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PhD THESIS

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Essays on Monetary Policy Before and After the Crisis

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Abstract

In Chapter 1 I briefly introduce the issues that will be studied in Chapter 2 and 3. In Chapter 2 I introduce a macroprudential policy for the cap on debt-to-income (DTI) ratio in a model which is estimated over the period of the build-up of household debt occurred in US before the financial crisis. The optimal macroprudential policy requires a more important role for labor income in credit supply decision and a strong countercyclical response of the cap on DTI to household debt. I find that this optimal macroprudential policy is successful in stabilizing household debt, is beneficial in terms of social welfare and is desirable as a complement for monetary policy, when this is enforced as a standard Taylor rule. I then consider also a monetary policy that can "lean against the wind" of a credit boom to pursue financial stability. It turns out that this policy is welfare-dominated by the strategy of assigning this goal to a macroprudential authority committing to optimally implementing the cap on DTI. However, the best-performing policy is a combination of "leaning against the wind" strategy and macroprudential policy.

In Chapter 3 I study optimal government spending and monetary policy in an economy hit by a liquidity shock, which may generate recession and deflation. I find that the optimal policy mix implies a money-financed fiscal stimulus, which is shaped as a one-period countercyclical fiscal stimulus along with a prolonged central bank's balance-sheet expansion. By comparing this optimal policy with other suboptimal policies we uncover several facts. First, an unconventional monetary policy performs unambiguously better when accompanied by a fiscal stimulus. Second, financing the stimulus with only public debt brings about long-lasting recession and deflation. Third, "active" monetary policies, like the standard Taylor rule, "inflation targeting" and "nominal GDP targeting" are efficient policies if the increase in money supply brought about by these policies is complemented with an optimal fiscal stimulus.

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I wish to dedicate my efforts to my whole family and all my dearest friends, in particular to Luca.

All errors are and remain my own.

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Chapter 1

Introduction

The thesis is composed by two papers, each dealing with the optimal conduct of monetary policy. However, the time of the two is different, as they refer to two distinct phases of the financial crisis. Specifically, Chapter 2 deals with the phase occurring prior to the burst of the crisis, and thus is related to the policy prescriptions that monetary authority is called upon to undertake in order to avert harmful consequences when the crisis actually explodes. Chapter 3 instead focuses on the phase subsequent to the crisis explosion and therefore when monetary policy is called upon to put in place policies aimed at dampening the negative effects caused by the crisis, that are recession and deflation.
Hence, my research deals with two policy prescriptions that, if undertaken, could have prevented the worst of the crisis to happen (in Chapter 2) and might help to end recession and bring the economy back to growth (in Chapter 3).

The actual financial crisis has given rise to many debates concerning the opportunity of introducing a particular policy aimed at preserving the economic system as a whole, in order to protect it against systemic risks: this policy is better known with the name of "macroprudential"[1]. In fact, since the years immediately following the burst of the financial crisis, many prestigious economists (for instance Blanchard et al. (2013)[10] have urged the necessity to "rethink" macroeconomic policy so that it may encompass even "prudential" issues[2]. Therefore,

[1] Macroprudential policy is defined "a policy that uses primarily prudential tools to limit systemic or system-wide financial risk, thereby limiting the incidence of disruptions in the provision of key financial services that can have serious consequences for the real economy, by dampening the build-up of financial imbalances and building defences that contain the speed and sharpness of subsequent downswings and their effects on the economy; identifying and addressing common exposures, risk concentrations, linkages and interdependencies that are sources of contagion and spillover risks that may jeopardize the functioning of the system as a whole" (Financial Stability Board, Bank for International Settlements and International Monetary Fund (2011) [27]).

[2] There exists a large number of papers originated from the increasing interest in the measures studied to avoid a new burst of a financial crisis like the one experienced some years ago. Many of them analyse the measures put ahead by the Basel III committee, established with the primary goal of ensuring the stability of the banking system. See Introduction in Chapter 2 for some references.

in light of the detrimental consequences of the crisis, it has been evenly acknowledged that primary goal of authorities should be that of monitoring financial stability. However, there is no unanimous consensus about which should be the authority in charge of financial stability.
In this regard, many commentators have called into question the standard role of monetary policy, arguing that central banks could do also the job of safeguarding financial stability. According to this view, central bank could "lean against the wind" of a credit boom, raising the policy rate to cushion the leveraging growth and thus prevent systemic risk.
Oppositely, there is the view suggesting that the role of financial stability should be assigned to a macroprudential authority designed on purpose, which establishes macroprudential rules with the goal of monitoring financial stability. In this case, macroprudential policies would be a complement of the standard conduct of monetary policy.
In Chapter 2, I challenge this latter view, investigating the impact of a specific macroprudential policy like the countercyclical cap on debt-to-income ratio (DTI). Drawing an economy that may experience a credit boom as occurred in US before the crisis, where the extraordinary high level of household debt has been driven by growth in house prices, I assess whether this macroprudential instrument would help pursue financial stability or if instead this task would be better achieved by monetary policy. With my research I seek to judge whether, in case of a credit boom, complementing monetary policy with a well-designed macroprudential policy like the countercyclical cap on DTI would be beneficial or not for the economy.

In Chapter 3 I study the stabilization effect of a specific macroeconomic policy - a money-financed fiscal stimulus - a policy that has now regained new interest and appeal.
After the turmoil caused by the burst of the financial crisis and in order to bolster the economy, central banks decided to promptly intervene by adopting large scale asset purchases, injecting huge amount of monetary base in the system in order to stimulate the economy. However, it is common knowledge that these measures have not been as successful as expected in bringing output back to potential levels and inflation to the target, in countries like Japan and also in the Eurozone[3].
Unconventional monetary policies worldwide have not been accompanied with expansionary fiscal policy, owing to the increasing fears of too-high levels of public debt, that instead have led to pursue policies aimed at reducing it[4]. This occurs in spite of the fact that a large strand of

[3] In this regard, Bank of Japan's chief Harada argued that, in spite of the large-scale implementation of unconventional monetary policies, Japanese recovery is still weak, as output has increased little and inflation has stagnated (see "Economic Activity and Prices in Japan, and Monetary Policy", speech at a meeting with business leaders in Yamaguchi Prefecture, April 13th 2013). Furthermore, a recent Bank of Japan's document reports that the expected year-to-year CPI is likely to be negative or close to zero for time being until the end of 2016 ("Outlook for Economic Activity and Prices (July 2016)").

[4] As for Japan, one one of the "arrows" of Prime minister Abe's program is indeed fiscal consolidation.

the prominent monetary economics' literature has unambiguously proven that when policy rates have reached the zero lower bound fiscal policy becomes more effective (Christiano, Eichenbaum and Rebelo (2011)[15], Eggertson (2010)[22] Eggertson and Krugman (2012)[23]).

Therefore, as a consequence of this gloomy scenario, some commentators have begun advocating another unorthodox monetary policy measure, consisting in financing a fiscal stimulus, and thus the implied increase in deficit, with issuance of money, without relying on public-debt financing[5]. This idea, which dates back to the famous "helicopter money" (Friedman (1969)[28]), was revived more recently, when Japan fell in a liquidity trap in the 90's. One of the most prominent advocate of this idea at that time was the former Fed chief Ben Bernanke [8], who had a speech in 2003 in which he argued for an expansionary fiscal policy *"explicitly coupled with incremental Bank of Japan purchases of government debt"*, so that the expansion of deficit would be in effect financed by money creation[6]. He also said that a fiscal stimulus which is *"accommodated by a program of open-market purchases to alleviate any tendency for interest rates to increase, would almost certainly be an effective stimulant to consumption and hence to prices"*. According to this view, this policy measure should have a "double effect", in the sense that the stimulus to demand brought about by the expansionary fiscal policy would be reinforced by a specific monetary policy pointed to keep the interest rate low so as to exacerbate the aggregate demand stimulus[7]. This fascinating idea has been recently formalized by Galì (2017)[30], who has built a standard New Keynesian DSGE model to show that this policy has indeed beneficial effects, both in terms of the fiscal multiplier and in terms of welfare. In his paper, money-financed fiscal stimulus is introduced as a policy rule, which is implicitly based on a strong commitment treasury-central bank.

Building on these insights, my research question is to investigate whether a money-financed fiscal stimulus becomes, in fact, an optimal policy, among the various alternatives a policymaker can make use of. In other words, I am interested in evaluating whether an optimal commitment treasury-central bank involves indeed a money-financed fiscal stimulus, in case of a shock that may lead to deep recession and deflation.

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[5] Although object of the chapter is not that of discussing the way it could be implemented, I point out that, due to the recent facts, the most viable way to adopt this policy to date might be the case of a specific commitment between government and central bank, such that the fiscal stimulus put ahead by the government is financed by issuance of government bonds in the secondary market and these are purchased by central bank according to a program of "unconventional monetary policy" (this policy can be called "quantitative easing plus fiscal stimulus"). The final outcome of this process is that the increase in fiscal deficit is in fact accompanied by expansion of monetary base. Galì (2017)[30] stressed that a money-financed fiscal stimulus may give rise to legal problems, because such policy may undermine central bank's independence, one the soundest pillars of the current economic system.

[6] In another speech in 2002 Bernanke said that a *"money-financed tax cut is essentially equivalent to Friedman's famous "helicopter drop" of money"* (Bernanke (2002) [7]).

[7] For more references on the idea of money-financed fiscal stimulus see the Introduction of Chapter 3.

Chapter 2

Macroprudential Cap on Debt-to-Income Ratio and Monetary Policy

2.1 Introduction

A wide range of papers in the post-crisis literature argue that one of the most important cause of the financial crisis is the huge build-up in households debt which occurred before the burst of the crisis. Several studies have also underlined the connection of high levels of households debt with the turmoil subsequent to the crisis[1].

This fact has called into question the stance of monetary policy over the years prior to the crisis. It is known that the Fed did not react to the extraordinarily high accumulation of household debt that took place before the crisis. Contrasting this stance, many commentators argue that central bank should have done something to avoid such large accumulation of debt: thus, it came the proposal of "leaning against the wind" of over-borrowing, according to which Fed should have raised the interest rate to break the credit boom. Some others instead believe that the best option would be the implementation of macroprudential policies designed with the purpose of safeguarding financial stability, so that central bank would remain engaged in its primary goal of inflation targeting ("two instruments for two goals").

In this regard, several macroprudential policies can be used to pursue financial stability[2]. In this

[1]For instance, Glick and Lansing (2010) [32] prove that there exits a negative correlation between the overall amount of households debt prior to the crisis and consumption levels thereafter, emphasising the burden of private debt on the recovery prospectives. Some other, as Mian and Sufi (2010) [56], instead observe the positive correlation arising in US between growth in household debt and unemployment rate, thus demonstrating the detrimental effect of excessive values of household debt.

[2]For a complete treatment of macroprudential instruments and its related literature refer to IMF(2013)[29], Bank of England(2011) [61] and Financial Stability Board, Bank for International Settlements and International

paper we study the macroeconomic consequences of the using the cap on debt-to-income ratio as a macroprudential instrument. The idea to move the cap on DTI in a countercyclical way arises precisely with the purpose of avoiding a new build-up of private leveraging that would undermine the financial system and presumably lead to a new financial crisis. This explains why this tool is "macroprudential", in the sense that it aims at protecting against systemic risks. Some recent empirical evidence has documented that this macroprudential policy is effective in restricting the amount of loans that can be requested and is expected to smooth the credit cycle so as to yield greater resilience of households and a lower probability of default[3].

With this paper we investigate several aspects of this macroprudential instrument. Firstly, we ask whether the implementation of the macroprudential cap on debt-to-income (DTI) ratio is beneficial for the economy and ensures financial stability. Secondly, we focus on the relation with monetary policy, assessing whether implementing a macroprudential cap on DTI increases the effectiveness of the conduct of monetary policy. Finally, we answer to the main question of the research: should we adopt macroprudential policy during a credit boom or instead we can call for a more important role of monetary policy ?

We proceed as follows. We estimate a DSGE model over the years before the crisis, when the growth of household debt-to-GDP ratio has taken place. The model is built so as to explain the credit boom, so that big emphasis is on one of the main determinant of the credit boom, i.e. the housing prices boom. In fact, it is widely recognized in the literature that the soar in household debt has been accompanied by a huge increase in house prices. This is shown in Figure 2.1: it can be clearly seen that the growth of households debt as a percentage of gross disposable income (solid blue line) has been tracked quite well by the increase in house prices (dashed black line) until the end of 2005. Crucially, since we are interested in the credit boom phase, in the paper we consider data series ranging from 1991:3 through 2005:4. In this sample, the correlation between household debt-to-disposable income and house prices amounts to 0.9953, implying that these two series have strongly moved in the same direction over the period considered[4]. Importantly, the soar of these two variables noticeably outweighed the increase in both consumption and real wages, thus rendering the credit boom a clear event.

As in other papers dealing with the credit boom, we model an economy featuring a collateral borrowing constraint. The borrowing constraint we propose represents a novelty. In the liter-

Monetary Fund (2009) [26].

[3]Refer, among the others, to Jacome and Mitra (2015) [38] Lim at al. (2011) [50], IMF (2013) [29], Vandenbussche et al.(2012) [78], Shim et al. (2013) [69].

[4] Concerning the strong relation between house prices and household debt, several authors (Justiniano et al. (2015) [41], Dynan (2012) [20], Mian and Sufi (2011) [57], Reinhart and Rogoff (2008) [64], Hatzius (2008) [35], Shiller (2007) [68]) argue that main cause of the credit boom is the huge appreciation of real estate values that, through appreciation of the collateral, fuelled credit supply. According to this view, main responsible of the soar in household debt is therefore the "valuation" of real estate values. In this paper, we consider this factor as the one which gives rise to the credit boom phase.

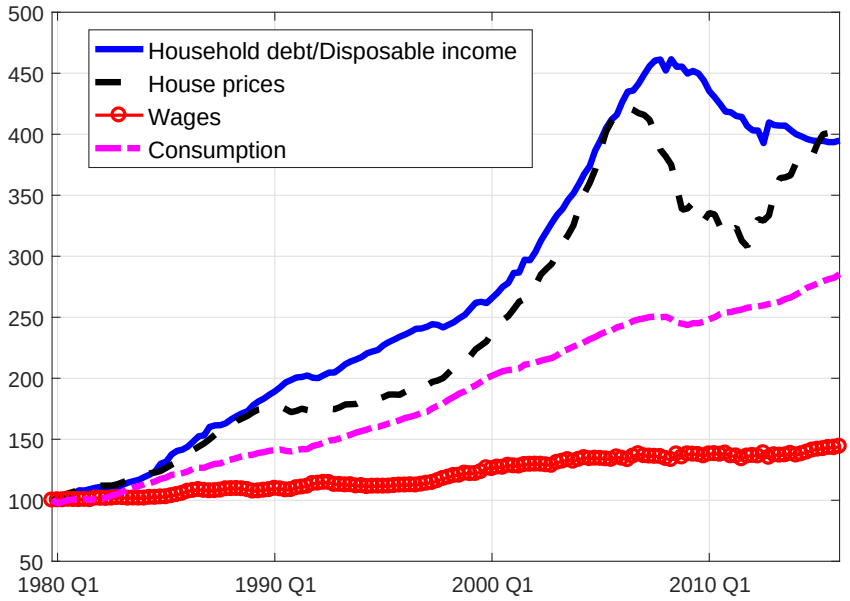

Figure 2.1: Households debt as a percentage of gross disposable income is obtained by summing the series "Households and Nonprofit Organizations; Consumer Credit" (HCCSDODNS) to "Households and Nonprofit Organizations; Home Mortgages" (HMLBSHNO) and dividing the sum by "Real Disposable Income" (DPIC96). The series of house prices is the Standard and Poor/Case-Shiller U.S. National Home Price Index (CSUSHPINSA). Consumption is the series Real Personal Consumption Expenditure (PCECC96). Wages is the series Nonfarm Business Sector: Real compensation per hours (COMPRNFB). Both are indexed so that 1980=100. 1979:Q4-2016:Q1. Source: Federal Reserve Bank of St. Louis.

ature, liquidity constraints are usually designed such that borrowing is either collateralized to real estate values (see Iacoviello (2005) [36]) or instead is backed by current-period labor income (Mendoza (2002) [55]). Importantly, in this paper we combine the two, so that the amount of debt borrowers are allowed to take on is tied to both their real estate values and their labor income. We think that this is a realistic feature as it more closely resembles what happens in practice in credit supply decision. Estimating the model over the years of the pre-crisis credit boom reveals that borrowers has been almost entirely tied to real estate values, leaving only a marginal role to borrowers' labor income. This implies that the amount of household debt has been strictly connected to the upswings occurred in the housing market before the crisis.

Building on this scenario, we introduce the macroprudential policy of our interest. Importantly, we depart from the literature in the sense that the macroprudential policy involves two policy instruments. The first policy instrument is the standard countercyclical reaction of the macroprudential instrument to the deviation of financial variables around the steady state. In our framework this implies that the cap on debt-to-income is assumed to react to deviation of household debt in a countercyclical fashion. The second instrument has to do instead with the way credit supply is implemented in the economy. Specifically, we assume that the authority may establish to what extent borrowers' variables can be pledged when requesting new debt.

In other words, the authority may affect whether household debt has to be tied to standard collateral values, like real estate values, or instead to borrowers' labor income. By this virtue, we conjecture that, given the pre-crisis experience, it may well be the case the that a macroprudential authority may find it desirable to reduce the incidence of collateral values and therefore increase the one of labor income[5].

Thus, by optimizing these two macroprudential instruments, we find that the optimal macroprudential policy involves a larger incidence of labor income in credit supply decision which leads to a less pronounced impact of the financial amplification mechanism. Importantly, this policy produces an improvement in social welfare and greater financial stability with respect to the estimated model. We show that a compelling general equilibrium effect is the fact that the presence of this macroprudential policy reflects into an increase in the "shadow price" of borrowing (or the marginal cost of borrowing), which discourages households from requesting new debt.

Our analysis allows to focus on the coexistence of macroprudential policy and monetary policy, and to draw important implications concerning the conduct of the latter during a credit boom. In particular, we consider three cases of interaction monetary-macroprudential policy. In the first, monetary policy is in form of a standard Taylor rule, such that central bank may react to movements in inflation rate and output. We therefore assume that both authorities coordinate so as to optimally search for their respective policies. Result is that a standard Taylor-type monetary policy rule performs unambiguously better in stabilizing the economy when accompanied with the macroprudential cap on DTI. In the second exercise, we allow monetary policy to take into account also financial variables, so that central bank can implement the policy of "leaning against the wind", namely raising the policy rate to tackle the soar of household debt. In this way we are able to compare the "leaning against the wind" policy with the strategy of "two instruments for two goals". Remarkably, we obtain that this latter strategy delivers higher social welfare. Finally, we consider the case of a combined policy that puts bigger effort in stabilizing household debt: the policy of "leaning against the wind" is complemented with macroprudential policy. This policy turns out to be the best-performing one in terms of social welfare and in stabilizing household debt. Importantly, the most effective stabilization of household debt comes at the cost of larger variability in output gap and inflation, thus implying that in a credit boom the standard objectives of monetary policy should "lend a hand" to financial stability.

The chapter is organized as follows. Section 2.2 presents our benchmark model, that is the one in which there is no macroprudential policy. Section 2.3 reports the estimation details. The macroprudential instrument of our interest is illustrated in Section 2.4, where we show also

[5]From a practical point of view, we speculate that this can be achieved by a specific regulation of banking sector.

its welfare implications. Section 2.5 focuses on the interaction between monetary policy and macroprudential policy. Section 2.6 discusses other issues related to our macroprudential policy. Section 2.7 summarizes results and concludes.

2.1.1 Related literature

The paper is mostly related to works that investigate the welfare effects of macroprudential policies, also in interaction with the optimal conduct of monetary policy. Kannan et al. (2012) [42] build a model with financial accelerator mechanism that produces credit boom and growth in house prices and show that implementing a macroprudential policy improves macroeconomic stability, if the macroprudential instrument is specifically targeted to reduce credit cycles and the economy is hit by financial shocks. They use a macroprudential policy designed as a rule according to which central bank reacts to increasing levels of debt. Lambertini et al. (2013) [49] study the potential benefits of rules for the loan-to-value (LTV) in a model in which the build-up of borrowing is motivated by news shocks. Main result is that for this macroprudential policy to be optimal, the loan-to-value must react to financial variables in a countercyclical way. Carrasco-Galego and Rubio (2014) [14] show with a general equilibrium model that the coordination between a macroprudential policy like the countercyclical LTV rule and monetary policy is welfare-improving for the society. Angelini et al. (2012) [1] use ad-hoc loss function to show that when an economy is affected by financial shocks (i.e. when the economy is not in "normal times") a countercyclical LTV rule (or countercyclical capital requirement) is effective in stabilizing output if this policy is implemented in cooperation with monetary policy. They conclude that macroprudential policies may complement the stabilization role provided by monetary policy.

Importantly, none of these papers deals with the macroprudential instrument of our interest: the countercyclical cap on debt-to-income ratio. With the present paper we also seek to fill this gap. Furthermore, the framework we propose allows also a comparison and a combination between this macroprudential policy and the standard LTV policy, discussed in the above papers.

The idea of adopting macroprudential policies has been also deeply motivated in general equilibrium models. The prominent literature that has been studying this issue argues that collateral constraints drive to a "pecuniary externality", as agents do not internalize the consequences of borrowing on the aggregate economy (see, among the others, Bianchi (2011) [9], Mendoza (2002) [55], Benigno et al. (2011) [4]). This negative externality can be tackled by adopting macroprudential policies, as these measures help prevent over-borrowing.

From a modelling perspective, the paper is related to works that assign a role to housing both as a durable good and as collateral for borrowers. In this field, Iacoviello (2005)[36] builds a mon-

etary business cycles model featuring housing in the utility function and heterogeneous agents, where entrepreneurs are subject to a liquidity constraint tied to real estate values. His estimates show that housing preference shock leads to a financial accelerator mechanism through the collateral constraint, which allows to match the positive response of nominal spending observed on US data. Iacoviello and Neri (2010)[37] estimate a DSGE model with housing in the utility function to show that housing demand shock is an important driver of the business cycle. By the same token, Guerrieri and Iacoviello (2016)[34] add occasionally-binding liquidity constraint and zero lower bound to a similar model, showing that housing preference shocks lead to an asymmetric impact, depending on whether the shock is positive or negative. We propose a model that shares some important features with these works, like housing in the utility function and the presence of savers and borrowers. However, main departure is that we include a role for labor income in credit supply decision. Liu et al. (2013)[51] introduce a collateral constraint in firms' investment decision to account for the positive comovement between land prices and business investment. Favilukis et al. (2010)[25] combine collateral constraint with both idiosyncratic and aggregate business cycle risk to show the impact of relaxing the collateral constraint on the propagation of shocks.

Finally, this paper is more broadly related to works that have assigned a primary role to financial shocks and financial friction in explaining the Great Recession. Some relevant examples include Del Negro et al. (2011)[19], Jermann and Quadrini (2012),[40], Christiano, Motto and Rostagno (2014)[16].

2.2 The model

In this section we describe the baseline model, which does not involve the macroprudential policy object of our analysis[6]. This model represents the benchmark of our analysis, which is needed in order to compare the effect of the implementation of the macroprudential instrument.

We set up a model that is able to replicate the progressive accumulation of debt and the soar in house prices that took place in US in the years pre-crisis (see Figure 2.1). The model is a New Keynesian DSGE model featuring borrowers and savers, where borrowers face a credit constraint in which the amount of borrowing depends on real estate value (standard lending collateral) and borrowers' labor income. This last feature makes a role for the macroprudential policy of our interest[7].

In the economy there exists a continuum of households, split between savers and borrowers.

[6]The macroprudential cap on debt-to-income (DTI) ratio is introduced in Section 2.4.

[7]For a seminal work on collateral credit constraint see Kiyotaki and Moore (1997) [45].

Throughout we label with "s" the savers and "b" the borrowers. They differ for the discount factor, as borrowers are more impatient so that they discount future at a lower discount factor: $\beta_b < \beta_s$. Savers and borrowers' maximization problem are object of the following subsections.

2.2.1 Savers

Savers are provided with the following life-time expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta_s^t z_t \left[\Gamma^s \log\left(C_t^s - \gamma^s C_{t-1}^s\right) + j_t \log(H_t^s) - \frac{(N_t^s)^{1+\psi}}{1+\psi} \right] \tag{2.1}$$

where C_t^s is a standard Dixit-Stiglitz aggregator for consumption, which aggregates the savers' demand for goods over all i goods in the economy. In order to account for the path of consumption observed in data we introduce habit in consumption, that is captured by the parameter γ^s. The parameter $\Gamma^s \equiv 1 - \gamma^s$ instead ensures that the steady state of the marginal utility of consumption is $1/C^s$. Variable N_t^s denotes hours of labor supplied by savers whereas H_t^s represents housing services, which enter the utility function scaled by the exogenous process j_t. This can be interpreted as an institutional or exogenous modification of resources that leads agents to purchase houses with respect to other goods (see Iacoviello and Neri (2010) [37]). Another source of shock, common in the literature, is the preference shock z_t, hitting the whole utility function. We assume that these shocks follow AR(1) processes:

$$\log(j_t) = (1-\rho_j)j + \rho_j \log(j_{t-1}) + u_t^j \tag{2.2}$$

$$\log(z_t) = (1-\rho_z)z + \rho_z \log(z_{t-1}) + u_t^z \tag{2.3}$$

where u_t^z, u_t^j are i.i.d. innovations with variance σ_z^2, σ_j^2.
Savers' per-period budget constraint is written in the following way:

$$P_t C_t^s + P_t I_t + B_t^s + P_t Q_t (H_t^s - H_{t-1}^s) = W_t^s N_t^s + B_{t-1}^s R_{t-1} + P_t R_t^k K_{t-1} + \int_0^1 \Psi_t dj \tag{2.4}$$

where $B_t^s > 0$ are one-period assets in nominal terms held at the end of period "t", and B_{t-1}^s are assets carried over from the previous period, which accrued the gross nominal interest rate R_{t-1}. Savers earn labor income, where W_t^s is nominal wage, paid by firms. Therefore, for this level of wage savers stand ready to supply all the amount of labor that is demanded by firms. We assume that savers solely own firms, so that Ψ_t are profits gained by owing firm's shares (a continuum of size 1 of firms). Savers accumulate capital from the previous period K_{t-1} and invest resources I_t in new capital, where R_t^k is the gross return of capital. Each period savers

purchase new housing H_t^s, so that $(H_t^s - H_{t-1}^s)$ is the variation of housing services within the period. Q_t is real housing price in unit of consumption and P_t is the aggregate price level. Capital evolves over time according to this law of motion:

$$K_t = a_t \left(I_t - \frac{\phi}{2} \left(\frac{I_t - I_{t-1}}{I} \right)^2 \right) + (1-\delta) K_{t-1} \tag{2.5}$$

where a convex adjustment cost is required to increase capital from one period to another. Capital depreciates at a constant rate δ. We include an exogenous disturbance a_t in capital accumulation that allows capital to increase (or decrease) for a given level of investment. This technology shock is also assumed to follow an AR(1) process:

$$\log(a_t) = (1-\rho_a)a + \rho_a \log(a_{t-1}) + u_t^a \tag{2.6}$$

where $u_t^a \sim N(0, \sigma_a^2)$.

Hence, savers' maximization problem consists of maximizing utility (2.1) under the budget constraint (2.4) and the law of capital accumulation (2.5). This problem is solved by taking the first-order conditions with respect to the control variables, that are consumption, housing, labor, investment, capital and assets. These can be combined so as to yield:

$$u'(C_t^s) = \beta_s R_t E_t \left[\frac{u'(C_{t+1}^s)}{\Pi_{t+1}} \right] \tag{2.7}$$

$$Q_t u'(C_t^s) = j_t h'(H_t^s) + \beta_s E_t \left[Q_{t+1} u'(C_{t+1}^s) \right] \tag{2.8}$$

$$\frac{v'(N_t^s)}{u'(C_t^s)} = w_t^s \tag{2.9}$$

$$u'(C_t^s) q_t^k = \beta_s E_t \left[u'(C_{t+1}^s)((R_{t+1}^k + q_{t+1}^k) \right] \tag{2.10}$$

$$a_t q_t^k u'(C_t^s) \left(1 - \frac{\phi \Delta I_t}{I} \right) = u'(C_t^s) - \beta_s E_t \left[u'(C_{t+1}^s) a_{t+1} q_{t+1}^k \frac{\phi \Delta I_{t+1}}{I} \right] \tag{2.11}$$

where $u'(C_t^s) = \Gamma^s z_t / (C_t^s - \gamma^s C_{t-1}^s)$, $h'(H_t^s) = 1/H_t^s$, $v'(N_t^s) = (N_t^s)^\psi$, defining the gross inflation rate as $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and real wage $w_t^s = W_t^s / P_t$.

Equation (2.7) is the standard Euler equation, that characterizes the intertemporal substitution of consumption. Equation (2.8) is the housing demand equation, whereas (2.9) is the marginal rate of substitution between consumption and labor. Equations (2.10)-(2.11) describe, respectively, the equilibrium conditions for the price of capital and the demand for investment. We turn now to the borrowers' problem.

2.2.2 Borrowers

Borrowers maximize the following life-time expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta_b^t z_t \left[\Gamma^b \log\left(C_t^b - \gamma^b C_{t-1}^b\right) + j_t \log\left(H_t^b\right) - \frac{(N_t^b)^{1+\psi}}{1+\psi} \right] \tag{2.12}$$

where C_t^b is the Dixit-Stiglitz aggregator defined as in the savers' problem. As for savers, there is habit in consumption and $\Gamma^b \equiv 1-\gamma^b$ implies a steady state value of the borrowers' marginal utility of consumption equal to $1/C^b$. Further, borrowers' total utility function is affected by the same preference shock specified above.

Borrowers' budget constraint shows:

$$P_t C_t^b + B_{t-1}^b R_{t-1}^b + P_t Q_t (H_t^b - H_{t-1}^b) = W_t^b N_t^b + B_t^b \tag{2.13}$$

where $B_t^b > 0$ denotes one-period debt (loans), received by the private sector (savers) in period "t". Borrowers purchase an amount H_t^b of houses each period.

Crucially, borrowers are liquidity constrained. The constraint implies that at any period borrowers can request an amount not greater than a fraction (loan-to-value ratio) of the current-period value of their real estate value and of a given value (cap on debt-to-income ratio) of their current-period labor income:

$$B_t^b \leq \gamma m P_t Q_t H_t^b + (1-\gamma)\tau W_t^b N_t^b \tag{2.14}$$

where m is the fraction set as loan-to-value ratio of the collateral and τ is the cap on debt-to-income ratio (DTI). This constraint implies that the amount of new borrowing in the period is directly connected to both the standard collateral for mortgages, namely real estate value, and labor income, representing the only source of income in the model.

This constraint is a novelty and so deserves an explanation. In the literature liquidity constraint are designed in a way that the amount of borrowing is no greater than either a fraction of real estate values, as in Iacoviello (2005)[36], or a fraction of current period labor income (Mendoza (2002)[55]). In this paper we put them together, so that borrowing depends on both. This is somewhat consistent with reality, as banks and financial companies supply loans taking into account both the collateral pledged by those requesting the loans (if this is a mortgage) and the borrower's current labor income, revealing this a good mirror of the solvency of the applicant. In this regard, Jappelli (1990) [39] is an empirical study unveiling that the credit supply is highly connected with borrowers' personal income. Further, after the financial crisis there is an

increasing trend towards using labor income as necessary criterion in the credit supply decision. As an example, in US the "Consumer Financial Protection Bureau" have established that, since January 2014, borrowers who already have a debt-to-income (DTI) ratio higher than 43% may not get a "Qualified Mortgage", so that large importance is assigned to this measure for judging whom is really worthy of getting such loan. This ratio is computed taking into account debt in a broad sense, composed by all the monthly recurring payments, for principal and interest of mortgages, property taxes, credit cart payments, car loan payments, student loan payments and other minor payments[8].

The borrowing constraint (2.14) can also be interpreted in terms of all loans being split between mortgages, which are typically supplied requesting housing as collateral, and unsecured loans, for which main guarantee for the lender is borrowers' labor income. Equivalently, one may think of all borrowers being divided in those pledging real estate as collateral and those that instead back the supplied loan with labor income.

A particular role is played by the parameter $0 \leq \gamma \leq 1$, which regulates the breakdown of how much of borrowing is tied to real estate and to labor income. We will see that the effectiveness of the macroprudential cap on DTI strongly hinges on this parameter.

Therefore, borrowers maximize utility (2.12) under the budget constraint (2.13) and the collateral liquidity constraint (2.14). First-order conditions yield:

$$u'(C_t^b)(1-\omega_t) = \beta_b E_t \left[\frac{u'(C_{t+1}^b) R_t}{\Pi_{t+1}} \right] \tag{2.15}$$

$$Q_t u'(C_t^b)\left(1-\gamma m \omega_t\right) = j_t h'(H_t^b) + \beta_b E_t \left[Q_{t+1} u'(C_{t+1}^b) \right] \tag{2.16}$$

$$\frac{v'(N_t^b)}{u'(C_t^b)} = w_t^b \left(1 + (1-\gamma)\tau \omega_t\right) \tag{2.17}$$

where $u'(C_t^b) = \Gamma^b z_t / (C_t^b - \gamma^b C_{t-1}^b)$, $h'(H_t^b) = 1/H_t^b$, $v'(N_t^b) = (N_t^b)^\psi$ and defining real wage as $w_t^b = W_t^b / P_t$.

Since now the maximization problem encompasses a borrowing constraint, there is also a complementary slackness condition, which is given by:

$$\omega_t \geq 0 \qquad \omega_t \left[\gamma m P_t Q_t H_t^b + (1-\gamma)\tau W_t^b N_t^b - B_t^b \right] = 0$$

where the variable ω_t represents the "shadow" price of borrowing, that is the marginal cost of making the constraint binding or equivalently the marginal revenue of relaxing it.

[8] According to the Consumer Financial Bureau, the debt-to-income ratio is "one way lenders measure your ability to manage the payments you make every month to repay the money you have borrowed." (from Consumer Financial Bureau website.)

The shadow price of borrowing enters borrowers' Euler equation, given by equation (2.15), and creates a "wedge" that can impede borrowers to smooth consumption over time. Besides, shadow price affects also the relation between real wage and marginal rate of substitution between consumption and labor. Regarding this, a sensible implication arises: the disutility of working one more hour at the margin gives rise to one more benefit, which sums to the wage earned by working more; this additional benefit is the opportunity to borrow more and so further increase their consumption.

Throughout the paper the borrowing constraint is treated as always binding, so that the shadow price of borrowing is always a positive equilibrium variable. This is reasonable to the extent to which we focus on the credit boom, so that it is likely that during this phase borrowers are, in fact, liquidity constrained.

2.2.3 Firms

The production side of the economy features perfectly competitive firms producing final goods and monopolistically competitive firms producing intermediate goods. Specifically, there exists a continuum of size 1 of perfectly competitive final good firms, in which each of them purchases an intermediate differentiated good in order to produce a final good, using the CES technology:

$$Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\epsilon_t}{\epsilon_t - 1}} di \right]^{\frac{\epsilon_t - 1}{\epsilon_t}}$$

where Y_t is the final good produced and $Y_t(i)$ is the intermediate good used as an input (i denotes the variety of the good). ϵ_t is the time-varying elasticity of substitution among different varieties of goods demanded and is supposed to be greater than one: $\epsilon_t > 1$. We allow for exogenous shifts of this elasticity, according to an AR(1) equation:

$$\log(\epsilon_t) = (1 - \rho_\epsilon)\epsilon + \rho_\epsilon \log(\epsilon_{t-1}) + u^{\epsilon_t} \tag{2.18}$$

with $u_t^\epsilon \sim N(0, \sigma_\epsilon^2)$. This represents the mark-up shock of our economy.

Final goods firms compete in a perfectly competitive market, so that they maximize profits by taking as given the aggregate price level and the price of any intermediate good. Hence, their profit maximization problem yields the equation demand of $Y_t(i)$:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t \tag{2.19}$$

where $P_t(i)$ is the price of intermediate good i and P_t is the aggregate price level. As to the latter, one can prove that by imposing the zero profit condition (since the market of final

goods is perfectly competitive) the aggregate price level P_t is given by the following aggregator: $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon_t} di\right]^{\frac{1}{1-\epsilon_t}}$.

Turning to intermediate goods, each good is produced by a firm competing with other firm in a monopolistic fashion. The firm producing the good i is endowed with the production function:

$$Y_t(i) = (K_t(i))^{\alpha} \left((N_t^s(i))^{\chi} \left(N_t^b(i)\right)^{1-\chi}\right)^{1-\alpha} \tag{2.20}$$

where χ represents the labor income share of unconstrained agents (or their contribution to production). Therefore, due to this assumption on labor, cost minimization problem delivers:

$$MC_t = \frac{w_t^s N_t^s(i)}{(1-\alpha)\chi Y_t(i)} \tag{2.21}$$

$$MC_t = \frac{w_t^b N_t^b(i)}{(1-\alpha)(1-\chi) Y_t(i)} \tag{2.22}$$

where MC_t is the real marginal cost. Cost minimization with respect to capital yields:

$$MC_t = \frac{R_t^k K_{t-1}(i)}{\alpha Y_t(i)}. \tag{2.23}$$

Intermediate good firms set price as in Calvo's model, that is a only a generic fraction $1-\theta$ of firms with $0 < \theta \leq 1$ can change its price at any future period T, whereas the remaining fraction θ of firms do not change the price and so anchor this to the inflation target $\bar{\Pi}$. Profit maximization problem of intermediate good firm allows to choose the optimal price level:

$$\max_{P_t^*} E_t \left\{ \sum_{T=t}^{\infty} (\beta_s \theta)^{T-t} \lambda_T \left[\bar{\Pi}^{T-t} P_t(i) \frac{Y_T(i)}{P_T} - MC_T Y_T(i) \right] \right\}$$

where $\lambda_t \equiv \Gamma^s / (C_t^s - \gamma^s C_{t-1}^s)$ is the savers' marginal utility of consumption, at which future profits are discounted[9]. Firms maximize this profit function subject to the demand equation, given by (2.19). First order condition of the above problem delivers:

$$\frac{P_t^*}{P_t} = \frac{\epsilon_t}{\epsilon_t - 1} \frac{E_t \left\{ \sum_{T=t}^{\infty} (\theta\beta_s)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\epsilon_t} MC_T Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\theta\beta_s)^{T-t} \lambda_t \left(\frac{P_T}{P_t} \frac{1}{\bar{\Pi}^{T-t}} \right)^{\epsilon_t - 1} Y_T \right\}} \tag{2.24}$$

where in equilibrium $P_t(i) = P_t^*$, since all firms set their price equal to the optimal one. Due to Calvo assumption, the aggregate price level follows the law of motion:

$$P_t^{1-\epsilon_t} = (1-\theta) P_t^{*1-\epsilon_t} + \theta P_{t-1}^{1-\epsilon_t} \bar{\Pi}^{1-\epsilon_t}. \tag{2.25}$$

[9] Recall that firms are only owned by savers.

Furthermore, in order to aggregate across all firms it is possible to define the index of price dispersion $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_t} di$. From equation (2.25), one can prove that price dispersion follows the law of motion:

$$\Delta_t \equiv \theta \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\epsilon_t} \Delta_{t-1} + (1-\theta) \left(\frac{1-\theta\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\epsilon_t - 1}}{1-\theta}\right)^{\frac{\epsilon_t}{\epsilon_t - 1}}.$$

2.2.4 Central bank

The monetary authority is responsible of setting the nominal interest rate. We assume that this is set according to the Taylor-like monetary rule:

$$R_t = R_{t-1}^{r_r} R^{(1-r_r)} \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{(1-r_r)r_\pi} \left(\frac{Y_t}{Y}\right)^{(1-r_r)r_y} \zeta_t \tag{2.26}$$

where variable without index are at their respective steady state. r_r is the parameter governing the persistence of the response of central bank to the previous period interest rate and r_y, r_π are parameters that govern the central bank's reaction to, respectively, output and inflation. We allow for a monetary policy shock ζ_t, designed as

$$\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + u_t^\zeta \tag{2.27}$$

with $u_t^\zeta \sim N(0, \sigma_\zeta^2)$.

2.2.5 Aggregation and market clearing

To close the model we impose market clearing conditions. Resource constraint (in real terms) of our economy reads:

$$Y_t = C_t^s + C_t^b + I_t. \tag{2.28}$$

Aggregate labor for savers and borrowers are:

$$N_t^s = \int_0^1 N_t^s(i) di \tag{2.29}$$

$$N_t^b = \int_0^1 N_t^b(i) di. \tag{2.30}$$

Aggregating capital units yields:

$$K_t = \int_0^1 K_t(i) di. \tag{2.31}$$

Then, market clearing condition of financial markets must hold, so that loans supplied to borrowers must be tantamount to the level of assets held by savers:

$$B_t^s - B_t^b = 0. \tag{2.32}$$

Finally, we assume that housing supply is in fixed amount and equal to $\bar{H}$, so that housing market clearing condition is written as

$$H_t^s + H_t^b = \bar{H}. \tag{2.33}$$

The complete set of equilibrium conditions is reported in Appendix.

2.3 Estimation

We estimate the baseline model with standard Bayesian techniques. The model is linearised around the deterministic steady state so that a state space representation of the model is derived. This representation enables to acquire the likelihood function through a standard Kalman filter. Then, we obtain the moments of the posterior distribution of the parameters object of estimation through the Metropolis-Hastings algorithm, by drawing from the posteriors a large enough number of times. Convergence of the algorithm is properly verified through standard Geweke's diagnostics test[10].

A detail of data used for estimation is reported in Appendix. We consider five observable variables: consumption, investment, interest rate, inflation and house prices[11]. As in Iacoviello and Guerrieri (2016) [34] we do not include household debt as an observable. The reason is that the map between this variable in the model and its correspondent on data is not perfect, because in the model debt can only be assumed by constrained agents, thus precluding the fact that asset holders may also borrow. Notwithstanding, we then show that the simulation of the estimated model draws a path of household debt that mirrors the one actually experienced. In order to remove the trend from raw data we apply the one-sided HP filter with smoothing parameters set to 100,000[12].

Series of data consist in quarterly observations ranging from 1991:3 - 2005:4. The reason for choosing this range is that we focus on the credit boom that has led to the financial crisis. As pointed out by Justiniano et al. (2015)[41], it would be unfeasible to replicate a credit boom

[10] We use the software platform Dynare 4.4.3 to perform the model estimation.

[11] Recall that the model features five structural shocks.

[12] For a discussion about using this method to remove the long-term component see Guerrieri and Iacoviello (2016)[34].

starting from a point in time in 2000's, as it would require a too large deviation from the steady state. Thus, we can think of the steady state of the model being the period after the Great Moderation, namely the beginning of 90's, so that variations of leveraging from these values can be understood in terms of likely deviations from the steady state.

INSERT TABLE 2.1

We first calibrate some parameters, that are reported in Table 2.1. Specifically, savers' discount factor $\beta_s = 0.9975$ implies an annual interest rate equal to 3% in steady state. We set the loan-to-value ratio to 85% as in Iacoviello and Neri (2010)[37]. Values of other coefficients α, δ, ϵ are quite standard in the literature. Note that setting $\epsilon = 6$ implies that the frictionless markup is equal to 1.2. We assume an inverse Frisch elasticity of labor equal to $\psi = 1$ and that agents are equally shared in production, so that $\chi = 0.5$. Finally, $\bar{\Pi} = 1.005$ implies a 2% annual inflation target rate. Importantly, this calibration implies that consumption accounts for 75% of GDP (C/Y=0.75) whereas the investment share is the remaining 25% (I/Y=0.25). We calibrate the weight of housing in utility function and the supply of houses so as to obtain a ratio between housing wealth and GDP as close as possible to Iacoviello and Neri (2010) [13].

INSERT TABLE 2.2

Regarding the priors used for our Bayesian estimation, they can be seen in Table 2.2. Overall, they are consistent with previous studies. The novelty is the prior chosen for parameters γ and τ, namely the share of collateral in the borrowing constraint and the cap on debt-to-income ratio. For both we consider a prior mean equal to 0.50. Notice that a debt-to-income to ratio of 50% is quite large and is consistent with the lax constraint on borrowing observed prior to the crisis. Indeed, according to Fed's Office of the Controller of the Currency before the crisis, a "subprime" loan was characterized by a loan with counterpart having a debt-to-income ratio larger than 50%[14]. Furthermore, Jacome and Mitra (2015) [38] study the implementation of the macroprudential cap on debt-to-income ratio in six countries (Brazil, Hong Kong SAR, Korea, Malaysia, Poland and Romania), reporting evidence that the range along which the cap on DTI has been moved when facing standard borrowers is between 30% and 65%. In Mendoza [55] τ is calibrated so that borrowers can borrow up to 40% of their labor income, however he does not focus on the credit boom phase.

[13] In their model this ratio is set to 1.36. In ours, this is slightly above (1.53).

[14] See the "Expanded Guidance for Subprime Lending Programs", Office of the Comptroller of the Currency Board of Governors of the Federal Reserve System.

INSERT TABLE 2.3

Table 2.3 reports the estimates. Specifically, habit in consumption is larger for savers (0.86) than for borrowers (0.34). The estimated cost of installing capital (11.05) is far more sizeable than prior mean. Furthermore, parameters of Taylor rule are equal to 0.51 for the interest rate inertia and to 2.11 and 0.11 for the central bank's responses to, respectively, inflation and output. As for structural shocks, one can notice a very large persistence for housing preference shocks (0.99). Remarkably, the cap on debt-to-income ratio is around 0.51, implying that agents have borrowed up to a fraction of 51% of their labor income. As discussed above, this is quite large and thus becomes consistent with the credit boom observed in the years 2000-2006. Crucially, estimates show that borrowing is almost entirely collateralized with real estate value: in fact, estimate of γ reads 0.98, so that borrowers' labor income only marginally affects the supply of new loans.

2.3.1 Model fit, variance decomposition and identifiability

The model described in the previous section appears to line up data pretty well. To observe this we match the moments obtained simulating the model with those computed on data. Specifically, we set parameters to their posterior mean, apart from those calibrated, and then simulate 100000 artificial time series from the model. In table 2.4 we report standard deviation and autocorrelation of the observables in comparison with those obtained by feeding the model with artificial data. It clearly turns out that moments of simulated time series are quite close to those computed on data.

INSERT TABLE 2.4

Further, we inspect the role of the model structural shocks in accounting for the observable variables. Figure 2.2 displays the historical variance decomposition for two key variables: consumption and house price. Importantly, as shown also in Iacoviello and Neri [37], housing preference shock almost entirely explains the path of house prices, whereas little is due to the role of monetary policy shock. Therefore, we can claim that the huge increase in house prices, which in turn fuelled the credit boom, has been originated in the housing market, thorough a sudden increase in the demand for housing.

Finally, we want to verify the quality of the estimation, investigating whether parameters are correctly identified. To do this, we perform the same exercise as in Schmitt-Groh and Uribe (2012) [66]. We first calibrate the model with the posterior mean of all parameters and simulate

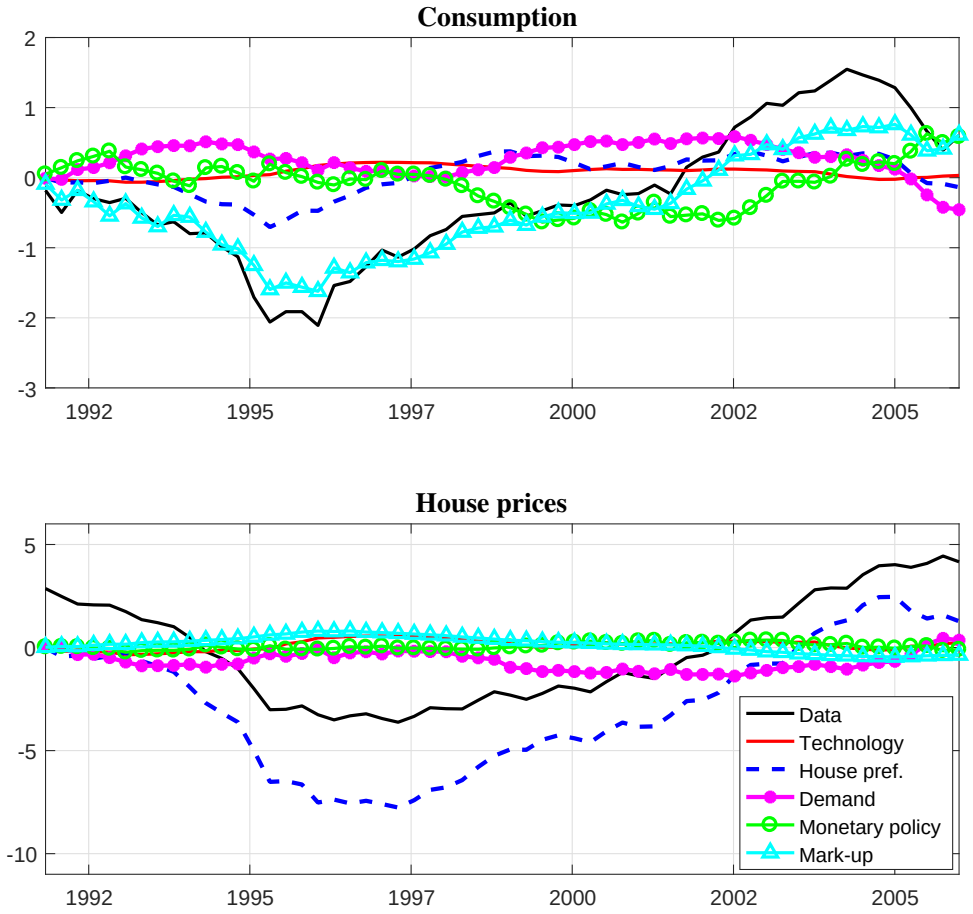

Figure 2.2: Historical variance decomposition of aggregate consumption and house prices. Variables are expressed in percentage deviation from the steady state.

artificial time series with length equal to the data set (58 observations). Then, we estimate again the model with the same specification prior, considering the artificial series for consumption, investment, house prices, inflation and interest rate as new observable variables, without exploiting the knowledge about the true parameters.

Objective of this exercise is to check whether the repeated estimation delivers the true underlying parameters, that is those estimated through real data.

INSERT TABLE 2.5

Overall, in Table 2.5 it can be seen that the new estimates are quite close to the true parameters. This is particularly remarkable for the two key parameters of our analysis, γ and τ, which report value that are surprisingly close to the true value ($\gamma = 0.95$, $\tau = 0.50$).

2.4 The impact of the macroprudential cap on debt-to-income (DTI) ratio

2.4.1 Cap on debt-to-income ratio as a macroprudential policy

Macroprudential policies are implemented with the specific role of safeguarding financial stability, maintaining an adequate level of household debt in the financial system [15]. This implies that macroprudential authority should be assigned specific policy instruments to accomplish this task.

As discussed in the Introduction, we assume that a macroprudential policy is featured by two policy instruments. They are the cap on debt-to-income ratio τ and the breakdown of the borrowing constraint between collateral and labor income, captured by the parameter γ. We now explain the reason for which these parameters are macroprudential instruments.

We begin with the cap on debt-to-income ratio. In the literature it is recommended that macroprudential policies should follow specific rules, so that policy instruments react to all contingencies in a predetermined way, that is following a rule. Further, they should be set in a countercyclical way so as to smooth credit cycles and to avert "over-heating" of macro variables, like private debt. Therefore, we assume that macroprudential authority sets the cap on debt-to-income ratio according to a countercyclical rule written in this way:

$$\tau_t = \tau \left(\frac{b_t^b}{b^b} \right)^{-\tau_b} \tag{2.34}$$

where τ is the estimated value of the cap on debt-to-income ratio, b_t^b is household debt (in real terms), where b^b is its steady state[16]. Thus, the cap on DTI is no longer a fixed parameter, but becomes a policy variable, which reacts to a given contingency of the economy. The underlying assumption is that the macroprudential authority is empowered to specify this policy variable to achieve financial stability. To this end, $\tau_b \geq 0$ is the parameter which rules the action of the authority, so that larger values of this parameter imply that the authority is responding to a larger extent. The sign of this parameter is non-negative to ensure that the instrument reacts in a countercyclical fashion to the deviation of household debt, so that an increase of the latter

[15] According to Bank of England (2009) [60] a macroprudential policy should react against the financial cycles in order "to avoid the type of boom and bust cycles in the supply of credit and liquidity that has marked the recent financial crisis". The use of instruments "effective in leaning against both the upswing and the downswing in the financial cycle" has been recommended also by the Basel Committee on the Global Financial System (2010) [62].

[16] We have also studied the case of a macroprudential policy responding to movements in house prices, rather than household debt. Main results of the paper do not differ. Results for the case of house prices are available upon request.

above the steady state would activate a reduction of the cap on debt-to-income ratio.

This first instrument of macroprudential policy, entailing a change in the polucy instrument (cap on debt-to-income ratio in our case) around the deterministic steady state is standard the literature. We now turn to the other macroprudential instrument, the parameter γ. Importantly, this parameter affects the steady state, thus implying that changing γ produces also a steady-state effect on welfare.

The reason for considering γ as a macroprudential instrument is the following. A remarkable result of our estimation is the fact that the impact of borrowers' labor income in the credit supply decision is quite poor, as borrowing is almost completely (98%) collateralized to real estate value. This implies that the amount of debt outstanding has been strongly affected by swings in real estate values. As a matter of fact, owing to the detrimental legacies caused by the burst of the bubble in the real estate sector, a fact that broke the credit boom phase and therefore ignited the financial crisis, authorities have called into question the strong linkage between real estate values and credit supply. Therefore, we can conjecture that a macroprudential authority may well want to lower the incidence of the collateral so as to dampen the financial amplification mechanism. By this virtue we assume that the macroprudential authority may set the parameter γ, which regulates the weight of the collateral (or , equivalently, the one of labor income) in credit supply decision.

Hence, a "macroprudential policy" can make use of *both* the parameter γ and a countercyclical rule described by equation (2.34). We stress the fact that the macroprudential authority is endowed with two instruments to pursue a specific macroprudential policy is of practical relevance, as distinguishes our paper from other dealing with the optimal conduct of macroprudential policies.

2.4.2 The welfare effect of the macroprudential cap on DTI

Main argument to support the effectiveness of a particular macroprudential policy is the effect that this policy produces in terms of welfare. To this end, we build a social welfare measure obtained by aggregating welfare measures of savers and borrowers. We numerically solve the model with standard perturbation method and then report all welfare measures, i.e. those for both agents and also social welfare. Welfare is calculated from the second-order approximation of welfare measure (defined below) and model equilibrium conditions [17].

We define the agent-specific unconditional welfare as the expected discounted sum of utility from

[17] It is well known in the literature that standard perturbation methods with first-order approximation of equilibrium conditions around the deterministic steady state of the model is not an accurate way to evaluate different policies in terms of welfare. Schmitt-Groh and Uribe (2004) [67] argue that in order to properly perform a second order approximation to the welfare objective function a second order approximation to the policy function is required. See also Kim and Kim (2003) [44] .

period 0 onwards:

$$V^s \equiv E_0 \sum_{t=0}^{\infty} \beta_s^t \left[\Gamma^s \log\left(C_t^s - \gamma^s C_{t-1}^s\right) + j_t \log(H_t^s) - \frac{(N_t^s)^{\psi}}{1+\psi} \right]$$

$$V^b \equiv E_0 \sum_{t=0}^{\infty} \beta_b^t \left[\Gamma^b \log\left(C_t^b - \gamma^b C_{t-1}^b\right) + j_t \log\left(H_t^b\right) - \frac{(N_t^b)^{\psi}}{1+\psi} \right]$$

where V^s and V^b denote, respectively, savers' welfare and borrowers' welfare.
These agent-specific welfare measure are aggregated so as to obtain a social-welfare function, which is written as:

$$V \quad \equiv \quad \mu V^s + (1-\mu) V^b. \tag{2.35}$$

where μ is the weight assigned to savers and therefore $(1-\mu)$ is the borrowers' one. We assume that the policymaker assign an equal weight to agents' welfare: $\mu = 0.5$. Later in the papers we consider also the case of different weights.
In the welfare analysis that follows we will evaluate welfare under different policies. To facilitate the comparison across them, we compute the consumption equivalent, which is the fraction of consumption that an agent should obtain with respect to the estimated model in order to be compensated for the implementation of a given policy. Thus, a positive value of this measure represents a gain whereas a negative one is a welfare loss.
As a first step, we compare welfare between the estimated model, that is when macroprudential policy is completely absent (and the cap on DTI remains at its estimated value ($\tau = 0.51$)) and the case in which macroprudential policy is active and optimized. In this latter case the macroprudential authority is concerned about finding the optimal response, captured by τ_b. We search for this optimal parameter, under the constraints imposed by the equilibrium conditions of the model. To do this, we apply the grid search method, building a grid of plausible values for the parameters γ and τ_b in order to find the "optimal" ones, that is the combination of the two that yields the maximum value of social welfare. This maximization problem can be written as:

$$\max_{\gamma^\star \in [0,1]; \tau_b^\star \in \left[\underline{\tau_b}, \bar{\tau_b}\right]} V \tag{2.36}$$

where V is social welfare, which is defined below, and $\underline{\tau_b}$ and $\bar{\tau_b}$ are, respectively, the lower and the upper bound of the grid of plausible values for the parameter in the rule for the cap on DTI. The grid of γ is clearly $[0,1]$, whereas for τ_b it is set as $[0,3]$, allowing the case of a large response to household debt. We will see that a reaction of the size $\tau_b = 3$ implies that, under the sequence of estimated shocks, the cap on debt-to-income is lowered by a maximum of 30% in the period considered.

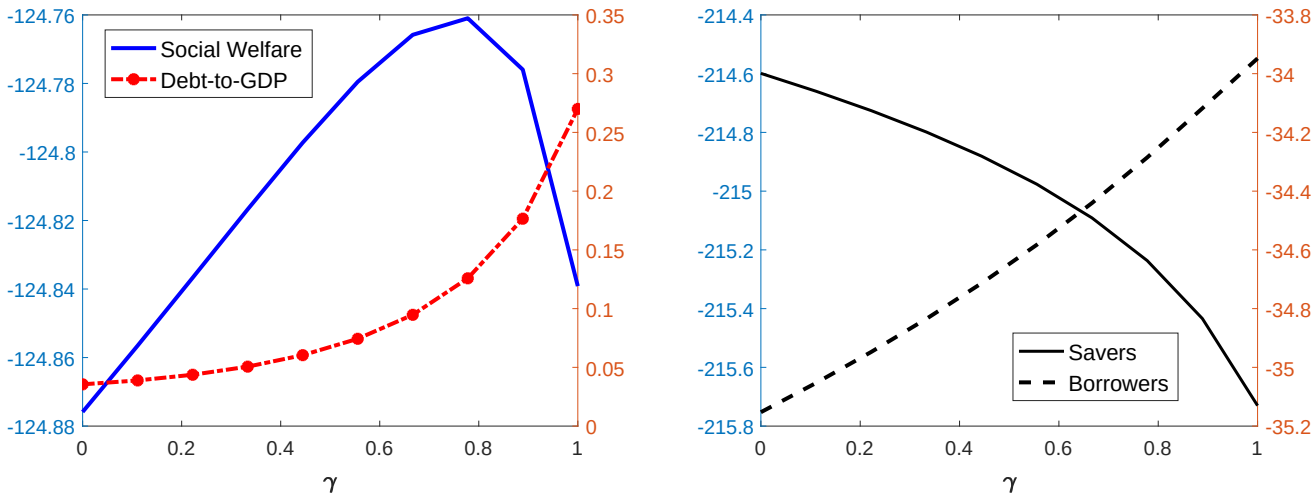

Figure 2.3: Social welfare, savers' and borrowers' welfare, unconditional mean of household debt-to-GDP ratio for $\gamma \in [0, 1]$.

INSERT TABLE 2.6

Results are displayed in Table 2.6. Firstly, it clearly emerges that the macroprudential policy is Pareto-improving, as it delivers social welfare equal to -124.7534 with respect to -124.8217 as in the estimated model. Secondly, it is evident that there is a considerable gain from adopting two instruments of macroprudential policy (see third and fourth row in Table 2.6). Thirdly, the optimal macroprudential policy involves a reduction of the parameter γ from the estimated model ($\gamma^\star = 0.79$) and the highest possible response of the cap on debt-to-income ratio around the steady state ($\tau_b^\star = 3$). Therefore, the simulation unambiguously reveals that the macroprudential cap on DTI produces a Pareto-improvement to the extent to which the cap on debt-to-income responds strongly to deviation of household debt and the weight of labor income in the borrowing constraint is large enough (that is, when γ is progressively smaller).

This latter result can be further observed in the left panel of Figure 2.3, where we display social welfare for all values of γ. It is apparent that social welfare is maximized in correspondence of $\gamma = 0.78$. Moreover, the same figure reveals that for this level of γ the economy experiences a large contraction in the ratio between household debt and GDP (the dashed red line represents its unconditional mean). Thus, it turns out that lowering the parameter γ from its estimated value leads to higher social welfare and more financial stability.

It is also interesting to note that the optimal macroprudential policy provides a strong stabilization effect of financial variables. In this regard, in Table 2.6 we also report percentage standard deviation of the main variables. It can be seen that that implementing an optimal macroprudential DTI leads to considerably lower standard deviation for household debt and house prices, along with a modest decline in variability of output and inflation.

Finally, we turn to the welfare changes specific to the agents. It turns out that, as in other papers dealing with macroprudential policies, (for instance, see Angelini et al. (2012)[1], Lambertini et al. (2013)[49], Carrasco-Galego and Rubio (2014)[14]) it appears a "trade-off" between savers and borrowers. However, in our framework there is a "reversal" of the trade-off documented in this literature: savers benefit from the implementation of the macroprudenial policy, whereas borrowers lose. We investigate better this tendency in the next subsection.
Summarizing, an optimal macroprudential DTI is welfare-enhancing with respect to the estimated model. The efficiency of the macroprudential policy stems from reducing the incidence of the collateral in credit supply decision (thus enlarging the role for labor income) and letting the cap on debt-to-income to respond to household debt in a countercylical fashion.

2.4.3 The credit boom and a counterfactual scenario

The effect of a macroprudential policy can be markedly observed in relation to the pre-crisis scenario, characterized by excessive level of leveraging. In this regard, the model presented above can be used to replicate this scenario, tracking the most relevant macroeconomic aggregate variables. Then, it is compelling to illustrate a different scenario, which, as we will see, features instead a more sustainable growth of household debt. This latter is the scenario where an optimal macroprudential debt-to-income ratio is adopted in the economy.
In order to replicate first the path of over-borrowing, we implement the following exercise. We simulate the estimated model, feeding the model with the series of smoothed shocks obtained from the Bayesian estimation. Model parameters are therefore set at the posterior mean.
The circled red line in Figure 2.4 draws the path response of consumption, output, aggregate labor income, house prices and household debt. Obviously, consumption and house prices report same path observed in data. Since house prices started growing significantly at the beginning of 1996, to emphasise the rise of household debt we report the variables as a difference with the respect to the value observed at the end of the year 1995. In this way, variables draw a path similar to a trend, although this has been removed with the HP filter. Therefore, we can see that from 1996 through the end of 2005 house prices increased by almost 100%, whereas consumption reported only a small increase (around 5%). Importantly, the simulation shows that household debt soared by almost 100% as well, which is overall consistent with the build-up of leveraging observed in data (see Figure 2.1)[18] By contrast, with the solid blue line we display the case of an optimal macroprudential policy, which is attained by setting $\gamma^{\star} = 0.79$ and $\tau_b^{\star} = 3$ in the rule (2.34). Most noticeably, when macroprudential policy is active household debt increases moderately, in spite of the same increase in house prices. This mainly occurs because the

[18]Recall that we do not include household debt as an observable variable.

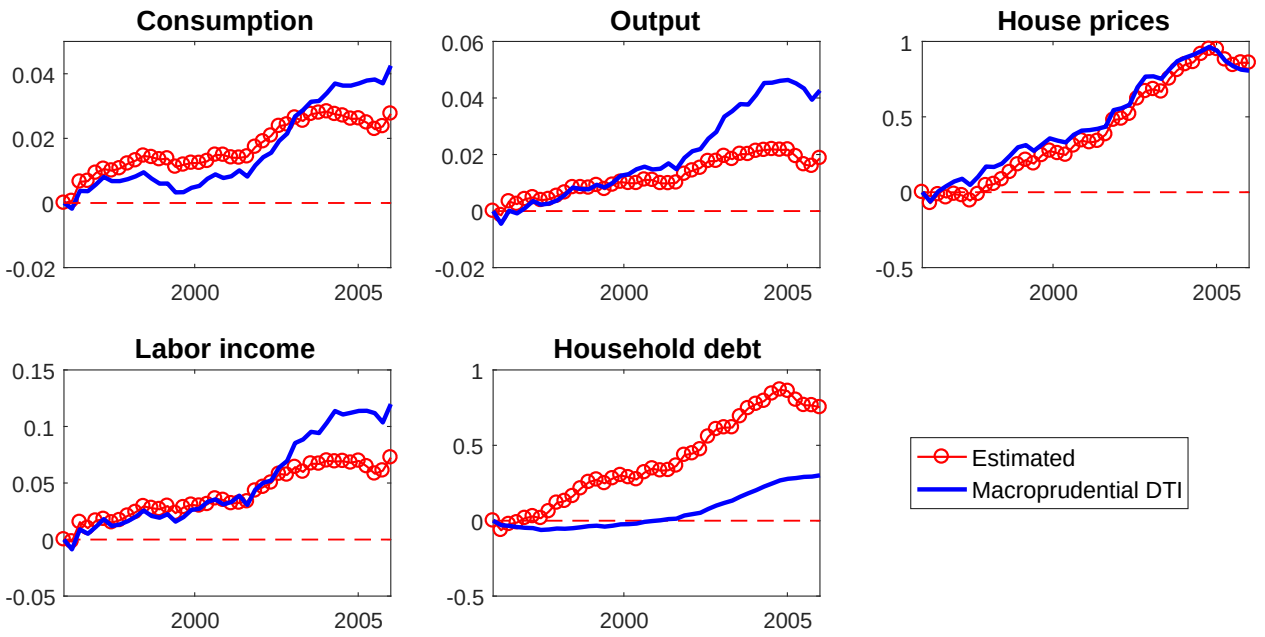

Figure 2.4: Estimated model vs. model with macroprudential policy. Responses of the main model variables, under the sequence of estimated shocks. Variables are computed as difference with respect to the level in 1995:4.

appreciation of the collateral now impacts less strongly on borrowing decision. Notice also the in our counterfactual simulation the cap on DTI is lowered until it reaches -30% by the end of 2005.

Importantly, the milder increase in household debt has no cost in terms of consumption and output, which instead show a larger increase over the years approaching the end of the sample. Figure 2.4 provides an intuition for why macroprudential policy leads to a reduction in borrowers' welfare. It is immediately apparent that borrowers experience a smaller increase in consumption and housing and a more sizeable rise in hours worked. Importantly, this general equilibrium effect can be largely explained by the behaviour of the "shadow price" of borrowing (right-down panel). In fact, in the estimated model ($\gamma = 0.98$) this variable goes down, pointing towards a relaxation of the borrowing constraint. This effect is a consequence of the appreciation of real estate values, that makes the borrowing constraint less binding and therefore borrowers are induced to assume more debt.

Conversely, when the macroprudential policy is in place, the marginal cost of a binding constraint rises, so that borrowers borrow less, consume less and are instead led to supply more work. Thus, the increase of the shadow price of borrowing, which amplifies the effect of the borrowers' marginal rate of substitution between consumption and labor, generates a mechanism such that borrowers are forced to "sacrifice" a portion of consumption and to supply more work. This effect can be seen by equilibrium condition (2.17), which is the marginal rate of substitution between consumption and labor. Clearly, an increase in ω_t, the "shadow" price of borrowing,

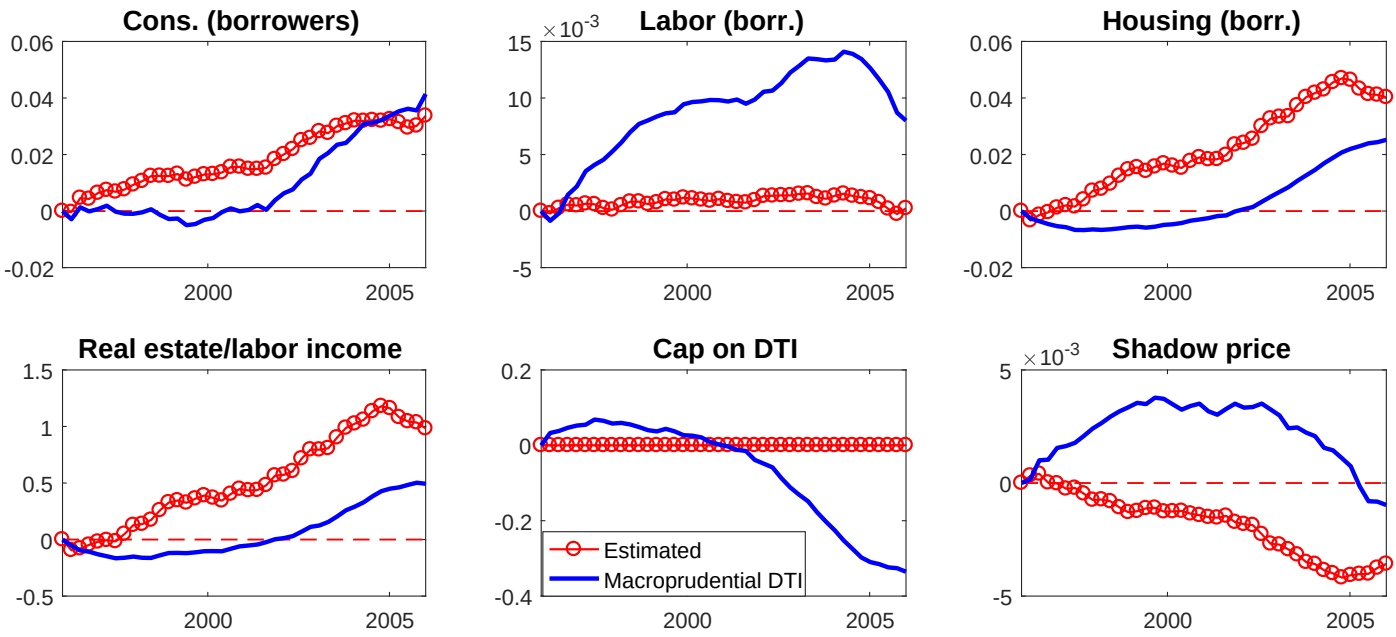

Figure 2.5: Estimated model vs. model with macroprudential policy. Responses of the main model variables, under the sequence of estimated shocks. Variables are computed as difference with respect to the level in 1995:4.

implies that the wedge between consumption and labor broadens, so that for a given level of consumption and wage borrowers are induced to work more, or for a given level of labor and wage borrowers consume less. Notice that this effect amplifies the lower is γ, that is the larger is the weight of labor income in credit supply decision. Therefore, it seems that the macroprudential policy, by lowering γ, acts through an amplification of the impact of the wedge in the MRS of borrowers. Crucially, this action, which is found to be Pareto-improving, leads to an increase in a wedge in the economy and a simultaneous stabilization of household debt. Thus, it arises a particular "trade-off", according to which a social-welfare maximizing policymaker tends to increase a wedge in the economy with the purpose of fulfilling financial stabilty.

The stabilization effect of the macroprudential policy can be also observed in the left-down panel of Figure 2.5, where we report the simulated real estate-to-labor income ratio, computed as: $Q_t H_t^b / w_t^b N_t^b$. Apparently, in the estimated model this ratio increase dramatically, more than doubling. When the model features the optimal macroprudential policy things are substantially different, as the ratio remains closer to zero level. Thus, although house prices worryingly soar, the action of the optimal macroprudential policy leads to a stabilization of the ratio between borrowers' real estate values and labor income. This is a fascinating consequence of the reduction in house purchases and the simultaneous increase in hours supplied.

2.5 The role of monetary policy during a credit boom

Particular attention is devoted in the literature to the role of monetary policy in case of a credit boom. In fact, the US monetary policy stance over the years of the credit boom prior to the crisis has been largely questioned[19]. In particular, the strategy advocated by former Fed' chief Alan Greenspan and labelled "mopping up after the crash", consisting in leaving the credit boom ending on its own, without undertaking any monetary policy action ("post hoc interventionism") has been criticized and blamed to be too lax[20].

Consequently, since the aftermath of the crisis there has been a wide debate concerning how monetary policy should be conducted when the credit boom poses a plausible threat. Two opposite views have been facing each other. The first view suggests that central bank should adopt a "reactive" stance, so that it should raise interest rate in the case of a credit boom. This strategy of - how is called - "leaning against the wind" of a credit boom, is coherent with the view that central bank is able to pursue both the goals of inflation targeting and financial stability[21]. The alternative view reckons that central bank should only pursue its primary objective of inflation targeting, leaving the goal of financial stability to the macroprudential authority ("two instruments for two goals", according to the famous Tinbergen principle)[22]. To this end, the macroprudential authority should be in charge of adopting ex-ante measures ("macroprudential" measures) with the precise intention of preventing the credit boom.

As we will see in the sequel, our framework allows to provide a comparison in terms of welfare between these opposing cases and, ultimately, a combination of the two.

2.5.1 Optimal Taylor rule and "leaning against the wind"

In the previous section, we showed that the macroprudential policy of our interest is welfare-enhancing. The welfare analysis has been run holding the parameters governing the monetary policy rule fixed at the estimated value. Thus, we studied the optimal macroprudential policy for a given monetary policy rule. In this section we examine the optimal conduct of monetary policy, when this is enforced as a standard Taylor rule. This allows us to infer how the presence

[19] Most important criticism lifted in the aftermath of the crisis is that policy rate has been "too low for too long" (Taylor (2009)[75]) .

[20] This strategy, that was dominant before the crisis, was based on the presumption that the monetary institution would stick to its main goal of inflation targeting, even during a credit boom. On this point, see Blinder and Reis (2005)[11].

[21] See, among the others, Woodford (2012) [81].

[22] Svensson (2012) [74] argues that central bank and macroprudential authority should have distinct objectives, with the monetary authority focusing exclusively on its goal of inflation stabilization and leaving macroprudential instruments to address financial instability. Svensson (2017)[73] shows that the benefits of "leaning against the wind" fall short of the costs.

of macroprudential policy affects the welfare impact of a standard Taylor rule.

To do this, we first explore only the optimal conduct of monetary policy, i.e. absent any macroprudential policy. Specifically, we assume that central bank maintains same inertia as observed in data ($r_r = 0.51$) and we search for the optimal (social welfare-maximizing) parameters of the Taylor rule (r_y, r_π) in the estimated model, solving the following problem:

$$\max_{r_y^\star, r_\pi^\star} V \tag{2.37}$$

where V is social welfare as defined above. Regarding the parameters of the Taylor rule, we consider a value within the grid $[1.01; 6]$ for r_π, and $[0; 3]$ for r_y. Notice that the grid for the parameter that governs the reaction of central bank to inflation (r_π) starts from a value larger than 1 to ensure determinacy of equilibrium.

Secondly, we allow central bank to respond also to variation in financial variables. In fact, the burst of the crisis has led many economists to think of the need for central banks to take into account also financial variables alongside inflation and output in their decisions concerning interest rate movements. This is a proposal in line with the view supporting that central bank could also be in charge of financial stability. According to this strategy, when an excess of borrowing threatens the economy, central bank should "lean against the wind" of a credit boom, by raising the interest rate in order to discourage borrowers and thus preserve financial stability[23].

In order to configure this latter strategy, we consider an alternative characterization of the Taylor rule:

$$R_t = R_{t-1}^{r_r} R^{(1-r_r)} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{(1-r_r)r_\pi} \left(\frac{Y_t}{Y}\right)^{(1-r_r)r_y} \left(\frac{b_t^b}{b^b}\right)^{(1-r_r)r_b} \zeta_t, \tag{2.38}$$

i.e. the standard Taylor rule is augmented with a reaction to deviation of household debt from the steady state, with $r_b \geq 0$[24]. Accordingly, now the problem becomes:

$$\max_{r_y, r_\pi, r_b} V$$

so that there is a further parameter r_b in social-welfare maximization. For the parameters of the augmented Taylor rule, we consider a value within the grid $[0, 3]$ for r_b, whereas for the other parameters r_y, r_π grids are set as in the previous section.

[23] Smets (2013) [70] provides an exhaustive survey of the literature dealing with the policy of "leaning against the wind".

[24] This specification of "debt-adjusted" Taylor rule can be found, among the others, in Curdia and Woodford (2010)[17], Lambertini et al. (2013)[49]

INSERT TABLE 2.7

Table 2.7 reports the welfare measures obtained in the two cases described above, that is in case of the "optimized Taylor rule (TR)" and when instead we consider a debt-adjusted Taylor rule so as to pursue the "leaning against the wind" (LAW) strategy. It can be noted that augmenting the Taylor rule with a reaction to household debt ($r_b^\star = 0.42$) delivers an improvement in social welfare with respect to the optimized standard Taylor rule.

Table 7 also reveals that the "leaning against the wind" policy brings about a larger stabilization of debt. However, lower standard deviation of household debt is accompanied with larger deviation of output and inflation, implying that greater financial stability comes at the cost of bearing more variability of output and inflation[25]. Thus, we find that implementing "leaning against the wind" policy generates a "trade-off" between stabilization of financial variables and stabilization of output and inflation.

2.5.2 Macroprudential DTI and optimized Taylor rules

We now investigate how the optimal response of the central bank changes once this acts in coordination with a macroprudential authority that can also attain an optimal policy. Therefore, it becomes a problem of joint optimization, so that both authorities are assumed to be optimizing at the same time:

$$\max_{\gamma,\tau_b,r_y,r_\pi} V \tag{2.39}$$

where parameters (r_y, r_π) are choice of the monetary authority, and parameters (γ, τ_b) is the choice of the macroprudential authority. This joint policy problem reflects the strategy of "two instruments for two goals", because central bank is engaged in the maximizing the parameters related to price stability and the macroprudential authority is instead concerned about finding the optimal response to variations that undermine financial stability.

We can derive two results from this exercise. First, an optimized Taylor rule performs better when accompanied with an optimal macroprudential policy. Second, by comparing the strategy of "leaning against the wind", previously characterized, with the strategy of "two instruments for two goals" we note that the latter is unambiguously welfare-dominating, as it delivers higher social welfare (-124.7088) than the strategy of "leaning against the wind" (-124.7185). Concerning the stabilization effect of these two policies, we can see that the strategy of "leaning against the wind" produces a stronger stabilization of household debt (11.70), which, nevertheless, is

[25] Woodford (2012) [81] argues that allowing more variability in output and inflation in order to purse financial stability may be desirable.

not considerably superior than in case of optimal macroprudential policy (12.18). However, the former policy leads to a noticeably larger variability of inflation and output, thus implying a "trade-off" between stabilization of financial variables and standard macroeconomic variables.
Finally, our framework allows to study the case of a combination between the "leaning against the wind" policy and the policy of "two instruments for two goals". Therefore, we impose that the coordinated maximization problem now becomes:

$$\max_{\gamma,\tau_b,r_y,r_\pi,r_b} V \tag{2.40}$$

so that both central bank (r_y, r_π, r_b) and macroprudential authority (γ, τ_b) may target financial stability.
Crucially, this exercise reveals that this combined policy is the best-performing, as it delivers the highest value of social welfare under all simulations. Furthermore, this policy seems to attain the best stabilization of household debt, albeit the stabilization of output and inflation worsens with respect to the estimated model[26]. Importantly, this result suggests that the best policy tends to favour the stabilization of household debt, or, in other words, that a departure from the standard goal of price stability is desirable when the economy is undergoing a credit boom[27].
Summarizing, we have found that the macroprudential cap on debt-to-income is an efficient complement of monetary policy. Importantly, a strategy of "two instruments for two goals" - financial stability assigned to a well-designed macroprudential policy and inflation targeting to central bank - is a more efficient strategy than the case in which the objective of financial stability is undertaken by central bank which "leans against the wind". However, the best performing policy, both in terms of social welfare and stabilization of private debt is a combination of the "leaning against the wind" policy and macroprudential policy.

2.6 Other issues

2.6.1 A comparison with a macroprudential loan-to-value (LTV) ratio

Most of the works analysing the interaction between monetary and macroprudential policy focus on the implementation of a macroprudential policy for the loan-to-value (LTV) ratio, rather than

[26]The fact that the best-performing policy leads to the best stabilization of household debt can be also found in Angelini et al. (2012)[1], Lambertini et al. (2013)[49]. Likewise, in these works the improvement in stabilization of private debt comes at the cost of bearing more variability in output and inflation.
[27]See discussion in Woodford (2012)[81].

the cap on debt-to-income ratio[28]. This way the macroprudential authority may target more directly the real estate sector, where the over-valuation of real estate values takes place. By this virtue, an increase in household debt is tackled with a reduction of the loan-to-value ratio.
Our framework allows to compare the performance of the LTV policy with the policy on the cap on DTI and, ultimately, to combine the two so as to design a joint macroprudential policy. To this end, we introduce in the our model a countercyclical rule for the LTV which is specular to the one adopted for the cap on DTI:

$$m_t = m \left(\frac{b_t^b}{b^b} \right)^{-m_b} \tag{2.41}$$

with $m_b \geq 0$. We therefore optimize the parameter m_b over the range $[0, 3]$.

INSERT TABLE 2.8

Results displayed in Table 2.8 clearly uncover that a LTV policy grants a welfare-gain with respect to the estimated model. Interestingly enough, with the LTV policy the stabilization of household debt is greatly attained. However, the macroprudential cap on DTI seems to perform unambiguously better than the LTV policy. Thus, we find the the macroprudential DTI is a more efficient policy than the standard LTV policy.
Finally, we explore the case when these two macroprudential policies are jointly used to achieve a better performance in terms of financial stability. It turns out that a combined use of the two policies improves on the performance of the LTV although, surprisingly enough, macroprudential DTI alone represents the best policy outcome.

2.6.2 Coordination vs. Non Coordination

In the previous section we found the optimal combination of the social-welfare maximizing parameters of the monetary policy rule and the macroprudential policy. In this regard, we assumed that the two authorities were maximizing jointly, under perfect coordination. Nonetheless, nobody would guarantee that perfect coordination allows to obtain the best policy outcome. In fact, in the literature there is no unanimous consensus on whether the two policies should be adopted in coordinated or non-coordinated way. As an example, Bean et al. (2010) [3] and Angelini et al. (2012)[1] argue for a coordination between the two policies, whereas Svensson

[28] The rule for the loan-to-value ratio as a macroprudential policy has been extensively studied in the literature (see, among the others, Lambertini et al. (2013)[49], Carrasco-Galego and Rubio (2014)[14]). Furthermore, empirical studies show that this instrument has been widely used in the aftermath of the crisis, also in combination with a macroprudential cap on DTI. An active countercyclical policy for the loan-to-value (LTV) has been first applied in Asia since the housing boom of mid-2000s. Refer to Lim et al (2011)[50] for a comprehensive focus on the application of the countercyclical LTV rule.

(2012) [74] and Carrasco-Galego and Rubio [14] find that non-coordinated policies are associated to a better policy outcome.

INSERT TABLE 2.9

In Table 2.9 we compare the case in which the two authorities coordinate with the case of non-coordination, namely when one authority acts as "leader" and therefore optimizes first, followed by the other authority, which instead behaves as "follower". Interestingly enough, it emerges that the case of coordination is Pareto-improving, delivering higher social welfare (-124.7088) than when central bank optimizes first (-124.7110) and when instead the leader is the macroprudential authority (-124.7095).

2.6.3 Social welfare function

Results drawn so far are based on a social welfare criterion, according to which welfare measures specific to both type of agents are aggregated. Fundamental is the way through which these measures enter the social welfare function (2.35). Thus far we have assumed that these two are equally weighted, so that $\mu = 0.5$. We now investigate to what degree the welfare analysis changes as long as the different weights are assigned to agents' welfare measure. Table 10 reports the cases in which savers' welfare is assumed to have a larger ($\mu = 0.75$) and a smaller ($\mu = 0.25$) weight. Importantly, the latter case reflects the case considered by Lambertini et al. (2013)[49] and Carrasco-Gallego and Rubio (2014)[14], where borrowers' welfare is given a more sizeable weight in social welfare function[29].

INSERT TABLE 2.10

It clearly arises that when savers' welfare matters more in social welfare function, namely when $\mu = 0.75$, the optimal macroprudential policy implies that the financial amplification mechanism that works through the collateral is fully cushioned, as household debt is only related to borrowers' labor income ($\gamma^{\star} = 0$). As a result, the countercyclical DTI has now the largest impact on the economy. By contrast, when borrowers' welfare matters to a larger extent ($\mu = 0.25$), credit supply decision would be entirely driven by real estate values ($\gamma^{\star} = 0$), and thus a countercyclical rule for the cap on DTI would play no role. Obviously, in this scenario financial

[29] In particular, these works assume that weights to savers and borrowers' welfare are assigned in a way that the planner equalizes utility across savers and borrowers in case of an equal constant consumption stream. Thus, social welfare function is written as: $V \equiv (1-\beta_s)V^s + (1-\beta_b)V^b$. In our framework, adopting this formulation for social welfare would imply that borrowers' welfare is given a weight which is approximately equal to (1-μ) = 0.92.

stability would become problematic, as shocks in the housing market fully impact on household debt.

2.7 Concluding remarks

The recent financial crisis has sparked many debates concerning the main cause that presumably led the financial system to collapse in such tremendous way. Undoubtedly, one of this cause is the credit boom that occurred prior to the crisis. Regarding this, it has become common belief that in order to safeguard the stability of the financial system and to keep under control systemic risks an excessive growth of households debt must be averted. We have found that, in principle, this task can be better attained by imposing that credit supply must depend on labor income to a larger extent than how much occurred in US before the burst of the crisis, based on our estimates. Thus, a macroprudential authority should first establish that credit supply be more tied to labor income. Then, it follows that a macroprudential cap on DTI, according to which the authority responds to contingencies in a rule-based manner, may deliver a good performance.

Crucially, the implementation of an efficient macroprudential policy leads to redefine the role of monetary policy during a credit boom. In this regard, we have inquired the conduct of monetary policy when central bank shares the stage with a macroprudential authority that implements the policy of our interest (rule for cap on DTI). Therefore, a standard Taylor-like monetary policy rule performs better when accompanied by a macroprudential rule for the cap on DTI, when this is designed to stabilize household debt.

Should central bank be also concerned about financial stability during a credit boom ? According to many commentators, central bank may well be able to stabilize financial variables by implementing a policy called "leaning against the wind", that is raising the policy rate when financial variables become over-heated. Contrasting this view, our analysis leads to conclude that the strategy in which central bank "leans against the wind" of a likely credit boom is welfare-dominated by the one in which the goal of financial stability is uniquely assigned to a macroprudential authority that implements the optimal rule for the cap on DTI, whereas central bank is only left its primary goal of price stability. Nonetheless, the best-performing policy turns out to be a combination of "leaning against the wind" and macroprudential policy, so that private debt is extraordinarily stabilized, at the cost of bearing bigger variability of inflation and output.

To conclude, the analysis has revealed that, to avoid a new credit boom to occur again credit supply must take into account borrowers' labor income to a larger extent than before the cri-

sis. Recent trends in macroprudential regulation seem to suggest that this represents a quite plausible way for years to come.

Tables

Table 2.1: Model calibration.

Parameter (description)	*Value*
β_s (savers discount factor)	0.9975
β_b (borrowers discount factor)	0.97
$\bar{\Pi}$ (inflation target)	1.005
ψ (inverse of Frisch elast. of labor)	1
χ (savers' share in production)	0.5
ϵ (s.s. elasticity among goods)	6
m (loan-to-value ratio)	0.85
j (housing weight in utility function)	0.03
$\bar{H}$ (housing supply)	1
α (capital share in production)	0.33
δ (capital depreciation)	0.025
Steady state ratios	*Value*
C/Y (consumption/GDP)	0.75
I/Y (investment/GDP)	0.25
$QH/4Y$ (housing wealth/GDP)	1.53

Table 2.2: Prior distribution of model parameters and shocks.

Parameter (description)	*Distribution*	*Mean*	*St.Deviation*
γ^s (savers' consumption habit)	Beta	0.7	0.1
γ^b (borrowers' consumption habit)	Beta	0.7	0.1
θ (frequency of price adjustment)	Beta	0.5	0.075
ϕ (capital adjustment cost)	Gamma	5	2
τ (cap on debt-to-income ratio)	Beta	0.5	0.25
γ (weight of collateral in borr. constr.)	Beta	0.5	0.25
r_r (Taylor rule)	Beta	0.75	0.1
r_π (Taylor rule)	Normal	1.5	0.25
r_y (Taylor rule)	Normal	0.125	0.025
ρ^z (persistence demand shock)	Beta	0.7	0.1
ρ^a (persistence techn. shock)	Beta	0.7	0.1
ρ^j (persistence hous. pref. shock)	Beta	0.7	0.1
ρ^ζ (persistence mon.pol. shock)	Beta	0.7	0.1
ρ^ϵ (persistence mark up shock)	Beta	0.7	0.1
σ^z (st.dev. demand shock)	Inv. Gamma	0.01	1
σ^a (st.dev. techn. shock)	Inv. Gamma	0.01	1
σ^j (st.dev. hous.pref. shock)	Inv. Gamma	0.01	1
σ^ζ (st.dev. mon.pol. shock)	Inv. Gamma	0.01	1
σ^ϵ (st.dev. mark up shock)	Inv. Gamma	0.01	1

Table 2.3: Posterior distribution estimates of model parameters and shocks.

Parameter (description)	*Mean*	*10 %*	*Median*	*90 %*
γ^s (savers consumption habit)	0.86	0.73	0.85	0.93
γ^b (borrowers consumption habit)	0.34	0.26	0.35	0.48
θ (frequency of price adjustment)	0.60	0.53	0.61	0.71
ϕ (capital adjustment cost)	11.07	7.20	11.15	14.85
τ (cap on debt-to-income ratio)	0.51	0.08	0.53	0.89
γ (weight of collateral in the borr. constr.)	0.98	0.96	0.98	1.00
r^R (Taylor rule)	0.51	0.43	0.51	0.61
r^π (Taylor rule)	2.11	1.81	2.13	2.47
r^Y (Taylor rule)	0.11	0.07	0.11	0.16
ρ^z (persistence demand shock)	0.73	0.66	0.72	0.82
ρ^a (persistence technology shock)	0.82	0.74	0.82	0.89
ρ^j (persistence housing pref. shock)	0.99	0.98	0.99	1.00
ρ^ζ (persistence mon.pol. shock)	0.61	0.52	0.60	0.70
ρ^ϵ (persistence mark up shock)	0.87	0.83	0.86	0.91
σ^z (st.dev. demand shock)	0.006	0.005	0.006	0.007
σ^a (st.dev. techn. shock)	0.006	0.006	0.006	0.007
σ^j (st.dev. hous. pref. shock)	0.016	0.014	0.016	0.018
σ^ζ (st.dev. mon. pol. shock)	0.001	0.001	0.001	0.001
σ^ϵ (st.dev. mark up shock)	0.064	0.044	0.069	0.083

Table 2.4: Data vs. model: moments matching. Moments are obtained by simulating 100000 artificial time series, setting parameters to the posterior mean.

	Stand. deviation (%)		***Autocorrelation***	
	Data	Model	Data	Model
Consumption	0.96	0.79	0.97	0.91
Investment	0.86	0.93	0.98	0.99
Inflation	0.09	0.13	0.68	0.88
Int. rate	0.15	0.16	0.89	0.94
House prices	2.55	2.60	0.95	0.98

Table 2.5: Identifiability exercise (SGU (2012)). Estimates of posterior mean using data (in second column) and artificial simulated time series (third column) as model observable. In the latter case, simulated series are obtained by setting parameter to the posterior mean and simulating the model for a number of periods equal to the dataset (58 obsevations).

Parameter (description)	*Data*	*Simulated series*
γ^s (savers consumption habit)	0.86	0.58
γ^b (borrowers consumption habit)	0.34	0.71
θ (frequency of price adjustment)	0.60	0.45
ϕ (capital adjustment cost)	11.07	7.18
τ (cap on debt-to-income ratio)	0.51	0.50
γ (weight of collateral in the borr. constr.)	0.98	0.95
r^R (Taylor rule)	0.51	0.24
r^π (Taylor rule)	2.11	2.37
r^Y (Taylor rule)	0.11	0.15
ρ^z (persistence demand shock)	0.73	0.75
ρ^a (persistence technology shock)	0.82	0.73
ρ^j (persistence housing pref. shock)	0.99	0.98
ρ^ζ (persistence mon.pol. shock)	0.61	0.64
ρ^ϵ (persistence mark up shock)	0.87	0.70
σ^z (st.dev. demand shock)	0.006	0.004
σ^a (st.dev. techn. shock)	0.006	0.004
σ^j (st.dev. hous. pref. shock)	0.016	0.032
σ^ζ (st.dev. mon. pol. shock)	0.001	0.001
σ^ϵ (st.dev. mark up shock)	0.064	0.066

Table 2.6: Welfare measures and standard deviation (in percentage) of main variables. In parenthesis consumption equivalent is reported. For the estimated model, all parameters are set to their posterior mean, apart from those that are calibrated. Standard deviation of shocks is set to the posterior mean.

	Welfare			**Stand. deviation (%)**			
	V	V^s	V^b	Y	π	Q	b^b
Estimated	-124.8217	-215.6659 (-)	-33.9775 (-)	1.74	0.14	48.68	29.24
Macroprudential DTI ($\gamma^\star = 0.79, \tau_b^\star = 3$)	**-124.7534**	-215.2349 (0.7726)	-33.2719 (-1.3293)	1.67	0.13	44.85	**12.04**
Macropr. (only γ) ($\gamma^\star = 0.79$)	-124.8035	-215.2692 (0.7109)	-33.3377 (-1.6239)	1.71	0.13	44.96	15.01
Macropr. (only $\tau_b^\star$) ($\tau_b^\star = 3$)	-124.8150	-215.5817 (0.1505)	-34.0434 (-0.2991)	1.73	0.14	48.66	28.66

Table 2.7: Welfare measures and standard deviation (in percentage) of main variables. In parenthesis consumption equivalent is reported. Standard deviation of shocks is set to the posterior mean.

	Welfare			**Stand. deviation (%)**			
	V	V^s	V^b	Y	π	Q	b^b
Optim. Taylor Rule (TR) $(r_y^\star = 0, r_\pi^\star = 4.34)$	-124.7813	-215.5657 (0.1791)	-33.9770 (-0.0886)	1.90	**0.07**	48.79	29.75
LAW $(r_y^\star = 0, r_\pi^\star = 6.00, r_b^\star = 0.42)$	-124.7185	-215.3877 (0.4980)	-34.0492 (-0.3254)	3.96	0.36	59.33	11.70
Optim. TR + DTI $(\gamma^\star = 0.78, \tau_b^\star = 3,$ $r_y^\star = 0, r_\pi^\star = 3.23)$	-124.7088	-215.1063 (1.0043)	-34.3113 (-1.5058)	1.91	0.08	44.75	12.18
LAW + DTI $(\gamma^\star = 0.78, \tau_b^\star = 3$ $r_y^\star = 0, r_\pi^\star = 6.00, r_b^\star = 0.22)$	**-124.6492**	-214.9475 (1.2911)	-34.3508 (-1.7004)	3.03	0.25	51.77	**7.65**

Table 2.8: Welfare measures and standard deviation (in percentage) of main variables. In parenthesis consumption equivalent is reported. Standard deviation of shocks is set to the posterior mean.

	Welfare			**Stand. deviation (%)**			
	V	V^s	V^b	Y	π	Q	b^b
LTV $(\gamma^\star = 0.78, m_b^\star = 3.00)$	-124.7559	-215.2250 (0.7904)	-34.2868 (-1.3961)	1.69	0.13	42.66	1.74
DTI $(\gamma^\star = 0.79, \tau_b^\star = 3)$	**-124.7534**	-215.2349 (0.7726)	-33.2719 (-1.3293)	1.67	0.13	44.85	12.04
LTV + DTI $(\gamma^\star = 0.78, m_b^\star = 3.00,\ \tau_b^\star = 3.00\ )$	-124.7551	-215.2231 (0.7938)	-34.2871 (-1.3974)	1.69	0.13	42.69	**1.69**

Table 2.9: Welfare measures and standard deviation (in percentage) of main variables. In parenthesis consumption equivalent is reported. Standard deviation of shocks is set to the posterior mean.

	Welfare			**Stand. deviation (%)**			
	V	V^s	V^b	Y	π	Q	b^b
Coordination	**-124.7088**	-215.1063 (1.0043)	-34.3113 (-1.5058)	1.91	0.08	44.75	12.18
DTI first	-124.7095	-215.1259 (0.9794)	-34.2933 (-1.4283)	1.88	0.05	44.85	12.81
TR first	-124.7110	-215.1119 (0.9942)	-34.3100 (-1.5000)	1.91	0.08	44.75	12.18

Table 2.10: Optimal macroprudential policy for different values of μ.

	Optimal Macropr. Policy
$\mu = 0.5$	$\gamma^\star = 0.79, \tau_b^\star = 3$
$\mu = 0.75$	$\gamma^\star = 0, \tau_b^\star = 3$
$\mu = 0.25$	$\gamma^\star = 1$

Chapter 3

Money-Financed Fiscal Stimulus as an Optimal Stabilization Policy

3.1 Introduction

Despite it has been considered a taboo for a long time, the idea of financing an increase in deficit with a permanent increase in monetary base has returned back to the center of the public debate. This is an old-fashioned idea, that dates back to the original proposal of "helicopter money" (Friedman (1969) [28]) and regained new interest after Japan fell into a "liquidity trap" at the end of 90's[1]. This idea is now revived, as a consequence of prolonged stagnation experienced by some country. Although since the end of the crisis central banks have engaged in wide programs of unconventional monetary policies, in some country output is still below the potential and deflation is still far from disappearing[2]. Therefore, some commentators have begun advocating another unorthodox monetary policy measure, consisting in financing a fiscal stimulus, and thus the implied increase in deficit, with emission of money, without depending on public-debt financing[3].

In this regard, in the present paper we build a model that provides a role for both unconventional monetary policy and a potential simultaneous fiscal expansion. Crucial for the implementation of a money-financed fiscal stimulus is the commitment of the cooperation government-central bank. We highlight the importance of this, by characterizing a Ramsey problem under full commitment. Remarkably, we find that the optimal policy mix under full commitment involves

[1] One of the most prominent advocate of this idea at that time was the former Fed chief Ben Bernanke, who had a speech in 2003 in which he argued for an expansionary fiscal policy "explicitly coupled with incremental Bank of Japan purchases of government debt" so that the expansion of deficit is in effect financed by money creation (see Bernanke (2003)[8]).

[2] As discussed in Chapter 1, this is the case of Japan and Eurozone.

[3] This policy has been greatly supported by Turner (2013 [76], 2015 [77]). See also related discussions in Reichlin et al. (2013)[63], Giavazzi and Tabellini (2014)[31]

a money-financed fiscal stimulus, which enables to stabilize the economy in a more efficient way than other policies do. In other words, the optimal commitment says that central bank expands the amount of reserves supplied to finance a simultaneous rise in government spending.

The optimal money-financed fiscal stimulus is implemented in a specific way: public spending increases in a countercyclical fashion the period in which the recessionary shock takes place, and then reverts back to the steady state the subsequent period, when output gap in closed; money supply instead is expanded to a larger extent than government spending and then is sluggishly withdrawn.

We discuss the properties of this optimal money-financed fiscal stimulus by looking at alternative policies that have been commonly adopted in the real world or widely suggested in the literature. Particularly fascinating, in light of the recent unconventional policies implemented in some country, is the comparison with the case of an optimal monetary policy in combination with constant government spending. The rationale of this analysis is the fact that, in the current scenario, unconventional monetary policies have not been accompanied by expansionary fiscal policy because of concerns of already-high levels of public debt. Interestingly enough, we show that when the unconventional monetary policy is accompanied by a fiscal stimulus the optimal stabilization improves considerably. In this case, the policymaker would commit to expand money supply and reduce the amount of public debt held by the private sector.

We then compare our optimal policy with the case in which optimal government spending is fully financed with public debt, with no use of money. We find that in this case of "passive" monetary policy the economy may experience persistent recession and deflation. Thus, debt-financing a fiscal stimulus turns out to be a noticeably sub-optimal policy.

Importantly, "active" monetary policies that feature an increase in money supply in order to attain a pre-specified target perform far better. In particular, in case of a standard Taylor rule regime, complementing the monetary injection with an optimal fiscal stimulus attains, surprisingly, as an efficient stabilization effect as in the case of optimal policy. Furthermore, since a money-financed fiscal stimulus prevents deflation, the policy rate is not cut to the zero lower bound but rather remains quite stable. Two other "active" monetary policies are "inflation-targeting" policy and "nominal GDP targeting" policy. Under these regimes, as long as the increase in money supply, required to achieve the monetary policy target, is complemented with an optimal increase in public spending, output gap and inflation are greatly stabilized and the interest rate on reserves is not necessarily lowered. However, this occurs at the cost of engineering a more pronounced increase of government spending, which offsets the fall in private spending.

The scenario is an economy hit by a "liquidity shock". "Liquidity" is intended as in Lagos (2010)[48] and Benigno and Nisticò (2017)[5], that is the degree by which assets are accepted

to purchase goods. When the property of exchengeability of these assets worsens, we are in presence of a "liquidity shock", which translates into a contraction in nominal spending[4].

A liquidity shock of this sort reflects what happened in the recent financial crisis, especially in US and Eurozone, where a large fraction of assets have lost the property of being store of value and having perfect resaleability[5]. Particularly in the Eurozone, the liquidity crisis has regarded those assets deemed as safest, like sovereign bonds (International Monetary Fund (2012), ch. 3 [29]).

Accordingly, our model features a financial friction in which both money and a given fraction of government bonds are accepted to purchase goods, but the latter may face a sudden deterioration of their value, making them lose value when purchasing goods. This perturbation sharply impairs the economy, producing recession and deflation, and the zero lower bound becomes a constraint for monetary policy. In this framework, the optimal policy mix turns out to be a money-financed fiscal stimulus.

The rest of the paper is structured as follows. Section 3.2 presents the representative-agent model used for our analysis. Section 3.3 characterizes the optimal monetary-fiscal policy. Section 3.4 discusses about the performance of several monetary policies. Finally, Section 3.5 summarizes main results and concludes.

3.1.1 Related literature

The paper closest to our work is Galì (2017) [30]. The author analyses the effects of a money-financed fiscal stimulus in a standard textbook New Keynesian model. He compares the effects of money-financing an increase in government spending or a reduction in taxes with the effects of financing these stimuli by issuing public debt. He shows that, both in normal times and in case of a liquidity trap, a money-financed fiscal stimulus produces a stronger stimulative effect on the economy. Importantly, to configure a money-financed fiscal stimulus Galì (2017)[30] assumes that there is a rule according to which government keeps public debt unchanged. We depart from this paper in that we find money-financed fiscal stimulus to be an optimal policy mix, that is the best policy outcome of a Ramsey problem with full commitment, rather than a policy rule. In our paper, the policymaker can choose to finance government spending either with money or with public debt, and it eventually opts for using money. Therefore, both government spending and money are endogenous, depending on the choice of the Ramsey policymaker. From a model perspective, our model differs from Galì (2017)[30] in two important aspects. Firstly, he provides

[4]The idea that shortage of safe assets and therefore a disequilibrium in the financial market reflects into shortage of demand for goods has been discussed, among the others, by De Long (2010) [18], following the original idea by Mill (1829) [58].

[5]See Caballero (2010) [13], Credit Suisse (2011)[71].

a role for money by introducing it in the utility function, whereas we consider a cash in advance constraint, according to which money is used to purchase goods in a goods market. Secondly, in Galì (2017)[30] the shock that triggers the liquidity trap is a shock to the natural rate of interest, whereas we focus on a liquidity shock.

Buiter (2014) [12] is another work that deals with the effects of "helicopter money" in a DSGE model. He argues that this policy has a positive impact on nominal spending because of "irredeemability" of money. Auerbach and Obstfeld (2005)[2] discuss the impact of unconventional monetary policy when the economy is in liquidity trap. He finds that a permanent increase of money supply raises output and inflation to the extent to which the increase of money is perceived as permanent by agents.

Our paper is also related to works that study optimal monetary and fiscal policy under commitment at the zero lower bound, in particular when fiscal policy involves the choice of government spending. Eggertson (2001) [21] proves that in a standard New Keynesian model real government spending can be a powerful tool when the economy falls in a liquidity trap. When interest rate is at zero level, a benevolent government can in principle use real spending to close the output gap. He underscores the fact that government spending can be used to stabilize the economy also when the policymaker cannot credibly commit to future policy actions. Similarly, Schmidt (2013) [65] shows that in a standard New Keynesian model optimal policy under commitment calls for a countercyclical role of government spending, that allows to mitigate recession and subsequent overshooting of the target. This last argument is strong enough to claim that fiscal policy can be preferred to monetary policy for stabilizing output. By the same token, Nakata (2013) [59] argues that the increase in government spending required to close the output gap under commitment is more sizeable in case of uncertainty rather than in a deterministic model. All these works acknowledge an effective countercylical role for public spending when the interest rate reaches the zero level. In our paper we show that this result is even stronger when the fiscal stimulus is financed with money.

Regarding the model presented in the paper, we draw an economy which is hit by a liquidity shock. We model liquidity shock in the same vein as in Benigno and Nisticò (2017)[5]. They use a model featuring liquidity shocks to analyse the implications of monetary policy. Main result is that there is an important role for the expansion of central bank's balance sheet to make up for the shortage of safe assets. A significant departure from their model is that they draw an economy split by borrowers and savers, where borrowing occurs through financial intermediation. Thus, the loss of value of assets that give rise to the liquidity crisis regards assets that are issued by the private sector. Conversely, we model a standard representative agent model, where bonds are supplied by the treasury. This implies that, unlike their paper, the liquidity

shock impacts on government bonds.

Other papers include a liquidity shock in their models. In particualar, Kiyotaki and Moore (2012)[46] and Del Negro et al. (2012)[19] build an economy in which money and equity have different degree of pledgeability, as equity can suddenly face a deterioration of value. However, in these models assets are used by entrepreneurs to finance their investment decision rather than in the goods market.

Finally, related works are also those who study fiscal policy at the zero lower bound. In this strand of literature, Krugman (1998)[47], Christiano, Eichenbaum and Rebelo (2010)[15], Eggertson (2010)[22] represent some prominent works documenting that fiscal policy is more effective when interest rate are at zero level.

3.2 The model

We present an almost standard New Keynesian DSGE model, featuring households, firms competing in monopolistic competitive firms, government and central bank. A crucial feature of the model is the presence of a financial friction, designed as a cash-in-advance constraint, which guarantees a motive (liquidity properties) for holding money[6].

3.2.1 Households

There is a continuum of households of size one. A representative household is provided with the following life-time expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \omega_g \frac{G_t^{1-\sigma^g}}{1-\sigma^g} - \frac{\int_0^1 N_t^{1+\eta}(i)di}{1+\eta} \right] \tag{3.1}$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\epsilon}{\epsilon-1}} di\right]^{\frac{\epsilon-1}{\epsilon}}$ and $G_t \equiv \left[\int_0^1 G_t(i)^{\frac{\epsilon}{\epsilon-1}} di\right]^{\frac{\epsilon-1}{\epsilon}}$ are Dixit-Stiglitz aggregator for consumption and public spending, which aggregate the demand for goods, from households and government, over all i goods in the economy. Government spending enters utility function scaled by the parameter ω_g. The elasticity of substitution among the different varieties of goods demanded is given by ϵ and assumed to be greater than one ($\epsilon > 1$). The representative household supplies all i's good-specific unit of labor $N_t(i)$.

The timing of the economy says that goods market opens first, followed by assets market. As in Benigno and Nisticò (2017)[5], we assume that at generic time "t" in the goods market

[6] A financial friction modelled in this way is in line with seminal works by Lucas (1982, 1984)[52] [53].

households use money and a fraction $0 \leq \xi_t \leq 1$ of government bonds carried from the previous period to purchase goods[7]. Thus, this cash-in-advance constraint is assumed:

$$M_{t-1}(1 + i^m_{t-1}) + \xi_t B_{t-1}(1 + i_{t-1}) \geq P_t C_t \tag{3.2}$$

where M_{t-1} is nominal money held from the previous period, paying the interest rate i^m_{t-1}, and B_{t-1} are nominal one-period government bonds issued in the secondary market and paying the interest rate i_{t-1}. Both money and government bonds are safe assets, in the sense that they are perfect store of value, as they are remunerated at their risk-free nominal interest rate. Moreover, money and a fraction ξ_t of bonds are also liquid assets, because they are accepted in the goods market as a medium of exchange to purchase goods. Thus, they differ in the fact that not all government bonds are liquid assets, but only a fraction ξ_t of them.

Furthermore, we assume that this fraction of bonds may experience sudden deterioration of their value. This reflects the "liquidity shock" in our economy: a shock to the variable ξ_t implies that fewer safe assets are accepted to purchase goods in the goods market and therefore we have a contraction of liquidity[8]. We assume that the variable ξ_t follows an AR(1) process in deviation from the steady state:

$$\hat{\xi}_t = \rho \hat{\xi}_{t-1} + \varepsilon_t \tag{3.3}$$

with $\hat{\xi}_t \equiv \log(\xi_t/\xi)$, where ξ is the steady state value and ε_t is an i.i.d. innovation with $\varepsilon_t \sim N(0, \sigma^2_\xi)$.

After goods market closes assets market opens, so that households can allocate their resources (money unspent in the goods market, labor income and profits' shares) to purchase assets and to save some money, which is needed to buy consumption goods in the next period, when the goods market will be open again. Hence, per-period budget constraint is written in the following way:

$$M_t + B_t = (1 - \xi_t)B_{t-1}(1 + i_{t-1}) + \left[M_{t-1}(1 + i^m_{t-1}) + \xi_t B_{t-1}(1 + i_{t-1}) - P_t C_t\right] + \int_0^1 W_t(i)N_t(i)di + \Gamma_t - P_t\bar{T} \tag{3.4}$$

where Γ_t are nominal profits obtained by holding firms' shares. We assume that the government levies a constant lump-sum tax $\bar{T}$ on households. This is required in order to guarantee a primary surplus for government in steady state and thus does not play any role in the dynamics of the

[7] In this paper we consider as "money" central bank's reserves and therefore we call "money" and "reserves" interchangeably. This is equivalent to assume that currency, which pays no interest, is negligible.

[8] As in Lagos (2010)[48] and Benigno and Nisticò (2017)[5] "liquidity" here has the interpretation of the "degree of acceptance" of the assets, which can be an intrinsic time-varying property of the asset itself or a feature attributed by the "market".

model.

It must be noted that an important feature of the model is that central bank's money pays an interest rate. This is consistent with reality: since 2008 central banks in countries like US and Japan have been paying an interest rate on current account balances (reserves or "high-powered" money) held at central bank. In our model, this interest rate corresponds to the policy rate[9]. Concerning the relation with the interest rate on bonds, the following inequalities hold:

$$i_t \geq i_t^m \geq 0. \tag{3.5}$$

In fact, since central bank "high-powered" money is the safest asset in the economy, nobody should be allowed to borrow at a rate lower than the one central bank pays on its liabilities, consisting this in a clear arbitrage opportunity. Besides, we take into account the zero lower bound on the interest rate on reserves.

Hence, households utility maximization problem consists of maximizing utility (3.1) under the budget constraint (3.4) and the cash-in-advance constraint (3.2). Complementary slackness condition related to the cash-in-advance constraint is given by:

$$\phi_t \geq 0 \qquad \phi_t \left[M_{t-1}(1 + i_{t-1}^m) + \xi_t B_{t-1}(1 + i_{t-1}) - P_t C_t \right] = 0. \tag{3.6}$$

where $\phi_t \geq 0$ is a Lagrange multiplier. The problem is solved by taking the first-order conditions with respect to the control variables, that are consumption, bonds, money and labor. We can neglect corner solution and denote with $\lambda_t > 0$ the Lagrange multiplier associated to the budget constraint, therefore we get:

$$C_t^{-\sigma} = (\lambda_t + \phi_t) P_t \tag{3.7}$$

$$\lambda_t = \beta(1 + i_t) E_t(\lambda_{t+1} + \xi_{t+1}\phi_{t+1}) \tag{3.8}$$

$$\lambda_t = \beta(1 + i_t^m) E_t(\lambda_{t+1} + \phi_{t+1}) \tag{3.9}$$

$$N_t^{\eta}(i) = \lambda_t W_t(i). \tag{3.10}$$

We can now combine (3.7)-(3.9) so as to yield the equations characterizing the spread:

$$\frac{i_t - i_t^m}{1 + i_t} E_t \left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right] = E_t \left\{ (1 - \xi_{t+1}) \gamma_{t+1} \left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right] \right\} \tag{3.11}$$

[9] This implies that there is no corridor between interest rates, so that the interest rate that central bank pays on its liabilities coincides with the policy rate and also with the interest rate on marginal lending facility. This is not of practical relevance in our framework.

$$\gamma_t = 1 - \beta(1 + i_t^m) E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right\}. \tag{3.12}$$

Due to the timing of the economy, the spread at time "t" affects the intertemporal subistution of consumption across time "$t+1$" and "$t+2$". By looking at equations (3.8) and (3.9), in periods when $\phi_{t+1} = 0$ (when the complementary slackness condition (3.6) is relaxed), we get $i_t = i_t^m$, so that there is no differential between the interest rate on bonds and the interest on reserves. This implies that money and bonds become perfect substitutes and their total amount held by the representative agent exceeds the level required to purchase goods in the goods market. In this case equation (3.12) boils down to a standard Euler equation. Interestingly, this case occurs also when $\xi_{t+1} = 1$, namely when all assets can be used for transaction purposes.
Using conditions (3.7),(3.9),(3.10) and (3.12) we obtain the marginal rate of substitution between consumption and labor:

$$N_t^{\eta}(i) C_t^{\sigma} = (1 - \gamma_t) \frac{W_t(i)}{P_t}. \tag{3.13}$$

The variable $(1 - \gamma_t)$ is a consequence of the monetary friction and represents a wedge between the marginal rate of substitution and real wage. In order to fully characterize the equilibrium of the model we need a transversality condition that prevents household from dying with bonds and money left over:

$$\lim_{T \to \infty} E_t \left[\beta^{T-t} \lambda_T \left(B_T + M_T \right) \right] = 0 \tag{3.14}$$

where $\lambda_T = C_T^{-\sigma} / P_T - \phi_T$ is the marginal utility of nominal wealth.

3.2.2 Firms

We assume that the economy features a continuum of firms of size one, each producing one differentiated good. The production function is linear in labor: $Y_t(i) = N_t(i)$.
Firms set price as in Calvo's model, that is a only a generic fraction $1 - \theta$ of firms with $0 < \theta \leq 1$ can change its price at any future period T with probability θ^{T-t}, whereas the remaining fraction θ of firms do not change the price. Profit maximization problem is set up as:

$$E_t \left\{ \sum_{T=t}^{\infty} (\theta\beta)^{T-t} \lambda_T \left[\frac{P_t(j)}{P_T} Y_T(i)(1 + \tau) - \frac{W_T(i)}{P_T} Y_T(i) \right] \right\}$$

where τ is a government subsidy aimed at eliminating the distortions of the economy. Firms maximize this profit function subject to the demand function, which is:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{3.15}$$

where aggregate output in real terms is given by:

$$Y_t = C_t + G_t. \tag{3.16}$$

First order condition of the above problem delivers:

$$\frac{P_t^*}{P_t} = \mu \frac{E_t \left\{ \sum_{T=t}^{\infty} (\theta\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \right)^{\epsilon} \frac{W_T(i)}{P_T} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\theta\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \right)^{\epsilon-1} Y_T \right\}} \tag{3.17}$$

where $\mu \equiv \epsilon/((\epsilon-1)(1+\tau))$ and in equilibrium $P_t(i) = P_t^*$, because all firms set their price equal to the optimal one.
In this last condition we can use (3.13) along with the demand function (3.15) and the resource constraint (3.16) so as to get:

$$\left(\frac{P_t^*}{P_t} \right)^{1+\epsilon\eta} = \mu \frac{E_t \left\{ \sum_{T=t}^{\infty} (\theta\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \right)^{\epsilon(1+\eta)} (Y_T(i))^{\eta} Y_T \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\theta\beta)^{T-t} \lambda_T \left(\frac{P_T}{P_t} \right)^{\epsilon-1} Y_T (C_T)^{-\sigma} (1-\gamma_T) \right\}} \tag{3.18}$$

where it can be noted the impact of the financial friction through the variable $(1-\gamma_t)$. Moreover, Calvo assumption implies that the aggregate price level follows the law of motion:

$$P_t^{1-\epsilon} = (1-\theta) P_t^{*1-\epsilon} + \theta P_{t-1}^{1-\epsilon}. \tag{3.19}$$

From the equation above we can obtain the law of motion for the index of price dispersion:

$$\Delta_t = \theta \left(\Pi_t \right)^{\epsilon(1+\eta)} \Delta_{t-1} + (1-\theta) \left(\frac{1-\theta \left(\Pi_t \right)^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon(1+\eta)}{\epsilon-1}} \tag{3.20}$$

where price dispersion is defined as: $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon(1+\eta)} di$. Finally, using (3.15) and the definition of production function and price dispersion we obtain that aggregate labor and output are related as:

$$Y_t \Delta_t = N_t \tag{3.21}$$

so that the policymaker has to keep track of price dispersion when studying welfare effects of different policies.

3.2.3 Government and Central Bank's budget constraint

We assume that government purchases consumption goods and subsidizes firms' revenues. In order to pay for this expenditure, government levies a constant lump-sum tax $\bar{T}$ and issues public debt in form of short-term government bonds (in nominal terms) B_t^G, paying the interest rate i_t. These bonds are issued in the secondary market and so are purchased by central bank and households.

We denote the total amount held by household as B_t whereas bonds held by central bank are B_t^C. Therefore, market clearing for bonds reads:

$$B_t^G = B_t + B_t^C. \tag{3.22}$$

Furthermore, government receives a real transfer S_t from central bank, through which this latter rebates its profits to the government. Thus, we implicitly assume that all central bank's profits are rebated to the government through the transfer S_t. Budget constraint of government is therefore given by:

$$P_t(G_t - \bar{T} + \tau Y_t) + B_{t-1}^G(1 + i_t) = B_t^G + P_t S_t. \tag{3.23}$$

Regarding central bank, its budget constraint in nominal terms reads:

$$B_t^C + M_{t-1}^C(1 + i_{t-1}^m) = B_{t-1}^C(1 + i_{t-1}) + M_t^C - P_t S_t \tag{3.24}$$

where B_t^C denotes central bank's holding of public debt and M_t^C is central bank money. Thus, the central bank's balance sheet features government bonds as the only asset and money the only liability. We can combine the two budget constraints by substituting for the transfer and using market clearing condition for bonds (3.22) so as to get the consolidated budget constraint:

$$P_t(G_t - \bar{T} + \tau Y_t) = B_t - B_{t-1}(1 + i_{t-1}) + M_t^C - M_{t-1}^C(1 + i_{t-1}^m). \tag{3.25}$$

On the left we have the deficit of the government whereas on the right the ways of financing it, namely variation in nominal money (seigniorage revenues) and variation of public debt issued to the private sector. Further, we can define the deficit $D_t \equiv (G_t - \bar{T} + \tau Y_t)$ and impose the transversality condition (3.14) so as to obtain the intertemporal budget constraint of the consolidated government-central bank:

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{\lambda_T}{\lambda_t} P_T D_T = E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{\lambda_T}{\lambda_t} \frac{i_T - i_T^m}{1 + i_T^m} M_T - B_{t-1}(1 + i_{t-1}) - M_{t-1}^C(1 + i_{t-1}^m). \tag{3.26}$$

This intertemporal equation allows to underscore the fact that the present value of deficit must be financed by the present value of future seigniorage revenues minus the initial stock of bonds held by private sector and central bank's money.

3.2.4 Market clearing and equilibrium conditions

In the market for reserves demand and supply must equalize, so that:

$$M_t = M_t^C. \tag{3.27}$$

We recall also the condition of equilibrium in the bonds market:

$$B_t^G = B_t + B_t^C. \tag{3.28}$$

We now report a summary of the equilibrium conditions of the model. On the demand side, we have the equations characterizing the spread between the market interest rate and the interest rate on reserves:

$$\frac{i_t - i_t^m}{1+i_t} E_t \left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right] = E_t \left\{ (1-\xi_{t+1}) \gamma_{t+1} \left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}} \right] \right\} \tag{3.29}$$

$$\gamma_t = 1 - \beta(1+i_t^m) E_t \left\{ \frac{(C_{t+1})^{-\sigma}}{(C_t)^{-\sigma} \Pi_{t+1}} \right\}. \tag{3.30}$$

The cash-in-advance constraint in real terms is expressed as:

$$\frac{m_{t-1}(1+i_{t-1}^m)}{\Pi_t} + \frac{\xi_t b_{t-1}(1+i_{t-1})}{\Pi_t} = C_t. \tag{3.31}$$

where $\Pi_t \equiv P_t/P_{t-1}$, $b_t \equiv B_t/P_t$, $m_t \equiv M_t/P_t$. As for the supply side of our economy, we obtain the following conditions:

$$\left(\frac{1-\theta\left(\Pi_t\right)^{\epsilon-1}}{1-\theta} \right)^{\frac{1+\epsilon\eta}{\epsilon-1}} = \frac{F_t}{K_t} \tag{3.32}$$

$$F_t = \mu \lambda_t Y_t^{1+\eta} \Delta_t^{\eta} + \theta\beta E_t \left\{ F_{t+1} \left(\Pi_{t+1}\right)^{\epsilon(1+\eta)} \right\} \tag{3.33}$$

$$K_t = \lambda_t Y_t C_t^{-\sigma} (1-\gamma_t) + \theta\beta E_t \left\{ K_{t+1} \left(\Pi_{t+1}\right)^{\epsilon} \right\}. \tag{3.34}$$

Price dispersion evolves as:

$$\Delta_t = \theta \left(\Pi_t\right)^{\epsilon(1+\eta)} \Delta_{t-1} + (1-\theta) \left(\frac{1-\theta\left(\Pi_t\right)^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon(1+\eta)}{\epsilon-1}}. \tag{3.35}$$

The resource constraint is given by:

$$Y_t = C_t + G_t. \tag{3.36}$$

Finally, consolidated budget constraint of treasury and central bank in real terms becomes:

$$G_t - \bar{T} + \tau Y_t = b_t - \frac{b_{t-1}(1+i_{t-1})}{\Pi_t} + \frac{M_t^C}{P_t} - \frac{M_{t-1}^C(1+i_{t-1}^m)}{P_t} \tag{3.37}$$

where we have used market clearing condition (3.27) of reserves.
Therefore, given the exogenous processes of $\{\xi_t\}$ and initial value of variables $\left\{m_{t_0-1}, b_{t_0-1}, i_{t_0-1}, i_{t_0-1}^m, \Delta_{t_0-1}\right\}$, an equilibrium of the model is a set of stochastic processes

$$\{Y_t, C_t, \Pi_t, P_t, F_t, K_t, \Delta_t, \gamma_t, i_t, i_t^m, m_t, b_t, G_t\}_{t=t_0}^{\infty}$$

that solve the set of equations (3.29)-(3.37) along with transversality condition (3.14) and the definition of inflation $\Pi_t = P_t/P_{t-1}$. Thus, there are nine equations and twelve unknowns, so that we are left with three variables object of policy. We can now spell out our definition of policies, that we are going to use from here onwards.

- A **monetary policy** is a choice of two of the following three instruments $\{i_t, i_t^m, M_t\}_{t=t_0}^{\infty}$.
- A **government spending policy** is a choice of the process of real government spending $\{G_t\}_{t=t_0}^{\infty}$.
- Therefore, a **monetary-government spending policy** is the choice of two of the following three instruments $\{i_t, i_t^m, M_t\}_{t=t_0}^{\infty}$ along with the choice of $\{G_t\}_{t=t_0}^{\infty}$.

The specification of monetary policy represents a sharp departure from the standard assumption of a central bank steering only the interest rate object of monetary policy. Under the standard assumption, there is only one monetary policy instrument: the policy rate. In this case, open market operations allow central bank to achieve the target for the policy rate and so they must be consistent with the chosen level of the policy rate. In other words, money supply is strictly dependent on the policy rate set by central bank.
In our model, instead, central bank can set both the policy rate and money supply, without this latter being dependent from the former. This pattern resembles the current scenario, where central banks are contemporaneously steering the policy rate (conventional monetary policy) and implementing balance-sheet expansions aimed at expanding monetary base (unconventional monetary policy), so that the supply of reserves is set independently of the policy rate[10].

[10]Some central bank is now paying reserves at a rate that is equal or very close to the target for the overnight rate. This is - how is called - the "floor system", which allows to "divorce" the supply of reserves from the policy rate. For a discussion on this point, see Keister et al. (2008) [43] and Goodfriend (2002) [33].

Importantly, this framework provides a role for unconventional monetary policy.

3.3 Optimal stabilization policy

We evaluate optimal government spending-monetary policy with full commitment by solving a standard Ramsey problem where the welfare criterion is given by the discounted sum of households' utility:

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{C_t^{1-\sigma}}{1-\sigma}+\omega_g\frac{G_t^{1-\sigma^g}}{1-\sigma^g}-\frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right] \tag{3.38}$$

where we have used (3.21) so that now price dispersion affects our welfare measure[11]. Using the linear-quadratic approach developed by Benigno and Woodford (2003) [6] we can derive a quadratic loss function starting from (3.38) and imposing that the Ramsey policy approximates an efficient steady state. Then, to compute the Ramsey policy we approximate equilibrium conditions (3.29)-(3.37) around the deterministic steady state of the model which, under some restriction, is the efficient one.

It is shown in Appendix that the quadratic loss function becomes:

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right\}$$

$$L_t\equiv\lambda_c(\hat{C}_t)^2+\lambda_g(\hat{G}_t)^2+\eta(\hat{Y}_t)^2+\lambda_\pi(\pi_t)^2 \tag{3.39}$$

where $\lambda_c,\lambda_g,\lambda_\pi$ are made up by structural parameters, as shown in Appendix.

Below we show that the first best can be achieved by implementing a particular subsidy to firms' revenues.

3.3.1 Steady state properties and numerical calibration

The efficient allocation is obtained by a planner maximizing welfare criterion under the resource constraint (3.36). This problem delivers the efficient allocation in steady state:

$$C^{-\sigma}=\omega_g G^{-\sigma^g}=Y^{\eta}. \tag{3.40}$$

We now show that with an appropriate choice of the subsidy this allocation can be attained also in our decentralized economy, when there is no inflation ($\Pi=1$). In fact, from the steady state

[11] We assume therefore that the consolidated government-central bank is "benevolent".

version of equation (3.18) and using condition (3.40) we obtain:

$$1 = \frac{\mu}{(1-\gamma)} = \frac{\epsilon}{(\epsilon-1)(1+\tau)(1-\gamma)}. \tag{3.41}$$

From this, it emerges that the level of the subsidy required to make the steady state an efficient one is:

$$\tau = \frac{\epsilon}{(\epsilon-1)(1-\gamma)} - 1. \tag{3.42}$$

It is important to note that the subsidy is set to remove both the distortion caused by monopolistic competition and the one given by the monetary friction. We can now discuss some properties of this steady state. From equations (3.29) and (3.30), and imposing no inflation ($\Pi = 1$) we can obtain:

$$\frac{i - i^m}{1 + i^m} = (1-\xi)\gamma \tag{3.43}$$

$$\gamma = 1 - \beta(1 + i^m). \tag{3.44}$$

Therefore, in case in which there is no monetary friction in steady state, that is when $\gamma = 0$, the interest rate differential between money and bonds no longer appears: $i = i^m$. This would correspond to the Friedman rule, where there is no distortion across assets in the economy. We notice that the Friedman rule case is nested in the efficient steady state of our economy as well as the case in which a differential between the two rates actually exists. Indeed, combining equations (3.43) and (3.44) and recalling that $0 \leq \xi \leq 1$, we note that the interest rate on reserves can attain any level within the range $[0, 1/\beta - 1]$. Accordingly, the first best encompasses both the case in which the interest on reserves is equal to the rate on bonds ($i = i^m$) and the case where the former is lower ($i > i^m$)[12].

The model is solved by taking a log-linearization of the above equilibrium conditions around the efficient steady state. The full log-linear model is shown in Appendix. In the simulation of the following sections we use the parameters reported in Table 3.1. The interest rate on government bonds is set to 4% per annum whereas the interest rate of reserves to 0.75%, which corresponds to the value was set by Fed when this rate was introduced in 2008. The discount factor is set to $\beta = 0.99$. We consider a logarithmic utility function in consumption and public spending, so that $\sigma = \sigma^g = 1$. The value of the parameter of the Calvo model of price adjustment (θ) and the elasticity of substitution across goods (ϵ) are quite standard in the literature. They imply, respectively, an average duration of prices of four quarters and a frictionless mark-up of 1.2. The inverse of the Frisch elasticity η is set equal to 1 as in Justiniano et al. (2015)[41] As in Galì (2017)[30], public debt-to-GDP is set to annual 60%, which implies b_y= 0.6 *4. To

[12]This is a consequence of the fact that we allow the existence of a subsidy that can be set in a way to eliminate the monetary friction.

Parameter	*(description)*	*Value*
β	(discount factor)	0.99
σ	(consumption intert. elasticity)	1
σ^g	(government sp. intert. elast.)	1
σ^g	(weight of gov.sp. in utility)	0.25
η	(inverse Frisch elasticity)	1
ϵ	(elasticity among goods)	7.66
θ	(Calvo frequency price adj.)	0.75
$1+i$	(annual int. rate on bonds)	1.04
$1+i^m$	(annual int. rate on reserves)	1.0075
s_c	(consumption share in GDP)	0.8
m_y	(money-to-GDP ratio)	0.125*4
b_y	(public debt-to-GDP ratio)	0.6*4

Table 3.1: Model calibration for numerical solution.

calibrate the steady state money-to-GDP ratio we consider the average velocity of M1 money in US over the years of the Great Moderation (1984-2007), so that we set m_y= 0.125*4. Notice that this calibration implies a fraction of assets usable for transactions in the goods market that amounts to around 12%. The share of consumption on GDP in US accounts for around 80 per cent, therefore we set $s_c = 0.8$. Clearly, the share of public spending on GDP is the remaining 20% ($s_g = 0.2$). Accordingly, the weight of public spending in utility function must be set equal to ω_g =0.25. This calibration implies that the steady state percentage level of lump-sum tax in GDP amounts to 44%.

3.3.2 Optimal monetary-government spending policy with a liquidity shock

In this section we show the optimal responses of the Ramsey policymaker to a liquidity shock[13]. As a first exercise we examine the impulse response functions under an optimal monetary-government spending policy, where all three policy instruments are used. We simulate a one-period liquidity shock, which causes a 1% deterioration of government bonds[14]. To investigate the impact of the magnitude of the shock we report also cases of 2% and 3% liquidity shocks. Figure 3.1 displays the impulse response functions of the optimal policy.

We can immediately notice that optimal policy entails three features: first, a one-period fiscal

[13]The complete Ramsey problem is described in Appendix.

[14]To solve the model, taking into account an occasionally binding zero lower bound, we employ the Occbin algorithm developed by Guerrieri and Iacoviello (2016)[34].

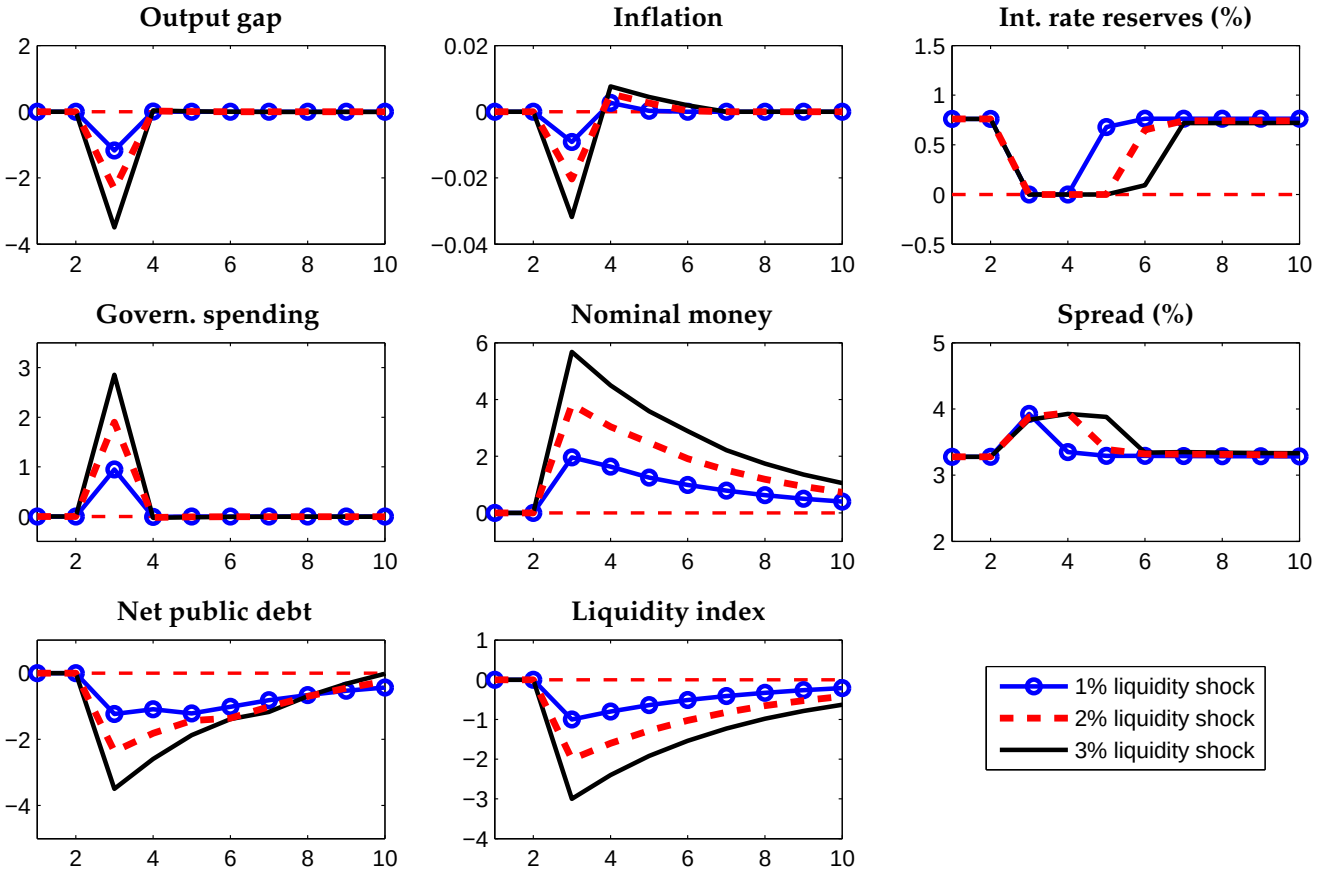

Figure 3.1: Optimal monetary-government spending policy for 1%, 2% and 3% liquidity shock. Liquidity process $\hat{\xi}_t$ is assumed to have persistence $\rho = 0.8$. Variables are in percentage deviation from the steady state, apart from the interest rates and the spread between them, which are expressed in annual percentage points.

stimulus that looks symmetric to the fall in output; second, a large balance-sheet expansion, which reflects into a sizeable money injection; third, the interest rate on reserves is pushed downwards to reach the zero lower bound.

Concerning government spending, it can be clearly seen how this draws a path which is countercyclical with respect to the fall in output gap. Indeed, we note that in case of a 1% liquidity shock the increase in government spending amounts to around 1%, which offsets the fall in output of quite the same magnitude. In the period after the shock, output is back to potential level and the fiscal stimulus is therefore fully withdrawn. This countercyclical movement of government spending when the policy rate has reached the zero lower bound can be found also in Eggertson (2001)[21], Schmidt (2013)[65] and Nakata (2013)[59]. However, an important difference arises: unlike these works, here output does not overshoot but rather goes back to potential the period after the shock.

It must be noted that, in principle, the increase in government spending works against the minimization of the loss function (3.39), as can be clearly noted through the second term in the same

loss function. However, it turns out that the Ramsey policymaker tends to "bear" an increase in government spending with the aim of stabilizing the other gaps, given by other terms in the loss function (3.39). Indeed, in the next counterfactual exercise we will show that this fiscal stimulus allows output gap and inflation to fall less. Nonetheless, the fact that a fiscal stimulus is costly for the policymaker can be inferred from the reduction of public spending to the target the period after the shock, when also the output gap closes.

As for the second feature, it emerges a long-lasting balance-sheet expansion which draws a path that appears to track symmetrically the liquidity process. In particular, money supply increases by around 2% on impact and then reverts back slowly, with seemingly the same pace as the liquidity index. Remarkably, the increase in monetary base is more sizeable than the one in government spending: in fact, the increase in monetary base doubles that in public spending.

A striking aspect of this optimal money-financed fiscal stimulus is that whereas the fiscal stimulus lasts only the period of the shock, as it reverts back the period later, the injection of monetary base is prolonged, as it persists until the liquidity shock dies out[15]. Therefore, we can argue that this balance-sheet expansion seems to be implemented so as to apparently restore liquidity in financial system, after the deterioration of bonds in the goods market. This result can be somehow validated when we let the persistence of the liquidity shock take other values with respect to the calibrated one ($\rho = 0.8$). In Figure 3.2, we display also the case of a shock with low persistence ($\rho = 0.4$) and the case of very high persistence ($\rho = 0.99$). It turns out that there is no impact on the variables of the loss function, that report the same path under all different degree of persistence. However, there is a sharp difference in the response of nominal money: when the liquidity shock is highly persistent, the policymaker puts in place a more sizeable and prolonged increase in monetary base. This feature seems to confirm the fact that money is supplied so as to restore the liquidity eroded by the loss of value of government bonds.

The increase of monetary base implied by the optimal policy is associated with a sizeable endogenous decrease in public debt held by private sector: in this way, the representative household can reduce the amount of assets that have lost their value for purchasing goods, and this is replaced with central bank's money. This implies that, if the total amount of public debt was constant, central bank would get to hold ("monetise") a larger share of public debt. Indeed, by market clearing condition for bonds market, one can see that, for a given level of public debt issued by the government, a lower amount held by the private sector translates into a larger portion held by central bank[16]. Thus, the commitment government-central bank would be such that the amount of public debt issued to finance the fiscal stimulus be purchased by central bank.

[15] Differently, in Galì (2017)[30] public spending is exogenous, following an AR(1) process.

[16] This can be seen by repeating the above simulation setting $B_t^G = B^G \quad \forall t$. In this case, it is clear that the mirror image of a fall of the amount of public debt held by the private sector is an increase of this purchased by central bank.

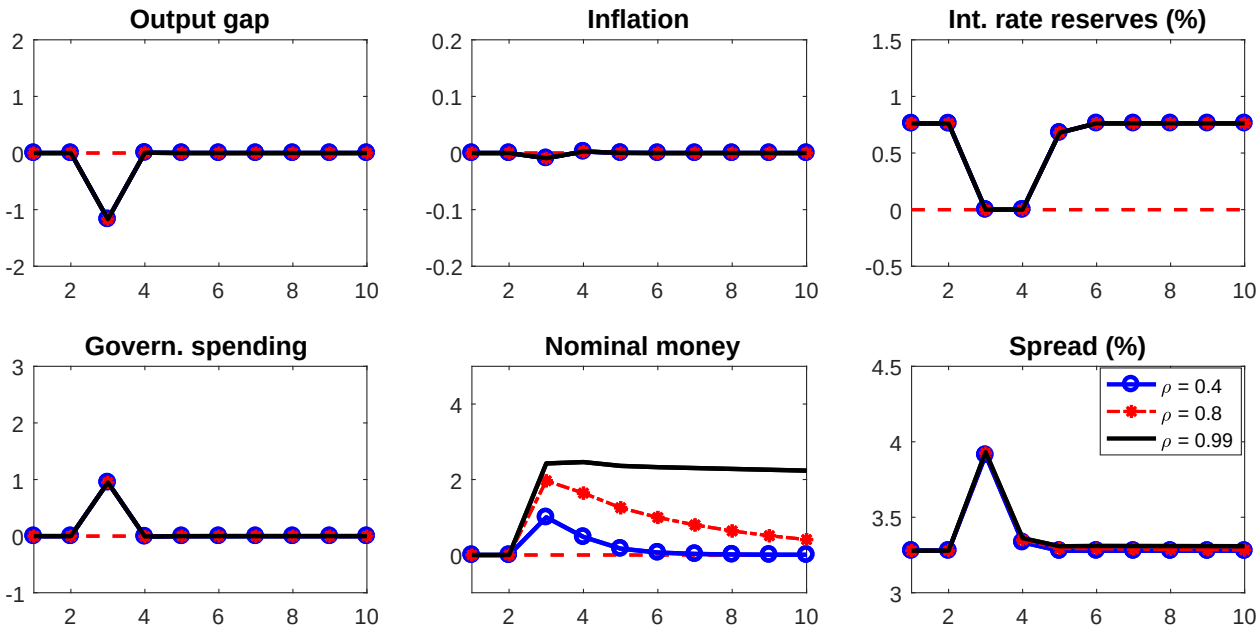

Figure 3.2: Optimal monetary-government spending policy for different degree of persistence of the (1%) liquidity shock. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are expressed in annual percentage points.

The third feature of the optimal monetary-government spending policy has to do with the interest rate on reserves. This is cut down so as to reach the zero lower bound in the period when the shock hits the economy. However, it remains at the zero lower bound for a longer time than the shock. This is a standard result in the literature dealing with optimal monetary policy at the zero lower bound (see Eggertson and Woodford (2003)[24]). The striking difference with this literature is that this result is here combined with a balance-sheet expansion[17]. Interestingly, due to the cut of the interest rate on reserves, the liquidity shock brings about an increase in the liquidity premium, namely the spread between the interest rate on government bonds and the rate on money[18]. Thus, as in Benigno and Nisticò (2017)[5] the optimal unconventional monetary policy does not work through the reduction of the liquidity premium, but rather thorough restoring liquidity, compensating the shortage in safe assets. However, in our model the increase in the liquidity premium produces two contrasting effects[19]. On one hand, higher liquidity premium reflects into lower expected next-period consumption, according to the Euler equation. One the other hand, an increase in the liquidity premium implies also that the government is putting in place a sort of additional balance-sheet expansion to stabilize the

[17] In Eggertson and Woodford (2003)[24] balance-sheet expansions are neutral at the zero lower bound.
[18] The interest rate on bonds remains very close to the steady state (not shown in Figure 3.1).
[19] Clearly, the general equilibrium dynamics of the model is more complex than a standard New Keynesian model because of the transaction friction and the role for the consolidated government-central bank. This implies that the model can hardly be handled analytically and therefore we rely on numerical methods.

economy[20]. In fact, in the goods market households are not only provided with new money, but also with an additional transfer of resources inherited from the previous period, stemming from a larger spread between government bonds and money.

Overall, the joint effect of these three features of the optimal monetary-government spending policy delivers a good performance in terms of stabilization of output gap and inflation. Output gap is fully stabilized in one quarter, owing to the action of government spending. Regarding inflation, we note that the shock produces only quite negligible fall of inflation (around -0.03% in case of a 3% liquidity shock) and subsequent overshooting, before stabilizing completely after two quarters.

Table 3.2 reports the government spending multiplier and the "monetary multiplier", indicating, the impact variation of output due to, respectively, the fiscal stimulus and the monetary expansion. It can be seen that the government spending multiplier is larger than 1 in absolute value, in line with literature suggesting a multiplier larger than 1 when interest rates are at zero level (see, for instance, Christiano, Eichenbaum and Rebelo (2010)[15], Eggertson (2010)[22]). Monetary multiplier is instead lower than 1 in absolute value. Notice that the value of this is increasing in the magnitude of the shock, whereas the fiscal multiplier is decreasing in it.

	$\left\lvert \Delta \hat{Y}_T / \Delta \hat{G}_T \right\rvert$	$\left\lvert \Delta \hat{Y}_T / \Delta \hat{M}_T \right\rvert$
1% shock	1.2315	0.5942
2% shock	1.2272	0.6080
3% shock	1.2251	0.6123

Table 3.2: Government spending and monetary multiplier under optmimal policy.

3.3.3 Optimal unconventional monetary policy with and without fiscal stimulus

We have shown that the optimal monetary-government spending policy in presence of a liquidity shock is a money-financed fiscal stimulus. In this section we compare this optimal policy mix with the case of optimal monetary policy, when government spending is kept constant. This analysis allows to appreciate the role of the fiscal stimulus in the optimal policy mix.

[20]In Benigno and Nisticò (2017)[5] the increase of the liquidity premium gives rise to a redistribution effect within the private sector.

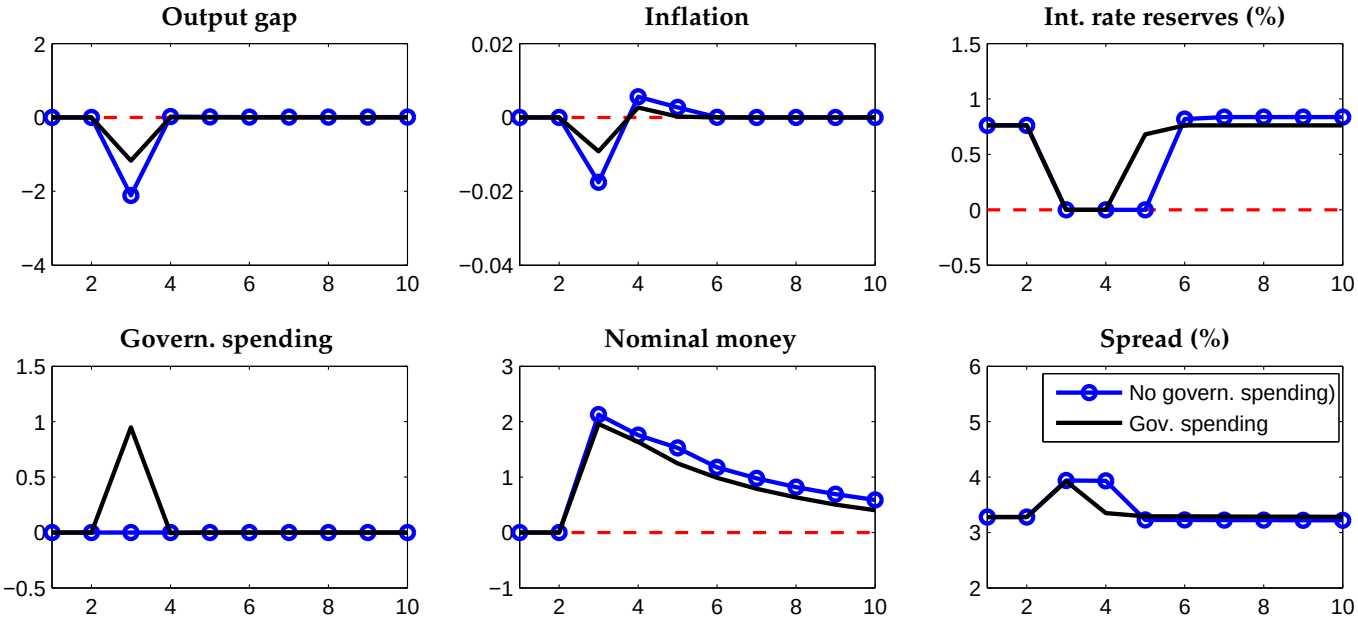

Figure 3.3: The role of government spending. IRFs under optimal monetary-government spending policy (solid black line) and optimal monetary policy with constant fiscal policy (dashed blue line) for a liquidity shock $\hat{\xi}_t$ with persistence $\rho = 0.8$. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are is expressed in annual percentage points.

In this exercise, we assume that public spending is held fixed at its steady state value:

$$\hat{G}_t = 0 \qquad \forall t. \tag{3.45}$$

This implies that government spending is completely stabilized in the loss function (3.38). Now the policymaker has only two variables of monetary policy to be used in order to stabilize the economy. It can be noted the important role played by the fiscal stimulus: in presence of this, the economy experiences a lower fall in output and inflation (see Figure 3.3). Output declines by around one-half of the amount under optimal unconventional monetary policy. Inflation declines to a lesser extent and then stabilizes completely one quarter before the case of monetary policy alone.

It is crucial the role played by government spending which increases in the shock period and subsequently reverts to the steady state. As previously discussed, it is remarkable that the policymaker uses a fiscal stimulus despite this is costly, as can be seen in the loss function (3.39). [21]. To emphasise further this result we evaluate the loss function under both the cases

[21] Notice that when government spending is kept fixed at the first best level, that is when condition (3.45) is satisfied, the loss function loses the second term, which disappears, and the in the first term we can use $\hat{Y}_t = \hat{C}_t$ Thus, loss function in this case is:

$$L_t \equiv \lambda_c(\hat{Y}_t)^2 + \eta(\hat{Y}_t)^2 + \lambda_\pi(\pi_t)^2.$$

of optimal policy and optimal monetary policy alone, namely under the case characterized by (3.45). In Table 3.3 emerges that under the optimal policy mix loss function is largely smaller

	Loss (%)
Optimal Policy	0.0240
$\hat{G}_t = 0$	0.2027

Table 3.3: Loss function value under optimal monetary-government spending policy and under optimal monetary policy with constant government spending.

than in case in which government spending is maintained on target.

As for the other policy variables, we observe that the expansion of monetary base occurs to a larger extent than in the optimal monetary-government spending policy. Thus, an optimal money-financed fiscal stimulus implies that the injection of monetary base can be less important than in case of unconventional monetary policy with no role for government spending.

Another key feature stems from the path of interest rate on reserves: remarkably, in case of fiscal stimulus accompanied by monetary expansion, the stay at the zero lower bound becomes shorter. We can therefore argue that a money-financed fiscal stimulus leads to an earlier exit from the zero lower bound.

Overall, this exercise has shown that a money-financed fiscal stimulus enables to stabilize the economy in a more efficient way than an optimal unconventional monetary policy with no role for government spending.

3.3.4 The sub-optimality of debt-financing the fiscal stimulus

We further stress the role of the optimal money-financed fiscal stimulus, comparing this optimal policy with a policy regime in which the stimulus is not financed with money but only with public debt. In this exercise we therefore assume that monetary policymaker is completely "passive", keeping unchanged the amount of nominal reserves and the interest rate paid on them. As a result, policymaker can only use government bonds and their interest rate to accommodate a liquidity shock.

Hence, we call the "debt-financing" regime a regime in which:

- $M_t = M \qquad \forall t.$
- $i_t^m = i^m \qquad \forall t.$

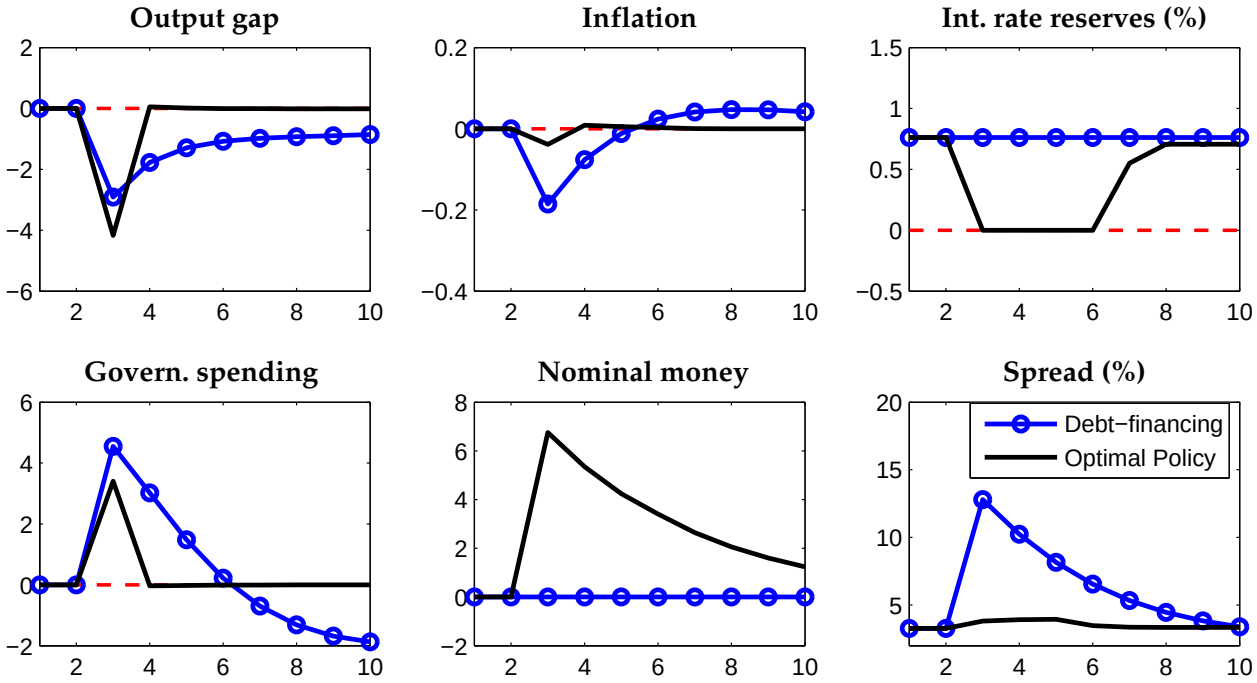

Figure 3.4: Optimal monetary-government spending policy vs. "debt financing" regime. IRFs for a 3.5% liquidity shock $\hat{\xi}_t$ with persistence $\rho = 0.8$. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are expressed in annual percentage points.

where M and i^m are the steady state value of nominal reserves and interest rate on reserves[22]. . The underlying assumption in this regime is that central bank can target the interest rate on reserves by implementing open market operations, even though the amount of money is kept unvaried. This implies that in each period government bonds purchased by central bank B_t^C adjust in accordance with $i_t^m = i^m$[23]. Therefore, the government is now forced to issue public debt to the private sector to finance public spending.

Figures 3.4-3.5 show what happens in this scenario of "fully" passive monetary policy under a 3.5% liquidity shock. Notice that, since nominal money is kept constant, nominal public debt soars progressively, to more than 20% (see Figure 3.5). Moreover, the interest rate on government bonds is now raised to a very large extent, so that liquidity premium increases substantially. Because of both the missing role of money in restoring liquidity and the fact that the interest rate on reserves cannot be cut, the policymaker is forced to increase the interest on bonds[24].

[22] The case in which the interest rate on bonds is targeted and no money is used is shown ahead in the paper (Figure 3.9).

[23] This regime is different from the "debt-financing" regime adopted in Galì (2017)[30]. He considers a regime in which central bank commits to a rule such that inflation is fully targeted in every period, that is $\pi_t = 0$, and money is adjusted endogenously. We call instead this regime "inflation targeting", as we will see in the next section. With "debt-financing" regime instead we mean that money is not used at all, and so is kept fixed. Notice also that an important difference with Galì (2017)[30] is that in our paper we have two monetary policy instruments, rather than one.

[24] As discussed above, an increase of the liquidity premium implies also that households are provided with more resources to purchase goods in the next period, when the goods market will be open again.

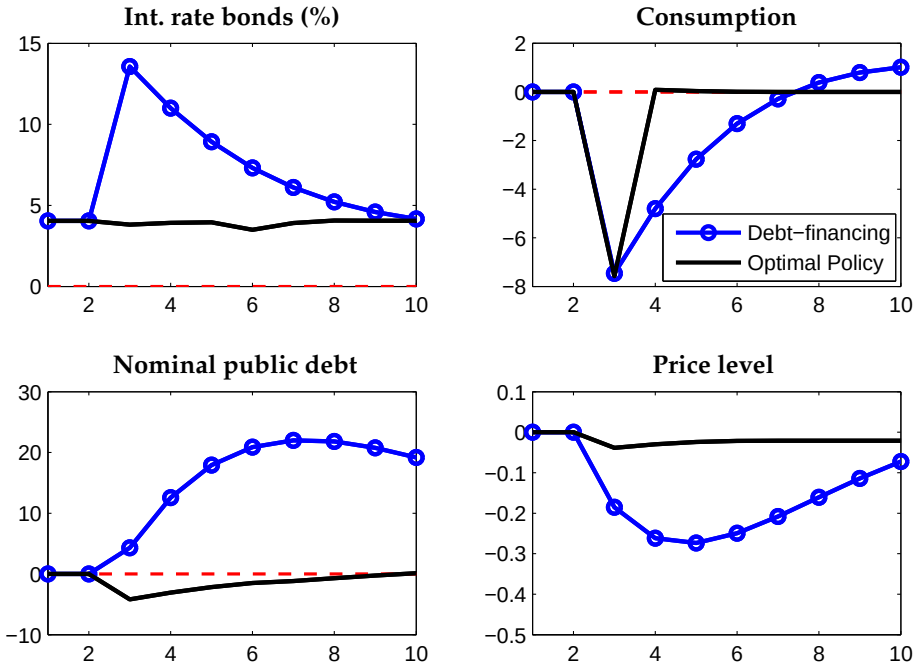

Figure 3.5: Optimal monetary-government spending policy vs. "debt financing" regime. IRF of the price level for a 1%liquidity shock with persistence $\rho = 0.8$. Price level is expressed in percentage variation from the steady state.

Importantly, in the "debt-financing" regime the liquidity shock brings about a larger deflation and a long-lasting recession. As to the former, inflation reaches around -0.2% on impact and then starts increasing, overshooting the target some quarter later. Further, Figure 3.5 displays that the price level experiences a larger and persistent decline. On the other hand, output declines by around 3% and then increases, but overall falls considerably short of the potential level. Therefore, it is unambiguously clear that under the debt-financing regime a liquidity shock is both recessionary and deflationary.

It can be noted that when the policymaker is not allowed to use money to finance public spending the optimal intervention with government spending becomes more pronounced, as public spending has to rise to a larger extent. Surprisingly, after this larger increase, government spending is taken back to a level below the target, implying a flawed stabilization of this.

Finally, Table 3.4 shows that the "debt-financing" regime implies a larger than 1% welfare loss.

To sum up, we note that when the fiscal stimulus is financed with issuance of debt instead

	Loss (%)
Optimal Policy	0.0240
Debt-financing	1.6014

Table 3.4: Loss function value under optimal monetary-government spending policy and under "debt-financing" regime.

of money, the liquidity shock leads to a long-lasting recession and a sharp deflation. Both net public debt and the interest rate paid on government bonds increase significantly to finance a larger fiscal stimulus and to restore the liquidity deteriorated.

3.4 Taylor Rule, Inflation Targeting, GDP targeting with a money-financed fiscal stimulus

In the previous section we first illustrated that the optimal policy mix in this framework involves a money-financed fiscal stimulus. Then, we went on showing that this optimal policy markedly outperforms a debt-financed fiscal stimulus, where reserves and their policy rate are kept constant. This latter policy can be labelled a "passive" monetary policy, to emphasise the fact the monetary policy instruments (like money and the policy rate) are not allowed to vary and therefore impact on the model equilibrium dynamics.
In this section we study the case of "active" monetary policies, where central bank deliberately uses its instruments to achieve a particular target. Therefore, we will demonstrate that these policies perform well when the active monetary policy is combined with an optimally implemented fiscal stimulus, such that the joint policy is in fact configured as a money-financed fiscal stimulus.

3.4.1 Taylor Rule

Since its proposal by John B. Taylor in 1993, a milestone in the vast majority of models with nominal rigidities is the Taylor rule. Because it has approximated fairly well post-war data, this policy has been widely used as a benchmark to study optimal monetary policy in different models. We characterize the "Taylor rule" regime in this way:

- $i_t^m = \phi_\pi \pi_t \qquad \forall t.$
- $i_t = i \qquad \forall t.$

with $\phi_\pi = 1.5$. Thus, we assume that policy rate i_t^m reacts positively to the changes in inflation, according to the well-known Taylor principle. Further, to ensure the determinacy of the equilibrium we impose that the response is more than one-to-one (see Woodford (2003) [79]). As for the other monetary policy instrument, we assume that the interest rate on government bonds remains at the steady state.
Figure 3.6 displays the responses under the Taylor rule regime and optimal government spending (solid black line). Importantly, to highlight the role of this latter we report also the responses

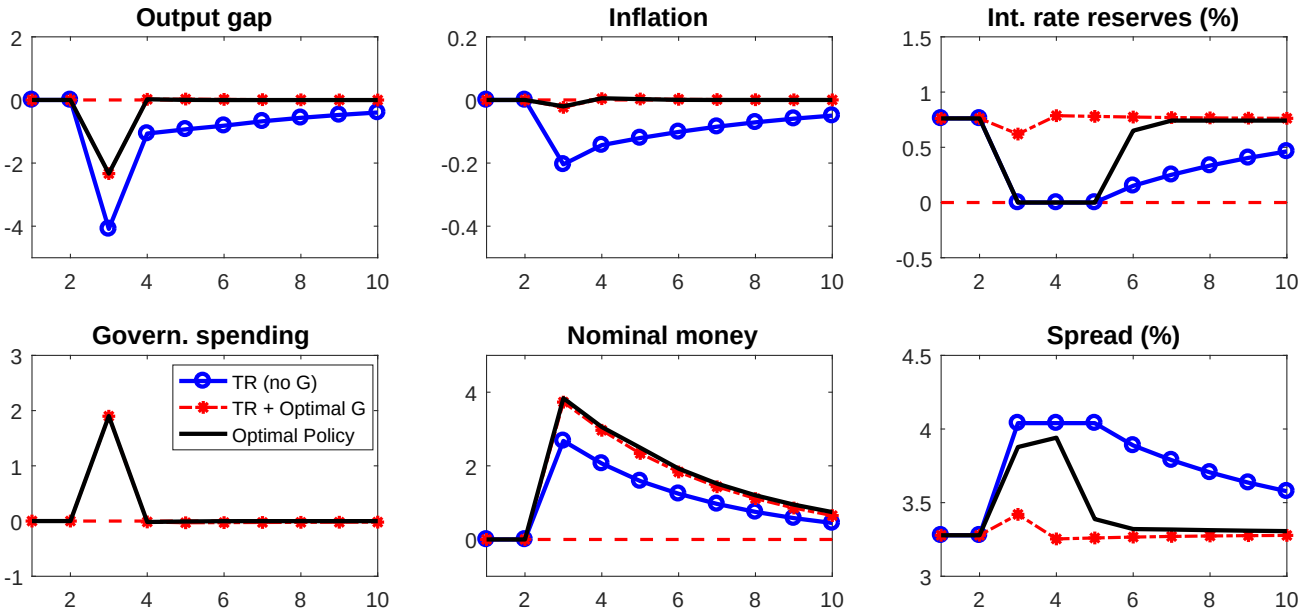

Figure 3.6: Taylor Rule regime: optimal government spending vs. model without government spending.IRFs of main variables for a 1%liquidity shock with persistence ρ=0.8. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are expressed in annual percentage points.

when government spending is constant (circled blue line)[25]. In this latter case, a liquidity shock draws a scenario that, ignoring the size of the variables' deviation, resembles the one really occurred after the crisis. Indeed, we can observe that, absent government spending, the liquidity shocks leads to sharp recession and a prolonged deflation. In particular, the liquidity shock drives to a 4% fall in output, which ameliorates the following period but never returns to the target level in the period displayed. Similarly, inflation drops by 0.2% and remains persistently in negative territory. As a consequence, the interest rate on reserves is cut down until the zero lower bound becomes a constraint. The commitment to a Taylor rule implies that the interest rate stays at the zero lower bound for three quarters. As a consequence, the liquidity premium goes up. Thus, the expansion of monetary base is not only an outcome of the optimal monetary policy, but is also implicated by an "active" monetary policy, like the Taylor rule.

Importantly, when the model features optimal government spending the Taylor rule (dashed red line with asterisks) regime performs unambiguously better, because the output gap is closed after only one period and inflation remains anchored to the target throughout. Hence, the role of government spending in complementing the "active" monetary policy so as to minimise output gap and deflation is decisive.

The striking feature lies in that when the standard Taylor rule is accompanied with an optimal fiscal stimulus the performance in terms of stabilization of macroeconomic variables is as good

[25]In the model without government spending we adjust the steady state value of transfer-to-GDP to preserve the same steady state.

as in the case of the optimal policy (solid black line), as can be seen by the perfect coincidence of the responses in the two cases. Most noticeably, because of the money-financed fiscal stimulus successfully prevents deflation to happen, the Taylor rule does not call for a reduction of the policy rate, and thus, in contrast with optimal policy, the ZLB does not pose a constraint. Therefore, this result highlights the powerful role of the money-financed fiscal stimulus in stabilizing the economy, which does not hinge on cutting the policy rate until the ZLB binds. As we will discuss next, this result is common with other "active" monetary policies.

3.4.2 Inflation-targeting

The fact that optimal government spending improves the effectiveness of an "active" monetary policy in stabilizing inflation and output can be appreciated also when monetary policy is configured as an "inflation-targeting" policy. In this exercise, we refer to "inflation-targeting" as a regime in which central banks seeks to achieve full price stability:

- $\pi_t = 0 \qquad \forall t.$
- $i_t = i \qquad \forall t.$

Further, we assume that in this regime the interest rate on bonds is kept fixed at the steady state. Thus, in this "active" monetary policy regime central bank may nonetheless steer the interest rate on reserves to achieve inflation targeting. As a consequence, the zero lower bound can potentially become a binding constraint[26].

Figure 3.7 displays impulse response functions under optimal monetary-government spending policy in comparison with the case of optimal government spending policy combined with a monetary policy under "inflation-targeting" regime. As in the Taylor rule regime, to achieve full inflation-targeting central bank undergoes a sizeable and enduring balance-sheet expansion. Surprisingly, it clearly comes out that when this policy is combined with the optimal fiscal stimulus, delivers a satisfying stabilization. In fact, under this regime output is almost completely stabilized, denoting a better performance than optimal policy if one has to only look to the output gap. However, as observed also in the Taylor rule regime, the counterpart is the behaviour of government spending: under "inflation-targeting" government spending has to increase to a larger extent than under optimal policy. Thus, it seem that, in order to pursue a policy of zero inflation, the policymaker commits to a large one-time fiscal stimulus, which is financed by an endogenous persistent balance-sheet expansion[27]. Importantly, with this policy in place, the

[26]Notice that the assumption of targeting the interest rate on bonds is not critical: results that follow hold even in the case in which central bank targets the interest rate on reserves. These are available upon request.

[27]By contrast, in Galì (2017)[30] a policy of inflation-targeting (called "debt-financing is his paper) leads to an endogenous reduction of money.

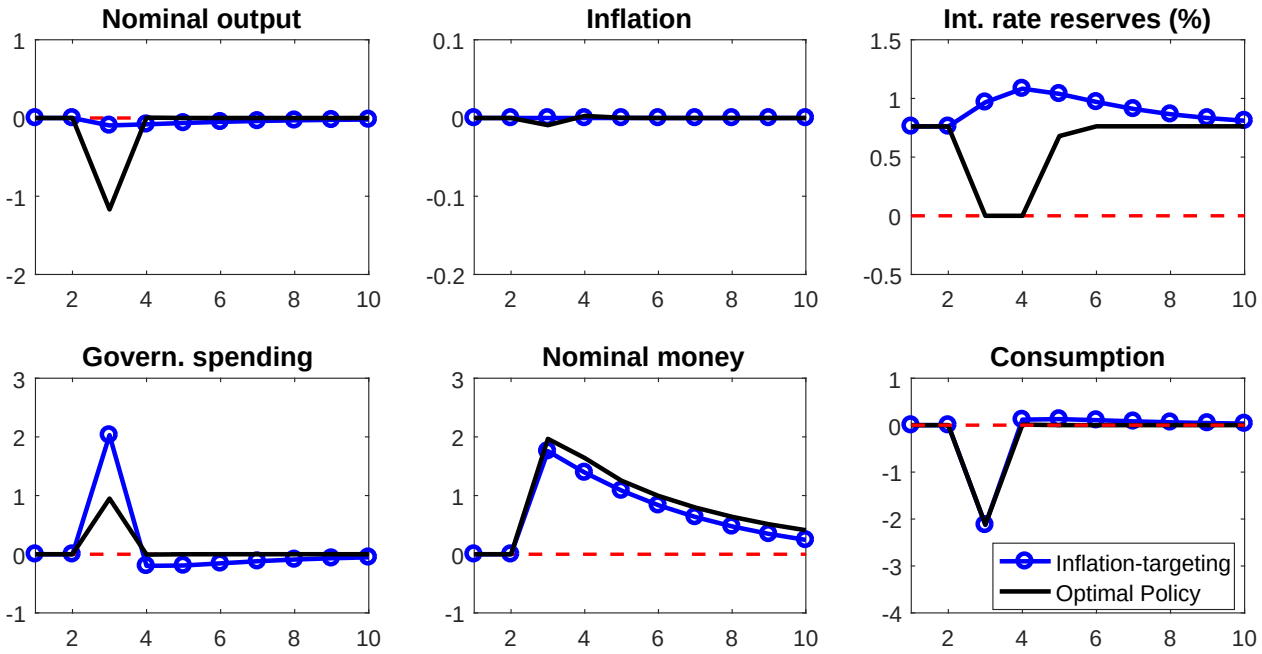

Figure 3.7: Optimal monetary-government spending policy vs. "inflation targeting" regime. IRFs of main variables for a 1%liquidity shock with persistence $\rho = 0.8$. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are expressed in annual percentage points.

increase in government spending perfectly makes up for the reduction in consumption caused by the liquidity shock, so that real output remains very close to potential output. In other words, the fall in private spending (consumption) is compensated by the increase in public spending.

A compelling feature of this "active" monetary policy is the fact that the interest rate on reserves is not cut, but rather remains quite stable. This implies that, when the policymaker can rely on a money-financed fiscal stimulus, is not forced to accommodate the liquidity shock by cutting the policy rate. Remarkably, this is a common feature with the policy of targeting nominal output, as we will show next.

Hence, the exercise of this section reveals that inflation-targeting policy enables to stabilize output gap extremely well when government and central bank coordinate so as to implement a money-financed fiscal stimulus involving a one-quarter above-target fiscal stimulus together with a prolonged monetary base expansion.

3.4.3 Nominal GDP Targeting

We now investigate the behaviour of another "active" monetary policy: the policy of targeting nominal output. This policy is based on an old idea (Meade (1978)[54]), which has now regained a broad support in the midst of the post-crisis recession[28]. According to the theory, this sort of active monetary policy is particularly recommended when central banks lose their power

[28]See, among the others, Sumner (2012) [72], Woodford (2012)[80] and references therein.

to influence the economy through movements of the policy rate. One of the key advantage is therefore the fact that the policymaker can pursue its policy regardless of the level of the interest rate. In other words, targeting nominal output would be (close to) optimal, irrespective of the economy being in a liquidity trap or not. Thus, we configure the "nominal GDP targeting" regime in this way:

- $P_t Y_t = PY \qquad \forall t.$
- $i_t = i \qquad \forall t.$

We show the responses obtained under this policy in Figure 3.8. Remarkably, we observe that targeting nominal output delivers an excellent stabilization of inflation. However, as in case of inflation targeting, this occurs at the expense of a much larger fiscal stimulus: the increase in government spending amounts to 2% in our simulation, doubling the increase in the case of our optimal monetary-government spending policy. Likewise, the increase in government spending exactly offsets the fall in consumption caused by the liquidity shock and the interest rate on reserves remains quite stable, without being taken to the zero lower bound.[29]. Thus, steering the policy rate turns out to be unnecessary when the policymaker is allowed to implement a money-financed fiscal stimulus. It is ultimately required that a balance-sheet expansion is accompanied by a larger increase in public spending.

To stress further that the excellent stabilization of inflation is due to the implementation of the money-financed fiscal stimulus, rather than cutting the policy rate, in Figure 3.9 we consider a regime in which central bank targets both nominal GDP and the interest rate on reserves. We plot this policy together with the case of a regime in which central bank keeps the amount of money unchanged and targets the interest rate on bonds, so that the interest rate on reserves can eventually be lowered. Clearly, this latter policy entails that the fiscal stimulus can only be financed with public debt. It is remarkable the role played by the injection of monetary base: in fact, when this is not allowed, inflation, nominal output and public spending diverge considerably from their target, and the interest rate on reserves stays at zero level for very long time.

Hence, we argue that an "active" policy of targeting nominal output performs strikingly well for the stabilization of inflation. This occurs because the one-time increase in government spending, which is coupled with a persistent increase in money supply, becomes more pronounced in order to counteract the large fall in consumption. Importantly, this policy is not dependent on a reduction of the interest rate on reserves.

[29] In Benigno and Nisticò (2017)[5] nominal GDP targeting is optimal, but the interest rate on reserves is lowered until the zero lower bound.

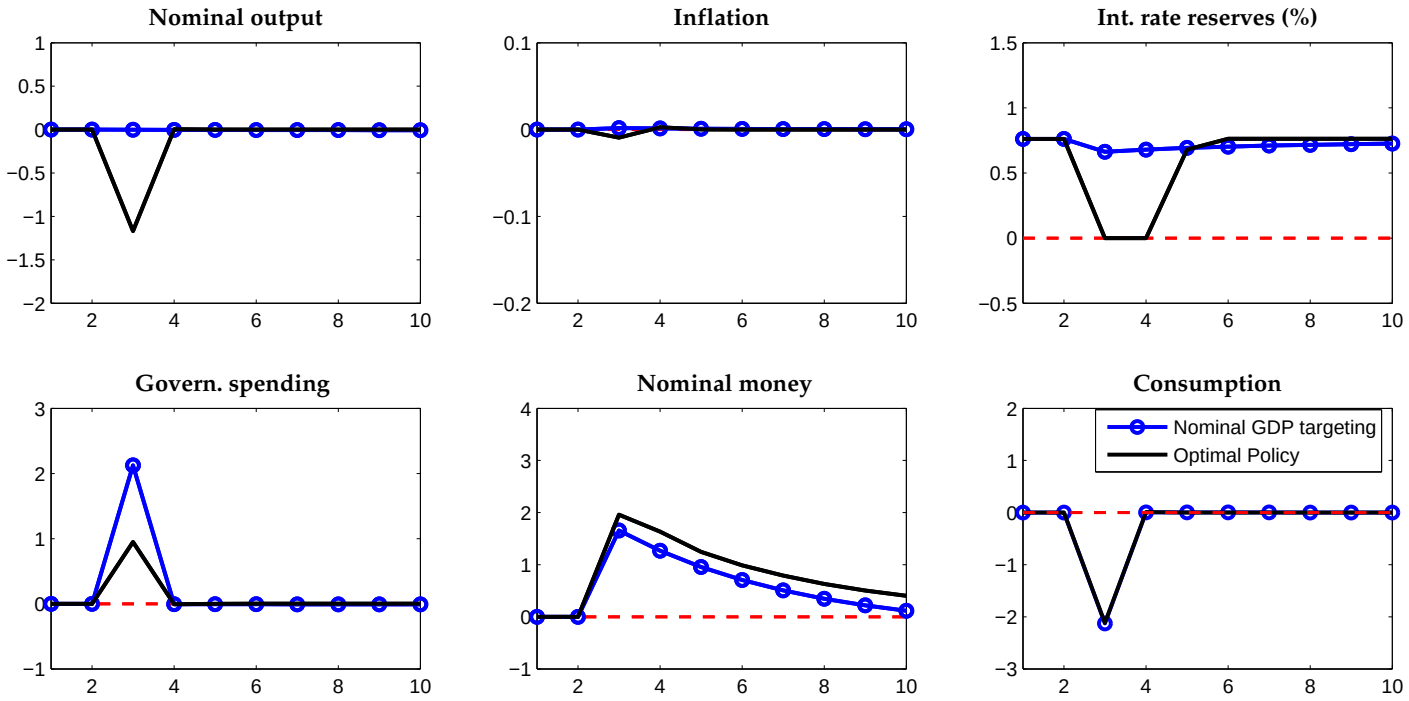

Figure 3.8: Optimal monetary-government spending policy vs. "targeting nominal GDP" regime. IRFs of main variables for a 1% liquidity shock with persistence $\rho = 0.8$. Variables are in percentage deviation from the steady state, apart from the interest rate on

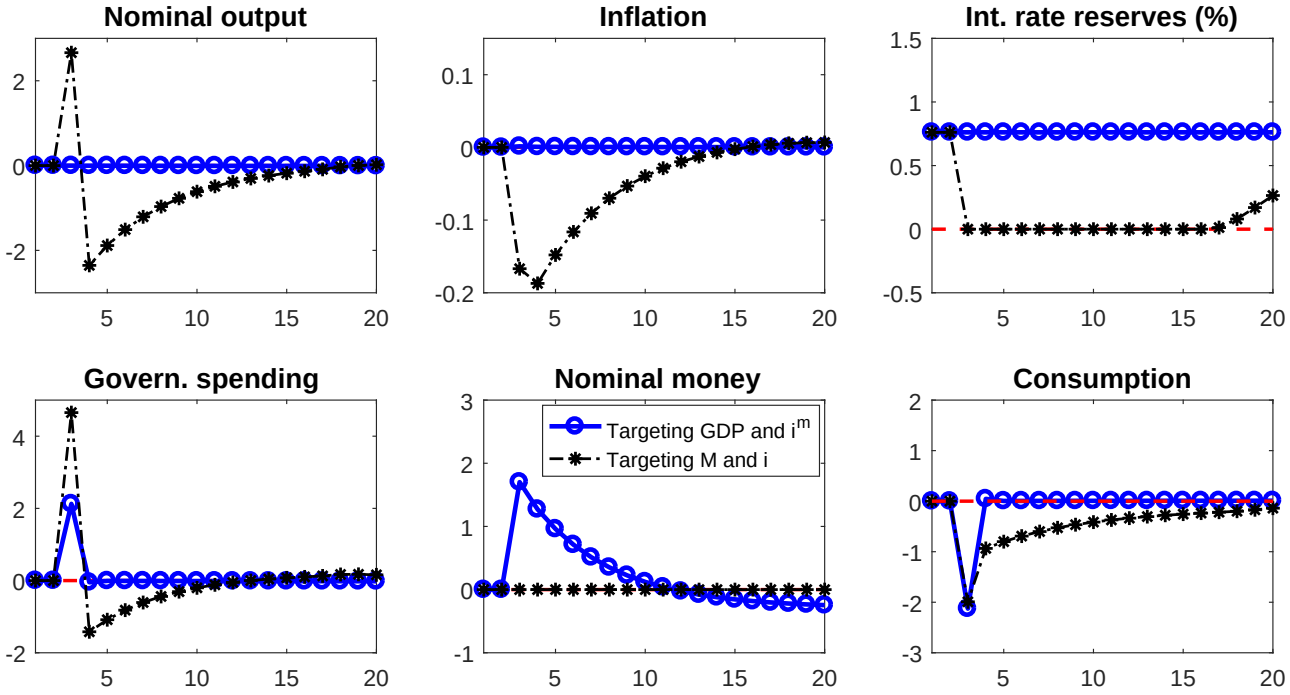

Figure 3.9: Targeting money and the interest rate on bonds vs. targeting nominal output and the interest rate on reserves. IRFs of main variables for a 1%liquidity shock with persistence $\rho = 0.8$. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves, which is expressed in annual percentage points.

3.5 Concluding Remarks

We have analysed the optimal mix of monetary policy and government spending policy in an economy hit by a liquidity shock, which brings about a fall in both output and inflation, and pushes the policy rate to the zero lower bound. It turns out that optimal policy prescribes a money-financed fiscal stimulus, which is capable of stabilizing completely the economy quite shortly. Thus, in our general equilibrium framework it is found that, for a given level of public debt, it is optimal for the policymaker financing a fiscal stimulus with purchase of public debt from the private sector, so that the amount of public debt in private hands reduces whereas money supply instead grows. In other words, the optimal commitment would be to "monetise" the increase in deficit brought about by the expansionary fiscal policy.

Crucial in the paper is the the aspect of the optimal money-financed fiscal stimulus, which entails a one-time countercyclical fiscal stimulus accompanied by a central bank's balance-sheet expansion. Importantly, the increase of money is not withdrawn at the same time of the fiscal stimulus, but instead is maintained in the financial system with large persistence, implying a slow tapering from the uncoventional monetary policy.

A money-financed fiscal stimulus affects also the optimal conduct of the interest rate object of monetary policy. It turns out that when optimal policy is in form of a money-financed fiscal stimulus, the exit from the zero lower bound occurs earlier than in case of an optimal unconventional monetary policy without the support of government spending.

The comparison of the optimal money-financed fiscal stimulus with the policy of financing the stimulus with public debt is striking: when optimal government spending is entirely financed by public debt (so that monetary policy is completely "passive") the economy experiences a long-lasting recession and deflation, and stabilization of output and inflation becomes problematic.

By contrast, when money supply increases so as to support "active" monetary policies, like the standard Taylor rule, "inflation-targeting" and "nominal GDP targeting" results change significantly. In this case, as long as the "active" monetary policy is complemented with an optimal fiscal stimulus the stabilization is astonishingly improved. In particular, a standard Taylor rule attains as an efficient stabilization as the joint optimal policy. When "inflation-targeting" or "nominal GDP targeting" are in place, stabilization of, respectively, output gap and inflation are greatly attained even without lowering the interest rate, although the policymaker is forced to exert a large fiscal expansion that, however, is shortly withdrawn.

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Appendix A

Appendix to Chapter 2

A.1 Competitive equilibrium

Here we lay out a summary of all equilibrium conditions needed to characterize the competitive equilibrium of the model.

Savers' equilibrium conditions are given by:

$$\frac{\Gamma_s z_t}{C_t^s - \gamma_s C_{t-1}^s} = \beta_s R_t E_t \left[\frac{\Gamma_s z_{t+1}}{(C_{t+1}^s - \gamma_s C_t^s)\Pi_{t+1}} \right] \tag{A.1}$$

$$\frac{(N_t^s)^\psi (C_t^s - \gamma_s C_{t-1}^s)}{\Gamma_s} = w_t^s \tag{A.2}$$

$$\frac{\Gamma_s Q_t}{C_t^s - \gamma_s C_{t-1}^s} = \frac{j_t}{(\bar{H} - H_t^b)} + \beta_s E_t \left[\frac{\Gamma_s Q_{t+1} z_{t+1}}{(C_{t+1}^s - \gamma_s C_t^s) z_t} \right] = 0 \tag{A.3}$$

$$\frac{\Gamma_s z_t}{C_t^s - \gamma_s C_{t-1}^s} q_t^k = \beta_s E_t \left[\frac{\Gamma_s z_{t+1}}{C_{t+1}^s - \gamma_s C_t^s} ((R_{t+1}^k + q_{t+1}^k) \right] \tag{A.4}$$

$$a_t q_t^k \frac{\Gamma_s z_t}{C_t^s - \gamma_s C_{t-1}^s} \left(1 - \frac{\phi \Delta I_t}{I} \right) = \frac{\Gamma_s z_t}{C_t^s - \gamma_s C_{t-1}^s} - \beta_s E_t \left[\frac{\Gamma_s z_{t+1}}{C_{t+1}^s - \gamma_s C_t^s} a_{t+1} q_{t+1}^k \frac{\phi \Delta I_{t+1}}{I} \right] \tag{A.5}$$

where in equation (A.3) we have substituted the housing market clearing condition. Capital accumulation equation is written as:

$$K_t = a_t \left(I_t - \frac{\phi}{2} \left(\frac{I_t - I_{t-1}}{I} \right)^2 \right) + (1 - \delta) K_{t-1}. \tag{A.6}$$

Equilibrium conditions for borrowers are:

$$\frac{\Gamma_b z_t}{C_t^b - \gamma_b C_{t-1}^b} (1 - \omega_t) = \beta_b E_t \left[\frac{\Gamma_b R_t z_{t+1}}{(C_{t+1}^b - \gamma_b C_t^b)\Pi_{t+1}} \right] \tag{A.7}$$

$$\frac{(N_t^b)^{\psi}(C_t^b - \gamma^b C_{t-1}^b)}{\Gamma_b} = w_t^b \left(1 + (1-\gamma)\tau\omega_t\right) \tag{A.8}$$

$$\frac{\Gamma_b Q_t}{C_t^b - \gamma_b C_{t-1}^b}\left(1 - (1-\gamma)m\omega_t\right) = \frac{j_t}{H_t^b} + \beta_b E_t\left[\frac{\Gamma_b Q_{t+1} z_{t+1}}{(C_{t+1}^b - \gamma_b C_t^b) z_t}\right] = 0. \tag{A.9}$$

Borrowers' budget constraint is:

$$C_t^b + Q_t(H_t^b - H_{t-1}^b) = b_t^b - \frac{b_{t-1}^b R_{t-1}}{\Pi_t} + w_t^b N_t^b, \tag{A.10}$$

and the borrowing constraint:

$$b_t^b \leq \gamma m Q_t H_t^b + (1-\gamma)\tau w_t^b N_t^b \tag{A.11}$$

holding with equality if $\omega_t > 0$.

As for the supply side of our economy, we can rewrite equation (2.24) in recursive way by defining two additional variables F_t and L_t, so that:

$$\left(\frac{1 - \theta\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\epsilon_t - 1}}{1-\theta}\right)^{\frac{1}{\epsilon_t - 1}} = \frac{F_t}{L_t} \tag{A.12}$$

$$F_t = \frac{\Gamma^s Y_t}{C_t^s - \gamma^s C_{t-1}^s} + \theta\beta_s E_t\left\{F_{t+1}\left(\Pi_{t+1}\right)^{\epsilon_t - 1}\right\} \tag{A.13}$$

$$L_t = \frac{\epsilon_t}{\epsilon_t - 1}\frac{\Gamma^s Y_t MC_t}{C_t^s - \gamma^s C_{t-1}^s} + \theta\beta_s E_t\left\{L_{t+1}\left(\Pi_{t+1}\right)^{\epsilon_t}\right\} \tag{A.14}$$

$$MC_t = \frac{W_t^s N_t^s / P_t}{(1-\alpha)\chi\Delta_t Y_t} \tag{A.15}$$

$$MC_t = \frac{W_t^b N_t^b / P_t}{(1-\alpha)(1-\chi)\Delta_t Y_t} \tag{A.16}$$

$$MC_t = \frac{R_t^k K_{t-1}}{\alpha\Delta_t Y_t} \tag{A.17}$$

using the definitions of aggregate labor and capital. Price dispersion evolves as:

$$\Delta_t = \theta\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\epsilon_t}\Delta_{t-1} + (1-\theta)\left(\frac{1 - \theta\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\epsilon_t - 1}}{1-\theta}\right)^{\frac{\epsilon_t}{\epsilon_t - 1}}. \tag{A.18}$$

By equalizing aggregate production function and resource constraint we get:

$$\Delta_t Y_t = K_t^{\alpha}\left((N_t^s)^{\chi}\left(N_t^b\right)^{1-\chi}\right)^{(1-\alpha)} \tag{A.19}$$

Finally, resource constraint is given by:

$$Y_t = C_t^s + C_t^b + I_t. \tag{A.20}$$

Therefore, we can now characterize the competitive equilibrium of the model. Assuming exogenous stochastic processes $\{\zeta_t, j_t, a_t, z_t, \epsilon_t\}_{t=0}^{\infty}$ a competitive equilibrium of the baseline model is a set of stochastic processes

$$\left\{Y_t, C_t^b, C_t^s, H_t^b, N_t^s, N_t^b, b_t^b, w_t^s, w_t^b, \omega_t, q_t, Q_t^k, F_t, L_t, \Delta_t, MC_t, \Pi_t, R_t^k, K_t, I_t\right\}$$

given initial value of variables $\left\{C_{-1}^b, C_{-1}^s, H_{-1}^b, b_{-1}^b \Delta_{-1}, K_{-1}, I_{-1}, R_{-1}\right\}$ and a rule of monetary policy $\{R_t\}_{t=0}^{\infty}$ specified by equation (2.26), so that equations (A.1)-(A.20) are all contemporaneously satisfied.

A.2 Data and observation equations

All data series are quarterly, with range 1991:3 - 2005:4. Series are all retrieved from the FRED website.

- **Real consumption**. Real personal consumption expenditure, chained 2009 dollars, logged. Series "PCECC96".
  Model variable: $\tilde{C}_t = (C_t/C) - 1$.

- **Nominal short-term interest rate**. Effective Federal Funds Rate, annualized percent, divided by 400 to express in quarterly units, then logged. Series "FEDFUNDS".
  Model variable: $\tilde{R}_t = R_t - R$.

- **Inflation**. Log difference of the implicit price deflator. Series "GDPDEF".
  Model variable: $\tilde{\Pi}_t = \log(\Pi_t/\Pi)$.

- **Real Home Prices**. Standard and Poor/Case-Shiller U.S. National Home Price Index (series "CSUSHPINSA"), deflated by the GDP deflator and then logged.
  Model variable: $\tilde{Q}_t = (Q_t/Q) - 1$.

- **Investment**. Real private fixed investment: Nonresidential (chain-type quantity index), series "B008RG3Q086SBEA", seasonally adjusted, logged.
  Model variable: $\tilde{I}_t = (I_t/I) - 1$.

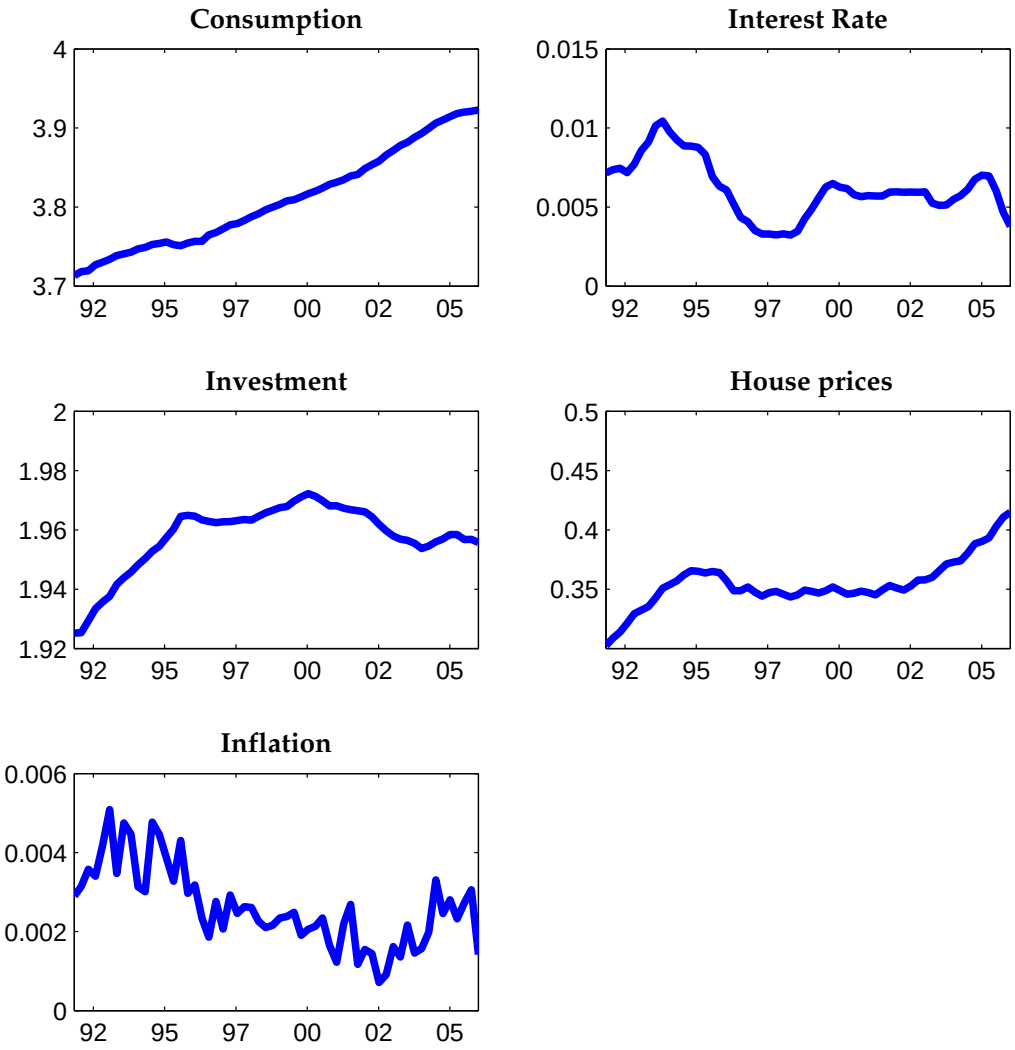

Figure A.1: Raw data of model observables. 1991:3 - 2005:4.

Appendix B

Appendix to Chapter 3

B.1 Log-linear model

In this section we report the log-linear equations that characterize the model. They are obtained by approximating the non-linear equilibrium conditions (3.29)-(3.37) around the non-stochastic efficient steady state.

Imposing $\Pi = 1$ and taking a log-linear approximation of (3.29)-(3.30) we obtain:

$$\hat{i}_t - \hat{i}_t^m + \frac{i - i^m}{1 + i^m} E_t \hat{\xi}_{t+1} = \Lambda \left[\lambda_c E_t (\hat{C}_{t+2} - \hat{C}_{t+1}) + E_t \pi_{t+2} - E_t \hat{i}_{t+1}^m \right]. \tag{B.1}$$

where $\pi_t \equiv \log(\Pi_t)$, $\hat{i}_t \equiv \log((1 + i_t)/(1 + i))$, $i_t^{\hat{m}} \equiv \log((1 + i_t^m)/(1 + i^m))$, $\hat{\xi_{t+1}} \equiv (\xi_{t+1} - \xi)/(1 - \xi)$, $\hat{C}_t \equiv (C_t - C)/Y$, $\Lambda = (1 - \xi)\beta(1 + i)$, $\lambda_c \equiv \sigma Y/C$ and all variables without the subscript are steady state variables.

On the supply side, we obtain a variation of the standard linear New Keynesian Phillips curve:

$$\pi_t = \kappa \left[(\lambda_c + \eta) \hat{Y}_t - \lambda_c \hat{G}_t \right] + \kappa \left[\lambda_c E_t (\hat{C}_{t+1} - \hat{C}_t) + E_t \pi_{t+1} - \hat{i}_t^m \right] + \beta E_t \pi_{t+1}. \tag{B.2}$$

with $\hat{Y}_t \equiv \log(Y_t/Y)$, $\hat{G}_t \equiv (G_t - G)/Y$ and $\kappa \equiv (1-\theta)(1-\theta\beta)/((1+\eta\epsilon)\theta)$. The cash-in-advance constraint in log-linear terms becomes:

$$\vartheta_m \left(\frac{\hat{m}_{t-1}}{m_y} + \hat{i}_{t-1}^m \right) + \xi \vartheta_b \left(\frac{\hat{b}_{t-1}}{b_y} + \hat{i}_{t-1} \right) + (1 - \xi) \vartheta_b \hat{\xi}_t = \hat{C}_t + (\vartheta_m + \xi \vartheta_b) \pi_t \tag{B.3}$$

defining $\hat{m_t} \equiv (m_t - m)/Y$, $m_y = m/Y$, $b_y = b/Y$, $\vartheta_b \equiv b_y(1 + i)$, $\vartheta_m \equiv m_y(1 + i^m)$.

The resource constraint is:

$$\hat{Y}_t = \hat{C}_t + \hat{G}_t \tag{B.4}$$

whereas the government-central bank budget constraint:

$$\hat{G}_t + \tau \hat{Y}_t = \hat{b}_t - \hat{b}_{t-1}(1+i) + \hat{m}_t - \hat{m}_{t-1}(1+i^m) - \vartheta_b \hat{i}_{t-1} - \vartheta_m \hat{i}^m_{t-1} + (\vartheta_b + \vartheta_m)\pi_t. \qquad \text{(B.5)}$$

Therefore, an equilibrium in the log-linear model consists in a sequence of variables $\left\{\hat{Y}_t, \hat{C}_t, \pi_t, \hat{i}_t, \hat{i}^m_t, \hat{m}_t, \hat{b}_t, \hat{G}_t\right\}$ satisfying equations (B.1)-(B.5), given the exogenous stochastic process (3) for $\hat{\xi}_t$ and with three policy variables to be specified.

B.2 Derivation loss function

We need to prove how the loss function (3.39) is derived. First, recall that in the efficient steady state the following conditions hold:

$$C^{-\sigma} = \omega_g G^{-\sigma^g} = Y^{\eta} \qquad \text{(B.6)}$$

By approximating (3.38) up to a second order we get:

$$U_t = U + \left[C^{-\sigma}(C_t - C) - \frac{\sigma}{2}C^{-\sigma-1}(C_t - C)^2\right] + \omega_g \left[G^{-\sigma^g}(G_t - G) - \frac{\sigma^g}{2}G^{-\sigma^g-1}(G_t - G)^2\right]$$
$$- \left[Y^{\eta}(Y_t - Y) + \frac{\eta}{2}Y^{\eta-1}(Y_t - Y)^2\right] - \frac{Y^{1+\eta}}{1+\eta}(\Delta_t - 1) + t.i.p. + \mathcal{O}\left(||\xi_t||\right)^3$$

where it must be noted that Δ_t is already of second-order.
This equation can be simplified to get:

$$U_t = U + C^{-\sigma}\left[(C_t - C) - \frac{\sigma}{2}C^{-1}(C_t - C)^2\right] + \omega_g G^{-\sigma^g}\left[(G_t - G) - \frac{\sigma^g}{2}G^{-1}(G_t - G)^2\right]$$
$$- Y^{\eta}\left[(Y_t - Y) + \frac{\eta}{2}Y^{-1}(Y_t - Y)^2\right] - \frac{Y^{1+\eta}}{1+\eta}(\Delta_t - 1) + t.i.p. + \mathcal{O}\left(||\xi_t||\right)^3.$$

For a generic variable Z_t we have:

$$Z_t = Z\left(1 + \hat{Z}_t + \hat{Z}_t^{\,2}\right) + \mathcal{O}\left(||\xi_t||\right)^3$$

with $\hat{Z}_t \equiv \log(Z_t/Z)$. Thus, we can obtain:

$$U_t = U + Y^{1+\eta}\left[-\frac{\sigma s_c^{-1}}{2}(\hat{C}_t)^2 - \frac{\sigma^g s_g^{-1}}{2}(\hat{G}_t)^2 - \frac{\eta}{2}(\hat{Y}_t)^2\right] - \frac{Y^{1+\eta}}{1+\eta}(\Delta_t - 1) + t.i.p. + \mathcal{O}\left(||\xi_t||\right)^3$$

where $s_c \equiv C/Y$, $s_g \equiv G/Y$ and we have used condition (51) and the resource constraint (3.36). We now approximate (3.35) and then integrate it across time to yield:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_t = \frac{\theta}{(1-\theta)(1-\theta\beta)} \epsilon(1+\eta)(1+\eta\epsilon) \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{(\pi_t)^2}{2} + t.i.p. + \mathcal{O}\left(||\xi_t||\right)^3.$$

Finally, we obtain our quadratic loss function:

$$W_{t_0} = -\frac{Y^{1+\eta}}{2} E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[\lambda_c (\hat{C}_t)^2 + \lambda_g (\hat{G}_t)^2 + \eta (\hat{Y}_t)^2 + \lambda_\pi (\pi_t)^2 \right] \right\} + t.i.p. + \mathcal{O}\left(||\xi_t||\right)^3$$

with:

$$\lambda_c \equiv \sigma s_c^{-1}$$

$$\lambda_g \equiv \sigma^g s_g^{-1}$$

$$\lambda_\pi \equiv \frac{\epsilon}{\kappa}.$$

B.3 Ramsey problem

Here we report the Ramsey problem under full commitment. We assume therefore that initial values of Lagrange multipliers are their steady state value.
The full problem is set up in the following way:

$$\begin{aligned}
E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Big\{ & \left[\lambda_c (\hat{C}_t)^2 + \lambda_g (\hat{G}_t)^2 + \eta (\hat{Y}_t)^2 + \lambda_\pi (\pi_t)^2 \right] + \\
& \lambda_{1,t} \left(\hat{i}_t - \hat{i}_t^m + \frac{i - i^m}{1 + i^m} E_t \hat{\xi}_{t+1} - \Lambda \left[\lambda_c E_t (\hat{C}_{t+2} - \hat{C}_{t+1}) + E_t \pi_{t+2} - E_t \hat{i}_{t+1}^m \right] \right) + \\
& \lambda_{2,t} \left(\kappa \left[(\lambda_c + \eta) \hat{Y}_t - \lambda_c \hat{G}_t \right] + \kappa \left[\lambda_c E_t (\hat{C}_{t+1} - \hat{C}_t) + E_t \pi_{t+1} - \hat{i}_t^m \right] + \beta E_t \pi_{t+1} - \pi_t \right) + \\
& \lambda_{3,t} \left(\vartheta_m \left(\frac{\hat{m}_{t-1}}{m_y} + \hat{i}_{t-1}^m \right) + \xi \vartheta_b \left(\frac{\hat{b}_{t-1}}{b_y} + \hat{i}_{t-1} \right) + (1-\xi) \vartheta_b \hat{\xi}_t - \hat{C}_t - (\vartheta_m + \xi \vartheta_b) \pi_t \right) + \\
& \lambda_{4,t} \left(\hat{b}_t - \hat{b}_{t-1}(1+i) + \hat{m}_t - \hat{m}_{t-1}(1+i^m) - \vartheta_b \hat{i}_{t-1} - \vartheta_m \hat{i}_{t-1}^m + (\vartheta_b + \vartheta_m) \pi_t - \hat{G}_t - \tau \hat{Y}_t \right) + \\
& + \lambda_{5,t} \left(\hat{i}_t^m + \log(1 + i^m) \right) \Big\}.
\end{aligned}$$

Hence, given the exogenous stochastic process for $\hat{\xi}_t$ a Ramsey optimal policy solves for the set of variables $\left\{ \hat{Y}_t, \hat{C}_t, \pi_t, \hat{i}_t, \hat{i}_t^m, \hat{m}_t, \hat{b}_t, \hat{G}_t \right\}_{t=0}^{\infty}$ and Lagrange multipliers $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}\}_{t=0}^{\infty}$.

B.4 Robustness: the case of wasteful government spending

A fascinating result of the analysis laid out so far is that complementing an "active" monetary policy, which involves a balance sheet expansion, with an optimal fiscal stimulus delivers a great stabilization of output and inflation. In this regard, a key feature of the model described in section 3.2 is the presence of government spending in the utility function. In fact, assuming that government provides utility to the representative agent implies that the Ramsey policymaker has also the objective of stabilizing government spending around the target level, which in our model corresponds to the first best[1]. However, we showed that policymaker makes large use of deviation of government spending from the target in order to reduce the gap of other variables, like output and inflation.

We can now show that this result is even more pronounced when the fiscal stimulus is not "costly" for the policymaker, in the sense that pushing public spending above the target does not directly increase the value of the loss function. Therefore, we relax the assumption of the presence of government spending in the utility function, assuming instead that government spending is wasteful. In Figure B.1 we plot the responses of the optimal policy under this last scenario together with the optimal policy described in the previous sections. It is evident that when government spending is not in the utility function the policymaker can fully stabilize output and inflation by implementing a money-financed fiscal stimulus. The reason is simple: in this case expanding government spending far above the steady state level is not costly for the policymaker, so that government spending can be pushed up to the point such that output gap remains closed. Equivalently, the increase of government spending fully offsets the fall in consumption caused by the sudden liquidity shortage. Interestingly enough, when government spending is not in the utility function the interest rate on reserves never reaches the zero bound[2].

[1] This can be clearly seen in the loss function (3.39).

[2] Also the arguments for supporting "active" monetary policies, discussed in the main text, are robust to a model characterization in which government spending does not enter utility function. Details of these simulations are available upon request.

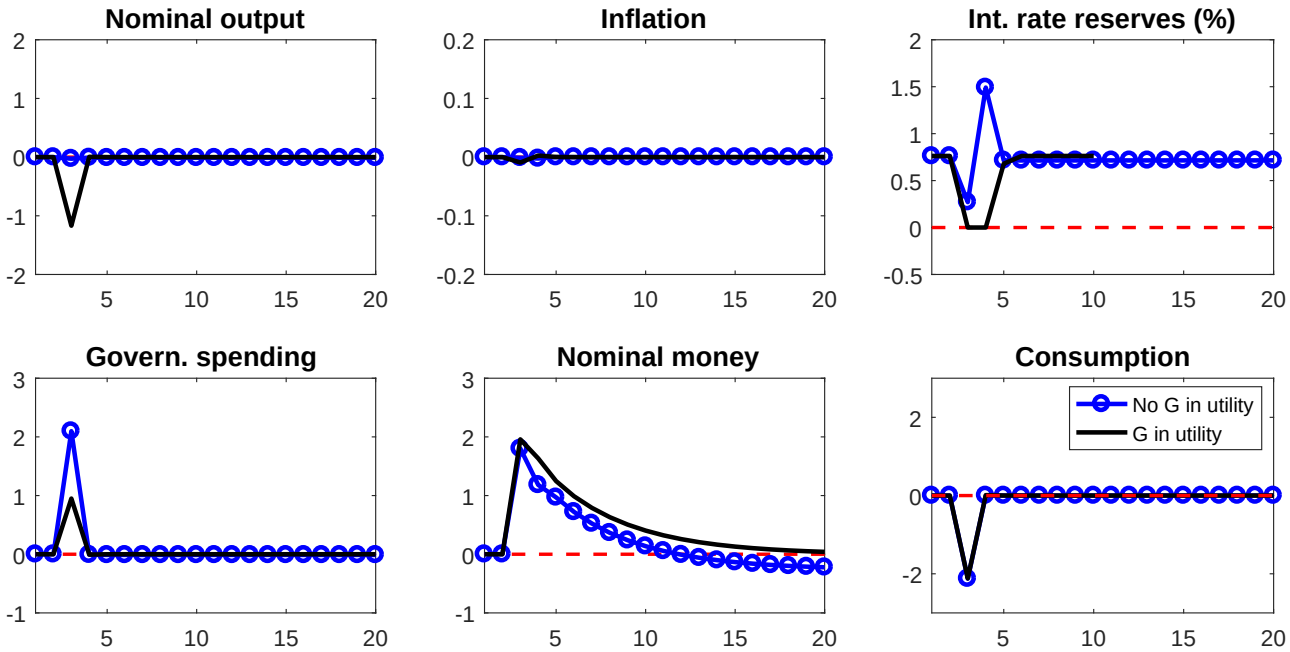

Figure B.1: Optimal policy: wasteful vs. useful government spending.IRFs of main variables for a 1%liquidity shock with persistence ρ=0.8. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are expressed in annual percentage points.

B.5 Sensitivity analysis

B.5.1 The role of price rigidities

The model described in section 3.2 is a New Keynesian model augmented with a financial friction, like the cash-in-advance constraint, and a role for central banking. It is well-known that main feature of the class of New Keynesian models is given by price rigidity, which affects the transmission of policies and introduces the trade-off between stabilization of output gap and inflation. Thus, we now investigate how the effect of the optimal money-financed fiscal stimulus depends on different degrees of price rigidity. In the baseline calibration we have set $\theta = 0.75$, implying that the average duration of prices is four quarters. In Figure B.2 we consider also the case of an average duration of three quarters ($\theta = 0.66$) and an average of ten quarters ($\theta = 0.90$).

The optimal response of the policy variables is the same in all the three cases considered. In particular, public spending rises by around 1% and then returns to the steady state level the following period. Money supply goes up by 2% and then reverts back in a way symmetric to the liquidity process, whereas the interest rate on reserves stays at the zero lower bound for one more quarter after the shock. Furthermore, output gap reports the same path under the three cases considered.

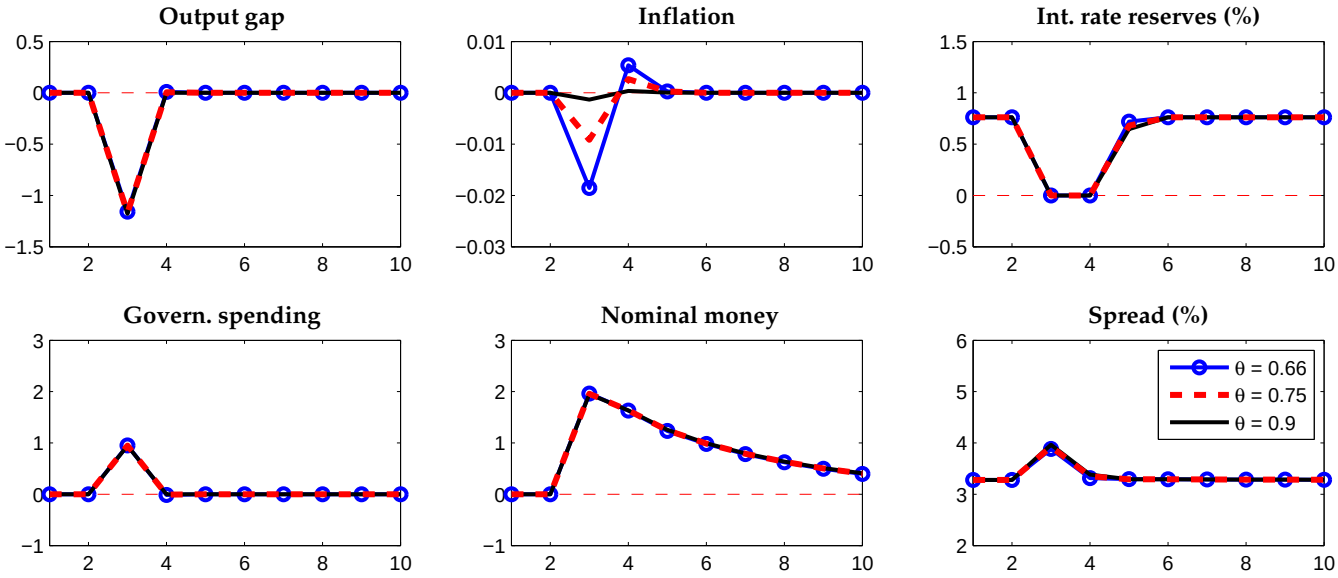

Figure B.2: Optimal unconventional monetary-government spending policy for different frequency of price adjustment. IRFs of main variables for a 1%liquidity shock with persistence $\rho = 0.8$. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves, which is expressed in annual percentage points.

What changes significantly is the response of inflation: importantly, when prices are more rigid inflation is much closer to the target level (price stability). This is a consequence of the fact that a lower θ implies that the cost of inflation in the loss function has a smaller weight and therefore the policymaker can afford a larger deviation of inflation. The bottom line is that the stabilization effect of the money-financed fiscal stimulus is more effective the more prices are rigid.

B.5.2 The role of steady state debt-to-GDP ratio

In some country where unconventional monetary policies are being largely implemented, one general concern is the high level of public debt. This is, for instance, the case of Japan, where fiscal policy has been carefully restrained because of diffuse concerns about the outstanding level of its public debt. In the analysis developed so far we have assumed that the steady state amount of net public debt-to-GDP amounts to 60%, as in Galì (2017)[30][3]. However, it might be of interest analysing the case of different value of net public debt-to-GDP, so as to consider also the case of a very large amount of debt, like in Japan today[4].

In this regard, in Figure **??** we report impulse response functions under optimal monetary-government spending policy for different steady state levels of net public debt-to-GDP. In comparison with the calibrated value of 60% we consider a low value of 20% and a very high value

[3]Recall that this is public debt issued to private sector, ruling out the share of public debt held by central bank.

[4]These change are obtained by slightly varying the amount of the steady state lump-sum tax $\bar{T}$.

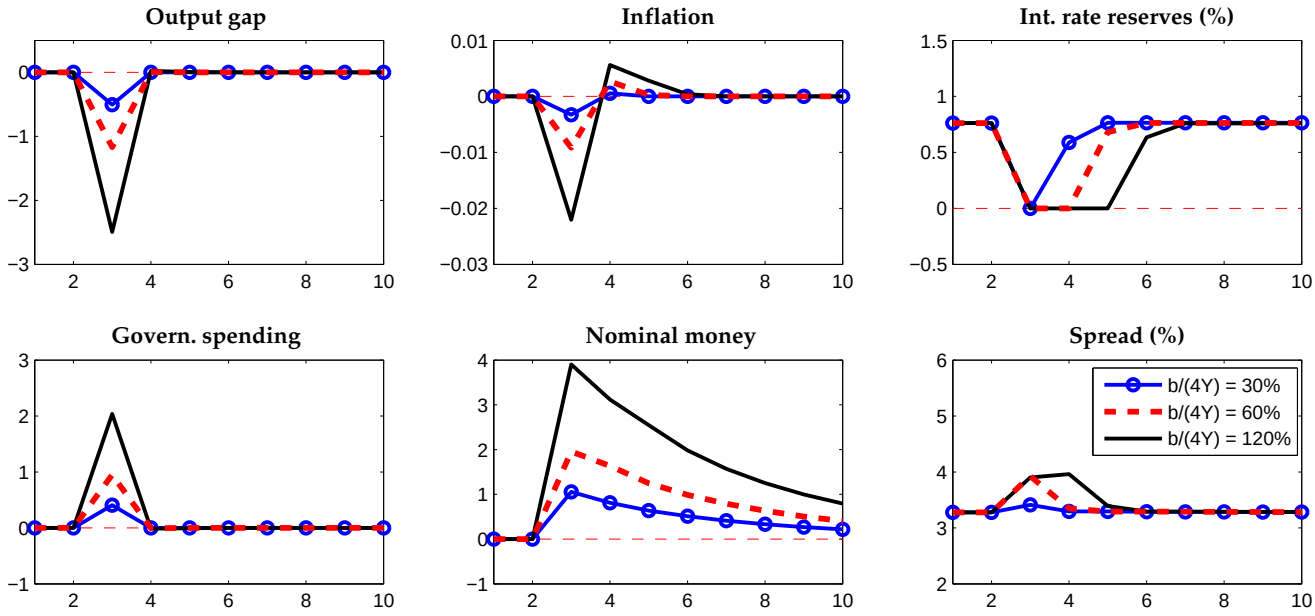

Figure B.3: Optimal monetary-government spending policy with different steady state value of net public debt-to-GDP. IRFs of main variables for a 1%liquidity shock with persistence ρ=0.8. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are expressed in annual percentage points.

of 120%, as in Japan today. It turns out that the level of net public debt in steady state affects substantially the conduct of the optimal monetary-government spending policy. In particular, it can be observed that when the level of public debt-to-GDP is high, the economy experiences a more sizeable fall in output and inflation. This occurs because the liquidity shock impacts on a larger portion of assets.

Importantly, this leads to a stronger response of the policymaker, that is forced to implement more important increase in public spending and monetary base expansion. Furthermore, the stay at the zero lower bound becomes longer, the larger is the steady state value of net public debt. Interestingly enough, in case of 20% level of net public debt the interest rate on reserves does not reach the zero level and a much smaller money-financed fiscal stimulus is required to stabilize output gap and inflation.

To sum up, the long-run value of net public debt-to-GDP is an important variable for the effectiveness of a money-financed fiscal stimulus in stabilizing the economy. When public debt is high, output gap and inflation deviate from the target to a larger extent, and therefore a more pronounced money-financed fiscal stimulus is needed, along with a longer stay at the zero lower bound.

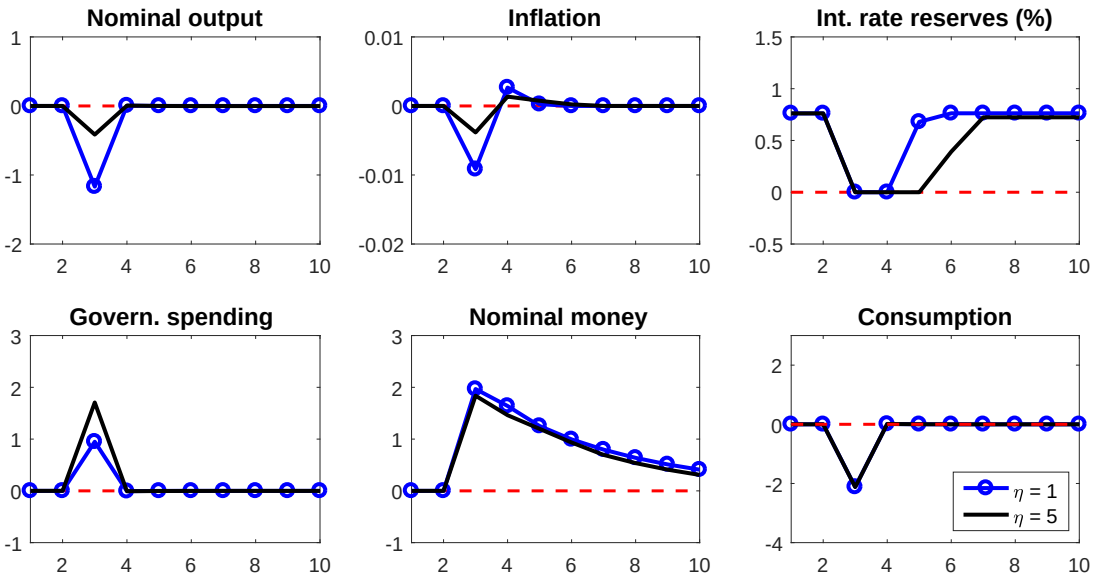

Figure B.4: $\eta = 1$ **vs.** $\eta = 5$**.** IRFs of main variables for a 1%liquidity shock with persistence ρ=0.8. Variables are in percentage deviation from the steady state, apart from the interest rate on reserves and the spread, that are expressed in annual percentage points.

B.5.3 The role of the inverse of Frisch elasticity

A controversial parameter in the monetary economics' literature is the inverse of the Frisch elasticity, captured by the parameter η in the model of this paper. In our baseline calibration this value is set equal to 1, as in Justiniano et al. (2015)[41], among the many others. However, in the literature it is also assumed that this value can take an higher value, implying that labor supply becomes less elastic to changes in real wage. Thus, Figure B.4 displays the case of $\eta = 1$, as in our calibration with the case $\eta = 5$, as in Galì (2017)[30]. It can be seen that varying this parameter is not irrelevant: in case of $\eta = 5$, the economy experiences a smaller contraction in output and inflation, which, however, is accommodated by a stronger response of the policymaker, involving a longer stay at the ZLB and a more pronounced fiscal stimulus. The reason has to do with the weight assigned to output gap in the loss function that the policymaker is trying to minimize. An higher value of η implies that the policymaker cares about output gap to a much larger extent, so that she becomes very willing to engage in a stronger money-financed fiscal stimulus, with the aim of ultimately reducing the output gap.