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Structural properties in the diffusion of the solar photovoltaic in Italy: individual people/householder vs firms

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Abstract

This paper develops two mathematical models to understand subjects' behavior in response to the urgency of a change and inputs from governments e.g., (subsides) in the context of the diffusion of the solar photovoltaic in Italy. The first model is a Markovian model of interacting particle systems. The second one, instead, is a Mean Field Game model. In both cases, we derive the scaling limit deterministic dynamics, and we compare the latter to the Italian solar photovoltaic data. We identify periods where the first model describes the behavior of domestic data well and a period where the second model captures a particular feature of data corresponding to companies. The comprehensive analysis, integrated with a philosophical inquiry focusing on the conceptual vocabulary and correlative implications, leads to the formulation of hypotheses about the efficacy of different forms of governmental subsidies.

Keywords: Green Energy Transition, Solar Photovoltaic, Individual based modeling, Procrastination, Markovian model, Mean Field Games.

AMS Subject Classification: 60K35, 91A16, 60J27.

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I. INTRODUCTION

Green Energy Transition (GET, henceforth) has, in general, many facets. One is the problem of understanding subjects' reactions to the urgency of a change and inputs from governments (e.g., subsidies). The latter is a fascinating question at the intersection of several cultural fields. This paper investigates this interrogative through the lens of mathematicians and philosophers by examining a specific case study, the diffusion of solar photovoltaic panels in Italy, to identify concrete issues and have some data comparing our proposed models. More precisely, in the present paper, the term "subjects" indicates either individual people and householders or companies. The purpose is to understand which type of structural modeling – to be detailed below – better captures the behavior of these subjects in order to formulate a hypothesis about the efficacy of different forms of governmental subsidies. We stress right away that we are not interested in investigating the ability of models to simulate emerging behaviors, and we postpone this latter problem to future research. Here, the validation of models consists of the latter's capability to fit specific empirical data. In particular, we construct two mathematical models aiming to describe different (structural) features of the considered GET and see the consequences of these choices; we now describe the characteristics of these models from a behavioral and a mathematical perspective.

From a behavioral perspective, the first model comprises subjects with a clear orientation who favor installing solar photovoltaic panels but often do not complete the action; individual people or householders inspire this model. Indeed, several studies have found that many householder do not invest in energy-efficient technologies, even if cost-benefit analyses have consistently shown that such technologies provide a high internal rate of return and are socially beneficial (e.g., [CHCV$^+$09, CH01, WGK15]). Reasons for the just-mentioned energy-efficiency gap comprise investment inefficiencies and behavioral factors, such as imperfect information, uncertainty about the benefits of the investments, and decision-making biases. One of these decision-making biases is the so-called "bounded rationality". Human beings are boundedly rational: they have limited cognitive resources, which naturally constrains optimal decision-making; see, e.g., [FSH15] and reference therein. In particular, empirical evidence from psychology and behavioral economics shows that consumer choices and actions often deviate systematically from the neoclassical economic assumptions of rationality, and certain fundamental and persistent biases in human decision-making regularly produce behavior that these assumptions cannot account for; see, e.g., [WD07, PS13]. A comprehensive review and modeling of all cognitive biases and behavioral anomalies are beyond the scope of the present paper. We defer the reader to [FSH15], Page 1386, for a nice description of all the most powerful and pervasive biases that influence consumer's patterns of energy usage (e.g., status quo bias, loss, and risk aversion, sunk-cost effects, temporal and spatial discounting, and the availability bias). In the present paper, instead, we summarize these cognitive biases in a state named *Deliberating*, which indicates individual people or householders that can articulate a complex process lying behind the assumption of a specific pattern of action, namely the solar panels' installation, but that they have not done the latter action yet. The second model, instead, comprises subjects who make more *rational decisions* based on optimization rules. This model is more inspired by the behavior of companies/firms. Indeed, some existing research works in the literature treat firms as economic agents with a rational maximization behavior; among these, we cite here [JP86], [BP88], [Got21], [SDBFB22], and [CS17].

Despite these differences, both models hinge on the existence of an *interaction* among

subjects. The present paper summarizes this interaction as dependence on specific rules of the mathematical models on the *fraction of subjects that have already installed the solar photovoltaic panels at time* t, out of a population of size N, denoted below by $\frac{N_G(t)}{N}$. How the term $\frac{N_G(t)}{N}$ enters is very different between the two models. More precisely, in the first model, the motivation for introducing $\frac{N_G(t)}{N}$ is a sort of *imitation*, the orientation to participate more confidently in the GET when we observe that more people have done it. In the second model, instead, there are subsidies to participate in the GET with a limited global amount, and when more companies have installed the panel, less accessible to get the subsidy.

From a mathematical point of view, in the first model, we end up with an interacting particle system of Markovian type, in which the randomness corresponds to the uncertainty that an individual installs solar photovoltaic panels due to bounded rationality. We analyze its scaling limit when the number of subjects goes to infinity and get certain deterministic differential equations. Notice that this type of modeling is also confirmed by the extensive literature on opinion dynamics in which the evolution of opinions in society is modeled through Markov chains; see, e.g., [HL75, LNL92, SLST17]. In the second model, we end up with a *game*, precisely a Mean Field Game (MFG, henceforth) since it is an optimization problem where certain elements depend on the global mean quantity $\frac{N_G(t)}{N}$. We defer the interested reader to the nice books of [CD$^+$18] for a presentation of the MFG theory. Here, we get a backward-forward system of differential equations in the scaling limit. MFG models linked to GET can be found in, e.g., [ADT20, DLT22], and references therein.

We compare the two models – precisely, their scaling limits deterministic equations – with a proper fit of the parameters on data of solar photovoltaic taken from [GSE20]. It is essential to observe that the solar photovoltaic cumulative capacity of the Italian time series is the sum of four time series, corresponding to "domestic", "companies", "services", and "agriculture" sectors. We focus on the domestic and the company time series, where our individual-based modeling looks more appropriate (together, they represent around 70% of the total, so their interpretation is a substantial problem). These time series immediately display a subdivision into two periods, separated roughly speaking by the year 2012. Before this year, Italy had a firm subsidy policy, the so-called Feed-in-Tariff (FiT, henceforth), later a weaker one. Both the time series of domestic and companies show, roughly speaking, a relatively exponential solid increase (clearly identified by linear trends on a logarithmic scale) in the period before 2012 and a weaker exponential increase after 2012. In addition, the company time series had a linear increase around 2011, essentially absent in the domestic time series. Therefore, we find it interesting to explain, by mathematical models, the following two issues:

(I) the periods of exponential increase;

(II) the single period of a linear trend, with a large derivative, around 2011, for companies.

The Markovian model easily fits the exponential increase periods. Still, it is not natural for an explanation of the solid linear increase of 2011. On the contrary, the MFG model, totally unsuitable for the exponential increase periods, explains well the linear increase of 2011 observed for companies. This empirical evidence leads to the following conclusion. Companies, around 2011, underwent a *game*, in contrast to all other periods and single people or householders. The planning ability of companies is superior to the domestic one. Hence it is more natural to use control theory arguments in explaining company behavior.

Nevertheless, the cost function depended on $\frac{N_G(t)}{N}$ because the global amount of subsidies was limited. Therefore, the structure of the problem is a game, not simply of control.

It is important to emphasize a structural element of the game (or control) theory, opposite to a Markovian model: the presence of a "time zero", an initial instant of time. In the Markovian model, any time is a new starting time due to memory loss. In a game or control problem, there is a functional to be minimized: at time zero, the agent should plan the future activity to minimize the functional. This planning is not repeated as in a Markovian model that continuously restarts. So, why 2011? Looking at the documents [GSE20], 2011 came after some years of moderate-size FiT subsidies, which prepared the ground, maintained alerted the companies. In 2011, Italy proposed a much more substantial FiT subsidy. Companies were prepared and acted in a game, with the initial time of the subsidy call as the initial time of the game. Householders were not prepared; slowly, they reacted but not with the game's logic.

The solar photovoltaic, as an example of GET has been investigated by many authors in the literature through the lenses of different approaches. We here mention the following works, which do not represent a comprehensive list. (i) The agent-bases approach of, e.g., [ZMCS11], [PSM15] and [PBOL22]. The Agent-based approach offers a framework to explicitly model the adoption decision process of the agent of a heterogeneous social system based on their individual preferences, behavioral rules, and interaction/communication within a social network. (ii) Versions of the popular Bass model ([Bas69]); see, e.g., [dSUL20], which state that the knowledge about such a system highly influences the diffusion of solar photovoltaic systems in Brazil. (iii) Finite element methods to account for spatial heterogeneity; see, e.g., [Kar16], which applies the finite element methods to forecast the diffusion of solar photovoltaic systems in southern Germany. (iv) Survey-based analysis; see, e.g., [CDM21]. Interestingly, some references in the literature analyze whether localized imitation drives the adoption of photovoltaic systems (a phenomenon captured by the ratio $\frac{N_G(t)}{N}$ in our model); see, e.g., [RW16], [BCP$^+$17], [CRBB22], [CG17], [CHB$^+$18], among (many) others. This issue and modeling related to interacting agents can be found in more general literature, like [Hum10]. However, we have not found a comparison between a Markovian modeling and a MFG one to understand certain differences observed in solar photovoltaic data.

The paper is organized as follows. In Section II, we give some preliminary remarks and data which motivate the mathematical models presented in Section III (the Markovian model) and IV (the MFG model). Section V concludes.

II. CONTEXT ELEMENTS

This section is devoted to preliminary remarks and data which motivate the mathematical models of the subsequent sections. It gathers both empirical and theoretical elements, and it constitutes the interdisciplinary seminal collection of data and intuitions from whom the entire rationale of the paper derived its plausibility from the authors' point of view.

We start with a short description of solar photovoltaics in Italy to summarize some of the central subsidies which have driven the growth. Thus, we introduce the main conceptual element of our approach, bounded rationality, which represents the main triggering factor in favor of choosing a Markovian model. Finally, we introduce two data sets that illustrate the problem and that we will use below to test our models. All three elements and the series

of considerations presented below should be considered as the constitutive framework for the subsequent development of the mathematical models and the correlative discussion and justification.

A. A brief overview on the development of the photovoltaic systems in Italy

In this subsection, for the sake of completeness, we briefly review the initiatives implemented by the Italian government encouraged the diffusion of solar photovoltaic systems (PVs, henceforth) from 2005 until today. These initiatives are called "Conto Energia" (CE); each CE guarantees contracts with fixed conditions for 20 years for grid-connected PVs with at least 1kW of peak power. Local electricity providers are required by law to buy the electricity that is generated by PVs. The first CE started in 2005, and it was a net metering plan ("scambio sul posto") designed for small PVs. The plan was meant to favor the direct use of self-produced electricity. Besides payment for each produced kWh of electricity, the consumer received additional rewards for directly consuming the self-generated energy. The CE2 was available to all PVs, but it was designed for larger plants with no or limited direct electricity self-consumption. The electricity produced was sold to the local energy supplier, for which the CE guarantees an additional FiT. It is essential to mention that in each new version of the CE, the FiT was decreased (from 0.36 €/kWh in 2006 to 0.20 €/kWh in 2012). With the introduction of CE4 (2011) direct consumption was rewarded financially. The CE5, unlike the CE4, provided incentives based on the energy fed into the grid and a premium rate for self-consumed energy.

After the end of the fifth CE program, FiT and premium schemes were dropped, and a tax credit program was implemented in 2013. After six years, in 2019, a new incentive decree for photovoltaic systems (RES1) was reintroduced, reserved for systems with a more than 20 kW capacity but not more than 1 MW. Subsidies are paid based on net electricity produced and fed into the grid. The unit incentive varies according to the size of the plant. An incentive is provided for plants that replace asbestos or eternity roofing, and a bonus on self-consumption of energy (provided it is more significant than 40% and the building is on a roof) is issued. For residential customers, a subsidized tax deduction is set at 50% instead of 36%.

However, in May 2020, the Italian government issued the "Revival Decree" (Decree Law 34/2020), introducing a further increase to 110%. Depending on whether the installation is connected to energy-saving measures or not, the 110% tax deduction can be applied to the entire investment (max 2400 €/kW) or only to a part of it (max 1600 €/kW). In addition, the energy not consumed directly is transferred free of charge to the grid. In addition, the Revival Decree provides for the subsidized tax at 110% and also for the implementation of battery energy storage systems up to an amount of 1000 €/kWh. In addition, the Relaunch Decree provides for the subsidized tax at 110% and the implementation of storage systems up to 1000 €/kWh.

Finally, an additional policy measure was introduced by the Ministerial Decree of September 16, 2020, which provides incentives for the configuration of collective self-consumption and renewable energy communities equal to 100 €/MWh and 110 €/MWh, respectively. The incentive lasts for 20 years, does not apply to plants exceeding a power of 200 kW, and has a duration of 60 days from the entry into force of the decree.

B. Bounded rationality and GET

As mentioned in the introduction, a large body of research has shown that even where cost-benefit calculations would suggest more advantageous choices, people persist in displaying seemingly irrational tendencies because they are "boundedly rational". Among these tendencies, we mention the following ones, which do not represent a comprehensive list[1].

(i) Retain the *status quo*, stick to *default setting*, or defer decision-making entirely (*inertia*), especially as the amount or the complexity of information increases; see, e.g., [KKT91, PK08].

(ii) *Satisficing*, which means that people typically process only enough information to reach a satisfactory decision rather then processing all available information to reach an optimal decision, as the latter demands much more time, effort and resources than would ordinarily seem justified by the prospective increase in utility or satisfaction; see, e.g., [SST93].

(iii) Be *loss averse*, which means that when faced with making a decision, people perceive the dis-utility of losing something as far greater than the utility of gaining something (i.e., they feel the pain of losses far more than the pleasure of gains); see, e.g., [ST08].

(iv) People prefer to avoid risk given the prospect of positive outcomes (i.e., gains), but the reverse is true given the prospect of negative outcomes (i.e., losses). In other words, people are more *risk averse* when faced with certain (high probability) gains of uncertain (low probability) losses, but more *risk-seeking* when faced with certain losses or uncertain gains; see, e.g., [Kah79].

(v) Perceive things as less valuable or significant if further away in time (*temporal discounting*) or space (*spatial discounting*), even if such things afford long-term benefits. For instance, many people prefer \$100 in six years rather than \$50 in four years – the discount rate of future utility, in other words, makes the value of \$100 two years later lower than the value of \$100 two years earlier, but not so lower as to be lower than the value of \$50 two years earlier. As many people, however, prefer \$50 today to \$100 two years from now. This means that the choice between the two options is not only influenced by the utility units and their temporal detachment, but also by how close the option with the closer gains is to us. This tendency to be short-sighted and make time-inconsistent judgements often leads to *procrastination*[2], inertia and decreased cooperation in group settings; see, e.g., [JHH+13].

(vi) Attitudes and behaviours of others influences people, which tend to follow norms reflecting what is socially approved (i.e., injunctive norms, which motivate by providing social rewards/punishment) and/or common (i.e., descriptive norms, which motivate by providing suggestions about effective and adaptive behaviour); see, e.g., [Fel84, CT98].

The present paper is, admittedly, not specific in modelling the just-mentioned tendencies – in particular (i)–(v) – and they are somehow summarized in a state named *Deliberating*,

[1] We defer – again – the reader to [FSH15], Page 1386, for a more complete description.

[2] Notice that procrastination is distinguished from two other psychological phenomena: hypocrisy and akrasia.

which indicates individuals or groups that can articulate a complex process lying behind the assumption of a specific (pattern of) action, e.g., the solar panels installation, but that they have not done the latter action yet; cfr., also, Section I. On the other hand, we are a little bit more precise, in capturing attitude (vi) via the ratio $\frac{N_G(t)}{N}$. However, this ratio better captures (vi) in the Markovian model since it is a sort of imitation, the orientation to participate more confidently in the GET when we observe that more people have done it. In the second model, instead, it captures the fact there are subsidies to participate in the GET with a limited global amount, and when more subjects have installed the panel, less accessible to get the subsidy.

C. Examples of time series

To have a graphical intuition, let us start by showing the time series of Italy, our main case study, and also Germany and France (just here for a short comparison), in the period 2006-2020, showing us the global power of photovoltaic installation, independently of any subdivision in categories. Figure 1 displays the three time series in linear scale and also, in the smaller window, in logarithmic scale. The first common element is the regime change around 2012, described above for Italy, in Section II A. The second common element is the (approximate) exponential growth in each of the two periods, visible both in linear scale (but there it could be confused with a power law) and more clearly in logarithmic scale, where the graphs are almost piecewise linear.

We insisted on Figure 1 only for the similarities to stress the fact that a mathematical modeling going in the direction of a two-period exponential growth is just natural. The differences between the three countries are not our concern in this work, except for some remarks in Section V.

The second relevant picture for our analysis is the decomposition, for Italy, of the total power in four categories, following [GSE20] where the data are taken from: Industry, which alone is more than half of Italian power, Domestic, Services and Agriculture. The time series are displayed in Figure 2. In the sequel, as discussed in the Introduction, we concentrate in industry and domestic.

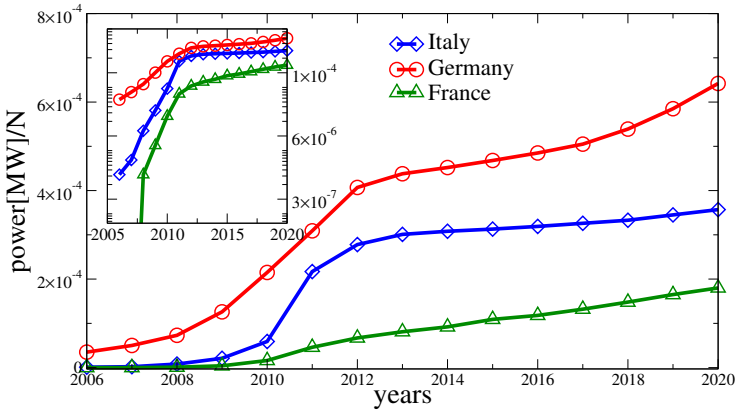

Figure 1. Global photovoltaic power in Italy, Germany and France.

III. THE MARKOVIAN MODEL

In this section, we present the Markovian model describing the individual people and householders' reaction to the urgency of a change and inputs from governments. In particular, this first model is based on N subjects, each one characterized by a state X_t^i at time t taking one of the two *qualitative* values

$$X_t^i \in \{D, G\}\,.$$

The state D means that the subject is *Deliberating*. With this expression we indicate individual people or householders that are able to articulate a complex process lying behind the assumption of a specific (pattern of) action. In the specific case, the action is the installation of the solar photovoltaic panels. Such a process gathers first the most precise assessment of the starting conditions as well as of the appropriateness of the time for acting and of the effective means at the subject's disposal. Last but not least, that process contemplates an overall and at the same time analytical evaluation of the consequences that the adoption of – or the refusal of adopting – that specific (pattern of) action may produce. We have in mind people or householders interested in the installation of solar panel photovoltaic; in principle, they want to participate in it but they have not done so yet. Notice that a complete model of the whole population should include other classes, like those who do not have in mind solar panels or are even oppositors. For instance, one can think to the so-called *Sinus-Milieus*® categories mentioned in [PSM15]. Their dynamics are different, the rates of change of their opinion should be introduced and analyzed. We simplify the model and consider only people involved in the above-described situation, that we call *Deliberating*. The state G means *Green*, namely that the subject has installed the solar photovoltaic panels. The state G is absorbing: a subject may jump from D to G but cannot jump back from G to D.

Let us now describe the dynamics of the states X_t^i, $i = 1, ..., N$. First we give the intuitive idea, then we show the natural mathematical formalism. If at some time t the individual i is in state $X_t^i = G$, it remains G forever, as we have already remarked. If it is in state $X_t^i = D$, it may remain in state D or jump to G, with prescribed probabilities of remaining or jumping. And the story repeats at later time (in case the subject remained in D), without memory, in a Markovian fashion, with the same probabilities of remaining D or jumping to G (this is the mathematical modeling assumption corresponding to the idea of bounded

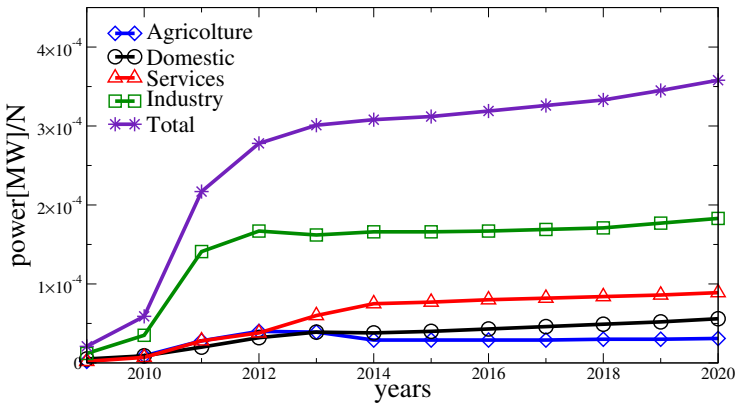

Figure 2. Decomposition of Italy time series in main sectors.

rationality). We could use a discrete-time Markov chain to model this phenomenon, with suitable transition probabilities $p_{D\to D}$ of remaining D and $p_{D\to G}$ jumping to G (obviously $p_{D\to D}=1-p_{D\to G}$, hence it is sufficient to specify $p_{D\to G}$). However, since we look for mean field equations satisfied in the limit when $N\to\infty$, the Mathematics is cleaner if we use continuous-time Markov models, since they have better analytical rules like the so-called Dynkin formula (see Subsection III A). Therefore we introduce *rates of transition*, instead of transition probabilities. A transition rate $\lambda_{D\to G}$ is (the limit as $t\to 0$ of) the probability of transition in time t divided by t. We have to specify the transition rate $\lambda_{D\to G}$ instead of the transition probability $p_{D\to G}$. The mathematical formalism needs a few more elements. First, since individuals will be interconnected, we have to consider the Markov process of the full family of individuals, $X_t=\left(X_t^1,...,X_t^N\right)$. When we write X in the sequel we mean a full family of states. Second, the transition rate $\lambda_{D\to G}$ of a single individual is a function of the state X of all the family of individuals, hence we prefer to denote it by $\lambda_N(X,t)$, to emphasize the dependence on the number N of individuals, the state X of the full family and also the time t, because we shall include in the model the possibility that the transition rules change in time, for instance as a consequence of new governmental rules about solar panels. We choose a very simple but specific transition rate formula

$$\lambda_N(X,t)=a(t)\frac{N_G(t)}{N}$$

where

$$N_G(t)=\sum_{j=1}^{N}1_{\left\{X_t^j=G\right\}}$$

is the number of green subjects at time t. Except for the factor $a(t)$, the transition rate $\lambda_N(X,t)$ is given by the percentage of green subjects, following the intuition that more people have become green, larger is the probability for a new individual to choose to become green too. The factor $a(t)$ modulates the rate in order to give importance to external factors like subsidies. With these introductory notions in mind, let us define the so-called infinitesimal generator of the continuous-time Markov process:

$$\mathcal{L}_tF(X)=\sum_{i=1}^{N}1_{\{X^i=D\}}\lambda_N(X,t)\left(F\left(X^{i\to G}\right)-F(\eta)\right)$$
$$\lambda_N(X,t)=a(t)\frac{N_G(X)}{N}$$

for all observables $F:\mathcal{S}\to\mathbb{R}$, $\mathcal{S}=\{D,G\}^N$. The intuitive meaning of the generator is that each subject i which is Deliberating (the sum is restricted to Deliberating ones by the factor $1_{\{X^i=D\}}$) has a rate $\lambda_N(X,t)$ to become Green. The rate $\lambda_N(X,t)$ depends on the fraction of Greens $\frac{N_G(X)}{N}$, with a time dependent coefficient $a(t)$. Heuristically, we could write in the limit as $\Delta t\to 0$:

$$\text{Prob}\left(D\to G|X,t\right)\sim\lambda_N(X,t)\cdot\Delta t,$$

where Prob denotes the probability.

The logic of this model is manifold. The Markovianity, without any decision/optimization aspect (opposite to the second model below) reflects, in the simplest possible modeling way, the idea of bounded rationality; for the sake of presentation only, in what follows, we will

speak about procrastination; see Subsection II B, item (v). Each subject, at every time instant, has a rate to install, instead of procrastinating. The dependence of $\lambda_N(X,t)$ from $\frac{N_G(X)}{N}$ corresponds either to the idea that we have a tendency to imitate the others, or to the more impersonal fact that, when a larger number of Greens exist, the Market is more developed - maybe lower prices, better distribution network, more information and experience - and the probability to install increases. The time dependent coefficient $a(t)$ contains a modulation related to subsidies: our tendency to imitate is depleted if the cost is high, encouraged if the cost is low.

Remark 1. *Clearly, at the abstract level, we may include other elements in the model, like personal features of subject which may influence the decision to install panels, an additive term of the form $\lambda_N(X,t) = a(t)\frac{N_G(X)}{N} + b(t)$ in the rate, a stochastic process representing external inputs subject to a dynamics instead of incorporating them just in the time-dependence of $a(t)$ and $b(t)$. We chose above the simplest model since it is sufficient to fit data with success.*

A. Dynkin equations

Let

$$p_G^N(t) := \frac{N_G(X(t))}{N}$$

be the percentage of green subjects. The advantage of the continuous-time model is the possibility to write an identity, similar to an integral equation, for the quantity $p_G^N(t)$. The identity reads

$$p_G^N(t) = p_G^N(0) + \int_0^t a(s)\left(1 - p_G^N(s)\right) p_G^N(s)\, ds + M_t/N. \tag{1}$$

It states that the change $p_G^N(t) - p_G^N(0)$, in a time interval $[0,t]$, of the percentage of green subjects is given by time-integral of a logistic term, $\left(1 - p_G^N(s)\right) p_G^N(s)$, modulated by the factor $a(s)$, plus a remainder M_t/N, where M_t is a certain stochastic process (it is a martingale); it is not easy to write explicitly the form of M_t but its average is known to be equal to zero and its variance can be estimated, proving in particular that the variance of M_t/N goes to zero as $N \to \infty$. When we consider the observable

$$F(X) = N_G(X)$$

we have, for every X and i,

$$1_{\{X^i=D\}}\left(F\left(X^{i\to G}\right) - F(X)\right) = 1_{\{X^i=D\}}$$

and therefore

$$\begin{aligned}\mathcal{L}_t F(X) &= \sum_{i=1}^N 1_{\{X^i=D\}}\lambda_N(X,t) = (N - N_G(X))\,\lambda(X,t)\\ &= a(t)\,\frac{(N - N_G(X))\,N_G(X)}{N}.\end{aligned}$$

By Itô-Dynkin equation (see [KL98], Appendix A), we deduce

$$N_G\left(X\left(t\right)\right) = N_G\left(X\left(0\right)\right) + \int_0^t a\left(s\right) \frac{\left(N - N_G\left(X\left(s\right)\right)\right) N_G\left(X\left(s\right)\right)}{N} ds + M_t$$

where M_t is a martingale, of which we know certain features by the second Itô-Dynkin equation (see [KL98], Appendix A). We have thus proved Equation (1).

B. Limit model

In the limit as $N \to \infty$, one can prove that M_t/N is infinitesimal in mean square (also its supremum in time over finite intervals). Assuming that $p_G^N\left(0\right)$ converges in probability to a number $p_G^0 \in [0,1]$, with elementary arguments one can prove that $p_G^N\left(t\right)$ converges uniformly in time on compact intervals, in probability, to a deterministic function $p_G\left(t\right)$, solution of the equation

$$p_G\left(t\right) = p_G^0 + \int_0^t a\left(s\right)\left(1 - p_G\left(s\right)\right) p_G\left(s\right) ds$$

namely

$$\begin{aligned} \frac{dp_G\left(t\right)}{dt} &= a\left(t\right)\left(1 - p_G\left(t\right)\right) p_G\left(t\right) \\ p_G\left(0\right) &= p_G^0. \end{aligned}$$

C. Simulations

As described in the Introduction, we apply the Markovian model to the Domestic data, since it is natural to conjecture that procrastination, which is at the foundation of the choice to use a Markov chain, is proper of individuals like single people or householders (instead of companies, as discussed below in the section on MFGs).

The Domestic time series requires a specification of the way we have obtained the data, from [GSE20]. In the period 2010-2020 we have explicit values of the Domestic data; the total national data is divided in the four categories described in a previous section. In 2009 there is a different subdivision in categories from which we deduce that Domestic is approximately given by a certain percentage p_{2009} of the total; we have used this datum in the plot. Concerning 2006-2008 we do not even have the percentage.

We have used the mean field model of the previous section, namely the ordinary differential equation for the quantity $p_G\left(t\right)$, with the following values:

$$\begin{aligned} a\left(t\right) &= 0.627 \text{ for } t \leq 2012.3 \\ a\left(t\right) &= 0.0637 \text{ for } t \geq 2014 \end{aligned}$$

and a constant connection in between, in absence of a better understanding of the intermediate period between the two different subsidy regimes. The result is shown in Figure 3.

The values of $a(t)$ have been fitted, by an exponential fit with minimum mean square error, considering the two separate time periods; then they have been used in the differential equation model, with suitable initial condition.

Since the fit in the period 2009-2012 may look arbitrary due to the smallness of the time series (only 4 values), we extrapolated values of the Domestic time series by using the proportion coefficient p_{2009} also in the years 2006-2008. The fit of the first period is surprisingly the same (first three digits) hence it looks particularly stable. The result is shown in Figure 4.

If we rely on this stable fit, we may guess that the increase of Domestic in the next few years would lead to an excellent performance, as the green extrapolation line of Figure 4 shows. Unfortunately, it was decided to change the structure of the subsidies.

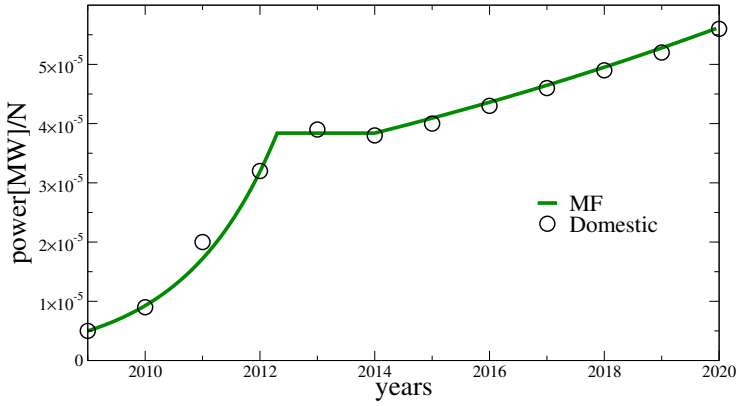

Figure 3. Mean field model (MF) is the continuous curve in green, over the data (small circles).

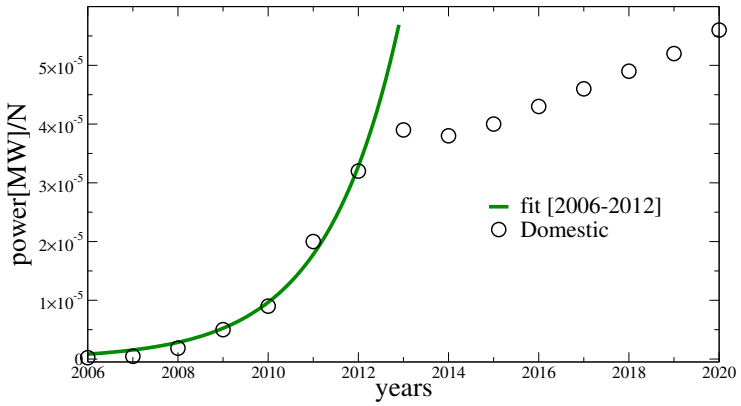

Figure 4. The fit of the first period, based on extrapolated initial data, and its potential continuation.

IV. MFG APPROACH

A. The control problem

Before we introduce the game, it is convenient to describe a simpler model, a control problem for a single agent, without interaction between different agents. Then the game will be a variant, easy to understand. Assume an agent is in state $X(t) = D$. In the previous sections we prescribed the transition rate to state G. Here we consider this rate *as a control function*: the agent may choose the rate, in order to accelerate or slow down the transition process, depending on the advantages that different strategies produce, according to a certain payoff. Thus call $\lambda(t)$ the transition rate, at time t, from state D to state G; write λ to denote the full function $\lambda(t)$, $t \in [0,T]$. The cost $J(\lambda)$ of the choice of a certain λ is assumed to be

$$J(\lambda) = \mathbb{E}\int_0^T \left(h(\lambda(t)) + p_G(t)\, 1_{\{X(t)=D\}}\right) dt.$$

where $h(\cdot) \geq 0$ is an increasing function with $h(0) = 0$, and $p_G(\cdot) \geq 0$ is also an increasing function. The meaning is: choosing a transition rate $\lambda(t) > 0$ has a cost $h(\lambda(t))$ (this produces a tendency to procrastinate); $p_G(t)$ corresponds to the proportion of Green, so that, being D has higher cost if $p_G(t)$ is bigger. We could interpret the term $\mathbb{E}\int_0^T p_G(t)\, 1_{\{X(t)=D\}} dt$ as an imitation tendency. But in fact it has also another possible meaning. Assume there is a limited amount of resources of a certain subsidy. If $p_G(t)$ is large, namely several subjects have already taken advantage of the subsidy, it is more difficult to get it, hence an higher cost. The single-agent state $X(t)$ is now a continuous-time Makov process with state space $\{D, G\}$ and infinitesimal generator $\mathcal{L}_t^\lambda$, depending on the rate λ, given by

$$\mathcal{L}_t^\lambda F(X) = 1_{\{X=D\}}\lambda(t)\left(F(G) - F(X)\right)$$

which means that if $X = G$, it remains G, if $X = D$ the rate of transition to G is $\lambda(t)$. General control theory states that the solution of a certain Hamilton-Jacobi-Belmann (HJB) equation should be found, for the so-called value function $v(t, X)$, where $t \in [0,T]$ and $X \in \{D, G\}$. Being the state discrete with two values, the value function is given by two function $v_D(t)$ e $v_G(t)$, that are the costs corresponding to the initial conditions (t, D) e (t, G) respectively. The HJB equation is, in general, of the form

$$\min_{\lambda \geq 0}\left\{\partial_t v(t,X) + \left(\mathcal{L}^\lambda v(t)\right)(X) + h(\lambda) + p_G(t)\, 1_{\{X=D\}}\right\} = 0$$

and thus, in the particular case of the function $v_D(t)$,

$$\min_{\lambda \geq 0}\left\{\partial_t v_D(t) + \mathcal{L}^\lambda v_D(t) + h(\lambda) + p_G(t)\right\} = 0.$$

We do not need to study also the equation for $v_G(t)$ because, starting from G, it is sufficient to use the null control to get the minimum value $J(0) = 0$, hence $v_G(t) = 0$. Hence only the function $v_D(t)$ counts. Since

$$\mathcal{L}^\lambda v(t,X) = 1_{\{X=D\}}\lambda\left(v(t,G) - v(t,X)\right)$$

we have

$$\mathcal{L}^\lambda v_D(t) = -\lambda v_C(t).$$

Hence the HJB equation for $v_C(t)$ is

$$\partial_t v_D(t) + p_G(t) + \min_{\lambda \geq 0} \{h(\lambda) - \lambda v_D(t)\} = 0.$$

Consider the simple choice $h(\lambda) = \sigma^2 \lambda^2$. The minimum of the function

$$\lambda \mapsto \sigma^2 \lambda^2 - \lambda v_D(t) = \lambda \left(\sigma^2 \lambda - v_D(t)\right)$$

is the solution of $2\sigma^2 \lambda = v_D(t)$, namely (call it $\lambda_*(t)$)

$$\lambda_*(t) = \frac{1}{2\sigma^2} v_D(t)$$

which will be the optimal control. The minimum value is

$$-\frac{1}{4\sigma^2} v_D^2(t).$$

Hence the HJB equation is

$$\partial_t v_D(t) + p_G(t) - \frac{1}{4\sigma^2} v_D^2(t) = 0.$$

Theorem 2. *Let $v_D(t)$ be the solution on $[0,T]$ of the backward Cauchy problem*

$$\begin{aligned} \partial_t v_D(t) + p_G(t) - \frac{1}{4\sigma^2} v_D^2(t) &= 0 \\ v_D(T) &= 0. \end{aligned}$$

then

$$\lambda_*(t) = \frac{1}{2\sigma^2} v_D(t)$$

is the optimal transition rate from D to G.

B. MFG

Assume now we have N agent, with states $X^1, ..., X^N \in \{D, G\}$. Again we consider a *controlled* Markov chain, but now for the full state $X = (X^1, ..., X^N)$, with generator

$$\mathcal{L}_t F(X) = \sum_{i=1}^{N} 1_{\{X^i = D\}} \lambda^i(t) \left(F(G) - F(X)\right)$$

where the rate $\lambda^i(t)$ is the control of subject i. The individual payoff of subject i, depending on other agents, is:

$$J_i(\lambda) = \mathbb{E} \int_0^T \left(\underbrace{\sigma^2 \lambda^i(t)^2}_{\text{transition cost}} + \underbrace{\frac{N_G(X(t))}{N} 1_{\{X_t^i = D\}}}_{\text{late arrival cost}} \right) dt$$

where $\lambda = \left(\lambda^1, ..., \lambda^N\right)$. Let us make a few remarks.

- In the Markov model (without control), with Green-transition rate

$$\lambda_N(X,t) = a(t)\frac{N_G(X)}{N}$$

subjects react time-by-time, based on *observed environment* $\frac{N_G}{N}$.

- In the game model with payoff, agents are asked to make a decision at the *same initial time* (here $t=0$), depending on an *unknown future environment* $\frac{N_G}{N}$.

- In the first case, the consequence is a *slow (but exponential) adaptation* to the environment.

- Opposite, in the second case, the consequence is a *fast reaction* to a potential environment.

Let us apply, in the limit as $N \to \infty$, the mean field closure of the previous problem. It means that each agent, say X^1, wants to solve the same problem of the previous section, but where $p_G(t)$ is replaced by the probability $\widetilde{p}_G(t)$ that subject X^1 is in state G at time t, when the optimal strategy is chosen. Hence the dynamic of subject X^1 is given by the infinitesimal generator

$$\widetilde{\mathcal{L}}_t F(X) = 1_{\{X=D\}}\frac{1}{2\sigma^2}\widetilde{v}_D(t)\left(F(G) - F(X)\right)$$

where the function $\widetilde{v}_D(t)$ solves the backward HJB equation associated to $\widetilde{p}_G(t)$.

We have $\widetilde{p}_G(t) = \mathbb{E}\left[1_{\{X(t)=G\}}\right]$. Dynkin formula for the observable $F(X) = 1_{\{X=G\}}$ is

$$1_{\{X(t)=G\}} = 1_{\{X(0)=G\}} + \int_0^t \widetilde{\mathcal{L}}_s F(X(s))\,ds + M_t$$

where M_t is a martingale. We have

$$\begin{aligned}\widetilde{\mathcal{L}}_t F(X) &= 1_{\{X=D\}}\frac{1}{2\sigma^2}\widetilde{v}_D(t)\left(1_{\{X=G\}}(G) - 1_{\{X=G\}}(X)\right)\\ &= 1_{\{X=D\}}\frac{1}{2\sigma^2}\widetilde{v}_D(t)\end{aligned}$$

hence

$$1_{\{X(t)=G\}} = 1_{\{X(0)=G\}} + \int_0^t 1_{\{X(s)=D\}}\frac{1}{2\sigma^2}\widetilde{v}_D(s)\,ds + M_t$$

which implies

$$\mathbb{E}\left[1_{\{X(t)=G\}}\right] = \mathbb{E}\left[1_{\{X(0)=G\}}\right] + \int_0^t \mathbb{E}\left[1_{\{X(s)=D\}}\right]\frac{1}{2\sigma^2}\widetilde{v}_D(s)\,ds$$

namely

$$\widetilde{p}_G(t) = \widetilde{p}_G(0) + \int_0^t \left(1-\widetilde{p}_G(s)\right)\frac{1}{2\sigma^2}\widetilde{v}_D(s)\,ds.$$

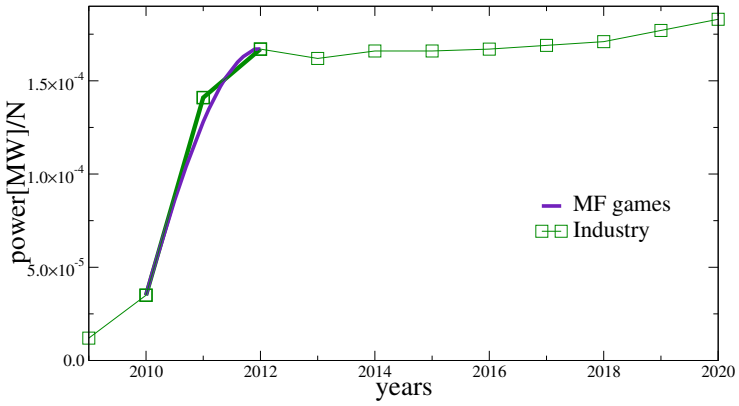

Figure 5. Sudden increase in 2011 of the Industrial compartment.

Theorem 3. *The mean-field-game system is*

$$\begin{aligned} \partial_t \widetilde{v}_D(t) + \widetilde{p}_G(t) - \frac{1}{4\sigma^2}\widetilde{v}_D^2(t) &= 0 \\ \partial_t \widetilde{p}_G(t) &= (1 - \widetilde{p}_G(t))\frac{1}{2\sigma^2}\widetilde{v}_D(t) \\ \widetilde{v}_D(T) = 0, \qquad \widetilde{p}_G(0) &= p_0 \end{aligned}$$

with $p_0 \in [0,1]$ *given by the initial proportion of subjects in state* G.

C. Simulations

Simulating the forward-backward mean field system above is less straightforward than a usual initial value problem. We have used a shooting method, namely looking for the initial condition of the variable $\widetilde{v}_D(t)$ which, along with the initial condition p_0, solving forward the system, we get $\widetilde{v}_D(T) = 0$.

The result, illustrated in Figure 5 is that the curve $\widetilde{p}_G(t)$ is concave. Namely, the result is completely different from the exponential growth of the Markovian model. Tuning the parameters, it may fit the data, as shown in the figure.

Let us stress the intuitive reason for the concavity, opposite to exponential growth: it stands in the intimate structure of a game, opposite to a Markovian mechanism. In a game, agents have a tendency to act in advance, to anticipate the move of the other players. The reason is the limited amount of subsidies, which does not allow to get them if they are exhausted by other players before. This is the opposite of procrastination, in a sense.

V. CONCLUSIONS

The conclusion we deduce from this analysis, keeping in mind the efficacy of the 2011 governmental action, is that a similar structure should be applied to the domestic compartment in the future, to trigger a stronger increase of domestic photovoltaic - which on the contrary is under stagnation. In order to push householders to act as a game, it is necessary to increase their planning ability and to prepare the outcome of the event, so that a "time

zero" will exist, with people conscious of it. Through advertisements and instruction of administrators of houses, maybe it is possible to do this. A second and more comprehensive avenue that might be explored revolves around the possibility of increasing the capability to "deliberate", meaning, strenghting the capability of the consumer in rationally evaluating the different options at stake. Educational paths aimed at this goal might be made available by public authorities in different forms as a kind of (public and free-of-charge) cultural and educational activity (for adults) or game activity (for citizens of schooling age).

However, to show that one should avoid easy prescriptions, let us compare the time series of Italy and Germany, see Figure 1 in Section II C. Both clearly show the two periods (the same) of exponential increase, with different rates. Only Italy has the very strong increase around 2011. We want to remark that in spite of it, in the long run what maybe counts more is the rate of exponential increase. Italy should increase the value of the imitation coefficient $a(t)$. Whether it is more important to act on it or trigger a shock like the 2011 one, it is an issue that deserve further research.

DECLARATIONS

Conflict of interest. The authors declare that they have no conflict of interest.

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