Discussion on “Sparse graphs using exchangeable random measures” by F. Caron and E.B. Fox, written by Roberto Casarin[†], Matteo Iacopini[†§*] and Luca Rossini[†‡]([†] University Ca’ Foscari of Venice, [§] Université Paris 1 - Panthéon-Sorbonne and [‡] Free University of Bozen-Bolzano).

Caron and Fox are to be congratulated on their excellent research, which has culminated in the development of a new class of random-graph models. The node degree and the degree distribution fail in giving a unique characterisation of network complexity (Estrada (2010)). For this reason global connectivity measures, such as communicability (Estrada and Hatano (2008, 2009)) and centrality (Borgatti and Everett (2006)) are used to analyse a graph. In this discussion we contribute to the analysis of the generalized gamma process (GGP) model compared with the Erdős-Rényi and the preferential attachment (Barabási and Albert (1999)) models. Our analysis is far from being exhaustive but shows that more theoretical aspects of the GGP model are to be investigated.

A connected component of the n-nodes graph $G = (V, E)$ is a subgraph in which any two vertices v_i and v_j are connected by paths. The number of connected components equals the multiplicity of the null eigenvalue of the graph Laplacian L, where the (i, j) entry of L is

$$L_{ij} := \begin{cases} d(v_i) & \text{if } i = j, \\ -1 & \text{if } i \neq j \text{ and } (v_i, v_j) \in E, \\ 0 & \text{otherwise,} \end{cases}$$

with $d(v_i)$ the degree of v_i.

The global clustering coefficient measures the tendency of nodes to cluster and is defined as

$$C = \frac{\text{number of triangle loops}}{\text{number of connected triples of vertices}}.$$

The assortativity coefficient between pairs of linked nodes is given by

$$r = \frac{\sum_{j=1}^{n} \sum_{k=1}^{n} jk(e_{jk} - q_j q_k)}{\sigma_q^2},$$

where q_k and e_{jk} are the distribution and the joint excess degree probability of the remaining degrees respectively, for the two vertices v_j and v_k, and σ_q is the standard deviation of q_k.

Finally, given the partition of the network into two non-overlapping subgraphs (core and periphery) that maximizes the number or weight of within-core-group edges, we compute the share of nodes in the core.

According to Figure 1(a), the GGP couples with the preferential attachment model and performs slightly worse than the Erdős-Rényi random graph in terms of the number of connected components. Figure 1(b) and Figure 1(c) highlight that the clustering structure of GGP does not vary too much with σ. The clustering coefficient is in line with the two benchmarks while the assortativity of the Erdős-Rényi model is not attained. For $\sigma = 0.5, 0.8$ GGP exhibits a lower share of nodes in the core (Figure 1(d)) than in the benchmarks and mimics the preferential attachment model for $\sigma = 0$.

Overall, the GGP can replicate typical behaviours of real world sparse networks and some fundamental features of random graphs generated from the preferential attachment model, making it suitable for a variety of applications in different fields.

We are very pleased to thank the authors for their work.

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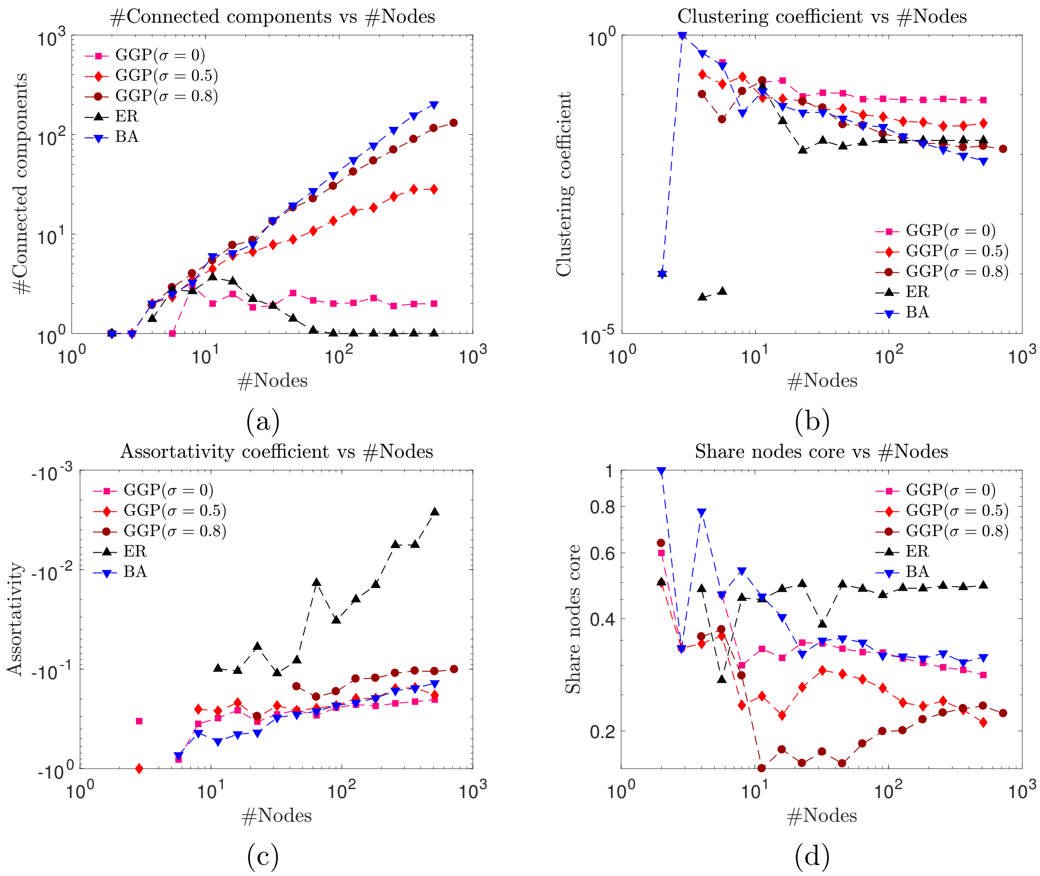

Figure 1: Network statistics versus number of nodes for the GGP undirected network, the Erdős-Rényi and the preferential attachment model of Barabási and Albert (1999): (a) number of connected components; (b) clustering coefficient; (c) assortativity coefficient; (d) share nodes core.

References

Barabási, A. L. and Albert, R. (1999). Emergence of scaling in random networks. *Science*, 286:509–512.

Borgatti, S. P. and Everett, M. G. (2006). A graph-theoretic perspective on centrality. *Social Networks*, 28(4):466–484.

Estrada, E. (2010). Quantifying network heterogeneity. *Physical Review E*, 82:066102.

Estrada, E. and Hatano, N. (2008). Communicability in complex networks. *Physical Review E*, 77(3):036111–12.

Estrada, E. and Hatano, N. (2009). Communicability graph and community structures in complex networks. *Applied Mathematics and Computation*, 214(2):500–511.