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The economics of helicopter money[☆]

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ABSTRACT

The economics of helicopter money is fundamentally tied to price-level determination in monetary models. In frameworks with intrinsically worthless currencies, the issuer's liabilities define the unit of account, and uniquely empower the issuer to implement helicopter money and escape liquidity traps. While traditional helicopter money requires cooperation between the treasury and the central bank — with the central bank critically guaranteeing treasury debt — we demonstrate that helicopter money can also be effectively executed independently by government or private currency issuers, without treasury involvement.

1. Introduction

"Let us suppose now that one day a helicopter flies over this community and drops an additional $1,000 in bills from the sky, which is, of course, hastily collected by members of the community. Let us suppose further that everyone is convinced that this is a unique event which will never be repeated." (Friedman, 1969)

Following the COVID-19 pandemic, the US government approved a two-trillion-dollar support package for the economy, and the Federal Reserve committed to unlimited quantitative easing, including the purchase of Treasury debt. The UK government announced it would extend the size of the government's bank account at the central bank, known historically as the "Ways and Means Facility". The European Central Bank also extended its asset purchase program to unprecedented levels.

A possible implementation of Friedman's proposal is for the government to transfer funds to citizens, financed by issuing debt, which is then purchased by the central bank through an increased supply of money or reserves. This suggestive idea received considerable attention in academia and policy circles, given that central banks across the globe, during the last 20 years, lost their conventional ammunition, having slashed the nominal interest rate down to zero. Helicopter money was discussed as a viable option to reflate the economy (see, among others, Bernanke, 2002, 2003; Galí, 2020a,b; Turner, 2013, 2016).

Since 2022, inflation has surged in many countries, and several commentators have argued that the extraordinary fiscal and monetary policy expansions are the culprits, particularly for the U.S. economy.[1]

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[1] See, among others, Summers (2021, 2022).

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This paper studies the economics behind the effectiveness of traditional helicopter money, linking it to the more general issue of price-level determination in monetary models, and characterizes a class of equivalent policies.

It describes an economy plagued by a slump due to an adverse demand shock, in which even cutting the nominal interest rate down to zero does not bring the economy to full capacity, as in the framework of Krugman (1998). The novelty of our analysis lies in going beyond the traditional concept of helicopter money, which emphasizes the effectiveness of an "unfunded" or "unbacked" fiscal expansion. Instead, we explore alternative approaches to implementing helicopter money that do not necessarily involve the treasury's participation or require an explicit or implicit central bank guarantee on treasury debt. This perspective is particularly relevant given recent developments in private cryptocurrencies, which are entirely detached from government oversight and have raised questions about their ability to maintain stable value.

In the traditional approach, the treasury's expansionary fiscal policy would put its finances in a state of insolvency, making it clear that the central bank is financing the unfunded fiscal expansion. Although the borders between monetary and fiscal policies are ambiguous in many jurisdictions, this full support by the central bank for treasury operations should not necessarily be taken for granted or not understood as such. This can be observed from the recent experience of the U.S. treasury debt downgrade or the market reactions to discussions about the debt limit. Instead, we consider situations in which the treasury cannot run unfunded fiscal expansions that are not subject to market valuation regarding their implications for solvency. In this way, our approach addresses institutional settings in which the boundaries between the currency issuer and the treasury are more clearly defined, such as those involving private currencies that operate entirely independently of the government. For these reasons, we outline the operational framework of a generic currency issuer rather than a government central bank.

One might then wonder how helicopter money can work without the treasury's involvement. In his writing, Friedman's hypothetical experiment was meant to show the effectiveness of monetary policy on inflation. It is, indeed, odd to think that the currency issuer cannot control the price level. At the end of the day, the Fed's liabilities define exactly what a dollar is. By virtue of this definition, the Fed has the power to print dollars at will without facing any constraints. Since the value of a dollar in terms of goods is the inverse of the price level, the Fed can really throw as many dollar bills from the sky as needed to lower the value of money and reflate the price level. Helicopter money should work, as discussed by Bernanke (2002).

In contrast to the properties and power of the currency issuer's liabilities, treasury debt, on the other hand, is in principle like the liabilities of any other agent in the economy. They are a promise to pay a given amount of dollars at maturity. As such, since the treasury cannot create dollars, the treasury's liabilities need to satisfy a solvency condition in order to be repaid and be nominally risk-free. A tax relief today should necessarily be offset by future taxes or by default on the treasury's debt. With the treasury out of the picture, however, the currency issuer can still rely on some policy options to reflate the economy, and all those options are equivalent to the "traditional" account of helicopter money. We discuss four alternatives.

First, the currency issuer can reduce its net worth in two ways. It can write a check to the treasury to be fully rebated to the private sector. This can also be done directly to the private sector, if the issuer is private or in the case of a central bank digital currency. Alternatively, without involving the treasury, the currency issuer could write off its credits, if any, to the private sector, thereby making a direct wealth transfer. In both cases, the private sector experiences an increase in wealth, which pushes up consumption, aggregate demand, and reflates the economy.

Second, the currency issuer could commit to lowering its seigniorage revenues by changing its inflation target. Since seigniorage is a real resource — mirroring the private sector's costs of holding money — the currency issuer reduces the real backing of money, thereby reducing the value of money and increasing the price level.

Third, when specifying policy in terms of the overall monetary base (reserves and cash), the currency issuer can reflate the economy by swapping money for reserves during a liquidity trap. It can then commit to a constant growth rate of the monetary base, which would increase remittances to the private sector and reflate the economy.

Fourth, if it holds real assets like gold, the currency issuer retains the ability to control the price level even in scenarios where seigniorage revenues are zero or entirely rebated to the private sector. By committing to actively use its gold holdings, the currency issuer can provide some real backing. In this scenario, the currency issuer could reduce the amount of gold it commits to mobilize, signaling its willingness to redeem a smaller amount of its liabilities for gold. This would deplete their exchange value and increase the price level.

This paper is related to the recent literature that has studied liquidity traps and policy options. Krugman (1998) is our main inspiration for describing a simple model of a slump at the zero lower bound. Building on his work, we characterize the long-run equilibrium and the policies that can reflate the economy, including helicopter money. Woodford (2000, 2001) serves as a reference for understanding the special role of the currency issuer's liabilities, as discussed in recent work by Buiter (2014) and Benigno (2020). Bassetto and Messer (2013), Benigno and Nisticò (2020), Del Negro and Sims (2015), and Hall and Reis (2015), among others, analyze the implications of separating the treasury and the currency issuer for controlling inflation through currency issuer balance-sheet policies.

Auerbach and Obstfeld (2005) and Buiter (2014) study experiments of helicopter drops in various models with different frictions. Along those lines, Galí (2020b) compares debt-financed versus money-financed fiscal cuts, as well as the role of government purchases, while Di Giorgio and Traficante (2018) examine the open-economy dimension of this comparison. Eggertsson and Woodford (2003) and Woodford (2012) emphasize the importance of forward guidance as an alternative way to reflate the economy out of a liquidity trap. More recently, Michau (2024) analyzes the effectiveness of helicopter drops in a secular stagnation, when households have a preference for wealth, while Amador and Bianchi (2023) argue that helicopter drops are irrelevant under commitment but rely on tax and transfer policies to neutralize them.

Jacobson et al. (2019) discuss the role of "unbacked" fiscal expansions as a driver of the reflation and recovery of 1933. Bianchi and Melosi (2019) study the lack of coordination between monetary and fiscal policy when there are large fiscal imbalances. In such contexts, inflation would eventually arise, causing a recession. Along the same lines, Bianchi, Faccini, and Melosi (2024) analyze how partially unfunded debt can explain persistent inflation, similar to what was observed after the pandemic episodes. Barro and Bianchi (2023) provide evidence that during the period 2020–2022, the fiscal expansion of 37 OECD countries was "financed" by unexpected inflation.

Finally, since the economics of helicopter money is fundamentally related to controlling the future price level, our work builds on the theory of price determination developed by Benigno (2020). In this framework, the currency issuer can control the currency's value by deploying a range of policy tools, including traditional instruments such as interest rates and money supply, as well as balance sheet-related measures such as remittances policies and the composition of assets and liabilities. We extend this approach by considering scenarios where currency backing is provided by seigniorage and gold holdings. We show that this allows the price level to be uniquely determined even in cases where the remittances policy is defined in nominal terms, which highlights that the real costs borne by consumers when holding cash and/or gold can serve as implicit resources backing intrinsically worthless liabilities, such as currency. Furthermore, we outline a set of policy tools that are both more realistic and better tailored to address the conditions of a liquidity trap. In this vein, we also connect with other theories of price determination, notably the fiscal theory of the price level proposed by Sims (1994) and Woodford (1995) and discuss how it can be embedded in our framework.

The remainder of the paper is organized as follows. Section 2 discusses the general theory of price-level determination and the theoretical foundations of our institutional perspective. We then use this theory in Section 3 to analyze the economics of helicopter money in a liquidity trap. Section 4 discusses the implications of an environment where a real asset like gold is available. Section 5 concludes.

2. The theory of price determination

As a starting point for understanding the economics of helicopter money and the control of inflation, we discuss the general theory of price determination within a simple model with flexible prices and derive the main insights. The theory presented here encompasses existing theories as special cases, as we will show with the fiscal theory of the price level in Section 3.4.

First, we model a simple endowment monetary economy in which consumers make optimal consumption and saving choices. The economy is monetary because there is a unit of account provided by an intrinsically worthless currency issued by the central bank or a generic private currency issuer. In this context, as discussed in Sims (1999), Woodford (2001) and Cochrane (2005), a currency is nothing more than a claim that makes no promises other than to pay in units of itself. This has striking and important consequences for understanding price determination. The currency issuer can print currency at will. It is not subject to any nominal default risk on any of its liabilities. These default-free special properties of its liabilities, by definition of currency, are not shared by any other agent in the economy. Other default-free claims issued by the other agents in the economy, private and public alike, are not claims to be paid in units of themselves but claims to be paid in units of a currency for which those agents have scarce resources. To be considered default-free, those claims must be appropriately backed by current and future resources.

Second, to emphasize our main point about the role of the treasury versus the currency issuer in price-level determination, we consider the case where the currency issuer holds no assets and issues two types of liabilities: cash (M) and reserves (X). The former does not bear an interest rate and provides non-pecuniary benefits to consumers. Reserves, on the other hand, pay an interest rate (i), which is the policy rate set by the currency issuer. Both types of liabilities can also be fully digital. Thus, M could represent digital tokens bearing no interest. Moreover, we also allow the currency issuer to distribute some or all of its remittances (T^C) directly to the private sector, bypassing the treasury's intermediation. This scenario, where the treasury has neither direct nor indirect connection with the currency issuer, is particularly relevant in monetary systems involving private currencies.

Here, we characterize all the relevant equilibrium conditions for price determination, leaving the details of the model and optimizing conditions to Appendix A. We take a somewhat unconventional approach by first outlining the constraints faced by the currency issuer and the treasury, and then discussing the implications of the household's optimality conditions.

We start with the flow budget constraint of the currency issuer, requiring to issue new liabilities at time t to pay back the amount outstanding in $t-1$ plus the time-t remittances:

$$\frac{X_t}{1+i_t} + M_t = X_{t-1} + M_{t-1} + T_t^C. \tag{1}$$

The key feature of a currency system with a worthless currency, as discussed above, is that the liabilities of the currency issuer, whether cash M or reserves X, are always paid in full, since they define the unit of account. Therefore, the currency issuer is not subject to any solvency constraint. Formally, this means that we do not need to impose any restriction on the issuer policy besides the flow budget constraint (1), which only limits the policies that can be implemented at any point in time. The currency issuer can thus choose only three out of the four sequences $\{X_t, M_t, T_t^C, i_t\}_{t=t_0}^{\infty}$.

The above characterization of the currency issuer's constraint can encompass various monetary frameworks, ranging from public to private issuance of currency. An interesting case arises when $X_t = 0$, leading to $M_t = M_{t-1} + T_t^C$. This could also describe systems like Bitcoin or other cryptocurrencies, where newly created digital tokens (M) are transferred to private individuals participating in the network to validate transactions. Additionally, it is possible to envision monetary systems in which digital tokens receive interest-rate payments each period. In this scenario, $M_t = 0$, and the relationship becomes $\frac{X_t}{1+i_t} = X_{t-1} + T_t^C$.

The uniqueness of the currency issuer becomes evident when contrasting it with the treasury. Its flow budget constraint, requiring to issue liabilities at time t to pay back the amount outstanding in $t-1$ net of the resources raised in t through taxes or remittances, is

$$\frac{B_t^F}{1+i_t} = B_{t-1}^F - T_t - T_t^F, \tag{2}$$

in which B^F is treasury's debt, T are the taxed levied, and T^F the remittances received from the currency issuer. Note that T^F is not necessarily equal to T_t^C, since the currency issuer is also allowed to make transfers to the private sector.

There is an important distinction to make in Eqs. (1) and (2) between the liabilities of the treasury and those of the currency issuer. In the former, the liabilities of the currency issuer are always paid in full and free of default, as the currency issuer has the ability to "print" those liabilities. Conversely, the treasury requires scarce resources — denominated in a currency it does not issue — to settle its debt. Therefore, for the treasury's debt to be considered free of default, it must satisfy a solvency constraint akin to that applied to consumer debt, as we will show later. This necessitates that:

$$\frac{B_{t_0-1}^F}{P_{t_0}} = \sum_{t=t_0}^{\infty} \Lambda_{t_0,t} \left\{ \frac{T_t}{P_t} + \frac{T_t^F}{P_t} \right\}, \tag{3}$$

in which $\Lambda_{t_0,t}$ is the real factor to discount units of goods at time t in terms of units of goods at time t_0, with $\Lambda_{t_0,t_0} = 1$. The above condition ensures the repayment with certainty of the stock $B_{t_0-1}^F$ outstanding at time t_0, insofar as current and future taxes, $\{T_t\}_{t=t_0}^{\infty}$, are adjusted given the path of remittances received from the central bank, $\{T_t^F\}_{t=t_0}^{\infty}$, and prices, $\left\{P_t, \Lambda_{t_0,t}\right\}_{t=t_0}^{\infty}$. Under condition (3), treasury debt is free of default and accordingly pays the risk-free interest rate i. Treasury debt would also be default-free if the treasury were to levy taxes in excess, satisfying (3) with a strict inequality, (<). We disregard this case.[2]

Furthermore, Eq. (3) is equivalent to the following condition:

$$\lim_{t\longrightarrow\infty} \left\{ \Lambda_{t_0,t} \frac{B_{t-1}^F}{P_t} \right\} = 0, \tag{4}$$

when considering the flow budget constraint (2). This states that the treasury does not accumulate a positive discounted value of assets nor debt in the long run.

The above consideration implies that the *intertemporal* budget constraint (3) is more restrictive for tax policy than the sole *flow* budget constraint (2), as it requires complementing the latter with the additional solvency condition (4).

The fiscal theory does not consider the solvency constraint (3) or, alternatively, (4), as it implicitly assumes that the treasury's debt is free of default regardless its tax policy—either because it is akin to currency issuer liabilities or because it is fully guaranteed by appropriate remittances from the currency issuer.[3] Our approach is more general, encompassing the fiscal theory as a special case, as discussed in Section 3.4. Within our framework, we also characterize monetary systems in which there is a clear separation between the issuer of the unit of account and the treasury. This framework applies equally to public or private currencies, such as cryptocurrencies.

The role of the treasury, which seems relevant in the formulation above, since remittances accrue directly to its budget, is not essential to our analysis. While the literature typically considers the case where all remittances go to the treasury (see, among others, Bassetto and Messer, 2013, and Benigno and Nisticò, 2020), i.e., $T^C = T^F$, we allow for a more general institutional configuration encompassing several special cases, as in Benigno (2020).

In particular, as we demonstrate below, the implications of our analysis hold even in the special case where all remittances are transferred directly to households via T^H, without any involvement of the treasury—i.e. $T_t^F = 0$ and $T_t^C = T_t^H$ for all t. This scenario characterizes monetary systems with private currencies, in which there is no connection between the issuer of the unit of account and any government agency.

The linkages between monetary and fiscal authorities are often subtle in practice. In certain jurisdictions, it might be tempting to classify the monetary authority as a branch of the government and to consider the treasury's debt as implicitly guaranteed by the currency issuer. However, even in these cases, one could argue that the treasury's debt is fundamentally different from currency issuer liabilities. The treasury's debt is subject to market scrutiny, with its creditworthiness evaluated by rating agencies, whereas the currency issuer's liabilities are immune to such evaluations.

On the one hand, this distinction seems relevant even in the case of the United States, where the Treasury's debt was downgraded by Fitch on August 1, 2023. On the other hand, it is a widely held belief in financial markets and policy circles that, should the U.S. Treasury face financing difficulties, the Federal Reserve would use the printing press to support it.

A relatively new central bank, like the European Central Bank, has been designed on the explicit principle (laid out in article 123 of TFEU) that it should be separated from the member states' treasuries and abstain from monetary financing.[4] The Greek debt restructuring — and more generally the sovereign debt crisis in the EMU, when interest-rate spreads opened wide after ten years

[2] By focusing on the case in which the treasury does not levy excessive taxes, we exclude scenarios where conflicts between the central bank and the treasury over price determination might arise, potentially resulting in a multiplicity of equilibria. Furthermore, we abstract from strategic interactions, as analyzed by Barthélemy et al. (2024).

[3] See the discussion in Benigno (2020, 2025).

[4] See also Sims (2013), who argues that individual country members of the EMU effectively borrow using real debt, since it is denominated in Euro, whose issuance they cannot control.

of essentially zero spreads — exemplifies these types of weak linkages. Moreover, even operations in the vein of the lender of last resort role, such as the Outright Monetary Transactions (OMT), are designed to be implemented under strict conditionality requiring sovereign debt solvency. On the other hand, the unconventional purchases of government debt by the ECB have created implicit linkages between monetary and fiscal policies that will be difficult to disentangle.

Let us now turn to the equilibrium conditions implied by the consumer problem. The consumption and saving choices of consumers are characterized by a standard Euler equation which, under the goods market equilibrium condition and the assumption of a constant endowment, simplifies to the Fisher equation:

$$1 + i_t = \frac{1}{\beta} \frac{P_{t+1}}{P_t}. \tag{5}$$

In the equation above, i is the nominal interest rate on default-free securities, which coincides with the policy rate; β denotes the utility discount factor, and P is the price level.

The consumer's demand for currency-issuer cash (tokens) is implied by:

$$V_m\left(\frac{M_t^d}{P_t}\right) = \frac{i_t}{1 + i_t}, \tag{6}$$

in which the function $V(\cdot)$ is concave and has a satiation point at a finite level of real cash balances $\bar{m}$, i.e. $V_m(\cdot) = 0$ for each $M_t^d/P_t \geq \bar{m}$.[5] Below this satiation point, the demand for real money balances is a negative function $L(\cdot)$ of the nominal interest rate i:

$$\frac{M_t^d}{P_t} \geq L(i_t), \tag{7}$$

where $L(i_t) \equiv V_m^{-1}(\frac{i_t}{1+i_t})$ and the strict equality holds for $M_t^d/P_t < \bar{m}$: the higher the opportunity cost of holding cash, measured by i, the lower its demand. Moreover, the nominal interest rate is non-negative at all times, because of the absence of arbitrage opportunities. In equilibrium cash demand is equal to its supply, $M_t^d = M_t$, and

$$\frac{M_t}{P_t} \geq L(i_t). \tag{8}$$

As it is well known from the literature (see e.g., (Sims, 1994; Woodford, 1995; Cochrane, 1999)), relying only on Eqs. (5) and (6) by positing a rule for either the supply of cash or an interest rate does not determine the price level. The key insight of the fiscal theory of the price level, which we are going to use in the subsequent discussion, is that characterizing equilibrium solely through Eqs. (5) and (6) is actually incomplete because it overlooks other relevant conditions. One such condition is the intertemporal budget constraint of the consumer, which, under optimality, is:

$$\sum_{t=t_0}^{\infty} \Lambda_{t_0,t} \left\{ C_t + \frac{i_t}{1+i_t} \frac{M_t}{P_t} \right\} = \frac{X_{t_0-1} + M_{t_0-1} + B_{t_0-1}}{P_{t_0}} + \sum_{t=t_0}^{\infty} \Lambda_{t_0,t} \left\{ Y - \frac{T_t}{P_t} + \frac{T_t^H}{P_t} \right\}. \tag{9}$$

Looking forward from any period t_0, the present-discounted value of consumption, denoted by C, and the cost of holding cash, i.e. the left-hand side of the above equation, should be equal to the real value of the assets the consumer holds at the beginning of t_0 and the present-discounted value of the real net income, i.e. the right-hand side of the expression. B denotes treasury debt, which also carries the interest rate i.[6] The endowment of goods, which is constant, is given by Y. T denotes taxes/transfers from the treasury and T^H direct transfers from the currency issuer. Finally, to complete the consumer problem, it is important to note that, using the flow budget constraint of the consumer, Eq. (9) is equivalent to the following limiting condition on the amount of outside assets held by the private sector:

$$\lim_{t \longrightarrow \infty} \Lambda_{t_0,t} \frac{B_{t-1} + M_{t-1} + X_{t-1}}{P_t} = 0. \tag{10}$$

To proceed, we substitute Eq. (3) into (9) to substitute for the path of taxes, use goods market equilibrium — i.e. $C_t = Y$, which also implies $\Lambda_{t_0,t} = \beta^{t-t_0}$ — asset market equilibrium — i.e. $B^F = B$ — and $T^C = T^H + T^F$ to obtain

$$\frac{X_{t_0-1} + M_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ V_m\left(\frac{M_t}{P_t}\right) \frac{M_t}{P_t} - \frac{T_t^C}{P_t} \right\}, \tag{11}$$

in which we also have used the equilibrium condition (6) to substitute for $i_t/(1+i_t)$.

Eq. (11) is the key condition for determining prices. Note that it is equivalent to

$$\lim_{t \to \infty} \beta^{t-t_0} \frac{X_{t-1} + M_{t-1}}{P_t} = 0, \tag{12}$$

once accounting for (1), and can therefore also be obtained combining (4) and (10).

We can now state the definition of equilibrium.

[5] In deriving Eq. (6), we have set the marginal utility of consumption equal to the unitary value when evaluated at the constant endowment.

[6] In this section, since the currency issuer holds no assets, B only includes treasury's debt, as the privately-issued securities must be in zero net supply within the private sector. We are going to relax this assumption in the next section.

Definition 1. An equilibrium is a set of non-negative sequences $\{P_t, i_t, X_t, M_t, T_t^C\}_{t=t_0}^{\infty}$ that satisfy conditions (1), (5) and (8) for each $t \geq t_0$, with (8) holding with equality whenever $i_t > 0$, and (11), given initial conditions X_{t_0-1} and M_{t_0-1}. The currency issuer can set policy by choosing two out of the four sequences $\{X_t, M_t, i_t, T_t^C\}_{t=t_0}^{\infty}$.[7]

The key insight of the approach presented here, in line with the fiscal theory of the price level, is that (11) (or (12)) is an equilibrium condition and not a solvency constraint. It is not a solvency constraint because, as already discussed, the currency issuer's policies are only constrained to satisfy the budget constraint (1), for its liabilities are default-free by definition. Indeed, we have not imposed (11) (or (12)) when writing the constraints faced by the currency issuer. On the contrary, it is an equilibrium condition. Thus, well-specified currency-issuer policies can help determine the price level through (11) (or (12)).

To further illustrate the point that the currency issuer is not subject to a solvency constraint, note that (1) implies that the currency issuer has only liabilities and no assets. Moreover, if the remittance policy is specified such that $T_t^C \geq 0$ for all t, it does not even receive any external resources from the treasury or the private sector. In this case, the currency issuer is indeed resourceless and it has zero profits; yet Eq. (11) still holds, in equilibrium. In the simple case in which $T_t^C = 0$ for all t, the equilibrium condition (11) holds in the following simplified version:

$$\frac{X_{t_0-1} + M_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ V_m\left(\frac{M_t}{P_t}\right) \frac{M_t}{P_t} \right\}. \tag{13}$$

In equilibrium, the real value of the outstanding liabilities of the currency issuer should be equal to the present-discounted value of the real cost of holding cash, which is borne by the consumer. This is an opportunity cost, not something which is paid to the currency issuer. However, it is interesting to read the above equation as the balance between the real value of the nominal liabilities distributed by the currency issuer and the real resources that can be implicitly extracted from the liquidity services that cash provides in the system. The policies of the currency issuer can be set to have those implicit resources provide a "backing" to currency, determining then its exchange value. This is what we show is possible in the analysis that follows, even if the currency issuer is resourceless. To be in line with this argument and with the fact that the above equation is not an intertemporal budget constraint, we find it appropriate to avoid the distinction made in the literature between active or passive policies, and just search for the specification of monetary policies, including also balance-sheet policies, that uniquely determine the price level within a global analysis.

Proposition 1. *Assume that the currency issuer has no assets and that it issues interest-bearing reserves X and non-interest-bearing cash (or tokens) M. Assume that it sets policy in terms of a constant and positive nominal interest rate $i_t = i > 0$ on X for each $t \geq t_0$ and a positive sequence $\{X_t\}_{t=t_0}^{\infty}$, and let $m \equiv M/P$.*

Under these assumptions, the price level at time t_0 is uniquely determined by

$$P_{t_0} = (1-\beta)\frac{X_{t_0-1} + M_{t_0-1}}{V_m(m)m}, \tag{14}$$

given the initial conditions $X_{t_0-1} > 0$ and $M_{t_0-1} \geq 0$, provided the following condition is satisfied:

$$\lim_{m_t \to 0} V_m\left(m_t\right) m_t > 0. \tag{15}$$

The sequence of prices $\{P_t\}_{t=t_0+1}^{\infty}$ is determined by (5) given i and P_{t_0}. In this equilibrium, the rate of inflation should be positive, and therefore $1 + i > 1/\beta$.

Proof. Please refer to Appendix B.1. ■

The equilibrium price level (14) is proportional to the overall liabilities of the currency issuer. Moreover, these liabilities grow over time at the gross rate of inflation, Π, as can be seen by writing Eq. (14) for two consecutive periods. Condition (15) is necessary to rule out the non-monetary solution in which currency becomes worthless in equilibrium. This condition ensures that currency remains essential, even when real cash balances are small.

Condition (15) was originally proposed by Obstfeld and Rogoff (1983) to exclude inflationary paths. However, they — and the subsequent literature — consider this assumption implausible, as it requires the liabilities of the currency issuer to remain essential even if prices become infinite (and, consequently, the exchange value of currency becomes negligible).[8]

In Section 4, we demonstrate that this assumption can be relaxed if the currency issuer appropriately manages a real asset, such as gold, on its balance sheet.

As shown in the proof of Proposition 1, the condition that the rate of inflation is positive, i.e., that gross inflation satisfies $\Pi > 1$, ensures that interest-bearing liabilities are positive at time t_0 and at any other point in time. A positive value of interest-bearing

[7] The requirement that $\{T_t^C\}_{t=t_0}^{\infty}$ is non-negative is not necessary for the characterization of the equilibrium, but it is required in currency systems in which the currency issuer does not receive transfers from the private sector or the treasury. Note that in the case in which $\{M_t\}_{t=t_0}^{\infty}$ is positive, the currency issuer can set either a balance-sheet policy $\{X_t\}_{t=t_0}^{\infty}$ or a remittance policy $\{T_t^C\}_{t=t_0}^{\infty}$, and either an interest-rate policy $\{i_t\}_{t=t_0}^{\infty}$ or a cash-supply policy $\{M_t\}_{t=t_0}^{\infty}$. Note also that whenever policy is set in terms of $\{i_t\}_{t=t_0}^{\infty}$ then the supply of reserves — which pay that interest rate — should be positive at all times.

[8] See, among others, Woodford (2003).

liabilities is required for the currency issuer to control, through the absence of arbitrage opportunities, the interest rate on any other default-free securities in the economy.

This condition imposes an important restriction that affects the range of admissible interest-rate policies and makes price stability unattainable. However, we will show later that a positive asset position relaxes this restriction and restores the attainability of price stability. This result is particularly noteworthy in a framework where issuers of currency set policy in terms of interest rates, aligning with the current practice of currency issuers holding assets in their portfolios.

The above analysis proves the possibility for the currency issuer to exert perfect control over the path of prices, regardless of its interaction or lack thereof with the fiscal authority. Before moving to the broader framework of the next section and addressing the challenges of exiting the liquidity trap, we present another simple example along similar lines to illustrate that the analysis by Obstfeld and Rogoff (1983) can be encompassed within this framework.

Proposition 2. *Assume that the currency issuer has no assets and issues only non-interest-bearing cash (or tokens) M. Assume also that its remittances are transferred only to the private sector, i.e. $T_t^C = T_t^H$ and $T_t^F = 0$ for each $t \geq t_0$, and that policy is set in terms of a constant growth rate of cash such that $M_t = (1+\mu)M_{t-1}$ with $\mu \geq 0$ for each $t \geq t_0$ and for a given $M_{t_0-1} > 0$.*

Under these assumptions, the price level is uniquely determined by

$$P_t = \frac{M_t}{V_m^{-1}\left(1 - \frac{\beta}{1+\mu}\right)}, \tag{16}$$

for each $t \geq t_0$, given initial conditions M_{t_0-1}, provided condition (15) is satisfied.

Proof. Please refer to Appendix B.2. ■

First, it is worth noting the realism of the mechanism outlined above for the creation of cash/tokens through transfers to households when characterizing certain cryptocurrencies like Bitcoin: in this case, new tokens are distributed to the people who performed the proof of work, with a time-varying μ_t that is expected to converge to zero around 2140.[9] Note also that under the assumptions of Proposition 2, the budget constraint (1) simplifies to

$$M_t = M_{t-1} + T_t^C.$$

The only tool available to the currency issuer to determine the price level is now the path of the cash (tokens) supply, which can be set by an appropriate remittance policy. In the above Proposition, we consider a simple policy in which $T_t^C = \mu M_{t-1}$, with $\mu \geq 0$, implying a constant growth rate of cash (tokens).

Three features of the solution are worth mentioning. First, as in Proposition 1, the equilibrium price level is proportional to the currency issuer's liabilities. Second, the uniqueness of the equilibrium again depends on the essentiality of the currency, captured by condition (15).[10] Holding some real assets, like gold, will ensure the same uniqueness properties. Finally, price stability is now achievable, unlike in Proposition 1, when the growth rate of cash is zero, i.e., $\mu = 0$.

This example effectively reformulates Obstfeld and Rogoff's (1983) analysis using insights from the fiscal theory of the price level and applies it to digital and private currencies as well. However, it is important to emphasize that there is no direct involvement of fiscal policy in determining equilibrium prices, as in the previous example. The fiscal policymaker is just one of the many actors subject to market conditions when issuing debt.

3. Liquidity trap

Using the model outlined in the previous Section and detailed in Appendix A, here we discuss the ways in which the currency issuer can reflate the economy out of a liquidity trap and bring it to potential. A liquidity trap is characterized by a condition under which at zero nominal interest rate there is an excess supply of goods, in the same spirit as Krugman (1998).

The model has an infinite horizon, but we can simplify it to a short and a long run. The short run for simplicity lasts one period, time t_0, the long run starts at time t_0+1. The economy is in a liquidity trap at time t_0 due to a fall of the natural rate of interest that cannot be accommodated by lowering the nominal interest rate, because of the zero-lower bound (ZLB) and sticky prices. The economy is in a slump in which consumption is below output. In the long run, instead, prices are flexible, and consumption is equal to output.[11]

[9] The creation of tokens through direct transfers to households would also apply in the case of a central bank digital currency, where households hold accounts at the central bank.

[10] Note, however, that differently from the results of Proposition 1, without condition (15) being satisfied, there can be multiple inflationary equilibria alongside the non-monetary equilibrium.

[11] For illustrative purposes, the short run lasts only one period. This is with no loss of generality, as we can make it longer by extending the duration of price rigidity and/or of the shock that brings the economy in a liquidity trap. For details see Benigno and Nisticò (2022).

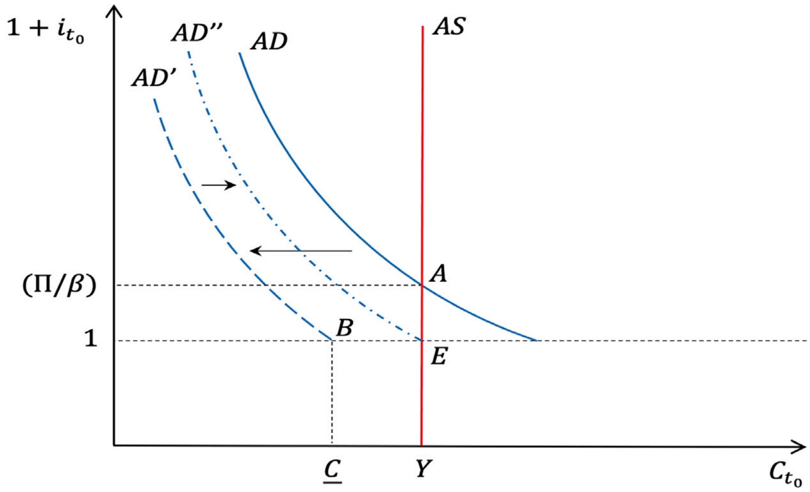

Fig. 1. The effects of a negative preference shock $\xi < \bar{\xi}$: AD shifts to the left into AD' and the economy is in a slump ($\underline{C} < Y$) due to the ZLB, unless the economy is reflated by shifting AD' to the right into AD''.

3.1. Short run

The action in the short run can be described by using the Euler equation

$$U_c(C_{t_0}) = \beta(1 + i_{t_0})\frac{P_{t_0}}{P_{t_0+1}}\frac{\xi_{t_0+1}}{\xi_{t_0}}U_c(C_{t_0+1}),$$

in which $U_c(\cdot)$ is the first derivative of the utility of consumption $U(\cdot)$, and ξ_{t_0} and ξ_{t_0+1} are preference shocks at the respective times.

To characterize the liquidity trap, we assume that prices are fully rigid in the short run and such that $P_{t_0} = P$, for a positive P. The preference shock at time t_0 is $\xi_{t_0} = \xi$, whereas $\xi_t = \bar{\xi}$ with $\bar{\xi} > \xi$ for $t \geq t_0 + 1$. Given all these assumptions, the goods market does not necessarily clear at time t_0 and consumption is determined by the Euler Equation as

$$U_c(C_{t_0}) = \beta(1 + i_{t_0})\frac{P}{P_{t_0+1}}\frac{\bar{\xi}}{\xi}U_c(Y), \tag{17}$$

where we have also used the goods market equilibrium in the long run, $C_{t_0+1} = Y$.

Let us leave aside for now the determination of the long-run price level P_{t_0+1} and assume that its equilibrium value is $\bar{P}$, with $\bar{P} > 0$. Further assume that the distance between $\bar{\xi}$ and ξ is large enough that, given P and $\bar{P}$, the following inequality holds:

$$\beta\frac{P}{\bar{P}}\frac{\bar{\xi}}{\xi} > 1. \tag{18}$$

If this is the case, Eq. (17) implies that the optimal short-run demand for consumption goods falls below output at any non-negative interest rate: the economy is in a slump.

Fig. 1 graphs this situation. In the space $(C_{t_0}, 1 + i_{t_0})$, the Euler Eq. (17) and the zero-lower bound on the nominal interest rate imply a downward-sloping aggregate demand curve (*AD*) that dies out at $i_t = 0$. The vertical red line displays the aggregate supply curve (*AS*), located at the level of the constant endowment Y. Starting from a stationary equilibrium where $C = Y$ and interest rate is at i (point A in the figure), a negative demand shock, which creates a gap such that $\xi < \bar{\xi}$ with (18) holding, shifts the AD curve to the left into AD', inducing a downward pressure on current consumption. The currency issuer can exploit the downward slope of aggregate demand and cut the nominal interest rate to stimulate consumption as much as possible.

To restore the equilibrium in the goods market, $C_{t_0} = Y$, the central bank would need to cut the nominal rate down to $1 + i_{t_0} = (\xi/\bar{\xi})(\bar{P}/(P\beta))$.[12] However, if the size of the shock satisfies (18), the required cut in the nominal rate would violate the zero lower bound. As a consequence, the currency issuer cannot descend the AD' schedule beyond point B, where the economy is in a slump and experiences a shortage of demand:

$$\underline{C} = Y U_c^{-1}\left(\beta\frac{P}{\bar{P}}\frac{\bar{\xi}}{\xi}\right) < Y.$$

[12] We assume decentralized markets and use the standard device that, within the household, the buyer of goods is distinct from the seller of goods. They reconvene within the household only at the end of the period—one with the goods purchased for consumption and the other with the revenues from selling part of the endowment. The unsold portion of the endowment, left in the market, is no longer useful for the seller.

Eq. (17) clarifies that the other possibility to restore the equilibrium in the goods market is to act on the future price level, reflating the economy, lowering the real rate and boosting consumption: in Fig. 1, indeed, raising $\bar{P}$ shifts the aggregate demand schedule to the right into AD'' and the economy can reach the full-employment equilibrium E.[13]

3.2. Long run

In the long run, i.e. for $t \geq t_0 + 1$, prices are flexible and the preference shock is at the high level $\xi_t = \bar{\xi}$. The goods market clears and consumption is equal to output. We now add the possibility for the currency issuer to hold assets, B^C, either issued by the treasury or the private sector that carries the interest rate i.[14] The flow budget constraint (1) modifies to

$$\frac{B_t^C - X_t}{1 + i_t} - M_t = B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C, \tag{19}$$

for each $t \geq t_0 + 1$.

The relevant equilibrium conditions to determine the path of prices are (5), (8) and (11), the latter modified to appropriately include the currency issuer's asset holdings:

$$1 + i_t = \frac{1}{\beta} \frac{P_{t+1}}{P_t}, \tag{20}$$

$$\frac{M_t}{P_t} \geq L(i_t) \equiv V_m^{-1} \left(\frac{i_t}{1 + i_t} \right), \tag{21}$$

for each $t \geq t_0 + 1$ and

$$\frac{X_{t_0} + M_{t_0} - B_{t_0}^C}{P_{t_0+1}} = \sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1} \left\{ V_m \left(\frac{M_t}{P_t} \right) \frac{M_t}{P_t} - \frac{T_t^C}{P_t} \right\}, \tag{22}$$

in which we normalized $U_c(Y) = 1$. We can define the long-run equilibrium.

Definition 2. The long-run equilibrium is a set of non-negative sequences $\{P_t, i_t, X_t, M_t, B_t^C, T_t^C\}_{t=t_0+1}^{\infty}$ that satisfy Eqs. (19), (20), (21) for each $t \geq t_0 + 1$ and (22) given X_{t_0}, M_{t_0} and $B_{t_0}^C$ inherited from the short run. The currency issuers can set policy by choosing three out of the five sequences $\{X_t, M_t, B_t^C, i_t, T_t^C\}_{t=t_0+1}^{\infty}$.

Let us recall that (22) is derived from the intertemporal budget constraint of the consumer using equilibrium conditions in goods and asset markets and the solvency constraint for treasury's liabilities. It resembles an intertemporal solvency constraint for the currency issuer, but it is not and should not be interpreted as such. As we have discussed in the simple example of the previous section, an equation like (22) would even hold when the currency issuer has only liabilities and no profits. It is then an equilibrium condition that can be used to determine the path of prices.

Proposition 3. *Assume that the currency issuer holds nominal assets B^C, issues interest-bearing reserves X and non-interest-bearing cash (or tokens) M. Assume that it sets policy in terms of a constant interest rate $i_t = i > 0$ on X for each $t \geq t_0 + 1$, a positive sequence $\{X_t\}_{t=t_0+1}^{\infty}$, and a constant nominal remittances policy $T_t^C = T^C \geq 0$ for each $t \geq t_0 + 1$.*

Under these assumptions, the price level at time $t_0 + 1$ is uniquely determined by

$$P_{t_0+1} = \frac{\mathcal{T}^C + X_{t_0} + M_{t_0} - B_{t_0}^C}{\mathcal{S}(\Pi, Y)}, \tag{23}$$

given non-negative $M_{t_0}, B_{t_0}^C$, provided the following conditions are satisfied

$$\mathcal{T}^C + X_{t_0} + M_{t_0} > B_{t_0}^C,$$

$$\lim_{m_t \longrightarrow 0} V_m(m_t) m_t > 0, \tag{24}$$

and given the definitions

$$\mathcal{S}(\Pi, Y) \equiv L\left(Y, \frac{\Pi}{\beta} - 1\right) \frac{\Pi - \beta}{(1 - \beta)\Pi} \tag{25}$$

and

$$\mathcal{T}^C \equiv \frac{\Pi}{\Pi - \beta} T^C. \tag{26}$$

[13] We should clarify that, although we analyze the policies to restart the economy once in a liquidity trap, the mechanisms we discuss are at work also for positive values of the nominal interest rates.

[14] Note that, if we allow the currency issuer to hold assets, B may also include privately-issued securities, as in this case they need not be in zero net supply within the private sector, unlike in the previous section.

Proof. Please refer to Appendix B.3. ■

The intuition behind Eq. (23) is simple and analogous to what discussed earlier. While in nominal terms the currency issuer's liabilities are default-free by definition, their *real* value depends instead on the real resources that can back the value of currency.

Although the currency issuer does not have taxation power, these resources can be derived, implicitly, from the households by issuing securities — namely cash — that households hold also for their non-pecuniary services. The cost of holding real cash balances is a real cost, as shown in Eq. (22), which corresponds to the term $\mathcal{S}(\Pi, Y)$. By setting a constant interest-targeting policy, the currency issuer can fix $\mathcal{S}(\Pi, Y)$ independently of the current price level. Accordingly, the long run price in Eq. (23) adjusts to satisfy the equilibrium condition that the cost of holding cash — or the present value of seigniorage revenues — $\mathcal{S}(\Pi, Y)$ matches the currency issuer net real liabilities $(\mathcal{T}^C + X_{t_0} + M_{t_0} - B^C_{t_0})/P_{t_0+1}$.[15] Furthermore, the non-monetary equilibrium is excluded by condition (24).

Note that, given $\mathcal{S}(\Pi, Y) > 0$, the numerator in Eq. (23) should be positive, to support a positive price level P_{t_0+1} at equilibrium. The remittance policy, therefore, should be set so as to ensure $\mathcal{T}^C > N_{t_0}$ regardless of the relative net financial position of the currency issuer versus the private sector, where the net worth is defined by

$$N_t \equiv \frac{B^C_t - X_t}{1 + i_t} - M_t, \tag{27}$$

noting that $i_{t_0} = 0$ in the liquidity trap.[16]

In Appendix B.3, we show that, in equilibrium, the amount of nominal assets at time $t_0 + 1$ required by a given, positive, level of nominal reserves is given by:

$$B^C_{t_0+1} = X^C_{t_0+1} + \mathcal{T}^C - \Pi\left(\frac{\Pi - 1}{\Pi - \beta}\right)\left(\mathcal{T}^C + X_{t_0} + M_{t_0} - B^C_{t_0}\right). \tag{28}$$

The above equation also clarifies that price stability ($\Pi = 1$) is now attainable, as long as the currency issuer holds a large enough portfolio of nominal assets, namely: $B^C_t = X^C_t + \mathcal{T}^C$, for each $t \geq t_0 + 1$, which implies that the equilibrium net worth of the currency issuer under price stability is constant, and equal to

$$N_t = \bar{N} \equiv N_{t_0} - \mathcal{T}^C, \tag{29}$$

for all $t \geq t_0 + 1$.

3.3. Reflationary policies and helicopter money

Having described the equilibrium in the short and the long-run, we can now discuss the policy options available to the currency issuer at t_0 to lift the economy from a liquidity trap, that are all equivalent to the traditional account of helicopter money.

Proposition 4. *Under the assumptions of Proposition 3, the issuer of currency can reflate the economy from the liquidity trap by* (i) *cutting nominal net worth at time* t_0*, which can be accomplished by increasing transfers* T^C_t *at time* $t = t_0$ *and/or writing off some assets* B^C_t *held at time* $t = t_0$*;* (ii) *increasing the present-discounted value of future remittances* $\mathcal{T}^C$*;* (iii) *lowering the long-run inflation rate* Π*, if the economy is on the left side of the Laffer curve.*

We note that the numerator of (23) is indeed equivalent to $\mathcal{T}^C - N_{t_0}$. The currency issuer can increase the price level at time $t_0 + 1$, and bring the economy out of the liquidity trap at time t_0 by cutting its nominal net worth, a lower value of N_{t_0}. This can be accomplished by increasing short-run transfers, either to the treasury or the households. Indeed, using (19) and the definition of N_t, and considering $i_{t_0} = 0$, we can write

$$N_{t_0} = N_{t_0-1} - T^C_{t_0}.$$

The policy of lowering N_{t_0} by increasing transfers to the private sector is similar to the traditional narrative of helicopter money. An alternative, but equivalent, way of cutting nominal net worth would be for the currency issuer to write-off some of the assets in its portfolio. In particular, by writing off private securities from its balance sheet, the currency issuer can trigger a positive and reflationary wealth effect directly on the private sector, without any involvement of the treasury.

We can visualize the intuition behind the above discussion by using an AD–AS logic. To this end, we can use the equilibrium condition (9) and exploit some simplifications on preferences as outlined in Appendix A.4 (namely log and separable preferences in consumption and real money balances), to write the long-run aggregate consumption demand as

$$C_{t_0+1} = \frac{1-\beta}{1+\theta}\left\{\frac{\widetilde{\mathcal{T}}^C - N_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} Y\right\}, \tag{30}$$

---

[15] From a quantitative perspective, the question whether seigniorage is significant enough to provide meaningful backing to the exchange value of currency is an empirical one. See Del Negro and Sims (2015) for an estimate, placing the present discounted value of seigniorage between 18% and 144% of private spending, depending on discount and real-growth rates, with a baseline estimate of 114%.

[16] If net worth N_{t_0} is negative, $\mathcal{T}^C$ can be also set equal to zero, as in the example of Section 2.

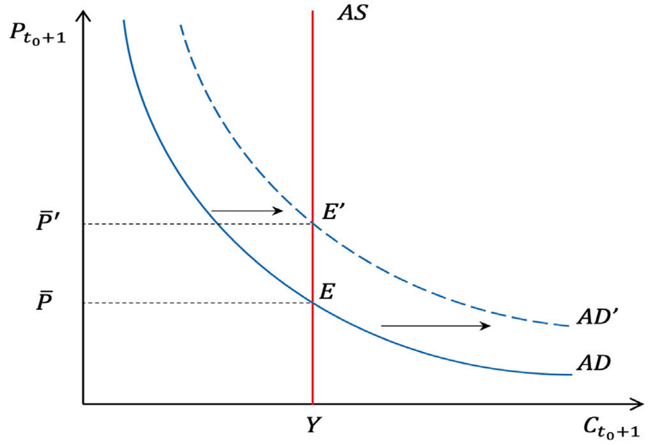

Fig. 2. The economics of helicopter money: the AD–AS logic.

for some positive parameter θ, and where we have defined:

$$\widetilde{\mathcal{T}}^C \equiv \frac{1+i}{i}\mathcal{T}^C. \tag{31}$$

This demand function emphasizes two implications. The first is that the net nominal asset position of the consumer, which can be considered as wealth, mirrors that of the currency issuer only, i.e. the term $(-N_{t_0})$ in Eq. (30). The second point is that what ultimately matters for the nominal wealth position of households also includes the human wealth component, which is indirectly affected by the interest rate and remittance policies of the currency issuer, through $\widetilde{\mathcal{T}}^C$. If the latter satisfies $\widetilde{\mathcal{T}}^C > N_{t_0}$, then the private sector is a net creditor with respect to the overall nominal wealth position that influences its spending decisions. Accordingly, an increase in the price level at t_0+1 reduces the demand for consumption through a negative wealth effect.[17]

This relationship is plotted in Fig. 2 as an AD equation together with the AS equation of constant long-run output. Cutting the currency issuer's net worth at time t_0 reflates the economy by expanding the demand for long-run consumption, and taking the economy into equilibrium E'.

Note that this proposal is different from those of Buiter (2014) and Galí (2020a,b). In their case, the mechanism operates as follows. First, the treasury lowers taxes, financing the cut with newly issued debt purchased by the currency issuer through an increase in its liabilities (reserves or money). Subsequently, the currency issuer writes off the treasury's debt or, equivalently, rolls it over permanently. The key aspect of their proposal is the lowering of taxes, ultimately financed by the increase in the currency issuer's liabilities.

Our analysis, however, clarifies that there is no need to increase the currency issuer's liabilities (money or reserves), and the treasury can remain completely uninvolved, provided the currency issuer writes off private securities from its balance sheet.

Moreover, we point to additional and alternative options. A second option to reflate the economy, indeed, is committing to an increase in the present-discounted value of future remittances in the numerator of (23). The effect of this policy action would be equivalent to a cut in nominal net worth, as the nominal wealth that is relevant for private sector's spending decisions expands — as shown by the first term in (30) — thus pushing up prices along the same lines as before.

Finally, a third policy option available to the currency issuer is to lower the present-discounted value of seigniorage by changing its inflation target Π, thus acting on the denominator of (23). In this latter case, indeed, the currency issuer is reducing the amount of real assets that are backing its nominal liabilities, thus depleting their exchange value. If the economy is on the left-hand side of the Laffer curve with respect to seigniorage revenues, this requires a lower inflation target Π, and thereby a lower nominal interest rate target i.[18]

It is also worth discussing the scenario in which the treasury defaults on debt held by the currency issuer or the latter forgives it. Such an action would result in a loss on the currency issuer's assets and an increase in the price level, as described by the mechanism above, without requiring an increase in the issuance of cash or reserves.[19] However, it is important to emphasize that, in the case of treasury default, this outcome critically depends on the constant remittances policy assumed in Proposition 3. As elaborated in Appendix B.5.1, indeed, the currency issuer can generally absorb the loss on its balance sheet resulting from a treasury default without affecting the equilibrium price level, provided it adopts a remittance policy towards the private sector that is contingent on default. Overall, this discussion underscores the currency issuer's ability to control the path of prices as desired.[20]

[17] The definition (31) clarifies that if the condition $\widetilde{\mathcal{T}}^C > N_{t_0}$ is satisfied, the private sector is a net creditor along the entire AD schedule, and not only in equilibrium. Clearly, in general equilibrium, where $1+i = \Pi/\beta$ — i.e. in allocations E and E' in Fig. 2 — we have $\widetilde{\mathcal{T}}^C = \mathcal{T}^C$, where the latter is defined by (26).

[18] If the economy is on the left side of the Laffer curve, a lower inflation target triggers both effects — on the seigniorage and the discounted path of future nominal remittances — consistently with each other.

[19] As we show in Appendix B.5.1, this is a fundamental difference between this case and the fiscal theory of the price level.

We now consider a second proposal based on a scenario in which the currency issuer sets policy by specifying the path of its liabilities and asset holdings. This approach corresponds to implementing a monetary-base growth rule, rather than relying on an interest-rate policy.

Proposition 5. *Assume that the currency issuer holds assets B^C, issues interest-bearing reserves X and non-interest-bearing cash (or tokens) M. Assume that for each time $t \geq t_0 + 1$ the currency issuer commits to the following constant growth rule for its liabilities:*[21]

$$M_t = (1+\mu)M_{t-1}, \tag{32}$$

$$X_t = (1+\mu)X_{t-1}. \tag{33}$$

with $\mu \geq 0$. Moreover, it sets the same growth rate for its nominal assets for each $t \geq t_0 + 1$:

$$B_t^C = (1+\mu)B_{t-1}^C. \tag{34}$$

Under these assumptions, the following holds:

(i) the price level at time $t_0 + 1$ is uniquely determined by

$$P_{t_0+1} = \frac{(1+\mu)M_{t_0}}{L\left(Y, \frac{\Pi}{\beta} - 1\right)}, \tag{35}$$

provided

$$\lim_{m_t \longrightarrow 0} V_m(m_t)m_t > 0 \tag{36}$$

and a non-zero net worth in the short run, i.e. $N_{t_0} \neq 0$;

(ii) a policy of swapping nominal money for reserves at time t_0, i.e. $\Delta M_{t_0} = -\Delta X_{t_0} > 0$, is equivalent to helicopter money and can bring the economy out of the liquidity trap.

Proof. Please refer to Appendix B.4. ■

To get further insights into the mechanism of helicopter money under the assumptions of Proposition 5, note that Eq. (35) can be written in a form analogous to Eq. (23):

$$P_{t_0+1} = \frac{\mathcal{T}_{t_0}^C - N_{t_0}}{S(\Pi, Y)}, \tag{37}$$

where $N_{t_0} = B_{t_0}^C - X_{t_0} - M_{t_0}$ is the currency issuer's nominal net worth in the short run, $S(\Pi, Y)$ is given by the definition (25), and $\mathcal{T}_{t_0}^C \equiv B_{t_0}^C - X_{t_0} + \frac{\mu}{1-\beta}M_{t_0}$ is the present discounted value of nominal remittances looking forward from time $t_0 + 1$, which depends on variables chosen at time t_0, and in particular on the level of currency, as shown in Appendix B.4.

Eq. (35) describes how a policy of swapping money for reserves works: the nominal currency outstanding at the beginning of time t_0+1, i.e., M_{t_0}, rises while the real seigniorage remains unchanged, thereby implying an increase in the price level. Moreover, Eq. (37) shows that, while swapping cash for reserves does not change the level of nominal net worth N_{t_0} outstanding at the beginning of period t_0+1 in the numerator of (37) because the interest rate is zero, the commitment implied by the constant-growth rules (32)–(33) raises the present discounted value of future remittances, $\mathcal{T}_{t_0}^C$, thereby increasing the nominal wealth relevant for households' spending decisions and, thereby, the equilibrium price level.

It is worth pointing out that a policy of swapping cash for reserves in the short-run only works under a cash-growth rule that fixes the rate of growth of nominal cash between t_0 and t_0+1. Under an interest-rate rule, such a policy would leave the time t_0+1 price level unchanged, as the increase in short-run cash M_{t_0} would be sterilized by the fall in cash growth between t_0 and t_0+1, and long-run money would be unchanged, as implied by Eq. (8).

3.4. Helicopter money and the fiscal theory of the price level

In this section, we discuss how the fiscal theory of the price level is nested in our framework.

As already argued, the fiscal theory does not make any distinction between treasury and currency issuer, and assumes that the treasury's debt is free of default irrespective of its tax policy. From the analysis of Section 2, it should be clear that the treasury's liabilities can only have this property if the currency issuer either (1) is also the treasury itself, or (2) is the guarantor of the treasury's debt, regardless of tax policy. When the currency issuer is not the treasury but another agent, either public or private — as in reality — case (2) above requires an implicit or explicit commitment to back the treasury if debt is not funded by taxes. We can think of this in two ways, that are formally analyzed in Appendix B.5.2: (i) the currency issue explicitly or implicitly commits to using reserves to

[20] See also Benigno and Nisticò (2020), which shows that the extent to which a currency issuer can adjust the intertemporal path of its remittances to render a default on its assets irrelevant for the equilibrium price level depends on the seigniorage revenues it can extract from the private sector, i.e., S.

[21] We thank an anonymous referee for pointing out this example. See also Auerbach and Obstfeld (2005) for an example of money supply reflating the economy in a liquidity trap, without any swap with reserves.

eventually purchase any amount of the treasury's debt that is not funded by taxes and remittances; or (ii) the currency issuer shapes its remittance policy in order to ensure, for any given tax policy, that the treasury can always pledge enough resources to repay any amount of outstanding debt. Formally, in case (1) or in case (2.i) above, the only restriction on the path of taxes and debt is given by (2). The solvency condition (3) does not apply since treasury's debt is effectively identical to the monetary base, and therefore default free by definition. In case (2.ii), the treasury is still subject to the solvency constraint (3), but the currency issuer is ready to fund the treasury, through appropriate remittances, to ensure its solvency. This means that the sequence of remittances $\{T_t^C\}_{t=t_0}^{\infty}$ is determined by $B_{t_0-1}^F$ for any tax policy specified by the fiscal authority $\{T_t\}_{t=t_0}^{\infty}$ and any feasible path of prices $\{P_t\}_{t=t_0}^{\infty}$.[22] In all the cases above, in equilibrium, the following condition holds at any time $t \geq t_0$:

$$\frac{X_{t-1} + M_{t-1} + B_{t-1}}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t} \left\{ V_m \left(\frac{M_T}{P_T} \right) \frac{M_T}{P_T} + \frac{T_T}{P_T} \right\}, \tag{38}$$

which for $t = t_0$ can be used to determine the price level at time t_0, under appropriate fiscal policies—a policy in which real taxes are constant over time being an example.

When it comes to helicopter money, the fiscal theory of the price level would imply that a helicopter drop can take the form of an unfunded fiscal expansion, of the kind discussed, among others, by Barro and Bianchi (2023), Bianchi et al. (2023) and Jacobson et al. (2019). In the traditional narrative of helicopter money, the government (treasury or currency issuer) increases permanently the long-run nominal liabilities — namely B_{t_0}, X_{t_0}, or M_{t_0} — in order to finance a tax cut in the short run. Note that, since the short-run nominal interest rate is zero, all these possibilities are equivalent, as implied by (38) at time $t = t_0 + 1$. Money- and debt-financed tax cuts are equivalent.[23] Indeed, given that all the government's liabilities have the special properties of the currency issuer's, B_{t_0}, X_{t_0} or M_{t_0} are always paid in full since they are guaranteed by the "printing press" of the currency issuer without any need to raise taxes or seigniorage revenues. And, indeed, for an increase in government debt to produce a reflationary effect in the long run, it is important that taxes and seigniorage not move (at least not proportionally). Results can be visualized using the simple AD–AS logic of Fig. 2, where the AD schedule now reads

$$C_{t_0+1} = \frac{1-\beta}{1+\theta} \left\{ \frac{B_{t_0} + X_{t_0} + M_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} \left(Y - \frac{T_t}{P_t} \right) \right\}, \tag{39}$$

and assuming that the private sector is a net creditor with respect to the government (i.e. $B_{t_0} + X_{t_0} + M_{t_0} > 0$). An increase in any of the government's nominal liabilities at time t_0 — that is not mirrored by an equivalent increase in the present real value of taxes — raises the nominal financial wealth that agents carry into period $t_0 + 1$ and creates an excess demand of goods at the initial price level: the demand curve shifts to the right into AD'. In order for consumption to fall back to the level of the constant endowment, such excess demand stimulates an increase in the price level that reduce the real value of the financial assets held by the consumer and restore equilibrium in E'.

4. An economy with gold

Here we extend the model of the previous sections to allow for a real asset like gold, denoted by G, which is in a fixed supply and provides direct utility benefits to the consumers through the function $Z(g_t)$, with standard concave properties; g_t is the time-t household's holdings of gold while g_t^C is the currency issuer's holdings, with $g_t + g_t^C = \bar{G}$ at all times. We present the details of the model in Appendix A.5.

We show that, in the scenario where the currency issuer does not back the treasury's liabilities, and therefore the solvency condition (3) holds, Eq. (22) is now replaced by

$$\frac{X_{t_0} + M_{t_0} - B_{t_0}^C}{P_{t_0+1}} - q_{t_0+1} g_{t_0}^C = \sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1} \left(V_m \left(\frac{M_t}{P_t} \right) \frac{M_t}{P_t} - Z_g(g_t) g_t^C - \frac{T_t^C}{P_t} \right), \tag{40}$$

where q_t is the real price of gold, which in general equilibrium satisfies[24]

$$q_t = \beta q_{t+1} + Z_g(g_t), \tag{41}$$

in which $Z_g(g_t)$ is the marginal utility of gold. The real price of gold depends on the discounted price in the next period and the marginal benefits that gold provides to the consumer.

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22 With no loss of generality, here we focus on the case where $T^C = T^F$.

23 This equivalence holds, from a qualitative — but not quantitative — perspective regardless of the fact that the economy is in a liquidity trap in period t_0: what is key is that the fiscal expansion is "unbacked".

24 As before, we use the normalization: $U_c(Y) = 1$.

Eq. (40) generalizes (22) by stating that, in equilibrium, the real value of the net liabilities of the currency issuer — including both nominal and real securities — must equal the present discounted value of the stream of real resources that the issuer can extract from the private sector through its policy. In this economy, the implications of the currency issuer's policy for gold holdings are key, in addition to those for cash supply and remittances. An important implication of Eq. (40) is that, once remittances policy is set in nominal terms, we can rule out non-monetary equilibria without the need to make any special assumption on the preference toward liquidity, and thus even in the case in which Eq. (15) does not hold. This is because seigniorage revenues are no longer essential for the equilibrium price level to be determinate and finite, as the currency issuer can extract real resources from the private sector by using its gold portfolio instead.

Proposition 6. *Assume that the currency issuer holds nominal assets B^C and gold g^C, issues interest-bearing reserves X and non-interest-bearing cash (or tokens) M. Assume that it sets policy in terms of a constant interest rate $i_t = i > 0$ on X for each $t \geq t_0 + 1$, a positive sequence $\{X_t\}_{t=t_0+1}^{\infty}$, a non-negative sequence $\{g_t^C\}_{t=t_0+1}^{\infty}$, and a constant nominal remittances policy $T_t^C = T^C \geq 0$ for each $t \geq t_0 + 1$.*

Under these assumptions, the price level at time $t_0 + 1$ is uniquely determined by

$$P_{t_0+1} = \frac{\mathcal{T}^C + X_{t_0} + M_{t_0} - B_{t_0}^C}{\mathcal{S}(\Pi, Y) + \mathcal{G}_{t_0}}, \tag{42}$$

given non negative $M_{t_0}, B_{t_0}^C$, provided the following conditions are satisfied

$$\operatorname{sgn}\left(\mathcal{T}^C + X_{t_0} + M_{t_0} - B_{t_0}^C\right) = \operatorname{sgn}\left(\mathcal{S}(\Pi, Y) + \mathcal{G}_{t_0}\right),$$

$$\mathcal{G}_{t_0} \neq 0,$$

given the definitions (25), (26) and

$$\mathcal{G}_{t_0} \equiv \sum_{t=t_0+1}^{\infty} \beta^{t-t_0-1}[Z_g(\bar{G} - g_t^C)(g_{t_0}^C - g_t^C)]. \tag{43}$$

Proof. The proof is the same as that of Proposition 3, discussed in Appendix B.3, once we use definition (43). ■

To rule out the non-monetary equilibrium it is key that $\mathcal{G}_{t_0} \neq 0$, which requires some trading in gold by the currency issuer, as (43) shows. Indeed, constant gold holdings by the currency issuer implies $\mathcal{G}_{t_0} = 0$, whereby Eq. (42) becomes identical to (23) and condition (15) is thus again necessary for the uniqueness of the equilibrium.

Let $\widetilde{N}_{t_0} \equiv B_{t_0}^C - X_{t_0} - M_{t_0}$ denote the currency issuer's net worth in nominal securities, having used $i_{t_0} = 0$. As we are going to show shortly, in the case where the household's aggregate demand is downward sloping it should be required that $\mathcal{G}_{t_0} > 0$. In this case, indeed, the real backing would be provided by the possibility for the currency issuer to mobilize its gold portfolio so as to redeem some of its nominal liabilities for gold, thereby extracting some real revenue from the private sector.

To clarify, consider the following example: the currency issuer starts with positive gold holdings, $g_{t_0}^C = \bar{g}^C > 0$, it keeps them constant until a generic period T, i.e. $g_t^C = \bar{g}^C$ for $t_0 \leq t < T$, and then sells a fraction $1 - \lambda$ of them at time T, i.e. $g_t^C = \lambda \bar{g}^C$ for $t \geq T$, with $\lambda \in [0, 1]$. In this scenario, we can show that the commitment to mobilize a fraction $1 - \lambda$ of the gold portfolio at time T implies

$$\mathcal{G}_{t_0}(\lambda, T) = \beta^{T-t_0-1} \frac{Z_g(\bar{G} - \lambda \bar{g}^C)}{Z_g(\bar{G} - \bar{g}^C)}(1 - \lambda)\bar{q}\bar{g}^C, \tag{44}$$

which is decreasing in T and, under certain conditions, in λ,[25] and where $\bar{q} \equiv \frac{1}{1-\beta} Z_g(\bar{G} - \bar{g}^C)$ denotes the equilibrium real price of gold if the currency issuer held its initial portfolio indefinitely, i.e. $g_t^C = \bar{g}^C$ for all $t \geq t_0$. Eq. (44) then shows that $\mathcal{G}_{t_0} > 0$ as long as the currency issuer commits to redeem its liabilities for gold (i.e. $\lambda < 1$) in a finite time (i.e. $T < \infty$) and even in a relatively small amount, i.e. λ close to one.

4.1. Reflationary policies with gold

To understand the policy options available to the currency issuer in this environment, we can build the intuition using again an AD–AS logic. Under the simplifying preference specification used in Appendix A.4, which we complement with $Z(g_t) = \vartheta \ln(g_t)$, we can write consumption demand, under the remittances policy $T_t^C = T^C$ for all $t \geq t_0 + 1$, as

[25] In particular, $\mathcal{G}_{t_0}$ is always decreasing in λ if preferences are logarithmic in gold holdings. This is also generally the case as long as $\gamma(g) < \frac{\bar{G} - \lambda \bar{g}^C}{(1-\lambda)\bar{g}^C}$, i.e. if the household's coefficient of relative risk aversion in gold holdings $\gamma(g)$ is not too large compared to the amount of gold the central bank commits to sell, $(1 - \lambda)\bar{g}^C / \bar{G}$.

$$C_{t_0+1} = \frac{1-\beta}{1+\theta-\vartheta}\left\{\frac{\widetilde{\mathcal{T}}^C - \widetilde{N}_{t_0}}{P_{t_0+1}} + q_{t_0+1}(\lambda, T)\bar{g} + \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} Y\right\}, \tag{45}$$

where $\widetilde{\mathcal{T}}^C$ is defined by (31), $\bar{g} = \bar{G} - \bar{g}^C$ and $q_{t_0+1}(\lambda, T)$ denotes the equilibrium real price of gold at time $t_0 + 1$, as a function of the share of gold holdings $1 - \lambda$ that the currency issuer commits to mobilize and the future date T at which it commits to do it, with:

$$\frac{q_{t_0+1}(\lambda, T)}{\bar{q}} \equiv 1 - \beta^{T-t_0-1}\left(1 - \frac{Z_g(\bar{G} - \lambda\bar{g}^C)}{Z_g(\bar{G} - \bar{g}^C)}\right) \tag{46}$$

and $q_{t_0+1} < \bar{q}$ for a finite T and $\lambda < 1$. The equilibrium real price of gold q_{t_0+1} is therefore increasing in both λ and T, given the concavity of $Z(\cdot)$, and is equal to $\bar{q}$ if the central bank does not commit to ever redeem its liabilities for gold (i.e. if either $\lambda = 1$ or $T \to \infty$). Moreover, it is also independent of the price level at $t_0 + 1$ and only depends on real factors, as implied by (41).

Eq. (45) then clearly shows that also in this environment, the same analysis of the previous section applies, from a qualitative perspective. Again, the relevant financial position of the household takes into account the component of human wealth influenced by the remittance policy of the currency issuer, through $\widetilde{\mathcal{T}}^C$. If the latter satisfies $\widetilde{\mathcal{T}}^C > \widetilde{N}_{t_0}$, the private sector is a net creditor with respect to the nominal wealth position that matters for its spending decisions. Accordingly, an increase in the price level at $t_0 + 1$ reduces the demand for consumption through a negative wealth effect, and the aggregate demand schedule is again a downward-sloping function, as in Fig. 2. Cutting the currency issuer's net worth at time t_0, or committing to reduce seigniorage revenues, reflates the economy by expanding the demand for long-run consumption, and bringing the economy in equilibrium E', along the lines discussed in Section 3.3.

In addition, in this environment the currency issuer has access to yet another policy option that acts on the denominator of (42). Specifically, a reduction in $\mathcal{G}_{t_0}$ signals a weaker willingness of the currency issuer to actively use its gold portfolio to back its nominal liabilities. The example introduced earlier illustrates that the currency issuer can achieve this in two ways: either by committing to mobilize a smaller share of its gold portfolio at a given future time (i.e., a higher λ), or by delaying the date at which it will use a given share of its gold holdings to redeem its nominal liabilities (i.e., a higher T).

In either case, the amount of real assets that the currency issuer is willing to allocate to backing its nominal liabilities decreases, thereby reducing their exchange value and reflating the economy. Within the AD–AS framework depicted in Fig. 2, lowering $\mathcal{G}_{t_0}$ by increasing either λ or T drives up the equilibrium real price of gold, q_{t_0+1} — as implied by Eq. (46) — and generates a positive wealth effect. This wealth effect shifts aggregate demand outward, pushing up the equilibrium price level, as described by Eq. (45).

5. Conclusion

This paper examines the economics behind the effectiveness of "helicopter money" in reflating an economy out of a slump and connects it to the broader issue of price-level determination in monetary models. A central feature of the analysis is the unique position of the currency issuer, which creates and distributes the unit of account and is the only agent in the economy not subject to a solvency constraint for its liabilities to remain default-free. In contrast, all other agents in the economy, including the treasury, are subject to such constraints. Consequently, the equilibrium condition relevant for determining the price level is, in principle, tied solely to the central bank's net liability position.

We explore a range of policy actions that are equivalent to the standard concept of "helicopter money" and characterize the mechanisms at work under various institutional arrangements between the currency issuer and the treasury. If the currency issuer guarantees the treasury's liabilities, "helicopter money" is equivalent to "unbacked fiscal expansions". Conversely, the currency issuer can implement balance-sheet or transfer policies that are effectively equivalent to "helicopter drops", even without meaningful economic involvement of the treasury.

To maintain simplicity and tractability, our model is intentionally streamlined. However, several extensions could address its limitations. First, incorporating a more dynamic framework could shed light on the medium-run effectiveness of policies and capture the endogenous duration of zero lower bound policies. This approach, inspired by Eggertsson and Woodford (2003), could reveal distinctions in the effectiveness of the policies discussed.

Second, the model assumes lump-sum transfers and taxes. While this assumption reflects constraints observed during certain shocks — such as the Great Lockdown — it limits the scope of fiscal tools analyzed. Exploring distortionary taxes or productive public spending could enhance the role of fiscal policy in lifting the economy out of a slump, as discussed by Eggertsson (2011). Comparing these alternative fiscal tools with those analyzed here would be a fruitful avenue for further research.

Finally, given the recent surge in prices, this framework could be adapted to study how the central bank might regain control of inflation by reversing the policies analyzed in this paper. Such an extension would require a more general framework that allows for a disconnect between the policy rate and the market interest rate, which drives consumption and saving decisions. This avenue is a promising direction for future research.

Appendix A. The model

A.1. Households

Households have preferences defined over consumption C and real money balances $m \equiv M/P$

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[U(C_t) + V\left(m_t\right) \right] \tag{A.1}$$

and are subject to a flow budget constraint of the form

$$P_t C_t + M_t + \frac{B_t + X_t}{1+i_t} \leq P_t Y + M_{t-1} + B_{t-1} + X_{t-1} - T_t + T_t^H, \tag{A.2}$$

where $M_t \geq 0$ and $X_t \geq 0$ whereas B can take a positive value (when saving), or a negative one (when borrowing). Borrowing possibilities are subject to a limit that prevents Ponzi schemes, which would otherwise allow for an infinite level of consumption. At each point in time t, a natural borrowing limit applies

$$-\left(B_{t-1} + X_{t-1} + M_{t-1} \right) \leq \sum_{T=t}^{\infty} \Lambda_{t,T}^{n} \left(P_T Y + T_T^H - T_T \right) < \infty, \tag{A.3}$$

which can be equivalently written in real terms as[26]

$$-\frac{B_{t-1} + X_{t-1} + M_{t-1}}{P_t} \leq \sum_{T=t}^{\infty} \Lambda_{t,T} \left(Y + \frac{T_T^H - T_T}{P_T} \right) < \infty, \tag{A.4}$$

where the real discount factor $\Lambda_{t,T}$ is related to real interest rates according to

$$\Lambda_{t,T} \equiv \prod_{j=t}^{T-1} \frac{P_{j+1}}{P_j(1+i_j)} = \frac{P_T}{P_t} \Lambda_{t,T}^{n} \tag{A.5}$$

for $T > t$ whereas the nominal discount factor is given by

$$\Lambda_{t,T}^{n} \equiv \prod_{j=t}^{T-1} \frac{1}{(1+i_j)} \tag{A.6}$$

with $\Lambda_{t,t}^{n} = 1$.

The borrowing limit (A.4) states that the real net debt position of households at time t (i.e. the term on the left hand side of the inequality) should not be larger than the present discounted value of their real net income. The latter should be finite, otherwise an infinite level of consumption would still be feasible. The constraint (A.4), or equivalently (A.3) in nominal terms, makes sure that at any point in time the household can pledge enough assets, together with current and future net income to pay back the debt. Thus, if commitment to obligations is not questionable, such debt is paid with certainty. This requirement is also coherent with attributing the default-free nominal rate to private debt in writing the budget constraint (A.2). Throughout, all default-free nominal debt has the same characteristics: it is repaid with certainty. Absence of arbitrage opportunities implies that $i_t \geq 0$.

Households choose sequences for consumption and portfolio holdings $\{C_t, M_t, B_t, X_t\}_{t=t_0}^{\infty}$, with $C_t, M_t, X_t \geq 0$, to maximize utility (A.1) under the sequence of flow budget constraints (A.2) and borrowing limits (A.4), taking as given the sequence of prices, endowment and taxes $\{P_t, Q_t, \Lambda_{t_0,t}, Y, T_t\}_{t=t_0}^{\infty}$ and initial conditions $M_{t_0-1}, B_{t_0-1}, X_{t_0-1}$.

Solution to the above optimization problem implies, for any period t, a standard Euler equation restricting the intertemporal path of consumption

$$\xi_t U_c(C_t) = \beta(1+i_t) \frac{P_t}{P_{t+1}} \xi_{t+1} U_c(C_{t+1}), \tag{A.7}$$

and the first-order condition with respect to cash holdings

$$\frac{\xi_t U_c(C_t)}{P_t} = \frac{\xi_t}{P_t} V_m\left(\frac{M_t}{P_t}\right) + \beta \frac{\xi_{t+1} U_c(C_{t+1})}{P_{t+1}}, \tag{A.8}$$

which, by using (A.7), implies the following demand for real cash balances

$$\frac{M_t^d}{P_t} \geq L(C_t, i_t) \tag{A.9}$$

where L is the liquidity-preference function $L(C_t, i_t) \equiv V_m^{-1}\left(U_c(C_t)\frac{i_t}{1+i_t}\right)$. Comparing (A.7) and (A.8), and considering the properties of $V(\cdot)$, it follows that the nominal interest rate cannot be negative, $i_t \geq 0$. Eq. (A.9) holds with equality whenever $i_t > 0$. Finally, optimization also requires households to exhaust all their resources, thus implying Eq. (9).

[26] We are assuming finite prices to have equivalence.

A.2. The policy authorities

The policy authorities include a currency issuer and a treasury. The currency issuer chooses a sequence for the nominal interest rate i, nominal remittances T^C to transfer to either or both the treasury and the household, its short-term liabilities M^C and X^C, and the assets to hold in its portfolio, B^C, so as to satisfy the following flow budget constraint

$$\frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C. \tag{A.10}$$

For future reference, let us also define the central bank's nominal net worth

$$N_t \equiv \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} \tag{A.11}$$

From an accounting perspective, M_t^C and X_t^C are the currency issuer's liabilities, but of a special type, as they define the unit of account. Thus, we do not need to require any solvency condition for the currency issuer to be sure that its liabilities are paid with certainty, as discussed in Section 2. This special property does not necessarily apply to the treasury, which chooses a sequence for taxes T and nominal liabilities B^F so as to satisfy its flow budget constraint

$$\frac{B_t^F}{1+i_t} = B_{t-1}^F - T_t - T_t^F, \tag{A.12}$$

taking as given asset prices and the nominal remittances T^F received by the currency issuer.

Like private debt, treasury bonds are also a claim on a given amount of the currency, which the treasury needs to raise to be solvent. It follows that the use of default-free interest rate in (A.12) is only accurate if treasury's debt is repaid with certainty, which requires a borrowing limit analogous to (A.3)

$$B_{t-1}^F \leq \sum_{T=t}^{\infty} \Lambda_{t,T}^n (T_T + T_T^F) \tag{A.13}$$

or, equivalently, in real terms

$$\frac{B_{t-1}^F}{P_t} \leq \sum_{T=t}^{\infty} \Lambda_{t,T} \left(\frac{T_T}{P_T} + \frac{T_T^F}{P_T} \right). \tag{A.14}$$

The nominal (real) value of treasury's debt at a certain point in time should not be greater than the nominal (real) present discounted value of taxes and remittances received from the currency issuer. The borrowing limit (A.13), or (A.14), prevents the treasury from running a Ponzi scheme and at the same time is coherent with the default-free properties of treasury's debt, as specified in budget constraint (A.12). Violation of the borrowing limit would allow for infinite spending possibilities for the treasury, and in the case of this model for infinite transfers to households implying infinite consumption. From this perspective, the treasury is not different from private borrowers in the economy: either it is solvent and repays its debt, or eventually it has to default on it, in which case Eq. (A.14) is modified to determine the endogenous default rate. Throughout, whenever constraint (A.14) applies, we assume that the treasury raises just enough resources to pay its obligations, and therefore (A.14) holds with equality.

A.3. Equilibrium

Equilibrium in goods and assets market implies:

$$\begin{aligned} C_t &= Y \\ B_t^F &= B_t + B_t^C \\ M_t^C &= M_t \\ X_t^C &= X_t, \end{aligned}$$

and finally the currency issuer's remittances are such that $T_t^C = T_t^H + T_t^F$.

Using goods market equilibrium we can write the Euler equation as

$$(1+i_t) = \frac{1}{\beta} \frac{\xi_t}{\xi_{t+1}} \frac{P_{t+1}}{P_t}, \tag{A.15}$$

Using goods and asset market equilibria, Eq. (A.9) becomes

$$\frac{M_t}{P_t} \geq L(Y, i_t), \tag{A.16}$$

considering that $i_t \geq 0$, and holding with equality whenever $i_t > 0$. The intertemporal budget constraint of the household, Eq. (9), once we impose the transversality condition (12), can be written as

$$\frac{B_{t_0-1} + X_{t_0-1} + M_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\xi_t}{\xi_{t_0}} \left(\frac{T_t}{P_t} - \frac{T_t^H}{P_t} + \frac{i_t}{1+i_t} \frac{M_t}{P_t} \right). \tag{A.17}$$

We can characterize two alternative scenarios. The first is when the currency issuer does not "back" the treasury's liabilities, in which case we can combine Eq. (A.17) with (A.14) — holding with equality — and get

$$\frac{B^C_{t_0-1} - X_{t_0-1} - M_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\xi_t}{\xi_{t_0}} \left(\frac{T^C_t}{P_t} - \frac{i_t}{1+i_t} \frac{M_t}{P_t} \right). \tag{A.18}$$

In this scenario, to complete the relevant equilibrium conditions, we add the flow budget constraint of the currency issuer

$$\frac{B^C_t}{1+i_t} - M_t - \frac{X_t}{1+i_t} = B^C_{t-1} - X_{t-1} - M_{t-1} - T^C_t. \tag{A.19}$$

An equilibrium in this case is a set of non-negative sequences $\{P_t, i_t, M_t, X_t, B^C_t, T^C_t\}_{t=t_0}^{\infty}$ satisfying (A.15), (A.16),(A.19) for each $t \geq t_0$ and (A.18), in which the currency issuer can specify three out of the following five sequences $\{i_t, M_t, X_t, B^C_t, T^C_t\}_{t=t_0}^{\infty}$, given the sequence of exogenous shocks $\{\xi_t\}_{t=t_0}^{\infty}$ and initial conditions $B^C_{t_0-1}, X_{t_0-1}, M_{t_0-1}$.[27] Note that the requirement that remittances are non-negative is not necessary for the characterization of the equilibrium, but it is useful to exclude any support from the treasury to the currency issuer.

In the second scenario, instead, the currency issuer "backs" the treasury, in which case the equilibrium condition (A.18) is replaced by

$$\frac{B_{t_0-1} + X_{t_0-1} + M_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\xi_t}{\xi_{t_0}} \left(\frac{T_t}{P_t} + \frac{i_t}{1+i_t} \frac{M_t}{P_t} \right), \tag{A.20}$$

and (A.19) is replaced by the consolidation of the flow budget constraints (A.10) and (A.12):

$$\frac{B_t}{1+i_t} + M_t + \frac{X_t}{1+i_t} = B_{t-1} + M_{t-1} + X_{t-1} - T_t. \tag{A.21}$$

In both equations above we made use of the simplifying assumption that, in this case, since treasury and currency issuer are effectively a single government agency, $T^H_t = 0$, for each t.

In this case, an equilibrium is a set of sequences $\{P_t, i_t, M_t, X_t, B_t, T_t\}_{t=t_0}^{\infty}$ of which the sequences $\{P_t, i_t, M_t, X_t, T_t\}_{t=t_0}^{\infty}$ are non-negative satisfying (A.15), (A.16), (A.21) at each $t \geq t_0$ and (A.17), given the sequence of shocks $\{\xi_t\}_{t=t_0}^{\infty}$ and initial conditions $B_{t_0-1}, M_{t_0-1}, X_{t_0-1}$. The currency issuer and the treasury can specify three out of the following five sequences $\{i_t, M_t, X_t, B_t, T_t\}_{t=t_0}^{\infty}$.

A.4. Derivation of Eq. (30)

We use the following preference specification:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\ln C_t + \theta \ln \frac{M_t}{P_t} \right].$$

Consider Eq. (9) evaluated at time $t_0 + 1$,

$$\sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} \left(C_t + \frac{i_t}{1+i_t} \frac{M_t}{P_t} \right) = \frac{M_{t_0} + X_{t_0} + B_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} \left(Y_t + \frac{T^H_t - T_t}{P_t} \right), \tag{A.22}$$

and recall that in the long run $\xi_t = \bar{\xi}$ for each $t \geq t_0 + 1$. This preference specification implies

$$\frac{M_t}{P_t} = \theta C_t \frac{1+i_t}{i_t}$$

and moreover: $\Lambda_{t_0+1,t} C_t = \beta^{t-t_0-1} C_{t_0+1}$. Using the above, we can write (A.22) as:

$$C_{t_0+1} = \frac{1-\beta}{1+\theta} \left\{ \frac{M_{t_0} + X_{t_0} + B_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} \left(Y_t + \frac{T^H_t - T_t}{P_t} \right) \right\}.$$

Under the scenario where the currency issuer does not back the treasury, and therefore using Eq. (A.14) — holding with equality — we obtain

$$C_{t_0+1} = \frac{1-\beta}{1+\theta} \left\{ \frac{M_{t_0} + X_{t_0} - B^C_{t_0}}{P_{t_0+1}} + \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} \left(Y_t + \frac{T^C_t}{P_t} \right) \right\}. \tag{A.23}$$

Using the assumption of constant endowment, and the assumptions and policies of Proposition 3, we can finally cast it in the form of Eq. (30):

$$C_{t_0+1} = \frac{1-\beta}{1+\theta} \left\{ \frac{M_{t_0} + X_{t_0} - B^C_{t_0}}{P_{t_0+1}} + \frac{T^C}{P_{t_0+1}} \sum_{t=t_0+1}^{\infty} \Lambda^n_{t_0+1,t} + \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} Y \right\}$$

[27] Note that the equilibrium condition (A.18) does not restrict the equilibrium variables at each point in time but only in the long run, which explains why there are three degrees of freedom to choose policy.

$$= \frac{1-\beta}{1+\theta}\left\{\frac{M_{t_0}+X_{t_0}-B^C_{t_0}}{P_{t_0+1}}+\frac{1+i}{i}\frac{T^C}{P_{t_0+1}}+\sum_{t=t_0+1}^{\infty}\Lambda_{t_0+1,t}Y\right\}$$

$$= \frac{1-\beta}{1+\theta}\left\{\frac{\widetilde{\mathcal{T}}^C-N_{t_0}}{P_{t_0+1}}+\sum_{t=t_0+1}^{\infty}\Lambda_{t_0+1,t}Y\right\}, \tag{A.24}$$

where in the first line we used Eq. (A.5) and the constant remittances policy of Proposition 3, in the second the constant interest-rate policy of Proposition 3, and in the last the definition of net worth (27), $N_{t_0}=B^C_{t_0}-M_{t_0}-X_{t_0}$ (considering $i_{t_0}=0$), and definition (31). Eq. (A.24) is thus the aggregate consumption demand, given income and policy, and for a given sequence of the real interest rate, captured by the discount factor $\Lambda_{t_0+1,t}$.

A.5. An economy with gold

Preferences are defined over consumption C, real money balances m, and gold holdings g

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[U(C_t)+V\left(m_t\right)+Z\left(g_t\right)\right] \tag{A.25}$$

where the private utility from gold, $Z(\cdot)$, is increasing and strictly concave. Households maximize (A.25) subject to a sequence of flow budget constraints of the form

$$P_tC_t+M_t+\frac{B_t+X_t}{1+i_t}+P_tq_t(g_t-g_{t-1})\leq P_tY+M_{t-1}+B_{t-1}+X_{t-1}-T_t+T^H_t, \tag{A.26}$$

where q_t is the real price of gold at time t. Solution of the optimization problem above requires, for any period t, the first-order conditions (A.7)–(A.8) plus the one for gold holdings, which implies the equilibrium real price of gold:

$$q_t=\frac{Z_g(g_t)}{U_c(C_t)}+\Lambda_{t,t+1}q_{t+1}. \tag{A.27}$$

The real price of gold depends on the utility benefits of gold in units of consumption goods and on the present-discounted value of its next-period level.

The other first-order condition that is affected compared to the benchmark model of the previous section is the intertemporal budget constraint of the household, which now reads as

$$\sum_{t=t_0}^{\infty}\Lambda_{t_0,t}\left(C_t+\frac{i_t}{1+i_t}\frac{M_t}{P_t}+(g_t-\bar{G})\frac{Z_g(g_t)}{U_c(C_t)}\right)=\frac{W_{t_0}}{P_{t_0}}+\sum_{t=t_0}^{\infty}\Lambda_{t_0,t}\left(Y+\frac{T^H_t-T_t}{P_t}\right) \tag{A.28}$$

in which nominal financial wealth at the beginning of period t is defined by:

$$W_t\equiv B_{t-1}+X_{t-1}+M_{t-1}+P_tq_t(g_{t-1}-\bar{G}).$$

We allow gold to be held as an asset by the central bank in its balance sheet. Let g^C_t denote the central bank's gold portfolio, with $\bar{G}=g_t+g^C_t$.

Under the assumption that the central bank does not back the treasury's liabilities, and therefore the solvency condition (3) holds, using goods and asset market equilibrium, we now obtain the key equation for price level determination at time t_0+1 as

$$\sum_{t=t_0+1}^{\infty}\beta^{t-t_0-1}\left(\frac{T^C_t}{P_t}+Z_g(g_t)g^C_t\right)=\mathcal{S}(Y,\Pi)+q_{t_0+1}g^C_{t_0}+\frac{B^C_{t_0}-X_{t_0}-M_{t_0}}{P_{t_0+1}}, \tag{A.29}$$

where we have used $U_c(Y)=1$. Using (A.27), we can further write it as

$$\sum_{t=t_0+1}^{\infty}\beta^{t-t_0-1}\left(\frac{T^C_t}{P_t}\right)=\mathcal{G}_{t_0}+\mathcal{S}(Y,\Pi)+\frac{B^C_{t_0}-X_{t_0}-M_{t_0}}{P_{t_0+1}} \tag{A.30}$$

where $\mathcal{G}_{t_0}$ is defined by (43).

Appendix B. Proofs of propositions

B.1. Proposition 1

Proof. First note that a policy of constant and positive interest rate on reserves, $i_t = i > 0$ implies, through (6), that real cash balances are constant, when the price level is finite, and therefore $m_t = m$ for each $t \geq t_0$, where $m_t \equiv M_t/P_t$.[28] Moreover, it also implies, through (5), that the inflation rate is constant at $\Pi_t = \Pi = \beta(1+i)$ for each $t \geq t_0 + 1$.

Plugging these results in Eq. (13), written for a generic time $t \geq t_0$ implies

$$\frac{X_{t-1} + M_{t-1}}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t} \left\{ V_m(m)\, m \right\} = \frac{1}{1-\beta} V_m(m) m, \tag{B.31}$$

finally yielding:

$$P_t = (1-\beta) \frac{X_{t-1} + M_{t-1}}{V_m(m) m}. \tag{B.32}$$

From the above, applied to time t and $t+1$, it follows that the growth rate of the monetary base is Π, for all $t \geq t_0$:

$$X_t + M_t = \Pi(X_{t-1} + M_{t-1}).$$

To show that inflation should be strictly positive, i.e. $\Pi > 1$, note that since real cash balances are constant over time, for all $t \geq t_0$, and $\beta(1+i_t) = \Pi_{t+1} = \Pi$, for all $t \geq t_0$, Eq. (6) implies

$$V_m(m_t) = \frac{i}{1+i} = \frac{\Pi - \beta}{\Pi} \tag{B.33}$$

for all $t \geq t_0$. Moreover, it further implies that, for $t > t_0$, the growth rate of cash is Π and therefore also reserves grow at the same rate for each $t > t_0$. At time $t = t_0$, instead, Eqs. (B.32) and (B.33) imply

$$P_{t_0} = (1-\beta) \frac{X_{t_0-1} + M_{t_0-1}}{m} \frac{\Pi}{\Pi - \beta}, \tag{B.34}$$

which, together with $m_{t_0} \equiv M_{t_0}/P_{t_0} = m$, implies that the equilibrium cash and reserves are, respectively:

$$M_{t_0} = \Pi \left(\frac{1-\beta}{\Pi - \beta} \right) \left(X_{t_0-1} + M_{t_0-1} \right) \tag{B.35}$$

$$X_{t_0} = \Pi \left(\frac{\Pi - 1}{\Pi - \beta} \right) \left(X_{t_0-1} + M_{t_0-1} \right). \tag{B.36}$$

Eq. (B.36), thus, requires, in order for the equilibrium reserves to be positive at t_0 and at any future date, a positive net rate of inflation, i.e. $\Pi > 1$. ■

B.2. Proposition 2

Proof. Evaluating (5) and (6) under the assumptions of the proposition implies the following difference equation in real cash balances

$$m_{t+1} = \frac{1+\mu}{\beta} \Big(1 - V_m(m_t) \Big) m_t. \tag{B.37}$$

As discussed in Obstfeld and Rogoff (1983), there are multiple solutions associated with the above equation. There can be deflationary solutions in which real cash balances tend towards infinity, or inflationary ones in which they approach zero, even within a finite period of time. Additionally, there is a stationary solution with a constant real cash balance, where prices grow at the same rate as cash.

In the stationary solution, (B.37) implies that

$$m_t = V_m^{-1} \left(1 - \frac{\beta}{1+\mu} \right), \tag{B.38}$$

provided $V_m(\cdot) > 0$. Since $m_t \equiv M_t/P_t$, therefore, the price level at time t is given by

$$P_t = \frac{M_t}{V_m^{-1}\left(1 - \frac{\beta}{1+\mu}\right)} = \frac{(1+\mu) M_{t-1}}{V_m^{-1}\left(1 - \frac{\beta}{1+\mu}\right)}, \tag{B.39}$$

where the second equality uses the constant cash-growth rule. Interestingly, the solution above shows similar features to price determination as in the fiscal theory of the price level, in which the price level is proportional to the beginning-of-period currency

[28] This is an important implication because pinning down a constant level of real balances through the cash demand (6) and cash-market equilibrium allows to rule out a continuum of inflationary solutions that would otherwise require additional assumptions.

issuer liabilities, here represented by only cash.[29] Therefore, for a given growth rate of cash μ for each $t \geq t_0$, a currency issuer's policy that injects more cash at time $t_0 - 1$, dropping it via a transfer policy, results in a higher price level at time t_0.

The key step is now to show that the stationary solution is an equilibrium and the only equilibrium. Using the previous insights, we should rely on Eq. (11), given the assumptions $X_t = 0$ and $T_t^C = \mu M_{t-1}$, and therefore on

$$\frac{M_{t-1}}{P_t} = \sum_{T=t}^{\infty} \beta^{T-t} \left\{ V_m \left(m_T\right) m_T - \mu \frac{(1+\mu)^{T-t} M_{t-1}}{P_T} \right\}. \tag{B.40}$$

First, it is straightforward to see that the stationary solution in which m_t is constant and given by (B.38) satisfies the above equation, considering that prices grow at the rate $1+\mu$. Therefore, the stationary solution is an equilibrium. Second, there are no equilibria in which real cash balances are satiated, as in the solutions of (B.37) where real cash balances grow monotonically. This is true provided μ is non-negative, as the equilibrium condition (B.40) necessarily requires that $V_m\left(m_t\right) > 0$ whenever prices are finite and $\mu \geq 0$. Third, condition (15) is sufficient to exclude equilibria with inflationary solutions since it will imply a positive value for the right-hand side of (B.40) providing a bound on how much real cash balances, on the left-hand side, can fall in any equilibrium. ■

B.3. Proposition 3

Proof. First note that, under the constant interest-rate policy of the proposition, we obtain — using Eq. (20) — a constant inflation rate $\frac{P_{t+1}}{P_t} = \Pi = \beta(1+i)$ and — using Eq. (21) — a constant level of real cash balances $m_t = m = L(Y, \Pi/\beta - 1)$, for each $t \geq t_0 + 1$.

Using these results, we can define the present discounted stream of equilibrium seigniorage revenues as

$$\mathcal{S}(\Pi, Y) \equiv \sum_{t=t_0+1}^{\infty} \Lambda_{t_0+1,t} \left\{ V_m\left(m_t\right) m_t \right\} = L\left(Y, \frac{\Pi}{\beta} - 1\right) \frac{\Pi - \beta}{(1-\beta)\Pi}. \tag{B.41}$$

Next note that, under the constant non-negative nominal remittance policy[30] and the constant positive interest-rate policy of the proposition, we can define the present discounted stream of nominal remittances as

$$\mathcal{T}^C \equiv \sum_{t=t_0+1}^{\infty} \Lambda^n_{t_0+1,t} \left\{ T_t^C \right\} = \frac{1+i}{i} T^C = \frac{\Pi}{\Pi - \beta} T^C, \tag{B.42}$$

where the last equality follows from equilibrium in the goods market, given the constant endowment. Using the definitions above, Eq. (22) can then be written as

$$\frac{X_{t_0} + M_{t_0} - B_{t_0}^C}{P_{t_0+1}} = \mathcal{S}(\Pi, Y) - \frac{\mathcal{T}^C}{P_{t_0+1}}. \tag{B.43}$$

Eq. (B.43) is again an equilibrium condition — not a solvency constraint — which then determines the long-run price level P_{t_0+1} at

$$P_{t_0+1} = \frac{\mathcal{T}^C + X_{t_0} + M_{t_0} - B_{t_0}^C}{\mathcal{S}(\Pi, Y)}. \tag{B.44}$$

Note, moreover, that solution (B.44) implies that the equilibrium law of motion of the net asset position in interest-bearing securities is

$$B_t^C - X_t = \Pi(B_{t-1}^C - X_{t-1}) - (\Pi - 1)\mathcal{T}^C,$$

for each $t \geq t_0 + 1$, where we have used the fact that cash grows at rate Π in the stationary solution. The above equation implies that the equilibrium net asset position in interest-bearing securities (i.e. $B_t^C - X_t$) should be held constant to support price stability, i.e. $\Pi = 1$. In addition, the currency issuer should have a positive asset position. Indeed, differently from Section 2, price stability is in fact attainable in this case using an interest-rate policy, as long as the initial asset position is sufficiently large. To see this, note that at time $t_0 + 1$ equilibrium cash is

$$M_{t_0+1} = \Pi \left(\frac{1-\beta}{\Pi - \beta} \right) \left(\mathcal{T}^C + X_{t_0} + M_{t_0} - B_{t_0}^C \right), \tag{B.45}$$

which implies that the equilibrium amount of nominal assets required by a given, positive, level of nominal reserves is given by Eq. (28), i.e.:

$$B_{t_0+1}^C = X_{t_0+1}^C + \mathcal{T}^C - \Pi \left(\frac{\Pi - 1}{\Pi - \beta} \right) \left(\mathcal{T}^C + X_{t_0} + M_{t_0} - B_{t_0}^C \right). \quad ■ \tag{B.46}$$

[29] Eq. (B.39) clarifies an important difference between cash-growth rules and interest-rate policies. On the one hand, a cash-growth rule directly determines the rate of growth of nominal cash at all times, therefore including the initial period, without the need to use the implications of equilibrium condition (11). An interest-rate rule, on the other hand, determines directly a constant growth rate of cash — equal to the inflation target — only starting in period t_0. In the initial period (i.e. between $t_0 - 1$ and t_0) the growth rate of cash is instead determined by the equilibrium condition (13), as implied by Eqs. (B.34) and (B.35).

[30] An alternative case is one where the currency issuer is able to control a stream of real remittances, by committing to make a (fully indexed) monetary transfer such that corresponds to a certain purchasing power in terms of consumption goods. For details, see Benigno and Nisticò (2022).

B.4. Proposition 5

Proof. We start by noting that under the policies of the proposition, the remittance policy implied residually by the flow-budget constraint of the currency issuer is then:[31]

$$T_t^C = \mu M_{t-1} + \frac{\mu - i_t}{1+i_t}(X_{t-1} - B_{t-1}^C). \tag{B.47}$$

Next we note that the cash-growth rule (32), together with (20) and (21), implies again that real cash balances follow Eq. (B.37), as in the simple example of Section 2:

$$m_{t+1} = \frac{1+\mu}{\beta}\Big(1 - V_m(m_t)\Big)m_t. \tag{B.48}$$

As a consequence, the constant real cash balances in (B.38) is still a stationary solution to the above difference equation, and therefore the equilibrium price level at time t_0+1 is

$$P_{t_0+1} = \frac{(1+\mu)M_{t_0}}{L\left(Y, \frac{\Pi}{\beta} - 1\right)}, \tag{B.49}$$

where we used the feature of a stationary solution that nominal currency and prices grow at the same rate for all $t \geq t_0+1$: $\frac{P_{t+1}}{P_t} = 1+\mu = \Pi$.

To prove that the stationary solution is an equilibrium and the only equilibrium we note the following. First, under the proposed policy, in which assets and liabilities all grow at rate $1+\mu$, the stationary solution with $m_t = m < \bar{m}$ for all $t \geq t_0+1$, in which also prices grow at rate $1+\mu$, is indeed a solution to (22), and it satisfies the transversality condition (12) which under the proposed policy reads

$$\lim_{T\to\infty} \beta^{T-t_0}\frac{X_T + M_T - B_T^C}{P_{T+1}} = \lim_{T\to\infty}\left(\frac{\beta(1+\mu)}{\Pi}\right)^{T-t_0}\frac{X_{t_0}+M_{t_0}-B_{t_0}^C}{P_{t_0+1}} = \lim_{T\to\infty}\beta^{T-t_0}\frac{X_{t_0}+M_{t_0}-B_{t_0}^C}{P_{t_0+1}} = 0$$

where in the last equality we used $1+\mu = \Pi$. This proves that the stationary solution is an equilibrium. Second, to prove that it is the only equilibrium, we need to rule out deflationary and inflationary solutions. Deflationary solutions where the economy is eventually satiated in liquidity and prices fall at rate β are ruled out by the proposed policy as long as μ is non-negative. Indeed, in this case the limit of the net liabilities of the currency issuer reads:

$$\lim_{T\to\infty} \beta^{T-t_0}\frac{X_T + M_T - B_T^C}{P_{T+1}} = \lim_{T\to\infty}(1+\mu)^{T-t_0}\frac{X_{t_0}+M_{t_0}-B_{t_0}^C}{P_{t_0+1}},$$

which is non-zero as long as $\mu \geq 0$ and unless $N_{t_0} = B_{t_0}^C - X_{t_0} - M_{t_0} = 0$. Finally, inflationary solution can be ruled out by condition (15), as before.

To get insights into the mechanism of a policy of swapping nominal cash for reserves, and see its implications for implementing helicopter money, note that Eq. (B.49) can be written in a form analogous to Eq. (B.44):

$$P_{t_0+1} = \frac{\mathcal{T}_{t_0}^C - N_{t_0}}{\mathcal{S}(\Pi, Y)}, \tag{B.50}$$

where

$$N_{t_0} = B_{t_0}^C - X_{t_0} - M_{t_0} \tag{B.51}$$

is currency issuer's nominal net worth in the short run, $\mathcal{S}(\Pi, Y)$ is given by the definition (B.41), and

$$\mathcal{T}_{t_0}^C \equiv B_{t_0}^C - X_{t_0} + \frac{\mu}{1-\beta}M_{t_0} \tag{B.52}$$

is the present discounted value of nominal remittances looking forward from time t_0+1, which depends on variables chosen at time t_0, and in particular on the level of currency. Indeed, in a stationary solution we can write:

$$\sum_{t=t_0+1}^{\infty}\beta^{t-t_0-1}\frac{T_t^C}{P_t} = \frac{B_{t_0}^C - X_{t_0} + \mu\sum_{t=t_0+1}^{\infty}\beta^{t-t_0-1}M_{t_0}}{P_{t_0+1}} = \frac{\mathcal{T}_{t_0}^C}{P_{t_0+1}}.$$

Thus, a policy of swapping nominal cash for reserves at time t_0, i.e. $\Delta M_{t_0} = -\Delta X_{t_0} > 0$, implies, through Eq. (B.51), $\Delta N_{t_0} = 0$ and, through Eq. (B.52),

$$\Delta\mathcal{T}_{t_0}^C = \Delta M_{t_0}\frac{1-\beta+\mu}{1-\beta} > 0,$$

where the inequality is satisfied as long as $\mu > \beta - 1$, which always holds under the assumption of non-negative μ. As a consequence, such a policy induces an increase in the price level at t_0+1, as implied by Eq. (B.50), and reflates the economy out of the liquidity trap. ■

[31] The appealing feature of this central-bank policy, also for practical relevance, is that it implies a constant level of real net worth in the stationary equilibrium, as it can be easily shown using (27) and (B.47).

B.5. Present-value budget constraints and price determination

Let $\mathcal{T}_t \equiv \sum_{T=t}^{\infty} \Lambda_{t,T}\left(\frac{T_T}{P_T}\right)$ denote the PDV of real taxes, $\mathcal{T}_t^C \equiv \sum_{T=t}^{\infty} \Lambda_{t,T}^n T_T^C$ the PDV of nominal remittances, with $\mathcal{T}_t^H \equiv \sum_{T=t}^{\infty} \Lambda_{t,T}^n T_T^H$ and $\mathcal{T}_t^F \equiv \sum_{T=t}^{\infty} \Lambda_{t,T}^n T_T^F$ the PDV of the remittances going to the households and the treasury, respectively, where $\mathcal{T}_t^C = \mathcal{T}_t^H + \mathcal{T}_t^F$, and $\mathcal{S}_t \equiv \sum_{T=t}^{\infty} \Lambda_{t,T}\left(\frac{i_T}{1+i_T}\frac{M_T}{P_T}\right)$ the PDV of real seigniorage revenues.

We can then write the present-value budget constraint of the treasury as:

$$B_{t-1}^F = P_t\mathcal{T}_t + \mathcal{T}_t^F + \ell_t^F \tag{B.53}$$

where $\ell^F \equiv \lim_{T\to\infty} \Lambda_{t,T}^n B_T^F$ is the PDV of treasury liabilities still outstanding in the infinitely distant future. Analogously, the present-value budget constraint of the currency issuer is:

$$X_{t-1} + M_{t-1} - B_{t-1}^C = P_t\mathcal{S}_t - \mathcal{T}_t^C + \ell_t^C \tag{B.54}$$

where $\ell^C \equiv \lim_{T\to\infty} \Lambda_{t,T}^n\left(X_T + M_T - B_T^C\right)$ is the PDV of currency issuer's net liabilities still outstanding in the infinitely distant future, and we have used asset-market equilibrium: $M = M^C$ and $X = X^C$.

Now note that the special properties of the currency issuer's liabilities stemming from their role as unit of account imply that ℓ^C is completely unrestricted, off equilibrium. The currency issuer can in principle run a policy that implies an exploding path of its net liabilities without any consequence whatsoever on the nominal risk-free properties of such liabilities.

Moreover, from the problem of the household, using the asset-market equilibrium, we have

$$\ell_t \equiv \lim_{T\to\infty} \Lambda_{t,T}^n\left(X_T + M_T + B_T\right) = \ell_t^F + \ell_t^C. \tag{B.55}$$

The solvency constraint (A.3) then implies the no-Ponzi Game condition $\ell_t \geq 0$, and the optimal consumption plan of the household requires exhaustion of its present-value intertemporal budget constraint which translates into the following transversality condition:

$$\ell_t = 0. \tag{B.56}$$

B.5.1. The currency issuer does not back the treasury

Consider now the case of separate treasury and currency issuer, where the former takes as given the path of remittances set independently by the latter. In this case, in order for the treasury's debt to be considered default-free, we impose the solvency constraint (4), which in the notation of this section reads:

$$\ell_t^F = 0. \tag{B.57}$$

This constraint restricts the set of tax policies that the treasury can run. Let $\overline{\mathcal{T}}_t$ denote the PDV of real taxes needed to back a given amount of outstanding debt B_{t-1}^F, for any given path of currency issuer's remittances accruing to the treasury $\mathcal{T}_t^F$ and any price level P_t:

$$\overline{\mathcal{T}}_t \equiv \frac{B_{t-1}^F - \mathcal{T}_t^F}{P_t}. \tag{B.58}$$

Given the definition above, a tax policy $\mathcal{T}_t$ with $\mathcal{T}_t = \overline{\mathcal{T}}_t$ satisfies the solvency constraint (B.57), and treasury's debt is therefore default-free. In this case, (B.55) and (B.56) imply that we can write the equilibrium condition relevant for price-level determination as

$$\ell_t^C = 0. \tag{B.59}$$

Imposing this condition on (B.54) then determines the price level, for a given remittance policy $\mathcal{T}_t^C = \mathcal{T}_t^H + \mathcal{T}_t^F$, a given balance-sheet policy setting either B_{t-1}^C or X_{t-1} and a given interest-rate policy determining M_{t-1} and seigniorage revenues $\mathcal{S}_t$:

$$P_t = \frac{\mathcal{T}_t^C - B_{t-1}^C + M_{t-1} + X_{t-1}}{\mathcal{S}_t}. \tag{B.60}$$

To scrutinize more thoroughly the implications for the equilibrium price level of imposing the solvency constraint (B.57) on the treasury, consider the case in which the treasury does not raise enough resources to back its outstanding stock of debt, i.e. $\mathcal{T}_t = \mathcal{T}_t^* < \overline{\mathcal{T}}_t$. In this case, when the currency issuer sets its remittance and balance-sheet policies independently of the tax policy, imposing constraint (B.57) means that the treasury has to default on its debt, whereby its present-value budget constraint must read

$$(1 - \varkappa_t^H)B_{t-1} + (1 - \varkappa_t^C)B_{t-1}^C = P_t\mathcal{T}_t^* + \mathcal{T}_t^F, \tag{B.61}$$

with $\varkappa_t^H$ and $\varkappa_t^C$ denoting the endogenous default rates on treasury's debt held by the private sector and the currency issuer, respectively. If the currency issuer is a central bank, it is typically a senior creditor of the treasury. Therefore, if the shortage in tax revenues is small enough, it can be absorbed by a default on the stock of debt held by the private sector only (meaning $\varkappa_t^H \in (0, 1]$ and $\varkappa_t^C = 0$), and has thus no effect on the equilibrium price level, which is still determined by (B.60).

If instead the currency issuer is not a senior creditor, as in the case of a private currency (i.e. $\varkappa_t^H = \varkappa_t^C$) or if a full default on privately held debt (i.e. $\varkappa_t^H = 1$) is not sufficient to absorb the shortage of tax revenues, the currency issuer experiences a loss on its nominal assets (i.e. $\varkappa_t^C \in (0, 1]$). Nevertheless, the currency issuer can still determine the equilibrium price level through (B.60), by appropriately adjusting the intertemporal path of its remittances to $\hat{\mathcal{T}}_t^C$, in order to cover the resulting capital loss, i.e.:

$$P_t = \frac{\hat{\mathcal{T}}_t^C - (1 - \varkappa_t^C)B_{t-1}^C + M_{t-1} + X_{t-1}}{\mathcal{S}_t} = \frac{\mathcal{T}_t^C - B_{t-1}^C + M_{t-1} + X_{t-1}}{\mathcal{S}_t}, \tag{B.62}$$

with $\hat{\mathcal{T}}_t^C = \mathcal{T}_t^C - \varkappa_t^C B_{t-1}^C$. In this case, the fall in currency issuer's net worth implied by the default on treasury's debt, $\varkappa_t^C B_{t-1}^C$, would only be temporary and compensated by an equivalent fall in remittances to the private sector, ruling out any possible wealth effect that could exert pressures on the price level, as shown by Eq. (30). Note that this adjustment needs not involve the treasury in any way, if the currency issuer transfers enough resources to the household directly, meaning that it can set

$$\hat{\mathcal{T}}_t^C = \hat{\mathcal{T}}_t^H + \mathcal{T}_t^F \tag{B.63}$$

$$\hat{\mathcal{T}}_t^H = \mathcal{T}_t^H - \varkappa_t^C B_{t-1}^C \tag{B.64}$$

with $\hat{\mathcal{T}}_t^H \geq 0$.

A different result would follow if, confronted with a default on the securities in its balance sheet, the currency issuer would not or could not adjust the path of its remittances to cover the losses.[32] In this case, at the price level consistent with (B.60), while the limiting condition (B.57) for the treasury always holds once we account for the default rates as just discussed, the one for the currency issuer is instead violated. The capital loss would indeed trigger a progressive reduction in currency issuer's net worth that would never be able to be reabsorbed, for an unchanged stream of remittances, implying $\ell_t^C > 0$. Therefore, since the equilibrium condition is still $\ell_t^C = 0$, given $\ell_t^F = 0$, an upward adjustment in the price level through Eq. (B.54) will occur in order to drive ℓ_t^C to zero.

Indeed, the fall in currency issuer's net worth, in the absence of a compensating adjustment in the path of remittances, would imply a positive wealth effect on the private sector, and the equilibrium price level would then increase at the level

$$P_t = \frac{\mathcal{T}_t^C - (1 - \varkappa_t^C)B_{t-1}^C + M_{t-1} + X_{t-1}}{\mathcal{S}_t} = \frac{M_{t-1} + X_{t-1}}{\mathcal{S}_t + \mathcal{T}_t^*} \tag{B.65}$$

where in the second equality we have used the equilibrium level of the endogenous default rate $\varkappa_t^C$. In this case, therefore, the lower is the stream of real taxes $\mathcal{T}_t^*$, the larger is the capital loss on the currency issuer's balance sheet $\varkappa_t^C B_{t-1}^C$, the stronger is the positive wealth effect on the private sector, and ultimately the higher is the equilibrium price level.

It is important at this point to emphasize the subtle but important differences between this case and the fiscal theory of the price level. Note indeed that, while the outcome in (B.65) is reminiscent of the fiscal theory of the price level, the transmission mechanism is profoundly different. Here it works through reductions in the net worth that are implied by the loss in the asset side of the currency issuer's balance sheet, rather than increases in its liability side that would follow the eventual monetization of unbacked treasury's debt. Moreover, even if in this case the equilibrium price level is determined also by tax policy, this is only possible insofar as the currency issuer holds some Treasury debt in its balance sheet, which is not a necessary requirement for the price level to be determinate in a fiat currency system, as we discussed in Section 2.

B.5.2. The currency issuer backs the treasury: the fiscal theory of the price level

Consider now the case in which the currency issuer explicitly backs the treasury, and extends the default-free properties of its own liabilities to the treasury's as well, as in the fiscal theory of the price level. Since the currency issuer guarantees the nominal liabilities of the treasury, there is no longer the need to impose the restriction $\ell_t^F = 0$.

The currency issuer can implement the backing by committing to appropriately adjust either its remittance policy or its balance-sheet policy. These two alternative ways are equivalent in terms of equilibrium price-level implications.

The first way is for the currency issuer to commit to adjust the path of nominal remittances in such a way that the treasury always has enough resources to repay its debt. Let $\overline{\mathcal{T}}_t^C$ denote the PDV of such path of remittances, for a given amount of outstanding debt B_{t-1}^F, any given path of real taxes $\mathcal{T}_t$ and any price level P_t

$$\overline{\mathcal{T}}_t^C \equiv B_{t-1}^F - P_t \mathcal{T}_t. \tag{B.66}$$

The commitment to back the treasury's liabilities in this case imposes a restriction on the nominal remittance policy, which needs to satisfy $\mathcal{T}_t^C = \overline{\mathcal{T}}_t^C$, for all t.

With this commitment, the currency issuer effectively implements $\ell_t^F = 0$ for any price level. The mirror image of the transversality condition, then, is again $\ell_t^C = 0$. Imposing this condition on (B.54) thus determines the price level, for the specific

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[32] Benigno and Nisticò (2020) show that the extent to which a currency issuer can adjust the intertemporal path of its remittances to make a default on its assets irrelevant for the equilibrium price level is related to the seigniorage revenues it can extract from the private sector, i.e. $\mathcal{S}_t$.

remittance policy $\overline{\mathcal{T}}_t^C$, for a given balance-sheet policy setting B_{t-1}^C and for a given "conventional" interest-rate policy determining M_{t-1} and seigniorage revenues $\mathcal{S}_t$:

$$P_t = \frac{\overline{\mathcal{T}}_t^C - B_{t-1}^C + M_{t-1} + X_{t-1}}{\mathcal{S}_t} = \frac{B_{t-1} + M_{t-1} + X_{t-1}}{\mathcal{T}_t + \mathcal{S}_t}, \tag{B.67}$$

where in the second equality we used (B.66) and asset-market clearing.

The second way to implement the backing of treasury's liabilities is for the currency issuer to commit to purchase any amount of treasury's debt or even indefinitely, issuing its own liabilities, which effectively imposes a restriction on the currency issuer's balance-sheet policy. Note a subtle but important difference with the previous scenario: the commitment to appropriately use the balance-sheet policy does not necessarily implement $\ell_t^F = 0$, and we can no longer rule out the case $\ell_t^F > 0$, for some price levels: the treasury can now run Ponzi schemes and accumulate debt even on an explosive path without jeopardizing the default-free properties of its liabilities, because the explicit currency issuer's backing can rule out the equilibrium with endogenous default discussed in the previous section. This backing then takes the form of the currency issuer's commitment to absorb the treasury's Ponzi scheme with one of its own, and accommodate the equilibrium condition $\ell_t = 0$, which now requires $\ell_t^C = -\ell_t^F$ and thus:

$$\ell_t^C = P_t \mathcal{T}_t + \mathcal{T}_t^C - B_{t-1}^F < 0. \tag{B.68}$$

Accordingly, the equilibrium price level in this case is

$$P_t = \frac{\mathcal{T}_t^C - B_{t-1}^C + M_{t-1} + X_{t-1} - \ell_t^C}{\mathcal{S}_t} = \frac{B_{t-1} + M_{t-1} + X_{t-1}}{\mathcal{T}_t + \mathcal{S}_t}, \tag{B.69}$$

where the second equality uses (B.68), and shows that the equilibrium price level is the same as (B.67), implying that the two alternatives to provide the backing are indeed equivalent.

Data availability

No data was used for the research described in the article.

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