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COVERT LEARNING AND DISCLOSURE*

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Abstract

I study a model of information acquisition and transmission in which the sender's ability to misreport her findings is limited. The sender learns covertly, so a key observation is that in equilibrium she must be deterred from undetectably *worsening the meaning* of the messages she sends. This force substantially disciplines equilibrium beliefs and behavior: the receiver is maximally skeptical of what the sender claims and learns all discovered information. I exploit these equilibrium properties to characterize the sense in which the sender benefits from her claims being *more verifiable*, showing that this is akin to increasing her commitment power. Finally, I identify sender- and receiver-optimal falsification environments.

Keywords: Verifiable disclosure, information acquisition, communication, partial verifiability, Bayesian persuasion, limited commitment.

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1 Introduction

To inform their choices, individuals typically rely on the advice of experts with access to superior information. In the presence of conflicting interests, the latter will attempt to influence the former's decisions in at least two ways: by *acquiring information selectively* and—as far as is feasible—by *misrepresenting their findings*. This paper studies the interplay between these two modes of influence, exploring how the expert's ability to misrepresent her discoveries influences what information she acquires and, ultimately, transmits to the receiver.

These two persuasion channels are central to several important environments. CEOs allocate resources within firms using information gathered and reported by empire-building managers, who favor investment in their own divisions in spite of the overall profitability for the firm. Prospective buyers base their purchasing decisions on reports by market analysts and advisors; in the presence of commissions (or other incentive schemes), such experts will obtain and report information in ways that maximize the chances of a sale. Voters evaluate politicians by reading investigative reports and analyses by partisan journalists, who are selective in their coverage and partial in their portrayal of events.

These settings share three key features, which are also the essential components of the model I study. First, there is a substantial conflict of interest between the 'sender' and the 'receiver'. Because of this conflict, the sender will use her ability to control information production and transmission to influence the receiver's decision in her favor, making the latter—in turn—wary of the advice.

Second, the sender can gather information privately, without substantial scrutiny by the receiver. In applications, this is the case because information production is often formally delegated (as in many manager-CEO relationships), difficult to monitor (as in the advisor-buyer case), or secret because of social norms (for example, journalistic sources are protected by anonymity).

Third, the sender faces constraints on the extent to which she can misrepresent her findings. This may be because some of her claims are *verifiable* by the receiver or by a trusted third party. She might also have reputational concerns, fearing that reports excessively different from what she finds may be ultimately discovered, damaging her reputation and future credibility. Additionally, she might be bound by disclosure regulation or contractual obligations.[1]

The model I develop incorporates these three ingredients by marrying the verifiable-disclosure literature's flexible approach to constrained communication with Bayesian persuasion's belief-based methods. My set-up can therefore be viewed from two different angles. It is a sender-receiver game with (partially) verifiable information, in which the sender's type distribution is determined by her own equilibrium covert information acquisition, rather than fixed and common knowledge. It is also a model of Bayesian persuasion with limited commitment, in which the sender can acquire information freely and covertly, but can only commit to sending a message from a fixed set for each realization of her private signal.

Combining these two approaches proves fruitful: I obtain an unraveling result across all possible misreporting constraints in a large class, which highlights how these constraints shape *what* information the sender chooses to acquire. More specifically, since the sender learns covertly, she must be deterred in equilibrium from undetectably *worsening the meaning* of the messages she is expected to send. This serves as a key disciplining force, ensuring that the sender seeks only information that she can then credibly transmit, given the misreporting constraints.

This insight, in turn, allows me to address comparative statics questions, delineating the connection between *more verifiability* of the sender's reporting and *more commitment power* in Bayesian persuasion problems with limited sender commitment. It also delivers a characterization of optimal verifiability environments: for the sender, this is a characterization of the constraints that allow her to obtain the Bayesian persuasion outcome; for

[1] For example, reputational concerns are the main (albeit imperfect) reason behind the credibility of credit-rating agencies (see Mathis, McAndrews, and Rochet (2009) for a discussion). Disclosure regulation plays a crucial role in financial reporting (see Leuz and Wysocki (2016) for a survey).

the receiver, they describe constraints that incentivize the sender to acquire (and transmit) full information in equilibrium.

1.1 Example and overview of the results

Two players, a sender and a receiver, are initially uninformed about a binary state, taking values in $\{0, 1\}$. They share a common prior belief $\overline{p} = 1/4$ that the state is 1. The sender can covertly and freely acquire information about the state by privately choosing a signal—that is: a mean-1/4 distribution of posterior beliefs—and observing its realization.

An important primitive of the model is the *verifiability structure*, which describes what the sender can say to the receiver following any given outcome of her information acquisition. Formally, if the sender discovers that the state is 1 with probability $s \in [0, 1]$, she can send a message from the set $M(s)$ to the receiver.[2]

For the purposes of this example, suppose that there are only two possible messages that the sender can use: call them $m_\emptyset$ and m_G. Assume that $m_\emptyset \in M(s)$ for all $s \in [0, 1]$, so that it can be interpreted as a 'silent' message, meaning that the sender can always choose to say nothing to the receiver. Assume instead that $m_G \in M(s)$ iff $s \in [0.5, 1]$, meaning that m_G proves to the receiver that the state has at least a 50% chance of being 1.

Assume that the sender is biased in favor of state 1 in the sense that she (weakly) benefits from inducing higher beliefs about the state in the receiver. Specifically, denote by $v(p)$ the payoff the sender obtains from inducing belief $p \in [0, 1]$ in the receiver and suppose that $v(p) = 0$ for $p < 0.4$, $v(p) = 1$ for $p \in [0.4, 0.8)$ and $v(p) = 3$ for $p \geq 0.8$.

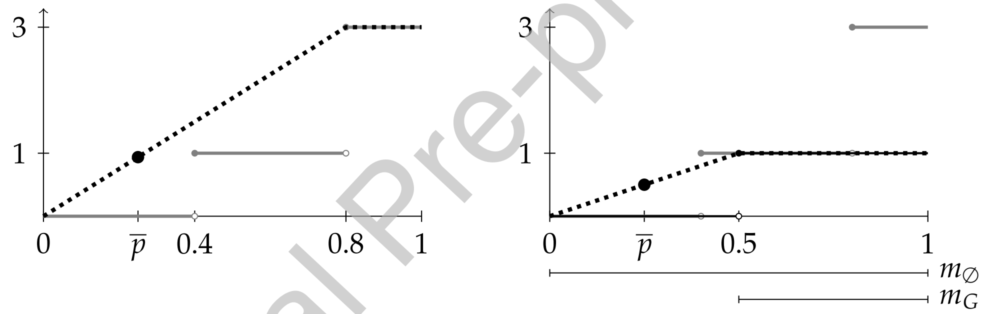
The payoff v is a reduced-form representation of the downstream interaction between the sender and the receiver.[3] The latter's behavior is not

[2]The fact that the sender's posterior belief uniquely determines what she can communicate is an assumption of the model, discussed in detail in Section 2.1.

[3]In this example, a standard microfoundation involves the receiver facing a decision problem under uncertainty with three actions: a_L, a_M and a_H. Her payoff is such that she optimally chooses a_L at beliefs strictly below 0.4, a_M at beliefs in $[0.4, 0.8)$ and a_H at beliefs at or above 0.8. The sender has state-independent preferences over the receiver's actions, obtaining payoffs of 0, 1 and 3 from actions a_L, a_M and a_H, respectively.

modeled explicitly: the receiver is an active player only insofar as we will require her posterior belief after every message to be always consistent with the verifiability structure (so, after observing m_G, she must believe the state has at least a 50% chance of being equal to 1) and formed using Bayes' rule on the equilibrium path.

Equilibrium unraveling and skepticism. Suppose first that the sender chooses to acquire information about the state through a signal 'splitting' her prior into posteriors 0 and 0.8. Note that this would be the equilibrium signal if the sender's information acquisition were public—as in Kamenica and Gentzkow (2011)—attaining an expected payoff of $(\operatorname{cav} v)(1/4)$, as depicted in Figure 1a.



(a) Bayesian persuasion. (b) Covert learning and disclosure.

Figure 1: On the left: the sender payoff function v (solid gray) and its concave envelope (thick, dotted) as a function of the receiver's belief. On the right: verifiability structure M (below the horizontal axis), the sender's skepticism-adjusted value function (solid black)—overlaid over v (solid gray)—and its concave envelope (thick, dotted). The large black dot corresponds to the sender's equilibrium expected payoff in each case.

This cannot be an equilibrium signal in my model. To see why, suppose indeed the sender chose such a signal in equilibrium, sending message $m_\varnothing$ when her type is 0 (i.e., when she privately discovers that the state is 0 for sure) and message m_G when her type is 0.8 (i.e., when the signal realization is $s = 0.8$). In this candidate equilibrium, the sender would obtain a payoff of 0 following message $m_\varnothing$, and a payoff of 3 following message m_G.

However, the sender could profitably deviate to a signal splitting the prior between 0 and $0.8 - \delta$ (for $\delta < 0.3$); in the former case sending message $m_\emptyset$ and in the latter sending message m_G. This deviation is not detectable by the receiver (since she does not observe the sender's choice of signal) and it increases the probability of sending m_G while decreasing the probability of sending $m_\emptyset$, thus strictly increasing the sender's expected payoff. This kind of deviation—which *worsens the meaning* of message m_G—plays a significant role in shaping equilibrium behavior.

Indeed, only a signal splitting the prior into posteriors 0 and 0.5 is not prone to such a deviation. These are the lowest sender types consistent with messages $m_\emptyset$ and m_G, respectively, and therefore these messages' meaning cannot be worsened any further. This signal is part of what I call an 'unraveling' equilibrium, in which sender type 0 sends message $m_\emptyset$, and sender type 0.5 sends m_G. Anticipating this behavior by the sender, the receiver is maximally skeptical, assigning to each message the lowest sender belief that is consistent with it: belief 0 following message $m_\emptyset$ and belief 0.5 following message m_G. These correspond precisely to the sender's beliefs, meaning that all acquired information is revealed to the receiver. This equilibrium is depicted in Figure 1b.

The three features of the unraveling equilibrium just described (on-path sender types being 'lowest-consistent' with the messages they send, the receiver's maximal skepticism and full revelation of acquired information) generalize beyond the specific payoff function v and verifiability structure M of the example. The first result of the paper (Theorem 1) shows that, when v is non-decreasing, all equilibria involving some variation in the receiver's belief which is payoff-relevant for the sender are 'unraveling' in the sense that they satisfy these three properties.

A direct consequence is that the sender's equilibrium expected payoff is the same across all unraveling equilibria and can be obtained as the concave envelope of a skepticism-adjusted value of posteriors. This function is depicted in Figure 1b in the context of the example, and this observation is generalized in Corollary 1.

Uniqueness of unraveling. In the example, the unraveling equilibrium just presented is, in fact, the only equilibrium. Theorem 2 provides a mild sufficient condition on the primitives of the environment ensuring that all equilibria are unraveling and that therefore the sender's equilibrium expected payoff is unique.

The condition is that the sender must be able to 'prove news better than the prior' (PNBP). This property holds if there exists a message that, even when interpreted most skeptically by the receiver, leads to a payoff for the sender strictly above what she would obtain at the prior. In the example, PNBP holds because message m_G—even if followed by a receiver belief of 0.5—leads to a payoff of 1 for the sender, which is strictly higher than her payoff of 0 at the prior.

At a high level, PNBP ensures that all equilibria are unraveling because a message guaranteeing the sender a payoff strictly above the prior always opens the door to a deviation that worsens the meaning of some message, unless the receiver is maximally skeptical on-path.

Theorem 2 also shows that, in contrast, if the sender cannot prove news better than the prior, there is always a sender-preferred equilibrium in which she acquires and transmits no information. This demonstrates how her bias makes information control worthless, absent some minimal amount of verifiability, as captured by PNBP.

More verifiability and more commitment power. A natural question to ask is whether (and in which sense) the sender benefits from being able to 'prove more'. For concreteness, suppose that in the example the sender has access to a new message, call it m_E, which she can only send if she discovers the *excellent* news that the state has at least a 90% chance of being high (i.e., $m_E \in M(s)$ iff $s \in [0.9, 1]$).

It can be easily verified that in equilibrium, the sender will now choose to split the prior between 0 and 0.9, sending messages $m_\emptyset$ and m_E, respectively. This leads to an increase in the sender's equilibrium expected payoff compared to the case in which only $m_\emptyset$ and m_G were available, as depicted in Figure 2.

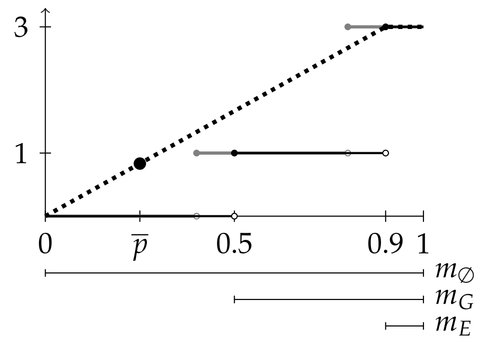


Figure 2: Adding message m_E makes the sender better off.

What is the appropriate notion of 'more verifiability' which ensures that the sender's payoff improves in equilibrium more generally? Theorem 3 provides a pre-order on the set of verifiability structures that characterizes those shifts that are beneficial for the sender when she can prove news better than the prior. The necessary and sufficient condition is easy to describe: the set of sender types that are able to separate from all lower types with a single message, regardless of the receiver's belief, must grow in the sense of set inclusion. This guarantees that any equilibrium distribution of receiver beliefs inducible in the lower-ranked verifiability structure is also inducible in the higher-ranked one.

If the sender cannot prove news better than the prior—meaning that unraveling equilibria need not exist—I show instead that shifts towards more verifiability may in fact hurt the sender ex ante (Section 4.3). This is caused by the lack of observability of the sender's information acquisition strategy. In a more constrained environment, the equilibrium meaning of some messages may worsen with more verifiability: if the sender is known to be able to prove good news, any claim of ignorance she makes will no longer be taken at face value by the receiver.

The comparative statics result is useful for two reasons. First, the pre-order makes it possible to compare a rich variety of common verifiability environments. For example, according to the pre-order, cheap talk is minimal, while full-verifiability structures are maximal. Interval-partition verifiability structures (Example 4)—capturing settings with 'grades' or 'ratings' that certifiably pool together sets of types—are ranked if and only if one is a refinement of the other (Example 7 and Remark 2). Adding a new message as in the example—thereby allowing the sender to credibly communicate her acquired information more finely—always leads to a higher-ranked structure (Example 8 and Remark 2).

Second, the result can be viewed as providing a tool for comparing Bayesian persuasion environments with different forms of sender commitment. In this context, it characterizes the appropriate notion of 'increasing commitment power' which is valuable for the sender.

Optimal verifiability. The comparative statics result also leads directly to a characterization of sender-optimal verifiability structures (Proposition 1), which allow the sender to attain the full-commitment value in equilibrium. The result therefore highlights how the full commitment assumption in the canonical Bayesian persuasion setting can be relaxed, without affecting the sender's equilibrium payoff. The necessary and sufficient condition states that all sender types must be able to separate from all lower ones by means of a single message, no matter what the receiver believes. Given the monotone bias of the sender, this guarantees that separation is possible for all types in equilibrium, allowing the sender to replicate the Bayesian persuasion outcome.

Finally, I characterize structures that lead to full information provision in equilibrium (Proposition 2). These are receiver-optimal, provided that the receiver values information about the state. They take a simple form: only the very best news (that is: claiming that the state is equal to 1 with certainty) is considered credible, while all inconclusive news is fully unverifiable. Every structure of this kind incentivizes perfect information acquisition by the sender, which she then fully reveals in equilibrium.

1.2 Literature

There is an extensive literature studying the role of verifiability (or lack thereof) in sender-receiver games. A number of papers have pointed out that the classical unraveling result (Viscusi (1978), Grossman and Hart (1980), Grossman (1981) and Milgrom (1981)) breaks down when the sender's private information is not fully verifiable, in the sense that some sender types do not have access to messages that allow them to sustain separation in equilibrium.[4] In this literature, the sender's type distribution is fixed and common knowledge; in my model, the players start equally uninformed and the sender can privately acquire information before communicating with the receiver. This paper therefore explores the role of (partially) verifiable information in a context in which what the sender privately knows is a covert strategic decision on her part.

In doing so, this paper contributes to a growing literature which augments sender-receiver games by endogenizing the sender's private information. This includes papers studying sender learning in cheap talk models,[5] settings with full verifiability[6] and gradual evidence acquisition.[7] This paper studies the case in which the sender's covertly acquired private information is only partially verifiable. For this reason, it is closest to a pair of papers studying covert information acquisition in Dye (1985)-like environments: Ben-Porath, Dekel, and Lipman (2018) and, in particular, DeMarzo, Kremer, and Skrzypacz (2019).[8]

As in these papers, my model studies how the lack of observability of

[4]This approach traces back to Dye (1985) and Jung and Kwon (1988) in sender-receiver games and to Green and Laffont (1986) in a mechanism design setting. Important contributions include Okuno-Fujiwara, Postlewaite, and Suzumura (1990), Lipman and Seppi (1995) and, more recently, Mathis (2008), Hagenbach, Koessler, and Perez-Richet (2014) and Rappoport (2024). Complete lack of verifiability is studied in the literature on cheap talk, initiated by Crawford and Sobel (1982).

[5]See, for example, Pei (2015) and Deimen and Szalay (2019). Earlier, Austen-Smith (1994) explored this question in a setting in which the sender either learns the value of the state perfectly, or not at all; in his setting, information acquisition is covert, but the sender can prove that she is informed.

[6]An early example is Matthews and Postlewaite (1985), in which the sender faces a binary choice between a fully informative and verifiable signal and a fully uninformative and unverifiable signal. More recently, Gentzkow and Kamenica (2017) show that under full verifiability the sender perfectly reveals all acquired information.

[7]For example, Argenziano, Severinov, and Squintani (2016), Felgenhauer and Loerke (2017), Janssen (2018), Herresthal (2022).

[8]Information acquisition with Dye evidence is also studied in Kartik, Lee, and Suen (2017) and Shishkin (2024).

the sender's information acquisition determines what she chooses to learn and communicate with the receiver. There are two substantial differences in our approaches.

The first concerns the class of verifiability structures studied. In the Dye setting these papers consider, the only unverifiable message is 'non-disclosure'. Its meaning must therefore be determined in equilibrium and will be shaped by the sender's ability to conceal negative evidence and to covertly change the way she decides to acquire information. In my model, the meaning of several messages—potentially of all of them—must be jointly determined in equilibrium.[9] My approach to partial verifiability allows me to capture Dye-style evidence (see Example 5) but also to speak directly to a variety of canonical settings in the communication literature which Dye evidence cannot capture. For example, my formalization of verifiability encompasses settings ranging from mandatory disclosure (and, more broadly, full verifiability) to cheap talk (see Examples 1 to 4).

The second key difference is that in these papers the sender's payoff is affine in the receiver's belief. It follows that the 'selective information acquisition' persuasion motive present in my model (and central to the Bayesian persuasion literature) is absent from these papers.

One important dimension in which my setting is less general than theirs is that they study problems in which the sender is constrained in the signals she can choose. In my model the sender is fully unrestricted in her information acquisition strategy. I discuss this in further detail in the context of Example 5 on p. 25.

This paper also contributes to the literature exploring the consequences of relaxing the commitment assumption in Kamenica and Gentzkow (2011). Some papers allow the sender to alter negative results with some positive probability (for example, Lipnowski, Ravid, and Shishkin (2022), Fréchette, Lizzeri, and Perego (2022) and Min (2021)) while others study models of Bayesian persuasion with lying costs. In this latter category, Nguyen and

[9]More formally, the receiver's belief following any m for which $M^{-1}(m)$ is not a singleton is determined in equilibrium. With Dye evidence, only the 'non-disclosure' message, call it $m_\emptyset$, is such that $M^{-1}(m_\emptyset)$ contains more than one element.

Tan (2021) analyze a setting in which the sender's information acquisition strategy is observable by the receiver while in Guo and Shmaya (2021)'s set-up, as in mine, it is not. Both provide conditions on the costs such that the sender-preferred equilibrium exhibits full information revelation.

In contrast to these papers, in my model the sender faces a constraint on what she can or cannot communicate depending on what she discovers. This distinct formalization of partial commitment allows me to bridge the gap between the Bayesian persuasion setting and models of verifiable disclosure. This, in turn, leads to a characterization of when 'more verifiability' of the sender's acquired information is akin to her having 'more commitment power'.

Finally, this paper is related to the three applied literatures mentioned in the introduction: information control within organizations,[10] acquisition and disclosure of product information,[11] and media bias.[12]

Roadmap. I introduce the model in the next section. Unpersuasive and unraveling equilibria are studied in Section 3, along with a discussion of the value of persuasion in the context of my model of disclosure with covert learning. Section 4 contains the comparative statics result, characterizing changes in verifiability that are beneficial for the sender, and the characterization of sender-optimal verifiability structures. Receiver-optimal structures are characterized in Section 5.

[10]Gibbons, Matouschek, and Roberts (2013) survey the literature on conflicting interests within organizations and their effect on the production and transmission of information.

[11]Dranove and Jin (2010) survey the product-quality disclosure literature. In the context of acquisition and disclosure of financial information, Lin and McNichols (1998) and Michaely and Womack (1999) provide empirical evidence of how financial analysts are biased in their recommendations when their employers have an underwriting relation with the firm under review. The financial reporting literature is surveyed in Leuz and Wysocki (2016).

[12]A large and active empirical literature documents media bias and its consequences for news production, reporting and electoral behavior. For example, Groseclose and Milyo (2005) highlight the *presence* of media bias and DellaVigna and Kaplan (2007) emphasize its influential role. Evidence of partial and selective coverage of events is also widely documented, for example in Puglisi and Snyder (2011) and Larcinese, Puglisi, and Snyder (2011). See Gentzkow, Shapiro, and Stone (2015) for a survey.

2 Model

There is an unknown binary state of the world ω, taking values in $\{0,1\}$. A sender and a receiver initially assign prior probability $\overline{p} \in [0,1]$ to $\omega = 1$. The sender can acquire information about the state by choosing a signal and observing its realization. Information acquisition is covert in the sense that the receiver observes neither the sender's chosen signal, nor its realization. Formally, a feasible signal is a pair $(\mathcal{S}, \pi)$ where $\mathcal{S}$ is a finite set of possible realizations and $\pi \in \Delta(\{0,1\} \times \mathcal{S})$ is a joint distribution such that the marginal distribution of the state coincides with the prior $\overline{p}$: $\sum_{s\in\mathcal{S}} \pi(1,s) = \overline{p}$.[13]

After acquiring information, the sender communicates with the receiver. Her acquired private information determines what she is able to say: if the sender's chosen signal is $(\mathcal{S}, \pi)$ and its realization is $s \in \mathcal{S}$, she must send a message from the non-empty finite set $M(\mathcal{S}, \pi; s)$ to the receiver. I refer to the mapping M, assigning sets $(M(\mathcal{S}, \pi; s))_{s\in\mathcal{S}}$ to every signal $(\mathcal{S}, \pi)$, as the *verifiability structure*, which is common knowledge among the two players. M describes how the sender's acquired private information constrains how she can communicate with the receiver. In doing so, it also captures what the sender can prove about what she learned to the receiver.

Having observed the sender's message, the receiver updates her belief about the state. The sender obtains a payoff $v(p)$ if the receiver holds a posterior belief $p \in [0,1]$. The function $v : [0,1] \to \mathbb{R}$ is assumed to be non-decreasing, capturing that the sender prefers inducing higher beliefs in the receiver, and upper semi-continuous. The sender maximizes the expected value of v.

The preferences of the receiver are not modeled explicitly as they play no role in the analysis. The receiver can be interpreted either as a literal player, who takes a payoff-relevant action following the sender's message, or as a 'market' in which the sender obtains return $v(p)$ at belief p.

[13]We will soon restrict $\mathcal{S}$ to be a subset of $[0,1]$, see Remark 1.

2.1 Beliefs determine messages

Operationalizing the notion of verifiability structure requires articulating how the sender's private information shapes what she can say and, vice versa, how the receiver's beliefs are restricted following a given message from the sender. I do so by restricting attention to verifiability structures in which *beliefs determine messages*, defined as follows.

Definition 1 (Beliefs determine messages)**.** *M is such that beliefs determine messages iff for every pair of signals $(\mathcal{S}, \pi)$ and $(\mathcal{S}', \pi')$, and every $s \in \mathcal{S}$, $s' \in \mathcal{S}'$, if $\pi(\cdot|s) = \pi'(\cdot|s')$ then $M(\mathcal{S}, \pi; s) = M(\mathcal{S}', \pi'; s')$.*

If beliefs determine messages, it is only the information *about the state* contained in the signal realization—that is: the sender's own private posterior belief—that determines what messages the sender can use.

The underlying assumption is that the sender cannot *directly* prove anything about how she chose to acquire information (that is: about the signal $(\mathcal{S}, \pi)$), but only about what she learned regarding the state. Observe that proving information about the state *indirectly* proves something about the sender's choice of signal. That is, if a certain message m proves to the receiver that the sender's belief lies in some set $S_m \subseteq [0, 1]$, the sender is also proving that she acquired information in a way that assigns positive probability to her obtaining a belief in S_m. The beliefs determine messages assumption imposes that such a message m proves nothing more than this to the receiver.

This assumption formalizes a feature of many applications: the details of the information-gathering process are typically difficult to verify while information about the state is—at least in part—verifiable. The reason why it is hard to provide credible information about the learning process may be technological: for example, the information-gathering procedure may be long and difficult to monitor. It may also be due to social norms or legal constraints: for example, journalistic sources are typically protected by anonymity.

In contrast, information about the state can more easily be verified, for example by a trusted third party (such as an auditor, a fact-checker or an accreditation body) who certifies what the sender reports about the payoff-relevant state. Importantly, such certification typically does not coincide with full disclosure of what the sender discovers: it may involve coarse grades or thresholds, and perhaps leave the sender the possibility of redacting a report if it reveals bad news. Section 2.2 below provides more detailed examples of such verifiability structures.

Importantly, abstracting away from this secondary layer of communication has a methodological payoff, as it allows the problem to be studied using a belief-based approach.

Remark 1. Given the restriction to verifiability structures in which beliefs determine messages, it follows that:

(i) It is without further loss to restrict the sender to choosing signals taking values in $[0,1]$ and such that each realization coincides with the sender's own private posterior belief about the state (i.e., such that $\pi(1|s) = s$ when $s \in [0,1]$ realizes). I shall frequently refer to s as the sender's *type*.

(ii) We can lighten notation further by omitting the dependence of M on the signal $(\mathcal{S}, \pi)$. Henceforth M will simply denote a mapping assigning to each $s \in [0,1]$ a non-empty finite set of messages $M(s)$.

Before turning to examples, I state a closure assumption on the verifiability structures considered, which is maintained throughout the analysis. For a given verifiability structure M, let $\mathcal{M} \equiv \cup_{s\in[0,1]} M(s)$ denote the set of all possible messages. For any given message $m \in \mathcal{M}$, let

$$M^{-1}(m) \equiv \{s \in [0,1] : m \in M(s)\}$$

denote the set of sender types that can send message m. We assume that this set always contains its infimum:

$$\inf M^{-1}(m) \in M^{-1}(m) \quad \text{for every} \quad m \in \mathcal{M}. \tag{1}$$

Therefore, for any given message, there is a well-defined lowest type that could have sent it. This condition ensures that the receiver's 'maximally skeptical' belief—which will be used throughout the analysis and is based on this lowest type—is always well-defined.

2.2 Examples of verifiability structures

The following examples illustrate some notable cases of verifiability structures in which beliefs determine messages.

Example 1 (Mandatory disclosure)**.** Suppose the sender is forced to reveal whatever she has discovered about the state. This could be due to a deliberate (ex ante) choice by the sender (e.g., by hiring a trusted third-party to certify that no acquired information is withheld) or to stringent disclosure regulation. Formally, we say M is the *mandatory disclosure* verifiability structure if $M(s) = \{s\}$ for each $s \in [0, 1]$, capturing that the sender is forced to truthfully report her belief about the state. It is immediate (for more details, see the discussion in Section 4) that this verifiability structure allows the sender to attain Kamenica and Gentzkow (2011)'s 'full-commitment' value in equilibrium. ◇

Example 2 (Full verifiability)**.** Mandatory disclosure as described in the previous example is a special case of a verifiability structure in which the sender is able to prove precisely what she discovered to the receiver. More generally, the sender might have the *option* of doing so, but may also be able to misreport or conceal her findings. For example, she might hire a trusted third-party certifier, but retain the option of preventing him from making any disclosure.[14]

Formally, we say verifiability structure M exhibits *full verifiability* if for each $s \in [0, 1]$, $s \in M(s)$ and $s \notin M(s')$ for $s' \neq s$. That is: each type of sender has the possibility of proving her identity to the receiver. This setting includes the mandatory disclosure setting of Example 1. It also describes a key property of the canonical verifiability assumptions in Grossman and

[14]We can describe this particular situation with the verifiability structure $M(s) = \{s, m_\varnothing\}$ for each $s \in [0, 1]$. Therefore, upon learning s, the sender can either allow the third-party certifier to make a (truthful) disclosure of s or can prevent him from reporting anything, sending the null message $m_\varnothing$.

Hart (1980), Grossman (1981) and Milgrom (1981). The outcome in this setting is discussed in Remark 4. ◇

The following environments exhibit instead *partial* verifiability in the sense that $M(s) \subseteq M(s')$ for some $s, s' \in [0, 1]$, meaning that type s' can always imitate type s.

Example 3 (Cheap talk)**.** At the other end of the spectrum from full verifiability, we have cheap talk situations, where the sender can claim anything regardless of what she discovered. This could be a consequence of the information acquisition technology: for example, it could be that learning only produces information which is 'soft' or easy-to-falsify. Formally, we have that $M(s) = \bar{M}$ for each $s \in [0, 1]$, for some given—perhaps very large—finite set $\bar{M}$. Section 3.2 discusses outcomes in environments of this kind. ◇

Example 4 (Partition)**.** Typically, what the sender can reveal is influenced by what she discovers (unlike in cheap talk) but she may be unable to unequivocally prove her findings. For example, third-party certification may only come in coarse grades (proving to the receiver that what the sender believes about the state lies in a certain set).

To formalize this notion, let $\mathcal{P}$ denote a partition of $[0, 1]$; a partitional verifiability structure $M_{\mathcal{P}}$ assigns to each $P \in \mathcal{P}$ a message m_P. That is: if type s lies in the partition element $P \subseteq [0, 1]$, she must reveal the partition element she belongs to by sending message m_P. In applications, partitional structures are typically of the *interval* form: each $P \in \mathcal{P}$ is an interval. In this case, the verifiability structure can be interpreted as a scoring protocol, with a higher score corresponding to a higher (non-overlapping) interval of types. Remark 2 provides a general result for comparing such structures from the perspective of the sender. ◇

Example 5 (Dye verifiability)**.** Suppose that the sender can always make a verifiable claim about what she discovers, unless she learns nothing, in which case she has no evidence to present. Suppose also that, regardless of what she learns, she always has the option of suppressing her verifiable information by claiming that she learned nothing instead. This simple verifiability structure was introduced by Dye (1985) and Jung and Kwon (1988) to show that unraveling can fail when verifiability is partial. In my model, this situation can be formalized by setting $M(s) = \{s, m_\emptyset\}$ for each $s \neq \overline{p}$, while $M(\overline{p}) = \{m_\emptyset\}$. Message $m_\emptyset$ can therefore be interpreted as the 'non-disclosure' message.[15]

This is a special case of a more general class of verifiability structures in which there is a set $S_v \subseteq [0,1]$ of verifiable sender types such that $M(s) = \{s, m_\emptyset\}$ for $s \in S_v$ while $M(s) = \{m_\emptyset\}$ for $s \notin S_v$. ◇

Example 6 (Certifiable thresholds)**.** The sender might be able to make certifiable claims of the form 'I discovered the state is high at least with probability s'. This report is credible provided no type $s' < s$ can also make such a claim. A special case of this is when the sender privately learns via a binary test and she can prove she obtained good news (e.g., she can prove her posterior is above the prior), but can always claim to have obtained no information.

Verifiability structures of this kind can be described by *certifiable thresholds*. Formally, they are defined by a sequence of sender types, $0 = s_0 < s_1 < \cdots < s_N \leq 1$, and a set of messages $\{m_0, \ldots, m_N\}$ such that $m_i \in M(s)$ iff $s \geq s_i$. In this environment, the sender can prove to the receiver that a threshold s_i (corresponding to a probability s_i of $\omega = 1$) has been reached. She can always claim to have attained a lower threshold, but not a higher one. One such example was solved in detail in Section 1.1. ◇

[15]To illustrate how unraveling fails with this verifiability structure in the case of a fixed (exogenous) signal, consider a strictly increasing v and a signal such that with probability $\alpha \in (0,1)$ the sender learns the value of the state and with complementary probability $1-\alpha$ learns nothing. This signal gives rise to three possible sender types: 0, $\overline{p}$ and 1. In equilibrium, sender type 0 will pool with type $\overline{p}$ by sending message $m_\emptyset$, while type 1 will separate by sending message 1. This signal cannot, in fact, be an equilibrium in my model, as it is subject to a deviation which worsens the meaning of $m_\emptyset$. See Theorem 2 and the discussion in the continuation of Example 5 on p. 25 for more details.

2.3 Strategies and solution concept

In light of Remark 1, rather than writing $(\mathcal{S}, \pi)$ for the sender's choice of signal, it creates no ambiguity to suppress the set of realizations and only denote it by the (finite-support) distribution π, where it is understood that the implied set of signal realizations is

$$\mathcal{S} = \operatorname{supp} \pi_S = \{s \in [0,1] : \pi(0,s) + \pi(1,s) > 0\},$$

where $s \mapsto \pi_S(s) = \pi(0,s) + \pi(1,s)$ denotes the marginal distribution of the signal.

We can therefore denote a strategy for the sender by a pair (π, μ), where π is the signal chosen and μ is her messaging mixed strategy: $(\pi, s) \mapsto \mu(\pi, s)$ assigns to every (π, s) an element of $\Delta(M(s))$. Denote by $\beta : \mathcal{M} \to [0,1]$ the receiver's posterior belief about the state. Given the triple (π, μ, β), say that type $s \in [0,1]$ is on-path iff $s \in \operatorname{supp} \pi_S$ and that message $m \in \mathcal{M}$ is on-path iff

$$m \in \operatorname{supp} \mu(\cdot|\pi, s) = \{m \in M(s) : \mu(m|\pi, s) > 0\}$$

for some $s \in [0,1]$ on-path.

The solution concept is in the spirit of perfect Bayesian equilibrium: the sender acquires and transmits information optimally given how the receiver forms beliefs and the receiver's posterior belief about the state is formed using Bayes' rule on the equilibrium path and is consistent with the verifiability structure off-path. As the sender's optimality conditions and the receiver's on-path Bayesian updating are standard, their formal statement is relegated to Appendix A. Off-path, consistency of the receiver's belief with the verifiability structure requires that

$$\beta(m) \in \operatorname{conv} M^{-1}(m) \quad \text{for every} \quad m \in \mathcal{M}, \tag{2}$$

where $\operatorname{conv} M^{-1}(m)$ denotes the convex hull of $M^{-1}(m)$.

To see why this is the appropriate equilibrium restriction on the receiver's off-path beliefs, notice that following any message $m \in \mathcal{M}$, the receiver's belief *about the sender's type* must be supported on $M^{-1}(m)$. Since the verifiability structure M is such that beliefs determine messages (Definition 1), the receiver must also believe that the sender's belief *about the state* is an element of $M^{-1}(m)$, so that her own belief about the state—$\beta(m)$—must reflect this and lie in conv $M^{-1}(m)$.

Finally, the following lemma provides a continuity condition on the verifiability structure ensuring the existence of an equilibrium.

Lemma 1 (Existence). *If $s \mapsto \max_{m \in M(s)} \min M^{-1}(m)$ for $s \in [0,1]$ is upper semi-continuous, an equilibrium exists.*

The proof is in Appendix A. The condition guarantees that the sender, when facing a skeptical receiver assigning belief $\min M^{-1}(m)$ to every message m, has an optimal choice of signal. A sufficient condition for this continuity requirement is that the set of all possible messages $\mathcal{M}$ is finite.

Since this assumption is not necessary to state the results in the next section, it is not maintained. It will be explicitly invoked in Section 4, where equilibrium existence is required to state the comparative statics results.

3 Unpersuasive and unraveling equilibria

Recall that—as discussed in the previous section—we restrict attention to non-decreasing and upper semi-continuous payoff functions v and verifiability structures M in which beliefs determine messages (Definition 1) and such that (1) holds. We will refer to parameters in this class as *admissible*.

Definition 2. For a given triple of admissible parameters $(v, \overline{p}, M)$, say that type $s \in [0,1]$ is *lowest-consistent* with message m iff $s = \min M^{-1}(m)$.

That is, sender type $s \in [0,1]$ is lowest-consistent with message m if no lower type can send m. Type s can therefore separate from every lower type—regardless of the receiver's belief—by sending m.[16] The maintained

[16]In the terminology of Seidmann and Winter (1997), sender type s is lowest-consistent with some message m if it is the lowest of the worst-case types for $M^{-1}(m)$.

assumption that beliefs determine messages further implies that the receiver's belief about $\omega = 1$ following message m must be at least s (both on- and off-path) as captured by equilibrium condition (2).

Note that Definition 2 does not rely on the verifiability structure M being such that beliefs determine messages.[17] If beliefs do not determine messages, however, even if sender type $s \in [0,1]$ is lowest-consistent with message m, the receiver could hold—off-path—any belief *about the state* following m. This is because the sender faces no constraints on the joint distribution of state and signal, other than the requirement that the state marginal must equal the prior. So, even if the receiver's belief *about the sender's type* would still have to be supported on types at least equal to s, this would not translate into any restriction on her belief about the state.

The following terminology will also prove useful in stating the results.

Definition 3. Given a sender strategy (π, μ) and receiver belief β, say that the receiver is *maximally skeptical* following message m iff $\beta(m) = \min M^{-1}(m)$. Say that the sender *reveals all acquired information* iff for every $s \in \operatorname{supp} \pi_S$ and every $m \in \operatorname{supp} \mu(\cdot|\pi, s)$, $\beta(m) = s$.

Equipped with these definitions, we can turn to studying the properties of equilibria.

Definition 4 (Unpersuasive equilibria)**.** An equilibrium (π, μ, β) is *unpersuasive* iff $v(\beta(m)) = v(\overline{p})$ for every message m on-path.

In unpersuasive equilibria, any variation in the receiver's belief is payoff-irrelevant for the sender: she obtains the same payoff she would achieve without information acquisition and transmission, with probability one. The first result shows that, in equilibria which are *not* unpersuasive, the sender's behavior and outcomes are tightly pinned down by the primitives of the model.

[17] It does, however, rely on the signal realizations being in $[0,1]$ and on the sets of available messages depending only on the signal realization (and not directly on the signal). As discussed in Remark 1, these restrictions entail no loss of generality once M is assumed to be such that beliefs determine messages.

Theorem 1 (Unraveling equilibria)**.** *Suppose that* (π, μ, β) *is an equilibrium which is not unpersuasive. Then:*

(i) *Every on-path sender type* s *is lowest-consistent with every* $m \in \operatorname{supp} \mu(\cdot|\pi, s)$*.*

(ii) *The receiver is maximally skeptical following every on-path message.*

(iii) *The sender reveals all acquired information.*

We call such equilibria unraveling*.*

The proof is in Appendix B. Notice that Theorem 1 (i) is saying two things at once. First, it states that only types that are lowest-consistent with *some* message can be on-path. The verifiability structure M therefore restricts the set of possible outcomes of the sender's equilibrium information acquisition to those that can then be credibly communicated to the receiver (i.e., to types that are lowest-consistent with some message). Second, it is saying that every on-path sender type will, indeed, only use messages she is lowest-consistent with in equilibrium.

Note also that the theorem is not making the stronger claim that the equilibrium set can be partitioned into unpersuasive and unraveling equilibria, as an equilibrium may be both.[18]

The logic behind the theorem is driven by the fact that, if the receiver were not maximally skeptical following an on-path message, the sender could deviate to a signal 'worsening the meaning' of that message, i.e., a signal such that the message ends up being sent by a lower type. This is profitable for the sender as the set of on-path messages is unchanged, but probability is shifted towards messages that lead to higher payoffs.

This deviation arises whenever the receiver's equilibrium beliefs vary in a way that affects the sender's payoff—i.e., in equilibria that are not unpersuasive. It is precisely this variation in payoff that gives the sender an incentive to shift probability from 'bad news' messages to 'good news' ones. This temptation evidently vanishes when all on-path messages yield the same payoff.

[18]To illustrate, adjust the example in Section 1.1 by increasing the prior to $\overline{p} = 0.5$. It can be verified that there is an equilibrium in which the sender acquires no information and sends message m_G on-path. This equilibrium is both unpersuasive and unraveling.

Notice how this reasoning relies on the assumption that beliefs determine messages, which aligns the receiver's skepticism about the sender's type with her skepticism about the state. If type s is lowest-consistent with message m, the sender cannot *arbitrarily* worsen its meaning, as she must believe that $\omega = 1$ with probability at least s when sending m. Consequently, the receiver's belief about the state following m cannot fall below s, as implied by equilibrium condition (2).

Finally, this intuition also hinges on the monotonicity and binary state assumptions, as they ensure that worsening the meaning of a message never corresponds to a decrease in the probability that a 'good news' message is sent. Outside of the binary-state case with monotone payoff, this need not be the case.

3.1 The sender's value in unraveling equilibria

Theorem 1 also directly implies that the sender obtains the same expected payoff across all unraveling equilibria. This expected payoff can be expressed as the concave envelope of a skepticism-adjusted version of v. Letting

$$\underline{v}_M(s) \equiv v\left(\max_{m \in M(s)} \min M^{-1}(m)\right), \tag{3}$$

for each $s \in [0,1]$, we have the following corollary of Theorem 1.

Corollary 1 (Unique value of persuasion)**.** *In every unraveling equilibrium the sender's expected payoff is* $(\operatorname{cav} \underline{v}_M)(\overline{p})$.

That is, the sender's equilibrium expected payoff is the concave envelope of the 'value of interim sender beliefs', when facing a receiver who is maximally skeptical following every message. In the example from Section 1.1, these objects are depicted in Figure 1b: $\underline{v}_M$ is the solid black line, while $\operatorname{cav} \underline{v}_M$ is the thick dotted line and $(\operatorname{cav} \underline{v}_M)(\overline{p})$ is the black dot.

Proof of Corollary 1. Consider any unraveling equilibrium (π, μ, β) and define $w_\beta(s) \equiv \max_{m \in M(s)} v(\beta(m))$, $s \in [0,1]$. The sender's equilibrium expected payoff is $(\text{cav}\, w_\beta)(\overline{p})$, since—for a given β and optimally chosen messages—the sender faces a canonical Bayesian persuasion problem with value of posteriors w_β. Since $w_\beta \geq \underline{v}_M$, we have that $(\text{cav}\, w_\beta)(\overline{p}) \geq (\text{cav}\, \underline{v}_M)(\overline{p})$.

Observe next that $\underline{v}_M = w_\beta$ on $\text{supp}\, \pi_S$. This holds because, for every $s \in \text{supp}\, \pi_S$, Theorem 1 (i) implies that s is lowest-consistent with some message, so that $\underline{v}_M(s) = v(s)$ by construction and Theorem 1 (iii) implies that $w_\beta(s) = v(s)$. It follows that, in an auxiliary Bayesian persuasion problem in which the sender faces value of posteriors $\underline{v}_M$, the sender can obtain an expected payoff of $(\text{cav}\, w_\beta)(\overline{p})$ by choosing signal π. Since $(\text{cav}\, \underline{v}_M)(\overline{p})$ is the highest expected payoff the sender can attain in this problem, it must be that $(\text{cav}\, w_\beta)(\overline{p}) \leq (\text{cav}\, \underline{v}_M)(\overline{p})$. □

3.2 Proving news better than the prior

We have so far established that any equilibrium which is not unpersuasive must be unraveling, meaning that it exhibits the properties described in Theorem 1. We now provide a sufficient condition on the primitives of the model which ensures that all equilibria are unraveling.

Definition 5 (PNBP). For a given triple of admissible parameters $(v, \overline{p}, M)$, say that the sender can *prove news better than the prior* iff $v(\min M^{-1}(m_h)) > v(\overline{p})$ for some $m_h \in \mathcal{M}$.

Being able to prove news better than the prior means that the sender can—by acquiring information in a way that makes sending message m_h possible—induce a belief in the receiver that she strictly prefers over the prior. Importantly, this is possible irrespective of the receiver's belief (i.e., even if she is maximally skeptical, assigning belief $\min M^{-1}(m_h)$ to message m_h).

PNBP can hold only if, in the terminology of Kamenica and Gentzkow (2011), there is *information the sender would share*. That is, only if the sender is not already obtaining her highest possible payoff at the prior. Notice that in full-verifiability settings (Examples 1 and 2) this condition is also sufficient for PNBP. In contrast, in cheap talk environments (Example 3) the sender cannot prove news better than the prior for any admissible v and $\overline{p}$ (so even if there is information she would share).

Theorem 2 (PNBP, unraveling and persuasion)**.** *If the sender can prove news better than the prior, every equilibrium is unraveling. If instead the sender cannot prove news better than the prior, there exists a sender-preferred unpersuasive equilibrium in which she acquires no information.*

The proof is in Appendix C. Theorem 2 implies that, if the sender can prove news better than the prior, her equilibrium expected payoff is unique, because of Corollary 1.[19] In contrast, if the sender cannot prove news better than the prior, any equilibrium which is not unpersuasive—i.e., which is unraveling, in light of Theorem 1—leads to an expected sender payoff not higher than $v(\overline{p})$.[20]

When PNBP fails, therefore, the sender's bias in favor of state 1 eliminates the possibility of valuable persuasion. That is, in equilibrium the sender can never obtain an expected payoff above the one she would achieve with no information acquisition (and transmission). An immediate consequence is that in 'cheap talk' environments (Example 3) covert learning has no value for the sender.

Why is PNBP sufficient for unraveling? In light of Theorem 1, it is enough to argue that if PNBP holds and an equilibrium is unpersuasive, then it must also be unraveling. The intuition is as follows. PNBP guarantees the existence of a message conveying 'good news'—that is, a message that yields the sender a payoff strictly above $v(\overline{p})$, even under maximal receiver skepticism. In an unpersuasive equilibrium, the sender must obtain $v(\overline{p})$ with probability one, so she must be deterred from deviating to a

[19]PNBP does not ensure that the sender's expected payoff in equilibrium is at least $v(\overline{p})$. For instance, in the example presented in Section 4.3 verifiability structure M'' leads to a unique equilibrium which is unraveling and such that the sender's expected payoff is strictly below $v(\overline{p})$.

[20]It is straightforward to construct examples of such unraveling equilibria. For example, modify the example in Section 1.1 by increasing the prior from 1/4 to 9/20, so that PNBP no longer holds. In this case, there is an unraveling equilibrium in which the receiver is maximally skeptical following every message and the sender obtains an expected payoff strictly below $v(\overline{p})$.

strategy in which she acquires information, sends the 'good news' message when she indeed discovers good news, and otherwise falls back on an on-path message that delivers $v(\overline{p})$.

Such a deviation can be ruled out only if, in equilibrium, the sender learns nothing, and the prior itself is lowest-consistent with some message m the sender uses. In this case, m can no longer serve as a fallback in the deviation just described. But then, the unpersuasive equilibrium is also an unraveling one.

To conclude this section and illustrate its results, it is informative to consider their implications in a setting with Dye verifiability, as introduced in Example 5.

Example 5 (Dye verifiability, continued)**.** Assume that $v(p) = p$, so that the sender obtains a payoff equal to the expected value of the state. This specialization of the model brings it closest to the set-up studied in DeMarzo et al. (2019). Assume, to rule out trivial cases, that $\overline{p} \in (0, 1)$. My results directly imply the following: (i) the sender's equilibrium expected payoff is equal to $v(\overline{p})$, (ii) the receiver's belief following the non-disclosure message $m_\emptyset$ is equal to 0 in all equilibria.

Point (i) holds because an equilibrium exists[21] and all equilibria are unraveling (because PNBP holds, so Theorem 2 applies) so they all lead to the same expected payoff for the sender, which is equal to $(\text{cav}\, \underline{v}_M)(\overline{p}) = v(\overline{p})$, by Corollary 1.

To see why (ii) holds, suppose otherwise that the receiver's belief following the non-disclosure message $m_\emptyset$ were equal to some $\beta(m_\emptyset) > 0$ in equilibrium, implying that the sender's payoff from sending message $m_\emptyset$ would strictly exceed $v(0)$. The sender could then deviate to acquiring full information, revealing what she learns when $\omega = 1$ (by sending message 1—recall that $M(1) = \{1, m_\emptyset\}$) and withholding her findings when $\omega = 0$ (by sending message $m_\emptyset$—recall that $M(0) = \{0, m_\emptyset\}$). This would

[21] Lemma 1 cannot be invoked for equilibrium existence, since the sufficient condition it relies on fails in this example. It is immediate that an equilibrium exists, however. For example, this is an equilibrium: the sender perfectly learns the value of the state (so $\pi_S(1) = \overline{p}$ and $\pi_S(0) = 1 - \overline{p}$) and sends message s when her type is $s \neq \overline{p}$; the receiver's belief is $\beta(m_\emptyset) = 0$ and $\beta(s) = s$ for $s \in [0, 1] \setminus \{\overline{p}\}$.

lead to an expected payoff equal to

$$\overline{p}v(1) + (1 - \overline{p})v(\beta(m_{\emptyset})) > \overline{p}v(1) + (1 - \overline{p})v(0) = v(\overline{p}),$$

meaning that there cannot be an equilibrium in which the sender's expected payoff is equal to $v(\overline{p})$, contradicting (i).

These two observations correspond to the 'minimum principle' in DeMarzo et al. (2019) (their Proposition 1). The most substantial difference between this specialization of my model and theirs is that they consider a setting in which the sender is *constrained* in her information acquisition choice in the sense that all available signals put positive probability on the unverifiable signal realization. Therefore, in their model the non-disclosure posterior belief may strictly exceed zero. They show that this belief must be minimal, in equilibrium. In contrast, my sender is able to choose any signal, so minimality of the non-disclosure posterior belief means that it must be equal to zero. ◇

4 Verifiability comparative statics

How does the sender's equilibrium payoff change as the extent to which she can misreport her acquired information varies? Which shifts towards 'more verifiability' are desirable from the sender's perspective?

To address these questions, I first provide a pre-order on the set of verifiability structures, formalizing an intuitive notion of 'more verifiability' in the context of the model. I then show that if the sender can prove news better than the prior, she obtains weakly better equilibrium outcomes under higher-ordered structures. I illustrate that the proposed pre-order is the appropriate notion for comparing verifiability structures in this context by providing a converse: unordered shifts may strictly hurt the sender, even when she can prove news better than the prior.

I then show that the result is tight: if the sender cannot prove news better than the prior, meaning that unpersuasive equilibria exist, increases in verifiability may—perhaps surprisingly—hurt her in equilibrium. I conclude the section by characterizing verifiability structures that are optimal for the sender.

4.1 More verifiability

The notion of 'more verifiability' that I introduce captures the idea that, for all sender types, separation possibilities from lower types do not decrease. More precisely, the requirement is that if a sender type can separate from all lower types by means of a single message, she must also be able to do so in the higher-ordered structure.

Given verifiability structure M, let the *lowest-consistent set* be the set of types that are lowest-consistent with some message (recall Definition 2) and denote it by $L_M \subseteq [0,1]$.

Definition 6 (Larger lowest-consistent set). Given verifiability structures M'' and M', say that M'' has a *larger lowest-consistent set than* M', and denote it by $M'' \succeq^{\text{lc}} M'$, iff $L_{M''} \supseteq L_{M'}$.

Notice that $\succeq^{\text{lc}}$ is a pre-order on the set of admissible verifiability structures.[22] I discuss next two examples of $\succeq^{\text{lc}}$-shifts.

Example 7 (Partition refinement)**.** As mentioned in Example 4, we can interpret interval-partition verifiability structures as scoring protocols: the sender can prove (perhaps via a third-party certifier) that her type lies in a certain interval, which corresponds to a score. It turns out that, if the sender chooses a certifier with a finer grid of scores (or if regulation imposes such a refinement), we have a $\succeq^{\text{lc}}$-increase in the verifiability structure. In this context, the comparative statics question can be cast as: when will the sender benefit, ex ante, from a finer certification technology?

Formally, for some given left-closed partition $\mathcal{P}$ of $[0,1]$ (that is: every $P \in \mathcal{P}$ is left-closed) let $M_{\mathcal{P}}$ denote a partitional verifiability structure as defined in Example 4. If $\mathcal{P}''$ is a refinement of $\mathcal{P}'$ then $M_{\mathcal{P}''} \succeq^{\text{lc}} M_{\mathcal{P}'}$. For

[22] $\succeq^{\text{lc}}$ is not anti-symmetric. For example, consider verifiability structure M' with $M'(s) = \{m_\emptyset\}$ for each $s \in [0,1]$ and M'' with $M''(s) = \{m_\emptyset, \hat{m}\}$ for each $s \in [0,1]$. Then $M'' \succeq^{\text{lc}} M' \succeq^{\text{lc}} M''$ but $M' \neq M''$.

interval partitions, also the converse holds: if $\mathcal{P}''$ is not a refinement of $\mathcal{P}'$ then $M_{\mathcal{P}''} \not\succeq^{\text{lc}} M_{\mathcal{P}'}$. $\diamond$

Example 8 (Adding a message)**.** Another natural way in which a verifiability structure may change is if a new message is added (or removed). To illustrate this starkly, suppose we start in a situation in which all talk is cheap: any message can be sent by all sender types (for example because all evidence is easily falsified, or because no hard evidence can be presented for legal or technological reasons).

The environment now changes: a non-falsifiable piece of evidence is discovered (and can be presented to the receiver) if and only if the sender learns that $\omega = 1$. This constitutes a 'new message', which the sender can use only at the degenerate belief $s = 1$, and that she can therefore exploit to credibly convey her acquired information to the receiver. Adding a message in this sense corresponds to a $\succeq^{\text{lc}}$-increase in the verifiability structure.

To illustrate further, a 'message' might also be added to an initial verifiability structure by allowing for 'the right to remain silent' when it was previously not permitted. This corresponds, for example, to the sender adding a clause to her contract with a third-party certifier, allowing her to prevent him from publishing the results of the test after having observed them.

More generally, fix some verifiability structure M', some 'new message' $m \notin \cup_{s\in[0,1]} M'(s)$ and a set of types $S_m \subseteq [0,1]$. Construct M'' as follows: $M''(s) \equiv M'(s)$ if $s \notin S_m$ and $M''(s) \equiv M'(s) \cup \{m\}$ if $s \in S_m$. So M'' is the same as M', except that types in S_m have access to the 'new message' m. It can be easily verified that $M'' \succeq^{\text{lc}} M'$. $\diamond$

4.2 Comparative statics result

Let $\underline{V}^*_M(v, \overline{p})$ and $\overline{V}^*_M(v, \overline{p})$ denote the lowest and highest equilibrium expected payoffs attainable for the sender at triple $(v, \overline{p}, M)$, respectively. To ensure that these values are well-defined, for the rest of this section we will restrict attention to verifiability structures satisfying the continuity requirement stated in the existence lemma (Lemma 1).

Recall that Theorem 2 and Corollary 1 together imply that $\overline{V}^*_M(v,\overline{p}) = \underline{V}^*_M(v,\overline{p}) = (\operatorname{cav} \underline{v}_M)(\overline{p})$ if PNBP holds and $\overline{V}^*_M(v,\overline{p}) = v(\overline{p})$ if it does not.

Theorem 3 (More verifiability). *Consider verifiability structures M' and M''. Then*

$$M'' \succeq^{\mathrm{lc}} M' \quad \Leftrightarrow \quad \overline{V}^*_{M''}(v,\overline{p}) \geq \overline{V}^*_{M'}(v,\overline{p})$$

for every admissible v and $\overline{p}$ such that at $(v,\overline{p},M')$ the sender can prove news better than the prior.

The proof is in Appendix D. The result can be directly applied to compare the verifiability structures introduced in the context of the examples.

Remark 2. If the sender can prove news better than the prior, 'adding a message' to any verifiability structure (as defined in Example 8) is beneficial for her. That is also the case for refining partitional structures (as introduced in Example 4 and discussed further in Example 7). For partitional structures, the converse applies to interval partitions (see, again, Example 4 for a definition): if an interval partition is not a refinement of another, the sender will obtain a strictly better equilibrium outcome in the latter, for some admissible v and $\overline{p}$.

Theorem 3 also characterizes which changes in the sender's commitment to reveal acquired information are valuable for her. In this interpretation, following signal realization s, the sender can commit to using a message from the set $M(s)$. Therefore, $\succeq^{\mathrm{lc}}$-increases in M are exactly the changes in the sender's commitment power which are beneficial for her.

Theorem 3 follows straightforwardly from the following lemma of independent interest (also proved in Appendix D).

Lemma 2. *Consider verifiability structures M' and M''. Then*

$$M'' \succeq^{\mathrm{lc}} M' \quad \Leftrightarrow \quad \underline{V}^*_{M''}(v,\overline{p}) \geq \underline{V}^*_{M'}(v,\overline{p})$$

for every admissible v and $\overline{p}$ such that at $(v,\overline{p},M')$ an unraveling equilibrium exists.

Therefore, whenever an unraveling equilibrium exists (regardless of whether PNBP holds or not) a $\succeq^{\text{lc}}$-increase in verifiability cannot lower the sender-worst equilibrium payoff. Conversely, changes in the verifiability structure that do not increase (in the set-inclusion sense) which types are lowest-consistent may do so.

Remark 3 (More separation possibilities)**.** An alternative (natural) notion of 'more verifiability' captures the idea that, for all sender types, separation possibilities increase. Formally, for a fixed verifiability structure M and message $m \in \mathcal{M}$, let $S^c_m \equiv [0,1] \setminus M^{-1}(m)$ denote the complement of $M^{-1}(m)$ in $[0,1]$. That is, S^c_m is the set of types that cannot send message m. For a given $s \in [0,1]$, let

$$K_M(s) \equiv \{S \in 2^{[0,1]} : S = S^c_m \text{ for some } m \in M(s)\}$$

denote the collection of type sets from which s can separate by means of a single message, no matter what the receiver's belief is. Given verifiability structures M'' and M', say that M'' exhibits *more separation possibilities than* M', and denote it by $M'' \succeq M'$, iff $K_{M''}(s) \supseteq K_{M'}(s)$ for all $s \in [0,1]$.

It can be easily verified that $M'' \succeq M' \Rightarrow M'' \succeq^{\text{lc}} M'$ while $M'' \succeq^{\text{lc}} M'$ $\not\Rightarrow M'' \succeq M'$. Shifts to a $\succeq$-higher structure are therefore *sufficient* to weakly improve the sender's equilibrium payoff (provided she can prove news better than the prior) but they are *not necessary* (as highlighted in Theorem 3) because of the monotonicity of v.

4.3 More verifiability may hurt the sender

At first glance, moving to a $\succeq^{\text{lc}}$-higher (or, perhaps, even $\succeq$-higher, as introduced in Remark 3) structure might appear (via a standard replication argument) to be *always* beneficial for the sender, regardless of whether she can prove news better than the prior or whether an unraveling equilibrium exists. The following example demonstrates that this is not the case: the mere existence of a verifiable 'good news' message can taint the meaning of 'claiming ignorance', turning the inability to produce good news into bad news—thereby hurting the sender.

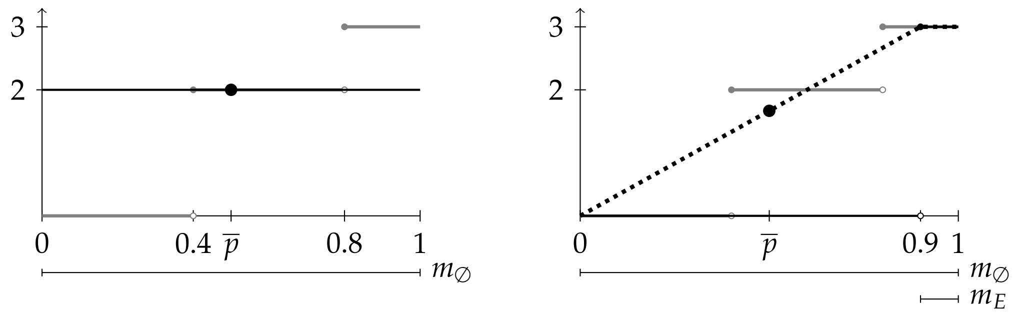
Fix some finite set of messages $\bar{M}$ and some message $m_E \notin \bar{M}$. Let verifiability structure M' be such that $M'(s) = \bar{M}$ for each $s \in [0,1]$, while M'' is such that $M''(s) = \bar{M}$ for $s < 0.9$ and $M''(s) = \bar{M} \cup \{m_E\}$ for $s \geq 0.9$. M' is therefore a cheap talk structure (as introduced in Example 3). M'' lets the sender use any of the cheap talk messages in $\bar{M}$, but also allows her to prove excellent news (namely: if she discovers that the state is more than 90% likely to be 1, she can prove it by sending message m_E). It is immediate that $M'' \succeq M'$ (and thus $M'' \succeq^{\text{lc}} M'$).

Finally, let v be defined as follows: $v(p) = 0$ for $p < 0.4$, $v(p) = 2$ for $p \in [0.4, 0.8)$ and $v(p) = 3$ for $p \geq 0.8$. Fix the prior to be $\overline{p} = 1/2$.

I now argue, applying directly the results from Section 3, that the (unique) equilibrium payoff with structure M'' is strictly lower than the (unique) equilibrium payoff with structure M'.

Since under M' the sender cannot prove news better than the prior, Theorem 2 implies that the sender's highest expected payoff in equilibrium is $v(\overline{p}) = 2$ (in this example, it is immediate that this is the sender's unique equilibrium payoff), as illustrated in Figure 3a for the case in which there is a single cheap talk message: $\bar{M} = \{m_\emptyset\}$. Under M'' however, the sender can prove news better than the prior, so Theorem 2 and Corollary 1 together imply that the sender's equilibrium expected payoff is $5/3 < 2$, as illustrated in Figure 3b.

The reason why a replication argument fails here is that the sender cannot commit to acquiring no information under M'', thereby replicating her payoff under M'. Informally, this is the case because 'claiming ignorance' is credible under M' (using any cheap talk message in $\bar{M}$), but it is not under M''. This is because under M'' the sender can prove good news (with m_E), so she will seek it, only claiming ignorance (using a message in $\bar{M}$) when she did not find said good news. Increasing separation possibilities may therefore worsen the meaning of messages in equilibrium (in this case: any message in $\bar{M}$ must lead to a payoff of 0 under M'', while it led to a payoff of 2 under M'). This can, in turn, lower the equilibrium expected payoff for the sender.



(a) Verifiability structure M' (b) Verifiability structure M''

Figure 3: On the left: the sender value function (solid black)—overlaid over v (solid gray). On the right: the sender's skepticism-adjusted value function (solid black)—overlaid over v (solid gray)—and its concave envelope (thick, dotted). In each case, the sender's equilibrium expected payoff is represented by the large black dot.

4.4 Value of commitment and sender-optimal verifiability

We showed that increasing verifiability, in the sense of enlarging the set of sender types that are lowest-consistent with some message, is both necessary and sufficient to make the sender ex ante weakly better off, provided that we are considering environments in which she can prove news better than the prior.

In light of this result, it is straightforward to characterize the structures that are optimal for the sender, in the sense that her equilibrium expected payoff is highest among all admissible ones. Note first that the sender can obtain her full-commitment payoff $(\text{cav}\, v)(\overline{p})$ (that is: the payoff she would obtain under public information acquisition, as in Kamenica and Gentzkow (2011)) with the 'mandatory disclosure' verifiability structure (the one of Example 1: $M(s) = \{s\}$, $s \in [0, 1]$). This follows from Theorem 2 and Corollary 1, if she can prove news better than the prior, and directly from Theorem 2, if she cannot.[23]

[23]In more detail: if with the mandatory disclosure verifiability structure the sender cannot prove news better than the prior, it must mean that $v(\overline{p}) = \max_{p \in [0,1]} v(p)$, so that $(\text{cav}\, v)(\overline{p}) = v(\overline{p})$. This payoff is therefore attained in an unpersuasive equilibrium, which exists by Theorem 2.

Therefore, characterizing sender-optimal structures is tantamount to characterizing structures that allow her to obtain such full-commitment payoff:

Proposition 1 (Sender-optimal verifiability)**.** *For a given verifiability structure M, the following are equivalent:*

(i) *In every equilibrium the sender obtains the full-commitment payoff for every admissible v and $\overline{p}$.*

(ii) *Every $s \in [0,1]$ is lowest-consistent with some message.*

The characterization therefore highlights precisely in which sense the full commitment assumption in Kamenica and Gentzkow (2011) can be relaxed without affecting the sender's equilibrium payoff. Provided that for every sender type there is a feasible message that no lower type can send, her equilibrium expected payoff matches the full-commitment one. If instead some type is not lowest-consistent with any message, it is possible to find a set of admissible parameters such that her equilibrium expected payoff is strictly lower than the full-commitment one.

Remark 4. Any full-verifiability environment as described in Example 2 is sender-optimal.

Proof of Proposition 1. Let M^{md} denote the mandatory disclosure verifiability structure ($M^{md}(s) = \{s\}$, $s \in [0,1]$). Consider first (i) $\Rightarrow$ (ii). Suppose that M is such that some $s \in [0,1]$ is not lowest-consistent with any message. Then $M \not\succeq^{\text{lc}} M^{md}$. It follows from Theorem 3 that there exist admissible v and $\overline{p}$ such that the sender obtains a payoff strictly below the full-commitment one, which is the unique outcome with M^{md}.

Consider now (ii) $\Rightarrow$ (i). If $v(\overline{p}) = v(1)$ the result is immediate, since the sender obtains the maximal payoff of $v(\overline{p})$ in every equilibrium. If $v(\overline{p}) < v(1)$, the sender can prove news better than the prior with M^{md}. Consider any structure M such that each $s \in [0,1]$ is lowest-consistent with some message. The sender can also prove news better than the prior with M, so her expected equilibrium payoff is unique (this follows from Theorem 2 and Corollary 1). Since $M \succeq^{\text{lc}} M^{md}$, she must obtain at least the same payoff with M as she does with M^{md} by Theorem 3. This payoff cannot be strictly higher than the full-commitment one, so they must coincide. □

5 Receiver-optimal verifiability

Which verifiability structures are desirable for the receiver? I show that a simple verifiability structure with two messages is such that the sender provides full information in equilibrium.[24] Provided that the receiver values information about the state, this structure will therefore be optimal for her across all admissible structures.[25] I also provide a converse: only structures in a specific binary class are such that, no matter what the (admissible) preferences and prior of the sender, she will provide full information in every equilibrium.

Proposition 2 (Full information provision)**.** *For a given verifiability structure M, the following are equivalent:*

(i) *In every equilibrium the sender provides full information for every $(v, \overline{p})$ such that $v(\overline{p}) < v(1)$.*

(ii) *Only types 0 and 1 are lowest-consistent with some message.*

$v(\overline{p}) < v(1)$ is a very mild requirement on what value the sender can potentially obtain from persuasion. If otherwise $v(\overline{p}) = v(1)$—recall that v is non-decreasing—the sender would be obtaining her highest possible payoff at the prior so, unsurprisingly, equilibria in which the sender does not provide full information persist.[26]

[24] I say that the sender *provides full information* in equilibrium if, conditional on $\omega = 1$, the receiver holds a posterior belief of 1 (with probability one) and, conditional on $\omega = 0$, the receiver holds a posterior belief of 0 (with probability one).

[25] In more detail: if the receiver obtains a payoff $u(p)$ at belief p, obtaining full information about the state is optimal (across all information structures) if u is convex. That is the case, for example, if $u(p)$ is the value of a decision problem under uncertainty with expected-utility preferences: there is an action set A and a continuous function $f : A \times \{0,1\} \to \mathbb{R}$ such that $u(p) = \max_{a \in A} pf(a,1) + (1-p)f(a,0)$ for each $p \in [0,1]$.

[26] If $v(\overline{p}) = v(1)$ and only types 0 and 1 are lowest-consistent with some message, it can be shown that there exists an equilibrium in which the sender provides full information. However, for the sender such an equilibrium may be (substantially) worse than an unper-

The class of verifiability structures leading to full information provision takes an intuitive form. It can be interpreted as the receiver committing to only checking the sender's claim of 'good news' if the latter produces evidence that news is *conclusively* good (i.e., if the sender proves that the state is 1).

In this situation, the only hope the sender has of swaying the receiver in her favor is to learn with maximal probability that the state is 1, which is achieved by also learning that the state is 0 with certainty. Since type 1 is lowest-consistent with some message and $v(\overline{p}) < v(1)$, the sender can prove news better than the prior. Therefore, Theorem 2 implies that all acquired information is revealed to the receiver, so the latter also learns the value of the state. For the receiver, committing to checking any other intermediate claim provides the sender with more commitment power, so may lead to less-than-full information provision.

Proof of Proposition 2. Consider first (ii) $\Rightarrow$ (i). Observe that the sufficient condition stated in Lemma 1 is satisfied, so an equilibrium exists. If type 1 is lowest-consistent with some message and $v(\overline{p}) < v(1)$ then the sender can prove news better than the prior, so all equilibria must be unraveling (Theorem 2). Therefore, in equilibrium, $\pi_S(1) = \overline{p}$ and $\pi_S(0) = 1 - \overline{p}$, since only 0 and 1 are lowest-consistent with some message. In any unraveling equilibrium the sender reveals all acquired information (Theorem 1), hence the sender provides full information.

Consider now (i) $\Rightarrow$ (ii). Suppose that some $s^* \notin \{0,1\}$ is also lowest-consistent with some message. Let $v(p) = 1_{\{p \geq s^*\}}$ and $\overline{p} = s^*/2$. The following is an unraveling equilibrium in which the sender acquires and provides less-than-full information.

The receiver is maximally skeptical, assigning belief $\beta(m) \equiv \min M^{-1}(m)$ to every $m \in \mathcal{M}$. The sender chooses a signal that splits the prior on $\{0, s^*\}$, sending any available message when 0 realizes and sending a message that type s^* is lowest-consistent with when s^* realizes. Letting

suasive equilibrium, which always exists and is sender-optimal, as she cannot prove news better than the prior (this follows from Theorem 2).

$w_\beta(s) \equiv \max_{m\in M(s)} v(\beta(m))$, $s \in [0,1]$, observe that the sender's strategy attains an expected payoff equal to $(\operatorname{cav} w_\beta)(\overline{p}) = (\operatorname{cav} v)(\overline{p}) = 1/2$, and is therefore optimal. The receiver's beliefs are updated using Bayes' rule on-path, and consistent with the verifiability structure by construction. This profile is therefore an unraveling equilibrium, as required. □

Declaration-interes

I wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Appendix A Equilibrium definition and existence

A.1 Equilibrium definition

Fix a prior $\overline{p} \in [0,1]$ and sender payoff function $v : [0,1] \to \mathbb{R}$. Let Π denote the set of signals (that is: Π is the set of all finite-support joint distributions of state and signal realizations with marginal distribution of the state coinciding with the prior $\overline{p}$). A strategy for the sender is a signal $\pi \in \Pi$ and a mixed messaging strategy $(\pi, s) \mapsto \mu(\pi, s)$ with $\mu(\pi, s) \in \Delta(M(s))$ for each $\pi \in \Pi$ and $s \in [0,1]$ (recall that $M(s)$ denotes the finite set of messages available to the sender at s). A triple (π, μ, β) is an equilibrium if:

(i) *Optimal information acquisition.* The sender chooses an optimal signal:

$$\int_{[0,1]} \int_{M(s)} v(\beta(m)) \mathrm{d}\mu(m|\pi, s) \mathrm{d}\pi_S(s) \geq \int_{[0,1]} \int_{M(s)} v(\beta(m)) \mathrm{d}\mu(m|\pi', s) \mathrm{d}\pi'_S(s)$$

for every $\pi' \in \Pi$.

(ii) *Sequentially rational communication.* The sender chooses an optimal message: $v(\beta(m)) \geq v(\beta(m'))$ for every $m \in \operatorname{supp} \mu(\cdot|\pi', s)$ and every $m' \in M(s)$, for every $\pi' \in \Pi$ and $s \in [0,1]$.

(iii) *Consistent receiver beliefs.* Beliefs are consistent with the verifiability structure: $\beta(m) \in \operatorname{conv} M^{-1}(m)$ for every $m \in \mathcal{M}$. If $m \in \mathcal{M}$ is on-path, it is further required that

$$\beta(m) \int_{[0,1]} \mu(m|\pi, s) \mathrm{d}\pi_S(s) = \int_{[0,1]} s\mu(m|\pi, s) \mathrm{d}\pi_S(s).$$

A.2 Equilibrium existence

A.2.1 Preliminaries

Consider a fixed receiver belief function β. Define

$$w_\beta(s) \equiv \max_{m \in M(s)} v(\beta(m)). \tag{4}$$

For any given bounded function $f : [0,1] \to \mathbb{R}$ and prior $\overline{p} \in [0,1]$ let

$$s_-(\overline{p}, f) \equiv \sup\{s \in [0, \overline{p}] : \operatorname{cav} f(s) = f(s)\} \tag{5}$$

and

$$s_+(\overline{p}, f) \equiv \inf\{s \in [\overline{p}, 1] : \operatorname{cav} f(s) = f(s)\}, \tag{6}$$

where $\operatorname{cav} f$ denotes the smallest concave function that majorizes f. The following two lemmas are immediate from Kamenica and Gentzkow (2011).

Lemma 3. *If w_β is upper semi-continuous, a sender-optimal signal exists and attains value* $(\operatorname{cav} w_\beta)(\overline{p})$. *The signal π with* $\operatorname{supp} \pi_S = \{s_-(\overline{p}, w_\beta), s_+(\overline{p}, w_\beta)\}$ *is sender-optimal.*

Lemma 4. $\operatorname{cav} w_\beta$ *is affine over* $[s_-(\overline{p}, w_\beta), s_+(\overline{p}, w_\beta)]$.

A.2.2 Proof of Lemma 1

The existence proof is divided into two parts: Lemma 5 and Lemma 6, dealing separately with the cases in which the sender can or cannot prove news better than the prior (Definition 5). Lemma 5 shows more than mere existence as it constructs an unpersuasive equilibrium in which the sender acquires no information, which is used to prove the second part of Theorem 2. Lemma 5 does not rely on the continuity assumption on M, namely upper semi-continuity of $s \mapsto \max_{m \in M(s)} \min M^{-1}(m)$, for $s \in [0,1]$, stated in Lemma 1. Lemma 6 does rely on it.

Lemma 5 (Existence without PNBP)**.** *If the sender cannot prove news better than the prior, an unpersuasive equilibrium in which the sender acquires no information exists.*

Proof. Fix some $m_0 \in M(\overline{p})$. The candidate equilibrium (π, μ, β) is defined as follows:

(i) The signal π is such that $\pi_S(\overline{p}) = 1$.

(ii) $\mu(m|\pi', s) = 1$ for some (arbitrary) m in $\arg\max_{m \in M(s)} \min M^{-1}(m)$, for every $\pi' \in \Pi$ and $s \in [0,1]$ for which $m_0 \notin M(s)$; $\mu(m_0|\pi', s) = 1$ for every $\pi' \in \Pi$ and $s \in [0,1]$ for which $m_0 \in M(s)$.

(iii) $\beta(m) = \min M^{-1}(m)$ if $m \neq m_0$ and $\beta(m_0) = \overline{p}$.

Observe first that, by construction, the sender's messaging strategy is optimal whenever m_0 is not available (since v is non-decreasing) and that the receiver's beliefs are consistent. It remains to show that for the sender it is optimal to acquire no information (item (i) above) and send message m_0 at signal realizations where m_0 is available. Observe that, since the sender cannot prove news better than the prior and v is non-decreasing, $v(\beta(m)) \leq v(\beta(m_0))$ for all $m \in \mathcal{M}$. This in turn implies both that sending m_0 is optimal, when feasible, and that acquiring no information, as specified by item (i), is an optimal information acquisition strategy, as message m_0 is then sent with probability one. □

Lemma 6 (Existence with PNBP)**.** *If the sender can prove news better than the prior and $s \mapsto \max_{m \in M(s)} \min M^{-1}(m)$, $s \in [0,1]$ is upper semi-continuous, an equilibrium exists.*

Proof. The candidate equilibrium (π, μ, β) is defined as follows.

(i) The signal π is such that $\operatorname{supp} \pi_S = \{s_-(\overline{p}, w_\beta), s_+(\overline{p}, w_\beta)\}$, where w_β, s_- and s_+ are defined in equations (4), (5) and (6).
(ii) $\mu(m|\pi', s) = 1$ for some (arbitrary) m in $\arg\max_{m \in M(s)} \min M^{-1}(m)$ for every $\pi' \in \Pi$ and $s \in [0, 1]$.
(iii) $\beta(m) = \min M^{-1}(m)$ for every $m \in \mathcal{M}$.

To streamline notation, write s_- (s_+) to denote $s_-(\overline{p}, w_\beta)$ ($s_+(\overline{p}, w_\beta)$) as we will be holding $\overline{p}$, β and w_β fixed throughout the rest of the proof. Suppose first that s_- (s_+) is lowest-consistent (Definition 2) with some message. Then, it follows from the definition of μ, that s_- (s_+) is lowest-consistent with every $m \in \operatorname{supp} \mu(m|\pi, s_-)$ ($m \in \operatorname{supp} \mu(m|\pi, s_+)$).

By construction, the receiver's belief is therefore correct following on-path messages and consistent off-path. Given β, the sender's messaging strategy is optimal by construction, as it maximizes the receiver's posterior belief and v is non-decreasing. Finally, notice that w_β is upper semi-continuous. This follows from two observations. First,

$$w_\beta(s) = \max_{m \in M(s)} v\big(\min M^{-1}(m)\big) = v\left(\max_{m \in M(s)} \min M^{-1}(m)\right),$$

where the first equality holds by construction (since $\beta = \min M^{-1}$) and the second equality follows from v being non-decreasing. Second, $s \mapsto v\big(\max_{m \in M(s)} \min M^{-1}(m)\big)$ is upper semi-continuous, since v is upper semi-continuous and non-decreasing and $s \mapsto \max_{m \in M(s)} \min M^{-1}(m)$ is upper semi-continuous by hypothesis.

Therefore, since w_β is upper semi-continuous, Lemma 3 implies that π is an optimal signal. It follows that, provided that each of s_- and s_+ is lowest-consistent with some message, the candidate is indeed an equilibrium.

To complete the proof we must therefore show that s_- and s_+ are indeed lowest-consistent with some message. The argument is divided into two separate cases.

Claim 1. *If $s_- = s_+ = \overline{p}$, then s_- and s_+ are lowest-consistent with some message.*

Proof. If $s_- = s_+ = \overline{p}$ then it must be that $(\text{cav}\, w_\beta)(\overline{p}) = w_\beta(\overline{p})$, by definition of s_- and s_+. Suppose otherwise that $\overline{p}$ is not lowest-consistent with any message. Then there exists some $\tilde{s} < \overline{p}$ such that $w_\beta(\tilde{s}) \geq w_\beta(\overline{p})$, since any message available at $\overline{p}$ (including optimal ones) is also available at some lower type. Recall also that, since the sender can prove news better than the prior, there exists some message m_h such that $w_\beta(\min M^{-1}(m_h)) > w_\beta(\overline{p})$ with $s_h \equiv \min M^{-1}(m_h) > \overline{p}$, since v is non-decreasing. Summarizing, we have that $\tilde{s} < \overline{p} < s_h$ with $w_\beta(\tilde{s}) \geq w_\beta(\overline{p})$ and $w_\beta(s_h) > w_\beta(\overline{p})$. Since $\text{cav}\, w_\beta$ is concave and majorizes w_β, it must be that $(\text{cav}\, w_\beta)(\overline{p}) > w_\beta(\overline{p})$, contradicting that $(\text{cav}\, w_\beta)(\overline{p}) = w_\beta(\overline{p})$. □

Claim 2. *If $s_- < \overline{p} < s_+$, then s_- and s_+ are lowest-consistent with some message.*

Proof. We make use of the following intermediate claim, proved after the main argument is complete.

Claim 3. $w_\beta(s_-) < w_\beta(s_+)$.

Suppose first that s_- is not lowest-consistent with any message. Then there exists a $\tilde{s} < s_-$ such that $w_\beta(\tilde{s}) \geq w_\beta(s_-)$. Since $\text{cav}\, w_\beta$ majorizes w_β we have that $(\text{cav}\, w_\beta)(\tilde{s}) \geq w_\beta(\tilde{s})$ and, by definition of s_-, $(\text{cav}\, w_\beta)(s_-) = w_\beta(s_-)$. It follows that $(\text{cav}\, w_\beta)(\tilde{s}) \geq (\text{cav}\, w_\beta)(s_-)$. Observe next that Claim 3 and the definition of s_- and s_+ together imply that $(\text{cav}\, w_\beta)(s_+) > (\text{cav}\, w_\beta)(s_-)$. Summarizing, we have that $\tilde{s} < s_- < s_+$ with $(\text{cav}\, w_\beta)(\tilde{s}) \geq (\text{cav}\, w_\beta)(s_-)$ and $(\text{cav}\, w_\beta)(s_+) > (\text{cav}\, w_\beta)(s_-)$, contradicting that $\text{cav}\, w_\beta$ is concave.

Suppose next that s_+ is not lowest-consistent with any message. Then there exists a $\tilde{s} < s_+$ such that $w_\beta(\tilde{s}) \geq w_\beta(s_+)$. Observe next that the definition of s_- and s_+ and Claim 3 imply that $(\text{cav}\, w_\beta)(s_+) = w_\beta(s_+) > w_\beta(s_-) = (\text{cav}\, w_\beta)(s_-)$. Since $w_\beta(\tilde{s}) \geq w_\beta(s_+)$, we have that

$$(\text{cav}\, w_\beta)(\tilde{s}) \geq (\text{cav}\, w_\beta)(s_+) > (\text{cav}\, w_\beta)(s_-).$$

There are three possibilities. If $\tilde{s} < s_-$, then $\text{cav}\, w_\beta$ is not concave, a contradiction. If $\tilde{s} = s_-$ then $(\text{cav}\, w_\beta)(s_-) > (\text{cav}\, w_\beta)(s_-)$, also a contradiction. If

$\tilde{s} \in (s_-, s_+)$ then $\operatorname{cav} w_\beta$ is not affine over $[s_-, s_+]$, contradicting Lemma 4. It follows that both s_- and s_+ are lowest-consistent with some message, completing the proof of the claim. □

Proof of Claim 3. Suppose otherwise that $w_\beta(s_-) \geq w_\beta(s_+)$. Since the sender can prove news better than the prior, there exists a message m_h such that $v(\min M^{-1}(m_h)) > v(\overline{p})$. Note that $s_h \equiv \min M^{-1}(m_h) > \overline{p}$, since v is non-decreasing. It follows from the definition of w_β that $w_\beta(s_h) > v(\overline{p})$.

Observe next that $w_\beta(s_-) \leq v(\overline{p})$, since for any $m \in M(s_-)$, $\beta(m) \leq s_-$ (recall that $\beta(m) = \min M^{-1}(m)$, by construction) and therefore $v(\beta(m)) \leq v(s_-) \leq v(\overline{p})$. This, in turn, implies that $w_\beta(s_h) > w_\beta(s_-)$. Hence, since $w_\beta(s_-) \geq w_\beta(s_+)$ by hypothesis, we have that $w_\beta(s_h) > w_\beta(s_+)$ also holds. Summarizing, we have established that $w_\beta(s_h) > w_\beta(s_-) \geq w_\beta(s_+)$.

Recall that, by definition, $(\operatorname{cav} w_\beta)(s_-) = w_\beta(s_-)$ and $(\operatorname{cav} w_\beta)(s_+) = w_\beta(s_+)$. There are three possibilities to consider. If $s_h < s_+$, $\operatorname{cav} w_\beta$ is not affine over (s_-, s_+), contradicting Lemma 4. If $s_h = s_+$ then $w_\beta(s_+) > w_\beta(s_+)$, a contradiction. If $s_h > s_+$ then, since

$$(\operatorname{cav} w_\beta)(s_h) > (\operatorname{cav} w_\beta)(s_-) \geq (\operatorname{cav} w_\beta)(s_+),$$

$\operatorname{cav} w_\beta$ is not concave, a contradiction. It follows that $w_\beta(s_-) < w_\beta(s_+)$. □

This completes the proof of Lemma 6. □

Appendix B Proof of Theorem 1

Let (π, μ, β) denote an equilibrium which is not unpersuasive and suppose otherwise that there is an $s'' \in \operatorname{supp} \pi_S$ and a message $m'' \in \mu(\cdot|\pi, s'')$ such that, for some $s' < s''$, $m'' \in M(s')$.

Since (π, μ, β) is not unpersuasive, some on-path message m must lead to a payoff $v(\beta(m)) \neq v(\overline{p})$ for the sender, meaning that $\beta(m) \neq \overline{p}$. It follows that the sender must be acquiring some information in equilibrium, so that $\operatorname{supp} \pi_S \neq \{\overline{p}\}$. Let $\underline{s} \equiv \min \operatorname{supp} \pi_S$ and $\overline{s} \equiv \max \operatorname{supp} \pi_S$; since $\operatorname{supp} \pi_S \neq \{\overline{p}\}$, $\sum_{s \in \operatorname{supp} \pi_S} \pi_S(s) s = \overline{p}$ implies that $\underline{s} < \overline{p} < \overline{s}$. Define the sender's equilibrium interim value from signal realization $s \in [0,1]$ as $w_\beta(s) \equiv \max_{m \in M(s)} v(\beta(m))$.

The argument relies on the following claim, proved after the main argument is complete.

Claim 4. *w_β is affine and strictly increasing on* $\operatorname{supp} \pi_S$.

Suppose first that $s'' \leq \overline{p}$. w_β being affine (from Claim 4) together with π being a feasible signal, meaning that $\sum_{s \in \operatorname{supp} \pi_S} \pi_S(s)s = \overline{p}$, imply that the sender's equilibrium expected payoff can be written as

$$\frac{\bar{s} - \overline{p}}{\bar{s} - s''} w_\beta(s'') + \frac{\overline{p} - s''}{\bar{s} - s''} w_\beta(\bar{s}). \tag{7}$$

Consider now a deviating signal supported on $\{s', \bar{s}\}$ only. By choosing this signal, the sender obtains an expected payoff equal to

$$\frac{\bar{s} - \overline{p}}{\bar{s} - s'} w_\beta(s') + \frac{\overline{p} - s'}{\bar{s} - s'} w_\beta(\bar{s}). \tag{8}$$

To show that (8) is strictly larger than (7) it suffices to note that: $w_\beta(s') \geq w_\beta(s'')$, since m'' is optimal at s'' and $m'' \in M(s')$, by hypothesis; $s' < s''$, by hypothesis; and $w_\beta(\bar{s}) > w_\beta(s'')$, since w_β is strictly increasing on $\operatorname{supp} \pi_S$, by Claim 4. This contradicts that π is an equilibrium signal.

Suppose next that $s'' > \overline{p}$. There are three cases to consider and, in each, a deviation analogous to the one in the previous paragraph can be constructed: (a) if $s' > \overline{p}$, a signal supported on $\{\underline{s}, s'\}$ is a profitable deviation; (b) if $s' = \overline{p}$, a signal supported on $\overline{p}$ only is a profitable deviation; (c) $s' < \overline{p}$, a signal supported on $\{s', s''\}$ is a profitable deviation. This establishes item (i) in the theorem.

To prove item (ii), consider any on-path message m. Item (i) implies that message m is sent by type $\min M^{-1}(m)$ only: $m \in \operatorname{supp} \mu(\cdot|\pi, \min M^{-1}(m))$ and $m \notin \operatorname{supp} \mu(\cdot|\pi, s)$ if $s \neq \min M^{-1}(m)$. As the receiver's beliefs are to satisfy Bayes' rule on the equilibrium path, it must be that $\beta(m) = \min M^{-1}(m)$.

For item (iii), consider any on-path sender type $s \in \operatorname{supp} \pi_S$ and a message $m \in \operatorname{supp} \mu(\cdot|\pi, s)$. Item (i) implies that $s = \min M^{-1}(m)$ and item (ii) implies that $\beta(m) = \min M^{-1}(m)$. Therefore, $\beta(m) = s$. $\square$

Proof of Claim 4. We first prove the 'affine' part. Recall that $\underline{s} \equiv \min \operatorname{supp} \pi_S$, $\bar{s} \equiv \max \operatorname{supp} \pi_S$ and that, since $\operatorname{supp} \pi_S \neq \{\bar{p}\}$, $\sum_{s \in \operatorname{supp} \pi_S} \pi_S(s)s = \bar{p}$ implies that $\underline{s} < \bar{p} < \bar{s}$. Let constants a and b be defined by $w_\beta(\underline{s}) = a + b\underline{s}$ and $w_\beta(\bar{s}) = a + b\bar{s}$.

Suppose that there is some $s \in \operatorname{supp} \pi_S$ with $w_\beta(s) < a + bs$. By appropriately shifting probability mass from s to $\{\underline{s}, \bar{s}\}$, we can construct a signal π' such that $\pi'_S(s) < \pi_S(s)$ which leads to a strictly higher expected payoff for the sender.

Suppose next that there is some $s \in \operatorname{supp} \pi_S$ with $w_\beta(s) > a + bs$. By appropriately shifting probability mass from $\{\underline{s}, \bar{s}\}$ to s we can construct a signal π' with $\pi'_S(s) > \pi_S(s)$ which leads to a strictly higher expected payoff for the sender.

We now turn to the 'strictly increasing' part. Suppose first that w_β is constant on $\operatorname{supp} \pi_S$. It must then equal $v(\bar{p})$, since if it were strictly larger (smaller) the sender would be inducing in the receiver a posterior belief strictly above (below) $\bar{p}$ with probability one (since v is non-decreasing). But if w_β is constant and equal to $v(\bar{p})$ on $\operatorname{supp} \pi_S$, the equilibrium must be unpersuasive, which does not hold by hypothesis.

Suppose next that there exist on-path signal realizations $s, s' \in \operatorname{supp} \pi_S$ such that $s' > s$ and $w_\beta(s') < w_\beta(s)$. Consider any $m' \in \operatorname{supp} \mu(\cdot|\pi, s')$ and any $m \in \operatorname{supp} \mu(\cdot|\pi, s)$. Since v is non-decreasing, it must be that $\beta(m') < \beta(m)$. Since the receiver uses Bayes' rule following on-path messages, it must also be that $m' \in \operatorname{supp} \mu(\cdot|\pi, \tilde{s}')$ for some $\tilde{s}' < s'$ in $\operatorname{supp} \pi_S$, or that $m \in \operatorname{supp} \mu(\cdot|\pi, \tilde{s})$ for some $\tilde{s} > s$ in $\operatorname{supp} \pi_S$, or both. In all cases, there exists a pair of on-path sender types with the same value of w_β. Since we established that w_β is affine on $\operatorname{supp} \pi_S$, it must then be constant on $\operatorname{supp} \pi_S$, contradicting that $w_\beta(s') < w_\beta(s)$. $\square$

Appendix C Proof of Theorem 2

Start with the first part of the statement: we want to show that if PNBP holds then all equilibria are unraveling. Exploiting Theorem 1, it is enough to show that if PNBP holds and an equilibrium is unpersuasive, then it is also an unraveling equilibrium. Let (π, μ, β) denote such an unpersuasive equilibrium. Observe that, since PNBP holds, there exists some m_h such that $v(\min M^{-1}(m_h)) > v(\overline{p})$ and, since v is non-decreasing, we also have that $s_h \equiv \min M^{-1}(m_h) > \overline{p}$.

Suppose first that the sender acquires information, so that $\text{supp}\, \pi_S \neq \{\overline{p}\}$. Let $\underline{s} \equiv \min \text{supp}\, \pi_S$. Since the sender acquires information, $\underline{s} < \overline{p}$. Since the equilibrium is unpersuasive, we have that $v(\beta(m)) = v(\overline{p})$ for every $m \in \text{supp}\, \mu(\cdot|\pi, \underline{s})$. Then the sender can profitably deviate by choosing signal π' with $\text{supp}\, \pi'_S = \{\underline{s}, s_h\}$. So, when PNBP holds, there cannot be unpersuasive equilibria in which the sender acquires information.

Consider now the case in which the sender acquires no information in equilibrium, so that $\pi_S(\overline{p}) = 1$. Suppose that there exists some $m'' \in M(\overline{p})$ such that $\mu(m''|\pi, \overline{p}) > 0$ for which we can find some $s' < \overline{p}$ such that $m'' \in M(s')$. That is: $\overline{p}$ is not lowest-consistent with message m''.

We will show that the sender can profitably deviate by choosing signal π' with $\text{supp}\, \pi'_S = \{s', s_h\}$. Following realization s', the sender obtains a payoff of at least $v(\overline{p})$ (since $m'' \in M(s')$, by construction) and following realization s_h the sender obtains a payoff strictly above $v(\overline{p})$ (since $m_h \in M(s_h)$, $\beta(m_h) \geq s_h$ and therefore $v(\beta(m_h)) \geq v(s_h) > v(\overline{p})$). Hence π' is a strictly profitable deviation, so if (π, μ, β) is an equilibrium in which the sender acquires no information, it must be that $\overline{p}$ is lowest-consistent with every $m \in \text{supp}\, \mu(\cdot|\pi, \overline{p})$. It follows that $\beta(m) = \overline{p} = \min M^{-1}(m)$ for every $m \in \text{supp}\, \mu(\cdot|\pi, \overline{p})$, so that the receiver is maximally skeptical following every on-path message and that the sender reveals all acquired information. The equilibrium is therefore unraveling.

Now to the second part of the statement. Such an unpersuasive equilibrium is constructed in Lemma 5, Appendix A. Note that Lemma 5 does not rely on the continuity requirement stated in Lemma 1, which is not assumed for Theorem 2.

To show that it is sender-preferred, we will show that there is no equilibrium in which the sender obtains an expected payoff strictly larger than $v(\overline{p})$. Suppose that (π, μ, β) is such an equilibrium. It cannot be unpersuasive by definition, so Theorem 1 implies that it must be unraveling. Therefore, Corollary 1 implies that the sender's expected payoff is equal to $(\text{cav}\,\underline{v}_M)(\overline{p})$, where $\underline{v}_M$ is defined in (3). However, if $(\text{cav}\,\underline{v}_M)(\overline{p}) > v(\overline{p})$ there must exist some $s \in [0,1]$ such that $\underline{v}_M(s) > v(\overline{p})$, implying in turn that there exists an $m_h \in \mathcal{M}$ such that $v(\min M^{-1}(m_h)) > v(\overline{p})$, i.e., that the sender can prove news better than the prior, a contradiction. $\square$

Appendix D Proof of Theorem 3

We prove first Lemma 2 and then turn to the proof of Theorem 3. Recall that in Section 4 we are maintaining the condition stated in Lemma 1, which ensures equilibrium existence.

D.1 Proof of Lemma 2

Start with the $\Rightarrow$ direction. We use a replication argument which exploits Theorem 1. Consider first the triple $(v, \overline{p}, M')$ and any unraveling equilibrium signal π', which exists by hypothesis and leads to payoff $\underline{V}^*_{M'}(v, \overline{p})$. This is because, if PNBP holds, every equilibrium is unraveling (by Theorem 2) and the sender's expected payoff is unique (by Corollary 1). If PNBP does not hold, the unraveling equilibrium expected payoff does not exceed the unpersuasive equilibrium payoff, from Theorem 2.

We will show that, in any equilibrium with parameters $(v, \overline{p}, M'')$, the sender's expected payoff from π' is at least as large as $\underline{V}^*_{M'}(v, \overline{p})$. This, in turn, means that the expected payoff in any equilibrium with parameters $(v, \overline{p}, M'')$ must also be at least as large as $\underline{V}^*_{M'}(v, \overline{p})$, completing the argument.

Observe that in any unraveling equilibrium at parameters $(v, \overline{p}, M')$, Theorem 1 implies that for any equilibrium signal π' at any $s \in \text{supp}\,\pi'_S$

the sender obtains a continuation payoff of $v(s)$. Since s is lowest-consistent with some message under M' and $L_{M''} \supseteq L_{M'}$, s is also lowest-consistent with some message under M''. This in turn implies that, for any equilibrium (unraveling or not) (π'', μ'', β'') when parameters are $(v, \overline{p}, M'')$, $\max_{m \in M''(s)} v(\beta''(m)) \geq v(s)$ for every $s \in \operatorname{supp} \pi'_S$. Hence the sender's expected payoff from π' with parameters $(v, \overline{p}, M'')$ is at least as large as $\underline{V}^*_{M'}(v, \overline{p})$, as required.

Now to the $\Leftarrow$ direction. We will prove the contrapositive, i.e., that $M'' \not\supseteq^{\text{lc}} M'$ implies $\underline{V}^*_{M''}(v, \overline{p}) < \underline{V}^*_{M'}(v, \overline{p})$ for some admissible v and $\overline{p}$ such that at $(v, \overline{p}, M')$ an unraveling equilibrium exists. Observe first that $M'' \not\supseteq^{\text{lc}} M'$ implies that $L_{M'} \neq \{0\}$, as $0 \in L_M$ for any M. Since $L_{M'} \neq \{0\}$, there exists some $s^* \in L_{M'}$, $s^* > 0$ such that $s^* \notin L_{M''}$.

Take $v(p) = 1_{\{p \geq s^*\}}$ and $\overline{p} = s^*/2$. Let $\underline{v}_{M'}$ and $\underline{v}_{M''}$ denote the value of posteriors with maximal receiver skepticism as defined in equation (3) when the verifiability structures are M' and M'', respectively. With parameters $(v, \overline{p}, M')$ PNBP holds, so all equilibria are unraveling (Theorem 2) and lead the sender to obtain an expected payoff equal to her full-commitment payoff of $(\operatorname{cav} v)(\overline{p}) = 1/2$ (this follows from observing that $(\operatorname{cav} v)(\overline{p}) = (\operatorname{cav} \underline{v}_{M'})(\overline{p})$ and applying Corollary 1).

This expected payoff is not attainable in any equilibrium under M''. If the sender cannot prove news better than the prior under M'', this follows from Theorem 2, as the largest expected payoff for the sender in equilibrium is $v(\overline{p}) = 0 < 1/2$ in this case. If the sender can prove news better than the prior under M'', this follows because all equilibria must be unraveling (from Theorem 2), so $s^* \notin L_{M''}$ implies that s^* cannot be on-path. However, in an unraveling equilibrium, s^* must be on-path if the sender is to obtain an expected payoff of $(\operatorname{cav} v)(\overline{p})$. □

D.2 Proof of Theorem 3

For the $\Rightarrow$ direction, observe that if at $(v, \overline{p}, M')$ the sender can prove news better than the prior, then an unraveling equilibrium exists (Theorem 2) and the equilibrium expected payoff is unique (Corollary 1), i.e., $\underline{V}^*_{M'}(v, \overline{p}) =$

$\overline{V}^*_{M'}(v,\overline{p}) = (\text{cav}\,\underline{v}_{M'})(\overline{p})$. Lemma 2 therefore implies that $\underline{V}^*_{M''}(v,\overline{p}) \geq \overline{V}^*_{M'}(v,\overline{p})$. As $M'' \succeq^{\text{lc}} M'$, the sender can also prove news better than the prior at $(v,\overline{p},M'')$, so that $\underline{V}^*_{M''}(v,\overline{p}) = \overline{V}^*_{M''}(v,\overline{p}) = (\text{cav}\,\underline{v}_{M''})(\overline{p})$ (from Theorem 2 and Corollary 1), completing this part of the argument.

Now to the $\Leftarrow$ direction. We will prove the contrapositive, i.e., that $M'' \not\succeq^{\text{lc}} M'$ implies $\overline{V}^*_{M''}(v,\overline{p}) < \overline{V}^*_{M'}(v,\overline{p})$ for some admissible v and $\overline{p}$ such that at $(v,\overline{p},M')$ PNBP holds. Observe that the construction used in the proof of Lemma 2 in fact considers a triple $(v,\overline{p},M')$ such that the sender can prove news better than the prior and shows that the unique equilibrium expected payoff at $(v,\overline{p},M')$ is not attainable in any equilibrium under M''. □

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