector of production could change the consumers' habits and, therefore, alter their demand of goods so much so that the firms in the sector affected by the lockdown could find no longer profitable to remain active even after the end of the pandemic or, alternatively, they could have an incentive to expand their production to respond to a strong demand.

The existing literature on the economic consequences of a lockdown has investigated several interesting issues (e.g. Alvarez et al. (2020), Caulkins et al. (2021), Giannitsaru et al. (2021), and Guerrieri et al. (2020)). Among these, Guerrieri et al. (2020) is probably the closest in scope to our contribution as their objective is to show how and under which conditions a supply shock may lead to a demand-deficient recession. Similar to their investigation we focus on the effects of a lockdown on goods' demand in a multi-sector economy without explicitly modeling the dynamics of the pandemics (see Literature Review for further details on this issue).

However, as far as we know, there is no contribution in the literature investigating how a change in the habits due to a lockdown may alter the consumption behavior after the pandemic. Could it be that the change in habits from old ones to new ones may lead an entire sector of the economy to disappear? If yes, how so? Could a government intervention avoid it? Could it be, instead, that the demand for goods not produced during the lockdown will expand after the pandemics? Could a change in the consumption composition push the good prices upward? Could prices rise above their pre-lockdown levels once the lockdown is over? The objective of this paper is to fill this gap in the literature and give an answer to these questions.

The first aim of our analysis is about understanding the mechanism(s) through which habits formed over the consumption of one good may affect the (inverse) demand of other goods independently

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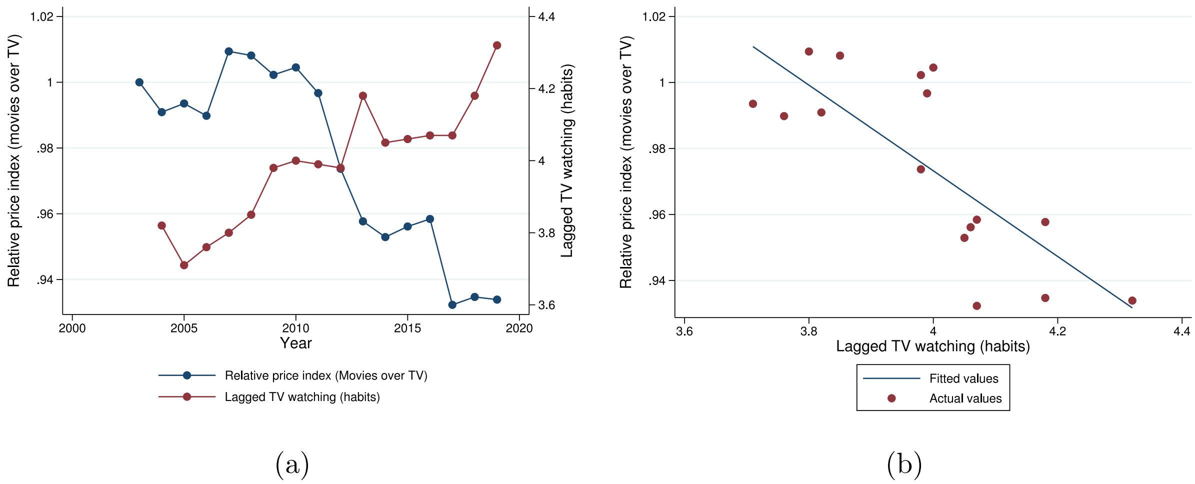


Fig. 1. Relative price index and lagged average TV watching in hours per day (habits) during weekends (a) Time series. (b) Scatter plot and correlation.

on the presence of a lockdown. Consider streaming services versus movie theater attendance as running example; the question could be rephrased as it follows: is it possible that the habits formed over watching streaming services has an influence on the relative prices of the two goods? If that is the case, what are the mechanisms linking habit formation to relative price movements?

A quick glance to the data suggests that such a relation exists and is significant. Coming back to our example, consider the CPI of admission to movies, theaters, and concerts and the CPI of cable and satellite television service in U.S. city average for all urban consumers in the period 2003–2019 with index 2003=100 (data from U.S. Bureau of Labor Statistics). The ratio of these two variables can be used as a proxy of the relative price of admission to theater which we indicate with RPI_t from now on.

On the other hand, we can use the American Time Use Survey (ATUS) to find the average hours per day in year t of TV watching across persons who engaged in the activity during the period 2003–2019.[1] Following Deaton (1986), habits can be defined as the lagged value of the consumption of an activity. Therefore in our case, TV watching at $t-1$ captures the habits.[2]

Plotting these time series, Fig. 1a, and their scatter plot, see Fig. 1b show a negative correlation between the relative price index and the habits formed over TV watching which does not seem driven by any particular outlier. Similar conclusions can be obtained by running some regressions which take into account also the presence of habits on other activities.[3]

These evidences as interesting as they can be, do not shed any light about the mechanisms behind the relation linking habits formed over one good and the relative price of another good. Our paper first objective is, therefore, to develop a model which can capture the relation between habits and relative prices as suggested by the empirical evidence, and in doing so to unveil the mechanisms behind it. This first result can be found in Section 5 and it was proved by assuming a quite general utility function. This represents our first contribution to the habit formation literature whose main focus has always been the relation between habits and the consumption and price of the same good.

Once, these mechanisms have been revealed, the next objective of this paper is to use the previously developed model to study how a lockdown on one sector of the economy may affect the demand and relative price of the good produced by that sector following the lockdown. This analysis was conducted using a linear–quadratic utility function, enabling us to examine the global dynamics of the model and to investigate the impacts of shocks that cause significant deviations from the steady state. Two possible outcomes exist: the demand for the unavailable good during lockdown could either increase or decrease relative to its pre-lockdown level. The former case represents a scenario of pent-up demand created during the lockdown. Once the lockdown ends, a mixture of this pent-up demand and labor market rigidities could drive up the relative price of the good that was unavailable during the lockdown. Conversely, in the second scenario, the economy may be characterized by a persistent lower demand and lower relative price of the good not available during the lockdown.

The next question is about which of the two alternatives will prevail. We prove that the way a lockdown affects the inverse demand of the goods and their relative price depends on (i) the habits speed of adjustment to changes in consumption, (ii) the habits speed of convergence to their steady state value, (iii) the cross-derivatives of utility between good 2 and good 1 consumption and between good 2 consumption and the habits as well as (iv) the lockdown duration.

Therefore, our model sheds light on the crucial role played by the habits in determining how the economy looks like after a lockdown with the habits characteristics affecting the direction of goods' demand changes. In fact, the same economy without habits would have experienced no changes after a lockdown with all the aggregate variables returning immediately to their pre-lockdown levels.[4]

We also propose a variation of this model that accounts for labor not immediately readjusting after the lockdown as this assumption seems more in line with the actual changes in labor force composition, with many workers not returning to their previous roles in sectors that were shut down during the lockdown. To this end, we examine the impact of a permanent shift in labor composition following the lockdown on our results.

The paper is organized as it follows. Section 2 provides a literature review and outlines the contributions most closely related to our analysis in terms of both modeling approaches and objectives. In Section 3,

[1] The data refers on the time spent on this activity during the weekend and holidays because the distribution of the time spent in consuming the other goods (movies etc.) suggests that more than 40% of it is concentrated in the weekend as for example emerges from the distribution of cinema attendance in the UK in 2013, by weekday reported by the UK Cinema Association (https://www.statista.com/statistics/296245/cinema-attendance-distribution-uk-by-weekday/).

[2] In particular, Deaton and many contributions on the empirical literature consider the case $h_{t+1} = c_t$ or equivalently $\Delta h_{t+1} = c_t - \delta_t h_t$ with $\delta = 1$. This is a quite extreme case as the habits fully depreciate every period.

[3] The interested reader can find these regressions in the Supplementary Material.

[4] This economy without lockdown is described in the Supplementary Material.

we present the model. In Section 4 we study the model and we show that the inverse demand of the good, unavailable during the lockdown, depends on the habits formed over the other good. Section 5 shows how our model can explain the empirical evidences between habits and relative prices presented in the Introduction. These results are obtained without choosing a specific utility function. Section 6, uses the previous results to describe how the economy is affected by a lockdown with a focus on the habits and good 2 price dynamics. The main result is summarized in Proposition 6 and it has been obtained by assuming a linear–quadratic utility function. Some numerical examples are also proposed to understand how the demand of the good unavailable during the lockdown can change after the lockdown. Finally, Section 7 concludes the paper. All the proofs are in Appendix.

2. Literature review

The literature in macroeconomics that explores the impacts of a pandemic can be categorized into three distinct streams, each with its unique objectives and modeling strategies.

The first stream of literature originates from the influential work by Goenka et al. (2014). The aim of this contribution was to incorporate a logistic equation, describing the pandemic's dynamics, into a neoclassical growth model. The authors demonstrated the existence of two steady states, dependent on whether the pandemic becomes endemic or is eradicated. An examination of transitional dynamics and of the optimality of the solution is conducted around each steady state. However, proving the optimality of the solution is notably complex in such models due to the presence of non-concavities in the state equation that describes the disease's dynamics. In particular, sufficiency conditions for optimality are far from obvious and often proved in the specific setting under study.

More recent studies, including those by d'Albis and Augeraud-Véron (2021) and Bosi et al. (2021), have adapted and broadened this model to the context of the COVID-19 pandemic. A significant focus of this literature is to comprehend how diverse policies can guide the economy towards one of the two identified steady states and influence the transitional dynamics.

This approach, along with the subsequent one, is especially significant when formulating policies during a pandemic as it may help to understand the mechanisms which facilitate the convergence and its speed toward one of the two steady states. However, its relevance decreases when the objective of the analysis is to compare an economy's state before and after a pandemic has run its course and has become endemic. We will discuss more this point later.

A second prolific stream of literature includes contributions by Giannitsaru et al. (2021), Alvarez et al. (2020) and Kaplan et al. (2020) among others. This literature focuses on designing an optimal lockdown with a specific emphasis to the COVID-19 epidemics and taking into account the demographic factors. The objective of this analysis is to show how the characteristics of the optimal lockdown and its effectiveness are affected by the nature of the disease including whether the immunity being waning or not. The difference with respect to the other two streams of literature is that the economy side of these models is quite thin while the modeling of the pandemic dynamics and of the demographics factors is generally richer.

The last stream of the macroeconomics literature has examined the impact of COVID-19 in multi-sectors economies. Notable contributions in this extensive body of literature include studies by Guerrieri et al. (2020), Beraja and Wolf (2021), and Baqaee and Farhi (2022) among others. In these studies, the authors model the effect of COVID-19 on the economy as exogenous shocks of similar magnitude to those observed during the pandemic. These shocks can take the form of lockdown measures, leading to the temporary cessation of certain sectors, as demonstrated in the work by Guerrieri et al. (2020). Alternatively, they can represent reduced production capabilities in specific sectors, as shown in Baqaee and Farhi's research (Baqaee and Farhi, 2022). These studies also takes into account exogenous shocks to the labor supply. These are generally represented as a significant labor reallocation across various sectors reflecting the scale of changes observed during the pandemic.

It is important to note that a common approach in this literature is to propose richer economic models with respect the previous streams of literature but to disregard the dynamics of the pandemic. This choice is consistently made, even in cases where macroeconomic models are analyzed solely through numerical simulations. The underlying viewpoint in these contributions is the following: given the observed lockdowns, the redistribution of labor across sectors, and the fact that the disease has become endemic (implying, in these models, a return to the pre-pandemic steady state), what are the implications in terms of demand for goods and production across sectors within the economy?

Our paper belongs to this last stream of literature, sharing analogous modeling choices and targeting similar economic questions. However, two distinguishing factors set our work apart from this body of literature: firstly, our research emphasizes the impact of consumption habits; secondly, it concentrates on global rather than local dynamics. This last departure is relevant in that the exogenous shocks both in term of output contractions and labor reallocations during the pandemic implied large deviations from the economy's steady state which should not be addressed only looking at the local dynamics.

3. Model setup

Consider an economy with two sectors producing two goods. Each sector has a unit mass of identical firms. Both goods are produced using a decreasing returns to scale production function $y = \ell^{\alpha}$ with labor, ℓ the only input. The profit maximization problem faced by a firm producing the final good i writes:

$$\max_{\ell_i} \ \pi_i \equiv p_i \ell_i^{\alpha} - w_i \ell_i,$$

with $i = 1, 2$, and $\alpha < 1$. The final good 1 is the numeraire whose price, p_1, is normalized to one, while $p_2 = p$ is endogenous and its value will be determined in equilibrium. The wage paid to the workers in sector i is denoted by w_i. As is typical with decreasing returns to scale in production, firms generate positive profits, which are then distributed to households, assumed to be the firm owners. Moreover, we assume that sector 2 becomes inactive during a lockdown, whereas sector 1 continues to operate, even in lockdown conditions.

The economy has also a unit mass of identical and infinitely lived households. The economy also admits an infinitely-lived representative household whose preferences are represented by the utility function

$$\int_0^{\infty} e^{-\rho t} u(c_1, c_2, h) dt,$$

where c_1, c_2 represent respectively the consumption of good 1 and good 2 and h indicates the stock of habits formed over the consumption of good 1 according to the equation

$$\dot{h} = \phi(c_1 - h),$$

with $\phi > 0$ and $h(0) = h_0$ exogenously given. In other words, the variable h measures how the agent is accustomed to good 1.[5]

Concavity in the consumption of good i implies that $u_{c_i c_i} < 0$. Habits can be harmful, $u_h < 0$, or beneficial, $u_h > 0$. This condition suggests a form of addiction. In order to explain this intuition, let us consider

[5] The assumption of habits formed over only one good is introduced to make the analysis less cumbersome. Intuitively similar results should be obtained assuming that habits are formed over both goods with state equations having same functional form and the ratio of initial habits, $\frac{h_{10}}{h_{20}}$, and $\frac{\phi_1}{\phi_2}$ sufficiently large. Finally, the assumption that habit formation is described by a linear ordinary differential equation in the variables consumption and habits is standard in the literature, see Carroll et al. (2000) among others.

for example a quadratic utility as the one we will focus on in Section 6 with $a_{hh} < 0$ and $a_{c_1h} > 0$. Note that habits are harmful (beneficial) provided the following inequalities do (do not) hold:

$$u_h < 0, \quad \Leftrightarrow \quad c_1 < -\frac{a_{hh}}{a_{c_1h}}h - \frac{a_h + a_{c_2h}y_2}{a_{c_1h}}.$$

Then, if consumption does not rise sufficiently in tandem with habits, the representative household's utility may decrease because u_h becomes negative. This latter effect amplifies the household's propensity to consume more of good 1. Suppose that during the lockdown households have increased the consumption of streaming services such as Netflix (good 1) as cinemas (good 2) were closed. An increase consumption of streaming services has implied a faster accumulation of the habits formed on this good. At the end of the lockdown, a reduction in consumption of streaming services may impact negatively the utility of the households also because of their addiction to the consumption of this good.

We also assume joint concavity of $u(.)$ in the three variables (c_1, c_2, h)[6]:

$$u_{hh} < 0, \qquad u_{c_1c_1}u_{hh} - (u_{c_1h})^2 > 0, \qquad and \quad |D^2u(c_1, c_2, h)| < 0, \tag{1}$$

where $D^2u(c_1, c_2, h)$ is the Hessian matrix of $u(.)$.[7] Adopting standard terminology from the literature, habits are deemed harmful if $u_h < 0$, and beneficial if $u_h > 0$.

We also assume that there is no Inada condition on the marginal utility of the two goods.[8] This assumption is unnecessary for an economy never affected by a lockdown and in fact the results of the next section hold independently on imposing or not the Inada conditions. However, this assumption becomes useful in the case of a lockdown because it allows the households to enjoy zero consumption and it prevents scarcity from driving the relative price to infinity.

Each household inelastically supplies her labor to the firms; in line with the assumptions made by Guerrieri et al. (2020), we assume that an exogenously given fraction $\xi \in [0, 1]$ of work time, $\bar{\ell}$, is supplied to sector 1 and the remaining $1 - \xi$ to sector 2 when both the sectors of production are active; on the other hand, if one sector is inactive (e.g. lockdown) then an exogenously given constant share $a \in (0, 1)$ of work allocated in the inactive sector is re-allocated to the active sector.[9] This labor re-allocation during the lockdown is consistent with the empirical evidence (see for example, Barrero et al., 2020).[10] Finally, the households' budget constraint when both sectors are active is

$$\dot{b} + c_1 + pc_2 = rb + w_1\xi\bar{\ell} + w_2(1 - \xi)\bar{\ell} + \pi_1 + \pi_2,$$

where b indicates the amount of foreign assets in positive-net supply and r the constant (exogenous) world interest rate. On the other hand, if only sector 1 is active than the inter-temporal budget constraint rewrites:

$$\dot{b} + c_1 = rb + w_1[\xi + a(1 - \xi)]\bar{\ell} + \pi_1,$$

Notice that in both cases we have indexed the bonds to good 1 consumption. As it will result clear later, this is done because we will focus on the case where good 1 is tradeable (e.g. streaming service) while good 2 is not (e.g. cinema). If you have a subscription to a streaming service such as Netflix then you can access it independently on your location. On the other hand, you cannot use a ticket of a cinema to see a movie in another cinema in another country.

4. Economy without or with lockdown

Before investigating the economic effects of a lockdown, it is convenient to study two different problems. The first is an economy where both sectors are active and the other where only sector one is active because a lockdown is imposed on the other or because it is no longer profitable to stay active.[11]

4.1. Economy with two active sectors of production

In this section, we consider the case of an economy with both sectors of production being active. In this context, we will show that our model is able to predict the relation between habits formed over one good and the relative price of another good as suggested in the introduction by our running example.

In this framework, the households' optimization problem writes

$$\max_{c_1, c_2, b, h} \int_0^\infty e^{-\rho t}u(c_1, h, c_2)dt,$$

subject to the following constraints

$$\dot{b} + c_1 + pc_2 = rb + w_1\xi\bar{\ell} + w_2(1 - \xi)\bar{\ell} + \pi_1 + \pi_2, \tag{2}$$

$$\dot{h} = \phi(c_1 - h), \tag{3}$$

$$b(0) = b_0, \quad h(0) = h_0, \quad given. \tag{4}$$

The inequality constraints, $h, c > 0$, also hold, along with a no-Ponzi scheme condition, which prevents households from perpetually accumulating debt.

A given state-control quadruple (c_1, c_2, h_1, b) is optimal if there exists absolutely continuous co-state functions μ and λ such that

$$u_{c_1} + \mu\phi - \lambda = 0, \tag{5}$$

$$u_{c_2} - p\lambda = 0, \tag{6}$$

$$\dot{\mu} = (\phi + \rho)\mu - u_h, \tag{7}$$

$$\dot{\lambda} = (\rho - r)\lambda, \tag{8}$$

$$\lim_{t\to\infty} h\mu e^{-\rho t} = 0, \tag{9}$$

$$\lim_{t\to\infty} b\lambda e^{-\rho t} = 0. \tag{10}$$

Observe also that $\lambda > 0$ while the sign of μ depends on the habits being harmful or beneficial. In the first case, it is negative while in the latter it is positive.

On the other hand, the profit-maximization problem of firm i leads to the following labor demand:

$$\ell_i = \left(\frac{p_i\alpha}{w_i}\right)^{\frac{1}{1-\alpha}}. \tag{11}$$

The labor market clearing conditions of sector 1 and sector 2 are respectively

$$\ell_1 = \xi\bar{\ell}, \qquad and \qquad \ell_2 = (1 - \xi)\bar{\ell}. \tag{12}$$

On the other hand, the goods market clearing conditions are

$$\dot{b} + c_1 = rb + y_1, \qquad and \qquad c_2 = y_2, \tag{13}$$

[6] In the numerical example we will check that these conditions are indeed respected in the different scenarios we will investigate.

[7] Observe that Bambi and Gozzi (2020) have shown that even without concavity the maximum principle may lead to an optimal and unique solution.

[8] A typical example is a linear–quadratic utility function which has been extensively used in the habit formation literature as well as in the seminal work on real business cycle by Kydland and Prescott (1982). Another context where the Inada conditions are not used is the structural change literature (e.g. Kongsamut et al. (2001)).

[9] In Section 6, we describe a similar economy populated with two types of agents and we will show that similar results can be obtained in the case of a linear–quadratic utility function.

[10] This quite simplistic modeling of the labor market well captures the rigidities in internal production which have been observed after a lockdown.

[11] It is worth noting that the pure exchange economy counterpart of our model (with no asset dynamics) leads to an equilibrium with households fully consuming what is produced internally and the habits dynamics adjusting accordingly.

where $y_1 = (\xi\bar{\ell})^\alpha$ and $y_2 = [(1-\xi)\bar{\ell}]^\alpha$. Observe that if there is a positive-net supply of the bond, $b > 0$, then the representative household will lend b to expand her future consumption of good 1 from the amount produced within the country, y_1, to a maximum $y_1 + rb$ by importing it. The opposite happens if there is a negative-net supply of the bond. In this case, the representative household may expand current consumption over the amount produced internally by borrowing from abroad but then she will repay this by a contraction of future consumption below y_1.[12]

On the other hand, the final good 2 is assumed to be not tradable and, therefore, its consumption is always equal to the amount produced within the country, y_2.

Definition 1 (*Decentralized Equilibrium*)**.** A decentralized equilibrium of the economy is an allocation $(c_1, c_2, h_1, b, \ell_1, \ell_2)_{t\geq 0}$ and a price path $(w_1, w_2, p)_{t\geq 0}$ such that

(i) Given $(w_1, w_2, p)_{t\geq 0}$, the representative household chooses a quadruple $(c_1, c_2, h_1, b)_{t\geq 0}$ to maximize her intertemporal utility subject to (1)–(3).
(ii) Given $(w_i, p_i)_{t\geq 0}$, the representative firm in sector i chooses ℓ_i to maximizes its profit subject to its production function, for $i = 1, 2$.
(iii) All markets clear in every period, i.e. (12) and (13) hold.

Observe that, at the decentralized equilibrium, we have that $u_{c_1} = u_{c_1}(c_1, h_1; y_2)$, $u_{c_2} = u_{c_2}(c_1, h_1; y_2)$, and $u_h = u_h(c_1, h_1; y_2)$. This dimension reduction simplifies considerably our analysis and several results can be derived without choosing a specific utility function. To make the model even more tractable we will also assume from now on that $r = \rho$ and therefore λ is constant. The only consequence of this assumption is that the economy will not grow over time.

Proposition 1. *A unique steady state exists with all the stationary variables function of the costate variable λ.*

It is worth noting that the existence and uniqueness of the steady state can be proved with and without the Inada conditions under some reasonable mild conditions on the parameters.[13] We can now linearize (5)–(7), the habits Eq. (3) and the final good 1 market clearing condition (13) around the steady state and eventually get the following result in the variables expressed as deviation from their steady state value, i.e. $\tilde{x} = x - x^*$.

Proposition 2. *The local dynamics of the economy around its steady state is described by the following system of equations in the variables $(\tilde{\mu}, \tilde{h}, \tilde{c}_1, \tilde{p}, \tilde{b}, \lambda)$:*

$$\dot{\tilde{\mu}} = \left[\left(1 + \frac{u^*_{c_1h}}{u^*_{c_1c_1}}\right)\phi + \rho\right]\tilde{\mu} + \frac{(u^*_{c_1h})^2 - u^*_{c_1c_1}u^*_{hh}}{u^*_{c_1c_1}}\tilde{h}, \tag{14}$$

$$\dot{\tilde{h}} = -\frac{\phi^2}{u^*_{c_1c_1}}\tilde{\mu} - \phi\left(1 + \frac{u^*_{c_1h}}{u^*_{c_1c_1}}\right)\tilde{h}, \tag{15}$$

$$\dot{\tilde{b}} = r\tilde{b} - \tilde{c}_1, \tag{16}$$

$$\tilde{c}_1 = -\frac{u^*_{c_1h}\tilde{h} + \phi\tilde{\mu}}{u^*_{c_1c_1}}, \tag{17}$$

$$\tilde{p} = \frac{u^*_{c_2c_1}\tilde{c}_1 + u^*_{c_2h}\tilde{h}}{\lambda}, \tag{18}$$

plus the transversality conditions.[14]

In Section 5 we will show how starting from Eq. (18) we can explain the relation between habits and relative prices as described by the empirical evidences in the Introduction. Moreover, in Section 6 we will also show that this relation will also help us understanding whether and under which conditions a lockdown may affect the relative price so that the final good 2 could become more or less profitable to be produced after the lockdown.

Let us proceed with our analysis and observe that a nice feature of our model is that (14) and (15) is a linear system of ODEs in the variables $(\tilde{\mu}, \tilde{h})$ which we can solve analytically.

Proposition 3. *Assume that $u^*_{c_1h} < \bar{u}^*_{c_1h}$. Then the solution of the system of linear ODEs (14) and (15) together with TVC (9) exists and has the following form:*

$$\tilde{h} = \tilde{h}_0 e^{\psi_1 t}, \tag{19}$$

$$\tilde{\mu} = -\frac{\phi u^*_{c_1h} + (\phi + \psi_1)u^*_{c_1c_1}}{\phi^2}\tilde{h}, \tag{20}$$

where ψ_1 is the real and negative eigenvalue whose value depends on λ (see Appendix A — Lemma 1).

The inequality at the beginning of the proposition, $u^*_{c_1h} < \bar{u}^*_{c_1h}$, guarantees a positive and a negative eigenvalue and it is a necessary condition for the TVC (9) to hold. The threshold value $\bar{u}^*_{c_1h}$ can be found in Appendix A — Lemma 1. Once $\tilde{h}$ and $\tilde{\mu}$ have been found, we substitute them into (17) to find how the dynamics of good 1 consumption and habits are related:

The last step of this analysis consists in finding the solution of b, and in doing so choosing the value of λ which makes the TVC (10) hold.

Proposition 4. *Assume that $a_{c_1h} < \bar{a}_{c_1h}$. Then the dynamic path of $\tilde{b}$ is*

$$\tilde{b} = \frac{\phi + \psi_1}{\phi(r - \psi_1)}\tilde{h}, \tag{21}$$

with λ equal to the value which makes the following equality hold

$$\tilde{b}_0 = \frac{\phi + \psi_1}{\phi(r - \psi_1)}\tilde{h}_0. \tag{22}$$

Without specifying an utility function it is not possible to find explicitly the value of λ. For this reason, we will choose, in Section 6, a specific functional form to assess the economic effects of a lockdown. In the next section, we briefly present an economy where only sector 1 is active.

4.2. Economy with one sector under lockdown

In this section, we consider the case where the sector producing good 2 is inactive because of a lockdown and a share $a \in (0,1)$ of labor, previously allocated in sector 2, is re-allocated in sector 1. We are also assuming that the length of the lockdown is unknown to the households; as such, the lockdown's end is perceived by them as an unanticipated shock. The households' optimization problem now writes

$$\max_{c_1,h,b} \int_0^\infty e^{-\rho t} u(c_1, h)dt,$$

subject to the following constraints

$$\dot{b} + c_1 = rb + w_1[\xi + a(1-\xi)]\bar{\ell} + \pi_1, \tag{23}$$

$$\dot{h} = \phi(c_1 - h), \tag{24}$$

$$b(0) = b_0, \quad h(0) = h_0, \quad \textit{given}. \tag{25}$$

The inequality constraints, $h, c > 0$, also hold,along with a no-Ponzi scheme condition. The maximum principle leads to the FOCs (5), (7)–(10).

The profit-maximization problem of the firms producing good 1 is also the same while the labor market clearing condition becomes

$$\ell_1 = [\xi + a(1-\xi)]\bar{\ell}, \tag{26}$$

[12] Notice that without foreign assets the model becomes trivial as in each period the households will consume the exogenously given amount $c_1 = y_1$ and $c_2 = y_2$ while the dynamics of the habits will be adjust accordingly by solving $\dot{h} = \phi(y_1 - h)$.

[13] The interested reader may look at the proof in Appendix for further details.

[14] Notice that, λ will be determined using a TVC.

where $y_1 = \{[\xi + a(1-\xi)]\bar{\ell}\}^{\alpha}$ with $a \in (0,1)$, meaning that the final good 1 production has expanded since a share a of labor previously allocated in sector 2 is now used in sector 1. The good market clearing condition is

$$\dot{b} + c_1 = rb + y_1. \tag{27}$$

Moreover, given the structure of our model, the functional form of the solution with one or two active sectors of production is the same. In other words, Lemma 1, Propositions 3, and 4 as well as Eqs. (19), (20), (29), (21), and (22) still hold in the case with only one sector. Of course, the path of the aggregate variables will be different since the absence of sector 2 and the labor reallocation from sector 1 to 2 will change the value of λ and, therefore, the steady state values of the main aggregate variables; moreover, it will also affect the transitional dynamics since the eigenvalue ψ_1 depends on λ. For the same reasons explained before, no further analysis is possible without assuming a specific utility function.

5. Habits and relative price dynamics (no lockdown)

In this section we explain how our model can generate the negative relation between relative prices and habits presented in Fig. 1a and b in the Introduction. Let us begin our discussion with Eq. (18) found in Proposition 2 which we rewrite below:

$$\tilde{p} = \frac{u^*_{c_2c_1}\tilde{c}_1 + u^*_{c_2h}\tilde{h}}{\lambda}.$$

Two things can be noted immediately. First, the two goods are substitutes if, when the price of good 2 rises, the demand for its substitute, good 1, does increase, formally:

$$\frac{\partial p}{\partial c_1} = \frac{u^*_{c_2c_1}}{\lambda} > 0 \qquad \Leftrightarrow \qquad u_{c_2c_1} > 0,$$

and complements otherwise. Second, and most importantly, the presence of the habits may reduce the price of good 2

$$\frac{\partial p}{\partial h} = \frac{u^*_{c_2h}}{\lambda} < 0 \qquad \Leftrightarrow \qquad u_{c_2h} < 0,$$

since $\lambda > 0$. This condition means that the price of the final good 2 may decrease if the marginal utility of consuming that good decreases as the habits accumulate, $u_{c_2h<0}$. In this case, although the habits are formed over the final good 1 consumption, they induce satiation in the consumption of good 2.

Taking into account these two relations we can derive the following condition on the price change:

$$dp < 0 \qquad \Leftrightarrow \qquad \underbrace{u^*_{c_2c_1}dc_1}_{Substitutability\ Effect} \quad < \quad \underbrace{-u^*_{c_2h}dh}_{Satiation\ Effect}, \tag{28}$$

with $u_{c_2c_1} > 0$ and $u_{c_2h} < 0$.[15] This relation is important because it gives an insight of the mechanism linking the habits formed over the consumption of one good and the relative price of another good. To find the equilibrium counterpart of (28), observe that combining (17) with (19) and (20) leads to

$$\tilde{c}_1 = \frac{\phi + \psi_1}{\phi}\tilde{h}. \tag{29}$$

Therefore, we have that the final good 1 is addictive if $\phi + \psi_1 > 0$ since its current consumption increases as the habits accumulate, $\frac{dc_1}{dh} > 0$. On the other hand, good 1 is satiating when $\psi_1 + \phi < 0$. Moreover, substituting $\tilde{c}_1$ into the relative price Eq. (18) leads to

$$\tilde{p} = \frac{(\phi + \psi_1)u^*_{c_2c_1} + \phi u^*_{c_2h}}{\phi\lambda}\tilde{h}. \tag{30}$$

[15] Numerical values for these two cross-derivatives can be found in Sections 6.3 and 6.4.

This equation provides an insight about the mechanism which may link habits formed over one good to the relative price of another good. For example, we can interpret the empirical evidences on our running example, see Fig. 1a and b, through the lens of this equation. First, it is worth noting that in equilibrium the relative price of good 2 (relative price of a movie theater ticket) depends on the habits formed over good 1 (TV watching). Second, a negative correlation between the two variables can be explained as it follows. Suppose that good 1 is addictive, $\phi + \psi_1 > 0$, then any increase in habits will push consumption up which will feed more habit accumulation. Since the two goods are substitute ($u_{c_2c_1} > 0$) any raise in demand of this good will be accompanied by an increase in the price of the other good ($p \uparrow$).

On the other hand, habits formed over good 1 may induce satiation on good 2, case $u_{c_2h} < 0$, and reduce its relative price. Suppose that agents have developed an habit of binge-watching TV series, then their utility of watching movies to the cinema could be reduced as a result of that and they would accept to go to the cinema only if the ticket price is sufficiently low ($p \downarrow$).

Overall, the negative correlation between habits formed over TV Watching and the relative price of a movie theater ticket can be explained by the following relation

$$\frac{dp}{dh} < 0 \qquad \Leftrightarrow \qquad \underbrace{u^*_{c_2c_1} \cdot \frac{\phi + \psi_1}{\phi}}_{Substitutability\ Effect} \quad < \quad \underbrace{-u^*_{c_2h}}_{Satiation\ Effect}, \tag{31}$$

which is the equilibrium counterpart of expression (28). Most importantly, this relation unveils which parameters in the model drive this mechanism. The substitutability effect depends in equilibrium on the (steady state) cross-derivative of utility between good 2 and good 1 consumption, $u^*_{c_2c_1}$, the habits speed of adjustment to a change in good 1 consumption, ϕ, and the habits speed of convergence to its steady state value, ψ_1. On the other hand the satiation effect depends on the (steady state) cross derivative of utility between good 2 consumption and habits, $u^*_{c_2h}$. Notice that in our model, the shadow price of a foreign asset, λ, is a positive constant and for this reason does not play any role in affecting the sign of the relation between relative prices and habits.

6. Lockdown and its effects on the economy

In this section, we will study the effect of a lockdown on the economy. In particular, we will study its effect on the economy during the lockdown as well as after its end. To do so, we assume a linear–quadratic utility[16]:

$$u(c_1, c_2, h) = a_{c_1}c_1 + a_{c_2}c_2 + a_h h + \frac{a_{c_1c_1}}{2}c_1^2 + \frac{a_{c_2c_2}}{2}c_2^2 + \frac{a_{hh}}{2}h^2 + a_{c_1c_2}c_1c_2 + a_{c_1h}c_1h + a_{c_2h}c_2h, \tag{32}$$

with the parameter conditions for concavity respected.[17] This functional form has been extensively used in the rational addiction literature (see among others Becker and Murphy, 1988, Dockner and Feichtinger, 1993, and Iannaccone, 1986) as well as in the business cycle literature when the emphasis was about big shocks or welfare analysis, see Kydland and Prescott (1982) and Benigno and Woodford (2012) among others. Moreover, this functional form has several advantages. First, we will be able to study the global dynamics and not just the local dynamics of the economy since an optimal control problem with linear–quadratic objective and linear states equations leads to a linear system of ODEs describing the dynamics of the economy. Second, it makes the model analytically tractable since it is possible to find the shadow price, λ. In fact, with this functional form, all the second

[16] It can be shown that a case with two-types of workers leads to similar results see Bambi et al. (2022).

[17] In the numerical example we will check that these conditions are indeed respected in the different scenarios we will investigate.

derivatives of the utility function are constant and, therefore, it is immediate to see that the eigenvalue, ψ_1, will be no more a function of the co-state variable λ whose value can be found using Proposition 4.

Before describing the timing of the shocks to the economy it is useful to find explicitly the steady state of an economy with two active sectors of production. Remember that in this case the output in the two sectors is $y_1 = (\xi\bar{\ell})^\alpha$ and $y_2 = [(1-\xi)\bar{\ell}]^\alpha$.

Proposition 5 (*Steady State with 2 Active Sectors*). *Assume that $a_{c_1h} < \bar{a}_{c_1h}$, $a_{c_1} > \underline{a}_{c_1}$, and $b_0 \in (\max\{\underline{b}_0, 0\}, \bar{b}_0)$. Then, the steady state values of the main aggregate variables are*

$$h^* = \frac{\phi(\psi_1 - r)}{(\phi + r)\psi_1}\left[rb_0 + y_1 + \frac{r(\phi+\psi_1)}{\phi(\psi_1 - r)}h_0\right], \tag{33}$$

$$c_1^* = h^*, \tag{34}$$

$$b^* = \frac{h^* - y_1}{r}, \tag{35}$$

$$p^* = \frac{a_{c_2} + a_{c_2c_2}y_2 + a_{c_1c_2}c_1^* + a_{c_2h}h^*}{\lambda}, \tag{36}$$

$$\lambda = m_0 + m_1\left[rb_0 + y_1 + \frac{r(\phi+\psi_1)}{\phi(\psi_1 - r)}h_0\right], \tag{37}$$

with $m_0 = a_{c_1} - \underline{a}_{c_1} > 0$, $m_1 = \frac{\phi(\psi_1 - r)}{(\phi+r)\psi_1}\cdot\frac{(\phi+\rho)a_{c_1c_1} + (\rho+2\phi)a_{c_1h} + \phi a_{hh}}{\phi+\rho} < 0$ and the threshold values $\bar{a}_{c_1h}$, a_{c_1}, $\underline{b}_0$, $\bar{b}_0$, and $\underline{a}_{c_2}$ reported in Appendix.[18]

Observe that conditions $a_{c_1h} < \bar{a}_{c_1h}$, $a_{c_1} > \underline{a}_{c_1}$, and $b_0 \in (\underline{b}_0, \bar{b}_0)$ are needed to have both h^* and λ strictly positive.

We can now describe how the lockdown is modeled. At $t = 0$ an unanticipated temporary lockdown is imposed on sector 2 which would have found profitable to be active otherwise. The duration of the lockdown, $\tilde{t}$, is unknown and the lifting of the lockdown is another unanticipated shock from the agents' perspective. Once the lockdown ends, sector 2 will reopen. A variant of this model, built on the more realistic assumption of a random lockdown duration, can be found in Bambi et al. (2022). Initial results from this model variation appear to align with the price dynamics predictions revealed in this paper.

We will now describe the dynamics of the economy in the different phases of the lockdown.

6.1. Arrival of the lockdown

Suppose that the lockdown on sector 2 is unanticipated, temporary, and implemented at $t = 0$. Since it is unanticipated, the agents will re-optimize following the problem setup explained in Section 4.2 with initial condition h_0 and b_0. The price dynamics is described by the following relation:

$$p_{NL} = p^* + \frac{(\phi+\psi_1)a_{c_2c_1} + \phi a_{c_2h}}{\phi\left\{m_0 + m_1\left[rb_0 + y_1 + \frac{r(\phi+\psi_1)}{\phi(\psi_1 - r)}h_0\right]\right\}}(h_0 - h^*_{NL})e^{\psi_1 t}, \tag{38}$$

with $h^*_{NL} = h^*$ as found in Eq. (33). We will also assume that $h_0 < h^*_{NL}$, although the other cases are also tractable as shown in Appendix A.

Since the duration of the lockdown, $\tilde{t}$, is unknown and the ending of the lockdown is modeled as another unanticipated shock, then the representative agent solves the same problem described in Section 4.2 and we can use the results found previously. In particular, we have that the dynamics during the lockdown, i.e. in $t \in [0, \tilde{t}]$, is described by the following equations:

$$h_L = h^*_L + (h_0 - h^*_L)e^{\psi_1 t}, \tag{39}$$

$$c_{1,L} = -\frac{\psi_1}{\phi}h^*_L + \frac{\phi+\psi_1}{\phi}h_L, \tag{40}$$

$$b_L = -\frac{\psi_1(\phi+r)}{r\phi(r-\psi_1)}h^*_L - \frac{y_{1,L}}{r} + \frac{\phi+\psi_1}{\phi(r-\psi_1)}h_L, \tag{41}$$

where the lockdown steady state habit stock is

$$h^*_L = \frac{\phi(\psi_1 - r)}{(\phi+r)\psi_1}\left[rb_0 + y_{1,L} + \frac{r(\phi+\psi_1)}{\phi(\psi_1 - r)}h_0\right], \tag{42}$$

with $y_{1,L} = \{[\xi + a(1-\xi)]\bar{\ell}\}^\alpha$ since a share $a \in (0,1)$ of labor in sector 2 has been re-allocated in sector 1. Notice also that for the same initial conditions, $h^*_L > h^*_{NL}$ since $y_{1,L} > y_{1,NL}$.

6.2. After the lockdown

At $t = \tilde{t}$ the agents re-optimize since the re-opening of the economy is again an unanticipated shock. Notice that the initial conditions for this problem are now $h_{\tilde{t}}$ and $b_{\tilde{t}}$ which can be found looking at Eqs. (39) and (41). In particular, we have that:

$$h_{\tilde{t}} = h^*_L + (h_0 - h^*_L)e^{\psi_1\tilde{t}}, \tag{43}$$

$$b_{\tilde{t}} = -\frac{\psi_1(\phi+r)}{r\phi(r-\psi_1)}h^*_L - \frac{y_{1,L}}{r} + \frac{\phi+\psi_1}{\phi(r-\psi_1)}h_{\tilde{t}}. \tag{44}$$

We also assume that labor readjusts immediately after the lockdown to its pre-lockdown levels so that $y_1 = (\xi\bar{\ell})^\alpha$ and $y_2 = [(1-\xi)\bar{\ell}]^\alpha$. This assumption will be relaxed later in Section 6.5 where the case of a permanent large change in the composition of the labor forces, as being observed across developed countries, will be studied.

The equilibrium path of the main variables in the case of a full readjustment of the labor force to its pre-lockdown level can be found by adapting the results of Section 4.1.

In particular, the equilibrium path of the habit stock in $t \in [\tilde{t}, \infty]$ is

$$h_{AL} = h^*_{AL} + (h_{\tilde{t}} - h^*_{AL})e^{\psi_1(t-\tilde{t})}, \tag{45}$$

or equivalently

$$h_{AL} = h^*_{AL} + \left[\underbrace{h^*_L - h^*_{AL}}_{>0} + \underbrace{(h_0 - h^*_L)}_{<0}e^{\psi_1\tilde{t}}\right]e^{\psi_1(t-\tilde{t})}, \tag{46}$$

where

$$h^*_{AL} = h^*_{NL} = h^*,$$

as proved in Appendix B with h^* as found in Proposition 5. Then, it is immediate to see that $h^*_L > h^*_{AL}$ because after the lockdown the labor market readjusts immediately to the pre-pandemic situation, i.e. $y_{1,AL} = (\xi\bar{\ell})^\alpha = y_{1,NL}$. In addition, we have also that $h_0 < h^*_L$ since we assumed before that $h_0 < h^*_{NL} = h^*_{AL}$. It can also be proved (see Proposition 5) that a full readjustment of the labor composition to its pre-lockdown level will imply the same shadow prices

$$\lambda_{NL} = \lambda_{AL} = \lambda.$$

Taking all these considerations into account, we can now write the equilibrium price dynamics of final good 2 in $t \in [\tilde{t}, \infty]$:

$$p_{AL} = p^* + \frac{(\phi+\psi_1)a_{c_2c_1} + \phi a_{c_2h}}{\phi\left\{m_0 + m_1\left[rb_0 + y_1 + \frac{r(\phi+\psi_1)}{\phi(\psi_1 - r)}h_0\right]\right\}} \times \left[\underbrace{h^*_L - h^*}_{>0} + \underbrace{(h_0 - h^*_L)}_{<0}e^{\psi_1\tilde{t}}\right]e^{\psi_1(t-\tilde{t})}. \tag{47}$$

We are now ready to prove the other main result of this paper. The following proposition describes the price dynamics when we are comparing two economies which are exactly identical at $t = 0$. However, the first economy never experiences a lockdown and parameters are chosen so that both sectors remain active, while the other economy experience a lockdown of $\tilde{t}$ periods. What will it happen after the lockdown? Will sector 2 firms profits expands or not as a result of the change in the relative price dynamics due to the lockdown? The next proposition provides a taxonomy of all possible scenarios.

[18] The steady state values depend on the initial conditions because of the linearity of the budget constraint, see Eq. (1). This is what usually happens in any endogenous growth model having in equilibrium an AK structure and choosing the parameters so that the growth rate is zero.

Proposition 6. *Consider the price dynamics in an economy without a lockdown (NL) and in an economy with a $\tilde{t}$-period lockdown (AL):*

$$p_{t,NL} = p^* + \frac{(\phi+\psi_1)a_{c_2c_1} + \phi a_{c_2h}}{\phi\lambda}(h_0 - h^*)e^{\psi_1 t} \quad with \quad t \in [0,\infty],$$

$$p_{t,AL} = p^* + \frac{(\phi+\psi_1)a_{c_2c_1} + \phi a_{c_2h}}{\phi\lambda} \times \left[h_L^* - h^* + (h_0 - h_L^*)e^{\psi_1\tilde{t}}\right]e^{\psi_1(t-\tilde{t})} \quad with \quad t \in [\tilde{t},\infty],$$

with λ, p^*, h^* as found in Proposition 5 and h_L^* as found in Eq. (42) and consider the case with $h_0 < h^*$. Then the following results hold:

- *if the satiation dominates the substitutability effect, $a_{c_2h} < \bar{a}_{c_2h}$, and the lockdown is sufficiently long, $\tilde{t} > \underline{\tilde{t}}$, then*

$$p_{t,AL} < p^* < p_{t,NL} \qquad with \quad t \in [\tilde{t},\infty]; \tag{48}$$

- *if the satiation dominates the substitutability effect, $a_{c_2h} < \bar{a}_{c_2h}$, and the lockdown is sufficiently short, $\tilde{t} < \underline{\tilde{t}}$, then*

$$p^* < p_{t,AL} < p_{t,NL} \qquad with \quad t \in [\tilde{t},\infty]; \tag{49}$$

- *if instead the substitutability dominates the satiation effect, $a_{c_2h} > \bar{a}_{c_2h}$, and the lockdown is sufficiently long, $\tilde{t} > \underline{\tilde{t}}$, then*

$$p_{t,AL} > p^* > p_{t,NL} \qquad with \quad t \in [\tilde{t},\infty]; \tag{50}$$

- *if the substitutability dominates the satiation effect, $a_{c_2h} > \bar{a}_{c_2h}$, and the lockdown is sufficiently short, $\tilde{t} < \underline{\tilde{t}}$, then*

$$p^* > p_{t,AL} > p_{t,NL} \qquad with \quad t \in [\tilde{t},\infty]; \tag{51}$$

where

$$\bar{a}_{c_2h} \equiv -\frac{\phi+\psi_1}{\phi}a_{c_2c_1}, \qquad and \qquad \underline{\tilde{t}} = \frac{\ln(h_L^* - h_0) - \ln(h_L^* - h^*)}{|\psi_1|}.$$

Similar results can be obtained for $h_0 > h^*$, see the proof in Appendix A for further details.

Several interesting facts emerge from this proposition. First, the result in (48) shows how deeply a lockdown may affect the economic activity.

The results in (49)–(50) show, for example, that the good 2 price at the end of the lockdown could be permanently higher (lower) than at its pre-lockdown level if the substitutability effect is higher (lower) than the satiation effect. The case of a price surge depends on the pent-up demand formed during the lockdown period as it will be explained in details in Section 6.4.

Another consideration is about the role of the habits initial condition, h_0. The condition $h_0 < h^*$ matters only for the position of p^* with respect to p_{NL} and p_{AL} while it has no role on the relation between these last two prices which is completely driven by the satiation and substitutability effect. The interested reader may find in the proof of Proposition 6 a more general formulation which consider also the case $h_0 > h^*$.

In the next two subsections, we will consider and discuss two scenarios. In the first we will show numerically how negatively the economic activity can be affected by a lockdown when the satiation dominates the substitutability effect. In the second, we will consider an opposite scenario where the substitutability effect dominates the satiation effect and show how the pent-up demand on good 2 may drive a strong economic recovery.

6.3. Price dynamics and demand contraction

We want now to illustrate through a numerical example the price dynamics as predicted by relation (48) and (49) and to provide an insight about good 2's demand change after the lockdown.[19]

Let us start with the production side of the economy. Assume that before the lockdown the two sectors receive an equal amount of work, $\xi = 0.5$, and that the work endowment is normalized to one, $\bar{\ell} = 1$. During the lockdown, thirty percent, $a = 0.3$, of labor in sector 2 is re-allocated in sector 1 consistently with the finding of Barrero et al. (2020). The labor share is $\alpha = 0.7$ implying a total production $y_1 = y_2 = 0.6156$ if both the sectors are active and $y_1 = 0.7397$ if sector 2 is inactive.

Looking now at the household problem, we set $a_{c_1} = a_{c_2} = 1$, $a_{c_1c_1} = a_{c_2c_2} = a_{hh} = -1$, and $a_h = -0.5$. We also consider the case of substitute goods with $a_{c_1c_2} = 0.3$. A positive change in the habits increases the marginal utility of good 1 consumption given that we have set $a_{c_1h} = 0.6$. Conversely, it decreases the marginal utility of good 2 consumption as we have assigned $a_{c_2h} = -0.1$. The last two parameter choices guarantee respectively a negative eigenvalue, so that there is convergence to the steady state, and satiation stronger than the substitutability effect. Finally we need to set the habits speed of adjustment to change in consumption, ϕ. The drastic change in habits documented in the previously mentioned literature (see the introduction) seems to suggest habits promptly adapting to the new lifestyle imposed by the lockdown. For this reason we set $\phi = 0.15$. This value implies that the half-life with which habits adjust toward a permanent change in c_1 is slightly more than one year which is a bit lower than the two years suggested in Carroll et al. (2000) but basically the same as the value proposed by the literature on the equity premium puzzle (see Abel, 1990 among others).[20]

The remaining parameters are set as it follows $r = \rho = 0.01$, $h_0 = 0.5$, and $b_0 = 1$. With this choice of the parameters we have that the final good 1 is addictive since $\frac{dc_1}{dh} = \frac{\phi+\psi_1}{\phi} = \frac{0.1-0.0868}{0.1} = 1.8679 > 0$.

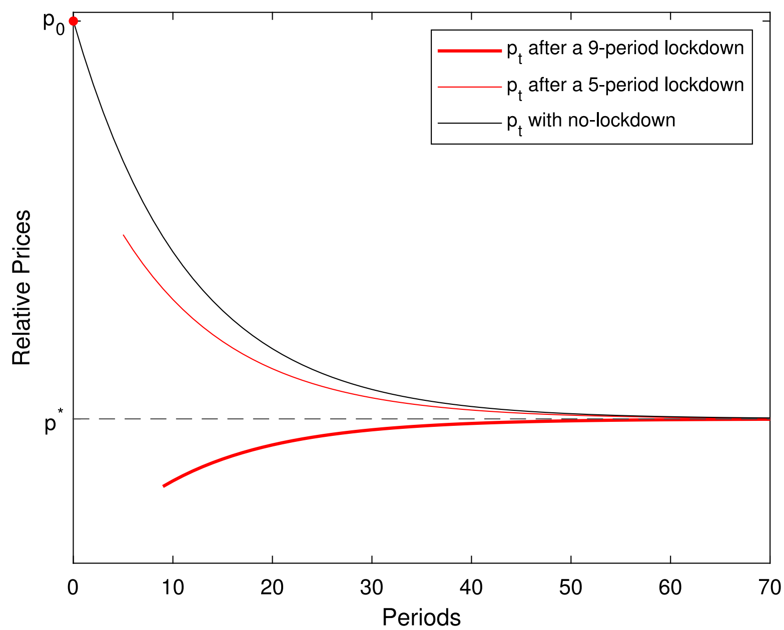
All inequalities for concavity in Proposition 5 are respected with this choice of the parameters. In particular, the utility function is strictly concave since $a_{c_1c_1}a_{hh} - a^2_{c_1h} = 0.64 > 0$ and $|D^2u(c_1,c_2,h)| = -0.576 < 0$ and we have that the thresholds are equal to $\bar{a}_{c_1h} = 1$, $\underline{a}_{c_1} = 0.3258$, $\bar{a}_{c_2h} = -0.0396$, $\underline{b}_0 = -60.8747$, and $\bar{b}_0 = 26.20$ meaning that all the parameters have been chosen within their constraints.

Once all the parameters were set, we have drawn in Fig. 2 the price dynamics under different lockdown durations. Consistently with the analytical findings, a dominating satiation effects implies that higher the lockdown and higher will be the negative effect on the price of good 2 $p = p_2$. The decrease in price is a result of the reduced demand for good 2. Nonetheless, given that the supply of good 2 is inelastic, the household will only be willing to consume the same quantity as before the lockdown if it is offered at a lower price. Notice that lower prices will imply lower profits in sector 2, $\pi_2 = (1-\alpha)p[\bar{l}(1-\xi)]^\alpha$. Intriguingly, a lockdown of sufficient length could drive prices and profits from above their pre-pandemic steady state level to below it by the end of the lockdown, as demonstrated by the bold red curve in the figure.

During the lockdown, any increase in the consumption and habits of watching TV (good 1) has reinforced each other due to the assumption of addictive habits ($dc_1/dh > 0$). Furthermore, the longer the lockdown, the more these habits have accumulated, implying increasingly higher levels of consumption. When the lockdown ends, the price of a movie theater ticket (good 2) must decrease since the satiation effect over the consumption of good 2 triggered by the built-up habits outweighs the substitutability effect.

[19] This part is also useful because we want to verify at least numerically that the several inequality constraints introduced throughout our analysis actually hold.

[20] Carroll et al. (2000) at page 345 explain that the choice of this value depends on the context examined. If the emphasis is on economic growth then lower values of ϕ can be chosen.



(a)

Fig. 2. Price dynamics when satiation dominates substitutability effect, $a_{c_2h} < \bar{a}_{c_2h}$.

6.4. Price dynamics and pent-up demand

The question we aim to address in this section is whether our model can shed light on the actual surge in goods' demand and prices seen across developed countries post-lockdown. To achieve this, we will now concentrate on the scenario where the effect of substitutability surpasses that of satiation, which is contrary to the case explored in the preceding section.

As shown by relations (49) and (50) in Proposition 6, a dominating substitutability effect implies that the after-lockdown prices, $p_{t,AL}$, are higher than before the lockdown, $p_{t,BL}$. In this section, we will show through a numerical example how this positive change in prices is related to the good 2 pent-up demand formed during the lockdown when consumption expenditures on good 2 were not possible. For this purpose, let us consider an economy which is at its steady state when suddenly a lockdown is imposed.[21] This means that

$$p_{t,BL} = p^*, \qquad and \qquad p_{t,AL} = p^* + SSE \cdot (h_L^* - h_0)(1 - e^{\psi_1 \tilde{t}})e^{\psi_1 (t-\tilde{t})},$$

where $SSE \equiv \frac{(\phi+\psi_1)a_{c_2c_1} + \phi a_{c_2h}}{\phi\lambda} > 0$ since the substitutability effect dominates the satiation effect, $a_{c_2h} > \bar{a}_{c_2h}$.

Let us consider now the following numerical example. Suppose that the parameters describing the production functions are chosen as in the previous exercise. However, we change the initial condition of b_0 from $b_0 = 1$ to $b_0 = \frac{h_0 - y_1}{r}$ so that the economy starts at its steady state. Notice that, the resulting value of b_0 is negative meaning that the debt will be repaid by running a good 1 trade surplus, $TB^* = y_1 - c_1^* > 0$. Moreover, we set $a_{c_2} = 0.8$, $a_{c_2c_2} = -1.257$, $a_{c_1c_1} = -0.7$, $a_{c_1c_2} = 0.6$, and $a_{c_2h} = -0.005$ in order to have (i) a lockdown recession implying a −20% deviation of GDP from its steady state level, (ii) a substitutability effect stronger than the satiation effect (exactly the opposite case of the previous numerical example), (iii) all the conditions for concavity satisfied, (iv) all the inequalities in Proposition 5 satisfied. In addition, we consider a monthly frequency and, therefore, we assume $r = \rho = 0.001$ and a lockdown length of 9 months.

[21] The economy is at its steady state when the initial habits condition is $h_0 = rb_0 + y_1$.

The drastic change in habits documented in the previously mentioned literature (see the Introduction) seems to suggest habits promptly adapting to the new lifestyle imposed by the lockdown. For this reason we set $\phi = 0.15$ consistently with the previous numerical exercise.

Using this choice of parameters we have computed the price dynamics before and after the lockdown. Fig. 3 shows the price deviations from its steady state level. The economy is characterized by a surge in p after the lockdown; as it emerges from the figure, the price overshoots its steady state level p^* by 1.6% points at the date $t = 9$ when the lockdown is lifted. This is driven by the pent-up demand built up during the lockdown and the rigidities in production. As previously explained, the positive change in price happens when the substitutability dominates the satiation effect with the magnitude of the price adjustment depending crucially by the lockdown duration, the habits speed of adjustment to changes in consumption, the habits speed of convergence and the cross-derivatives of utility between good 2 and good 1 consumption and between good 2 consumption and the habits.

To determine how large is the pent-up demand formed during the lockdown, we look at how much the good 2 demand has changed before and at the date of the reopening, $t = 9$. This is done in Fig. 3(b) where we have drawn the inverse demand functions $p = f(c_2; h^*)$ and $p = f(c_2; h_{\tilde{t},AL})$ with $f(p; h) = p = \frac{a_{c_2} + a_{c_2c_2}c_2 + SSE \cdot h}{\lambda}$ together with the supply curve. Then, the change in good 2 demand at the fixed price p^* is:

$$\Delta c_2^d(p^*) \equiv c_{2,AL}^d - c_2^{d*} = \frac{SSE}{-a_{c_2c_2}}(h_{AL} - h^*),$$

which, given our parameters' choice implies a 4.9% increase in the good 2 demand. To meet this demand, output needs to be expanded by an equal percentage. However, this is not possible in our economy as the good 2 supply curve is vertical and good 2 cannot be purchased from abroad. As a consequence the expansion in demand fully translates in a price surge.

Interestingly, also the demand and consumption of good 1 is higher at the end of the lockdown. Based on our parameters' choice, we find that good 1 consumption has expanded by 5.45% at date $t = 9$. As a result of good 1 output expansion during the lockdown, also the debt at date $t = 9$ is lower than at its steady state value. At the same time the

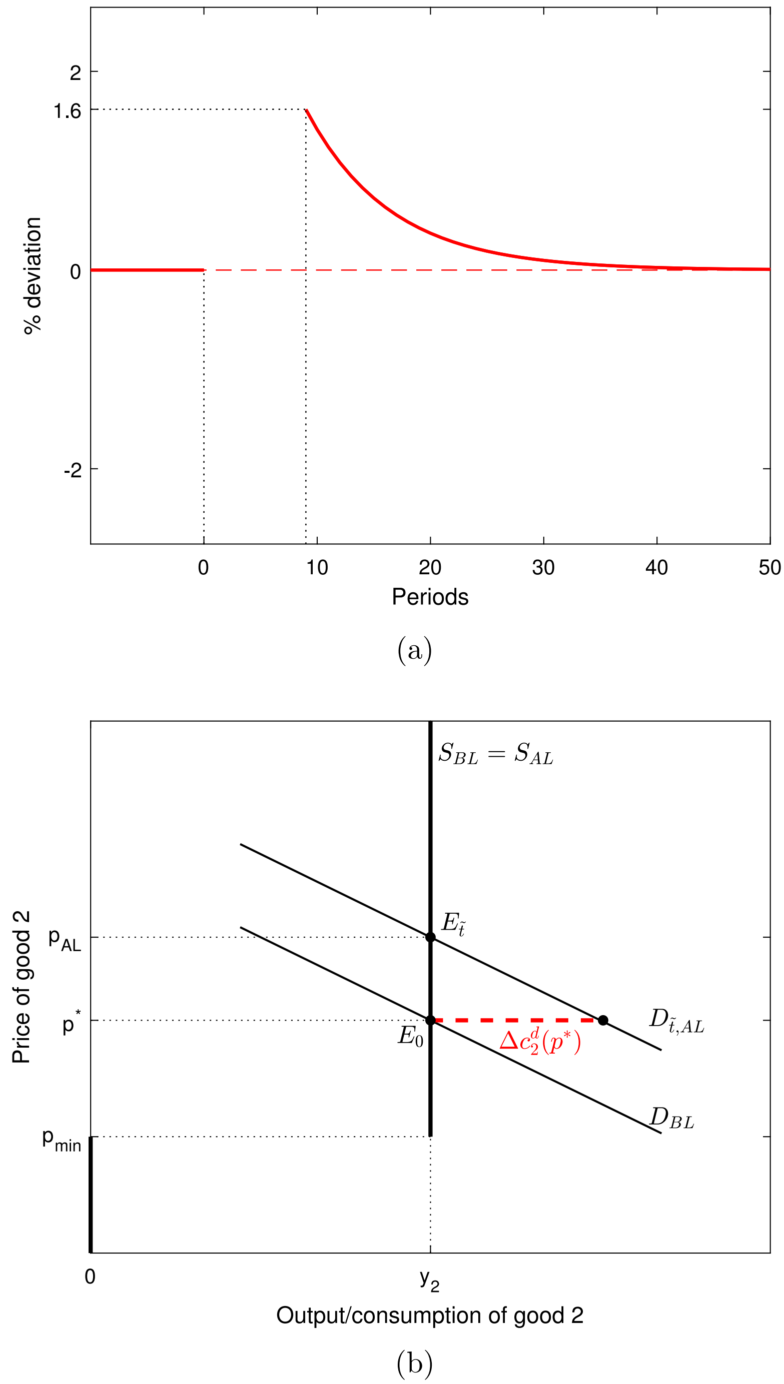


Fig. 3. Pent-up Demand. (a) Relative price percentage deviation from its steady state p^*. (b) Good 2's demand change at the fixed price p^*.

trade balance, $TB_{\tilde{t}} \equiv y_1 - c_{1,\tilde{t}} < y_1 - c_1^* = TB^*$. The remaining debt will be repaid over time till converging to the pre-lockdown steady state.

This prediction of the model seems a plausible channel to explain the actual goods' demand and price surge experienced across developed countries after the lockdown.

6.5. Permanent change in labor composition

We now aim to explore how our findings shift if we deviate from the assumption that labor immediately rebounds after the lockdown. This model variation considers the significant and permanent changes in labor force composition currently observed across developed nations.

In our model, a shift in the labor composition resulting in more time worked within firms of sector 1 implies a change where $d\xi > 0$. Since $y_1 = (\xi \bar{\ell})^\alpha$ and $y_2 = [(1-\xi)\bar{\ell}]^\alpha$ then sector 1 will expand and sector 2 will shrink. A shift in the labor composition will also influence the shadow price, λ, as in equilibrium it is dependent on the output from both sectors, as can be seen in Eq. (37).

For convenience, let us rewrite the equilibrium relative price equation for the model with linear–quadratic utility

$$p = \frac{\phi(a_{c_2} + a_{c_2c_2}c_2) + [(\psi_1 + \phi)a_{c_1c_2} + \phi a_{c_2h}]h}{\phi\lambda}. \tag{52}$$

Notice that this is also the inverse demand curve of good 2, $p = f(c_2; h)$. Using this equation and the inverse supply curve, we will study how the

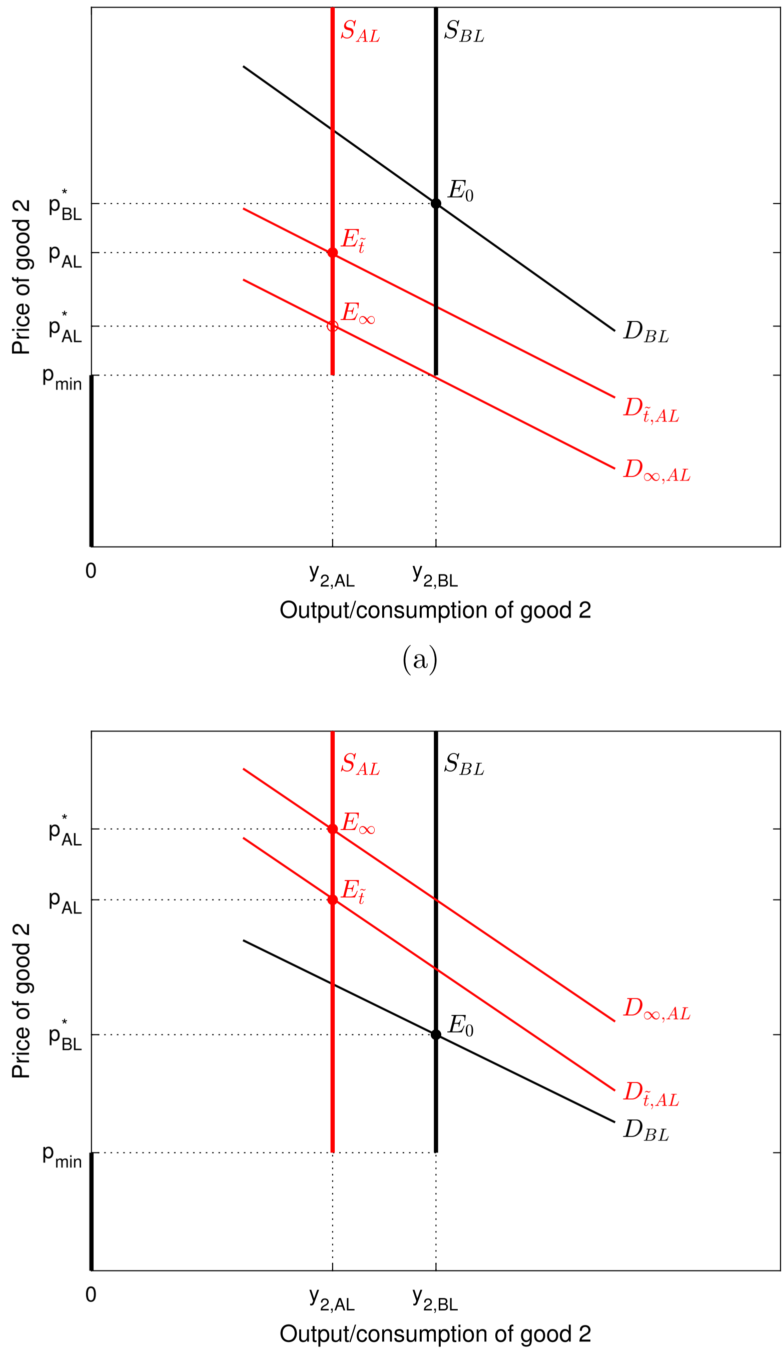


Fig. 4. Lockdown effect on sector 2 in two cases. (a) Sufficiently Strong Satiation, $a_{c_2h} \ll -\frac{\phi+\rho}{\phi} a_{c_2c_1}$. (b) Substitutability > Satiation.

good 2 market looks like at date $t = 0$ and at the date of the re-opening, $t = \tilde{t}$. As shown in Appendix B, the price change equation is equal to

$$dp = \underbrace{\left[\frac{1}{\lambda}\left(a_{c_2c_2}\frac{\partial y_2}{\partial \xi} - p\frac{\partial \lambda}{\partial \xi}\right)\right]}_{Labor\ Composition\ Multiplier\ (LCE)} \cdot d\xi + SSE \cdot dh, \tag{53}$$

where the first term on the right hand side is the labor composition effect while the last is the substitutability-satiation effect. We have also indicated with $SSE = \frac{(\psi_1+\phi)a_{c_1c_2}+\phi a_{c_2h}}{\phi\lambda}$, the multiplier of this last effect which is exactly the same as in the benchmark model. Differently from the benchmark case, the price dynamics is now also driven by the labor composition effect. Let us try to understand a bit more this new channel. Using the shadow price Eq. (37) and the habits path (45), we can rewrite the multiplier of this new effect as follows:

$$LCE = \frac{1}{\lambda}\left[\underbrace{a_{c_2c_2}\frac{\partial y_2}{\partial \xi}}_{>0} - p\left(a_{c_1c_2} + \frac{\phi}{\phi+\rho}a_{c_2h}\right)\underbrace{\frac{\partial y_2}{\partial \xi}}_{<0}\underbrace{-pm_1\frac{\partial y_1}{\partial \xi}}_{>0}\right]. \tag{54}$$

A decrease in production of good 2, $\frac{\partial y_2}{\partial \xi} < 0$, have the following effects. First, a shrink in production increases the price of that good by $a_{c_2c_2}\frac{\partial y_2}{\partial \xi} > 0$ by shifting the supply curve to the left, from S_{BL} to S_{AL} in Fig. 4. However, this effect on the equilibrium relative price can be mitigated or enhanced depending on the level of substitutability between the two goods and the role played by the habits.

In particular, the higher the level of substitutability between the two goods, term $pa_{c_1c_2}$ in the multiplier, the higher will be the positive effect on p as an expansion in good 1 consumption tilts the inverse demand curve up. On the other hand, the higher the degree of satiation on good 2 implied by habits accumulation on good 1, term $-\frac{p\phi}{\phi+\rho}a_{c_2h}$ in the multiplier, the lower will be the effect on p of an adjustment in production as now the inverse demand curve shifts down. Interestingly, the interaction between satiation and substitutability enters also in the labor composition multiplier and, in particular, we have that the former dominates the latter when $a_{c_2,h} < -\frac{\phi+\rho}{\phi}a_{c_2c_1} < -\frac{\psi_1+\phi}{\phi}a_{c_2c_1} \equiv \bar{a}_{c_2h}$ with $a_{c_2c_1} > 0$. A strong satiation effect once compared with the substitutability, mitigates the positive change in the relative price due to the labor reallocation across sectors.

Moreover, an increase in production of good 1, $\frac{\partial y_1}{\partial \xi} > 0$, has a positive effect, $-pm_1 > 0$, on dp as p is the inverse of its relative price. As a consequence it shifts the good 2's inverse demand curve up.

Clearly, the overall sign of the labor composition effect depends on the magnitude of these components. Interestingly, a sufficiently strong satiation effect, $a_{c_2,h} \ll -\frac{\phi+\rho}{\phi}a_{c_2c_1}$, will imply not only $SSE < 0$ but also $LCE < 0$, and similarly to expression (28) the relative price $p_{\tilde{t}}$ shrinks; for the same parameters' choice we can also prove that p^* shrinks due the labor composition change and that p may converges monotonically from above or from below to its new steady state value, $p^*_{AL} < p^*_{BL}$ with AL and BL meaning after and before lockdown respectively. Fig. 4(a) shows how the inverse demand curve, D, and the supply curve, S, adjusts. In particular, we have drawn these curves before the lockdown, i.e. $t = 0$, once the lockdown is over, i.e. $t = \tilde{t}$, as well as at their final position when the economy has reached its steady state, i.e. $t \to \infty$. In Fig. 4(a), the demand curve shifts down from D_{BL} to $D_{\tilde{t},AL}$ because the satiation effect is sufficiently strong. Then, assuming that p converges from above to its steady state level, the inverse demand curve will shift down even further to its position $D_{\infty,AL}$. Note that it could be instead that the convergence is from below and that the new steady state p^*_{AL} would lie between p_{AL} and p^*_{BL}.

On the other hand, if the substitutability dominates the satiation effect, $a_{c_2h} > \bar{a}_{c_2h}$, then, $LCE > 0$, $SSE > 0$, and the demand curve shifts/tilts up from D_{BL} to $D_{\tilde{t},AL}$ while the supply curve shifts to the left from S_{BL} to S_{AL}. As a result, $E_{\tilde{t}}$ will be the new equilibrium in the good 2 market once the lockdown is over. Such an equilibrium will be characterized by a higher relative price, $p_{\tilde{t}}$, than before the lockdown. In addition, also p^* adjusts positively and the relative price converge to its new steady state value $p^*_{AL} > p^*_{BL}$. Observe that this is a sufficient condition meaning that the relative price may adjust positively even when the satiation effect weakly dominates the substitutability effect. Therefore, a readjustment in the labor composition reduces the range of parameters consistent with a permanent depression of sector 2. These considerations and conclusions are illustrated in Fig. 4(b) where we shows the inverse demand curve and the supply curve adjustments before, and at the end of the lockdown $t = \tilde{t}$ as well as their final position when the economy has reached its steady state assumed to be above p_{AL}. The position of the steady state depends on several factors including the magnitude of the labor composition change.

In this last case, a labor composition adjustment affect the steady state prices, and in particular, we have that $p^*_{AL} > p^*_{BL}$. From Fig. 2(b), it is also clear that the raise in the price after the lockdown is due to a combination of a pent-up demand formed during the lockdown together with a shift of the supply curve to the left.

In particular, we prove the following Proposition.

Proposition 7. *If the substitutability is stronger than the satiation,* $a_{c_2h} > \bar{a}_{c_2h}$ *then*

$$\frac{dp^*}{d\xi} > 0.$$

If the satiation is sufficiently strong, $a_{c_2h} \ll -\frac{\phi+\rho}{\phi}a_{c_2c_1}$, *then*

$$\frac{dp^*}{d\xi} < 0.$$

Moreover, it is interesting to have an insight about what happens to the price dynamics when we are not in these two extreme cases, for example, when we choose the parameters so that we have a mild satiation effect. As shown in the Appendix B — Proposition 9, the economy may face the following price dynamics: first, at the end of the lockdown (at time $\tilde{t}$), the relative price could be lower than at its pre-lockdown level, $p_{\tilde{t},AL} < p^*_{\tilde{t},BL}$, however, the price may increase after the lockdown and eventually converges to its steady state which is higher than before the lockdown, $p^*_{AL} > p^*_{BL}$.

7. Conclusion

In this paper, we propose a model that illustrates the relationship between habits concerning one good (good 1) and the relative price of another good (good 2). The first aim of this work, within the context of the habit formation literature, is to explain certain empirical evidence connecting these two variables, as outlined in the Introduction. Our second contribution is to apply this model to comprehend the shifts in demand for goods due to habits formed during a lockdown. Specifically, we observe that the change in the (inverse) demand for the good unavailable during the lockdown depends on the strength of the habits and the length of the lockdown. We delve further to demonstrate that the direction of demand change is influenced by the magnitude of the substitutability effect and the satiation effect. If the latter outweighs the former, the change in demand is negative; if the former is greater, the change is positive.

Lastly, we explored the impacts on demand and price dynamics when there is a permanent change in labor composition following the lockdown. Our findings indicate that the two effects previously mentioned still influence the outcomes, but the dynamics become considerably more complex since a permanent shift in labor composition leads to a sustained change in the post-lockdown steady state. Additional extensions, including the scenario of a randomly determined lockdown duration, present intriguing avenues for future research.

CRediT authorship contribution statement

Mauro Bambi: Investigation. **Daria Ghilli:** Investigation. **Fausto Gozzi:** Investigation. **Marta Leocata:** Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jmateco.2023.102933.

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