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On Debt Neutrality in the Savers-Spenders Theory of Fiscal Policy*

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Abstract

According to the recent literature, heterogeneity across households does not invalidate debt neutrality in the long-run. The doctrinal view is at odds with the layman's view, on the basis of which government debt, by altering the intragenerational distribution of resources, may exert permanent effects on consumption, labor and therefore capital. This paper develops an intertemporal optimizing "savers-spenders" model of capital formation with endogenous labor choices to investigate if the intuitive view has some theoretical support. We discover that Ricardian equivalence is not an ineluctable law of a heterogeneous world. Two dimensions of heterogeneity matter for supporting debt nonneutrality: the savers-spenders distinction, on the one side, and the diversity in tastes, on the other. The dynamic effects of debt are large for some individual variables and factor prices, but may be reduced for aggregate demand and output. The paper shows that in a heterogeneous world also the hypothesis of recursive-time preferences undermines long-run debt neutrality.

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1 Introduction

Can intragenerational heterogeneity across households invalidate debt neutrality in the long-run? Or more clearly, can the non-exact coincidence within the same generation of government bond holders and taxpayers undermine the asymptotic equivalence between government debt and lump-sum taxes?

While we expect the answer of the layman to be yes, the theoretical doctrine provides a negative peremptory answer.

The layman's view is based on the idea that government debt, by altering the intragenerational distribution of resources, may exert permanent effects on consumption and labor decisions, and therefore on physical capital and output.

The doctrinal view, instead, stems from some recent contributions which show that long-run debt neutrality invariably holds in various macroeconomic frameworks that consider household heterogeneity. See, for example, Carmichael (1982), Aiyagari (1989), Evans (1991), Daniel (1993), Elmendorf-Mankiw (1999), Smetters (1999) and Mankiw (2000).[1]

These articles, despite the different models employed, provide an explanation of debt neutrality that ends up with the same basic mechanism: in a world of differentiated degrees of altruism, the discount rate of the most patient households pins down the long-run interest rate, therefore making capital stock independent of government debt.[2]

In this literature, however, two aspects have often been emphasized. First, government debt provokes large long-run redistributive effects across households and therefore neutrality is accompanied by an increase in inequality. Second, intragenerational heterogeneity matters for short-run debt nonneutrality as large transitional effects on aggregate demand occur, due to a

[1] The same conclusion, even if differently motivated, can be found in the survey of Seater (1993). Moreover, Smetters (1999) corroborates this result, demonstrating that the capital stock invariance is also satisfied when several conventional violations of Ricardian equivalence are at work along with heterogeneity.

[2] An exception is represented by Evans (1991), where it is demonstrated that Ricardian equivalence holds approximately in several versions of the Blanchard-Yaari model (modified also to include heterogeneity and imperfect annuity markets) for realistic parameter values.

momentary failure of the equivalence between debt and lump-sum taxes.[3]

The discrepancy between the short-run and long-run doctrinal findings, seen also through the layman's lens, seems to suggest that the intrinsic mechanics of the setups employed (rather than Ricardian equivalence) is actually driving the results. In fact, the redistributive effects of debt do not translate into permanent consequences on capital stock and output because the economy remains in the steady-state immobilized in the quick-sands of a "modified golden rule", having capital stock as a unique endogenous variable.

This paper develops a simple intertemporal optimizing model of capital formation to investigate the issue of debt neutrality and non-demographic heterogeneity with two main tasks. The first is to find out if the layman's view is theoretically correct. In particular, we want to establish whether by relaxing some restrictive features of the models employed in the literature debt neutrality still holds or the income redistribution brought about by government debt is able to produce irreversible real effects. The second related task is to explore explicitly the short-run effects of debt manipulations.

The model is based on the "savers-spenders" theory of fiscal policy formulated by Mankiw (2000). According to this theory, there are two types of agents in the economy: individuals who consume and accumulate capital on an intertemporal basis, i.e. Barro-Ramsey agents called "savers", and individuals having a short-time horizon who cannot accumulate any wealth and consume only the current after-tax income, i.e. Keynesian agents called "spenders".[4] This framework is studied under the hypothesis of endogenous labor-leisure choices as a potential way of breaking the "modified golden rule" entrapment mentioned above.[5]

We discover that Ricardian equivalence is not an ineluctable law of a heterogenous world. Government debt is in general nonneutral in the "savers-spenders" model when we allow for endogenous labor decisions. The violation

[3] These two aspects are particularly emphasized by Daniel (1993), Smetters (1999), and Mankiw (2000).

[4] I chose Mankiw's model mainly for three reasons. First, it is an easy-tractable and transparent model. Second, it has a robust empirical justification, being consistent with three stylized facts: i) highly imperfect consumption smoothing; ii) many households having almost no wealth and few having too much; iii) aggregate wealth accumulation that can largely be explained by intergenerational bequest motive (see, for the empirical adequacy of the model, Campbell-Mankiw, 1989, and Mankiw, 2000). Third, it focuses entirely on intragenerational aspects, abstracting from intergenerational considerations.

[5] Judd (1985, par. 5) develops a two-agents model where both agents supply labor and hold capital. His model is used only for the analysis of optimal capital income taxation.

of Ricardian neutrality is obtained because government debt through its financing scheme, i.e. lump-sum taxes levied on every agent, accomplishes a redistribution of income across Barro-Ramsey and Keynesian individuals, leading to irreversible changes in consumption, labor supply and consequently capital stock. If plausible values of taste parameters are considered, higher debt is associated with lower aggregate labor and capital stock.

The hypothesis of endogenous labor choices, however, is a necessary, but not sufficient, condition for the nonneutrality of a debt-for-tax-swap policy. In fact, capital stock invariance arises when labor supplies are endogenous, but agents have the same tastes. This result suggests that there are two dimensions of heterogeneity that matter to have debt nonneutrality. The conventional one is related to the distinction between Ricardian and Keynesian agents. The second type of heterogeneity, instead, considers differences in tastes among agents for the consumption-leisure trade-off.[6] Therefore the beliefs that the distribution effects of government debt are negligible (Seater, 1993) or that the labor supply endogeneity plays a minor role (Smetters, 1999) may implicitly incorporate the concept of similarity in individual tastes.

The second finding is that, when debt nonneutrality occurs, the dynamic effects of debt are large for some variables, like consumption of savers, labor and factor prices; other variables, like consumption of nonsavers, aggregate demand and output may exhibit a moderate variability. It is not necessarily true, as pointed out by several papers that support long-run Ricardian equivalence, that public debt has large transitional effects on aggregate demand. Moreover, contrary to the standard non-Ricardian view, the "savers-spenders" model predicts that an increase in government debt leads to a short-run fall of the interest rate. Notice that if the permanent rise in debt is accompanied by a sudden increase in lump-sum taxation, no short-run variability will occur when long-run neutrality prevails.

Finally, we show that the introduction of the hypothesis of time-recursive preferences into the "savers-spenders" model with inelastic labor choices invalidates debt neutrality and can give rise to a positive short-run effect of government debt on the real interest rate, resuscitating the conventional results.

[6] Similar types of behavioral differences across individuals have also been contemplated, in analyzing problems of aggregation, by Browning-Hansen-Heckman (1999) and Blundell-Stoker (2000). They highlight three types of heterogeneity: i) heterogeneity in market participation, ii) heterogeneity in individual tastes, and iii) heterogeneity in (uninsurable) risks faced by individuals.

The paper is organized as follows. Section 2 sets out the basic model. The steady-state effects of government debt are investigated in section 3, while its comparative dynamics are studied in section 4. Section 5 develops the case of an endogenous rate of time preference. Section 6 concludes.

2 The model

Consider a real economy populated by three sectors: households, firms and the government. There are two types of households: savers and nonsavers.[7] Savers and nonsavers belong to the same generation as they are both infinitely-lived. Savers decide on consumption, labor supply and wealth accumulation as well as portfolio composition. Financial wealth is given by real capital and government bonds. Nonsavers do not accumulate wealth and choose only consumption and labor effort. Both agents pay lump-sum taxes for financing government expenditures. Production is obtained by competitive firms by using capital and labor. Government decisions on debt and on how to split the burden of taxation between savers and nonsavers are considered to be exogenous. Competitive behavior of agents, perfect foresight and continuous time are assumed.

2.1 *Savers*

The representative agent of the saver-type makes consumption, c_S, labor, l_S, and savings, i.e. accumulation of nonhuman wealth a, decisions in order to solve the following intertemporal problem

$$\max \int_0^\infty U(c_S, 1 - l_S) \exp(-\rho t) dt \tag{1}$$

subject to the instantaneous budget constraint

$$c_S + \dot{a} = w l_S + ra - t_S \tag{2}$$

and the initial condition on wealth: $a(0) = a_0$,
where w is the real wage, r is the rate of return on wealth, t_S represents lump-sum taxes levied on savers, and ρ is the exogenous rate of time preference. The instantaneous utility function $U\,(,)$, defined over consumption and

[7] Although we will always speak of the "savers-spenders" model, we prefer to use the term nonsavers instead of spenders.

leisure, i.e. $1 - l_S$, is assumed to be increasing in its arguments, homothetic and concave.

The optimality conditions for the individual problem (1)-(2) are

$$U_c(c_S, 1 - l_S) = \lambda \tag{3a}$$

$$U_l(c_S, 1 - l_S) = -\lambda w \tag{3b}$$

$$\dot{\lambda} - \lambda\rho = -\lambda r \tag{3c}$$

where λ represents the shadow value of wealth. The flow budget constraint (2) and the transversality condition $\lim_{t\to\infty} \lambda a \exp(-\rho t) = 0$ must also be satisfied at the optimum. Total wealth is composed of two perfectly substitutable assets, i.e. physical capital, k, and government bonds (having instant maturity), b.

2.2 *Nonsavers*

Each member of the nonsaver group decides on consumption, c_N, and labor, l_N, in order to maximize the lifetime utility function

$$\int_0^\infty V(c_N, 1 - l_N) \exp(-\rho t) dt \tag{4}$$

subject to the static budget constraint

$$c_N = w l_N - t_N \tag{5}$$

where t_N represents lump-sum taxes paid by nonsaver households. $V(,)$ satisfies the conventional properties of regularity and is homothetic. Savers and nonsavers are paid the same wage as their labor is assumed to be perfectly substitutable.

The first order conditions characterizing the nonsaver's optimal program are (5) and

$$\frac{V_l(c_N, 1 - l_N)}{V_c(c_N, 1 - l_N)} = -w \tag{6}$$

2.3 *Firms*

Firms behave competitively in the output and factor markets. They produce output, y, by using capital and labor, l, as inputs according to the following production function:

$$y = F(k, l) = lf(\frac{k}{l}) \tag{7}$$

where f is the output-labor ratio. The production function (7) satisfies the conventional properties of regularity and is linearly homogeneous in its arguments.

First order conditions for maximum profit entail[8]

$$F_k(k, l) = r \tag{8a}$$

$$F_l(k, l) = w \tag{8b}$$

Total amount of labor employed by firms must be equal to the sum of labor supplied by the two types of individuals, that is

$$l = l_S + l_N \tag{9}$$

Equation (9) ensures the equilibrium on the labor market.

2.4 *Government budget constraint and output market equilibrium*

The government dynamic budget constraint is

$$\dot{b} = g + rb - t \tag{10}$$

where $t = t_S + t_N$ represents total lump-sum taxes levied on the two agents. The government deficit, given by government expenditure plus interest payments on government debt less lump-sum taxes, can be financed by issuing new debt. However we will assume that the government maintains a balanced budget through the endogenous adjustment of lump-sum taxes.

[8] Capital stock does not depreciate and capital accumulation does not involve adjustment costs.

Furthermore, we assume that each type of household pays a fixed proportion of total lump-sum taxes. This implies the following relationship between lump-sum taxes paid by savers and nonsavers

$$t_S = \frac{(1-\tau)}{\tau} t_N \tag{11}$$

where $\tau \in (0,1)$ represents the proportion of total lump-sum taxes paid by nonsavers. τ is exogenously determined by the policymaker.

Finally, the equilibrium on the goods market requires that potential output always equals aggregate consumption plus government spending plus investment:

$$y = c_S + c_N + g + \dot{k} \tag{12}$$

The complete macroeconomic model -obtained by combining the optimality conditions for savers, nonsavers, and firms with the government budget constraint and the markets clearing conditions- exhibits saddle-point stability as is shown below.

3 Long-run effects of government debt

In this paragraph, we study the long-run effects of an exogenous change in the level of government debt accompanied by the endogenous adjustment of lump-sum taxes levied on savers and nonsavers.

Using, without loss of generality, Cobb-Douglas preferences, the economy is described in the long-run equilibrium by the following system

$$1 - \bar{l}_S = \frac{(1-\alpha)}{\alpha\, \bar{w}} \bar{c}_S \tag{13a}$$

$$1 - \bar{l}_N = \frac{(1-\beta)}{\beta\, \bar{w}} \bar{c}_N \tag{13b}$$

$$\bar{c}_N = \bar{w}\bar{l}_N - \tau(g + \rho b) \tag{13c}$$

$$F_k(\bar{k}, \bar{l}) = \rho \tag{13d}$$

$$\bar{w} = F_l(\bar{k}, \bar{l}) \tag{13e}$$

$$F(\bar{k}, \bar{l}) = \bar{c}_S + \bar{c}_N + g \quad (13f)$$

$$\bar{l} = \bar{l}_S + \bar{l}_N \quad (13g)$$

where overbars denote steady-state endogenous variables, and $\alpha \in (0,1]$ and $\beta \in (0,1]$ are preferences parameters of savers and nonsavers, respectively.

In the long-run the marginal product of capital, i.e. the real interest rate, is fixed by the exogenous rate of time preference, equations (13d), thereby uniquely determining capital intensity since the production function is linearly homogeneous. Therefore the capital-labor ratio is independent of the government debt. This implies that total labor and capital can move only in the same direction and by the same proportion. The long-run wage rate, equation (13e), is also given.

Substituting $\bar{l}_N$ from (13b) into (13c) yields the following reduced-form solution for consumption of nonsavers

$$\bar{c}_N = \beta[\bar{w} - \tau(g + \rho b)] \quad (14)$$

This equation highlights the fact that higher government debt lowers nonsavers consumption by reducing their "full disposable income". In fact, a portion τ of the lump-sum taxes necessary to finance the higher interest payments on debt is levied on nonsavers, who do not hold government bonds and do not receive the "interest gift" from the government. Moreover, according to equation (13b), the reduction of nonsavers consumption increases their labor effort.[9]

What happens to consumption of savers is less obvious. Consumption of the Barro-Ramsey agents can be expressed, by combining (2) and (13a), as

$$\bar{c}_S = \alpha[\bar{w} + \rho\,\bar{k} + \tau\rho b - (1-\tau)g]$$

Here it is evident that the lump-sum taxes paid by savers for financing an additional dollar of government debt, $(1-\tau)\rho$, are lower than the benefits of public debt, ρ, as the other individuals with no government bonds are making their tax contributions of $\tau\rho$. Therefore, government debt raises the disposable income of savers as they obtain a net income of $\tau\rho b$, by holding an

[9] Obviously, in the special case $\tau = 0$, there are no effects of debt on consumption and labor of nonsavers.

amount b of bonds. However, the complete evaluation of savers "full disposable income" and consumption also requires the analysis of the comparative statics effect of b on the whole economy and in particular on capital stock.

A clear understanding of the model is obtained by plugging equation (14) into (13f) and then by eliminating total labor through the "modified golden rule" relationship $\bar{l} = \Gamma \bar{k}$, with $\Gamma = f'^{-1}(\rho) > 0$; we then get

$$\bar{c}_S = f\Gamma \bar{k} - \beta[\bar{w} - \tau(g + \rho b)] - g \tag{15a}$$

This equation represents the output market clearing condition, assuring that long-run output is equal to aggregate demand.[10]

By substituting equations (13a), (13b), and (14) for $\bar{l}_S$, $\bar{l}_N$ and $\bar{c}_N$ respectively into equation (13g), and eliminating total labor through the relationship $\bar{l} = \Gamma \bar{k}$ from (13d), we obtain

$$\bar{c}_S = \frac{\alpha \bar{w}}{(1-\alpha)} \left\{ 2 - \Gamma \bar{k} - \frac{(1-\beta)}{\bar{w}} [\bar{w} - \tau(g + \rho b)] \right\} \tag{15b}$$

Equation (15b) gives the combinations of consumption of savers and capital stock that ensure the equilibrium on the labor market, i.e. labor demand of firms equal to labor supplies of savers and nonsavers.[11]

The core macroeconomic equilibrium is given by the simultaneous equilibrium in the goods and labor markets. By substituting (15a) into (15b), the reduced form for capital stock is obtained

$$\bar{k} = \frac{\Lambda}{\Gamma} \left\{ 2 + \frac{(1-\alpha)}{\alpha \bar{w}} g + \frac{(\beta - \alpha)}{\alpha\beta \bar{w}} [\bar{w} - \tau(g + \rho b)] \right\} \tag{16}$$

where $\Lambda = \dfrac{\alpha \bar{w}}{\alpha \bar{w} + (1-\alpha) f} > 0$ and $\Gamma = f'^{-1}(\rho) > 0$.

[10] Equation (15a) describes a positive relationship between $\bar{c}_S$ and $\bar{k}$ as an increase in capital stock raises output and must therefore be associated with higher consumption of savers, i.e. higher aggregate demand, to maintain the output market in equilibrium, given nonsavers' consumption and public spending. For a given capital stock, a rise in b reducing consumption of nonsavers requires an increase in $\bar{c}_S$ to keep aggregate demand unchanged and guarantee the equilibrium in the goods market.

[11] According to this equation, a higher capital stock raises labor demand of firms and therefore needs, given $\bar{l}_N$, a higher labor supplied by, i.e. a lower consumption of, savers. For a given capital stock (and hence total labor), an increase in labor supply of nonsavers brought about by higher b calls for a drop in the labor effort, hence an increase in consumption, of savers.

The substitution of (16) into one expression for $\bar{c}_S$ above yields

$$\bar{c}_S = 2f\Lambda - \Lambda g - \frac{\Lambda}{\Pi}[\bar{w} - \tau(g + \rho b)] \tag{17}$$

where $\Pi = \dfrac{\beta\,\bar{w}}{\beta\,\bar{w} + (1-\beta)f} > 0$.

>From equations (16) and (17), an increase in public debt causes an unclear effect on capital stock, as $\mathrm{sgn}\left(\dfrac{d\,\bar{k}}{db}\right) = \mathrm{sgn}(\alpha - \beta)$ is ambiguous, and a rise in steady-state consumption of savers, as expected. Therefore the ultimate determinants of the capital stock multiplier are tastes, i.e. α and β.

The intuition behind these results is immediate. A rise in government debt, by modifying savers and nonsavers consumption in the opposite direction through the income redistribution, exerts antithetic effects on individual labor supplies.[12] The labor effort of savers is reduced, while that of nonsavers is increased. The overall effect on total labor, and hence capital, is ambiguous as it depends on whether or not the rise in $\bar{l}_N$ exceeds the reduction in $\bar{l}_S$. Which one of the two effects prevails depends on the impact of public debt on individual consumption and labor supply. This impact is measured by the taste parameters entering the efficiency conditions for the optimal consumption-leisure choices.

If $\alpha > \beta$ (including the extreme case $\alpha = 1$, i.e. inelastic labor supply of savers[13]), a rise in the government debt increases total labor supply and crowds in capital stock. Output and aggregate demand are increased as well. If otherwise $\alpha < \beta$ (including the extreme case $\beta = 1$, i.e. inelastic labor supply of nonsavers), government debt reduces labor, crowds-out capital and lowers output and aggregate demand.

Which one of the two cases is more plausible? As savers have a greater "full disposable income", we can reasonably assume that the value of their consumption of leisure as a ratio of disposable income, i.e. $1 - \alpha$, is greater than the corresponding one of nonsavers, i.e. $1 - \beta$. In other words, savers consume relatively more leisure as a percentage of their "full disposable income". This assumption implies that $\beta > \alpha$. Therefore the most plausible case is given by a government debt negatively affecting labor, capital for-

[12] Note that the experiment assumes that the tax burden is spread across agents.

[13] This case may also describe a Kaldorian economy populated by capitalists and workers. See Kaldor (1955) and Lansing (1999).

mation and output. However, the reduction of capital does not prevent consumption of savers to rise.

The redistribution of income across households also implies a welfare redistribution. In fact, welfare of nonsavers is unambiguously lowered by higher government debt, because of joint reduction in consumption and leisure. The opposite occurs for savers.

The following comments are in order. First, government debt can be nonneutral in the long-run, since by redistributing income among households it changes consumption and working efforts of agents thereby exerting permanent effects on capital and output. Nonneutrality of public debt is obtained when labor is supplied elastically by at least one agent, namely, according to our parameterization, savers.

Second, the result obtained in the literature of long-run debt neutrality in a heterogeneous world is based on the special case $\alpha = \beta = 1$, i.e. inelastic labor supply of savers and nonsavers. It is still satisfied however when labor supplies are endogenous and the two agents have the same behavioral parameters, i.e. $\alpha = \beta < 1$.

This feature of the model highlights that there are two types of heterogeneity. One type considers the distinction between altruistic no-liquidity-constrained agents and liquidity-constrained ones, while the other type considers differences in tastes among agents for the consumption-leisure trade-off. The higher the disparity in tastes between the two classes of agents, the less neutral is government debt.

Notice that government debt is nonneutral when both types of heterogeneity are included in the model. If any of this type of heterogeneity is lost, government debt will become neutral in the long-run.

Our findings can be summarized in the following proposition:

Proposition 1 *A necessary condition to invalidate long-run neutrality of government debt within an intragenerationally heterogeneous world of the savers-spenders type is to consider an elastic labor supply. However this condition is necessary, but not sufficient. Sufficiency requires different tastes between savers and nonsavers regarding consumption-leisure choices. If the above conditions are satisfied, under a plausible parameterization, high government debt lowers labor and capital formation, while redistributing income and welfare across households.*

4 The comparative dynamics of government debt

Most of the contributions that support long-run Ricardian neutrality in models with heterogenous-agents emphasize that government debt generates relatively large effects in the short-run, especially on aggregate demand.

Our purpose is to study whether government debt produces high short-run variability when steady-state nonneutrality occurs. With regard to this, the question to be ascertained is whether the short-run variability is greater or lower than the long-run one.

In order to simplify the analysis of the transitional dynamics considerably, we focus on the case of an inelastic labor supply of nonsavers, i.e. $\beta = l_N = 1$. The results of this pilot case are rather general as they carry over for other possible values of taste parameters (provided $\beta > \alpha$).

Figure 1 contains the phase diagram we use to illustrate the long-run equilibrium and the transitional dynamics. Analytical details regarding the stability condition of the model and the phase diagram are given in Appendix A.

The $\dot{c}_S = 0$ schedule represents the capital market equilibrium condition, assuring perfect consumption smoothing along the optimal path, i.e. $r = \rho$. This schedule is downward-sloping. The $\dot{k} = 0$ schedule describes the combinations of consumption of savers and capital stock that maintain the equilibrium on the goods market. This schedule has an ambiguous slope. In figure 1 we have assumed that the $\dot{k} = 0$ schedule is positively sloped.[14]

The system exhibits saddle-point stability. The saddle-path, labeled SS in figure 1, is upward sloping, and steeper than the $\dot{k} = 0$ schedule. For any values of k different from the steady-state, the value of c_S must be such as to place the economy on the unique converging path.

Consider an unanticipated permanent increase in b. The long-run effect is for $\bar{c}_S$ to rise and for $\bar{k}$ to fall. In figure 1 the initial equilibrium is at A_0 and the new one at A_1.

As soon as the unexpected rise in public debt, accompanied by higher taxes, takes place, the saddle-path shifts upward.[15] Consumption of savers rises on impact to bring the economy onto the new saddle-path S'S' at A_{01},

[14] Notice that the sign of the $\dot{k} = 0$ schedule slope does not affect qualitatively the short-run dynamics (see Appendix A).

[15] Both the $\dot{k} = 0$ and $\dot{c}_S = 0$ schedules shift up, but the $\dot{k} = 0$ schedule shifts up by more.

overshooting its new long-run equilibrium value. The upward jump of savers' consumption causes an instantaneous drop in labor effort. Also production falls on impact. Since capital stock is predetermined at its initial values, the fall in labor effort drives the real wage up and reduces the real interest rate. Both factor prices overshoot their constant steady-state values. The rise in wage dampens the labor drop, dictating whether labor overadjusts or not on impact. However, according to structural parameters, the most plausible case is given by an overshooting of labor.

In response to the unanticipated shock, consumption of nonsavers may either rise or fall as the reduction of disposable income due to the higher debt may be reverted by the short-run increase in real wage and the softer tax burden (compared to the steady-state one) due to the interest rate reduction. However, an initial drop in nonsavers' consumption is more likely to occur, implying undershooting. Aggregate demand may jump up or down when the shock takes place.

INSERT FIG. 1

Why does consumption of savers overadjust on impact, transmitting to the whole system a high potential volatility? The intuitive explanation is the following. Permanent income of the forward-looking agents is increased by higher government debt. Savers revise their consumption upward -since they anticipate higher future permanent consumption- and work less. The consequent impact reduction of the interest rate calls for expectations of future contraction of consumption in order to satisfy the Keynes-Ramsey intertemporal arbitrage condition. Therefore the extent of savers' consumption jump has just to bring about an anticipation of future reduction of consumption along the transition path so as to offset the reduced interest rate. The only way to realize this is to have an overresponse of savers consumption.

After the system has been placed on the new stable arm, the economy converges monotonically toward the long-run equilibrium. Capital stock begins to decumulate because of the reduction of the interest rate and the upward jump of consumption diminishes saving. Savers' and nonsavers' consumption, wage and aggregate demand fall along the convergent path.[16] Labor and interest rate increase and output may either fall or rise.

The variability of the whole system strictly depends on the high volatility of savers' consumption and on how much the wage reacts to changes in labor.

[16] If the impact effect on aggregate demand is positive, a perverse-shooting of aggregate demand will occur. This is a case of high volatility of aggregate demand.

If the wage does not move very much in the short-run, we have a labor overreaction which probably leads to an output overshooting. If instead there is a strong wage adjustment on impact, labor and output undershootings occur, while aggregate demand probably increases initially and diminishes along the transition path. In this case much of the variability is transferred to the demand side.

Three aspects on the transitional dynamics need to be emphasized. First, the model predicts that government debt lowers the interest rate on impact. This negative association is contrary to the conventional view of public debt.[17]

Second, when labor is inelastically supplied by both agents, i.e. we are in the neutrality case described by Mankiw (2000) and others, or agents supply labor elastically and share the same tastes, i.e. $\alpha = \beta < 1$, the model does not admit transitional dynamics. In response to the debt shock, consumption of savers and nonsavers immediately jumps to the new equilibrium, while capital stock remains unchanged at its original level. In this special case, government debt is neutral both in the short and long-run.[18]

Third, if an endogenous labor supply of nonsavers is assumed, government debt will increase labor effort of nonsavers on impact and in the steady-state, dampening the rise in the wage rate. In this circumstance, total labor may rise or fall on impact. Labor effort falls along the convergence toward the new equilibrium. In this case the variability induced by government debt would be dampened as the long-run multipliers, which govern the short-run impacts, are reduced.

The analysis of the short-run dynamics can be summarized as follows:

Proposition 2 *Provided that long-run debt is nonneutral, an unexpected permanent increase in government debt generates large transitional variability in consumption of savers, labor and factor prices. The variability of nonsavers' consumption, output and aggregate demand may be reduced in the short-run compared to the long-run. Contrary to the conventional view,*

[17] The reduction of the interest rate caused by higher government debt can also be found in one-sector no bequests OLG models having endogenous labor supply. See, for example, Phelps (1994).

[18] Notice that if a temporary reduction of lump-sum taxes, followed by an immediate government debt issuance and after some periods higher lump-sum taxes, were considered, a high short-run variability of aggregate demand would appear in the model. See Daniel (1993) and Mankiw (2000). Our experiment focuses on the case of a sudden adjustment of lump-sum taxes following the debt rise.

government debt reduces the real interest rate on impact. If, however, the long-run debt-taxes equivalence is satisfied, i.e. labor choices are inelastic or savers and nonsavers have the same tastes, the economy immediately jumps from the initial steady-state to the new one after a permanent debt shock takes place and no transitional dynamics occur.

5 Government debt and endogenous rate of time preference

In section 2, we have shown that when the assumption of inelastic labor decisions is relaxed, government debt is no longer equivalent to lump-sum taxes since the redistributive effects of debt generate irreversible effects. The reason for this result is that endogenous labor supply breaks the isolated capital stock determination implied by the "modified golden rule".

The purpose of this section is to present another simple way of obtaining the violation of long-run Ricardian equivalence. An additional motivation for doing this experiment is the short-run negative association between government debt and real interest rate, which is contrary to the traditional view of government debt. In order to provide such an extension of the savers-spenders model, we incorporate into the model the hypothesis of an endogenous rate of time preference, by letting the rate of time discount depend positively on the level of utility.[19] For the simplicity and transparency of the results of this experiment inelastic labor choices are assumed.

The representative saver maximizes the following functional

$$\int_0^\infty U(c_S)\exp(-\Phi)dt \tag{1'}$$

subject to the instantaneous budget constraint (2) and the initial condition on wealth. The discount factor in (1') is given by

$$\Phi = \int_0^t \rho[U(c_S)]dv \tag{1''}$$

where the endogenous discount rate ρ satisfies the properties: $\rho > 0$, $\rho' > 0$, $\rho'' > 0$, and $\rho - U\rho' > 0$.

[19] This is the case of the Uzawa (1968) preferences. This hypothesis is functionally equivalent to the case of the Epstein (1987) preferences, i.e. recursive but intertemporally dependent preferences. See Obstfeld (1990).

The first-order conditions are

$$1-\frac{\rho'[U(c_S)]}{\rho[U(c_S)]}\{U(c_S)+\lambda[w+ra-t_S-c_S]\}=\frac{\lambda}{U_c(c_S)} \tag{3a'}$$

$$\dot{\lambda}=\lambda\{\rho[U(c_S)]-r\} \tag{3b'}$$

together with the budget constraint (2) and the proper transversality condition.

The rest of the model is the same as before. The model is saddle-point stable as shown in Appendix B.

The long-run economy is described by the system

$$\bar{c}_N= F_l(\bar{k})-\tau[g+F_k(\bar{k})b] \tag{18a}$$

$$F_k(\bar{k})=\rho[U(\bar{c}_S)] \tag{18b}$$

$$F(\bar{k})=\bar{c}_S+\bar{c}_N+g \tag{18c}$$

Equation (18a) represents nonsavers' consumption function. Equation (18b), i.e. the "modified golden rule", gives the supply of capital of the forward-looking agents. It postulates a negative relationship between capital stock and consumption of savers. A higher consumption of savers increases the rate of time preference and the real interest rate, leading to a fall in capital. Equation (18c) represents the goods market equilibrium condition. This equation postulates a positive relationship between capital stock and savers consumption.

The permanent rise in government debt by reducing consumption of nonsavers tends, through equation (18c), to reduce capital stock.[20] But the lower capital stock implies higher consumption of savers as the discount rate and interest rate are increased. The increase of consumption of savers does not prevent capital from falling.

The transitional dynamics can be described through figure 1. The saddle-path also in the present case is upward-sloping. The dynamic behavior of consumption and capital is similar to that seen in the case of elastic labor

[20] If we plug equation (18a) into (18c), we obtain: $H(\bar{k},b)=\bar{c}_S+(1-\tau)g$, where $H_k>0$ (if the elasticity of substitution between labor and capital is nearly one), and $H_b>0$.

choices. Savers' consumption jumps up on impact, more than the long-run adjustment. There is an initial drop in nonsavers consumption, which undershoots the new long-run value. The interest rate, given $r = \rho[U(c_S)] + \frac{\dot{\lambda}}{\lambda}$, may either increase or decrease on impact as the subjective discount rate of savers is pulled up and expectations of a future contraction of consumption along the transition path, i.e. increase of the marginal utility of wealth, arise. If the first effect prevails, we obtain a positive association between debt and the interest rate as predicted by the conventional view.[21] Aggregate demand is increased on impact and reduced along the transition path. This is a case of high volatility of aggregate demand.

The convergence is characterized by a capital decumulation and consumption of both agents as well as aggregate demand reduction.

The findings of this section can be recapitulated as follows

Proposition 3 *The assumption of recursive but intertemporally dependent preferences generates government debt nonneutrality as it tackles the "modified golden rule" entrapment. Government debt reduces capital and increases the interest rate, while redistributing income across agents. Consumption of savers and aggregate demand are highly volatile in the short-run. The savers-spenders model amended to incorporate this hypothesis may predict a positive association between government debt and real interest rate in accordance with the traditional view.*

6 Conclusion

This paper has investigated the robustness of the repeatedly asserted long-run invariance of capital stock to government debt manipulations in a world of heterogenous households. The analysis is based on an intertemporal optimizing "savers-spenders" model of capital accumulation.

We have shown that by relaxing apparently innocuous hypotheses and conventional nonviolations of Ricardian equivalence, like inelastic labor choices

[21] This version of the "savers-spenders" model, despite the two-agents structure, has a close resemblance to the representative agent optimizing model of Devereux (1991). There it is shown that government spending and real interest rate may be negatively correlated, contrary to the prediction of the one sector neoclassical growth model with a fixed discount rate.

or a fixed rate of time preference, the results changed substantially as permanent departures from long-run debt neutrality are obtained.

In the "savers-spenders" model any hypothesis capable of breaking the isolated capital stock determination implied by the "modified golden rule" generates long-run debt nonneutrality, because of the income/wealth redistribution across agents brought about by government debt changes. Notice that within this context agents that do not hold government bonds play the same role in invalidating Ricardian equivalence that the "new entrants" play in nonaltruistic OLG models.

In this perspective government debt represents a way of redistributing welfare among agents of the same generations. Therefore public debt redistributes resources not only between different generations (as the classical literature on public debt has largely emphasized), but also within the same generation. In so doing, government debt may represent a way of subsidizing some individuals, i.e. the savers, at the expense of others, i.e. the spenders, and therefore exert permanent effects on capital formation through changes in labor effort or the endogenous discount rate.

Finally, the key findings of the analysis are the following: i) Ricardian equivalence can be invalidated by letting labor be endogenously chosen. Two dimensions of heterogeneity matter for supporting debt nonneutrality in this case: the savers-spenders distinction, on the one side, and the diversity in tastes, on the other; ii) the dynamic effects of debt are large for some individual variables and factor prices, but may be reduced for output and, contrary to the recent literature, aggregate demand; iii) the hypothesis of recursive-time preferences undermines long-run debt neutrality when labor choices are inelastic.

A Appendix

The analysis of the transitional dynamics is developed under the simplifying assumptions that nonsavers supply labor inelastically, i.e. $\beta = 1$ and $l_N = 1$, and both agents have logarithmic preferences.

The short-run model is given by

$$\frac{\alpha}{c_S} = \lambda \tag{Ia}$$

$$\frac{(1-\alpha)}{(2-l)} = \lambda F_l(k,l) \tag{Ib}$$

$$c_N = F_l(k,l) - \tau\left[g + F_k(k,l)b\right] \tag{Ic}$$

$$\dot{\lambda} = \lambda\left[\rho - F_k(k,l)\right] \tag{Id}$$

$$\dot{k} = F(k,l) - c_S - c_N - g \tag{Ie}$$

where $l = l_S + 1$.

After using equation (Ia) to eliminate λ from the model, equations (Ib) and (Ic) can be solved, once linearized around the steady-state, for l and c_N in terms of the dynamics variables (c_S and k) and the exogenous variable (b) to yield

$$l = l_S(c_S, k) \tag{IIa}$$

$$c_N = c_N(c_S, k, b) \tag{IIb}$$

where

$l_{c_S} = -\dfrac{(1-\alpha)}{\Theta} < 0$; $\quad l_k = \dfrac{\alpha(2-\bar{l})F_{kl}}{\Theta} > 0$;

$c_{N,c_S} = -\dfrac{(1-\alpha)(F_{kl} - \tau b F_{kl})}{\Theta} > 0$; $c_{N,k} = \dfrac{\alpha F_l(F_{kl} - \tau b F_{kk})}{\Theta} > 0$;

$c_{N,b} = -\tau\rho < 0$; and $\Theta = \alpha[F_l - (2-\bar{l})F_{ll}] > 0$.

By substituting out the values of l and c_N from equations (IIa) and (IIb) into equations (Id) and (Ie), the model can be reduced to the following pair of differential equations linearized around the steady-state

$$\begin{bmatrix} \dot{c}_S \\ \dot{k} \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} \begin{bmatrix} c_S - \bar{c}_S \\ k - \bar{k} \end{bmatrix} \tag{III}$$

where

$$j_{11} = -\frac{(1-\alpha)\ \bar{c}\ F_{kl}}{\Theta};$$

$$j_{12} = \frac{\alpha\ \bar{c}\ F_l F_{kk}}{\Theta};$$

$$j_{21} = \frac{\left[-F_l + (1-\alpha)(F_{ll} - \tau b F_{kl}) + \alpha(2-\ \bar{l}\ )F_{ll}\right]}{\Theta};$$

$$j_{22} = \frac{\left\{\alpha F_k[F_l - (2-\ \bar{l}\ )F_{ll}] + \alpha(2-\ \bar{l}\ )F_l F_{kl} - \alpha F_l(F_{lk} - \tau b F_{kk})\right\}}{\Theta}.$$

Notice that, while $j_{11} < 0$, $j_{12} < 0$ and $j_{21} < 0$, the sign of j_{22} is ambiguous.

The determinant of the above Jacobian is unambiguously negative as a required condition for saddle-point stability.[22]

The slope of the $\dot{c}_S = 0$ schedule in figure 1 is $-\dfrac{j_{12}}{j_{11}} < 0$, while the slope of the $\dot{k} = 0$ schedule is $-\dfrac{j_{22}}{j_{21}}$, which is ambiguous.

The equation of the stable manifold is

$$c_S = \bar{c}_S + \Xi(k - \bar{k})$$

where $\Xi = \dfrac{j_{12}}{\eta_1 - j_{11}} = \dfrac{\eta_1 - j_{22}}{j_{21}} > 0$ and $\eta_1 < 0$ denotes the stable eigenvalue of the Jacobian in (III).[23]

[22] In fact, the transition matrix must have one positive eigenvalue associated with the jump variable, c_S, and one negative eigenvalue associated with the predetermined variable, k. The determinant is equal to $\mid J \mid = -\dfrac{\left\{\bar{c}\ F_{kl}[(1-\alpha)\ \bar{y} + \alpha F_l\ \bar{l}\ ]\right\}}{\Theta\ \bar{k}} < 0$.

[23] Notice that the saddle-path is always positively sloped regardless the sign of j_{22} . In fact, if $j_{22} > 0$, we have that $\dfrac{\eta_1 - j_{22}}{j_{21}} > -\dfrac{j_{22}}{j_{21}} > 0$. While if $j_{22} < 0$, it can be easily demonstrated that $\eta_1 - j_{11} < 0$.

B Appendix

In the case of an endogenous rate of time preference, the complete short-run model is

$$\lambda = \frac{[\rho - U(c_S)\rho']U_c(c_S)}{\rho + [F_l(k) + F_k(k)k + \tau F_k(k)b - (1-\tau)g - c_S]\rho' U_c(c_S)} \qquad \text{(Ia')}$$

$$c_N = c_N(k, b) \qquad \text{(Ib')}$$

$$\frac{\dot{\lambda}}{\lambda} = [\rho - F_k(k)] \qquad \text{(Ic')}$$

$$\dot{k} = F(k) - c_S - c_N - g \qquad \text{(Id')}$$

where $\rho = \rho[U(c_S)]$ and $\rho' = \rho'[U(c_S)] > 0$ and $c_{N,k} = F_{kl} - \tau b F_{kk} > 0$, $c_{N,b} = -\tau\rho < 0$;

>From equation (Ia'), we get

$$\lambda = \lambda(c_S, k) \qquad \text{(IIa')}$$

where $\lambda_{c_S} = \dfrac{[(\rho - U\rho')U_{cc} - UU_c^2\rho'' + \bar{\lambda}\, U_c\rho']}{\rho} < 0$;

$\lambda_k = -\dfrac{[\bar{\lambda}\, U_c\rho'(F_k + \bar{k}\, F_{kk} + F_{kl} - \tau b F_{kk})]}{\rho} < 0.$

By using equation (IIa'), once linearized, to eliminate $\dfrac{\dot{\lambda}}{\lambda}$ from (Ic') and by substituting out c_N, the model can be reduced to the following dynamic system

$$\begin{bmatrix} \dot{c}_S \\ \dot{k} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} c_S - \bar{c}_S \\ k - \bar{k} \end{bmatrix} \qquad \text{(III')}$$

where

$h_{11} = \dfrac{(\bar{\lambda}\, U_c\rho' + \lambda_k)}{\lambda_c}$;

$h_{12} = -\dfrac{[\bar{\lambda}\, F_{kk} + \lambda_k(F_k + \bar{k}\, F_{kk} + F_{kl} - \tau b F_{kk})]}{\lambda_c}$;

$h_{21} = -1$;

$h_{22} = (F_k + \bar{k}\, F_{kk} + F_{kl} - \tau b F_{kk})$.

The determinant of the Jacobian is negative as required for a saddle-point property.[24] The equation of the saddle-path is given by

$$c_S = \bar{c}_S + \Theta(k - \bar{\bar{k}})$$

where $\Theta = -[\epsilon_1 - (F_k + \bar{k}\ F_{kk} + F_{kl} - \tau b F_{kk}) > 0$ and $\epsilon_1 < 0$ is the stable eigenvalue of the coefficient matrix.

The long-run multipliers are

$$\frac{d\ \bar{c}_S}{db} = -\frac{\tau F_k F_{kk}}{[\rho' U_c(F_k - F_{kl} + \tau b F_{kk}) - F_{kk}]} > 0;$$

$$\frac{d\ \bar{k}}{db} = -\frac{\tau \rho' U_c F_k}{[\rho' U_c(F_k - F_{kl} + \tau b F_{kk}) - F_{kk}]} < 0;$$

$$\frac{d\ \bar{c}_N}{db} = -\left\{\tau F_k + \frac{\tau \rho' U_c F_k(F_{kl} - \tau b F_{kk})}{[\rho' U_c(F_k - F_{kl} + \tau b F_{kk}) - F_{kk}]}\right\} < 0.$$

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[24] The determinant is equal to $\mid J' \mid = \frac{[(F_k + \bar{k}\ F_{kk} + F_{kl} - \tau b F_{kk})\ \bar{\lambda}\ U_c \rho' - F_{kk}\ \bar{\lambda}]}{\lambda_c} < 0.$

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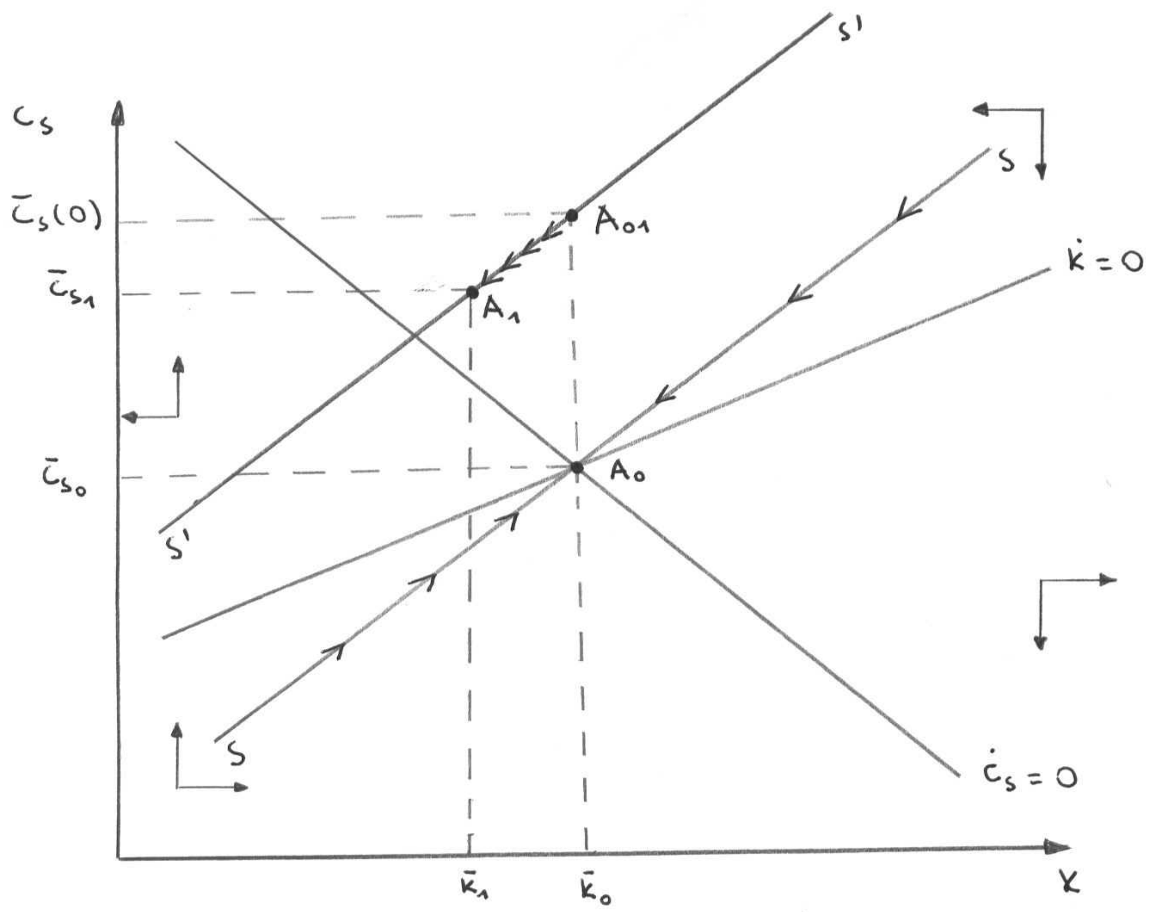

c_s
$\bar{c}_s(0)$
$\bar{c}_{s1}$
$\bar{c}_{s0}$
A_{01}
A_1
A_0
S'
S
$\dot{k} = 0$
$\dot{c}_s = 0$
$\bar{k}_1$
$\bar{k}_0$
k