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# Fiscal Policy Coordination and Government Debt Deleveraging in the EMU

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**Abstract**

In Chapter 1 I explain my motivation for the topic and review some literature on Fiscal Policy Coordination in Currency Unions, in particular in the European Economic and Monetary Union (EMU). The EMU is a perfect case study for this key issue, which has been covered more and more by the recent literature. Fiscal Policy Coordination was initially discussed by the seminal article of Mundell (1961), where one of the successful criteria of an Optimum Currency Area (OCA) was shown to be a risk-sharing system like fiscal transfers that redistribute money to areas adversely affected by shocks. This motivates the creation of a Fiscal Union inside a Currency Union like the EMU, and can be seen as an extreme case of Fiscal Policy Coordination. I also suggest some questions and avenues of research, which guide my work.

In Chapter 2 we build a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union to study the effects of fiscal policy coordination, by evaluating the stabilization properties of different degrees of fiscal policy coordination, in a setting where the union-wide monetary policy affects fiscal policies and viceversa, because of price rigidities and distortionary taxation. We calibrate the model to represent two groups of countries in the European Economic and Monetary Union and run numerical simulations of the model under a range of alternative shocks and under alternative scenarios for fiscal policy. We also compare welfare under the different scenarios, bringing to policy conclusions for the proper macroeconomic management of a Currency Union. We find that: a) coordinating fiscal policy, by targeting net exports rather than output, produces more stable dynamics, b) consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization, c) taxes on labour income are exponentially more distortionary than taxes on firm sales. Our policy prescriptions for the Eurozone are then to use fiscal policy to reduce international demand imbalances, either by stabilizing trade flows across countries or by creating some form of fiscal union or both, while avoiding the excessive use of labour taxes, in favour of sales taxes.

In Chapter 3 we build a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union with a debt-elastic government bond spread in an incomplete market setting, to study the effects of government debt deleveraging, by evaluating the stabilization properties of different deleveraging rules, in a setting where the union-wide monetary policy affects fiscal policies and viceversa, because of price rigidities and distortionary taxation. We calibrate the model to represent two groups of countries in the European Monetary Union and run numerical simulations under a range of alternative shocks and under alternative scenarios for government debt deleveraging. We also compare welfare under the different scenarios, bringing to policy conclusions for the proper government debt management in a Currency Union. We find that: a) backloading deleveraging or reducing its speed provides more stabilization to the economy, b) taxes are the instrument for deleveraging which stabilizes the economy the most, c) coordinating fiscal policy by targeting the net exports gap, instead of the output gap, increases volatility with incomplete markets, d) the joint movements of the fiscal instruments increases volatility with incomplete markets, although consolidating budget constraints might otherwise smooth distortions. Our policy prescriptions for the Eurozone are then to reduce the speed of deleveraging and to use taxes to achieve it, while reducing domestic demand imbalances and avoiding to form a fiscal union like the one we describe. Once financial markets are completely integrated though, these results are overturned.

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All errors are and remain my own.

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# Chapter 1

# Introduction & Literature Review

## 1.1 Introduction

European economic integration started in 1957 with free trade agreements and has brought now to about 19 countries sharing the same currency (the EURO) with a single monetary policy enacted by the European Central Bank (ECB). This means that each country in the European Economic and Monetary Union (EMU) has lost its monetary authority, leaving it in the hands of the ECB which can only enact *ONE monetary policy for all countries.* In other words the ECB can only target Euro-wide inflation and output, but cannot target any country-level variables (if not, which country should it target?). On the other hand, fiscal policy can and *should* target country-level aggregates. As a matter of fact, now each country can *only* manage its own fiscal policy, because there is still no fiscal authority at the EMU level. Since the two main macroeconomic stabilizers are one at the EMU level and the other at country level, many questions can arise regarding the possible interactions between the two. Would it be better to coordinate fiscal policies inside the monetary union? Would this create a scope for a central fiscal authority? Would that be an extreme example of fiscal policy coordination? I see it that way.

Although a single monetary policy cannot balance asymmetric shocks to different countries, a single fiscal policy can and *should*. Not only is it important, for the literature and for real life, to understand how monetary and fiscal policies interact, in a general open economy and specifically in a Currency Union, but it is very interesting and useful to study the creation of the Euro as the most modern and largest-scale case study of the engineering of an Optimum Currency Area, *an area in which it would maximize economic efficiency to have the region share a single currency*, as pioneered in the seminal work of Mundell (1961). The optimal management of a Currency Union has to do with many issues. I will focus on the interaction between monetary and fiscal policies and on the stabilization properties of different fiscal policy scenarios, following mainly Ferrero (2009) and Hjortsø (2012), while ignoring political issues, which in reality are also very important, if not necessary, for the proper functioning of a Currency Union.

The main interest in studying the interaction between monetary policy and fiscal policies in the Eurozone is to follow the ever increasing economic integration taking place in Europe, which started in 1957 and has gone a long way, but still has many unfinished tasks and unsolved problems. How should economic integration evolve in Europe? What are the main problems that can be addressed and how? On one hand this is a real-life problem with real-life consequences. On the other hand this topic has to do with the Optimum Currency Area literature and with the history of fixed exchange rate regimes, from Bretton Woods (1945-1971), where 44 countries decided to peg their currencies to the U.S. dollar, allowing them to fluctuate within bands, and where the International Monetary Fund (IMF) was created to manage temporary imbalances in the Balance of Payments, to the European Monetary System (1978-1992), the forerunner of the Euro, where many countries decided to peg their currencies to avoid exchange rate variability and achieve monetary stability. Both of these experiments failed miserably, but then the Euro was born in 1998 giving new hopes to the proper functioning of a Currency Union. Unfortunately, the recent global financial crisis and european sovereign debt crisis have showed many weaknesses of the EMU and the point is to understand why things went so bad and how things *could have* gone better and *could* go better in the future.

Fiscal and monetary policy are the two main macroeconomic stabilization tools that countries can use in the presence of adverse shocks to stabilize aggregate output and inflation. With the birth of the Euro, european countries have lost their monetary independence, because one currency for all countries implies one monetary policy for all countries, and have very constrained fiscal independence, because of the laws passed at EMU level limiting discretion in fiscal policy implementation (balanced budget rule, for example). There are different interaction mechanisms because of one union-wide monetary policy and different country-level fiscal policies. What are the effects of such economic integration and how can it be modified in the future to increase wealth and stability? Since the single monetary policy affects member countries in different ways, fiscal policy should be used to balance these asymmetric effects on different countries. As found in Beetsma and Jensen (2005), while monetary policy is concerned with stabilizing the union-wide economy, fiscal policy in a currency union should mainly aim at stabilizing inflation differences and the terms of trade (which is naturally a role for floating exchange rates), implying a greater role for fiscal policy the higher the difference in inflation and price rigidity between countries, which is the case for Europe. *In 2009 many economists and the IMF recommended that Europe establish a fiscal union with a bailout fund, a banking union, a mechanism for countries to pursue the same economic and fiscal policies, and a common issuance of Eurobonds.* Little has been done and many questions are still open.

My research is part of the Optimum Currency Area (OCA) literature, which started with Mundell (1961). Robert Mundell pioneered the theory of an OCA as an area in which it would maximize economic efficiency to have the region share a single currency. *The successful criteria for an OCA pointed out by Mundell are: labour mobility, openness with capital mobility and price and wage flexibility, participant countries with similar business cycles, and a risk-sharing system like fiscal transfers to redistribute money to areas adversely affected by shocks.* This last concept is in fact the equivalent of a fiscal union with a centralized fiscal authority that addresses country-level shocks with union-wide resources, as if it were transferring resources from other countries to the country hit by the shock.

My research is also part of the Open-Economy New-Keynesian DSGE literature, mainly adopted for the analysis of monetary policy, but lately adopted more and more for the analysis of fiscal policy and its interaction with monetary policy. A useful introduction to the analysis of New-Keynesian DSGE models is given in Galí (2009), and specifically in Galí and Monacelli (2005) regarding the Open Economy, although in a setting with a continuum of countries, which is not very realistic or tractable, and with a direct approach to log-linear approximation, often skipping non-linear analysis.

## 1.2 Literature Review

Several papers have analyzed the interactions between monetary policy and fiscal policies in a Currency Union, with differing consensus over the need for fiscal policy coordination or for a Fiscal Union inside a Currency Union.

My main reference is Ferrero (2009), which uses a two-country model with Calvo price-setting, monopolistic competition, distortionary taxation on income and sales, and nominal debt to study centralized monetary policy and decentralized fiscal policies and their effects in the union, from the viewpoint of a benevolent central planner maximizing a welfare criterion for the union as a whole. The paper mainly investigates the design of monetary and fiscal stabilization policies. Its main finding regards the substantial welfare gains from flexible fiscal policy, defined as flexible debt targeting, compared to balanced budget rules. This shows the great importance of the role of fiscal policy and its limitations induced by the Maastricht Treaty and Fiscal Compact. The paper studies optimal monetary and fiscal policies from the viewpoint of a benevolent central planner, while not considering strategic interactions between policymakers, home bias, incomplete markets or the zero lower bound on the nominal interest rate. Ferrero (2009) finds that the optimal equilibrium prescribes that monetary policy should achieve price stability at the union level through flexible inflation targeting, while fiscal policy should avoid creating inflationary expectations at the union level, and should stabilize idiosyncratic shocks allowing for permanent variations of government debt, through relative tax rates that track the evolution of the terms of trade.

Hjortsø (2012) uses a two-country model of a currency union with incomplete financial markets, so that full private insurance against country-specific shocks is not possible, to study optimal coordinated fiscal policy in a monetary union without fiscal transfers. The paper finds that, if internationally traded goods are complements, fiscal policy should optimally reduce intra-union current account imbalances at the expense of larger domestic output gaps. The model has home bias in private consumption and households that gain utility also from government consumption. It models financial intermediaries that can borrow at the nominal interest rate set by the central bank, but charge a rate which depends also on the aggregate debt of the borrowing country. This allows for yields on government bonds, which are increasing in the level of debt, to produce yield differences across countries. The paper does not consider fiscal transfers or distortionary taxation, abstracting from these fiscal policy considerations.

Ferrero (2009) also builds on Benigno and Woodford (2004), which applies the study of optimal policy in a closed economy to a framework with sticky prices and both monetary and fiscal policy, where the only source of government revenue is distortionary taxation, which breaks Ricardian equivalence, and allows for non-trivial interactions between monetary and fiscal policy. It analyzes the optimal policy, through a linear-quadratic approach, from the viewpoint of a benevolent central planner in a closed economy, and then derives policy rules with which the authorities can implement the optimal plan. The paper shows optimal policy and the rules to implement it in a closed economy with complete financial markets, while not considering the role of incomplete markets or open economy dynamics, nor the strategic interactions between the different policy authorities.

Benigno (2004) studies optimal monetary policy in a two country model with different price rigidities, but with money in the utility function and without fiscal policy. It assumes unitary elasticity of substitution across countries and no home bias in consumption. Financial markets are complete within a country and incomplete across countries, implying agents can only trade a risk-free bond across countries. The paper takes the optimal monetary policy approach and finds that an inflation targeting rule where a higher weight is given to the inflation of the country with a higher degree of nominal rigidity is nearly optimal. This could imply a greater role for fiscal policy in stabilizing the effects of different price rigidities.

Galí and Monacelli (2008) studies the interactions between the centralized monetary policy and the decentralized fiscal policies which have a country-specific stabilization role. It uses Calvo staggered price-setting, fiscal policy with country-specific levels of public expenditure which yields utility to domestic households and a currency union made up of a continuum of small open economies, subject to different productivity shocks. While it finds the usual union-wide inflation-targeting role for monetary policy, it argues that every country must simultaneously implement its fiscal policy part of the optimal plan, implying a need for fiscal policy coordination. The paper finds that fiscal policies have a stabilization role for the union as a whole and increasing importance with greater nominal rigidities, implying that external constraints on fiscal policy might not be desirable. It doesn't include distortionary taxation or government debt in the model, abstracting from their effects on policy interactions.

Farhi and Werning (2012*b*) studies the role of fiscal policy, in a currency union made of a continuum of small open economies, as a cross-country insurance tool in the presence of nominal price and wage rigidities. It finds that, in the presence of incomplete financial markets, there is a greater gain from macro insurance for countries inside a currency union. At the same time, even with complete markets, private insurance is inefficiently low, implying a role for macro insurance by governments. These findings motivate the creation of a fiscal union inside a currency union. It shows there is a larger role for macro insurance the larger the asymmetric shocks on member countries, the more persistent the shocks and the less open the member economies. The paper abstracts from banks and sovereign financing issues and finds no need for coordination of fiscal policy at the union level.

The structure of Galí and Monacelli (2008) and Farhi and Werning (2012*b*) with a continuum of countries means that more variables will be exogenous, compared to a two-country model, and that a single country, being one of an infinite continuum, as specified in Galí (2009), does not influence any world variable. This means that all world variables must be exogenous and that it is harder to see the interaction among countries, so that international trade has no role because any expenditure on goods from any one country has a value of zero, being one of infinitely many composing the integral, as written in Galí (2009) that an integral of any variable over all countries is the same as an integral of the same variable over all countries except one. This poses questions on the validity of such a model and pushes me to prefer a two-country model instead, where the interactions among the two countries (or two groups of countries) are more evident and the dynamics are thus clearer. I choose to follow an approach similar to that of Silveira (2006), where the structure of Galí and Monacelli (2008) is adapted to a two-country model setting, rather than a setting with a continuum of countries. At the same time, I focus on a currency union setting, following Ferrero (2009), where there is no nominal exchange rate to act as an automatic stabilizer, because both countries share the same currency.

## 1.3 Further Research

An incredible number of questions can be addressed in this field of research, following the above mentioned papers and trying to answer some of these questions with different models of a Currency Union:

- Is there a better way to target monetary policy in a Currency Union to favour equally all countries, rather than favoring the high inflation countries at the expense of the high unemployment countries?
- What should fiscal policies target at a country-level and how should they interact with each other and with the central monetary policy?
- Other than evaluating union-wide gains and losses from different policies, which are the countries that gain or lose more and why?
- What are the effects of high government debt and external fiscal policy limitations on the interaction between monetary policy and fiscal policies in a Currency Union?
- What are the effects of the only use of distortionary taxation to finance fiscal policy in a Currency Union and should countries harmonize them?
- Is there a scope for a central fiscal capacity in the EMU and if so how should fiscal policy be managed at the union level?

- Are there strategic motives that might bring national governments to deviate from the optimal fiscal policy at the union level?
- Must there be coordination between centralized monetary policy and country-level fiscal policies?

I will try to answer some of these questions using different Two-Country Open-Economy New-Keynesian DSGE models. The two countries have different sizes, so they can be used to model a currency union in different ways: two groups of countries in the EMU, one country and the rest of the EMU, or the EMU and the rest of the world. This allows to study the dynamics of the Eurozone economy from different points of view. I will focus on the first option, following Ferrero (2009), and so considering the two countries as two groups of countries in the EMU.

I will evaluate some of the following effects of the Fiscal Compact Rules:

- Government Budget Deficit/GDP ratio no higher than 3%.
- Government Debt/GDP ratio no higher than 60%.
- Debt Brake Rule: if Debt/GDP is more than 60% it should decrease by 5% of the excess per year.
- Current Account Balance/GDP ratio between −4% and +6%.
- Countries must adopt a balanced budget law into national legislation.

The analysis will be carried out evaluating how the ECB conducts its monetary policy:

- Its primary objective of price stability is formulated as a situation in which the one-year increase in the CPI for the Eurozone is less than, but close to, 2%.
- Other goals, like a high level of employment and sustainable growth, are formulated vaguely and can only be pursued if compatible with price stability.

# Chapter 2

# One EMU Fiscal Policy for the EURO[1]

## 2.1 Introduction

*Are there gains from fiscal policy coordination in a monetary union subject to alternative shocks? Does this create a scope for a centralized fiscal capacity in the EMU?*

Given a single monetary policy in the European Economic and Monetary Union (EMU), country-specific shocks cannot be addressed through monetary policy, but must be balanced by country-specific fiscal policies. Whether this calls for coordination or not is a much debated issue, and has been typically investigated by looking at fiscal multipliers, as Farhi and Werning (2012*a*) finds a greater output multiplier if government spending is financed by a foreign country rather than the home country. This would create a scope for a central EMU budget, as centrally financed government spending has larger effects than nationally financed government spending, but this is in a model without distortionary taxation, which is far from reality.

We focus on fiscal policy coordination with complete international financial markets and study the stabilization properties and the welfare implications of different fiscal policy scenarios. We study the effects of supply (productivity) and demand (preference) shocks hitting a single country under alternative fiscal policy specifications and compare uncoordinated fiscal policy (Pure Currency Union scenario), where each country chooses its government consumption, transfers and taxation, with fully coordinated fiscal policy (Full Fiscal Union scenario), where government consumption, transfers and taxation are chosen for each country by the union as a whole, with a consolidated budget. An intermediate case (Coordinated Currency Union scenario) shows the effects of coordinating fiscal policy without consolidating the budget.

We analyze the welfare gains from stabilization through variations in government consumption in the country hit by the shock, financed by the union as a whole, considering whether there is a scope for a fiscal capacity in the EMU to address asymmetric shocks to member countries, as a shock-absorption mechanism, as addressed in Van Rompuy et al. (2012). This was mentioned also in the more recent Juncker et al. (2015), where a Fiscal Union is seen as a Euro area-wide macroeconomic stabilization tool, over and above national fiscal policies needed to cushion country-specific shocks, which is thought to be key in avoiding procyclical fiscal policies at all times. We define two welfare criteria and evaluate the welfare gains from a common macroeconomic stabilization function which can better deal with shocks

[1] This chapter is co-authored with Chiara Guerello, LUISS Guido Carli, and Guido Traficante, Università Europea di Roma.

that cannot be managed at the national level alone. We compare welfare under the Full Fiscal Union, the Coordinated Currency Union and the Pure Currency Union scenarios, bringing to policy conclusions for the proper macroeconomic management of a Currency Union. A future Fiscal Union will need a Eurozone treasury for collective decision-making on fiscal policy, which will also need to be accountable and legitimated democratically, but this is out of the scope of the present paper.

We follow the open economy approach of Galí (2009), in a two-country setting like in Silveira (2006), but without taking the limit case of a small open economy. Our model follows the specifications of Ferrero (2009), which adapts the optimal approach of Benigno and Woodford (2004) to monetary and fiscal policy in a cashless closed economy without capital, where there are only distortionary taxes as sources of government revenue, to a two-country open-economy Currency Union setting. We follow also the Currency Union model of Benigno (2004), by adding a fiscal authority and including a government budget constraint, but without money in the utility function and concentrating on the cashless limit case. Our model adds home bias in consumption (or a degree of openness to international trade) and government transfers to households to the model in Ferrero (2009). The former allows for deviations from Purchasing Power Parity, while the latter is an additional instrument for fiscal policy.

As in Ferrero (2009), this model is structured to allow for spillovers from monetary to fiscal policy and viceversa, and from one country to another through country-specific fiscal policies. Nominal rigidities, in the form of staggered prices, generate real effects of monetary policy, while distortionary taxation generates non-Ricardian effects of fiscal policy. Monetary policy influences fiscal policy through its effect on the real value of debt, which depends on the interest rate and inflation. Fiscal policy influences monetary policy through its effect on prices and wages, which depend on the tax rates. This framework allows to study the interaction between country-specific fiscal policies, where in the absence of the nominal exchange rate as an automatic stabilizer, fiscal policies influence each other through their effects on output and the terms of trade.

The EMU is represented by two countries of different size forming a Currency Union. Each country has an independent Fiscal Authority, while the Currency Union shares a common Monetary Authority. The Central Bank sets the nominal interest rate for the whole Currency Union following an Inflation Targeting regime, where the target is on union-wide CPI inflation. This assumption reflects the current functioning of the ECB, whose primary objective of price stability is formulated as a situation in which the one-year increase in the CPI for the Eurozone is less than, but close to, 2%. Fiscal policy is designed following the Fiscal Compact Rules, by imposing that the Government Debt-to-GDP ratio is about 60% and that countries must adopt a balanced budget law in their national legislation. Following Hjortsø (2012), we add a stabilization rule for government consumption or alternatively for government transfers, but in a setting where distortionary taxes and government debt add the financing dimension to the fiscal policy problem. Governments choose the level of government consumption and transfers, which are financed by distortionary taxes on labour income and firm sales and by short-term government bonds. In particular:

- In a Pure Currency Union scenario governments choose the level of government consumption or transfers for domestic stabilization purposes by setting them alternatively to target the output gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while balancing the budget.

- In a Coordinated Currency Union scenario governments choose the level of local government consumption or transfers for international stabilization purposes by setting them to target the net exports gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while balancing the budget.

- A Full Fiscal Union scenario uses a consolidated budget constraint to finance local government consumption or transfers for international stabilization purposes by setting them to target the net exports gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while balancing the consolidated budget, just like in the Currency Union case, but varying equally the tax rates across countries, so as to use union-wide resources to finance the government expenditure.

Our main results show that coordinating fiscal policy by targeting net exports, rather than output, produces more stable dynamics, and that consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization, while taxes on labour income are exponentially more distortionary than taxes on firm sales. Our policy prescriptions for the Eurozone are then to use fiscal policy to reduce international demand imbalances, either by stabilizing trade flows across countries or by creating some form of fiscal union or both, while avoiding the excessive use of labour taxes, in favour of sales taxes.

The remainder of the paper is structured as follows. Section 2 describes the general model and the fiscal policy scenarios of a Pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union. Section 3 presents the calibration of the parameters and steady state stances of the model to two groups of countries in the EMU. Section 4 describes two welfare criteria and provides welfare rankings of the different fiscal policy scenarios. Section 5 provides numerical simulations under different scenarios, comparing different degrees of fiscal policy coordination, alternative fiscal policy instruments for stabilization and alternative government financing schemes. Section 6 shows the effects of fiscal shocks and describes fiscal policy multipliers and spillovers. Section 7 shows numerical simulations and welfare evaluations of the case for international goods as complements, rather than substitutes. Section 8 collects the main conclusions and provides possible extensions. The Mathematical Appendix provides all derivations, collects all equilibrium conditions of the model used for the simulations, and describes the steady state on which the model is calibrated.

## 2.2 A Two-Country Currency Union Model

The world economy is composed of two countries, which form a Currency Union. Both economies are assumed to share identical preferences, technology and market structure, but may be subject to different shocks, price rigidities, initial conditions and fiscal stances. The two countries are indexed by $H$ and $F$ for *Home* and *Foreign*. The world is populated by a continuum of infinitely-lived households of measure one, indexed by $i \in [0, 1]$. Each household owns a monopolistically competitive firm producing a differentiated good, indexed by $j \in [0, 1]$. The population on the segment $[0, h)$ belongs to country $H$ while the population on the segment $[h, 1]$ belongs to country $F$. This means that the relative size of country $H$ is $h \in [0, 1]$, while the relative size of country $F$ is $1 - h$. This is true for both households and firms.

Firms set prices in a staggered fashion following Calvo (1983) and use only labour for production. There is no capital and no investment. Labour markets are competitive and internationally segmented, so that labour supply is country-wide and not firm-specific. All goods are tradable and the Law of One Price (LOP) holds for all single goods $j$. At the same time deviations from Purchasing Power Parity (PPP) may arise because of home bias in consumption. Financial markets are complete internationally, allowing households to trade a full set of one-period state-contingent claims across borders, other than purchase one-period risk-free bonds issued by the two countries' governments.

The world economy comprises also two Governments or Fiscal Authorities (one for each country), each one choosing government consumption and transfers to stabilize the economy, financed by distortionary taxes on labour income and firm sales and by short-term government bonds, and a Central Bank or Monetary Authority (one for the whole EMU), which sets the nominal interest rate targeting union-wide inflation.

We denote variables referred to the *Foreign* country with a star (*).

### 2.2.1 Households

In each country there is a continuum of households, which gain utility from private consumption and disutility from labour, consume goods produced in both countries with home bias, supply labour to domestic firms and collect profits from those firms. Households can trade a complete set of one-period state-contingent claims across borders and purchase one-period risk-free bonds issued by the two countries' governments, subject to their budget constraint.

Each household in country $H$, indexed by $i \in [0, h)$, and each household in country $F$, indexed by $i \in [h, 1]$, seeks to maximize respectively the present-value utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \, U(C_t^i, N_t^i) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \xi_t^* \, U(C_t^{*i}, N_t^{*i}) \tag{2.1}$$

where $\beta \in [0, 1]$ is the common discount factor, which households use to discount future utility, $\xi_t$ is a preference shock to *Home* households and $\xi_t^*$ is a preference shock to *Foreign* households. These preference shocks are assumed to follow the AR(1) processes in logs:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \qquad \text{and} \qquad \xi_t^* = (\xi_{t-1}^*)^{\rho_\xi^*} e^{\varepsilon_t} \tag{2.2}$$

where $\rho_\xi \in [0, 1]$ and $\rho_\xi^* \in [0, 1]$ are measures of persistence of the shocks and $\varepsilon_t$ is a zero mean white noise process. $N_t^i$ denotes hours of labour supplied by households in country $H$, $N_t^{*i}$ denotes hours of labour supplied by households in country $F$. $C_t^i$ is a composite index for private consumption defined by[2]:

$$C_t^i \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} (C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t}^i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2.3}$$

for households in country $H$, while the analogous index for households in country $F$, $C_t^{*i}$, is defined by:

$$C_t^{*i} \equiv \left[ (1-\alpha^*)^{\frac{1}{\eta}} (C_{H,t}^{*i})^{\frac{\eta-1}{\eta}} + \alpha^{*\frac{1}{\eta}} (C_{F,t}^{*i})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2.4}$$

where $C_{H,t}^i$ is an index of consumption of domestic goods for households in country $H$, given by the constant elasticity of substitution (CES) function (also known as Dixit and Stiglitz (1977) aggregator function):

$$C_{H,t}^i \equiv \left( \left( \frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.5}$$

whereas, for households in country $F$ the same index of consumption of domestic goods, $C_{H,t}^{*i}$, is given by:

$$C_{H,t}^{*i} \equiv \left( \left( \frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 C_{H,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.6}$$

[2] In the special case in which $\eta = 1$, the consumption index is given by $C_t^i = \left( \frac{C_{H,t}^i}{1-\alpha} \right)^{1-\alpha} \left( \frac{C_{F,t}^i}{\alpha} \right)^{\alpha}$. For a derivation see Appendix A.1. The same applies to the respective index for country $F$, which in the special case in which $\eta = 1$ is given by $C_t^{*i} = \left( \frac{C_{H,t}^{*i}}{1-\alpha^*} \right)^{1-\alpha^*} \left( \frac{C_{F,t}^{*i}}{\alpha^*} \right)^{\alpha^*}$.

where $j \in [0,1]$ denotes a single good variety of the continuum of differentiated goods produced in the world economy. $C^i_{F,t}$ is an index of imported goods for households in country $H$, given by the analogous CES function:

$$C^i_{F,t} \equiv \left( \left( \frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 C^i_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.7}$$

while the same index for imported goods for households in country $F$, $C^{*i}_{F,t}$, is given by:

$$C^{*i}_{F,t} \equiv \left( \left( \frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h C^{*i}_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.8}$$

The parameter $\varepsilon > 1$ measures the elasticity of substitution between varieties produced within a given country. The parameter $\eta > 0$ measures the substitutability between domestic and foreign goods (international trade elasticity). The parameter $\alpha \in [0,1]$ is a measure of openness of the *Home* economy to international trade. Equivalently $(1-\alpha)$ is a measure of the degree of home bias in consumption in country $H$. When $\alpha$ tends to zero the share of foreign goods in domestic consumption vanishes and the country ends up in autarky, consuming only domestic goods. If $1-\alpha > h$ there is home bias in consumption in country $H$, because the share of consumption of domestic goods is greater than the share of production of domestic goods. The same applies to the *Foreign* parameter of openness to international trade $\alpha^* \in [0,1]$ for country $F$, except for the fact that if $1-\alpha^* > 1-h$ there is home bias in consumption in country $F$, because the share of consumption of domestic goods in country $F$ is greater than the share of production of domestic goods in country $F$.

We choose to specify additively separable period utility of the type with Constant Relative Risk Aversion (CRRA), so with constant elasticity of intertemporal substitution, and with constant elasticity of labour supply, which take the following form[3]:

$$U(C^i_t, N^i_t) \equiv \frac{(C^i_t)^{1-\sigma} - 1}{1-\sigma} - \frac{(N^i_t)^{1+\varphi}}{1+\varphi} \qquad U(C^{*i}_t, N^{*i}_t) \equiv \frac{(C^{*i}_t)^{1-\sigma} - 1}{1-\sigma} - \frac{(N^{*i}_t)^{1+\varphi}}{1+\varphi} \tag{2.9}$$

for countries $H$ and $F$ respectively, where $\sigma$ is the inverse of the elasticity of intertemporal substitution[4] (it is also the Coefficient of Relative Risk Aversion (CRRA)), and $\varphi$ is the inverse of the Frisch elasticity of labour supply[5].

Households in country $H$ maximize their present-value utility, equation 2.1, subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j) C^i_{H,t}(j) \, dj + \int_h^1 P_{F,t}(j) C^i_{F,t}(j) \, dj + D^i_t + B^i_t \leq \frac{D^i_{t-1}}{\mathcal{Q}_{t-1,t}} + B^i_{t-1}(1+i_{t-1}) + (1-\tau^w_t) W_t N^i_t + T^i_t + \Gamma^i_t \tag{2.10}$$

for $t = 0, 1, 2, \ldots$, where $P_{H,t}(j)$ is the price of domestic variety $j$, $P_{F,t}(j)$ is the price of variety $j$ imported from country $F$, $D^i_{t-1}$ is the portfolio of state-contingent claims purchased by the household in period $t-1$, $\mathcal{Q}_{t-1,t}$ is the stochastic discount factor, which is the same for households in both countries and represents the price of state-contingent claims or equivalently the inverse of the gross return on state-contingent claims, $B^i_t$ are risk-free government bonds (of either or both governments) purchased

[3]In the special case in which $\sigma = 1$, utility from private consumption is given by $U(C^i_t) = \ln C^i_t$ in country $H$ and $U(C^{*i}_t) = \ln C^{*i}_t$ in country $F$. For a derivation see Appendix A.2.

[4]The elasticity of intertemporal substitution measures the responsiveness of consumption growth to changes in the real interest rate, which is the relative price of consumption between different dates, and is defined as the percent change in consumption growth divided by the percent change in the gross real interest rate. For a derivation see Appendix A.3.

[5]The Frisch elasticity of labour supply measures the extent to which labour supply responds to a change in the nominal wage, given a constant marginal utility of wealth, and is defined as the percent change in the supply of labour divided by the percent change in the nominal wage. For a derivation see Appendix A.4.

by the household in period $t$, $i_{t-1}$ is the nominal interest rate set by the central bank in period $t-1$, which is also the net return on government bonds, $W_t$ is the nominal wage for households in country $H$, $T_t^i$ denotes lump-sum transfers from the government to households, $\Gamma_t^i$ denotes the share of profits net of taxes to households from ownership of firms and $\tau_t^w \in [0,1]$ is a marginal tax rate on labour income paid by households to the government.

Households in country $F$ analogously maximize their present-value utility, equation 2.1, subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}^*(j)C_{H,t}^{*i}(j)\,dj+\int_h^1 P_{F,t}^*(j)C_{F,t}^{*i}(j)\,dj+D_t^{*i}+B_t^{*i} \leq \frac{D_{t-1}^{*i}}{\mathcal{Q}_{t-1,t}}+B_{t-1}^{*i}(1+i_{t-1})+(1-\tau_t^{*w})W_t^*N_t^{*i}+T_t^{*i}+\Gamma_t^{*i} \tag{2.11}$$

for $t = 0, 1, 2, \ldots$, where all the starred $(^*)$ variables are the *Foreign* equivalent of the unstarred ones explained above.

All variables are expressed in units of the union's currency. Last but not least, households in countries $H$ and $F$ respectively, are subject to the following solvency constraints, for all $t$, that prevent them from engaging in Ponzi-schemes:

$$\lim_{T\to\infty} E_t\left\{\mathcal{Q}_{t,T}D_T^i\right\} \geq 0 \qquad \lim_{T\to\infty} E_t\left\{\mathcal{Q}_{t,T}D_T^{*i}\right\} \geq 0 \tag{2.12}$$

The first order conditions for the household's problem in country $H$ are[6]:

$$(C_t^i)^\sigma (N_t^i)^\varphi = (1-\tau_t^w)\frac{W_t}{P_t} \tag{2.13}$$

which is the household's intratemporal optimality condition, and:

$$\mathcal{Q}_{t,t+1} = \beta\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{P_t}{P_{t+1}} \tag{2.14}$$

which is the household's intertemporal optimality condition or Euler Equation, that holds for all states of nature at $t$ and $t+1$, and:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} \tag{2.15}$$

which is the no-arbitrage condition between riskless government bonds and state-contingent claims.

Aggregating the intratemporal optimality condition, equation 2.13, yields the aggregate labour supply equation for households in country $H$:

$$N_t = (h)^{1+\frac{\sigma}{\varphi}}(C_t)^{-\frac{\sigma}{\varphi}}\left[(1-\tau_t^w)\frac{W_t}{P_t}\right]^{\frac{1}{\varphi}} \tag{2.16}$$

where $N_t$ is aggregate labour supply and $C_t$ is aggregate consumption for households in country $H$, defined by:

$$N_t \equiv \int_0^h N_t^i\,di = hN_t^i \qquad C_t \equiv \int_0^h C_t^i\,di = hC_t^i \tag{2.17}$$

while aggregating the intertemporal optimality condition for households in country $H$, equation 2.14, taking conditional expectations on both sides of the equation and using the no-arbitrage condition

[6]See Appendix A.8 for all derivations for this section.

between government bonds and state-contingent claims, yields:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t \left\{ \frac{\xi_{t+1}}{\xi_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \tag{2.18}$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period riskless government bond paying off one unit of the union's currency in $t+1$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross CPI inflation in country $H$.

Specular equations for country $F$ can be obtained following the same procedure, where the results are the same, but with all the relevant starred ($^*$) variables instead of the unstarred ones.

The first order conditions for the household's problem in country $F$ are:

$$(C_t^{*i})^{\sigma}(N_t^{*i})^{\varphi} = (1-\tau_t^{*w})\frac{W_t^*}{P_t^*} \tag{2.19}$$

which is the household's intratemporal optimality condition, and:

$$\mathcal{Q}_{t,t+1} = \beta \frac{\xi_{t+1}^*}{\xi_t^*} \left( \frac{C_{t+1}^{*i}}{C_t^{*i}} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \tag{2.20}$$

which is the household's intertemporal optimality condition or Euler Equation, that holds for all states of nature at $t$ and $t+1$, and:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} \tag{2.21}$$

which is the no-arbitrage condition between riskless government bonds and state-contingent claims.

The aggregate labour supply equation for households in country $F$ is given by:

$$N_t^* = (1-h)^{1+\frac{\sigma}{\varphi}}(C_t^*)^{-\frac{\sigma}{\varphi}} \left[ (1-\tau_t^{*w})\frac{W_t^*}{P_t^*} \right]^{\frac{1}{\varphi}} \tag{2.22}$$

where $N_t^*$ is aggregate labour supply and $C_t^*$ is aggregate consumption for households in country $F$, defined by:

$$N_t^* \equiv \int_h^1 N_t^{*i}\, di = (1-h)N_t^{*i} \qquad C_t^* \equiv \int_h^1 C_t^{*i}\, di = (1-h)C_t^{*i} \tag{2.23}$$

while taking conditional expectations on both sides of the aggregate intertemporal optimality condition and using the no-arbitrage condition between government bonds and state-contingent claims yields:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t \left\{ \frac{\xi_{t+1}^*}{\xi_t^*} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{\Pi_{t+1}^*} \right\} \tag{2.24}$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period riskless government bond paying off one unit of the union's currency in $t+1$, which is the same for both countries in the currency union, and $\Pi_{t+1}^* \equiv \frac{P_{t+1}^*}{P_t^*}$ is gross CPI inflation in country $F$.

Aggregating the budget constraints of households in countries $H$ and $F$ respectively and considering that in optimality they hold with equality yields:

$$P_tC_t + D_t + B_t = (1+i_{t-1})(D_{t-1}+B_{t-1}) + (1-\tau_t^w)W_tN_t + T_t + \Gamma_t \tag{2.25}$$

$$P_t^*C_t^* + D_t^* + B_t^* = (1+i_{t-1})(D_{t-1}^*+B_{t-1}^*) + (1-\tau_t^{*w})W_t^*N_t^* + T_t^* + \Gamma_t^* \tag{2.26}$$

where aggregate contingent claims and aggregate bonds are respectively defined as:

$$D_t \equiv \int_0^h D_t^i \, di = hD_t^i \qquad D_t^* \equiv \int_h^1 D_t^{*i} \, di = (1-h)D_t^{*i} \tag{2.27}$$

$$B_t \equiv \int_0^h B_t^i \, di = hB_t^i \qquad B_t^* \equiv \int_h^1 B_t^{*i} \, di = (1-h)B_t^{*i} \tag{2.28}$$

while aggregate transfers and profits are respectively defined as:

$$T_t \equiv \int_0^h T_t^i \, di = hT_t^i \qquad T_t^* \equiv \int_h^1 T_t^{*i} \, di = (1-h)T_t^{*i} \tag{2.29}$$

$$\Gamma_t \equiv \int_0^h \Gamma_t^i \, di = h\Gamma_t^i \qquad \Gamma_t^* \equiv \int_h^1 \Gamma_t^{*i} \, di = (1-h)\Gamma_t^{*i} \tag{2.30}$$

and aggregate consumption and labour supply are as previously defined.

$P_t$ is the Consumer Price Index (CPI) for country $H$, given by[7]:

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.31}$$

while $P_t^*$ is the Consumer Price Index (CPI) for country $F$, given by:

$$P_t^* \equiv \left[(1-\alpha^*)(P_{H,t}^*)^{1-\eta} + \alpha^*(P_{F,t}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.32}$$

where $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country $H$ and $P_{F,t}$ is a price index for goods imported from country $F$, respectively defined by:

$$P_{H,t} \equiv \left(\frac{1}{h}\int_0^h P_{H,t}(j)^{1-\varepsilon} \, dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.33}$$

$$P_{F,t} \equiv \left(\frac{1}{1-h}\int_h^1 P_{F,t}(j)^{1-\varepsilon} \, dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.34}$$

while $P_{H,t}^*$ is the domestic price index or Producer Price Index (PPI) in country $F$ and $P_{F,t}^*$ is a price index for goods imported from country $H$, respectively defined by:

$$P_{H,t}^* \equiv \left(\frac{1}{1-h}\int_h^1 P_{H,t}^*(j)^{1-\varepsilon} \, dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.35}$$

$$P_{F,t}^* \equiv \left(\frac{1}{h}\int_0^h P_{F,t}^*(j)^{1-\varepsilon} \, dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.36}$$

Since one-period state-contingent claims can be traded freely between households within and across borders, they are in zero international net supply, so that the market clearing condition for these assets

[7] The Consumer Price Index (CPI) for country $H$, $P_t$, can be found by minimizing a given expenditure on both domestic and foreign goods under the constraint that equation 2.3 equals one:

$$\min_{C_{H,t}^i, C_{F,t}^i} P_{H,t}C_{H,t}^i + P_{F,t}C_{F,t}^i \qquad s.t. \qquad \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t}^i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} = 1$$

The same applies to the respective Consumer Price Index (CPI) for country $F$, $P_t^*$. In the particular case in which $\eta = 1$, the CPI in country $H$ takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$, while the CPI in country $F$ takes the form $P_t^* = (P_{H,t}^*)^{1-\alpha^*}(P_{F,t}^*)^{\alpha^*}$. For a derivation see Appendix A.7.

in every period $t$ is consequently given by:

$$\int_0^h D_t^i \, di + \int_h^1 D_t^{*i} \, di = hD_t^i + (1-h)D_t^{*i} = D_t + D_t^* = 0 \tag{2.37}$$

Adding up the two budget constraints and imposing the market clearing condition for state-contingent claims yields an equation linking consumption in the two countries:

$$\begin{aligned} P_t C_t + B_t + P_t^* C_t^* + B_t^* = (1+i_{t-1}) \left( B_{t-1} + B_{t-1}^* \right) \\ + (1-\tau_t^w) W_t N_t + T_t + \Gamma_t + (1-\tau_t^{*w}) W_t^* N_t^* + T_t^* + \Gamma_t^* \end{aligned} \tag{2.38}$$

### 2.2.2 International Identities and Assumptions

Several international identities and assumptions need to be spelled out in order to link the *Home* economy to the *Foreign* one and to be able to close the model.

The *terms of trade* are defined as the price of foreign goods in terms of home goods, for households in country $H$ and in country $F$, and are given respectively by:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} \qquad \text{and} \qquad \mathcal{S}_t^* \equiv \frac{P_{F,t}^*}{P_{H,t}^*} \tag{2.39}$$

Although deviations from *Purchasing Power Parity (PPP)* may arise because of home bias in consumption, we assume that the *Law of One Price (LOP)* holds for every single good $j$, which implies:

$$P_{H,t}(j) = P_{F,t}^*(j) \qquad \text{and} \qquad P_{F,t}(j) = P_{H,t}^*(j) \tag{2.40}$$

for all $j \in [0,1]$, where $P_{H,t}(j)$ (or $P_{F,t}(j)$ for goods imported from country $F$) is the price of good $j$ in country $H$ and $P_{F,t}^*(j)$ (or $P_{H,t}^*(j)$ for goods produced in country $F$) is the price of good $j$ in country $F$ in terms of the union's currency. Plugging the previous expressions into the definitions of $P_{H,t}$ and $P_{F,t}$ respectively yields:

$$P_{H,t} = P_{F,t}^* \qquad \text{and} \qquad P_{F,t} = P_{H,t}^* \tag{2.41}$$

Combining the previous result with the definition of the terms of trade for countries $H$ and $F$ yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P_{H,t}^*}{P_{F,t}^*} = \frac{1}{\mathcal{S}_t^*} \tag{2.42}$$

Dividing the CPIs in countries $H$ and $F$ by the relative price indices for domestic and imported goods and combining them with the definition of the terms of trade yields the following relationships[8]:

$$\frac{P_t}{P_{H,t}} = \left[ 1 - \alpha + \alpha (\mathcal{S}_t)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{2.43}$$

$$\frac{P_t}{P_{F,t}} = \frac{\left[ 1 - \alpha + \alpha (\mathcal{S}_t)^{1-\eta} \right]^{\frac{1}{1-\eta}}}{\mathcal{S}_t} \tag{2.44}$$

[8] In the special case in which $\eta = 1$ the CPIs take the form shown in Appendix A.7, so these ratios consequently take the form:

$$\frac{P_t}{P_{H,t}} = \left( \frac{P_{F,t}}{P_{H,t}} \right)^{\alpha} = \mathcal{S}_t^{\alpha} \qquad \text{and} \qquad \frac{P_t}{P_{F,t}} = \left( \frac{P_{H,t}}{P_{F,t}} \right)^{1-\alpha} = \mathcal{S}_t^{\alpha-1}$$

$$\frac{P_t^*}{P_{H,t}^*} = \left( \frac{P_{F,t}^*}{P_{H,t}^*} \right)^{\alpha^*} = (\mathcal{S}_t^*)^{\alpha^*} = (\mathcal{S}_t)^{-\alpha^*} \qquad \text{and} \qquad \frac{P_t^*}{P_{F,t}^*} = \left( \frac{P_{H,t}^*}{P_{F,t}^*} \right)^{1-\alpha^*} = (\mathcal{S}_t^*)^{\alpha^*-1} = (\mathcal{S}_t)^{1-\alpha^*}$$

$$\frac{P_t^*}{P_{H,t}^*} = \left[1 - \alpha^* + \alpha^*(\mathcal{S}_t^*)^{1-\eta}\right]^{\frac{1}{1-\eta}} = \left[1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{2.45}$$

$$\frac{P_t^*}{P_{F,t}^*} = \frac{\left[1 - \alpha^* + \alpha^*(\mathcal{S}_t^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\mathcal{S}_t^*} = \mathcal{S}_t\left[1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{2.46}$$

Computing the first price ratio for periods $t$ and $t-1$ and dividing one by the other yields:

$$\frac{P_t}{P_{H,t}}\frac{P_{H,t-1}}{P_{t-1}} = \frac{\left[1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\left[1 - \alpha + \alpha(\mathcal{S}_{t-1})^{1-\eta}\right]^{\frac{1}{1-\eta}}} \tag{2.47}$$

which rearranged yields a relationship between PPI inflation and CPI inflation in country $H$[9]:

$$\Pi_t = \Pi_{H,t}\left[\frac{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}}{1 - \alpha + \alpha(\mathcal{S}_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{2.48}$$

The same relationship can be derived for country $F$ and will be given by:

$$\Pi_t^* = \Pi_{H,t}^*\left[\frac{1 - \alpha^* + \alpha^*\mathcal{S}_t^{\eta-1}}{1 - \alpha^* + \alpha^*(\mathcal{S}_{t-1})^{\eta-1}}\right]^{\frac{1}{1-\eta}} \tag{2.49}$$

Dividing the terms of trade in period $t$ by the terms of trade in period $t-1$ yields a relationship showing the evolution of the terms of trade over time:

$$\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} = \frac{\Pi_{H,t}^*}{\Pi_{H,t}} \implies \mathcal{S}_t = \frac{\Pi_{H,t}^*}{\Pi_{H,t}}\mathcal{S}_{t-1} \tag{2.50}$$

as a function of PPI inflation in both countries $H$ and $F$.

The *Real Exchange Rate* between the *Home* country and country $F$ is the ratio of the two countries' CPIs, expressed both in terms of the union's currency, and is defined by:

$$Q_t \equiv \frac{P_t^*}{P_t} \tag{2.51}$$

Combining the previous results with the definition of the real exchange rate yields a relationship between the real exchange rate and the terms of trade[10]:

$$Q_t = \frac{P_t^*}{P_{F,t}^*}\frac{P_{H,t}}{P_t} = \frac{\left[1 - \alpha^* + \alpha^*(\mathcal{S}_t^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\mathcal{S}_t^*\left[1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}} = \mathcal{S}_t\left[\frac{1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{2.52}$$

where the difference between the real exchange rate and the terms of trade is given by the degree of openness of the two countries and the international trade elasticity. If the countries both have complete home bias ($\alpha = \alpha^* = 0$), then they are in autarky and the real exchange rate is exactly equal to the

[9] In the special case in which $\eta = 1$ the price ratios take the form shown above and the relationships between PPI inflation and CPI inflation in countries $H$ and $F$ respectively take the form:

$$\Pi_t = \Pi_{H,t}\left(\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}}\right)^{\alpha} \quad \text{and} \quad \Pi_t^* = \Pi_{H,t}^*\left(\frac{\mathcal{S}_t^*}{\mathcal{S}_{t-1}^*}\right)^{\alpha^*} = \Pi_{H,t}^*\left(\frac{\mathcal{S}_{t-1}}{\mathcal{S}_t}\right)^{\alpha^*}$$

[10] In the special case in which $\eta = 1$ the CPIs take the form shown in Appendix A.7 and the real exchange rate takes the form:

$$Q_t = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\alpha-\alpha^*} = \mathcal{S}_t^{1-\alpha-\alpha^*}$$

terms of trade, because the CPI and PPI are the same in each country.

### 2.2.3 International Risk-Sharing

The no-arbitrage condition for state-contingent claims and the assumption of complete markets implies that the price of the state-contingent claims must be the same for households in both countries. This in turn implies that we can link the consumption of households in the two countries by equating their Euler Equations through their Stochastic Discount Factor, which yields an international risk-sharing condition.[11]

The risk sharing condition linking consumption of households in country $H$ to consumption of households in country $F$, through their Euler Equations, is given by:

$$C^i_{t+1} = \left( \frac{\xi_{t+1}}{\xi_t} \frac{\xi^*_t}{\xi^*_{t+1}} \frac{Q_{t+1}}{Q_t} \right)^{\frac{1}{\sigma}} \frac{C^i_t}{C^{*i}_t} C^{*i}_{t+1} \tag{2.53}$$

By repeated substitution of *Home* consumption backward in time and moving backwards one period, the previous equation reduces to one linking the consumption of households in the two countries as a function of initial conditions, the real exchange rate and preference shocks:

$$C^i_t = \left( \frac{\xi^*_0}{\xi_0} \frac{1}{Q_0} \right)^{\frac{1}{\sigma}} \left( \frac{C^i_0}{C^{*i}_0} \right) \left( \frac{\xi_t}{\xi^*_t} Q_t \right)^{\frac{1}{\sigma}} C^{*i}_t \tag{2.54}$$

By assuming symmetric initial conditions for households in countries $H$ and $F$, the previous expression reduces to:

$$C^i_t = \left( \frac{\xi_t}{\xi^*_t} Q_t \right)^{\frac{1}{\sigma}} C^{*i}_t \tag{2.55}$$

where household consumption in the two countries differs only in the presence of asymmetric preference shocks and of deviations from purchasing power parity (when the real exchange rate is different from one).

Aggregating the previous condition across households in each country yields:

$$C_t = \frac{h}{1-h} \left( \frac{\xi_t}{\xi^*_t} Q_t \right)^{\frac{1}{\sigma}} C^*_t \tag{2.56}$$

where $C_t$ and $C^*_t$ are aggregate consumption for households in countries $H$ and $F$ respectively, as previously defined, and the difference in aggregate consumption across countries is also given by the relative size of the two countries.

Substituting the real exchange rate with equation 2.52 yields an international risk-sharing condition linking aggregate consumption in the two countries as a function of the terms of trade, the degree of openness of the two countries and the international trade elasticity, other than preference shocks and country size[12]:

$$C_t = \frac{h}{1-h} \left[ \frac{\xi_t}{\xi^*_t} \mathcal{S}_t \left( \frac{1-\alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}} \right)^{\frac{1}{1-\eta}} \right]^{\frac{1}{\sigma}} C^*_t \tag{2.58}$$

[11] All derivations for this section are in Appendix A.9.

[12] In the special case in which $\eta = 1$ the CPIs take the form shown in Appendix A.7 and the international risk-sharing condition takes the form:

$$C_t = \frac{h}{1-h} \left( \frac{\xi_t}{\xi^*_t} \mathcal{S}_t^{1-\alpha-\alpha^*} \right)^{\frac{1}{\sigma}} C^*_t \tag{2.57}$$

### 2.2.4 Firms

In country $H$ there is a continuum of firms indexed by $j \in [0, h)$, while in country $F$ there is a continuum of firms, indexed by $j \in [h, 1]$, each producing a differentiated good with the same technology represented respectively by the following production functions:

$$Y_t(j) = A_t N_t(j) \qquad Y_t^*(j) = A_t^* N_t^*(j) \tag{2.59}$$

where $A_t$ and $A_t^*$ represent the level of technology in countries $H$ and $F$, respectively, which evolve exogenously over time following the AR(1) processes in logs:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \qquad \text{and} \qquad A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \tag{2.60}$$

where $\rho_a \in [0, 1]$ and $\rho_a^* \in [0, 1]$ are measures of persistence of the shocks and $\varepsilon_t$ is a zero mean white noise process.

From the production functions we can derive labour demand for individual firms in countries $H$ and $F$ and the respective nominal and real marginal costs of production, which are equal across firms in each country and are given by[13]:

$$N_t(j) = \frac{Y_t(j)}{A_t} \implies MC_t^n = \frac{W_t}{A_t} \implies MC_t = \frac{W_t}{A_t P_{H,t}} \tag{2.61}$$

$$N_t^*(j) = \frac{Y_t^*(j)}{A_t^*} \implies MC_t^{*n} = \frac{W_t^*}{A_t^*} \implies MC_t^* = \frac{W_t^*}{A_t^* P_{H,t}^*} \tag{2.62}$$

Aggregating individual labour demand across firms in each country yields the aggregate labour demand for countries $H$ and $F$, respectively given by[14]:

$$N_t \equiv \int_0^h N_t(j)\, dj = \int_0^h \frac{Y_t(j)}{A_t}\, dj = \frac{Y_t}{A_t} \int_0^h \frac{1}{h} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj = \frac{Y_t}{A_t} d_t \tag{2.63}$$

$$N_t^* \equiv \int_h^1 N_t^*(j)\, dj = \int_h^1 \frac{Y_t^*(j)}{A_t^*}\, dj = \frac{Y_t^*}{A_t^*} \int_h^1 \frac{1}{1-h} \left( \frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} dj = \frac{Y_t^*}{A_t^*} d_t^* \tag{2.64}$$

where $Y_t$ and $Y_t^*$ are aggregate output in countries $H$ and $F$, respectively given by:

$$Y_t \equiv \left( \left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y_t^* \equiv \left( \left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.65}$$

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[13] Individual labour demand can be derived directly from the production function by dividing both sides by $A_t$, while nominal marginal costs can be derived by minimizing costs for firms subject to the production function, or labour demand:

$$\min_{Y_t(j)} W_t N_t(j) \qquad \text{s.t.} \qquad Y_t(j) = A_t N_t(j)$$

Real marginal costs are simply nominal marginal costs divided by the PPI, which is the relevant price for firms.

[14] The results are obtained by substituting in the demand functions for output in each country, given by:

$$Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \frac{Y_t}{h} \qquad \text{and} \qquad Y_t^*(j) = \left( \frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \frac{Y_t^*}{1-h}$$

as derived in Appendix A.10.

and where the terms:

$$d_t \equiv \int_0^h \frac{1}{h} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj \qquad \text{and} \qquad d_t^* \equiv \int_h^1 \frac{1}{1-h} \left( \frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} dj \tag{2.66}$$

represent relative price dispersion across firms in each country. In steady state and in a flexible price equilibrium these relative price dispersions are equal to one.

Firm $j$'s period $t$ profits net of taxes in country $H$ are given by:

$$\Gamma_t(j) = (1 - \tau_t^s) P_{H,t}(j) Y_t(j) - W_t N_t(j) \tag{2.67}$$

where $\tau_t^s$ is the marginal tax rate on firm sales in country $H$. Substituting in labour demand, marginal costs, substituting the output of firm $j$ with the demand function for output, integrating over all $j \in [0, h)$, using the definition of $P_{H,t}$, and substituting in price dispersion yields aggregate profits net of taxes in country $H$:

$$\Gamma_t = (1 - \tau_t^s) P_{H,t} Y_t - P_{H,t} MC_t Y_t d_t = P_{H,t} Y_t (1 - \tau_t^s - MC_t d_t) \tag{2.68}$$

Analogous results can be obtained for aggregate profits net of taxes in country $F$:

$$\Gamma_t^* \equiv \int_h^1 \Gamma_t^*(j)\, dj = (1 - \tau_t^{*s}) P_{H,t}^* Y_t^* - P_{H,t}^* MC_t^* Y_t^* d_t^* = P_{H,t}^* Y_t^* (1 - \tau_t^{*s} - MC_t^* d_t^*) \tag{2.69}$$

where $\tau_t^{*s}$ is the marginal tax rate on firm sales in country $F$, and where firm $j$'s period $t$ profits net of taxes in country $F$ are given by:

$$\Gamma_t^*(j) = (1 - \tau_t^{*s}) P_{H,t}^*(j) Y_t^*(j) - W_t^* N_t^*(j) = Y_t^*(j)[(1 - \tau_t^{*s}) P_{H,t}(j) - MC_t^{*n}] \tag{2.70}$$

Here we can see how aggregate profits are a function of the PPI, aggregate output, marginal cost and price dispersion for both countries $H$ and $F$, other than the tax rate on firm sales, while nominal aggregate labour income is shown to be a function of some of the same variables and equal to:

$$W_t N_t = W_t \frac{Y_t}{A_t} d_t = MC_t^n Y_t d_t = P_{H,t} Y_t MC_t d_t \tag{2.71}$$

in country $H$, and:

$$W_t^* N_t^* = W_t^* \frac{Y_t^*}{A_t^*} d_t^* = MC_t^{*n} Y_t^* d_t^* = P_{H,t}^* Y_t^* MC_t^* d_t^* \tag{2.72}$$

in country $F$.

Following Calvo (1983), each firm in country $H$ may reset its price with probability $1-\theta$ in any given period. Thus, each period a fraction $1-\theta$ of randomly selected firms reset their price, while a fraction $\theta$ keep their prices unchanged. As a result, the average duration of a price in country $H$ is given by $(1-\theta)^{-1}$, and $\theta$ can be seen as a natural index of price stickiness for country $H$. In country $F$ each firm may reset its price with probability $1-\theta^*$ in any given period. Thus, each period a fraction $1-\theta^*$ of randomly selected firms reset their price, while a fraction $\theta^*$ keep their prices unchanged. As a result, the average duration of a price in country $F$ is given by $(1-\theta^*)^{-1}$, and $\theta^*$ can be seen as a natural index of price stickiness for country $F$. This allows for the two countries to have different degrees of price rigidity.

A firm in country $H$ re-optimizing in period $t$ will choose the price $\bar{P}_{H,t}$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the problem:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \mathcal{Q}_{t,t+k} Y_{t+k|t}(j) \left[ (1-\tau^s_{t+k}) \bar{P}_{H,t} - MC^n_{t+k} \right] \right\} \tag{2.73}$$

subject to the sequence of demand constraints[15]:

$$Y_{t+k|t}(j) = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \frac{Y_{t+k}}{h} \tag{2.74}$$

for $k = 0, 1, 2, \ldots$, where $\mathcal{Q}_{t,t+k}$ is the households' stochastic discount factor in country $H$ for discounting $k$-period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{\xi_{t+k}}{\xi_t} \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \tag{2.75}$$

for $k = 0, 1, 2, \ldots$, and where $Y_{t+k|t}(j)$ is the output in period $t+k$ for firm $j$ which last reset its price in period $t$.

Analogously, a firm in country $F$ re-optimizing in period $t$ will choose the price $\bar{P}^*_{H,t}$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the problem:

$$\max_{\bar{P}^*_{H,t}} \sum_{k=0}^{\infty} \theta^{*k} E_t \left\{ \mathcal{Q}_{t,t+k} Y^*_{t+k|t}(j) \left[ (1-\tau^{*s}_{t+k}) \bar{P}^*_{H,t} - MC^{*n}_{t+k} \right] \right\} \tag{2.76}$$

subject to the sequence of demand constraints:

$$Y^*_{t+k|t}(j) = \left( \frac{\bar{P}^*_{H,t}}{P^*_{H,t+k}} \right)^{-\varepsilon} \frac{Y^*_{t+k}}{1-h} \tag{2.77}$$

for $k = 0, 1, 2, \ldots$, where $\mathcal{Q}_{t,t+k}$ is the households' stochastic discount factor in country $F$ for discounting $k$-period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{\xi^*_{t+k}}{\xi^*_t} \left( \frac{C^*_{t+k}}{C^*_t} \right)^{-\sigma} \frac{P^*_t}{P^*_{t+k}} \tag{2.78}$$

for $k = 0, 1, 2, \ldots$, which is equal to the stochastic discount factor for households in country $H$, and where $Y^*_{t+k|t}(j)$ is the output in period $t+k$ for firm $j$ which last reset its price in period $t$.

The optimal price chosen by firms in country $H$ can be expressed as a function of only aggregate variables[16]:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC^n_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1-\tau^s_{t+k}) \right\}} \tag{2.79}$$

Notice that in the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}_H = \frac{\varepsilon}{(\varepsilon - 1)(1-\tau^s)} MC^n \tag{2.80}$$

where $MC^n$ is the nominal marginal cost in steady state and in the flexible price equilibrium in country

[15] As shown in Appendix A.10 the derivation of the demand function for firms is much like the derivation of the demand function for consumption goods, except for the timing of price setting, which implies that $P_{H,t+k}(j) = \bar{P}_{H,t}(j)$ with probability $\theta^k$ for $k = 0, 1, 2, \ldots$, and the fact that all firms are the same and so they set the same price in any given period, which allows us to drop the $j$ index.

[16] See Appendix A.11 for the derivation of the optimal price-setting by firms.

$H$, and where the optimal price is shown to be set as a markup over nominal marginal costs.

Analogous results hold for firms in country $F$. The optimal price chosen by firms in country $F$ can be expressed as a function of only aggregate variables:

$$\bar{P}^*_{H,t} = \frac{\varepsilon}{\varepsilon-1}\frac{\sum_{k=0}^{\infty}(\beta\theta^*)^k E_t\left\{\frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}}\frac{Y^*_{t+k}}{(P^*_{H,t+k})^{-\varepsilon}}MC^{*n}_{t+k}\right\}}{\sum_{k=0}^{\infty}(\beta\theta^*)^k E_t\left\{\frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}}\frac{Y^*_{t+k}}{(P^*_{H,t+k})^{-\varepsilon}}(1-\tau^{*s}_{t+k})\right\}} \tag{2.81}$$

In the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}^*_{H} = \frac{\varepsilon}{(\varepsilon-1)(1-\tau^{*s})}MC^{*n} \tag{2.82}$$

where $MC^{*n}$ is the nominal marginal cost in steady state and in the flexible price equilibrium in country $F$, and where the optimal price is shown to be set as a markup over nominal marginal costs.

### 2.2.5 Net Exports, Net Foreign Assets and the Balance of Payments

*Net Exports* are defined as domestic production minus domestic consumption, which is equal to exports minus imports, and for country $H$ are given by:

$$NX_t \equiv P_{H,t}Y_t - P_tC_t - P_{H,t}G_t \tag{2.83}$$

In real terms (divided by $P_{H,t}$), Net Exports for country $H$ can be written as:

$$\widetilde{NX}_t = Y_t - \frac{P_t}{P_{H,t}}C_t - G_t = Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}C_t - G_t \tag{2.84}$$

where net exports are shown to be a function of the country's degree of openness and the terms of trade, other than domestic production and public and private domestic consumption.

Since exports for country $H$ are imports for country $F$ and viceversa, then net exports are in zero international net supply: $NX_t + NX^*_t = 0$. In real terms: $\widetilde{NX}_t + S_t\widetilde{NX}^*_t = 0$. This implies the following relationship between output in the two countries:

$$Y_t + S_tY^*_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}C_t + S_t\left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}}C^*_t + G_t + S_tG^*_t \tag{2.85}$$

*Net Foreign Assets* are given by the sum of private and public assets held abroad, and for countries $H$ and $F$ are given respectively by:

$$NFA_t \equiv D_t + B_t - B^G_t \qquad \text{and} \qquad NFA^*_t \equiv D^*_t + B^*_t - B^{*G}_t \tag{2.86}$$

In real terms (divided by $P_{H,t}$ and $P^*_{H,t}$ respectively), Net Foreign Assets for countries $H$ and $F$ can be written respectively as:

$$\widetilde{NFA}_t \equiv \tilde{D}_t + \tilde{B}_t - \tilde{B}^G_t \qquad \text{and} \qquad \widetilde{NFA}^*_t \equiv \tilde{D}^*_t + \tilde{B}^*_t - \tilde{B}^{*G}_t \tag{2.87}$$

Since foreign assets for country $H$ are domestic assets for country $F$, then net foreign assets are in zero international net supply: $NFA_t + NFA^*_t = 0$. In real terms: $\widetilde{NFA}_t + S_t\widetilde{NFA}^*_t = 0$.

The *Balance of Payments* is given by net exports plus interest accrued on net foreign assets:

$$BP_t \equiv NX_t + i_{t-1}NFA_{t-1} \tag{2.88}$$

In real terms (divided by $P_{H,t}$), the Balance of Payments for country $H$ can be written as:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + i_{t-1}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \tag{2.89}$$

Combining the relationships between net foreign assets and net exports in the two countries shows that the balance of payments of the two countries are related in the following way: $BP_t + BP_t^* = 0$. In real terms: $\widetilde{BP}_t + S_t\widetilde{BP}_t^* = 0$.

From the households' budget constraint, substituting in firm profits and labour income, and substituting in the expression for transfers backed out from the government budget constraint, yields the following relationship between assets and goods for country $H$:

$$D_t + B_t - (D_{t-1} + B_{t-1})(1 + i_{t-1}) = B_t^G - B_{t-1}^G(1 + i_{t-1}) + P_{H,t}Y_t - P_tC_t - P_{H,t}G_t \tag{2.90}$$

while substituting in the definitions of net exports and net foreign assets yields:

$$NFA_t = NFA_{t-1}(1 + i_{t-1}) + NX_t \tag{2.91}$$

while substituting in the definition of the balance of payments yields a relationship between net foreign assets and the balance of payments:

$$NFA_t = NFA_{t-1} + BP_t \tag{2.92}$$

which shows that the balance of payments is equal to the variation of net foreign assets over one period.

In real terms (divided by $P_{H,t}$), the previous equation can be rewritten as:

$$\widetilde{NFA}_t = (1 + i_{t-1})\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \tag{2.93}$$

### 2.2.6 Central Bank and Monetary Policy

The only central bank in the currency union sets monetary policy by choosing the nominal interest rate to target *union-wide* inflation through a Taylor rule.

Monetary policy follows an Inflation Targeting regime of the kind:

$$\beta(1 + i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \tag{2.94}$$

where union-wide output is defined as the population-weighted geometric average of per-capita output in the two countries:

$$Y_t^U \equiv \left(\frac{Y_t}{h}\right)^h \left(\frac{Y_t^*}{1-h}\right)^{1-h} \tag{2.95}$$

while the union-wide CPI is analogously defined:

$$P_t^U \equiv (P_t)^h (P_t^*)^{1-h} \tag{2.96}$$

so that consequently union-wide inflation is defined as the population-weighted geometric average of the CPI inflations in the two countries:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{2.97}$$

while variables without subscripts $t$ denote their respective steady state levels, $\phi_\pi$ represents the responsiveness of the interest rate to inflation and $\rho_i$ is a measure of the persistence of the interest rate over time (interest rate smoothing).

### 2.2.7 Government and Fiscal Policy in a Pure Currency Union

In a Pure Currency Union each government chooses the amount of government consumption and transfers, financed by marginal tax rates on labour income and firm sales and by short-term government bonds.

In country $H$ the government finances a stream of public consumption $G_t$ and transfers $T_t$ subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j)G_t(j)\,dj + \int_0^h T_t^i\,di + B_{t-1}^G(1+i_{t-1}) = B_t^G + \tau_t^s P_{H,t}Y_t + \int_0^h \tau_t^w W_t N_t^i\,di \tag{2.98}$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period $t$, including interest. $B_t^G$ are government bonds issued by country $H$ in period $t$, while all other variables are as explained above. Government consumption, $G_t$, is given by the following CES function, just like equation 2.74 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h G_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.99}$$

The optimal allocation of any given public expenditure within each category of goods yields the following demand function[17]:

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \frac{G_t}{h} \tag{2.100}$$

and total government expenditure on public consumption simplifies to:

$$\int_0^h P_{H,t}(j)G_t(j)\,dj = P_{H,t}G_t \tag{2.101}$$

The government budget constraint can be rewritten in aggregate form as:

$$P_{H,t}G_t + T_t + B_{t-1}^G(1+i_{t-1}) = B_t^G + \tau_t^s P_{H,t}Y_t + \tau_t^w W_t N_t \tag{2.102}$$

while dividing the government budget constraint by $P_{H,t}$ yields the government budget constraint in real terms:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.103}$$

where variables with a tilde ( $\tilde{}$ ) are in real terms (divided by $P_{H,t}$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the consumption scenario, fiscal policy in country $H$ chooses government consumption to stabilize

[17] As shown in Appendix A.5 for private domestic consumption, $C_{H,t}$.

the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{2.104}$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.105}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ through the following tax rule[18]:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{2.106}$$

where $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_y \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

In the transfer scenario, fiscal policy in country $H$ chooses real transfers to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{2.107}$$

while keeping government consumption constant and balancing the budget:

$$G_t = G \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.108}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{2.109}$$

where $\rho_t \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_y \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

In country $F$ the government finances a stream of public consumption $G_t^*$ and transfers $T_t^*$ subject to the following sequence of budget constraints:

$$\int_h^1 P_{H,t}^*(j) G_t^*(j)\, dj + \int_h^1 T_t^{*i}\, di + B_{t-1}^{*G}(1+i_{t-1}) = B_t^{*G} + \tau_t^{*s} P_{H,t}^* Y_t^* + \int_h^1 \tau_t^{*w} W_t^* N_t^{*i}\, di \tag{2.110}$$

where the right hand side represents government income from taxation and newly issued government

[18]If the overall tax rate is defined as:

$$\tau_t^o \equiv \tau_t^s + \tau_t^w$$

then the variation of the tax rates on labour income and on firm sales will be given respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ of the variation of the overall tax rate in the following way:

$$(\tau_t^w - \tau^w) \equiv \gamma(\tau_t^o - \tau^o)$$

$$(\tau_t^s - \tau^s) \equiv (1-\gamma)(\tau_t^o - \tau^o)$$

which implies the tax rule in the text.

bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period $t$, including interest. $B_t^{*G}$ are government bonds issued by country $F$ in period $t$, while all other variables are as explained above. Government consumption, $G_t^*$, is given by the following CES function, just like equation 2.77 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t^* \equiv \left( \left( \frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 G_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.111}$$

The optimal allocation of any given public expenditure within each category of goods yields the following demand function[19]:

$$G_t^*(j) = \left( \frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \frac{G_t^*}{1-h} \tag{2.112}$$

and total government expenditures on public consumption simplify to:

$$\int_h^1 P_{H,t}^*(j) G_t^*(j) \, dj = P_{H,t}^* G_t^* \tag{2.113}$$

The government budget constraint can be rewritten in aggregate form as:

$$P_{H,t}^* G_t^* + T_t^* + B_{t-1}^{*G}(1 + i_{t-1}) = B_t^{*G} + \tau_t^{*s} P_{H,t}^* Y_t^* + \tau_t^{*w} W_t^* N_t^* \tag{2.114}$$

while dividing the government budget constraint by $P_{H,t}^*$ yields the government budget constraint in real terms:

$$G_t^* + \tilde{T}_t^* + i_{t-1} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{2.115}$$

where variables with a tilde ( $\tilde{}$ ) are in real terms (divided by $P_{H,t}^*$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the consumption scenario, fiscal policy in country $F$ chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left( \frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_g^*)} \left( \frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \tag{2.116}$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{2.117}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma^* \in [0, 1]$ and $1 - \gamma^*$ through the following

[19] As shown in Appendix A.5 for private domestic consumption, $C_{H,t}$.

tax rule[20]:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{2.118}$$

where $\rho_g^* \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_y^* \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

In the transfer scenario, fiscal policy in country $F$ chooses real transfers to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{2.119}$$

while keeping government consumption constant and balancing the budget:

$$G_t^* = G^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{2.120}$$

which means that fiscal policy is financed by the variation of the tax rates on firm sales and labour income from their steady state levels respectively by a share $\gamma^* \in [0,1]$ and $1-\gamma^*$ through the following tax rule:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{2.121}$$

where $\rho_t^* \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_y^* \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

Since government bonds are traded freely within and across borders without frictions and are perfectly substitutable because they offer the same return, the total amount of bonds held by households in both countries must equal the total amount of bonds issued by the two countries' governments, so the market clearing condition for these assets in every period $t$ is given by:

$$B_t + B_t^* = B_t^G + B_t^{*G} \tag{2.122}$$

which can be rewritten in real terms (dividing by $P_{H,t}$) as:

$$\tilde{B}_t + S_t\tilde{B}_t^* = \tilde{B}_t^G + S_t\tilde{B}_t^{*G} \tag{2.123}$$

### 2.2.8 Government and Fiscal Policy in a Coordinated Currency Union

If the Governments of the two countries choose to coordinate, they will use their fiscal instruments to target a common objective, while maintaining independent budget constraints. Instead of using government consumption or transfers to stabilize the domestic output gap countercyclically, they will use the same fiscal instruments to stabilize the net exports gap procyclically. This represents the act of

[20] If the overall tax rate is defined as:

$$\tau^{*o} \equiv \tau_t^{*s} + \tau_t^{*w}$$

then the variation of the tax rates on labour income and on firm sales will be given respectively by a share $\gamma^* \in [0,1]$ and $1-\gamma^*$ of the variation of the overall tax rate in the following way:

$$(\tau_t^{*w} - \tau^{*w}) \equiv \gamma^*(\tau_t^{*o} - \tau^{*o})$$

$$(\tau_t^{*s} - \tau^{*s}) \equiv (1-\gamma^*)(\tau_t^{*o} - \tau^{*o})$$

which implies the tax rule in the text.

coordinating their policies on a common objective, which depends on the interactions between the two economies. The budget constraints of the two fiscal authorities instead remain unmodified.

In the consumption scenario, fiscal policy in country $H$ chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{2.124}$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.125}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{2.126}$$

where $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx} \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap.

In the same consumption scenario, fiscal policy in country $F$ chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*}\right)^{\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{2.127}$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{2.128}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma^* \in [0,1]$ and $1-\gamma^*$ through the following tax rule:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{2.129}$$

where $\rho_g^* \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx}^* \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap.

In the transfer scenario, fiscal policy in country $H$ chooses real transfers to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{2.130}$$

while keeping government consumption constant and balancing the budget:

$$G_t = G \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.131}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{2.132}$$

where $\rho_t \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx} \geq 0$ represents the responsiveness of real transfers to variations of the real net exports gap.

In the same transfer scenario, fiscal policy in country $F$ chooses real transfers to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*}\right)^{\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{2.133}$$

while keeping government consumption constant and balancing the budget:

$$G_t^* = G^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{2.134}$$

which means that fiscal policy is financed by the variation of the tax rates on firm sales and labour income from their steady state levels respectively by a share $\gamma^* \in [0,1]$ and $1-\gamma^*$ through the following tax rule:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{2.135}$$

where $\rho_t^* \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx}^* \geq 0$ represents the responsiveness of real transfers to variations of the real net exports gap.

### 2.2.9 Government and Fiscal Policy in a Full Fiscal Union

If instead of considering two fiscal authorities managing fiscal policy independently, one for each country, or coordinating their policies, but with two separate budget constraints, we consider only one fiscal authority managing fiscal policy for both countries at the same time in a coordinated way and with a consolidated budget constraint, then we can think of it as an extreme case of fiscal policy coordination and call it a Full Fiscal Union.

A Full Fiscal Union uses local government spending to manage fiscal policy at the union level with a consolidated budget constraint. The Fiscal Union finances streams of local public consumption, $G_t$ and $G_t^*$, and transfers, $T_t$ and $T_t^*$, subject to the consolidated budget constraint of the two national fiscal authorities:

$$P_{H,t}G_t + P_{H,t}^*G_t^* + T_t + T_t^* + \mathcal{B}_{t-1}^G(1+i_{t-1}) = \mathcal{B}_t^G + \tau_t^s P_{H,t}Y_t + \tau_t^{*s}P_{H,t}^*Y_t^* + \tau_t^w W_t N_t + \tau_t^{*w}W_t^*N_t^* \tag{2.136}$$

where $\mathcal{B}_t^G$ is overall nominal government debt at time $t$, defined by the sum of the government debts of countries $H$ and $F$:

$$\mathcal{B}_t^G \equiv B_t^G + B_t^{*G} \tag{2.137}$$

Dividing the government budget constraint by $P_{H,t}$ yields the government budget constraint in real terms (for country $H$):

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1}\frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} = (\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* + \tilde{\mathcal{B}}_t^G - \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \quad (2.138)$$

where variables with a tilde ( $\tilde{}$ ) are in real terms (divided by $P_{H,t}$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of overall government debt, which is given by:

$$\tilde{\mathcal{B}}_t^G = \tilde{B}_t^G + S_t \tilde{B}_t^{*G} \quad (2.139)$$

In the consumption scenario, union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \quad (2.140)$$

$$\frac{G_t^*}{G^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*}\right)^{\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.141)$$

while keeping real transfers constant and balancing the overall budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \text{and} \qquad \tilde{\mathcal{B}}_t^G = \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = S_t\left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G}\right) \quad (2.142)$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ in each country through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \quad (2.143)$$

while distributing equally among the two countries the cost of fiscal policy by varying jointly the tax rates in the following way:

$$(\tau_t^s - \tau^s) = (\tau_t^{*s} - \tau^{*s}) \quad (2.144)$$

$$(\tau_t^w - \tau^w) = (\tau_t^{*w} - \tau^{*w}) \quad (2.145)$$

where $\rho_g \in [0,1]$ for country $H$ and $\rho_g^* \in [0,1]$ for country $F$ are measures of persistence of the government consumption shocks in their AR(1) processes in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_{nx} \geq 0$ for country $H$ and $\psi_{nx}^* \geq 0$ for country $F$ represent the responsiveness of government consumption to variations of the real net exports gap.

In the transfer scenario, union-wide fiscal policy chooses real transfers in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \quad (2.146)$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*}\right)^{\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \quad (2.147)$$

while keeping government consumption constant and balancing the overall budget:

$$G_t = G \qquad G_t^* = G^* \qquad \tilde{\mathcal{B}}_t^G = \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G + S_t \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{2.148}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ in each country through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{2.149}$$

while distributing equally among the two countries the cost of fiscal policy by varying jointly the tax rates in the following way:

$$(\tau_t^s - \tau^s) = (\tau_t^{*s} - \tau^{*s}) \tag{2.150}$$

$$(\tau_t^w - \tau^w) = (\tau_t^{*w} - \tau^{*w}) \tag{2.151}$$

where $\rho_t \in [0,1]$ for country $H$ and $\rho_t^* \in [0,1]$ for country $F$ are measures of persistence of the transfer shocks in their AR(1) processes in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_{nx} \geq 0$ for country $H$ and $\psi_{nx}^* \geq 0$ for country $F$ represent the responsiveness of real transfers to variations of the real net exports gap.

## 2.3 Calibration

The model is calibrated[21] following mainly Ferrero (2009), so we consider the top 5 Eurozone countries, which account for more than 80% of Eurozone GDP and we divide them into the periphery (namely, France, Netherlands, Italy and Spain), country $F$, and the core (namely Germany), country $H$. The size of country $H$ is set according to the relative GDP size to $h = 0.4$, as Germany accounts for over 35% of Eurozone GDP.

As in Ferrero (2009) most of the parameters governing the economies of the two countries are set symmetrically, with the exception of the degree of price rigidity, which has been set such that in country $H$ the average duration of a price is 4 quarters while in country $F$ it is 5 quarters. The gross markup $\frac{\varepsilon}{\varepsilon-1}$ has been set to 1.1, which implies a net markup of 10%, and the discount factor has been chosen to match a compounded annual interest rate of 2%. The parameters for monetary policy follow common values used in the literature, so we set the response of the interest rate to inflation to $\phi_\pi = 1.5$, according to the Taylor principle, and the interest rate smoothing parameter to $\rho_i = 0.8$. Table 2.1 collects all calibrated parameters and steady state stances.

In the calibration, we set $\eta > \frac{1}{\sigma}$ so that $C_H$ and $C_F$ are substitutes and hence the substitution effect of a price change dominates the income effect. In the opposite case $\left(\eta < \frac{1}{\sigma}\right)$ $C_H$ and $C_F$ are complements and the income effect of a price change dominates the substitution effect. This implies that fiscal policy and spillovers from one country to the other have very different effects based on the two calibrations. In our analysis we focus on the case in which $C_H$ and $C_F$ are substitutes because we believe it is more realistic, especially for advanced economies, and more in line with the recent literature (See Ferrero (2009) and Blanchard, Erceg and Lindé (2015) for instance), but we also consider the case in which they are complements, as a sensitivity analysis for the effects of fiscal policy, as studied in Hjortsø (2012).

The calibration of the two countries mainly differs in the fiscal policy parameters. In particular, the government consumption-to-GDP ratios have been set respectively to 18.7% for country $H$ and 21.9% for country $F$, according to the average of the last 9 years (source ECB-SDW). The marginal tax rates

[21] The calibration is done on the steady state values described in Appendix A.15.

Table 2.1: Calibrated Parameters and Steady State Stances.

| Parameters | Description | Country $H$ | Country $F$ |
|---|---|---|---|
| $h$ | Relative size of domestic economy | .4 | .6 |
| $\beta$ | Discount factor | .995 | .995 |
| $\varepsilon$ | Elasticity of substitution of domestic goods | 11 | 11 |
| $\frac{\varepsilon}{\varepsilon-1}$ | Gross Price Mark-Up | 1.1 | 1.1 |
| $\eta$ | Elasticity of substitution foreign and domestic goods | [0.3, 4.5] | [0.3, 4.5] |
| $\sigma$ | Inverse Elasticity of intertemporal substitution | 3 | 3 |
| $\varphi$ | Inverse Frisch Elasticity of labour supply | 0.5 | 0.5 |
| $\theta$ | Degree of price rigidity | 3/4 | 4/5 |
| $\alpha$ | Openness of domestic economy | .52 | .361 |
| $\frac{\alpha}{h}$ | Relative openness of domestic economy | 1.3 | .6017 |
| $\frac{1-\alpha}{h}$ | Home bias | 1.2 | 1.065 |
| $\psi_y$ | Responsiveness of fiscal policy to output gap | 0.045 | 0.001 |
| $\psi_{nx}$ | Responsiveness of fiscal policy to net exports gap | 0.0697 | 0.0092 |
| $\phi_\pi$ | Responsiveness of monetary policy to inflation | 1.5 | 1.5 |
| $\rho_i$ | Interest Rate smoothing parameter | 0.8 | 0.8 |
| $\rho_\xi$ | Persistence of preference shock | 0.94 | 0.8 |
| $\rho_a$ | Persistence of technology shock | 0.58 | 0.70 |
| $\rho_g$ | Persistence of fiscal shock | 0.74 | 0.81 |
| $\sigma_\xi$ | Standard deviation preference shock | 0.0024 | 0.0086 |
| $\sigma_a$ | Standard deviation technology shock | 0.0087 | 0.0033 |
| $\sigma_g$ | Standard deviation fiscal shock | 0.0056 | 0.0031 |
| $corr_\xi$ | Correlation preference shock | 0.625 | 0.625 |
| $corr_a$ | Correlation technology shock | 0.418 | 0.418 |
| $corr_g$ | Correlation fiscal shock | 0 | 0 |
| Steady State Ratios | Description | Country $H$ | Country $F$ |
| $(1+i)^4-1$ | Annualized Interest Rate | 2% | 2% |
| $\tau^w$ | Tax Rate on wage income | 40.6% | 27.9% |
| $\tau^s$ | Tax Rate on firm sales | 2.5% | 19.5% |
| $\tau^w MC+\tau^s$ | Tax Revenues-to-GDP | 38.49% | 39.92% |
| $\frac{G}{Y}$ | Government consumption-to-GDP | 18.7% | 21.9% |
| $\frac{T}{Y}$ | Real Transfers-to-GDP | 18.58% | 16.81% |
| $\frac{\tilde{N}X}{Y}$ | Net Exports-to-GDP | 1.72% | -1.14% |
| $\frac{C}{Y}$ | Consumption-to-GDP | 79.58% | 79.24% |
| $\alpha^* \frac{C^*}{Y}$ | Exports-to-GDP | 43.1% | 27.47% |

on labour income have been set respectively to 40.61% for country $H$ and 27.94% for country $F$ in accordance to the average in the last 9 years of the labour income tax wedges, excluding social security contributions made by the employer, for the median individual, as reported in OECD (2015). The marginal tax rate on firm sales has been set to 19.5% for country $F$ according to the average in the last 9 years of the VAT for France, Italy, Spain and The Netherlands as reported in Eurostat, European-Commission et al. (2015), while it has been calibrated for country $H$ to match the average ratio of net exports-to-GDP of 1.73% observed over the past 9 years for Germany[22]. Although the observed VAT rate for Germany is 19%, we set its marginal tax rate on firm sales to 2.5%, as if there was a production incentive, to correct for the fact that country $H$ should have a greater productivity compared to country $F$, as Germany has a greater productivity than the periphery countries. This calibration implies a steady state tax revenue-to-GDP ratio of respectively 38.49% for country $H$ and 39.92% for country $F$, clearly in line with the data observed over the past decades for Germany (38.72%) and for France, Italy, Spain and The Netherlands (39.15%). Finally, the annualized steady state value of government debt-to-GDP in both countries is set to roughly 60% as stated in the Maastricht Treaty.

Since the two countries' fiscal policy ratios have been calibrated according to the data, the transfers-to-GDP ratios have been set such that the government deficit is zero in steady state:

$$\frac{\tilde{T}}{Y} = (\tau^s + \tau^w MC) - \frac{G}{Y} - \left(\frac{1}{\beta} - 1\right)\frac{\tilde{B}^G}{Y} \tag{2.152}$$

$$\frac{\tilde{T}^*}{Y^*} = (\tau^{*s} + \tau^{*w} MC^*) - \frac{G^*}{Y^*} - \left(\frac{1}{\beta} - 1\right)\tilde{B}^{*G} \tag{2.153}$$

Henceforth, the overall calibration of the fiscal sector implies a steady state ratio of transfers-to-GDP of respectively 18.58% for country $H$ and 16.81% for country $F$, and a steady state ratio of current expenditure-to-GDP of respectively 37.28% for country $H$ and 38.71% for country $F$. This calibration is broadly in line with the observed data over the last 10 years for the subsidies-to-GDP ratio (26.85% for Germany and 24.69% for the other countries) and the current expenditure (less interest)-to-GDP ratio (35.54% for Germany and 36.85% for the other countries).

The parameters of openness have been set to match an export-to-GDP ratio $\left(\frac{\alpha^* C^*}{Y}\right)$ of roughly 43% for country $H$[23] taken from the aggregate demand equation, while for country $F$ the parameter of openness is recovered by equating per-capita consumption across countries, which yields the following equation[24]:

$$\alpha^* = \frac{h}{1-h}\left[\alpha + \frac{\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}} - 1}{1 + \frac{h}{1-h}\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}}}\right] \tag{2.154}$$

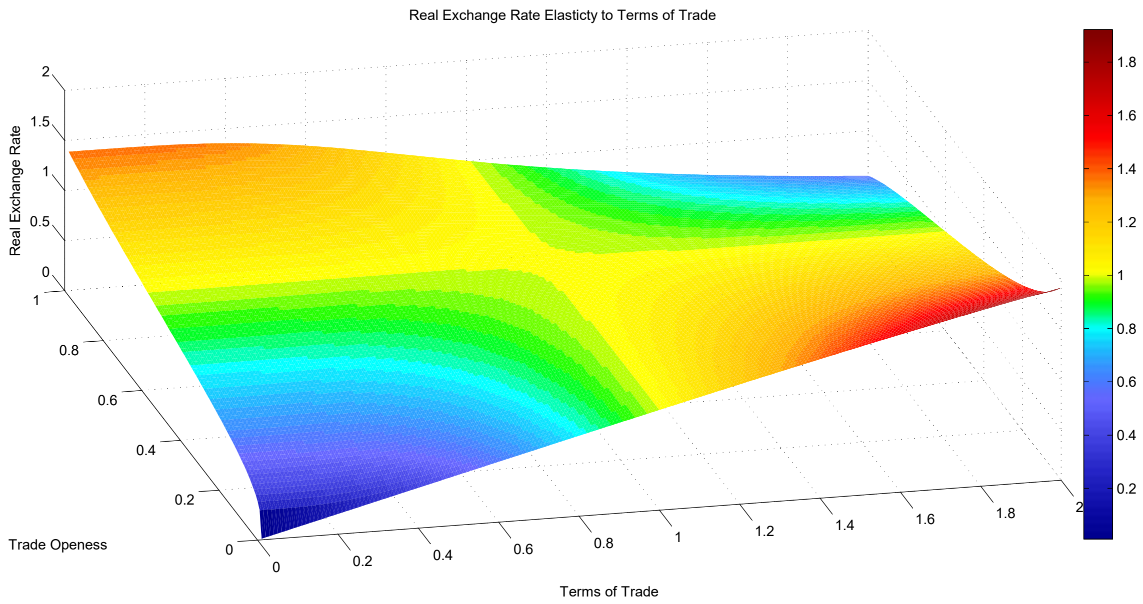
Consequently, home biases are given by $\frac{1-\alpha}{h} = 1.2$ and $\frac{1-\alpha^*}{1-h} = 1.065$. Since both home biases are larger than one it means that the share of consumption of domestic goods is higher than the share of production of domestic goods. This generates a gap between the relative production price indices and the relative consumption price indices based on the different composition of the households' consumption basket in the two countries. Hence, the dynamics of the real exchange rate follow the dynamics of the terms of trade in a non-linear way. As Figure 2.1 shows, the real exchange rate increases as the terms of trade increase if the degree of openness of country $H$ is less than the size of country $F$ $(1 - h = 0.6)$, which is the case for our calibration $(\alpha = 0.52)$, while the real exchange rate decreases when the terms of trade

[22] The average current account to GDP ratio observed over the past 9 years for Germany is roughly 6.36%. However, we adjust the data for the overall trade weight with France, Italy, Spain and The Netherlands (26%).

[23] The value recovered from the data as the average of the last 9 years is 43.5%.

[24] Appendix A.16 shows the derivation.

Figure 2.1: Elasticity of the Real Exchange Rate to the Terms of Trade as a function of Trade Openness

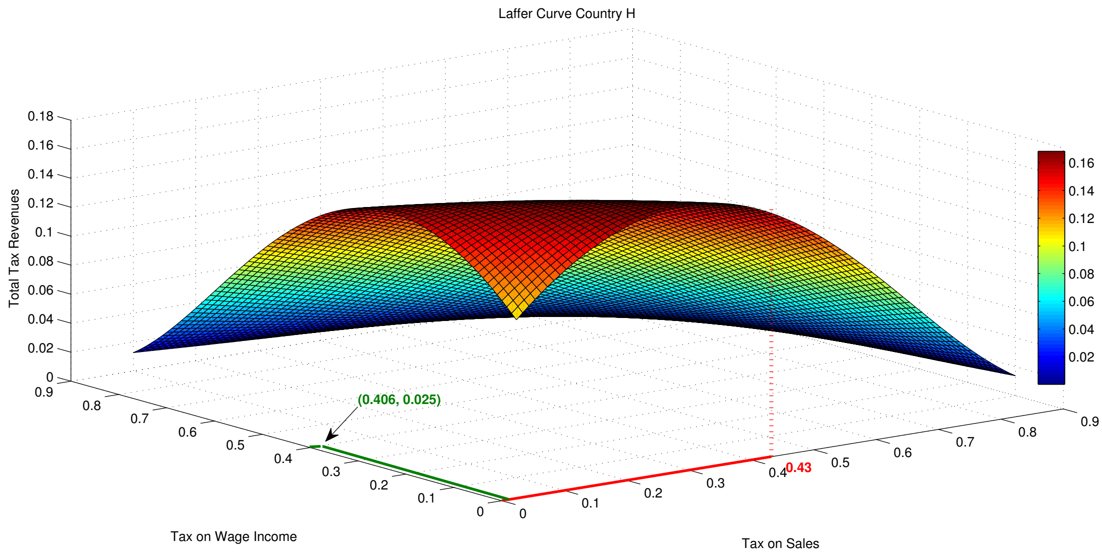



increase if the degree of openness of country $H$ is more than the size of country $F$ $(h = 0.4)$. These two conditions imply respectively, given equation 2.154, that the degree of openness of country $F$ is less than the size of country $H$ $(h = 0.4)$ for the real exchange rate to increase as the terms of trade increase, and that the degree of openness of country $F$ is more than the size of country $H$ $(h = 0.4)$ for the real exchange rate to decrease as the terms of trade increase.

Regarding the dynamic parametrization of the shocks, all three exogenous shocks are assumed to follow a $VAR(1)$ process that generally allows for both direct spillovers and second order correlation of the innovations. However, the structure has been restricted for both the technology shocks and the preference shocks to exclude direct spillovers and for the fiscal shocks to assume fully independent stochastic processes.

With the exception of the preference shocks, whose dynamics have been calibrated following Kollmann et al. (2014), the parameters characterizing the dynamics of both technology shocks and fiscal shocks have been estimated. For the estimation we have employed the time series for Germany, France, Italy and Spain of respectively final government consumption expenditure for the fiscal shocks and labour productivity per hours worked for the technology shocks. All the series are chain-linked volumes re-based respectively in 2005 (fiscal shocks) and 2010 (technology shocks), seasonally adjusted and filtered by means of a Hodrick-Prescott filter. The sample considered spans at quarterly frequency from 2002 Q1 to 2014 Q2 for the fiscal shocks and to 2015 Q3 for the technology shocks. Finally, despite a large debate on the high correlation between preference shocks in the Eurozone, there is no proper reference in the literature for its calibration. We decide to set this parameter according to the observed business

Figure 2.2: Laffer Curve for Country H




cycle correlation (which is roughly 0.5) and we pick the value that maximizes the simulated correlation between output in the two countries (which is roughly 0.42)[25].

### 2.3.1 Sensitivity Analysis

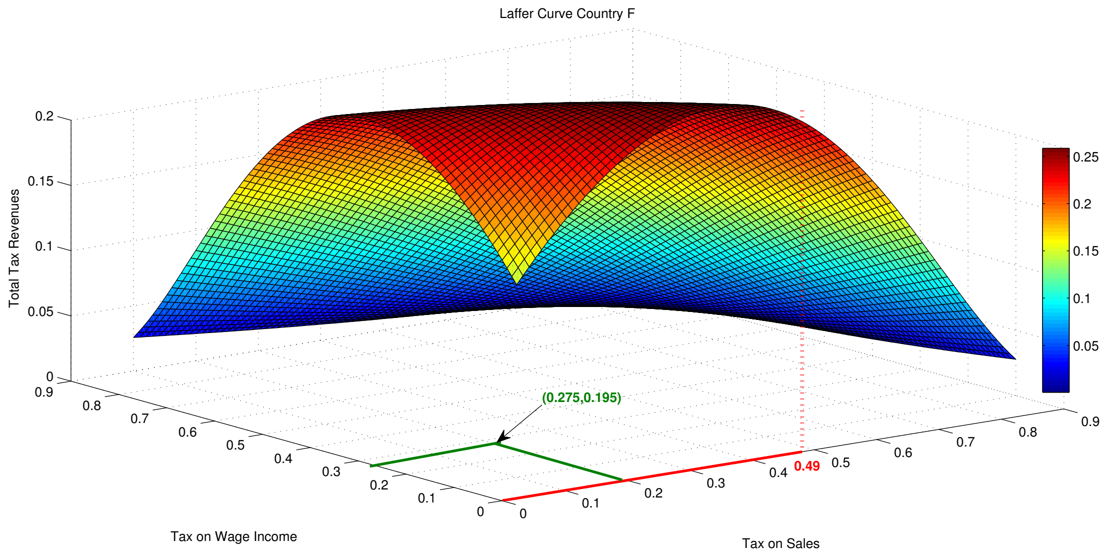
Figures 2.2 and 2.3 show a sensitivity analysis for the tax rates on labour income and firm sales in steady state for each of the two countries, where the tax rates on labour income and firm sales that maximize overall tax revenues in each country are shown in red, while the calibrated tax rates on labour income and firm sales are shown in green.

We can see from the graphs that the revenue maximizing value for the tax rate on labour income is zero because it is highly distortionary, which implies that the distortion it creates through a reduction in the tax base dominates the increase in the tax revenues. On the other hand the revenue maximizing value for the tax rate on firm sales is between 40% and 50%, which implies it is much less distortionary compared to the tax rate on labour income, so the burden of adjustment should be mainly borne by the tax rate on firm sales. Notice also that if the tax rate on firm sales exceeds the values in red, then it will reduce tax revenues and might as well have the opposite effects on the economy.

These graphs also show how relatively far the calibrations of the tax rates are from their revenue maximizing values, so that for the given calibration revenues are increasing in the tax rate on firm sales and decreasing in the tax rate on labour income. At the same time the revenue maximizing values are close for the two countries, because the dynamics are similar, while the calibrations are quite different because of the different structure of the two economies.

[25]The simulated values of the correlation of business cycles in our model, given our calibration, are always lower than the observed correlation. Therefore, we decide to select the correlation of preference shocks that maximizes the correlation of business cycles.

Figure 2.3: Laffer Curve for Country F




## 2.4 Welfare Analysis and Optimal Fiscal Policy Parameters

The selection of the optimal fiscal policy parameters follows from the analysis of the fiscal policy rules used in our model. In the search for the optimal fiscal policy parameters, we limit the analysis to an expenditure rule that sets government consumption to target either the output gap or the net exports gap. Specifically, these fiscal policy parameters have been selected to maximize the unconditional expectation of lifetime utility of the total population of households[26] under the condition that they induce a locally unique rational expectations equilibrium[27]. Therefore the fiscal policy rules in our model are judged optimal in their class of rules, because the fiscal policy parameters are chosen to yield the highest average level of welfare to the representative household compared to all other fiscal policy parameters.

As a measure of welfare we consider the weighted average of the second order approximation of the utility of households in each country, given by:

$$\widetilde{W_t} = hW_t + (1-h)W_t^* \tag{2.155}$$

where welfare for country $H$ is given by:

$$W_t = \xi_t \left( \frac{\left(\frac{C_t}{h}\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\frac{Y_t d_t}{A_t h}\right)^{1+\varphi}}{1+\varphi} \right) + \beta W_{t+1} \tag{2.156}$$

[26]Even if in the Pure Currency Union scenario the fiscal decisions are taken independently, we consider the results of the joint maximization of average aggregate welfare because it is in line with the results of a dynamic game between the two countries.

[27]Following Schmitt-Grohé and Uribe (2007), we discretize the policy space by means of a grid search, because welfare is a non monotonic function of the fiscal policy parameters and has several local maxima. We consider 100 different values for each target variable (i.e. either the output gap or the net exports gap) and limit the parameter space to lie between 0 and 0.1 or between -0.1 and 0 based on the expected sign of the parameter, because a larger parameter space would imply a non stationary equilibrium given by the distortionary effect of taxation overcoming the stabilizing effect of government spending.

and welfare for country $F$ is given by:

$$W_t^* = \xi_t^* \left( \frac{\left(\frac{C_t^*}{1-h}\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\frac{Y_t^* d_t^*}{A_t^*(1-h)}\right)^{1+\varphi}}{1+\varphi} \right) + \beta W_{t+1}^* \tag{2.157}$$

Although we select the fiscal policy parameters based on the unconditional expectation of lifetime utility, to compare welfare attained under alternative fiscal policy scenarios we prefer to rely on the expectation of lifetime utility conditional on the initial state being the non-stochastic steady state. In this way, the welfare ranking of alternative policies will depend on the assumed value and distribution of the initial state vector ($x_0$). This measure accounts for the transitional dynamics leading back to the stochastic steady state and, as the deterministic steady state is the same across all the scenarios considered, we ensure that the economy begins from the same initial point under all possible policies.

### 2.4.1 Welfare Gains based on Consumption Equivalent Variations

Following Schmitt-Grohé and Uribe (2007), we compute the welfare gain of a particular fiscal policy scenario relative to our benchmark scenario: the Pure Currency Union scenario with exogenous government consumption. We denote the benchmark policy scenario with $b$, the alternative scenarios with $a$, and the steady state scenario with 0, and we consider the welfare gain $\lambda$ as the percentage increase in the benchmark scenario's expected consumption that leaves the representative household as well off as in the alternative scenario. Therefore $\lambda$ can be recovered from the following identity:

$$E\{W_a\} = \frac{\xi_b}{(1-\beta)} \left( \frac{\left(\frac{(1+\lambda)C_b}{h}\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\frac{Y_b d_b}{A_b h}\right)^{1+\varphi}}{1+\varphi} \right) =$$

$$\left(E\{W_b\} - W_0\right)(1+\lambda)^{(1-\sigma)} + \frac{(1+\lambda)^{(1-\sigma)} - 1}{(1-\sigma)(1-\beta)} + W_0 \tag{2.158}$$

The welfare gain is then equal to:

$$\lambda = \left[ \frac{(1-\sigma)\left(E\{W_a\} - W_0\right) + (1-\beta)^{-1}}{(1-\sigma)\left(E\{W_b\} - W_0\right) + (1-\beta)^{-1}} \right]^{\frac{1}{1-\sigma}} - 1 \tag{2.159}$$

Note that in the equation above $\lambda$ is a function of both the initial conditions ($x_0$) and the expected variance ($\sigma_0$), because it is a function of the conditional expectations of welfare, which in turn depend on $x_0$ and $\sigma_0$. To compute the value of $\lambda$ we consider its Taylor expansion around the point $x = x_0$ and $\sigma_0 = 0$. Since we choose the initial state to be the deterministic steady state, we need to consider a second-order approximation of $\lambda$ because only the second derivatives of welfare with respect to $\sigma$ are non-zero. Indeed, since the steady state is the same across all scenarios, $\lambda$ vanishes around the point $(x_0, \sigma_0)$ and the first derivatives with respect to $\sigma$ are null. Totally differentiating twice the welfare gain $\lambda$ and evaluating the results at $(x_0, \sigma_0)$ yields:

$$\lambda \approx \left( \frac{\partial^2 W_b}{\partial \sigma^2} - \frac{\partial^2 W_a}{\partial \sigma^2} \right)(1-\beta) \tag{2.160}$$

The optimal fiscal policy parameters and the welfare gains based on Consumption Equivalent Variations are reported in Table 2.2. The optimal coefficients have been selected respectively under the Pure Currency Union scenario for the response of government consumption to the output gap and under the

Coordinated Currency Union scenario for the response of government consumption to the net exports gap[28].

Table 2.2: Optimal Fiscal Policy Parameters and Welfare Gains based on CEV

| Policy Scenarios | Optimal Parameters* | | Conditional Welfare Gains | | |
|---|---|---|---|---|---|
| | $\psi$ | $\psi^*$ | Country H | Country F | Average |
| PCU | 0 | 0 | 0% | 0% | 0% |
| PCU | 0.045 | 0.001 | 0% | 0.86% | 0.51% |
| CCU | 0.0697 | 0.0092 | 2.17% | 0.02% | 0.88% |
| FFU | 0.0697 | 0.0092 | 0.03% | 0.27% | 0.18% |
| FFU | 0 | 0 | 0.03% | -0.09% | -0.04% |

*The optimal parameters are selected looking at the maximum unconditional expected lifetime utility.

From Table 2.2 we can see that the average welfare gain is highest in the Coordinated Currency Union scenario compared to other scenarios, because stabilizing net exports is found to be welfare improving compared to stabilizing output. Second place for average welfare is held by the Pure Currency Union scenario, which has a greater welfare gain compared to the the Full Fiscal Union scenario, mainly because the latter compared to the former reduces a lot welfare in country $H$ and increases a little welfare in country $F$, because of the distributional effects created by the consolidation of budget constraints.

The highest welfare gain for country $H$ is in the Coordinated Currency Union scenario, because it is the country with positive net exports and thus has more to gain in stabilizing the net exports gap. Also, since country $H$ has a lower degree of price rigidity, after a shock prices move more than in country $F$, so that in country $H$ the direct effect on output (income effect) and the indirect effect through the terms of trade (substitution effect) move in the same direction, bringing the economy further away from the initial equilibrium. Thus, stabilizing net exports yields greater welfare gains because it counteracts the substitution effect, reducing the negative effects of the shock. Note that, although small, there is a welfare gain for country $H$ in the Full Fiscal Union scenario compared to the Pure Currency Union scenario, given by the distributional effects of the consolidation of budget constraints, that puts more burden of financing on the country with higher output (country $F$). At the same time Table 2.2 shows that targeting output rather than having stochastic government consumption produces no welfare gains for country $H$.

The highest welfare gain for country $F$ is instead in the Pure Currency Union scenario, because it is the country with negative net exports and thus has more to gain in stabilizing the output gap. Also, since country $F$ has a higher degree of price rigidity, after a shock prices move less than in country $H$, so that in country $F$ the income effect and the substitution effect move in opposite directions not allowing the economy to go far away from the initial equilibrium, and thus stabilizing output yields greater welfare gains because it allows country $F$ to partly offset the higher degree of price rigidity by letting the terms of trade and thus net exports fluctuate freely. Note that, although small, there is a welfare gain for country $F$ in the Coordinated Currency Union scenario and the Full Fiscal Union scenario, but only compared to the Pure Currency Union scenario with stochastic government consumption. At the same time, the Full Fiscal Union scenario with stochastic government consumption has a welfare cost for country $F$, which is given by the additional burden of financing fiscal policy given by the distributional effect of consolidating budget constraints, since country $F$ has a larger output.

[28] The welfare gains for government transfers as a policy instrument are not reported, and the selected coefficients for government transfers are set equal to the optimal coefficients for government consumption, to make them comparable and to focus more on government consumption as a policy instrument.

We can also see from Table 2.2 that using a targeting rule, whichever the target, rather than having exogenous government consumption, is always welfare improving for any country because rules are meant to stabilize a variable and thus reduce its volatility, which yields always a welfare gain. We can also notice that the scenario which minimizes the difference in welfare gains between countries is the Full Fiscal Union scenario, which is in fact meant to treat both countries as equally as possible. In the Full Fiscal Union scenario the welfare gains for country $H$ from targeting the net exports gap are coupled with the distributional effects of consolidated budget constraints, which reduce the welfare gains for country $H$ increasing the welfare gains for country $F$. Although country $F$ has practically no welfare gains from targeting the net exports gap or consolidating budget constraints separately, it attains quite a welfare gain from the two together, showing that the consolidation of budget constraints has a distributional effect on welfare gains too, so that country $F$ not only bears a higher financing burden, but also attains a higher welfare gain from consolidation.

### 2.4.2 Welfare Gains based on an ad hoc Loss Function

Blanchard, Erceg and Lindé (2015) argues that utility-based welfare measures probably underestimate the benefits of reducing the output gap in economies facing a high resource slack (negative net exports), as in the Euro Area periphery. Explicitly, the utility-based welfare measure shows less benefits from fiscal expansions than a simple ad hoc welfare measure because net exports play a substantial role in reducing the periphery's output gap and the increase in consumption in the periphery is delayed so that it has very small welfare effects.

We decide to compare the policy scenarios also based on an ad hoc loss function for the reasons stated above, as in Blanchard, Erceg and Lindé (2015). Since fiscal policy has a stabilizing function, it mimics the behavior of monetary policy, and together they reduce both the inflation gap and the output gap. Furthermore, there are gains in terms of consumption and unemployment related to closing the output gap that are underestimated by utility-based measures. Hence, the gains from fiscal spillovers between the core and the periphery lie between the ones based on consumption equivalent variations and the ones based on an ad hoc loss function.

Using a standard quadratic loss function, the policymakers are assumed to care only about minimizing the square of the output gap and of the inflation gap in both regions. Each region's loss function is, hence, simply the sum of the square of the inflation gap and the square of the output gap, with weights 3 and 1 respectively. The overall loss function is the weighted average of each region's loss function:

$$Loss = \sum_{j=0}^{\infty} \beta^j \left\{ h \left[ (\hat{\pi}_{t+j})^2 + \frac{1}{3} (\hat{Y}_{t+j})^2 \right] + (1-h) \left[ (\hat{\pi}^*_{t+j})^2 + \frac{1}{3} (\hat{Y}^*_{t+j})^2 \right] \right\} \tag{2.161}$$

Table 2.3: Welfare Gains based on an ad hoc Loss Function

| Policy Scenarios | Losses | | | Welfare Gains* | | |
|---|---|---|---|---|---|---|
| | Country H | Country F | Average | Country H | Country F | Average |
| PCU ($\psi = \psi^* = 0$) | 0.6226 | 0.1899 | 0.1388 | 0 | 0 | 0 |
| PCU | 0.8872 | 0.2696 | 0.1972 | -42.51% | -41.92% | -42.02% |
| CCU | 0.0416 | 0.0969 | 0.0748 | 93.31% | 94.90% | 94.61% |
| FFU | 0.0351 | 0.0744 | 0.0587 | 94.35% | 96.08% | 95.77% |
| FFU ($\psi = \psi^* = 0$) | 0.0409 | 0.0906 | 0.0707 | 93.42% | 95.23% | 94.91% |

*Welfare Gains are computed as $\frac{Loss_b - Loss_a}{Loss_b}$, with $Loss_b$ the loss of the PCU with $\psi = \psi^* = 0$.

From Table 2.3 we can see that the average welfare gain is greater in the Full Fiscal Union scenario compared to other scenarios, because targeting the net exports gap reduces the overall inflation gap and consolidating budget constraints reduces the output gap, providing overall stabilization for both countries. Second place for average welfare is held by the Coordinated Currency Union scenario, which has welfare gains compared to the Pure Currency Union scenario which has welfare losses, mainly because targeting the net exports gap reduces international spillovers by stabilizing overall output and inflation, with little difference in welfare gains compared to the Full Fiscal Union scenario. At the same time we can see that targeting the output gap is welfare reducing, compared to stochastic government consumption, which means that output is stabilized more by targeting net exports or nothing.

We can see from Table 2.3 that the welfare gains for both countries, both individually and on average, are increasing in the degree of coordination. The fact that both countries incur in big welfare losses in the Pure Currency Union scenario, while incurring in welfare gains in the other scenarios, shows the big welfare gains from either targeting the net exports gap compared to the output gap or consolidating budget constraints. As a matter of fact, adding one dimension to the other yields very small additional welfare gains, as most of the gains take place by *either* targeting net exports *or* consolidating budget constraints. What is quite surprising is that, according to this welfare measure, even only consolidating budget constraints yields welfare gains similar and a little greater to those achieved by targeting the net exports gap compared to stochastic government consumption. This is because a consolidated budget constraint stabilizes the inflation gap and the output gap on its own somehow, just like targeting the net exports gap does.

## 2.5 Numerical Simulations

We simulate the model numerically using Dynare[29], which takes a second-order approximation of the model around its symmetric non-stochastic steady state with zero inflation and constant government debt. We compare the Impulse Response Functions of the main variables to negative shocks of one standard deviation of different nature, under a range of fiscal policy specifications, to study the stabilization properties of different fiscal policy instruments, financing schemes and coordination strategies and to study the international transmission of fiscal policy shocks.

In the following graphs we compare the impulse responses to a negative technology shock in country $H$ and the impulse responses to a negative preference shock in country $F$. These two shocks represent better than other shocks the relevant dynamics in the Eurozone, and give the most amplified dynamics. This is because a supply shock is more relevant in a country like Germany (country $H$), which is a main producer and exporter of goods and services in the EMU, while a demand shock is more relevant for periphery countries (country $F$), which are mainly consumers and importers in the EMU. We also compare impulse responses to fiscal shocks in both countries.

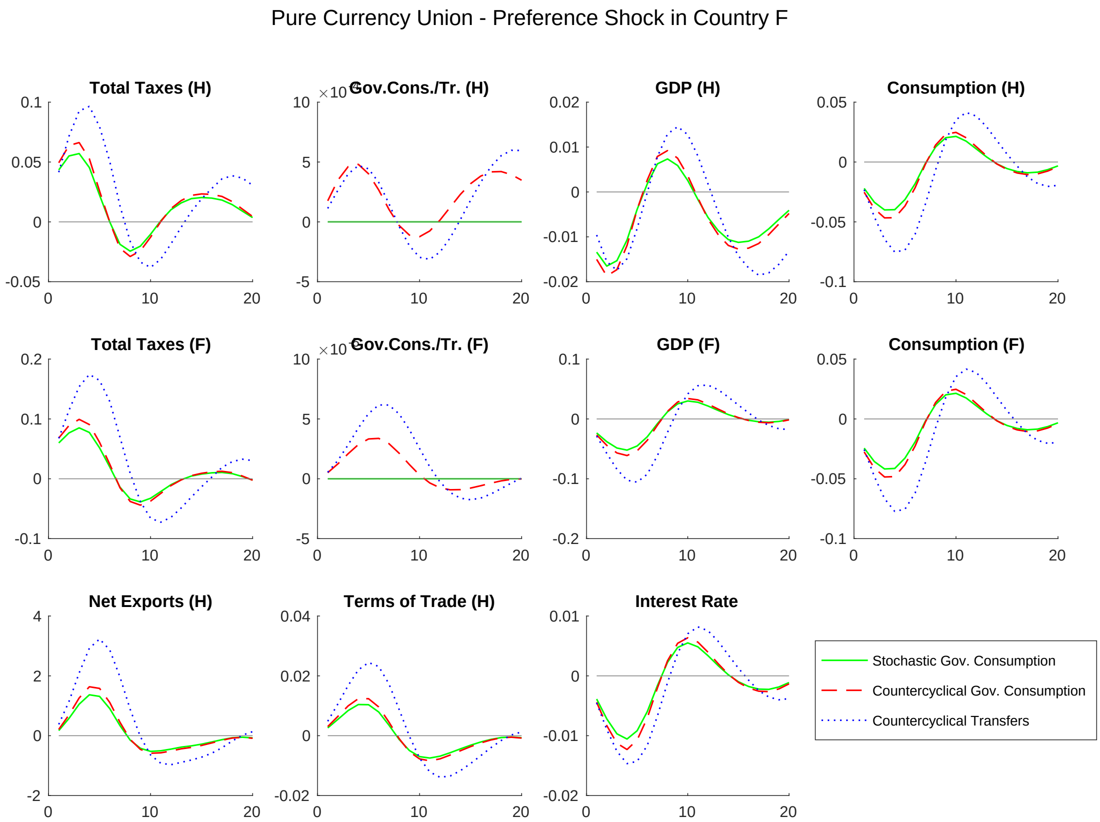
### 2.5.1 Fiscal Policy Instruments

In the first set of graphs we compare different fiscal policy instruments in both the Pure Currency Union scenario and the Full Fiscal Union scenario: countercyclical government consumption and countercyclical real transfers with respect to stochastic government consumption.

Figure 2.4 shows the impulse responses with different fiscal instruments to a negative preference shock in country F under the Pure Currency Union scenario. Because of the assumption of complete markets, domestic and foreign consumption move in a similar way, and they reduce on impact after a negative

[29]All the equilibrium conditions of the model used for the simulations are shown in Appendix A.14.

Figure 2.4: Fiscal Instruments - Pure Currency Union - Preference Shock in Country F

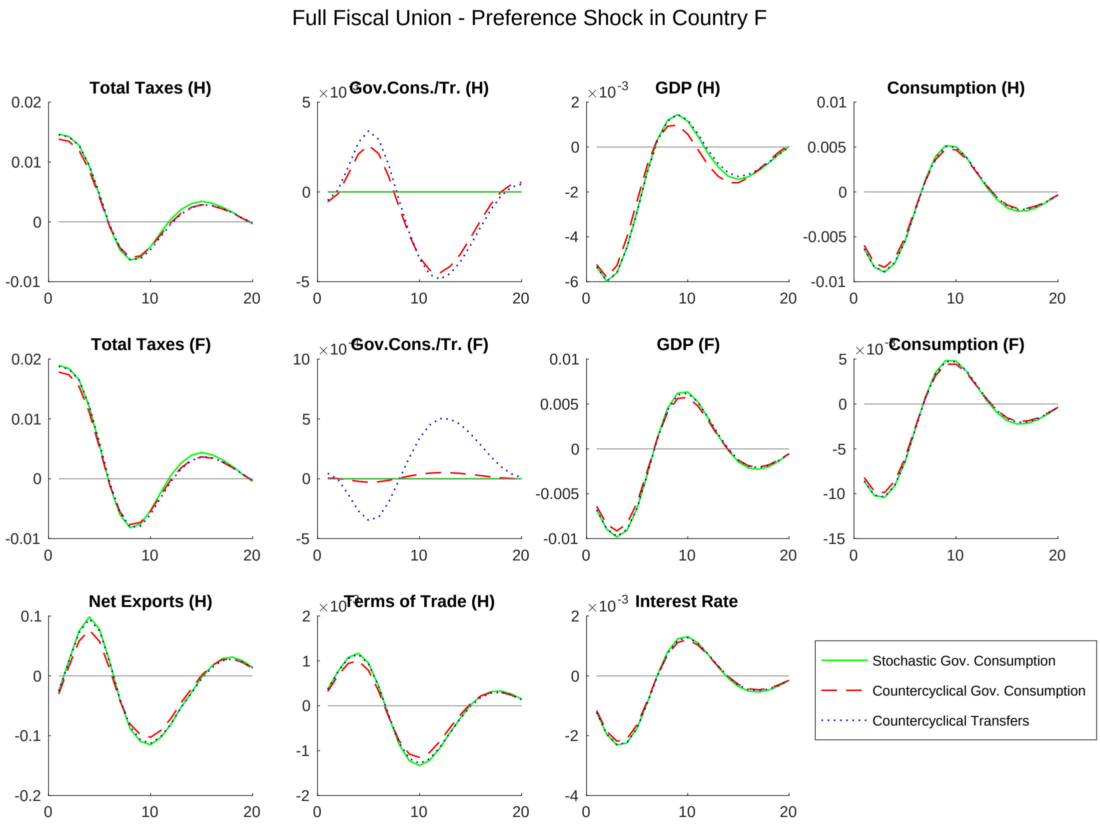



preference shock. As a consequence real GDP goes down, while inflation diminishes bringing to an increase in the terms of trade because of the difference in price rigidity, inducing a variation of net exports. The reduction in inflation in both countries induces the common central bank to lower the interest rate, while the response of national fiscal authorities varies according to the scenario. Countercyclical fiscal policy implies an increase of government consumption or transfers, which calls for higher taxation to keep the budget balanced. Therefore, private consumption and output are below their steady state values for about ten quarters, with a more severe recession in country $F$ because net exports increase in country $H$ (which means they decrease in country $F$). Interestingly, most variables' responses still follow a similar path when fiscal policy uses countercyclical government consumption as when it uses stochastic government consumption, with the latter creating slightly more stabilized dynamics. Using countercyclical government transfers though, after a preference shock, brings to more amplified dynamics for all variables.

After a technology shock[30], which reduces labour supply and income, using countercyclical transfers reduces the effects of the shock on prices and thus the interest rate, because it affects consumption which is aimed at both domestic and foreign goods, while using countercyclical government consumption affects prices more and directly, because it is aimed only at domestic goods. Instead, after a preference shock

[30]The model has been simulated after both a technology and a preference shock, but to keep it short we report and discuss only the most relevant cases. The graphs related to the other shocks are available upon request.

Figure 2.5: Fiscal Instruments - Full Fiscal Union - Preference Shock in Country F

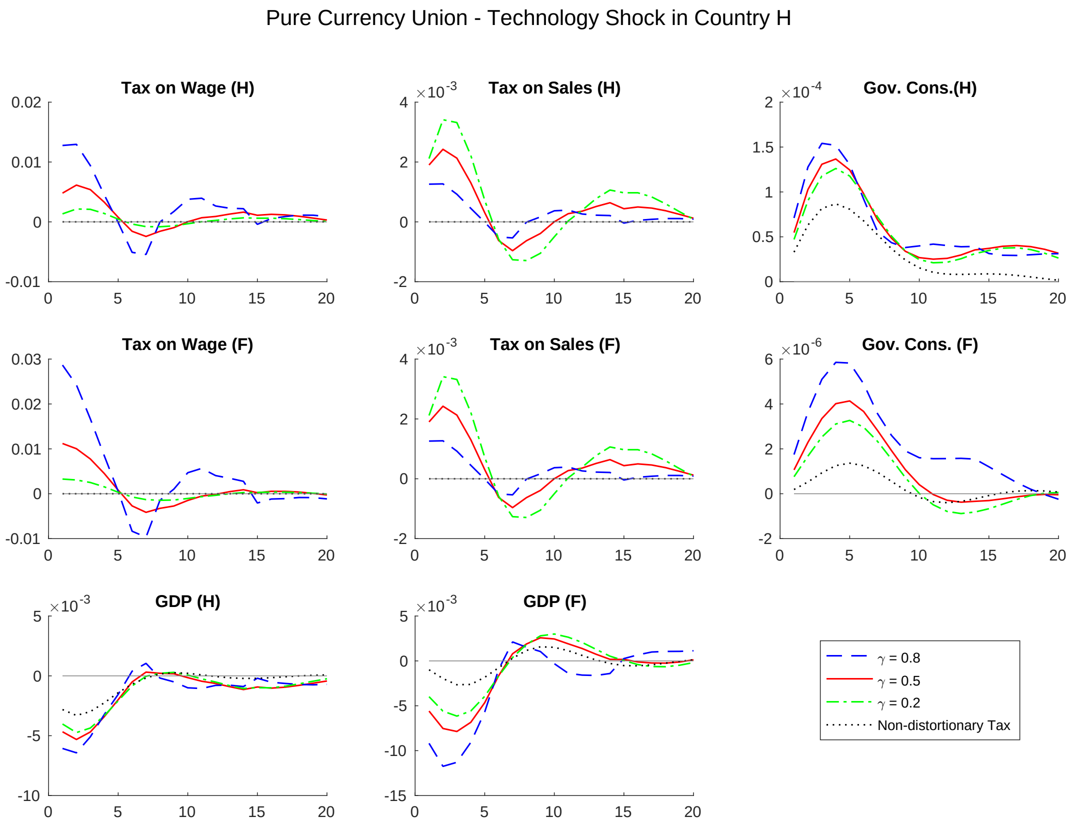



which reduces consumption and thus prices and the interest rate, government transfers are not able to counteract the decrease in consumption and the fall in prices right away, but only partially and with a delay, compared to government consumption which, since it is only aimed at domestic goods, counteracts the fall in prices and thus the fall in the interest rate, stabilizing consumption more.

This implies that the best instrument for fiscal policy in the Pure Currency Union scenario depends on the type of shock: government consumption is better in stabilizing consumption after a preference shock, while transfers are slightly better in stabilizing consumption after a technology shock. To abstract from the nature of the shock, we use countercyclical government consumption as an instrument for most simulations, because it is generally less volatile than using government transfers.

Figure 2.5 shows the impulse responses with different fiscal instruments to the same negative preference shock in country F of figure 2.4, under the Full Fiscal Union scenario. The qualitative response of the variables after the shock is similar to the Pure Currency Union scenario, but now the dynamics are not significantly affected by the fiscal instrument used by the government, because consolidating budget constraints provides on its own almost all the stabilization, leaving little job for the fiscal policy instrument. At this point only countercyclical government consumption can provide a little more stabilization, by targeting net exports and purchasing only domestic goods, compensating directly international demand imbalances. To achieve as much stabilization as possible the best fiscal instrument to use is countercyclical government consumption, especially in the Full Fiscal Union case, even if the gain

Figure 2.6: Fiscal Financing - Pure Currency Union - Technology Shock in Country H




is not that big.

### 2.5.2 Financing Fiscal Policy

Here we compare different financing schemes for fiscal policy, varying the percentage financed by the tax rate on labour income and the tax rate on firm sales. More in detail, we simulate the model under three combinations of $\tau^s$ and $\tau^w$:

- $\gamma = 0.2$, financed roughly 20% by varying the tax rate on labour income and 80% by varying the tax rate on firm sales.
- $\gamma = 0.5$, financed roughly by varying equally the two tax rates. This can be considered as the baseline financing scheme, followed in all other sections.
- $\gamma = 0.8$, financed roughly 80% by varying the tax rate on labour income and 20% by varying the tax rate on firm sales.

We also compare the outcomes of financing fiscal policy with lump-sum taxes, which do not produce distortions in the economy. Figure 2.6 shows the impulse responses with different financing schemes to a negative technology shock in country $H$ under the Pure Currency Union scenario[31]

After a technology shock in country $H$, most variables' responses are quite more amplified when fiscal policy is financed by varying more the tax rate on labour income ($\gamma = 0.8$), because it is more distortionary than the tax rate on firm sales. The most stabilized dynamics are given by varying mainly the tax rate on firm sales ($\gamma = 0.2$), which is less distortionary than the tax rate on labour income. At the same time, varying equally the two tax rates ($\gamma = 0.5$) creates little more distortion compared to $\gamma = 0.2$ and much less distortion compared to $\gamma = 0.8$. Thus, the amplification of the shocks is increasing exponentially in $\gamma$, starting from non-distortionary lump-sum taxes, which follow similar but much less amplified dynamics, to the most amplified dynamics given by the massive use of the tax rate on labour income to finance fiscal policy.

This implies that fiscal policy has a greater stabilization role when it is mainly financed with taxation on firm sales because it is less distortionary. In fact the tax rate on labour income affects very much the equilibrium level of output because of its impact on labour supply, while the tax rate on firm sales affects primarily prices and inflation, with secondary effects on output. We could reduce $\gamma$ to a minimum, but we prefer to use balanced financing ($\gamma = 0.5$) in all other simulations, as it appears to be more consistent with the empirical evidence. Notice that the tax mix does not affect qualitatively the dynamics, even with lump-sum taxes (non distortionary taxation).

### 2.5.3 Fiscal Policy Coordination

In the following graphs we compare the three different degrees of fiscal policy coordination, Pure Currency Union, Coordinated Currency Union, and Full Fiscal Union, after a negative technology shock in country $H$ and after a negative preference shock in country $F$[32]. The impulse responses are shown in Figure 2.7 and Figure 2.8 respectively.

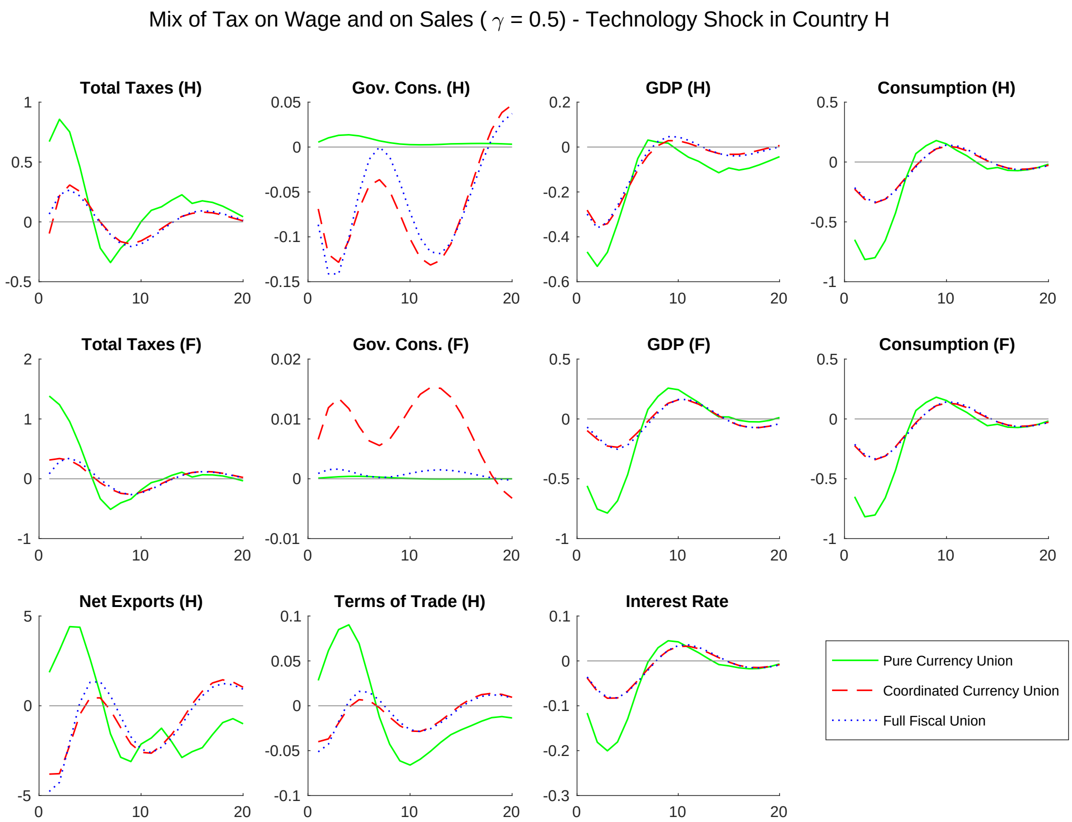
Looking at Figure 2.7 and 2.8, we can see how the dynamics are much more amplified under the Pure Currency Union scenario and much less amplified under the Coordinated Currency Union scenario. This is because when government consumption targets net exports rather than output, international spillovers are reduced and the economy is more stable. Specifically, net exports fluctuate highly after a shock, due to the re-balancing of the household consumption baskets following movements in the terms of trade. By stabilizing net exports the terms of trade are consequently more stable, reducing the substitution effect of a price change, and making consumption and output less volatile than in the Pure Currency Union scenario.

Furthermore, since government consumption is financed with distortionary tax rates, the stabilizing effect of government consumption on output is small because of the opposite effect on consumption of the movements in the tax rates. This is especially true in the Pure Currency Union scenario, where fiscal policy cannot achieve much output stabilization. Whereas, rebalancing domestic demand to be in line with domestic supply, by stabilizing net exports, is a target that fiscal policy can achieve. The Full Fiscal Union scenario presents dynamics which are very close to those in the Coordinated Currency Union scenario, because in both cases government consumption targets net exports. As a consequence, we provide evidence of a significant gain in terms of stabilization when the government targets net exports, while if we also consolidate budget constraints we obtain very small improvements in terms of stabilization. However, government consumption reacts less in the Full Fiscal Union scenario than in the Coordinated Currency Union scenario, because consolidating budget constraints produces international stabilization on its own.

[31] The results are similar after a preference shock in country $F$, and also in the Full Fiscal Union scenario, so we don't show the impulse responses, although they are available on request.

[32] The financing scheme for these simulations is given by a balanced mix of the two tax rates, corresponding to the case $\gamma = \gamma^* = 0.5$ analyzed in the previous section.

Figure 2.7: Fiscal Coordination - Government Consumption - Technology Shock in Country H



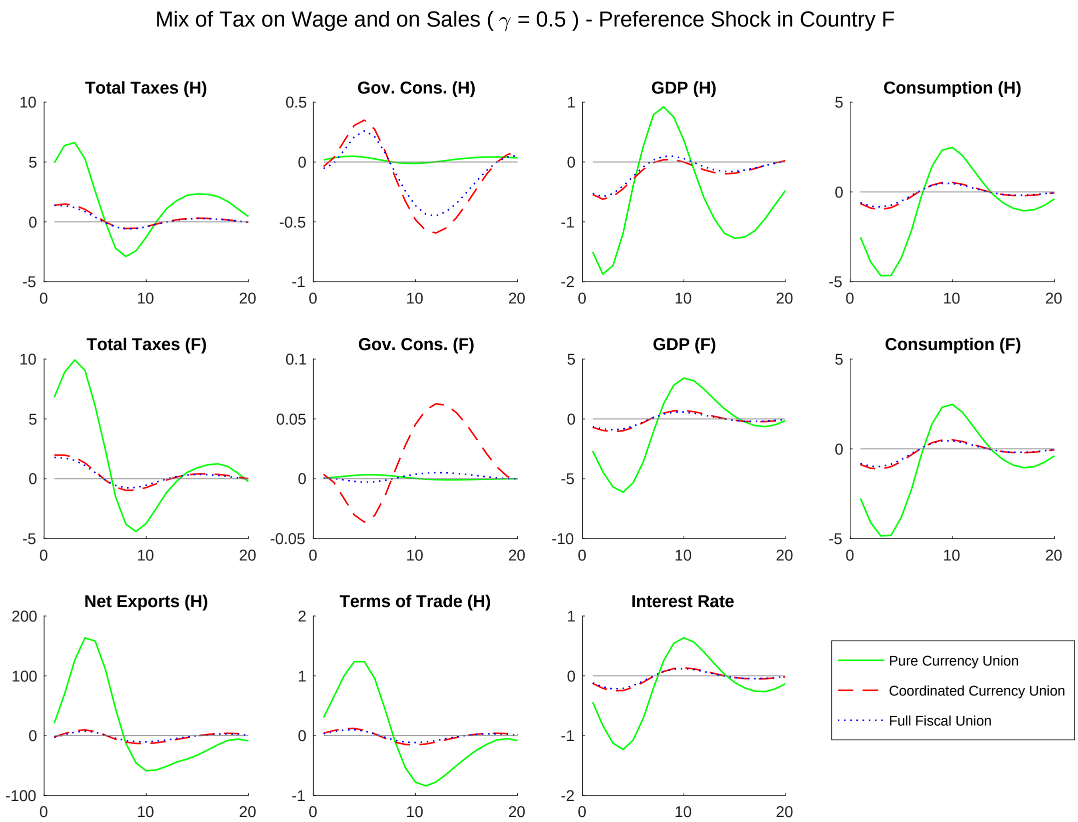

## 2.6 Fiscal Shocks and Fiscal Multipliers and Spillovers

In the previous sections we assessed the stabilizing properties of different degrees of fiscal policy coordination, but we can also evaluate the effects of a fiscal stimulus under the same scenarios. In this section we study fiscal multipliers and spillovers, which describe the effects of exogenous variations of government consumption on domestic and foreign output respectively. Although in Europe fiscal multipliers are typically smaller than in the U.S., Coenen et al. (2010), using several macroeconomic models, found that in the Eurozone the effects of government spending are sizable but require a large fiscal space, especially in periods of economic downturn, and monetary support (i.e. holding the interest rate constant for some periods). Furthermore, in a monetary union, a fiscal stimulus implemented in a country with larger fiscal space (such as Germany) can create positive international spillovers.

### 2.6.1 Steady State Fiscal Multipliers and Spillovers

Fiscal policy multipliers in steady state are defined as the effect of $G$ on $Y$, while fiscal policy spillovers in steady state are defined as the effect of $G^*$ on $Y$. Fiscal policy multipliers and spillovers in steady

Figure 2.8: Fiscal Coordination - Government Consumption - Preference Shock in Country F




state in country $H$ are given respectively by the following equations:

$$\frac{\partial Y}{\partial G} = \left[1 - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} \frac{1-h}{h} \left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^{s})(1-\tau^{w})}\right]^{\frac{1}{\varphi}}\right]^{-1} \tag{2.162}$$

$$\frac{\partial Y}{\partial G^*} = -\left[\frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\right] \left[1 - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} \frac{1-h}{h} \left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^{s})(1-\tau^{w})}\right]^{\frac{1}{\varphi}}\right]^{-1} \tag{2.163}$$

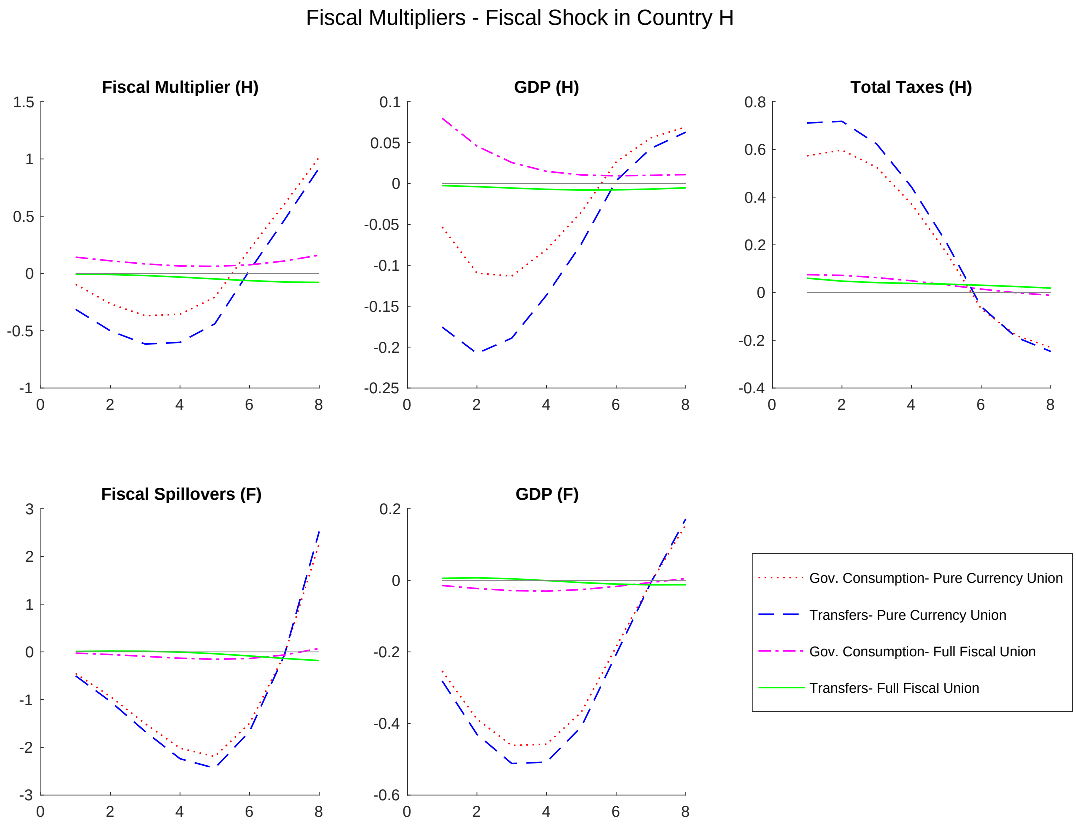
These equations show that fiscal multipliers mainly depend on the degree of openness of the two countries and, hence, international demand imbalances are a key determinant in the dynamics of output after a fiscal shock. Specifically, from the previous equations, sufficient, but not necessary, conditions for fiscal multipliers to be positive and greater than 1 and at the same time for fiscal spillovers to be negative are:

$$\frac{\alpha^*}{\alpha} < \frac{h}{1-h} \tag{2.164}$$

$$\tau^{*s} + \tau^{*w} - \tau^{*s}\tau^{*w} > \tau^{s} + \tau^{w} - \tau^{s}\tau^{w} \tag{2.165}$$

This tells us that when fiscal multipliers are positive and greater than 1, fiscal spillovers are negative, and viceversa. Given our calibration, the conditions listed above are satisfied with opposite inequality

Figure 2.9: Fiscal Multipliers and Spillovers - Fiscal Shock in Country H




signs and, hence, as reported in Table 2.4, in country H in steady state the fiscal multiplier is negative and the fiscal spillover is positive. This implies that in country F in steady state the fiscal multiplier is positive and the fiscal spillover is negative. This is because the degree of openness of country H is much higher than that of country F.

Table 2.4: Fiscal Multipliers and Spillovers in Steady State

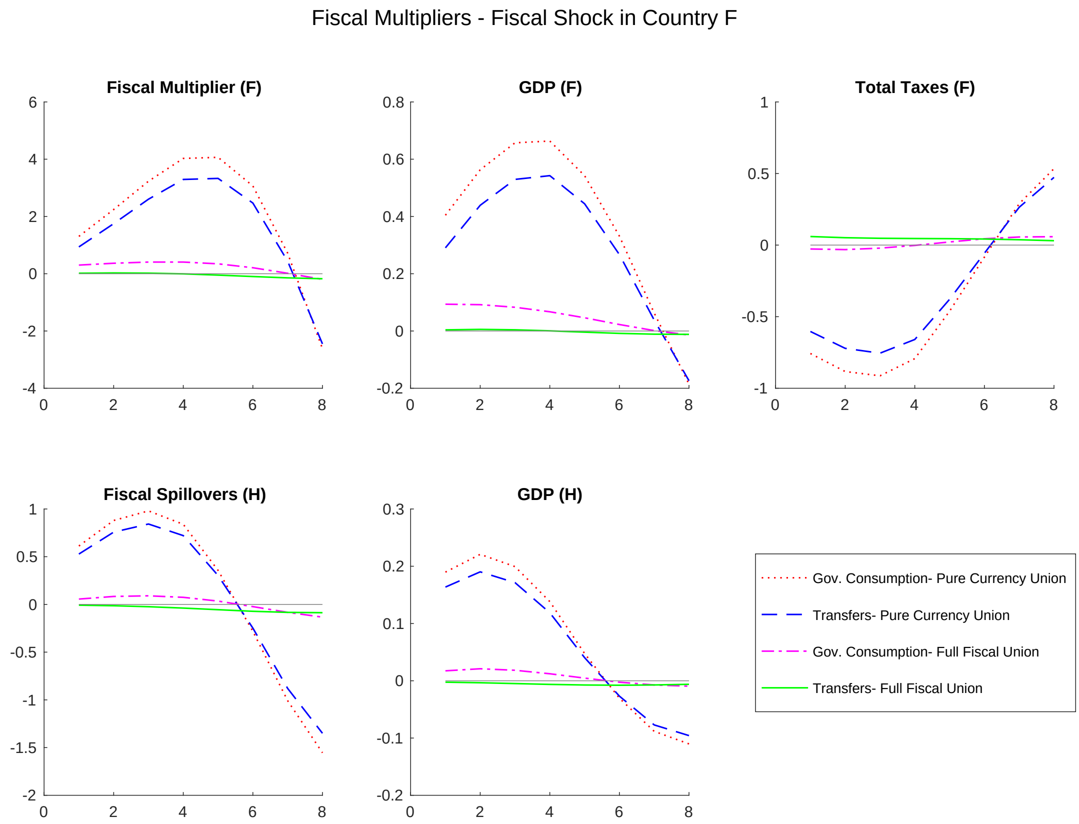
| | Fiscal Multipliers | Fiscal Spillovers | G/Y |
|---|---|---|---|
| Country H | -24.4062 | 16.8643 | 0.187 |
| Country F | 25.4062 | -36.7682 | 0.219 |

Notice that these fiscal multipliers and spillovers do not consider the effects of how the government consumption is financed, which implies variations of the tax rates as a function of the variations of $G$ and $G^*$, but they do include both supply and demand factors.

### 2.6.2 Short-Run Fiscal Multipliers and Spillovers

In this section, we compare the short-run fiscal policy multipliers and spillovers for both countries $H$ and $F$ in the Pure Currency Union scenario and in the Full Fiscal Union one. The short-run fiscal

Figure 2.10: Fiscal Multipliers and Spillovers - Fiscal Shock in Country F




policy multiplier is defined as the percentage deviation in domestic output given by a 1% deviation in domestic government consumption (or transfers), while the fiscal policy spillover is defined as the percentage deviation in foreign output given by a 1% deviation in domestic government consumption (or transfers)[33]. To study the effects of fiscal policy shocks, we show the dynamics of output and taxes after these shocks in each country. Notice that in order to compute the fiscal policy multipliers and spillovers we have to consider government consumption (or transfers) as purely stochastic, so that the only difference between the Pure Currency Union scenario and the Full Fiscal Union one is given by the consolidation of budget constraints.

Figure 2.9 shows fiscal policy multipliers and spillovers, output and taxes, after a positive fiscal policy shock in country $H$ financed by a mix of taxes ($\gamma = 0.5$), comparing the Pure Currency Union scenario with the Full Fiscal Union one. Figure 2.10 reports the same analysis by considering a positive fiscal policy shock in country $F$.

The figures show that there is a relationship between fiscal policy multipliers and spillovers: the bigger the fiscal policy multiplier in one country over time, the bigger the fiscal policy spillover on the other country. The graphs also show that fiscal policy multipliers and spillovers are smaller in absolute terms in the Full Fiscal Union scenario compared to the Pure Currency Union scenario, after both a shock to

[33]The fiscal multiplier is computed as the log deviation of domestic output over the log-deviation of domestic government consumption (or transfers) after a positive shock of one standard deviation to domestic government consumption (or transfers), while the fiscal spillover is computed as the log-deviation of foreign output to a fiscal policy shock in the home country.

government consumption or transfers. This is achieved through the consolidation of budget constraints, as it allows to distribute tax distortions across countries, leading in turn to more stable dynamics of taxes and output. Moreover, in the Pure Currency Union scenario, the fiscal policy multiplier and spillover change sign after about 6 quarters, given by the persistence of the fiscal policy shock. This path is driven mainly by the distortionary taxation, whose effect overturns the stimulus given by the expansionary fiscal policy.

Another important result is that fiscal multipliers and spillovers are positive after a fiscal shock in country $F$, while they are negative after a fiscal shock in country $H$. This is mainly given by the difference in price rigidity and trade openness between the two countries and, in turn, the different effects that government spending has on the terms of trade. In country $H$, where there is lower price rigidity, an increase in government spending brings about an increase in the terms of trade and in net exports. As a consequence, consumption and output decrease in both countries, because of complete markets. On the other hand, in country $F$, where prices are more rigid, an increase in government spending induces a decrease in the terms of trade and in net exports, fostering higher consumption and output in both countries, also because of complete markets. The dynamics of taxes is another key factor explaining why output in country $H$ increases only in the Full Fiscal Union scenario, reversing the effect of government consumption on output compared to the Pure Currency Union scenario.

All the simulations conducted suggest that generally a Full Fiscal Union produces more stabilization in the economy, through the reduction of international spillovers attained by targeting a common international variable such as net exports and/or by consolidating budget constraints, as we have seen in this section. This also implies that fiscal policy multipliers and spillovers are greatly reduced in the Full Fiscal Union scenario compared to the Pure Currency Union scenario. The stabilization is also induced by the larger fiscal capacity observed under the Full Fiscal Union scenario, which allows one country to benefit from the other country financing part of its expenditure, as if it received transfers from it or as if it were allowed to increase its government debt.

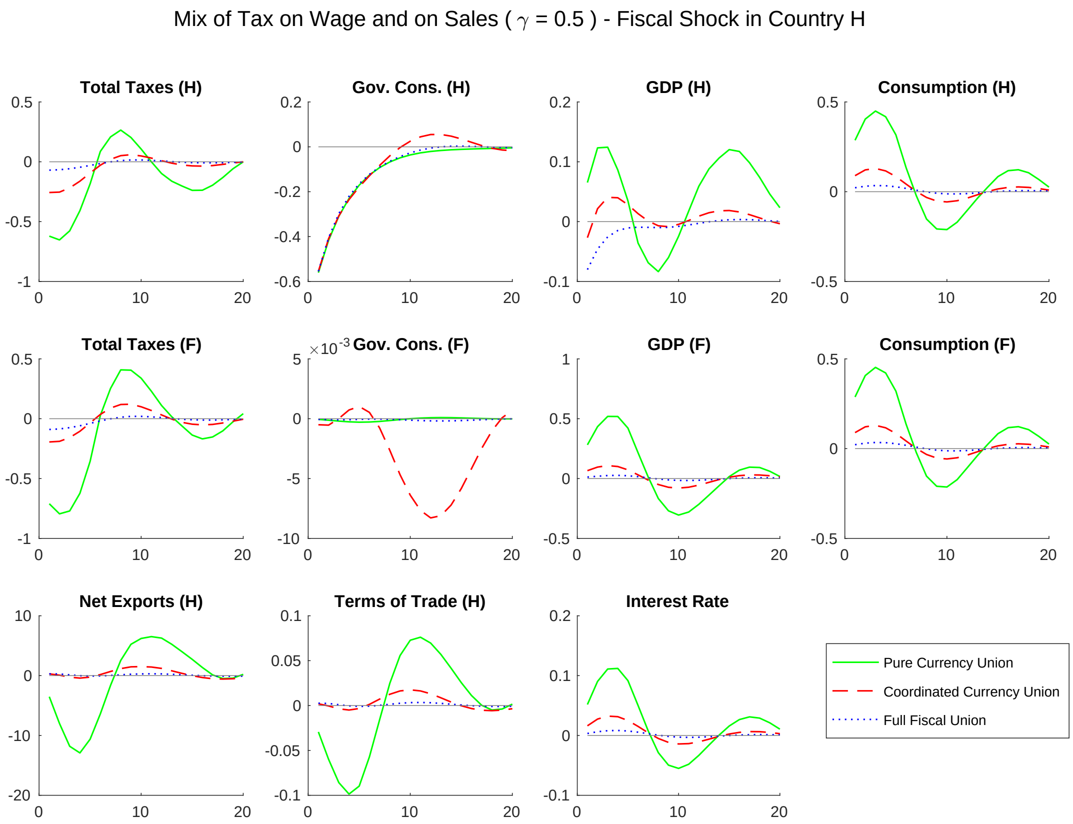
### 2.6.3 Fiscal Shocks and Fiscal Policy Coordination

Figures 2.11 and 2.12 show the impulse responses to a negative fiscal policy shock in country $H$ and $F$, respectively, financed by a mix of taxes ($\gamma = 0.5$), for different degrees of fiscal policy coordination, comparing the three scenarios of Pure Currency Union, Coordinated Currency Union, and Full Fiscal Union.

Looking at Figure 2.11, we see that the transmission of a fiscal shock acts through the consequent tax and price adjustments. In the Pure Currency Union scenario a decrease in government spending in country $H$ allows to reduce taxes, which spurs private consumption and consequently domestic output. Higher private consumption has also inflationary effects, which brings net exports down. The drop in net exports then concurs to revert the path of real output that goes below its steady state value, and then back above it after net exports become positive. The dynamics of foreign variables follow closely the domestic ones due to the presence of complete financial markets.

In the other two scenarios (Coordinated Currency Union and Full Fiscal Union), where government consumption targets net exports, the international transmission of the fiscal policy shock is much more muted, since the terms of trade and consequently net exports are much more stable by construction. As a consequence, the overall volatility is consistently reduced with respect to the Pure Currency Union scenario. The dynamics in the Full Fiscal Union scenario are more stabilized compared to the Coordinated Currency Union scenario because of the international stabilization brought about by consolidating budget constraints and distributing tax distortions. This implies that fiscal shocks are best managed by both targeting net exports and consolidating budget constraints, because they jointly push most variables to

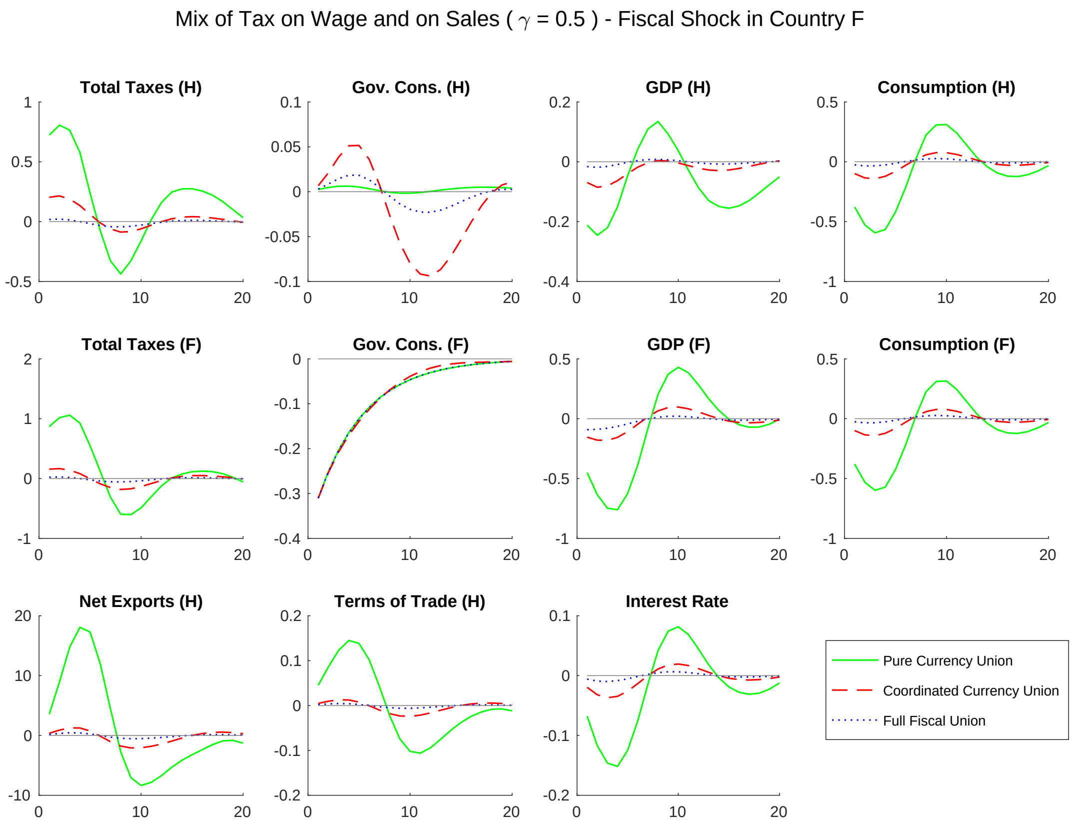
Figure 2.11: Fiscal Policy Coordination - Government Consumption - Fiscal Shock in Country H




be more stable over time, counteracting the spillovers coming from fiscal policy shocks. At the same time, although it is not so clear which of the two dimensions is more important for stabilization purposes, fiscal policy can take advantage of both the benefits from coordinating on the same target (net exports) and those from consolidating budget constraints, so that if for some reason the stabilizing power of one of the two is missing, the other can make up for it.

The same conclusions about the stabilizing properties of a scenario in which fiscal policy targets net exports and governments consolidate budget constraints can be drawn by looking at Figure 2.12. Notice that, however, a negative fiscal policy shock in country $F$ reduces domestic and foreign output, consequently increasing overall taxes. These opposite dynamics are mainly due to the structural differences between the two countries and, in particular, due to the higher price rigidity and lower trade openness of country $F$, that decreases the sensitivity of output and consumption to international adjustments. As a consequence, net exports increase substantially to compensate for the weaker response of producer prices in country $F$. The surge in net exports, however, is not sufficient to increase output in country $H$, because of the opposite effect that taxation has on private consumption and output. These findings are consistent with the different signs of the fiscal policy multipliers in country $H$ and in country $F$.

Figure 2.12: Fiscal Policy Coordination - Government Consumption - Fiscal Shock in Country F



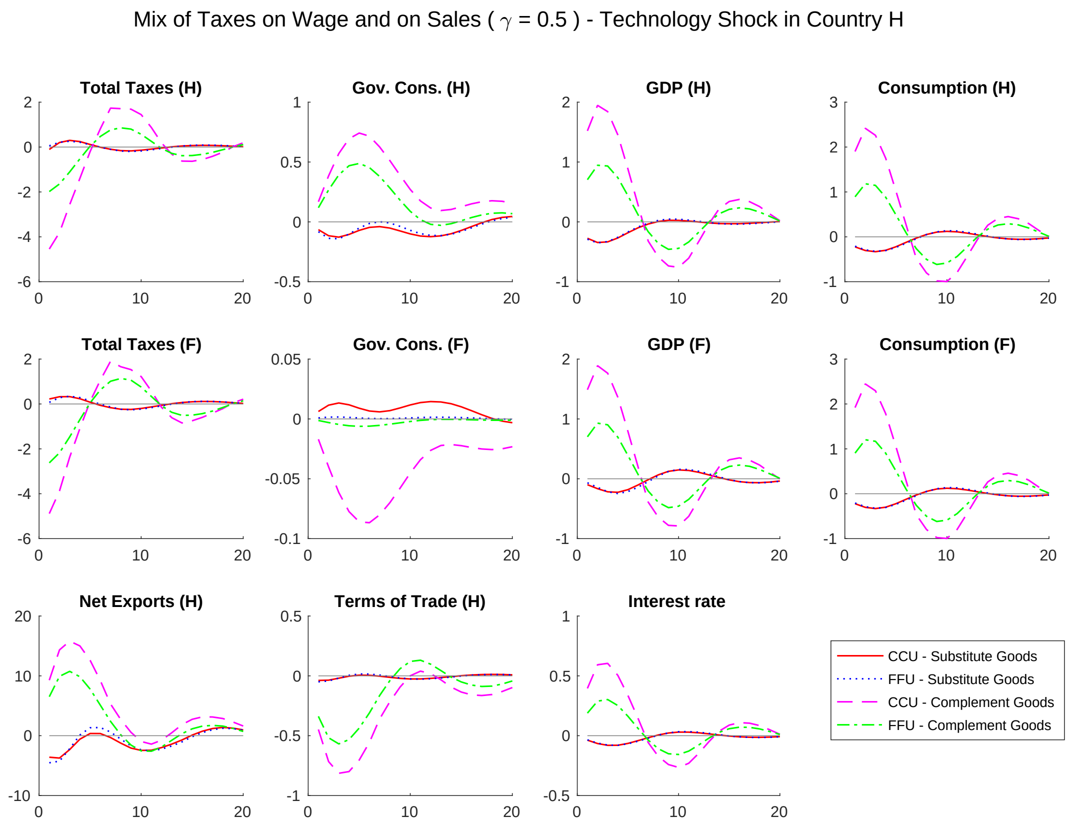

## 2.7 The Case for International Goods as Complements

In the discussion above we highlighted the role played by international demand gaps in shaping the transmission of either demand, supply or fiscal shocks, where the substitution effect of a price change dominates the income effect. However, Hjortsø (2012) shows that this latter result holds only in the case of a high international trade elasticity ($\eta$), while the opposite holds in the case of a low international trade elasticity, which is when goods are complements ($\eta \leq \frac{1}{\sigma}$). More importantly, this affects the ability of fiscal policy to reduce international demand imbalances, given by fluctuations in the terms of trade and in net exports. For these reasons, we assess how much the previous results are sensitive to the alternative assumption of international goods as complements, instead of substitutes.

### 2.7.1 Fiscal Policy Coordination and Welfare: a Comparison

Figure 2.13 shows the impulse responses to a negative technology shock in country $H$ in the Coordinated Currency Union scenario and in the Full Fiscal Union one, comparing the two cases of international goods as substitutes and complements. When goods are substitutes PPI inflation becomes relatively higher in country $H$ compared to country $F$. As a result the terms of trade deteriorate and the substitution effect dominates the income effect, lowering net exports and output. When domestic and foreign goods are instead complements, the income effect dominates the substitution effect, spurring net exports and output in country $H$.

Figure 2.13: International Substitutes vs Complements - Technology Shock in Country H




Notice that in the Full Fiscal Union scenario the overall volatility of the economy is substantially lower, in particular that of output and net exports. This finding is consistent with Hjortsø (2012), who highlights the fact that the cross-country insurance role of fiscal policy is smaller when the traded goods are substitutes instead of complements. More in detail, Figure 2.13 shows that, although with substitute goods the dynamics are less amplified altogether, with complement goods the Full Fiscal Union scenario produces quite more stabilization than the Coordinated Currency Union scenario.

The domination of the income effect on the substitution effect could also affect the welfare analysis, both in terms of Consumption Equivalent Variations and in terms of an ad hoc loss function. When the traded goods are complements, relative prices react more aggressively to shocks in response to international demand imbalances, and the welfare gains in targeting net exports are quite inferior.

As Table 2.5 reports, the highest welfare gains in terms of Consumption Equivalent Variations are attained in the Coordinated Currency Union scenario, but since the income effect dominates the substitution effect, the largest welfare gains from demand stabilization are for country $F$. Even if there is a welfare gain from stabilizing both the output gap and the net exports gap with a low elasticity of substitution, the income effect and the substitution effect move in the same direction for the country featuring the highest price rigidity (country $F$), while they move in opposite directions for country $H$. Therefore,

Table 2.5: Optimal Fiscal Policy Parameters and Welfare Gains based on CEV - Complements

| Policy Scenarios | Optimal Parameters* | | Conditional Welfare Gains | | |
|---|---|---|---|---|---|
| | $\psi$ | $\psi^*$ | Country H | Country F | Total |
| PCU | 0 | 0 | 0% | 0% | 0% |
| PCU | 0.0692 | 0.0373 | 0.01% | 0.04% | 0.03% |
| CCU | 0.0729 | 0.0373 | 0.02% | 0.23% | 0.15% |
| FFU | 0.0729 | 0.0373 | 0.02% | -0.40% | -0.23% |
| FFU | 0 | 0 | 0.02% | -0.12% | -0.06% |

*The optimal parameters are selected looking at the maximum unconditional expected lifetime utility.

since the stabilization of net exports partially offsets the substitution effect, there are additional welfare gains for country $F$ from stabilizing international demand imbalances.

Finally, Table 2.5 shows that the consolidation of budget constraints leads to welfare losses, also when the elasticity of substitution is low. Additionally, stabilizing net exports, when budget constraints are consolidated, amplifies the effects of distributed tax adjustments, because it offsets price adjustments that would otherwise counterbalance the tax adjustments, bringing to welfare losses for country $F$.

Table 2.6: Welfare Gains based on an ad hoc Loss Function - Complements

| Policy Scenarios | Loss Functions | | | Welfare Gains* | | |
|---|---|---|---|---|---|---|
| | Country H | Country F | Total | Country H | Country F | Total |
| PCU ($\psi = \psi^* = 0$) | 0.0226 | 0.0220 | 0.0222 | 0% | 0% | 0% |
| PCU | 0.0222 | 0.0215 | 0.0218 | 1.62% | 2.06% | 1.88% |
| CCU | 0.2228 | 0.2029 | 0.2108 | -887% | -822% | -849% |
| FFU | 0.1132 | 0.1015 | 0.1062 | -401% | -361% | -377% |
| FFU ($\psi = \psi^* = 0$) | 0.0382 | 0.0375 | 0.0378 | -69.31% | -170.54% | -70.04% |

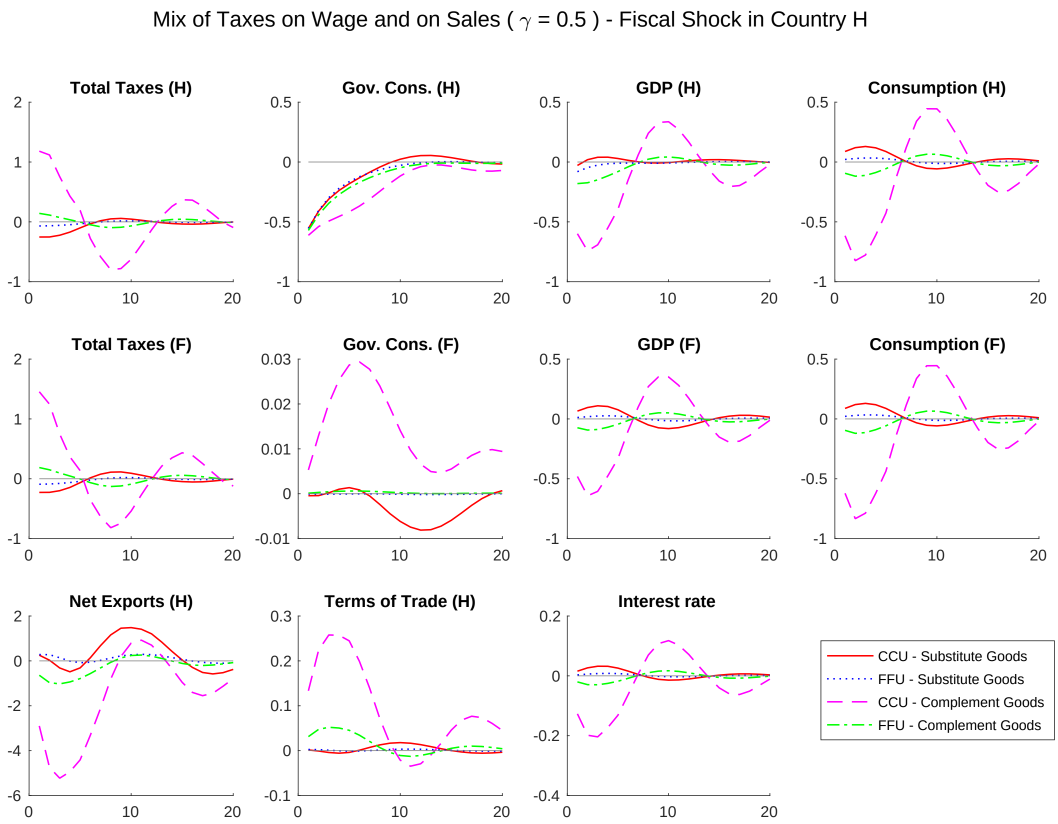
* Welfare Gains are computed as $\frac{Loss_b - Loss_a}{Loss_b}$, with $Loss_b$ the loss of the PCU with $\psi = \psi^* = 0$.

Since there is no trade off between price and output stabilization if the internationally traded goods are complements, fiscal policy is able to successfully stabilize the output gap. For these reasons, Table 2.6 reports that only in the Pure Currency Union scenario with countercyclical government consumption there is a gain in terms of an ad hoc loss function, compared to the case of exogenous government consumption. Since with a low international trade elasticity there is a trade off between output stabilization and international demand stabilization, targeting net exports has a cost in terms of both output and price stabilization for both countries, which is why there are welfare losses in most cases. Finally, consolidating budgets partially offsets the amplification effect due to the stabilization of net exports, which is why the Full Fiscal Union scenario, despite being less stabilizing than the Pure Currency Union scenario, has a lower cost in terms of the ad hoc loss function than the Coordinated Currency Union scenario, as can be observed in Table 2.6.

### 2.7.2 Fiscal Shocks and Fiscal Multipliers and Spillovers

Hjortsø (2012) argues that when goods are complements fiscal policy is more effective on consumption and output. In Figure 2.14 we document similar evidence for the case of a fiscal tightening in country $H$. Private consumption and output are more volatile, with the effect of the initial drop in the PPI amplified by the reduction in net exports for country $H$, due to the substitution effect. On the other hand, a negative fiscal shock in country $F$, shown in Figure 2.15, triggers a higher output in country $H$ due

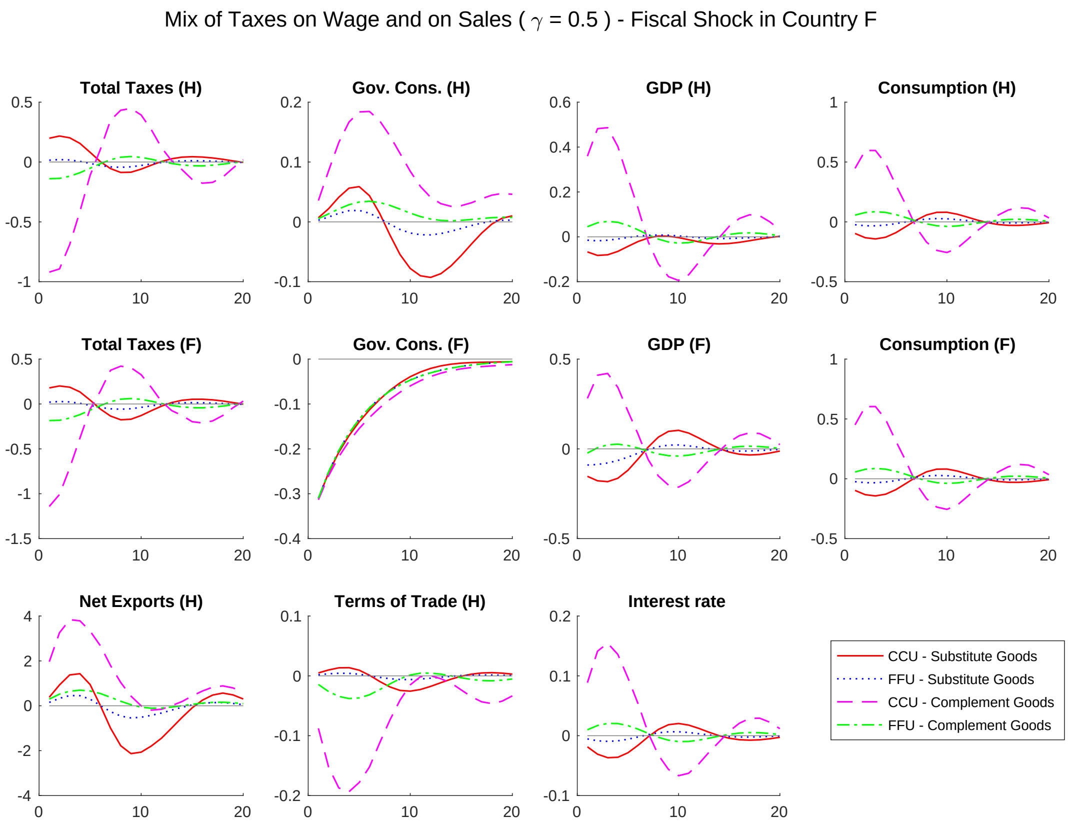
Figure 2.14: Substitutes vs Complements - Government Consumption - Fiscal Shock in Country H




to the increase in net exports. Consistently with the different reactions of net exports after a negative fiscal policy shock in countries $H$ and $F$, Figures 2.16 and 2.17 show that the sign of the fiscal policy multipliers do not only depend on the degree of fiscal policy coordination, but also on the international trade elasticity. The fiscal policy multiplier is positive for country $H$ with a high international trade elasticity. Moreover, also price rigidity plays a relevant role since it determines the sign of the substitution effect, while the international trade elasticity determines which dominates between the substitution effect and the income effect.

In Figures 2.14 and 2.15 we observe the opposite reactions of net exports after a fiscal policy shock, while from Figures 2.16 and 2.17 it is clear that the effect of a fiscal shock on output is the opposite if the internationally traded goods are complements rather than substitutes. Since this fact implies a positive fiscal policy multiplier for country $H$ when international goods are complements and a negative one when international goods are substitutes, while for country $F$ it is the contrary, we can infer that the fiscal policy multiplier depends strongly and non-linearly on the relative degree of openness to international trade and on the international trade elasticity. Indeed, together they determine the sign of the substitution effect and whether the substitution effect or the income effect dominates.

Figure 2.15: Substitutes vs Complements - Government Consumption - Fiscal Shock in Country F

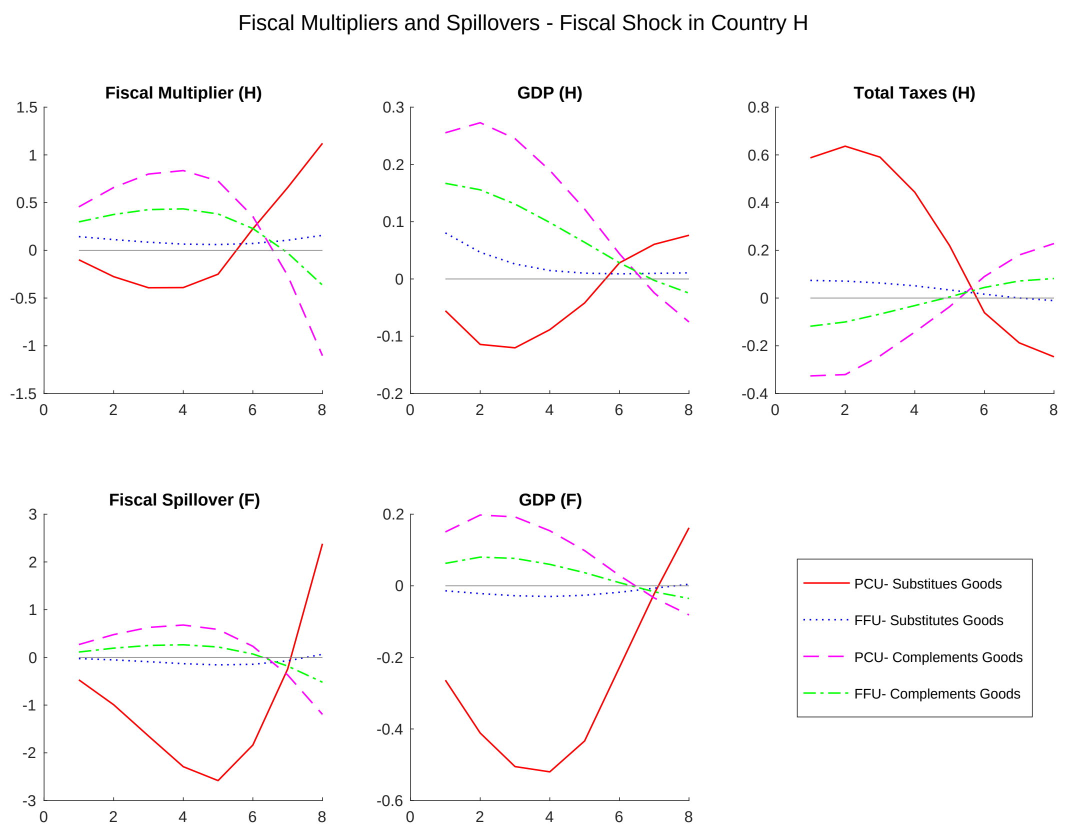



## 2.8 Conclusions and Possible Extensions

This paper provides a characterization of the stabilization properties of different fiscal policy scenarios in a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union. We consider three fiscal policy scenarios. In the Pure Currency Union scenario, each fiscal authority moves government spending countercyclically in response to deviations of real output from steady state. In the other two scenarios – Coordinated Currency Union and Full Fiscal Union respectively – each government targets procyclically a common variable, the net exports gap, but in the Full Fiscal Union scenario the two government budget constraints are consolidated and the tax rates in the two countries move together to finance the government expenditure. In all three scenarios, the fiscal authorities have access only to distortionary taxation and must balance the budget. The presence of distortionary taxation and price rigidity implies that fiscal policy and monetary policy are interconnected and produce real effects in the economy.

The main difference between our work and the previous literature is that we consider jointly the stabilization properties of targeting rules for fiscal policy and its financing with distortionary taxation, while most literature assumes stochastic government spending and lump-sum taxes, which imply very different dynamics. Based on the strength of the targeting rule, distortionary taxation might reverse the qualitative effects of fiscal policy on the economy, bringing to very different results.

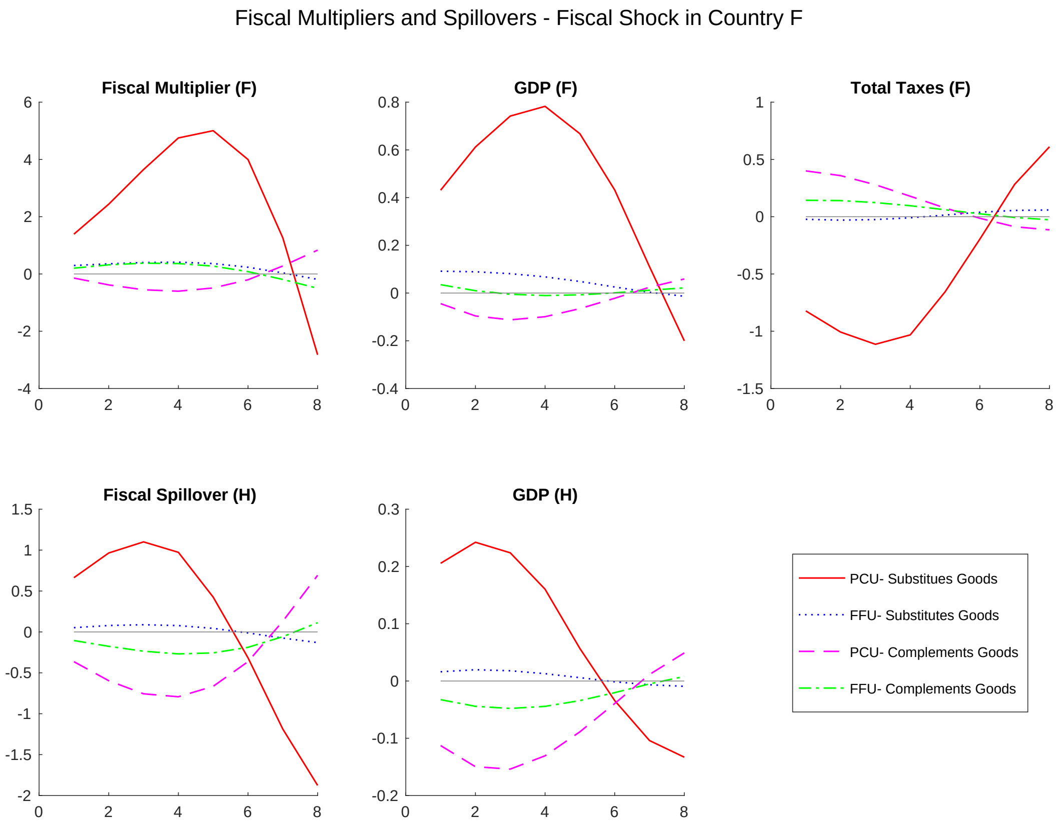
Figure 2.16: Substitutes vs Complements - Fiscal Multipliers and Spillovers in Country H




Our main result is that coordinating fiscal policy by targeting the net exports gap produces much more stabilization in the economy than targeting the output gap, which provides even less stabilization on output itself. By also consolidating budget constraints we attain the most stabilized dynamics between countries through the joint adjustment of the tax rates. Actually, even only consolidating budget constraints with stochastic government consumption produces about as much stabilization as only targeting the net exports gap without consolidating budget constraints, which implies that adding one dimension to the other barely produces more stabilization. This can be viewed as two instruments for stabilization that can be used jointly, while one can make up for the temporary lack of the other, as a sort of insurance mechanism.

Our analysis also highlights that taxes on labour income entail exponentially more distortionary effects than taxes on firm sales, which implies that the latter should be generally preferred to the former. This has a significant effect on the sign of fiscal policy multipliers and spillovers. A fiscal expansion generally implies a negative multiplier in the Core country ($H$) of our model, which is characterized by lower price rigidity and trade openness. On the contrary, the multiplier is positive in the Periphery country ($F$), where prices are stickier and a fiscal expansion induces a deterioration in the terms of trade, fostering bigger consumption and output, which then spills over to the other country through complete financial markets. This also implies that the signs of the fiscal spillovers on the other country are the same as the signs of the fiscal multipliers in the first country.

Figure 2.17: Substitutes vs Complements - Fiscal Multipliers and Spillovers in Country F




In a robustness check, we also show that if the internationally traded goods are considered as complements, instead of substitutes, most of the dynamic effects of the shocks on the economy are reversed, consistently with the previous literature. Compared to the case of substitutes, the Full Fiscal Union scenario improves substantially the stability of the economy with respect to the Coordinated Currency Union, which is actually destabilizing in the case of complements, compared to the Pure Currency Union scenario. Consolidating budget constraints allows to partially offset this destabilization produced by targeting the net exports gap. Although with international goods as complements the dynamics of the economy are reversed, our result that the Full Fiscal Union scenario is more stabilizing than the Coordinated Currency Union scenario still holds and to a greater extent compared to the case of substitutes.

Our model and results have nonetheless some shortcomings, which also entail possible future avenues of research. We assume complete international financial markets, which is a quite strong assumption, and more of an objective than a reality. Nonetheless, if fiscal policy has a cross-country insurance role with *complete* international financial markets, it must have an even stronger insurance role with *incomplete* international financial markets, where private cross-country insurance is no more available. We do not take into consideration explicitly the Zero Lower Bound on the nominal interest rate, which is an important feature of the recent global liquidity trap. In this case too, if fiscal policy affects the economy without constraints on monetary policy, with these constraints fiscal policy should be even more effective. Last but not least, we allow for positive government debt, but without allowing it to vary over time.

Allowing for government debt dynamics would modify the evolution of most variables in the economy and would influence the government budget and the effects of fiscal policy. Also constraining households' access to financial markets would amplify the effects of fiscal policy, as households' consumption would be much more dependent on current income rather than permanent income.

This research was conducted to assess the stabilization properties of fiscal policy coordination in the Eurozone and to bring to policy conclusions for the proper macroeconomic management of a Currency Union. The policy prescriptions are that countries in the Eurozone should reduce international demand imbalances by either stabilizing trade flows across countries or by creating some form of fiscal union with a common budget and taxation strategy, or both, while avoiding the excessive use of labour taxes, in favour of sales taxes. This would dampen all shocks hitting different countries of the Eurozone and would reduce the volatility of output and consumption in every country. A future Fiscal Union would also need a Eurozone treasury for collective decision-making on fiscal policy, which would need to be accountable and legitimated democratically, but this is out of the scope of the present paper.

# Chapter 3

# Government Debt Deleveraging in the EMU[1]

## 3.1 Introduction

*What are the effects of government debt deleveraging in a monetary union? What are the different effects of deleveraging in the face of alternative shocks and which is the best timing for deleveraging?*

Given a situation of high government debt in the EMU and a request to reduce government debt positions to 60% of GDP, finding the best way and timing for deleveraging is an important issue. Should governments deleverage no matter what the macroeconomic conditions are or should they deleverage in such a way to leave room for fiscal policy stabilization in bad times? As Ferrero (2009) shows, there are important welfare gains in adopting a flexible fiscal policy, because by constraining fiscal policy, deleveraging creates a trade-off between discipline and stabilization. The effects of government debt deleveraging in the face of alternative shocks are evaluated both with respect to timing and with respect to other macroeconomic factors.

We focus on government debt deleveraging with *incomplete* international financial markets and a debt-elastic government bond spread and show the effects of deleveraging in the face of supply (productivity) and demand (preference) shocks hitting a single country under alternative fiscal policy scenarios and compare uncoordinated fiscal policy (Pure Currency Union scenario), where government debt deleveraging is managed independently, with coordinated fiscal policy (Full Fiscal Union scenario), where debt is aggregated and managed at the union level with a consolidated budget. An intermediate case (Coordinated Currency Union scenario) will show the effects of coordinating fiscal policy without consolidating the budget.

We study the effects of different paths for dynamic government debt deleveraging and analyze the best timing for countries' deleveraging and the optimal taxation and spending combinations, taking into account the possibility of a Redemption Fund, as outlined in Van Rompuy et al. (2012). The possibility for countries to finance their debt at a lower interest rate, and all at the same interest rate, and the possibility for countries in dire circumstances to obtain a partial debt relief through transfers by a central EMU government may be welfare improving for the union as a whole, especially in the face of asymmetric shocks.

We follow the open economy approach of Galí (2009), in a two-country setting like in Silveira (2006), but without taking the limit case of a small open economy. Our model follows the specifications of

[1]This chapter is co-authored with Chiara Guerello, LUISS Guido Carli, and Guido Traficante, Università Europea di Roma.

Ferrero (2009), which adapts the optimal approach of Benigno and Woodford (2004) to monetary and fiscal policy in a cashless closed economy without capital, where there are only distortionary taxes as sources of government revenue, to a two-country open-economy Currency Union setting. Ferrero (2009) follows also the optimal monetary approach of Benigno (2004) in a Currency Union, by adding a fiscal authority and including a government budget constraint, but without money in the utility function and concentrating on the cashless limit case. Our model adds home bias in consumption (or a degree of openness to international trade) and government transfers to households to the model in Ferrero (2009). Following Hjortsø (2012) we add a stabilization rule for government consumption or alternatively for government transfers, but in a setting where distortionary taxes and government debt add the financing dimension to the fiscal policy problem. We also add a debt-elastic government bond spread in an incomplete financial market setting, to account for international financial frictions, also following Hjortsø (2012).

As in Ferrero (2009), this model is structured so as to allow for spillovers from monetary to fiscal policy and viceversa, and from one country to another through country-specific fiscal policies. Nominal rigidities, in the form of staggered price adjustments, generate real effects of monetary policy, while distortionary taxation generates non-Ricardian effects of fiscal policy. Monetary policy influences fiscal policy through its effect on the real value of debt, which depends on the interest rate and inflation. Fiscal policy influences monetary policy through its effect on prices and wages, which depend on the tax rates. This framework allows to study the interaction between country-specific fiscal policies, where in the absence of the nominal exchange rate as an automatic stabilizer, fiscal policies influence each other through their effects on output and the terms of trade. Incomplete financial markets with a government bond spread and government debt deleveraging dynamics provide additional frictions in the economy which can be addressed by monetary or fiscal authorities, or by an additional macro-prudential authority.

The EMU is represented by two countries of different size forming a Currency Union. Each country has an independent Fiscal Authority, while the Currency Union shares a common Monetary Authority. The EMU is inhabited by a continuum of Households of measure one, which can trade a complete set of one-period state-contingent claims only within their own country, implying that international financial markets are *incomplete*. Additionally, only one government's debt is tradable internationally and has a debt-elastic government bond spread. This assumption implies that the international trade of foreign bonds is subject to intermediation costs, following mainly De Paoli (2009), which in turn follows Turnovsky (1985). As explained in Ghironi (2006) and Schmitt-Grohé and Uribe (2003) this assumption ensures stationarity of net foreign assets and is an assumption also used in Eggertsson, Ferrero and Raffo (2014). The Central Bank sets the nominal interest rate for the whole Currency Union, which is the interest rate paid on the government bonds which are not traded internationally. Monetary policy sets the nominal interest rate following an Inflation Targeting regime, where the target is on union-wide CPI inflation. This assumption reflects the current functioning of the ECB, whose primary objective of price stability is formulated as a situation in which the one-year increase in the CPI for the Eurozone is less than, but close to, 2%. Fiscal policy is designed following the Fiscal Compact Rules, by imposing that the Government Debt-to-GDP ratio is about 60% in steady state and that countries must either adopt a balanced budget law in their national legislation or adopt the Debt Brake Rule, which implies that if government debt-to-GDP is more than 60% it should decrease by 5% of the excess every year. Following Hjortsø (2012), we add a stabilization rule for government consumption or alternatively for government transfers, but in a setting where distortionary taxes and government debt add the financing dimension to the fiscal policy problem, and we also add a government bond spread, which depends on government debt-to-GDP. Governments choose the level of government consumption and transfers, which are financed by distortionary taxes on labour income and firm sales and by short-term government bonds. In particular:

- In a Pure Currency Union scenario governments choose the level of government consumption or transfers for domestic stabilization purposes by setting them alternatively to target the output gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while using the other to deleverage government debt.

- In a Coordinated Currency Union scenario governments choose the level of local government consumption or transfers for international stabilization purposes by setting them to target the net exports gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while using the other to deleverage government debt.

- A Full Fiscal Union scenario uses a consolidated budget constraint to finance local government consumption or transfers for international stabilization purposes by setting them to target the net exports gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while using the other to deleverage government debt, just like in the Currency Union case, but varying equally the fiscal instruments across countries, so as to use union-wide resources to finance the government expenditure and the deleveraging.

Our main results are the following. First, with incomplete markets targeting the output gap produces more stabilization than targeting the net exports gap, while under deleveraging it makes hardly a difference because the deleveraging shock drives the dynamics much more than other shocks. Second, consolidating budget constraints may provide more stabilization, but the joint movement of the fiscal instruments does not in an incomplete market setting, which is even more true under deleveraging. These are the two results which are overturned in a setting with *complete* international financial markets. Third, taxes are a better instrument for deleveraging compared to government consumption or transfers, because they counteract the deflationary effect of the deleveraging shock much more. Fourth, by backloading the deleveraging process one can achieve greater stabilization over time, because the fiscal instruments move less initially, as they need to deleverage less at the beginning and more over time. The policy prescriptions for the Eurozone in this case are that government debt should be reduced more gradually, to avoid excessive volatility in the economy, and should be reduced by varying distortionary taxes, while countries should concentrate on reducing their output gap and avoid creating a fiscal union like the one we describe, until financial markets are integrated completely, allowing for perfect international risk-sharing, like in our previous complete markets model.

The remainder of the paper is structured as follows. Section 2 describes the general model differentiating between a Pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union with different deleveraging scenarios and instruments. Section 3 presents the calibration of the parameters and steady state stances of the model to two groups of countries in the European Monetary Union. Section 4 provides numerical simulations under different deleveraging scenarios, comparing Pure Currency Union, Coordinated Currency Union and Full Fiscal Union outcomes, other than alternative fiscal policy instruments for deleveraging and alternative deleveraging schemes. Section 5 describes a welfare measure based on an ad hoc loss function and provides welfare evaluations of the different deleveraging scenarios and instruments. Section 6 collects the main conclusions and provides possible extensions. The Mathematical Appendix provides all derivations, collects all equilibrium conditions of the model used for the simulations, and describes the steady state on which the model is calibrated.

## 3.2 A Two-Country Currency Union Model

The world economy is composed of two countries, which form a Currency Union. Both economies are assumed to share identical preferences, technology and market structure, but may be subject to different shocks, price rigidities, initial conditions and fiscal stances. The two countries are indexed by $H$ and $F$ for *Home* and *Foreign*. The world is populated by a continuum of infinitely-lived households of measure one, indexed by $i \in [0,1]$. Each household owns a monopolistically competitive firm producing a differentiated good, indexed by $j \in [0,1]$. The population on the segment $[0,h)$ belongs to country $H$ while the population on the segment $[h,1]$ belongs to country $F$. This means that the relative size of country $H$ is $h \in [0,1]$, while the relative size of country $F$ is $1-h$. This is true for both households and firms.

Firms set prices in a staggered fashion following Calvo (1983) and use only labour for production. There is no capital and no investment. Labour markets are competitive and internationally segmented, so that labour supply is country-wide and not firm-specific. All goods are tradable and the Law of One Price (LOP) holds for all single goods $j$. At the same time deviations from Purchasing Power Parity (PPP) may arise because of home bias in consumption. Financial markets are incomplete internationally, allowing households to trade a full set of one-period state-contingent claims only within their own country. Households in country $H$ can purchase one-period bonds issued by both of the two countries' governments, while households in country $F$ can only purchase one-period bonds issued by their own country's government.

The world economy comprises also two Governments or Fiscal Authorities (one for each country), each one choosing government consumption and transfers to stabilize the economy, financed by distortionary taxes on labour income and firm sales and by short-term government bonds, and a Central Bank or Monetary Authority (one for the whole EMU), which sets the nominal interest rate targeting union-wide inflation.

We denote variables referred to the *Foreign* country with a star ($^*$).

### 3.2.1 Households

In each country there is a continuum of households, which gain utility from private consumption and disutility from labour, consume goods produced in both countries with home bias, supply labour to domestic firms, and collect profits from those firms. Households can trade a complete set of one-period state-contingent claims only within their own country. Households in country $H$ can purchase one-period bonds issued by both countries' governments, while households in country $F$ can only purchase one-period bonds issued by their own country's government, subject to their budget constraints.

Each household in country $H$, indexed by $i \in [0,h)$, and each household in country $F$, indexed by $i \in [h,1]$, seeks to maximize respectively the present-value utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \, U(C_t^i, N_t^i) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \xi_t^* \, U(C_t^{*i}, N_t^{*i}) \tag{3.1}$$

where $\beta \in [0,1]$ is the common discount factor, which households use to discount future utility, $\xi_t$ is a preference shock to *Home* households and $\xi_t^*$ is a preference shock to *Foreign* households. These preference shocks are assumed to follow the AR(1) processes in logs:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \qquad \text{and} \qquad \xi_t^* = (\xi_{t-1}^*)^{\rho_\xi^*} e^{\varepsilon_t} \tag{3.2}$$

where $\rho_\xi \in [0,1]$ and $\rho_\xi^* \in [0,1]$ are measures of persistence of the shocks and $\varepsilon_t$ is a zero mean white noise process. $N_t^i$ denotes hours of labour supplied by households in country $H$, $N_t^{*i}$ denotes hours of labour supplied by households in country $F$. $C_t^i$ is a composite index for private consumption defined by:

$$C_t^i \equiv \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t}^i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3.3}$$

for households in country $H$, while the analogous index for households in country $F$, $C_t^{*i}$, is defined by:

$$C_t^{*i} \equiv \left[(1-\alpha^*)^{\frac{1}{\eta}}(C_{H,t}^{*i})^{\frac{\eta-1}{\eta}} + \alpha^{*\frac{1}{\eta}}(C_{F,t}^{*i})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3.4}$$

where $C_{H,t}^i$ is an index of consumption of domestic goods for households in country $H$, given by the constant elasticity of substitution (CES) function (also known as Dixit and Stiglitz (1977) aggregator function):

$$C_{H,t}^i \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.5}$$

whereas, for households in country $F$ the same index of consumption of domestic goods, $C_{H,t}^{*i}$, is given by:

$$C_{H,t}^{*i} \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{H,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.6}$$

where $j \in [0,1]$ denotes a single good variety of the continuum of differentiated goods produced in the world economy. $C_{F,t}^i$ is an index of imported goods for households in country $H$, given by the analogous CES function:

$$C_{F,t}^i \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.7}$$

while the same index for imported goods for households in country $F$, $C_{F,t}^{*i}$, is given by:

$$C_{F,t}^{*i} \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{F,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.8}$$

The parameter $\varepsilon > 1$ measures the elasticity of substitution between varieties produced within a given country. The parameter $\eta > 0$ measures the substitutability between domestic and foreign goods (international trade elasticity). The parameter $\alpha \in [0,1]$ is a measure of openness of the *Home* economy to international trade. Equivalently $(1-\alpha)$ is a measure of the degree of home bias in consumption in country $H$. When $\alpha$ tends to zero the share of foreign goods in domestic consumption vanishes and the country ends up in autarky, consuming only domestic goods. If $1-\alpha > h$ there is home bias in consumption in country $H$, because the share of consumption of domestic goods is greater than the share of production of domestic goods. The same applies to the *Foreign* parameter of openness to international trade $\alpha^* \in [0,1]$ for country $F$, except for the fact that if $1-\alpha^* > 1-h$ there is home bias in consumption in country $F$, because the share of consumption of domestic goods in country $F$ is greater than the share of production of domestic goods in country $F$.

We choose to specify additively separable period utility of the type with Constant Relative Risk Aversion (CRRA), so with constant elasticity of intertemporal substitution, and with constant elasticity of labour supply, which take the following form:

$$U(C_t^i, N_t^i) \equiv \frac{(C_t^i)^{1-\sigma}-1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \qquad U(C_t^{*i}, N_t^{*i}) \equiv \frac{(C_t^{*i})^{1-\sigma}-1}{1-\sigma} - \frac{(N_t^{*i})^{1+\varphi}}{1+\varphi} \tag{3.9}$$

for countries $H$ and $F$ respectively, where $\sigma$ is the inverse of the elasticity of intertemporal substitution (it is also the Coefficient of Relative Risk Aversion (CRRA)), and $\varphi$ is the inverse of the Frisch elasticity of labour supply.

Households in country $H$ maximize their present-value utility, equation 3.1, subject to the following sequence of budget constraints:

$$\begin{aligned}&\int_0^h P_{H,t}(j)C^i_{H,t}(j)\,dj + \int_h^1 P_{F,t}(j)C^i_{F,t}(j)\,dj + D^i_t + B^i_{H,t} + B^i_{F,t}\\ &\quad \leq \frac{D^i_{t-1}}{\mathcal{Q}_{t-1,t}} + B^i_{H,t-1}(1+i_{t-1}) + B^i_{F,t-1}(1+i^*_{t-1})(1-\delta_{t-1}) + (1-\tau^w_t)W_tN^i_t + T^i_t + \Gamma^i_t + \mathcal{I}^{*i}_t \end{aligned} \tag{3.10}$$

for $t = 0, 1, 2, \ldots,$ where $P_{H,t}(j)$ is the price of domestic variety $j$, $P_{F,t}(j)$ is the price of variety $j$ imported from country $F$, $D^i_{t-1}$ is the portfolio of state-contingent claims purchased by the household in period $t-1$, $\mathcal{Q}_{t-1,t}$ is the stochastic discount factor for households in country $H$, which is different for households in the two countries and represents the price of state-contingent claims or equivalently the inverse of the gross return on state-contingent claims, $B^i_{H,t}$ are government bonds issued by country $H$ and purchased by the household in period $t$, $i_{t-1}$ is the nominal interest rate set by the central bank in period $t-1$, which is also the net return on government bonds issued by country $H$, $W_t$ is the nominal wage for households in country $H$, $T^i_t$ denotes lump-sum transfers from the government to households, $\Gamma^i_t$ denotes the share of profits net of taxes to households from ownership of firms, $\mathcal{I}^i_t$ denotes the share of profits to households in country $H$ from ownership of the financial intermediaries and $\tau^w_t \in [0,1]$ is a marginal tax rate on labour income paid by households to the government.

The only financial assets traded internationally are given by $B^i_{F,t}$, which are government bonds issued by country $F$ and purchased by households in country $H$ in period $t$, $i^*_{t-1}$ is the nominal interest rate for country $F$ in period $t-1$, which is also the net return on government bonds issued by country $F$, while $\delta_t \in [0,1]$ is a transaction cost for households in country $H$ on purchases of government bonds issued by country $F$, given by the AR(1) process:

$$\delta_t \equiv (1-\rho_\delta)\delta^B\left(\frac{B^{*G}_{t-1}}{P^*_{H,t-1}Y^*_{t-1}} - \frac{B^{*G}}{P^*_HY^*}\right) + \rho_\delta\delta_{t-1} + \varepsilon_t \tag{3.11}$$

where $\frac{B^{*G}_{t-1}}{P^*_{H,t-1}Y^*_{t-1}}$ is the overall real government debt-to-GDP for country $F$ in period $t-1$, variables without subscript $t$ are their respective steady state values, $\rho_\delta \in [0,1]$ is a measure of persistence of the government bond spread shock and $\varepsilon_t$ is a zero-mean white noise process. The previous equation shows how the transaction cost for households in country $H$ on purchases of government bonds issued by country $F$, or the government bond spread given by $(1+i^*_t)\delta_t$, is increasing in the deviation of government debt-to-GDP from steady state by a factor given by $\delta^B$, which represents the sensitivity of the government bond spread or of the transaction cost to deviations of government debt-to-GDP from steady state.

The financial intermediaries, owned by the households in country $H$, earn profits on all the internationally traded bonds $B^i_{F,t-1}$ by collecting savings from households in country $H$ at the interest rate set by the central bank $i_{t-1}$ and lending to the government in country $F$ at the interest rate paid on its government bonds $i^*_{t-1}$. The aggregate profits of these financial intermediaries are given by:

$$\mathcal{I}_t \equiv \int_0^h \mathcal{I}^i_t\,di \equiv h\mathcal{I}^i_t \equiv B_{F,t-1}\left[(1+i^*_{t-1}) - (1+i^*_{t-1})(1-\delta_{t-1})\right] = B_{F,t-1}(1+i^*_{t-1})\delta_{t-1} \tag{3.12}$$

where $B_{F,t-1} \equiv \int_0^h B^i_{F,t-1}\,di \equiv hB^i_{F,t-1}$ are aggregate bonds issued by the government in country $F$ and purchased by households in country $H$ and where the government bond spread for country $F$, on which financial intermediaries make profits, is given by $(1+i^*_{t-1})\delta_{t-1}$.

Households in country $F$ analogously maximize their present-value utility, equation 3.1, subject to the following sequence of budget constraints:

$$\int_0^h P^*_{H,t}(j)C^{*i}_{H,t}(j)\,dj + \int_h^1 P^*_{F,t}(j)C^{*i}_{F,t}(j)\,dj + D^{*i}_t + B^{*i}_{F,t} \\ \leq \frac{D^{*i}_{t-1}}{\mathcal{Q}^*_{t-1,t}} + B^{*i}_{F,t-1}(1+i^*_{t-1}) + (1-\tau^{*w}_t)W^*_t N^{*i}_t + T^{*i}_t + \Gamma^{*i}_t \quad (3.13)$$

for $t = 0, 1, 2, \ldots,$ where all the starred $(^*)$ variables are the *Foreign* equivalent of the unstarred ones explained above, so that $\mathcal{Q}^*_{t-1,t}$ is the stochastic discount factor for households in country $F$ and $B^{*i}_{F,t}$ are government bonds issued by country $F$ and purchased by the household in country $F$ in period $t$. As shown by the budget constraint, households in country $F$ can only purchase government bonds issued by their own country and, differently from households in country $H$, they do not pay any transaction costs on the purchase of these government bonds.

All variables are expressed in units of the union's currency. Last but not least, households in countries $H$ and $F$ respectively, are subject to the following solvency constraints, for all $t$, that prevent them from engaging in Ponzi-schemes:

$$\lim_{T\to\infty} E_t\left\{\mathcal{Q}_{t,T}D^i_T\right\} \geq 0 \qquad \lim_{T\to\infty} E_t\left\{\mathcal{Q}^*_{t,T}D^{*i}_T\right\} \geq 0 \quad (3.14)$$

The first order conditions for the household's problem in country $H$ are[2]:

$$(C^i_t)^\sigma (N^i_t)^\varphi = (1-\tau^w_t)\frac{W_t}{P_t} \quad (3.15)$$

which is the household's intratemporal optimality condition, and:

$$\mathcal{Q}_{t,t+1} = \beta\frac{\xi_{t+1}}{\xi_t}\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}} \quad (3.16)$$

which is the household's intertemporal optimality condition or Euler Equation, that holds for all states of nature at $t$ and $t+1$, and:

$$\frac{1}{(1+i^*_t)(1-\delta_t)} = \frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} \quad (3.17)$$

which is the no-arbitrage condition between government bonds issued by both countries and state-contingent claims.

Aggregating the intratemporal optimality condition, equation 3.15, yields the aggregate labour supply equation for households in country $H$:

$$N_t = (h)^{1+\frac{\sigma}{\varphi}}(C_t)^{-\frac{\sigma}{\varphi}}\left[(1-\tau^w_t)\frac{W_t}{P_t}\right]^{\frac{1}{\varphi}} \quad (3.18)$$

where $N_t$ is aggregate labour supply and $C_t$ is aggregate consumption for households in country $H$, defined by:

$$N_t \equiv \int_0^h N^i_t\,di = hN^i_t \qquad C_t \equiv \int_0^h C^i_t\,di = hC^i_t \quad (3.19)$$

[2]See Appendix B.1 for all derivations for this section.

while aggregating the intertemporal optimality condition for households in country $H$, equation 3.16, taking conditional expectations on both sides of the equation and using the no-arbitrage condition between government bonds and state-contingent claims yields:

$$\frac{1}{(1+i_t^*)(1-\delta_t)} = \frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t\left\{\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\Pi_{t+1}}\right\} \tag{3.20}$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period government bond issued by country $H$ paying off one unit of the union's currency in $t+1$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross CPI inflation in country $H$.

Specular equations for country $F$ can be obtained following the same procedure, where the results are similar, but with all the relevant starred ($^*$) variables instead of the unstarred ones.

The first order conditions for the household's problem in country $F$ are:

$$(C_t^{*i})^{\sigma}(N_t^{*i})^{\varphi} = (1-\tau_t^{*w})\frac{W_t^*}{P_t^*} \tag{3.21}$$

which is the household's intratemporal optimality condition, and:

$$\mathcal{Q}_{t,t+1}^* = \beta\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^{*i}}{C_t^{*i}}\right)^{-\sigma}\frac{P_t^*}{P_{t+1}^*} \tag{3.22}$$

which is the household's intertemporal optimality condition or Euler Equation, that holds for all states of nature at $t$ and $t+1$, and:

$$\frac{1}{1+i_t^*} = E_t\{\mathcal{Q}_{t,t+1}^*\} \tag{3.23}$$

which is the no-arbitrage condition between government bonds issued by country $F$ and state-contingent claims.

The aggregate labour supply equation for households in country $F$ is given by:

$$N_t^* = (1-h)^{1+\frac{\sigma}{\varphi}}(C_t^*)^{-\frac{\sigma}{\varphi}}\left[(1-\tau_t^{*w})\frac{W_t^*}{P_t^*}\right]^{\frac{1}{\varphi}} \tag{3.24}$$

where $N_t^*$ is aggregate labour supply and $C_t^*$ is aggregate consumption for households in country $F$, defined by:

$$N_t^* \equiv \int_h^1 N_t^{*i}\,di = (1-h)N_t^{*i} \qquad C_t^* \equiv \int_h^1 C_t^{*i}\,di = (1-h)C_t^{*i} \tag{3.25}$$

while taking conditional expectations on both sides of the aggregate intertemporal optimality condition and using the no-arbitrage condition between government bonds and state-contingent claims yields:

$$\frac{1}{1+i_t^*} = E_t\{\mathcal{Q}_{t,t+1}^*\} = \beta E_t\left\{\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{1}{\Pi_{t+1}^*}\right\} \tag{3.26}$$

where $\frac{1}{1+i_t^*} = E_t\{\mathcal{Q}_{t,t+1}^*\}$ is the price of a one-period government bond issued by country $F$ paying off one unit of the union's currency in $t+1$ and $\Pi_{t+1}^* \equiv \frac{P_{t+1}^*}{P_t^*}$ is gross CPI inflation in country $F$.

Aggregating the budget constraints of households in countries $H$ and $F$ respectively and considering that in optimality they hold with equality yields:

$$P_tC_t + D_t + B_{H,t} + B_{F,t} = (1+i_{t-1})(D_{t-1} + B_{H,t-1} + B_{F,t-1}) + (1-\tau_t^w)W_tN_t + T_t + \Gamma_t + \mathcal{I}_t \tag{3.27}$$

$$P_t^* C_t^* + D_t^* + B_{F,t}^* = \frac{1 + i_{t-1}}{1 - \delta_{t-1}} (D_{t-1}^* + B_{F,t-1}^*) + (1 - \tau_t^{*w}) W_t^* N_t^* + T_t^* + \Gamma_t^* \tag{3.28}$$

where aggregate contingent claims and aggregate bonds are respectively defined as:

$$D_t \equiv \int_0^h D_t^i \, di = h D_t^i \qquad D_t^* \equiv \int_h^1 D_t^{*i} \, di = (1-h) D_t^{*i} \tag{3.29}$$

$$B_{H,t} \equiv \int_0^h B_{H,t}^i \, di = h B_{H,t}^i \tag{3.30}$$

$$B_{F,t} \equiv \int_0^h B_{F,t}^i \, di = h B_{F,t}^i \qquad B_{F,t}^* \equiv \int_h^1 B_{F,t}^{*i} \, di = (1-h) B_{F,t}^{*i} \tag{3.31}$$

while aggregate transfers and profits are respectively defined as:

$$T_t \equiv \int_0^h T_t^i \, di = h T_t^i \qquad T_t^* \equiv \int_h^1 T_t^{*i} \, di = (1-h) T_t^{*i} \tag{3.32}$$

$$\Gamma_t \equiv \int_0^h \Gamma_t^i \, di = h \Gamma_t^i \qquad \Gamma_t^* \equiv \int_h^1 \Gamma_t^{*i} \, di = (1-h) \Gamma_t^{*i} \tag{3.33}$$

$$\mathcal{I}_t \equiv \int_0^h \mathcal{I}_t^i \, di = h \mathcal{I}_t^i \tag{3.34}$$

and aggregate consumption and labour supply are as previously defined.

$P_t$ is the Consumer Price Index (CPI) for country $H$, given by:

$$P_t \equiv \left[ (1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{3.35}$$

while $P_t^*$ is the Consumer Price Index (CPI) for country $F$, given by:

$$P_t^* \equiv \left[ (1-\alpha^*)(P_{H,t}^*)^{1-\eta} + \alpha^* (P_{F,t}^*)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{3.36}$$

where $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country $H$ and $P_{F,t}$ is a price index for goods imported from country $F$, respectively defined by:

$$P_{H,t} \equiv \left( \frac{1}{h} \int_0^h P_{H,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{3.37}$$

$$P_{F,t} \equiv \left( \frac{1}{1-h} \int_h^1 P_{F,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{3.38}$$

while $P_{H,t}^*$ is the domestic price index or Producer Price Index (PPI) in country $F$ and $P_{F,t}^*$ is a price index for goods imported from country $H$, respectively defined by:

$$P_{H,t}^* \equiv \left( \frac{1}{1-h} \int_h^1 P_{H,t}^*(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{3.39}$$

$$P_{F,t}^* \equiv \left( \frac{1}{h} \int_0^h P_{F,t}^*(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{3.40}$$

Since one-period state-contingent claims can be traded freely between households only within each country, they are in zero national net supply, so that the market clearing conditions for these assets in every period $t$ are consequently given by:

$$\int_0^h D_t^i\, di = hD_t^i = D_t = 0 \qquad \int_h^1 D_t^{*i}\, di = (1-h)D_t^{*i} = D_t^* = 0 \tag{3.41}$$

Rewriting the aggregate budget constraints by imposing the previous market clearing conditions yields:

$$P_t C_t + B_{H,t} + B_{F,t} = (1+i_{t-1})(B_{H,t-1} + B_{F,t-1}) + (1-\tau_t^w)W_t N_t + T_t + \Gamma_t + \mathcal{I}_t \tag{3.42}$$

$$P_t^* C_t^* + B_{F,t}^* = \frac{1+i_{t-1}}{1-\delta_{t-1}} B_{F,t-1}^* + (1-\tau_t^{*w})W_t^* N_t^* + T_t^* + \Gamma_t^* \tag{3.43}$$

### 3.2.2 International Identities and Assumptions

Several international identities and assumptions need to be spelled out in order to link the *Home* economy to the *Foreign* one and to be able to close the model.

The *terms of trade* are defined as the price of foreign goods in terms of home goods, for households in country $H$ and in country $F$, and are given respectively by:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} \qquad \text{and} \qquad \mathcal{S}_t^* \equiv \frac{P_{F,t}^*}{P_{H,t}^*} \tag{3.44}$$

Although deviations from *Purchasing Power Parity (PPP)* may arise because of home bias in consumption, we assume that the *Law of One Price (LOP)* holds for every single good $j$, which implies:

$$P_{H,t}(j) = P_{F,t}^*(j) \qquad \text{and} \qquad P_{F,t}(j) = P_{H,t}^*(j) \tag{3.45}$$

for all $j \in [0,1]$, where $P_{H,t}(j)$ (or $P_{F,t}(j)$ for goods imported from country $F$) is the price of good $j$ in country $H$ and $P_{F,t}^*(j)$ (or $P_{H,t}^*(j)$ for goods produced in country $F$) is the price of good $j$ in country $F$ in terms of the union's currency. Plugging the previous expressions into the definitions of $P_{H,t}$ and $P_{F,t}$, equations 3.37 and 3.38, respectively yields:

$$P_{H,t} = P_{F,t}^* \qquad \text{and} \qquad P_{F,t} = P_{H,t}^* \tag{3.46}$$

Combining the previous result with the definition of the terms of trade for countries $H$ and $F$ yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P_{H,t}^*}{P_{F,t}^*} = \frac{1}{\mathcal{S}_t^*} \tag{3.47}$$

Dividing the CPIs in countries $H$ and $F$ by the relative price indices for domestic and imported goods and combining them with the definition of the terms of trade yields the following relationships:

$$\frac{P_t}{P_{H,t}} = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{3.48}$$

$$\frac{P_t}{P_{F,t}} = \frac{\left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\mathcal{S}_t} \tag{3.49}$$

$$\frac{P_t^*}{P_{H,t}^*} = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t^*)^{1-\eta}\right]^{\frac{1}{1-\eta}} = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{3.50}$$

$$\frac{P_t^*}{P_{F,t}^*} = \frac{\left[1-\alpha^* + \alpha^*(\mathcal{S}_t^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\mathcal{S}_t^*} = \mathcal{S}_t\left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{3.51}$$

Computing the first price ratio for periods $t$ and $t-1$ and dividing one by the other yields:

$$\frac{P_t}{P_{H,t}}\frac{P_{H,t-1}}{P_{t-1}} = \frac{\left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\left[1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}\right]^{\frac{1}{1-\eta}}} \tag{3.52}$$

which rearranged yields a relationship between PPI inflation and CPI inflation in country $H$:

$$\Pi_t = \Pi_{H,t}\left[\frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{3.53}$$

The same relationship can be derived for country $F$ and will be given by:

$$\Pi_t^* = \Pi_{H,t}^*\left[\frac{1-\alpha^*+\alpha^*\mathcal{S}_t^{\eta-1}}{1-\alpha^*+\alpha^*(\mathcal{S}_{t-1})^{\eta-1}}\right]^{\frac{1}{1-\eta}} \tag{3.54}$$

Dividing the terms of trade in period $t$ by the terms of trade in period $t-1$ yields a relationship showing the evolution of the terms of trade over time:

$$\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} = \frac{\Pi_{H,t}^*}{\Pi_{H,t}} \implies \mathcal{S}_t = \frac{\Pi_{H,t}^*}{\Pi_{H,t}}\mathcal{S}_{t-1} \tag{3.55}$$

as a function of PPI inflation in both countries $H$ and $F$.

The *Real Exchange Rate* between the *Home* country and country $F$ is the ratio of the two countries' CPIs, expressed both in terms of the union's currency, and is defined by:

$$Q_t \equiv \frac{P_t^*}{P_t} \tag{3.56}$$

Combining the previous results with the definition of the real exchange rate yields a relationship between the real exchange rate and the terms of trade:

$$Q_t = \frac{P_t^*}{P_{F,t}^*}\frac{P_{H,t}}{P_t} = \frac{\left[1-\alpha^*+\alpha^*(\mathcal{S}_t^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\mathcal{S}_t^*\left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}} = \mathcal{S}_t\left[\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{3.57}$$

where the difference between the real exchange rate and the terms of trade is given by the degree of openness of the two countries and the international trade elasticity. If the countries both have complete home bias ($\alpha = \alpha^* = 0$), then they are in autarky and the real exchange rate is exactly equal to the terms of trade, because the CPI and PPI are the same in each country.

Since state-contingent claims cannot be traded internationally there is no full international risk-sharing, but the only assets traded internationally, the bonds issued by the government in country $F$, yield an equation linking the interest rates in the two countries, through the transaction cost or government bond spread, given by:

$$1+i_t = (1+i_t^*)(1-\delta_t) \implies 1+i_t^* = \frac{1+i_t}{1-\delta_t} \tag{3.58}$$

which implies that the interest rate paid on government bonds issued by country $F$ is increasing in the transaction cost $\delta_t$, or in the government bond spread $(1+i_t^*)\delta_t$, other than increasing in the interest rate set by the central bank and paid on government bonds issued by country $H$, $i_t$.

### 3.2.3 Firms

In country $H$ there is a continuum of firms indexed by $j \in [0,h)$, while in country $F$ there is a continuum of firms, indexed by $j \in [h,1]$, each producing a differentiated good with the same technology represented respectively by the following production functions:

$$Y_t(j) = A_t N_t(j) \qquad Y_t^*(j) = A_t^* N_t^*(j) \tag{3.59}$$

where $A_t$ and $A_t^*$ represent the level of technology in countries $H$ and $F$, respectively, which evolve exogenously over time following the AR(1) processes in logs:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \qquad \text{and} \qquad A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \tag{3.60}$$

where $\rho_a \in [0,1]$ and $\rho_a^* \in [0,1]$ are measures of persistence of the shocks and $\varepsilon_t$ is a zero mean white noise process.

From the production functions we can derive labour demand for individual firms in countries $H$ and $F$ and the respective nominal and real marginal costs of production, which are equal across firms in each country and are given by:

$$N_t(j) = \frac{Y_t(j)}{A_t} \implies MC_t^n = \frac{W_t}{A_t} \implies MC_t = \frac{W_t}{A_t P_{H,t}} \tag{3.61}$$

$$N_t^*(j) = \frac{Y_t^*(j)}{A_t^*} \implies MC_t^{*n} = \frac{W_t^*}{A_t^*} \implies MC_t^* = \frac{W_t^*}{A_t^* P_{H,t}^*} \tag{3.62}$$

Aggregating individual labour demand across firms in each country yields the aggregate labour demand for countries $H$ and $F$, respectively given by:

$$N_t \equiv \int_0^h N_t(j)\,dj = \int_0^h \frac{Y_t(j)}{A_t}\,dj = \frac{Y_t}{A_t}\int_0^h \frac{1}{h}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj = \frac{Y_t}{A_t} d_t \tag{3.63}$$

$$N_t^* \equiv \int_h^1 N_t^*(j)\,dj = \int_h^1 \frac{Y_t^*(j)}{A_t^*}\,dj = \frac{Y_t^*}{A_t^*}\int_h^1 \frac{1}{1-h}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} dj = \frac{Y_t^*}{A_t^*} d_t^* \tag{3.64}$$

where $Y_t$ and $Y_t^*$ are aggregate output in countries $H$ and $F$, respectively given by:

$$Y_t \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y_t^* \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.65}$$

and where the terms:

$$d_t \equiv \int_0^h \frac{1}{h}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj \qquad \text{and} \qquad d_t^* \equiv \int_h^1 \frac{1}{1-h}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} dj \tag{3.66}$$

represent relative price dispersion across firms in each country. In steady state and in a flexible price equilibrium these relative price dispersions are equal to one.

Firm $j$'s period $t$ profits net of taxes in country $H$ are given by:

$$\Gamma_t(j) = (1-\tau_t^s) P_{H,t}(j) Y_t(j) - W_t N_t(j) \tag{3.67}$$

where $\tau_t^s$ is the marginal tax rate on firm sales in country $H$. Substituting in labour demand, marginal costs, substituting the output of firm $j$ with the demand function for output, integrating over all $j \in [0, h)$, using the definition of $P_{H,t}$, and substituting in price dispersion yields aggregate profits net of taxes in country $H$:

$$\Gamma_t \equiv \int_0^h \Gamma_t(j)\,dj = (1-\tau_t^s)P_{H,t}Y_t - P_{H,t}MC_tY_td_t = P_{H,t}Y_t(1-\tau_t^s - MC_td_t) \tag{3.68}$$

Analogous results can be obtained for aggregate profits net of taxes in country $F$:

$$\Gamma_t^* \equiv \int_h^1 \Gamma_t^*(j)\,dj = (1-\tau_t^{*s})P_{H,t}^*Y_t^* - P_{H,t}^*MC_t^*Y_t^*d_t^* = P_{H,t}^*Y_t^*(1-\tau_t^{*s} - MC_t^*d_t^*) \tag{3.69}$$

where $\tau_t^{*s}$ is the marginal tax rate on firm sales in country $F$, and where firm $j$'s period $t$ profits net of taxes in country $F$ are given by:

$$\Gamma_t^*(j) = (1-\tau_t^{*s})P_{H,t}^*(j)Y_t^*(j) - W_t^*N_t^*(j) = Y_t^*(j)[(1-\tau_t^{*s})P_{H,t}(j) - MC_t^{*n}] \tag{3.70}$$

Here we can see how aggregate profits are a function of the PPI, aggregate output, marginal cost and price dispersion for both countries $H$ and $F$, other than the tax rate on firm sales, while nominal aggregate labour income is shown to be a function of some of the same variables and equal to:

$$W_tN_t = W_t\frac{Y_t}{A_t}d_t = MC_t^nY_td_t = P_{H,t}Y_tMC_td_t \tag{3.71}$$

in country $H$, and:

$$W_t^*N_t^* = W_t^*\frac{Y_t^*}{A_t^*}d_t^* = MC_t^{*n}Y_t^*d_t^* = P_{H,t}^*Y_t^*MC_t^*d_t^* \tag{3.72}$$

in country $F$.

Following Calvo (1983), each firm in country $H$ may reset its price with probability $1-\theta$ in any given period. Thus, each period a fraction $1-\theta$ of randomly selected firms reset their price, while a fraction $\theta$ keep their prices unchanged. As a result, the average duration of a price in country $H$ is given by $(1-\theta)^{-1}$, and $\theta$ can be seen as a natural index of price stickiness for country $H$. In country $F$ each firm may reset its price with probability $1-\theta^*$ in any given period. Thus, each period a fraction $1-\theta^*$ of randomly selected firms reset their price, while a fraction $\theta^*$ keep their prices unchanged. As a result, the average duration of a price in country $F$ is given by $(1-\theta^*)^{-1}$, and $\theta^*$ can be seen as a natural index of price stickiness for country $F$. This allows for the two countries to have different degrees of price rigidity.

A firm in country $H$ re-optimizing in period $t$ will choose the price $\bar{P}_{H,t}$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the problem:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t\left\{\mathcal{Q}_{t,t+k}Y_{t+k|t}(j)\left[(1-\tau_{t+k}^s)\bar{P}_{H,t} - MC_{t+k}^n\right]\right\} \tag{3.73}$$

subject to the sequence of demand constraints:

$$Y_{t+k|t}(j) = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}\frac{Y_{t+k}}{h} \tag{3.74}$$

for $k = 0, 1, 2, \ldots$, where $\mathcal{Q}_{t,t+k}$ is the households' stochastic discount factor in country $H$ for discounting

$k$-period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{\xi_{t+k}}{\xi_t} \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \tag{3.75}$$

for $k = 0, 1, 2, \ldots$, and where $Y_{t+k|t}(j)$ is the output in period $t + k$ for firm $j$ which last reset its price in period $t$.

Analogously, a firm in country $F$ re-optimizing in period $t$ will choose the price $\bar{P}^*_{H,t}$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the problem:

$$\max_{\bar{P}^*_{H,t}} \sum_{k=0}^{\infty} \theta^{*k} E_t \left\{ \mathcal{Q}^*_{t,t+k} Y^*_{t+k|t}(j) \left[ (1 - \tau^{*s}_{t+k}) \bar{P}^*_{H,t} - MC^{*n}_{t+k} \right] \right\} \tag{3.76}$$

subject to the sequence of demand constraints:

$$Y^*_{t+k|t}(j) = \left( \frac{\bar{P}^*_{H,t}}{P^*_{H,t+k}} \right)^{-\varepsilon} \frac{Y^*_{t+k}}{1-h} \tag{3.77}$$

for $k = 0, 1, 2, \ldots$, where $\mathcal{Q}^*_{t,t+k}$ is the households' stochastic discount factor in country $F$ for discounting $k$-period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}^*_{t,t+k} = \beta^k \frac{\xi^*_{t+k}}{\xi^*_t} \left( \frac{C^*_{t+k}}{C^*_t} \right)^{-\sigma} \frac{P^*_t}{P^*_{t+k}} \tag{3.78}$$

for $k = 0, 1, 2, \ldots$, which is equal to the stochastic discount factor for households in country $H$, and where $Y^*_{t+k|t}(j)$ is the output in period $t + k$ for firm $j$ which last reset its price in period $t$.

The optimal price chosen by firms in country $H$ can be expressed as a function of only aggregate variables:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC^n_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1 - \tau^s_{t+k}) \right\}} \tag{3.79}$$

Notice that in the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}_H = \frac{\varepsilon}{(\varepsilon - 1)(1 - \tau^s)} MC^n \tag{3.80}$$

where $MC^n$ is the nominal marginal cost in steady state and in the flexible price equilibrium in country $H$, and where the optimal price is shown to be set as a markup over nominal marginal costs.

Analogous results hold for firms in country $F$. The optimal price chosen by firms in country $F$ can be expressed as a function of only aggregate variables:

$$\bar{P}^*_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}} \frac{Y^*_{t+k}}{(P^*_{H,t+k})^{-\varepsilon}} MC^{*n}_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}} \frac{Y^*_{t+k}}{(P^*_{H,t+k})^{-\varepsilon}} (1 - \tau^{*s}_{t+k}) \right\}} \tag{3.81}$$

In the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}^*_H = \frac{\varepsilon}{(\varepsilon - 1)(1 - \tau^{*s})} MC^{*n} \tag{3.82}$$

where $MC^{*n}$ is the nominal marginal cost in steady state and in the flexible price equilibrium in country

$F$, and where the optimal price is shown to be set as a markup over nominal marginal costs.

### 3.2.4 Net Exports, Net Foreign Assets and the Balance of Payments

*Net Exports* are defined as domestic production minus domestic consumption, which is equal to exports minus imports, and for country $H$ are given by:

$$NX_t \equiv P_{H,t}Y_t - P_tC_t - P_{H,t}G_t \tag{3.83}$$

In real terms (divided by $P_{H,t}$), Net Exports for country $H$ can be written as:

$$\widetilde{NX}_t = Y_t - \frac{P_t}{P_{H,t}}C_t - G_t = Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{3.84}$$

where net exports are shown to be a function of the country's degree of openness and the terms of trade, other than domestic production and public and private domestic consumption.

Since exports for country $H$ are imports for country $F$ and viceversa, then net exports are in zero international net supply: $NX_t + NX_t^* = 0$. In real terms: $\widetilde{NX}_t + S_t\widetilde{NX}_t^* = 0$. This implies the following relationship between output in the two countries:

$$Y_t + S_tY_t^* = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t + S_t\left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} C_t^* + G_t + S_tG_t^* \tag{3.85}$$

*Net Foreign Assets* are given by the sum of private and public assets held abroad, and for countries $H$ and $F$ are given respectively by:

$$NFA_t \equiv B_{F,t} \qquad \text{and} \qquad NFA_t^* \equiv B_{F,t}^* - B_t^{*G} \tag{3.86}$$

In real terms (divided by $P_{H,t}$ and $P_{H,t}^*$ respectively), Net Foreign Assets for countries $H$ and $F$ can be written respectively as:

$$\widetilde{NFA}_t \equiv \tilde{B}_{F,t} \qquad \text{and} \qquad \widetilde{NFA}_t^* \equiv \tilde{B}_{F,t}^* - \tilde{B}_t^{*G} \tag{3.87}$$

Since foreign assets for country $H$ are domestic assets for country $F$, then net foreign assets are in zero international net supply: $NFA_t + NFA_t^* = 0$. In real terms: $\widetilde{NFA}_t + S_t\widetilde{NFA}_t^* = 0$.

The *Balance of Payments* is given by net exports plus interest accrued on net foreign assets and income from abroad (from financial intermediaries):

$$BP_t \equiv NX_t + i_{t-1}NFA_{t-1} + \delta_{t-1}(1+i_{t-1}^*)B_{F,t-1} = NX_t + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}NFA_{t-1} \tag{3.88}$$

In real terms (divided by $P_{H,t}$), the Balance of Payments for country $H$ can be written as:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + i_{t-1}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \delta_t(1+i_{t-1}^*)\frac{B_{F,t-1}}{\Pi_{H,t}} = \widetilde{NX}_t + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \tag{3.89}$$

Combining the relationships between net foreign assets and net exports in the two countries shows that the balance of payments of the two countries are related in the following way: $BP_t + BP_t^* = 0$. In real terms: $\widetilde{BP}_t + S_t\widetilde{BP}_t^* = 0$.

Since government bonds issued by country $H$ are not traded internationally, while government bonds issued by country $F$ are traded internationally, the market clearing conditions for these bonds are given by:

$$B_{H,t} - B_t^G = 0 \qquad B_{F,t} + B_{F,t}^* - B_t^{*G} = 0 \tag{3.90}$$

From the households' budget constraint, substituting in firm profits and labour income, substituting in the expression for transfers backed out from the government budget constraint and substituting in the market clearing condition for government bonds issued by country $H$, yields the following relationship between assets and goods for country $H$:

$$B_{F,t} - \frac{i_{t-1} + \delta_{t-1}}{1 - \delta_{t-1}} B_{F,t-1} = P_{H,t} Y_t - P_t C_t - P_{H,t} G_t \tag{3.91}$$

while substituting in the definitions of net exports and net foreign assets yields:

$$NFA_t = \frac{i_{t-1} + \delta_{t-1}}{1 - \delta_{t-1}} NFA_{t-1} + NX_t \tag{3.92}$$

while substituting in the definition of the balance of payments yields a relationship between net foreign assets and the balance of payments:

$$NFA_t = NFA_{t-1} + BP_t \tag{3.93}$$

which shows that the balance of payments is equal to the variation of net foreign assets over one period.

In real terms (divided by $P_{H,t}$), the previous equations can be rewritten as:

$$\widetilde{NFA}_t = \frac{1 + i_{t-1}}{1 - \delta_{t-1}} \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \tag{3.94}$$

### 3.2.5 Central Bank and Monetary Policy

The only central bank in the currency union sets monetary policy by choosing the nominal interest rate to target *union-wide* inflation through a Taylor rule.

Monetary policy follows an Inflation Targeting regime of the kind:

$$\beta(1 + i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \tag{3.95}$$

where union-wide output is defined as the population-weighted geometric average of per-capita output in the two countries:

$$Y_t^U \equiv \left(\frac{Y_t}{h}\right)^h \left(\frac{Y_t^*}{1-h}\right)^{1-h} \tag{3.96}$$

while the union-wide CPI is analogously defined:

$$P_t^U \equiv (P_t)^h (P_t^*)^{1-h} \tag{3.97}$$

so that consequently union-wide inflation is defined as the population-weighted geometric average of the CPI inflations in the two countries:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{3.98}$$

while variables without subscripts $t$ denote their respective steady state levels, $\phi_\pi$ represents the responsiveness of the interest rate to inflation and $\rho_i$ is a measure of the persistence of the interest rate over time (interest rate smoothing).

### 3.2.6 Government and Fiscal Policy in a Pure Currency Union

In a Pure Currency Union each government chooses the amount of government consumption and transfers, financed by marginal tax rates on labour income and firm sales and by short-term government bonds.

In country $H$ the government finances a stream of public consumption $G_t$ and transfers $T_t$ subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j)G_t(j)\,dj + \int_0^h T_t^i\,di + B_{t-1}^G(1+i_{t-1}) = B_t^G + \tau_t^s P_{H,t}Y_t + \int_0^h \tau_t^w W_t N_t^i\,di \tag{3.99}$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period $t$, including interest. $B_t^G$ are government bonds issued by country $H$ in period $t$, while all other variables are as explained above. Government consumption, $G_t$, is given by the following CES function, just like equation 3.74 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h G_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.100}$$

The optimal allocation of any given public expenditure within each category of goods yields the following demand function:

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\frac{G_t}{h} \tag{3.101}$$

and total government expenditure on public consumption simplifies to:

$$\int_0^h P_{H,t}(j)G_t(j)\,dj = P_{H,t}G_t \tag{3.102}$$

The government budget constraint can be rewritten in aggregate form as:

$$P_{H,t}G_t + T_t + B_{t-1}^G(1+i_{t-1}) = B_t^G + \tau_t^s P_{H,t}Y_t + \tau_t^w W_t N_t \tag{3.103}$$

while dividing the government budget constraint by $P_{H,t}$ yields the government budget constraint in real terms:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{3.104}$$

where variables with a tilde ( ˜ ) are in real terms (divided by $P_{H,t}$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the transfer scenario, fiscal policy in country $H$ chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)}\left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{3.105}$$

while keeping real transfers constant and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \tag{3.106}$$

as the government does not need to deleverage its government debt, and varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{3.107}$$

where $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_y \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

In the consumption scenario, fiscal policy in country $H$ chooses real transfers to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{3.108}$$

while keeping government consumption constant and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad G_t = G \tag{3.109}$$

as the government does not need to deleverage its government debt, and varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{3.110}$$

where $\rho_t \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_y \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

In the distortionary tax scenario, fiscal policy in country $H$ chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{3.111}$$

while keeping real transfers constant and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \tag{3.112}$$

as the government does not need to deleverage its government debt, and varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{3.113}$$

where $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_y \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

In country $F$ the government finances a stream of public consumption $G_t^*$ and transfers $T_t^*$ subject to the following sequence of budget constraints:

$$\int_h^1 P_{H,t}^*(j)G_t^*(j)\,dj + \int_h^1 T_t^{*i}\,di + B_{t-1}^{*G}(1+i_{t-1}^*) = B_t^{*G} + \tau_t^{*s}P_{H,t}^*Y_t^* + \int_h^1 \tau_t^{*w}W_t^*N_t^{*i}\,di \tag{3.114}$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period $t$, including interest. $B_t^{*G}$ are government bonds issued by country $F$ in period $t$, while all other variables are as explained above. Government consumption, $G_t^*$, is given by the following CES function, just like equation 3.77 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t^* \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 G_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.115}$$

The optimal allocation of any given public expenditure within each category of goods yields the following demand function:

$$G_t^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} \frac{G_t^*}{1-h} \tag{3.116}$$

and total government expenditures on public consumption simplify to:

$$\int_h^1 P_{H,t}^*(j)G_t^*(j)\,dj = P_{H,t}^*G_t^* \tag{3.117}$$

The government budget constraint can be rewritten in aggregate form as:

$$P_{H,t}^*G_t^* + T_t^* + B_{t-1}^{*G}\frac{1+i_{t-1}}{1-\delta_{t-1}} = B_t^{*G} + \tau_t^{*s}P_{H,t}^*Y_t^* + \tau_t^{*w}W_t^*N_t^* \tag{3.118}$$

while dividing the government budget constraint by $P_{H,t}^*$ yields the government budget constraint in real terms:

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s}Y_t^* + \tau_t^{*w}MC_t^*d_t^*Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{3.119}$$

where variables with a tilde ( $\tilde{}$ ) are in real terms (divided by $P_{H,t}^*$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the transfer scenario, fiscal policy in country $F$ chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{3.120}$$

while using real transfers $\tilde{T}_t^*$ to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^*\left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{3.121}$$

and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^*) - (\tau^{*s} Y^* + \tau^{*w} MC^* Y^*) = G_t^* - G^* \tag{3.122}$$

where $\rho_g^* \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_y^* \geq 0$ represents the responsiveness of government consumption to variations of the output gap, $\gamma_t^* \in [0,1]$ is the desired share of reduction per period of the excess real government debt with respect to steady state. If $\gamma_t^*$ increases over time then the deleveraging is fast and backloaded, while if $\gamma_t^*$ decreases over time then the deleveraging is slow and frontloaded. If $\gamma_t^* = \gamma^*$ then the deleveraging is constant and linear in the desired fraction of reduction per period of the excess real government debt, as the fraction of deleveraging does not vary over time.

In the consumption scenario, fiscal policy in country $F$ chooses real transfers to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{3.123}$$

while using government consumption $G_t^*$ to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{3.124}$$

and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^*) - (\tau^{*s} Y^* + \tau^{*w} MC^* Y^*) = \tilde{T}_t^* - \tilde{T}^* \tag{3.125}$$

where $\rho_t^* \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_y^* \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

In the distortionary tax scenario, fiscal policy in country $F$ chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{3.126}$$

while keeping real transfers constant and varying equally the tax rates on labour income and firm sales to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \qquad \tilde{T}_t^* = \tilde{T}^* \tag{3.127}$$

and also to finance the remaining government expenditure, through the tax rule:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \tag{3.128}$$

where $\rho_g^* \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_y^* \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

### 3.2.7 Government and Fiscal Policy in a Coordinated Currency Union

If the Governments of the two countries choose to coordinate, they will use their fiscal instruments to target a common objective, while maintaining independent budget constraints. Instead of using government consumption or transfers to stabilize the domestic output gap countercyiclically, they will use the same fiscal instruments to stabilize the net exports gap procyclically. This represents the act of coordinating their policies on a common objective, which depends on the interactions between the two economies. The budget constraints of the two fiscal authorities instead remain unmodified.

In the transfer scenario, fiscal policy in country $H$ chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{3.129}$$

while keeping real transfers constant and maintaining a balanced budget, through the debt rule:

$$\tilde{B}^G_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \tag{3.130}$$

as the government does not need to deleverage its government debt, and varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau^w_t - \tau^w = \tau^s_t - \tau^s \tag{3.131}$$

where $\rho_g \in [0, 1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_{nx} \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap.

In the same transfer scenario, fiscal policy in country $F$ chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G^*_t}{G^*} = \left(\frac{\widetilde{NX}^*_t}{\widetilde{NX}^*}\right)^{\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} \tag{3.132}$$

while using real transfers $\tilde{T}^*_t$ to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}\right) \tag{3.133}$$

and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau^{*w}_t - \tau^{*w} = \tau^{*s}_t - \tau^{*s} \qquad (\tau^{*s}_t Y^*_t + \tau^{*w}_t MC^*_t d^*_t Y^*_t) - (\tau^{*s} Y^* + \tau^{*w} MC^* Y^*) = G^*_t - G^* \tag{3.134}$$

where $\rho^*_g \in [0, 1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi^*_{nx} \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap, $\gamma_t^* \in [0,1]$ is the desired share of reduction per period of the excess real government debt with respect to steady state. If $\gamma_t^*$ increases over time then the deleveraging is fast and backloaded, while if $\gamma_t^*$ decreases over time then the deleveraging is slow and frontloaded. If $\gamma_t^* = \gamma^*$ then the deleveraging is constant and linear in the desired fraction of reduction per period of the excess real government debt, as the fraction of deleveraging does not vary over time.

In the consumption scenario, fiscal policy in country $H$ chooses real transfers to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{3.135}$$

while keeping government consumption constant and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad G_t = G \tag{3.136}$$

as the government does not need to deleverage its government debt, and varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{3.137}$$

where $\rho_t \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx} \geq 0$ represents the responsiveness of real transfers to variations of the real net exports gap.

In the same consumption scenario, fiscal policy in country $F$ chooses real transfers to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{3.138}$$

while using government consumption $G_t^*$ to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{3.139}$$

and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^*) - (\tau^{*s} Y^* + \tau^{*w} MC^* Y^*) = \tilde{T}_t^* - \tilde{T}^* \tag{3.140}$$

where $\rho_t^* \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx}^* \geq 0$ represents the responsiveness of real transfers to variations of the real net exports gap.

In the distortionary tax scenario, fiscal policy in country $H$ chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{3.141}$$

while keeping real transfers constant and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \tag{3.142}$$

as the government does not need to deleverage its government debt, and varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{3.143}$$

where $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx} \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap.

In the same distortionary tax scenario, fiscal policy in country $F$ chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{3.144}$$

while keeping real transfers constant and varying equally the tax rates on labour income and firm sales to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \qquad \tilde{T}_t^* = \tilde{T}^* \tag{3.145}$$

and also to finance the remaining government expenditure, through the tax rules:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \tag{3.146}$$

where $\rho_g^* \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx}^* \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap.

### 3.2.8 Government and Fiscal Policy in a Full Fiscal Union

If instead of considering two fiscal authorities managing fiscal policy independently, one for each country, or coordinating their policies, but with two separate budget constraints, we consider only one fiscal authority managing fiscal policy for both countries at the same time in a coordinated way and with a consolidated budget constraint, then we can think of it as an extreme case of fiscal policy coordination and call it a Full Fiscal Union. In this case government debt will be aggregated across countries and both countries will contribute to the deleveraging of government debt. Nonetheless, given that financial markets are still incomplete, there continue to be two separate government bonds for the two countries, which pay different interest rates and so have different bond yields.

A Full Fiscal Union uses local government spending to manage fiscal policy at the union level with a consolidated budget constraint. The Fiscal Union finances streams of local public consumption, $G_t$ and $G_t^*$, and transfers, $T_t$ and $T_t^*$, subject to the consolidated budget constraint of the two national fiscal authorities:

$$P_{H,t}G_t + P^*_{H,t}G^*_t + T_t + T^*_t + B^G_{t-1}(1+i_{t-1}) + B^{*G}_{t-1}\frac{1+i_{t-1}}{1-\delta_{t-1}} =$$
$$B^G_t + B^{*G}_t + \tau^s_t P_{H,t}Y_t + \tau^{*s}_t P^*_{H,t}Y^*_t + \tau^w_t W_t N_t + \tau^{*w}_t W^*_t N^*_t \quad (3.147)$$

Dividing the government budget constraint by $P_{H,t}$ yields the government budget constraint in real terms (for country $H$):

$$G_t + \tilde{T}_t + S_t(G^*_t + \tilde{T}^*_t) + i_{t-1}\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{S_{t-1}\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} =$$
$$(\tau^s_t + \tau^w_t MC_t d_t)Y_t + (\tau^{*s}_t + \tau^{*w}_t MC^*_t d^*_t)S_t Y^*_t + \tilde{B}^G_t - \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} + S_t\tilde{B}_t^{*G} - \frac{S_{t-1}\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} \quad (3.148)$$

where variables with a tilde ( $\tilde{}$ ) are in real terms (divided by $P_{H,t}$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of overall government debt.

In the transfer scenario, union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)}\left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \quad (3.149)$$

$$\frac{G^*_t}{G^*} = \left(\frac{\widetilde{NX}^*_t}{\widetilde{NX}^*}\right)^{\psi^*_{nx}(1-\rho^*_g)}\left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_g)}\left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} \quad (3.150)$$

while using real transfers equally in both countries to deleverage the government debt of country $F$, while country $H$ maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t\left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}\right) \qquad \tilde{B}^G_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \qquad \tilde{T}_t - \tilde{T} = \tilde{T}^*_t - \tilde{T}^* \quad (3.151)$$

and varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau^w_t - \tau^w = \tau^s_t - \tau^s \qquad \tau^{*w}_t - \tau^{*w} = \tau^w_t - \tau^w \qquad \tau^{*s}_t - \tau^{*s} = \tau^s_t - \tau^s \quad (3.152)$$

$$(\tau^{*s}_t Y^*_t + \tau^{*w}_t MC^*_t d^*_t Y^*_t) - (\tau^{*s}Y^* + \tau^{*w}MC^*Y^*) = G^*_t - G^* \quad (3.153)$$

where $\rho_g \in [0,1]$ for country $H$ and $\rho^*_g \in [0,1]$ for country $F$ are measures of persistence of the government consumption shocks in their AR(1) processes in logs and $\varepsilon_t$ is a zero mean white noise process. Variables without subscripts $t$ represent their respective steady state level, while $\psi_{nx} \geq 0$ for country $H$ and $\psi^*_{nx} \geq 0$ for country $F$ represent the responsiveness of government consumption to variations of the real net exports gap, $\gamma^*_t \in [0,1]$ is the desired share of reduction per period of the excess real government debt with respect to steady state. If $\gamma^*_t$ increases over time then the deleveraging is fast and backloaded, while if $\gamma^*_t$ decreases over time then the deleveraging is slow and frontloaded. If $\gamma^*_t = \gamma^*$ then the deleveraging is constant and linear in the desired fraction of reduction per period of the excess real government debt, as the fraction of deleveraging does not vary over time.

In the consumption scenario, union-wide fiscal policy chooses real transfers in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{3.154}$$

$$\frac{\tilde{T}^*_t}{\tilde{T}^*} = \left(\frac{\widetilde{NX}^*_t}{\widetilde{NX}^*}\right)^{\psi^*_{nx}(1-\rho^*_t)} \left(\frac{\tilde{T}^*_{t-1}}{\tilde{T}^*}\right)^{\rho^*_t} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_t)} \left(\frac{\tilde{T}^*_{t-1}}{\tilde{T}^*}\right)^{\rho^*_t} e^{\varepsilon_t} \tag{3.155}$$

while using government consumption equally in both countries to deleverage the government debt of country $F$, while country $H$ maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}\right) \qquad \tilde{B}^G_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \qquad G_t - G = G^*_t - G^* \tag{3.156}$$

and varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau^w_t - \tau^w = \tau^s_t - \tau^s \qquad \tau^{*w}_t - \tau^{*w} = \tau^w_t - \tau^w \qquad \tau^{*s}_t - \tau^{*s} = \tau^s_t - \tau^s \tag{3.157}$$

$$(\tau^{*s}_t Y^*_t + \tau^{*w}_t MC^*_t d^*_t Y^*_t) - (\tau^{*s} Y^* + \tau^{*w} MC^* Y^*) = \tilde{T}^*_t - \tilde{T}^* \tag{3.158}$$

where $\rho_t \in [0,1]$ for country $H$ and $\rho^*_t \in [0,1]$ for country $F$ are measures of persistence of the transfer shocks in their AR(1) processes in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx} \geq 0$ for country $H$ and $\psi^*_{nx} \geq 0$ for country $F$ represent the responsiveness of real transfers to variations of the real net exports gap.

In the distortionary tax scenario, union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{3.159}$$

$$\frac{G^*_t}{G^*} = \left(\frac{\widetilde{NX}^*_t}{\widetilde{NX}^*}\right)^{\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} \tag{3.160}$$

while keeping real transfers constant and varying equally across countries the tax rates on labour income and firm sales to deleverage the government debt of country $F$, while country $H$ maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}\right) \qquad \tilde{B}^G_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \qquad \tilde{T}^*_t = \tilde{T}^* \tag{3.161}$$

and also varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau^w_t - \tau^w = \tau^s_t - \tau^s \qquad \tau^{*w}_t - \tau^{*w} = \tau^w_t - \tau^w \qquad \tau^{*s}_t - \tau^{*s} = \tau^s_t - \tau^s \tag{3.162}$$

where $\rho_g \in [0,1]$ for country $H$ and $\rho^*_g \in [0,1]$ for country $F$ are measures of persistence of the government consumption shocks in their AR(1) processes in logs and $\varepsilon_t$ is a zero mean white noise process, while $\psi_{nx} \geq 0$ for country $H$ and $\psi^*_{nx} \geq 0$ for country $F$ represent the responsiveness of government consumption to variations of the real net exports gap.

## 3.3 Calibration

The model is calibrated[3] following mainly Ferrero (2009), so we consider the top 5 Eurozone countries, which account for more than 80% of Eurozone GDP and we divide them into the periphery (namely, France, Italy, Spain and The Netherlands), country $F$, and the core (namely Germany), country $H$. The size of country $H$ is set according to the relative GDP size to $h = 0.4$, as Germany accounts for over 35% of Eurozone GDP.

As in Ferrero (2009) most of the parameters governing the economies of the two countries are set symmetrically, with the exception of the degree of price rigidity, which has been set such that in country $H$ the average duration of a price is 4 quarters while in country $F$ it is 5 quarters. The gross markup $\frac{\varepsilon}{\varepsilon-1}$ has been set to 1.1, which implies a net markup of 10%, and the discount factor has been chosen to match a compounded annual interest rate of 2%. The parameters for monetary policy follow common values used in the literature, so we set the response of the interest rate to inflation to $\phi_\pi = 1.5$, according to the Taylor principle, and the interest rate smoothing parameter to $\rho_i = 0.8$. We estimate the sensitivity of the transaction cost, or of the government bond spread, $\delta_t$ to deviations of government debt-to-GDP from steady state and find that for every ten percentage points increase in government debt-to-GDP the government bond spread increases by 0.7 percentage points[4], according to which we set $\delta^B = 0.07$. Table 3.1 collects all calibrated parameters and steady state stances.

In the calibration, we set $\eta > \frac{1}{\sigma}$ so that $C_H$ and $C_F$ are substitutes and hence the substitution effect of a price change dominates the income effect. In the opposite case $\left(\eta < \frac{1}{\sigma}\right)$ $C_H$ and $C_F$ are complements and the income effect of a price change dominates the substitution effect. This implies that fiscal policy and spillovers from one country to the other have very different effects based on the two calibrations. In our analysis we focus on the case in which $C_H$ and $C_F$ are substitutes because we believe it is more realistic, especially for advanced economies, and more in line with the recent literature (See Ferrero (2009) and Blanchard, Erceg and Lindé (2015) for instance), although it is quite different from the assumption in Hjortsø (2012).

The calibration of the two countries mainly differs in the fiscal policy parameters. In particular, the government consumption-to-GDP ratios have been set respectively to 18.7% for country $H$ and 21.9% for country $F$, according to the average of the last 9 years (source ECB-SDW). The marginal tax rates on labour income have been set respectively to 40.61% for country $H$ and 27.94% for country $F$ in accordance to the average in the last 9 years of the labour income tax wedges, excluding social security contributions made by the employer, for the median individual, as reported in OECD (2015). The marginal tax rate on firm sales has been set to 19.5% for country $F$ according to the average in the last 9 years of the VAT for France, Italy, Spain and The Netherlands as reported in Eurostat, European-Commission et al. (2015), while it has been calibrated for country $H$ to match the average ratio of net exports-to-GDP of 1.73% observed over the past 9 years for Germany[5]. Although the observed VAT rate for Germany is 19%, we set its marginal tax rate on firm sales to 2.5%, as if there was a production incentive, to correct for the fact that country $H$ should have a greater productivity compared to country $F$, as Germany has a greater productivity than the periphery countries. This calibration implies a steady state tax revenue-to-GDP ratio of respectively 38.49% for country $H$ and 39.92% for country $F$, clearly in line with the data observed over the past decades for Germany (38.72%) and for France, Italy, Spain

[3]The calibration is done on the steady state values described in Appendix B.6.
[4]This result is in line with Hjortsø (2012), which finds a similar sensitivity to be 0.5 instead of 0.7.
[5]The average current account to GDP ratio observed over the past 9 years for Germany is roughly 6.36%. However, we adjust the data for the overall trade weight with France, Italy, Spain and The Netherlands (26%).

Table 3.1: Calibrated Parameters and Steady State Stances.

| Parameters | Description | Country $H$ | Country $F$ |
|---|---|---|---|
| $h$ | Relative size of domestic economy | 0.4 | 0.6 |
| $\beta$ | Discount factor | 0.995 | 0.995 |
| $\varepsilon$ | Elasticity of substitution of domestic goods | 11 | 11 |
| $\frac{\varepsilon}{\varepsilon-1}$ | Gross Price Mark-Up | 1.1 | 1.1 |
| $\eta$ | Elasticity of substitution foreign and domestic goods | 4.5 | 4.5 |
| $\sigma$ | Inverse Elasticity of intertemporal substitution | 3 | 3 |
| $\varphi$ | Inverse Frisch Elasticity of labour supply | 0.5 | 0.5 |
| $\theta$ | Degree of price rigidity | 3/4 | 4/5 |
| $\delta^B$ | Sensitivity of bond spread to debt-to-GDP deviations | 0.07 | 0.07 |
| $\gamma$ | Desired reduction of excess government debt-to-GDP | 0 | 0.05 |
| $\alpha$ | Openness of domestic economy | 0.52 | 0.361 |
| $\frac{\alpha}{h}$ | Relative openness of domestic economy | 1.3 | 0.6017 |
| $\frac{1-\alpha}{h}$ | Home bias | 1.2 | 1.065 |
| $\psi_y$ | Responsiveness of fiscal policy to output gap | 0.045 | 0.001 |
| $\psi_{nx}$ | Responsiveness of fiscal policy to net exports gap | 0.0697 | 0.0092 |
| $\phi_\pi$ | Responsiveness of monetary policy to inflation | 1.5 | 1.5 |
| $\rho_i$ | Interest Rate smoothing parameter | 0.8 | 0.8 |
| $\rho_\xi$ | Persistence of preference shock | 0.94 | 0.8 |
| $\rho_a$ | Persistence of technology shock | 0.58 | 0.70 |
| $\rho_\delta$ | Persistence of spread shock | 0.95 | 0.95 |
| $\sigma_\xi$ | Standard deviation preference shock | 0.0024 | 0.0086 |
| $\sigma_a$ | Standard deviation technology shock | 0.0087 | 0.0033 |
| $\sigma_\delta$ | Standard deviation spread shock | 0.14 | 0.14 |
| $corr_\xi$ | Correlation preference shock | 0.625 | 0.625 |
| $corr_a$ | Correlation technology shock | 0.418 | 0.418 |
| **Steady State Ratios** | **Description** | **Country $H$** | **Country $F$** |
| $(1+i)^4-1$ | Annualized Interest Rate | 2% | 2% |
| $\tau^w$ | Tax Rate on wage income | 40.6% | 27.9% |
| $\tau^s$ | Tax Rate on firm sales | 2.5% | 19.5% |
| $\tau^w MC + \tau^s$ | Tax Revenues-to-GDP | 38.49% | 39.92% |
| $\frac{G}{Y}$ | Government consumption-to-GDP | 18.7% | 21.9% |
| $\frac{\tilde{T}}{Y}$ | Real Transfers-to-GDP | 18.58% | 16.81% |
| $\frac{\tilde{NX}}{Y}$ | Net Exports-to-GDP | 1.72% | -1.14% |
| $\frac{C}{Y}$ | Consumption-to-GDP | 79.58% | 79.24% |
| $\alpha^* \frac{C^*}{Y}$ | Exports-to-GDP | 43.1% | 27.47% |

and The Netherlands (39.15%). Finally, the annualized steady state value of government debt-to-GDP in both countries is set to roughly 60% as stated in the Maastricht Treaty. In the simulations, country $F$ starts with a higher level of government debt-to-GDP, equal to roughly 90%, in line with the average level of government debt-to-GDP for France, Italy, Spain and The Netherlands. The desired fraction of reduction of excess government debt is set to $\gamma = 0.05$ for country $F$, corresponding to a 5% yearly reduction of excess government debt[6], to comply with the Debt Brake Rule in the Fiscal Compact, and 0 for country $H$, as only country $F$ needs to deleverage in our simulations. The calibration of the fiscal policy parameters, $\psi_y$ and $\psi_{nx}$, follows the estimations done in our previous work, Cole, Guerello and Traficante (2016).

Since the two countries' fiscal policy ratios have been calibrated according to the data, the transfers-to-GDP ratios have been set such that the government deficit is zero in steady state:

$$\frac{\tilde{T}}{Y} = (\tau^s + \tau^w MC) - \frac{G}{Y} - \left(\frac{1}{\beta} - 1\right) \frac{\tilde{B}^G}{Y} \tag{3.163}$$

$$\frac{\tilde{T}^*}{Y^*} = (\tau^{*s} + \tau^{*w} MC^*) - \frac{G^*}{Y^*} - \left(\frac{1}{\beta} - 1\right) \tilde{B}^{*G} \tag{3.164}$$

Henceforth, the overall calibration of the fiscal sector implies a steady state ratio of transfers-to-GDP of respectively 18.58% for country $H$ and 16.81% for country $F$, and a steady state ratio of current expenditure-to-GDP of respectively 37.28% for country $H$ and 38.71% for country $F$. This calibration is broadly in line with the observed data over the last 10 years for the subsidies-to-GDP ratio (26.85% for Germany and 24.69% for the other countries) and the current expenditure (less interest)-to-GDP ratio (35.54% for Germany and 36.85% for the other countries).

The parameters of openness have been set to match an export-to-GDP ratio $\left(\frac{\alpha^* C^*}{Y}\right)$ of roughly 43% for country $H$[7] taken from the aggregate demand equation, while for country $F$ the parameter of openness is recovered by equating per-capita consumption across countries, which yields the following equation[8]:

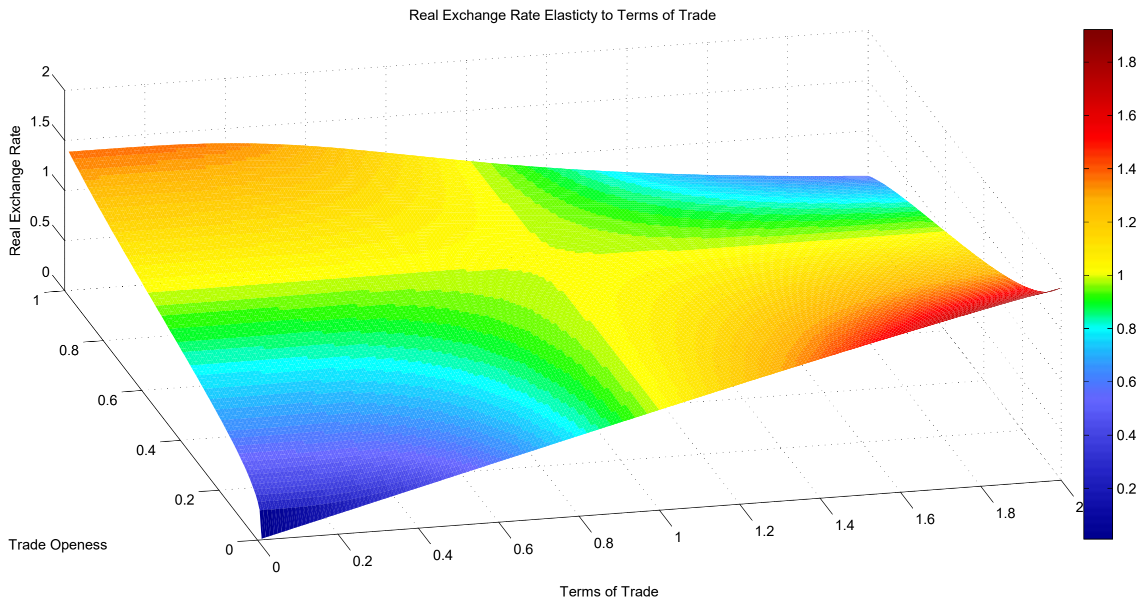
$$\alpha^* = \frac{h}{1-h}\left[\alpha + \frac{\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}} - 1}{1 + \frac{h}{1-h}\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}}}\right] \tag{3.165}$$

Consequently, home biases are given by $\frac{1-\alpha}{h} = 1.2$ and $\frac{1-\alpha^*}{1-h} = 1.065$. Since both home biases are larger than one it means that the share of consumption of domestic goods is higher than the share of production of domestic goods. This generates a gap between the relative production price indices and the relative consumption price indices based on the different composition of the households' consumption basket in the two countries. Hence, the dynamics of the real exchange rate follow the dynamics of the terms of trade in a non-linear way. As Figure 3.1 shows, the real exchange rate increases as the terms of trade increase if the degree of openness of country $H$ is less than the size of country $F$ $(1-h = 0.6)$, which is the case for our calibration $(\alpha = 0.52)$, while the real exchange rate decreases when the terms of trade increase if the degree of openness of country $H$ is more than the size of country $F$ $(h = 0.4)$. These two conditions imply respectively, given Equation 3.165, that the degree of openness of country $F$ is less than the size of country $H$ $(h = 0.4)$ for the real exchange rate to increase as the terms of trade increase, and that the degree of openness of country $F$ is more than the size of country $H$ $(h = 0.4)$ for the real exchange rate to decrease as the terms of trade increase.

Regarding the dynamic parametrization of the shocks, both technology and preference shocks are

[6] This corresponds to a similar reduction in government debt-to-steady state GDP.
[7] The value recovered from the data as the average of the last 9 years is 43.5%.
[8] Appendix B.7 shows the derivation.

Figure 3.1: Elasticity of the Real Exchange Rate to the Terms of Trade as a function of Trade Openness




assumed to follow a $VAR(1)$ process that generally allows for both direct spillovers and second order correlation of the innovations. However, the structure has been restricted for all shocks to exclude direct spillovers.

With the exception of the preference shocks, whose dynamics have been calibrated following Kollmann et al. (2014), the parameters characterizing the dynamics of the technology shocks have been estimated. For the estimation we have employed the time series for Germany, France, Italy and Spain of labour productivity per hours worked for the technology shocks. All the series are chain-linked volumes re-based respectively in 2010, seasonally adjusted and filtered by means of a Hodrick-Prescott filter. The sample considered spans at quarterly frequency from 2002 Q1 to 2015 Q3. Finally, despite a large debate on the high correlation between preference shocks in the Eurozone, there is no proper reference in the literature for its calibration. We decide to set this parameter according to the observed business cycle correlation (which is roughly 0.5) and we pick the value that maximizes the simulated correlation between output in the two countries (which is roughly 0.42)[9]. The transaction cost $\delta_t$ responds to the debt-to-GDP ratio in deviation from Maastricht treaty's objective of 60%. It is assumed to follow an autoregressive AR(1) process. The parameters defining the AR(1) process and the response to the debt-to-GDP ratio are calibrated looking at the data for the spread between the long run interest rates on government bonds[10] for France, Italy, Spain and The Netherlands and the same yield for Germany. These series

[9]The simulated values of the correlation of business cycles in our model, given our calibration, are always lower than the observed correlation. Therefore, we decide to select the correlation of preference shocks that maximizes the correlation of business cycles.

[10]The data is collected by Eurostat and reported in the ECB statistical data warehouse. It is harmonized to assess the

are combined with data on the government debt-to-GDP ratio (as a difference from Maastricht treaty's objective) using a panel VAR technique and taking the estimated parameters in the bond spread equation and the estimated variance of the residuals. In order to account for the effects of the european sovereign debt crisis, the data on the spread has been demeaned and country fixed effects have been introduced in the model to account for the initial conditions (i.e. few countries show historically high levels of government debt).

## 3.4 Numerical Simulations

We simulate the model numerically using Dynare[11], which takes a first-order approximation of the model around its symmetric non-stochastic steady state with zero inflation and constant government debt[12]. We compare the Impulse Response Functions of the main variables to negative shocks of one standard deviation of different nature under deleveraging by country $F$ from 90% government debt-to-GDP to its steady state value of 60% government debt-to-GDP, under a range of fiscal policy specifications, to study the stabilization properties of different fiscal policy instruments for deleveraging, different fiscal policy coordination strategies, and different deleveraging schemes, and to study the international transmission of shocks with incomplete financial markets.

### 3.4.1 The role of Incomplete Markets

In our previous work, Cole, Guerello and Traficante (2016), we studied different fiscal policy instruments for stabilization purposes and different degrees of fiscal policy coordination in a model with *complete* markets. We found that coordinating fiscal policy by targeting net exports, rather than output, produces more stable dynamics, and that consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization. This means that a Full Fiscal Union scenario with *complete* financial markets produces more stabilization than a Coordinated Currency Union one, which in turn is more stable than a Pure Currency Union scenario.

Now we turn to *incomplete* financial markets, as described in the present model, to see if the same results we found with *complete* financial markets still hold.

Looking at Figure 3.2 we can see that with incomplete markets in a Pure Currency Union, since households cannot fully insure themselves against country-specific shocks, consumption in the two countries follows two very different paths. This is the main difference with complete markets, where consumption in both countries follows a similar path, because there is perfect international risk-sharing. This difference in consumption is mainly given by the deviation of government debt in country $F$ and by the consequent interest rate spread, shown by the government bond spread. The assumption of incomplete markets gives rise to two different interest rates for the households in the two countries, which in turn gives rise to the differences in consumption and asset allocation in the two countries, because of the deviation of government debt in country $F$ from its steady state. This deviation occurs only because government debt is kept constant in nominal terms, which implies movements of real government debt given by inflation fluctuations.

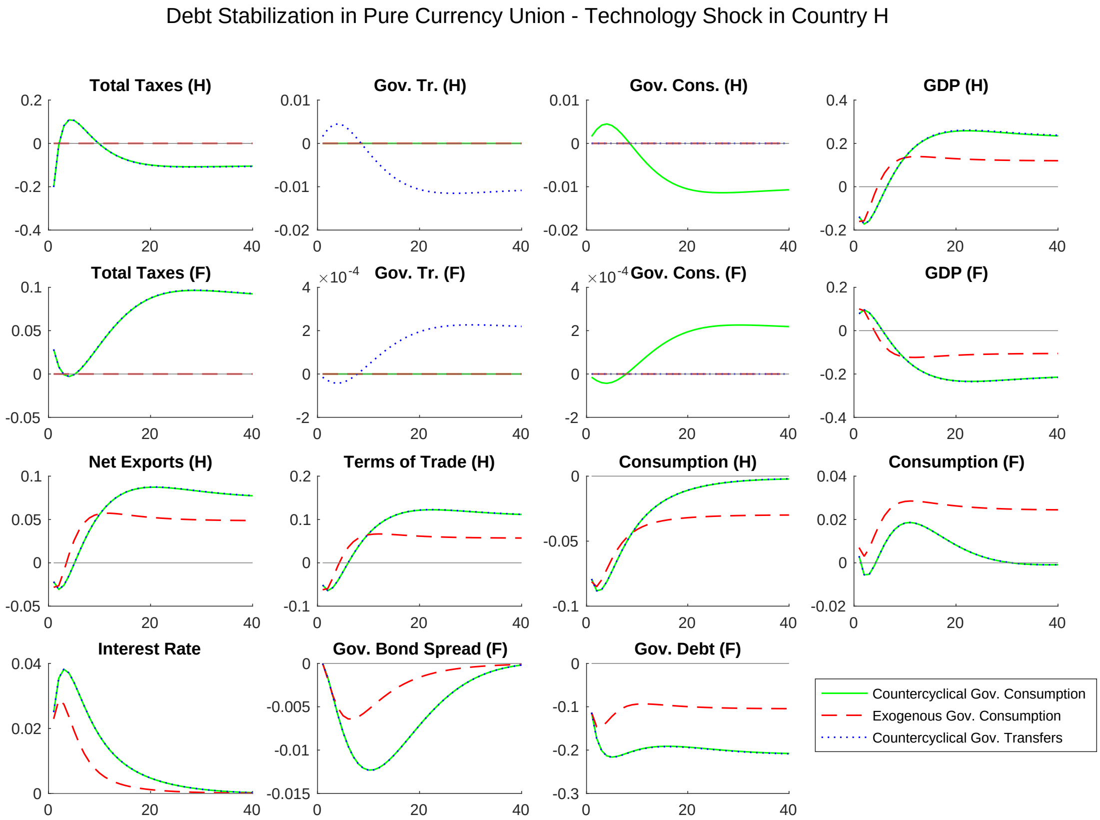
After a negative technology shock in country $H$, the increase in marginal costs for firms brings output down and prices up on impact, bringing down consumption in country $H$ and the terms of trade. As the shock fades away over time, consumption in both countries, the interest rate and the government bond

convergence of the Member Countries. The sample spans between 2002 and 2015 and features quarterly frequency.

[11] All the equilibrium conditions of the model used for the simulations are shown in Appendix B.5.

[12] All simulations are given by first-order approximations of the equilibrium conditions, except for Section 3.4.2, where the simulations are given by second-order approximations of the equilibrium conditions.

Figure 3.2: Instruments in Incomplete Markets - Pure Currency Union - Technology Shock in Country H



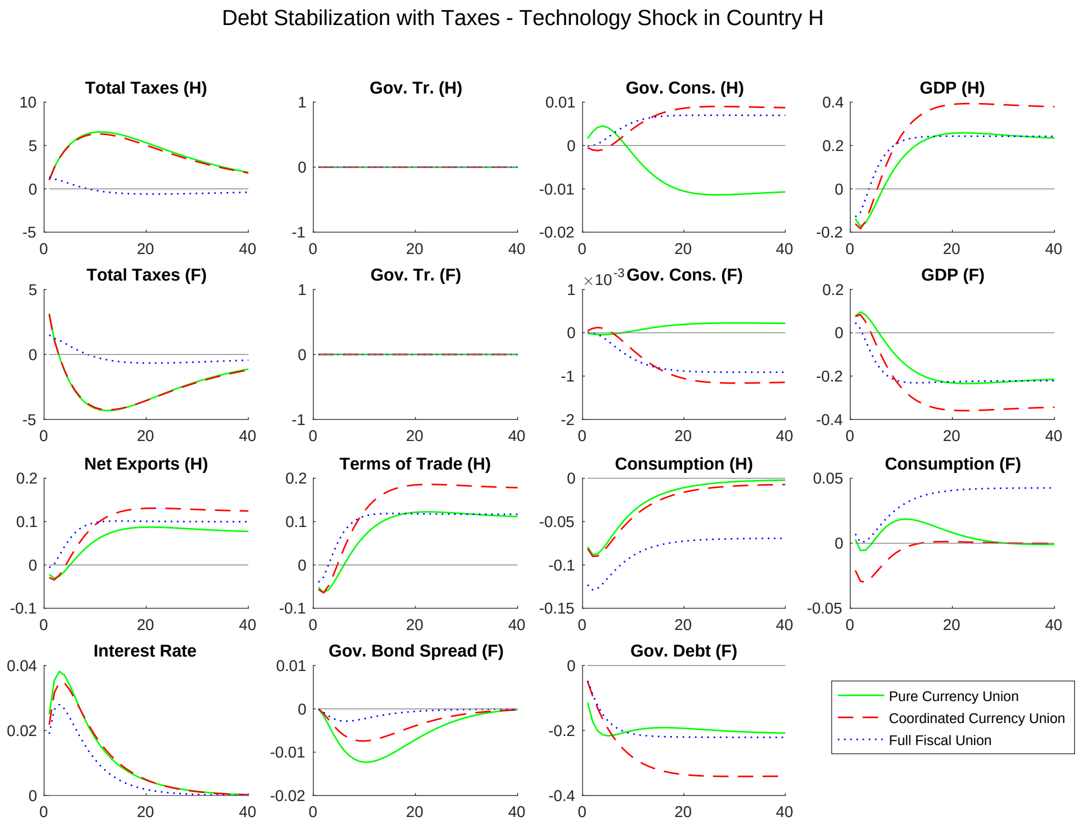

spread go back to their steady state, while the increase in the terms of trade makes net exports increase, which creates the gap in the GDPs of the two countries that we can see in the graph, whose persistence depends on that of the terms of trade and net exports.

Figure 3.2 shows that using government consumption rather than government transfers to target the output gap makes little difference, but it is more stabilizing than having exogenous government consumption, at least for household consumption, meaning that in the presence of a technology shock in country $H$ in the Pure Currency Union scenario there is always a gain for household consumption in stabilizing the economy, whichever instrument we use. This can be seen because consumption follows a much more stable path in both countries when using a targeting rule for fiscal policy, although output is less stabilized. This is because the opposite movements in the tax rates bring consumption in both countries back to steady state faster. These results hold qualitatively also in the presence of a preference shock in country $F$ or in the presence of a shock to the bond spread in country $F$, in all scenarios, although it less evident in the Full Fiscal Union scenario[13].

Figures 3.3 and 3.4 show the comparison between different degrees of fiscal policy coordination with incomplete financial markets, where government debt is stabilized using taxes, after a negative technology shock in country $H$ and after a positive shock to the bond spread in country $F$, respectively.

[13] The other simulations are available upon request.

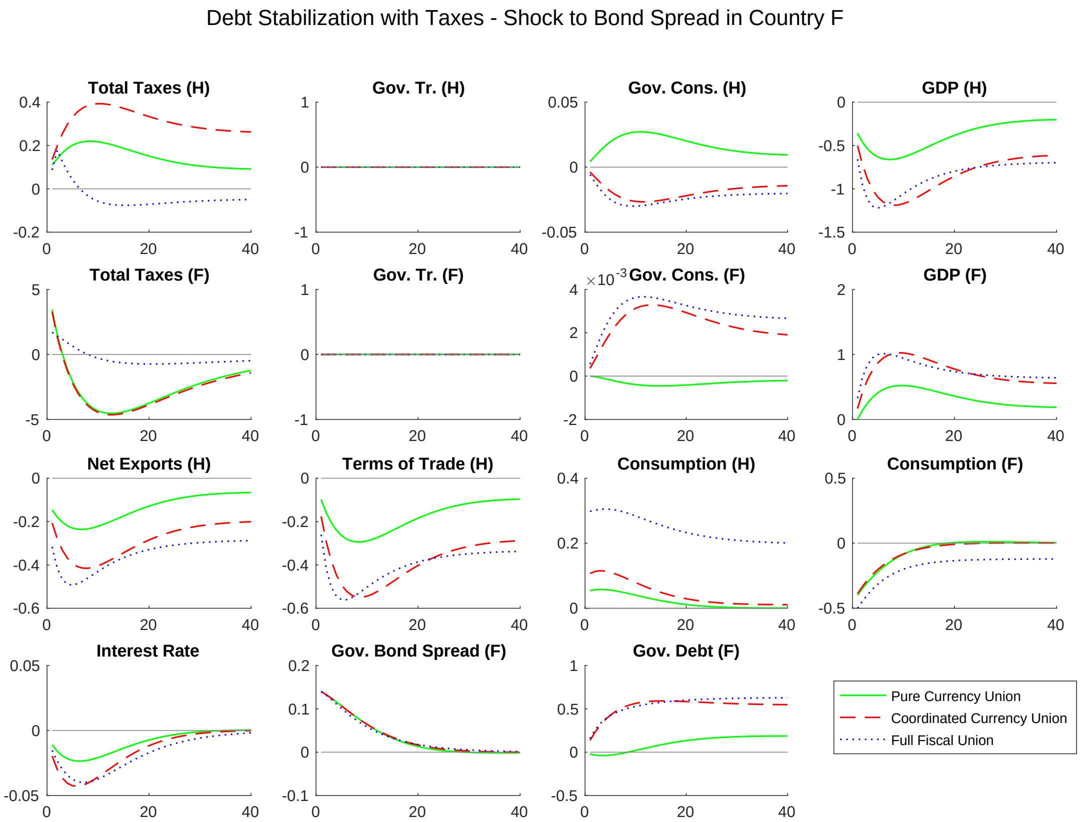
Figure 3.3: Coordination in Incomplete Markets with Taxes - Technology Shock in Country H




The simulations after a negative preference shock in country $F$ give opposite dynamics compared to a negative technology shock in country $H$, although the ranking of the scenarios by degree of stabilization is more or less the same. Looking at the figures we can see how in both cases most variables, especially consumption and GDP, are more stabilized in the Pure Currency Union scenario compared to the other scenarios. GDP is more volatile in the Coordinated Currency Union scenario after a technology shock in country $H$, showing that with incomplete markets targeting output rather than net exports stabilizes more output and net exports, the opposite result compared to the one obtained with complete financial markets. However, consumption is more volatile in the Full Fiscal Union scenario compared to the other two scenarios, because moving the tax rates jointly in the two countries amplifies the effects of shocks, due to the fact that with incomplete markets consumption in the two countries follows different paths, and would probably need opposite movements of the tax rates in the two countries to produce more stabilization. Additionally in the Full Fiscal Union scenario the loss in stabilization given by targeting net exports rather than output is counteracted by the gain in stabilization given by the consolidation of budget constraints. This can be seen in Figure 3.3, because most variables are more stabilized in the Full Fiscal Union scenario compared to the Coordinated Currency Union scenario.

After a shock to the government bond spread in country $F$ the dynamics are a little different, which brings to a slightly different ranking in the stabilization properties of different degrees of coordination. In Figure 3.4 we can see that after a shock to the bond spread in country $F$ the Full Fiscal Union scenario

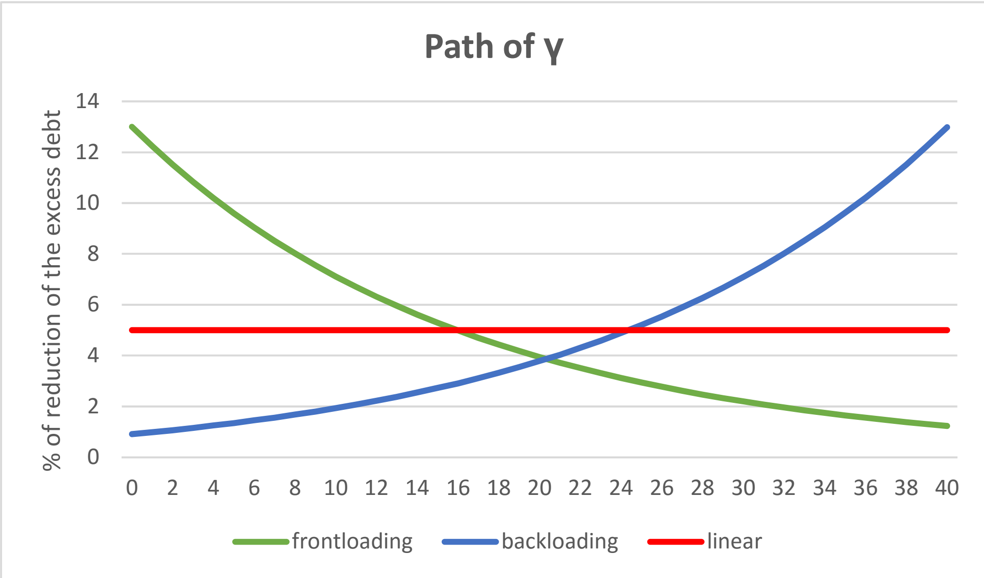
Figure 3.4: Coordination in Incomplete Markets with Taxes - Shock to Bond Spread in Country F




follows dynamics which are much more similar to the Coordinated Currency Union scenario, compared to after a technology shock in country $H$, although in most cases the Full Fiscal Union scenario is worst in terms of stabilization. This is because consolidating budget constraints always reduces inflation and the terms of trade which, after a shock to the bond spread in country $F$, amplifies the negative movements of the interest rate and terms of trade.

After analyzing the stabilization properties of different fiscal instruments for debt stabilization and of different degrees of fiscal policy coordination in a setting with incomplete financial markets, we find that stabilizing output after a shock using government consumption or transfers produces more stabilization than without using a targeting rule, while stabilizing net exports rather than output actually amplifies most dynamics. Also, although consolidating budget constraints might provide more stabilization, moving tax rates jointly does not, which implies that the Full Fiscal Union scenario might be more stabilizing than the Coordinated Currency Union scenario, but is surely not more stabilizing than the Pure Currency union scenario. These results are opposite to those obtained under complete financial markets, showing the important role of the assumption of *incomplete* financial markets in our present model.

Figure 3.5: Paths for $\gamma_t$ with different Deleveraging Schemes



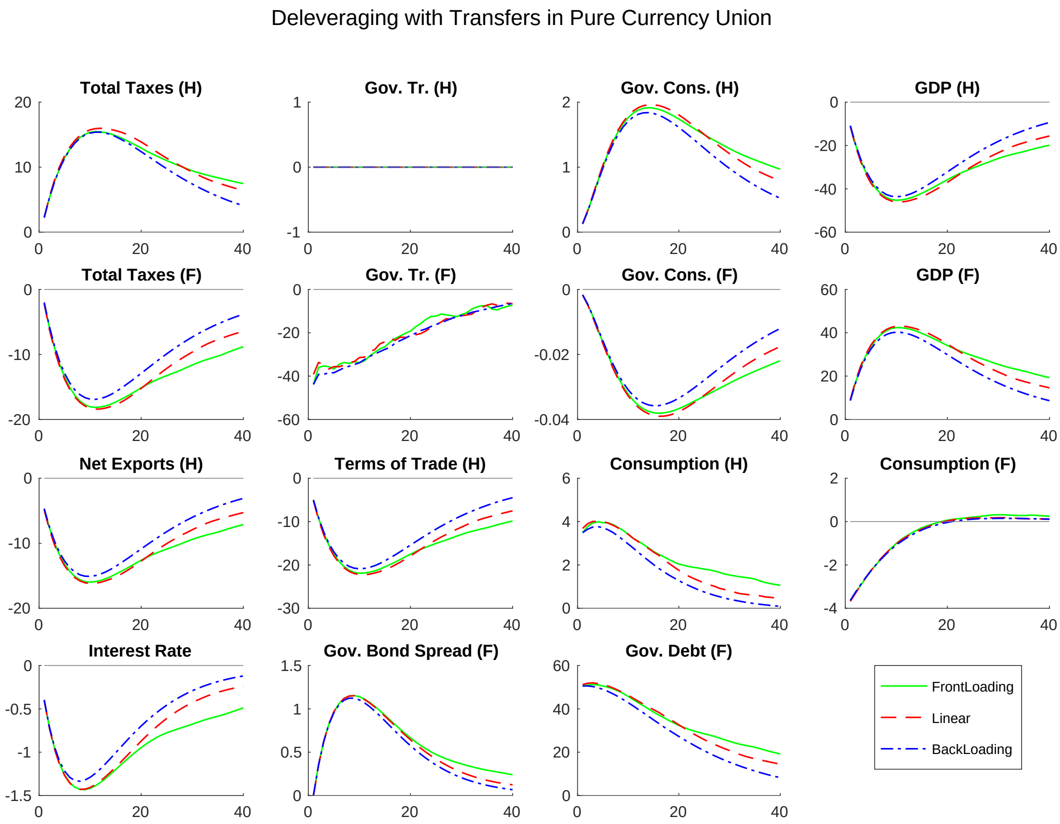

### 3.4.2 Deleveraging Schemes

Country F features an initial level of government debt-to-GDP of about 90%, which is quite higher than its steady state level of 60%. The deleveraging scheme implies that country F has to gradually reduce its government debt over time by using a fiscal instrument, either government transfers or taxes. In the baseline calibration, country F has to reduce its government debt by 5% of the excess each quarter until it goes back to steady state, as stated in the Maastricht Treaty. However, to fully understand the effects of a deleveraging scheme in an economy, it is interesting to analyze how a different speed of deleveraging affects the transmission of the shock on government debt, which brings its level from a steady state value of 60% to 90% before deleveraging. Specifically, we assume the coefficient governing the deleveraging rule, $\gamma_t$, to be time-varying and its process to be determined by the following alternative paths:

- Frontloading: the amount and consequent cost of deleveraging is higher initially and decreases over time as the level of excess government debt goes down. This is achieved through a path for $\gamma_t$ that starts from a level of roughly 13% and is reduced to 0.1% in 10 years (40 quarters). However, for this path to be comparable with the baseline case, it is designed such that the average reduction of excess government debt is 5%, as in the baseline calibration.

- Backloading: the amount and consequent cost of deleveraging is more evenly distributed over time, as the percentage of desired reduction of excess government debt increases over time, while the excess government debt decreases. The features of this path are symmetric to the one assumed for the frontloading case. Specifically, $\gamma_t$ starts from a level of roughly 1% and increases to 10% in 10 years (40 quarters), while the average value for $\gamma_t$ is roughly 5%.

Figure 3.6: Deleveraging Schemes - Government Transfers - Pure Currency Union

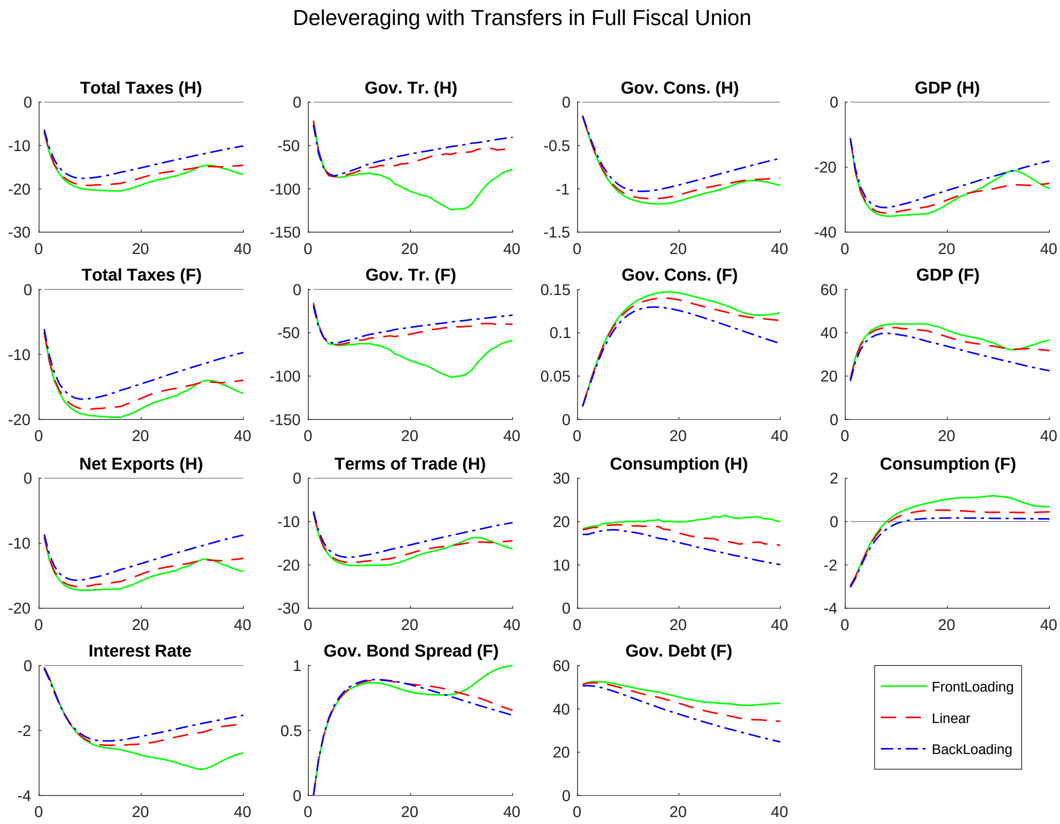



The three paths for $\gamma_t$ are shown in Figure 3.5. We can notice that the frontloading case implies a higher cost of deleveraging than the baseline case in the first 4 years and the backloading case implies a lower speed of convergence for roughly 5 years.

Figures 3.6 and 3.7 compare different deleveraging schemes in respectively the Pure Currency Union and the Full Fiscal Union scenarios, where transfers are used to deleverage the government debt of country $F$. The positive shock to the government debt of country $F$ at time zero implies that from the first quarter onwards government transfers are not kept constant any longer, but they adjust to reduce the government debt by the desired amount. Therefore, the cost of deleveraging affects negatively the economy through a wealth effect on households' consumption, in country F in the Pure Currency Union scenario and in both countries in the Full Fiscal Union scenario.

Looking at Figure 3.6, we can observe that the government transfers strongly decrease by roughly 40% on impact and persist below their steady state for a very long period, due to the deleveraging. The first and main effect is that also consumption in country F strongly decreases and, even if the negative wealth effect of a reduction in government transfers is smoothed by international adjustments, consumption stays below the steady state for at least 5 years. Notice that the wealth effect on consumption is strongly amplified by the response of the bond spread to a higher debt-to-GDP. As we observe, the transaction cost $\delta_t$ slowly increases over time for roughly 10 quarters because, although the government debt constantly decreases, the effect of the initial positive shock on government debt persists because of

Figure 3.7: Deleveraging Schemes - Government Transfers - Full Fiscal Union




the high persistence of the bond spread process. Therefore, the interest rate for country $F$, approximately $i_t^* \approx i_t + \delta_t$, increases, although the central bank decreases the interest rate for country $H$, $i_t$, to contrast deflationary pressures. The higher interest rate in country F brings households in the country to postpone consumption, while the lower interest rate in country H brings households to increase current consumption.

Comparing the different deleveraging schemes, we can notice that as long as the government bond spread increases, the effect of the interest rate on consumption, driven by the increase in the bond spread, dominates the wealth effect that follows from the decrease in government transfers. In the first 12 quarters the difference in the speed of deleveraging can hardly be seen. However, after roughly 3 years the government bond spread decreases, converging back to its steady state value, following the decrease of government debt in country $F$. Therefore, most of the variables converge back to steady state, mainly driven by the reduction in excess government debt, although with different persistence given by the different deleveraging schemes. Indeed, a real government debt higher than steady state implies a deflationary pressure on the economy that is reduced as the excess government debt goes down. It is possible to observe this phenomenon in Figure 3.6 looking at the path of the interest rate, as well as household consumption in country $H$ and GDP in both countries, while household consumption in country $F$ and the bond spread converge back to steady state. Specifically, we can see that with the backloading scheme, due to a higher implied deleveraging rate after 5 years, when the interest rate effect and the wealth effect on household consumption in country $F$ vanish, deflationary pressures reduce faster, implying more stabilization in the economy. Symmetrically, as the speed of deleveraging in the frontloading scheme slows down compared to both the linear and the backloading schemes after 4 years, the strong persistence of the excess government debt observed after 20 quarters at values far above the steady state (20% higher) implies that inflation in country $F$, the interest rate in country $H$ and, hence, household consumption in country $H$ persist at values far from the steady state as well.

Looking at Figure 3.7, it is possible to distinguish the role played by distortionary taxes in the Full Fiscal Union scenario in increasing the persistence of the debt shocks. Specifically, in this scenario the cost of deleveraging is shared between the two countries' government transfers. However, since the tax rates are assumed to move jointly between the two countries, the tax rates in country $H$ decrease rather than increase, as in the Pure Currency Union scenario, following the path of the tax rates in country $F$. Therefore, to keep government debt constant in country H, while decreasing taxes and GDP, it is necessary to decrease transfers in both countries even more. This implies higher persistence for most variables in the economy, especially for consumption in country $H$, because in this country distortionary taxes move in the opposite direction with respect to the Pure Currency Union scenario. For instance, consumption in country H in the Full Fiscal Union scenario, as in Figure 3.7, persists at values above the steady state for more than 10 years, while in the Pure Currency Union scenario, as in Figure 3.6, it converges back to steady state in the same period of time. Although the higher persistence in the economy makes both the movements in the interest rates and in the deleveraging instruments relevant even at longer horizons, the differences in the speed of convergence are enhanced in the Full Fiscal Union scenario because the inflation channel, which is the main driver of these differences, is amplified in country H by the effect of lower taxes on consumption. In Figure 3.7 it is mainly possible to notice the much slower convergence of the variables with the frontloading scheme because in this case the deflationary pressure persists in the medium-to-long run.

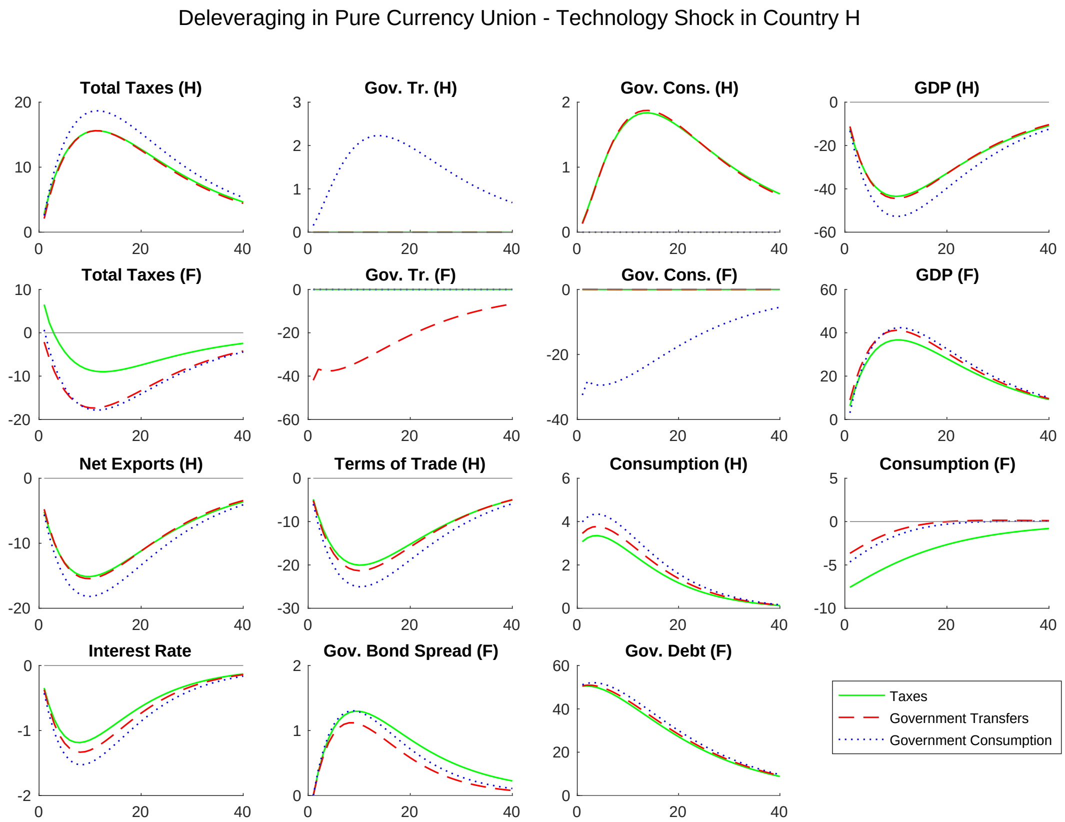
In summary, this analysis shows two main results. First, that the speed of convergence matters only in the medium-to-long run, after household consumption in country $F$ has gone back to steady state. Therefore the backloading scheme is more stabilizing than both the linear and frontloading schemes in all scenarios. Second, it is clear that the Full Fiscal Union scenario is less stabilizing than the Pure Currency Union scenario because of the opposite movements in the tax rates of country $H$.

### 3.4.3 Instruments for Deleveraging

Figures 3.8 and 3.9 show the comparison between different fiscal instruments for deleveraging after a negative technology shock in country $H$ in the Pure Currency Union and in the Full Fiscal Union scenario, respectively. First, the effects of the technology shock, or any other shock, are marginal compared to the much stronger deleveraging shock, as can be seen looking at government consumption and transfers for country $F$. In fact, in Figure 3.8 we only see the dynamics of the fiscal policy instrument in country $F$ when it is used to deleverage government debt, but not when it is used to stabilize the output gap, because the former has a much greater order of magnitude compared to the latter. Moreover we can see the huge volatility of GDP, which is given by the strong deflationary pressure produced by the deleveraging shock, bringing all variables to fluctuate much more than we saw in the simulations without deleveraging. Therefore, it is quite indifferent to analyze the response of the economy to one shock or another while deleveraging, as the deleveraging shock itself produces most of the dynamics that can be seen in the model. For this reason we show most simulations after a technology shock in country $H$, which is one of the most commonly used shocks to analyze business cycles, as most results hold also after other shocks when deleveraging.

Looking at Figures 3.8 and 3.9 we can see that, as explained in the previous section, the shock to the

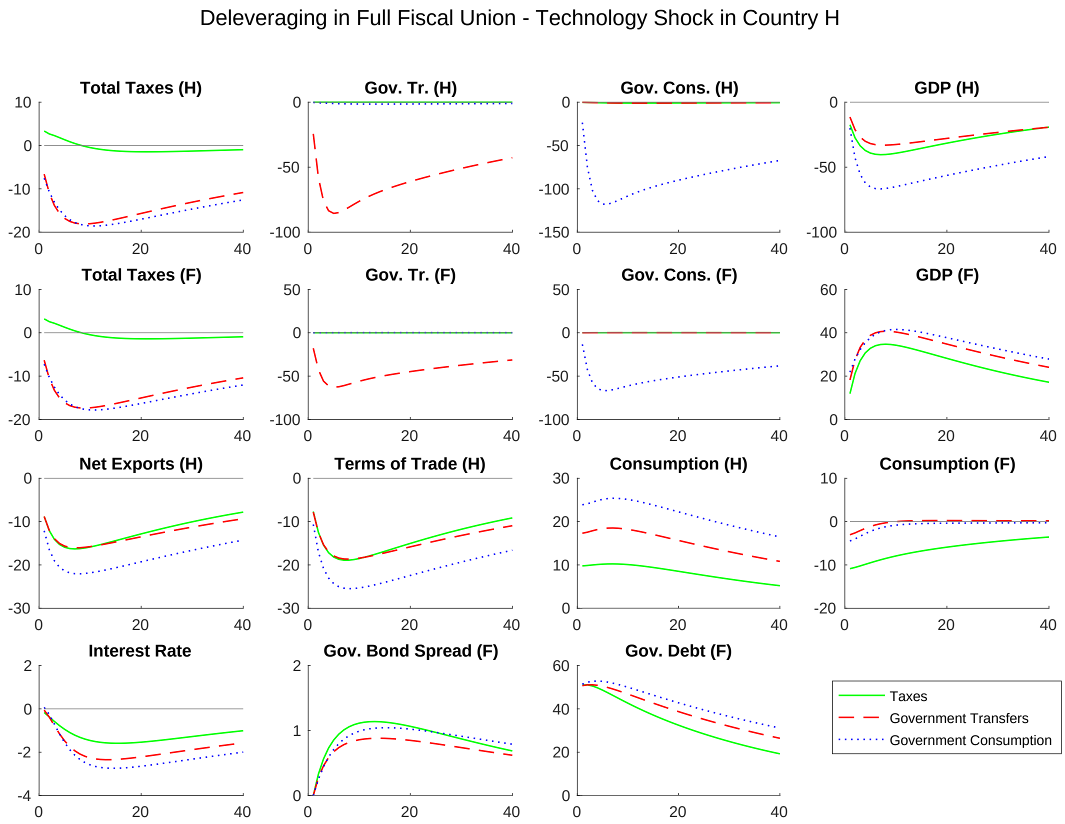
Figure 3.8: Instruments for Deleveraging - Pure Currency Union - Technology Shock in Country H




government debt at time zero, which brings it to 90% of GDP before deleveraging, creates deflationary pressures in country $F$. This can be seen by the movements of the interest rate, which tracks average CPI inflation in the Currency Union, and by the movements of the terms of trade, which show that prices decrease more in country $F$ than in country $H$. The path of net exports is given by movements in the terms of trade, and in turn affects both output in countries $H$ and $F$. The level of government debt affects the path of the government bond spread, which in turn affects consumption in country $F$.

Looking at Figure 3.8 we can see that there are no great differences in the stabilization properties of different fiscal instruments for deleveraging, although using distortionary taxes to deleverage government debt produces more stable dynamics for most variables, except for consumption in country $F$ and the government bond spread. Compared to using government consumption, government transfers produce more stabilization for all variables, mainly because they affect prices less. At the same time distortionary taxes affect the dynamics of the economy more than other fiscal instruments, but depending on the direction of their movement and consequent effect on consumption, they can reduce the deflationary pressure much more than other fiscal instruments for deleveraging, bringing in the end to a gain in stabilization. Figure 3.8 shows that most variables follow qualitatively very similar paths, except for the fiscal policy instruments and household consumption in country $F$. The fiscal policy instruments behave differently by construction. Taxes move much more when government transfers are used to deleverage, because they need to counteract the effect of the smaller increase in GDP on the government budget

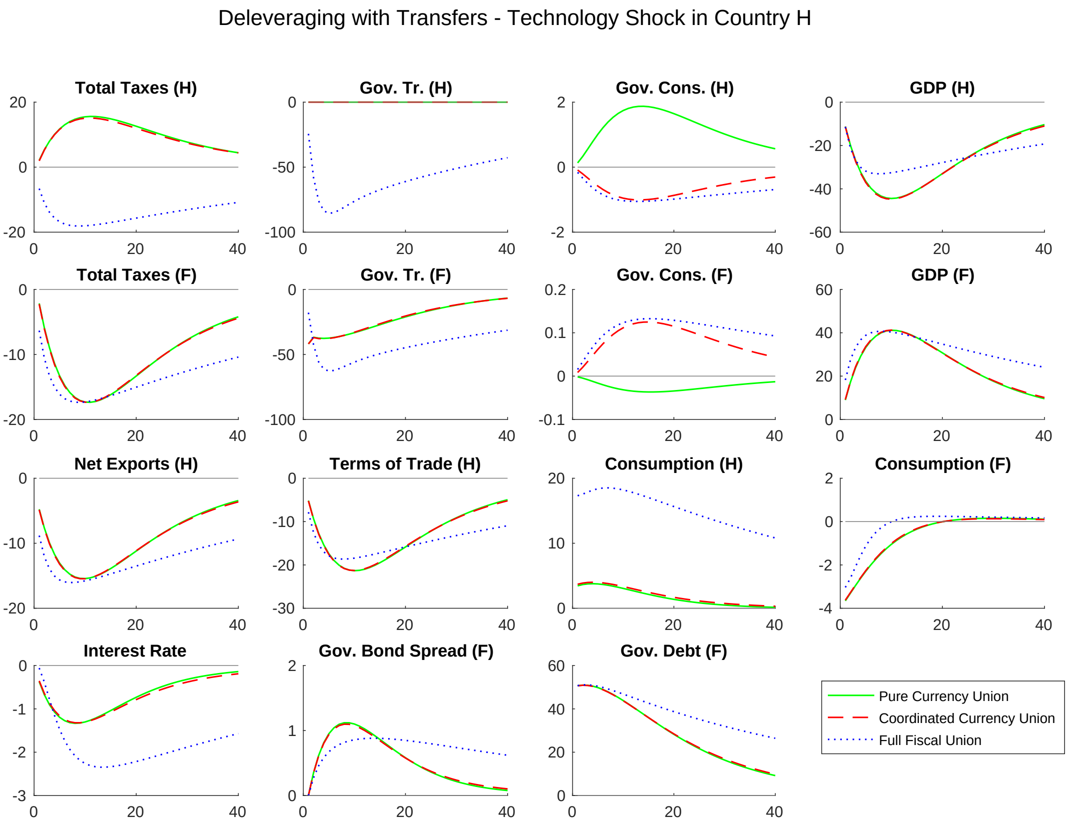
Figure 3.9: Instruments for Deleveraging - Full Fiscal Union - Technology Shock in Country H




constraint. Instead, when deleveraging with taxes, the stronger decrease in consumption in country $F$ makes output in country $F$ increase less, so that taxes don't need to decrease as much to balance the effect of the smaller increase in GDP in country $F$ on the government budget constraint. Taxes also decrease less because they need to finance the deleveraging directly, compared to when the deleveraging is financed by government transfers.

Figure 3.9 shows the same deleveraging dynamics, but in the Full Fiscal Union scenario. The first thing that can be noticed is that there is much more divergence in the paths given by the different instruments for deleveraging, compared to the Pure Currency Union scenario. This is mainly because the tax rates and the fiscal instruments for deleveraging move jointly across countries, which amplifies most dynamics in an incomplete market setting, while the dynamics for different fiscal instruments are more similar to one another in the Pure Currency Union scenario, where taxes move in opposite directions in the two countries. This affects also the persistence of most variables, which take longer to return to their steady state in the Full Fiscal Union scenario, compared to the Pure Currency Union one. Nonetheless, the ranking by stabilization property of the fiscal instruments for deleveraging does not change in the Full Fiscal Union scenario. As in the Pure Currency Union scenario, distortionary taxes provide more stabilization than other fiscal instruments for most variables, except for consumption in country $F$ and GDP in country $H$ in this case. In general, in the Full Fiscal Union scenario there is more deflationary pressure on the economy with respect to the Pure Currency Union scenario, because in this case country

Figure 3.10: Coordination of Deleveraging - Government Transfers - Technology Shock in Country H

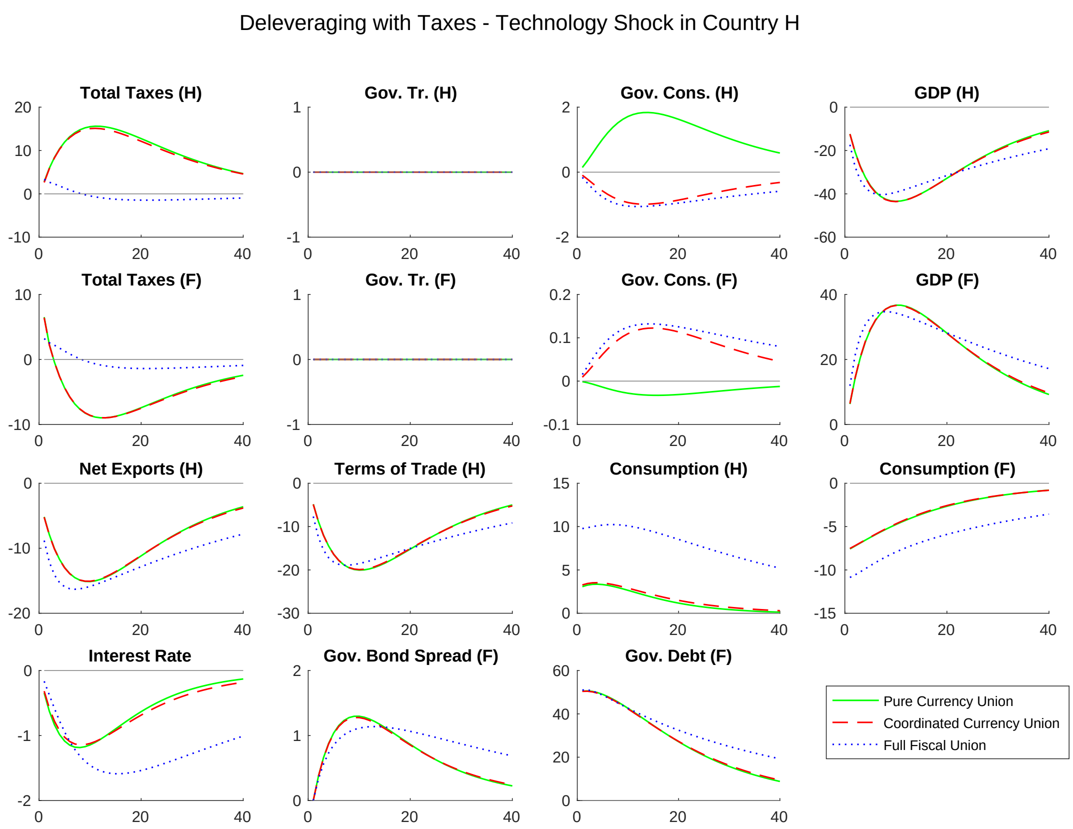



$H$ bares much more of the deflationary pressure and consequent cost of deleveraging, following the consolidation of budget constraints and the joint movement in the tax rates and in the deleveraging instruments. This is why in the Full Fiscal Union scenario GDP in country $H$ is not stabilized as well by deleveraging with taxes. Once again, as in the Pure Currency Union scenario, using government transfers for deleveraging provides more stabilization than using government consumption for all variables, for the same reasons explained above. In fact government transfers create the least distortion in the economy, which is the reason that makes them such a good instrument. However, distortionary taxes may be able to reduce the distortions in the economy if they move in the right direction, by counteracting the distortions created by other means. This is why they are a very good instrument, although more difficult to use. In the end taxes are the deleveraging instrument which stabilizes more the economy in any scenario.

### 3.4.4 Coordination of Deleveraging

Figures 3.10 and 3.11 compare the three different degrees of fiscal policy coordination, or the three different scenarios, while using government transfers or taxes, respectively, to deleverage the government debt of country $F$. We can see in both cases how there is practically no difference in the dynamics of all variables between the Pure Currency Union scenario and the Coordinated Currency Union one. In

Figure 3.11: Coordination of Deleveraging - Taxes - Technology Shock in Country H




fact the only difference is in the movements of government consumption which targets the output gap in the Pure Currency Union scenario and the net exports gap in the Coordinated Currency Union one. This tells us that stabilizing one variable or the other is irrelevant for the dynamics of all other variables. This is because the technology shock and consequent fiscal policy response is of a much smaller order of magnitude compared to the shock created by the high government debt and consequent deleveraging. Therefore the difference in the dynamics of all variables can be seen in the Full Fiscal Union scenario, where all variables evolve in a quite different way compared to the other scenarios. This effect is given by the consolidation of budget constraints, which makes country $H$ bear part of the deleveraging cost, and the joint movements in the tax rates which, as we saw before, generally amplify the movements of most variables. In fact we can see that the Full Fiscal Union scenario provides a greater amplification of the deleveraging shock than the other scenarios, as we can see by looking at the increased volatility of most variables. This is mainly due to the fact the financial markets are incomplete, which makes the joint movements of the tax rates exacerbate the deleveraging shock rather than smooth it, like taxes smooth shocks in the complete financial market setting of Cole, Guerello and Traficante (2016). Therefore, as in the simulations in incomplete markets without deleveraging, the Full Fiscal Union scenario is the worst in terms of stabilization of the deleveraging shock in incomplete markets.

Looking at Figure 3.10 we can see the different stabilization properties of the Full Fiscal Union scenario compared to the other two scenarios, which yield approximately the same dynamics. It is clear that the Full Fiscal Union scenario amplifies more than the other scenarios the deleveraging shock. It is also the scenario that yields the greatest persistence of most variables away from their steady state. The fact that both countries share the cost of deleveraging creates some perverse effects in incomplete markets, mainly given by the opposite movement in the tax rates in country $H$ because of the shared cost of fiscal policy. This creates a higher deflationary pressure, which can be seen by the more amplified movements in the interest rate, and brings real government debt to decrease at a slower rate, giving more persistence to the government bond spread. The lower taxes, lower interest rate and lower government bond spread push consumption higher in both countries, amplifying the movements and persistence of consumption in country $H$, but actually stabilizing more consumption in country $F$. In the Full Fiscal Union scenario, since real transfers and taxes in country $H$ move jointly with real transfers and taxes in country $F$, the lower taxes in country $H$ make consumption increase more and government debt decrease less, because of the effect of deflation, and thus GDP decreases less in country $H$. The higher GDP in both countries pushes taxes to decrease more to satisfy the budget constraint, while the greater deflation gives more persistence to government debt and increases the cost of deleveraging borne by real transfers in both country. This perverse effect makes it very costly and destabilizing to consolidate budget constraints, especially by imposing a joint movement of the fiscal policy instruments.

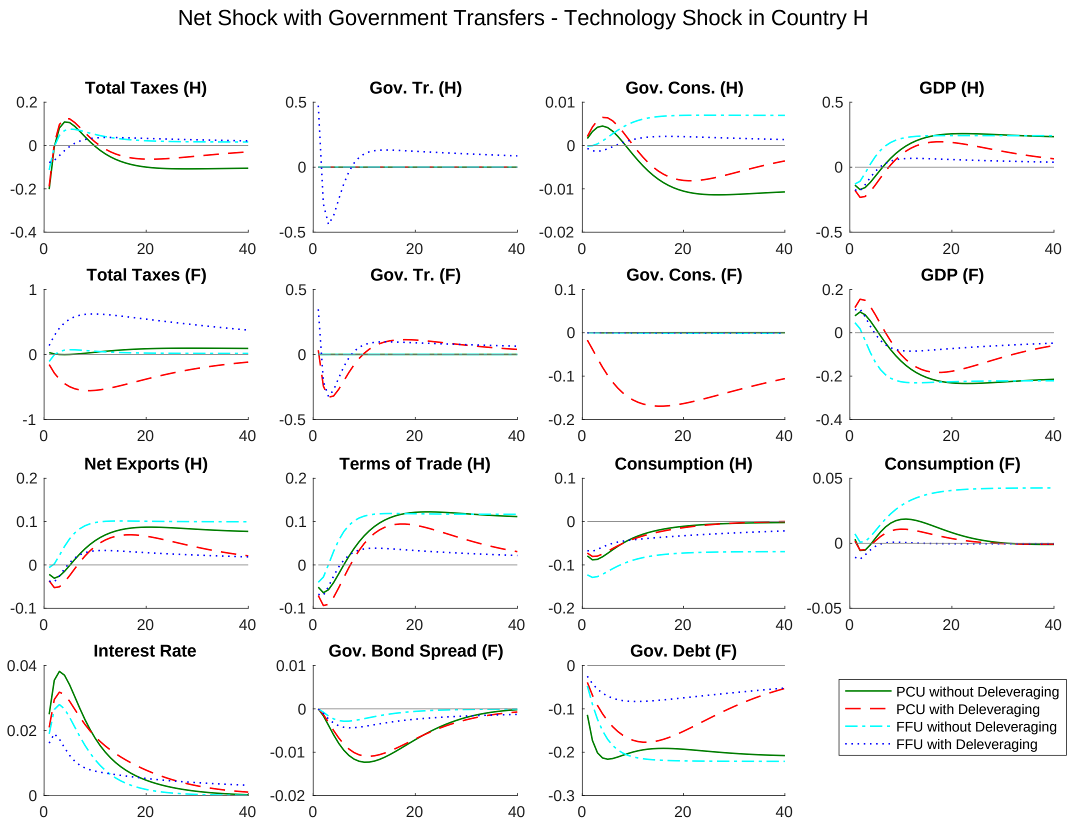
Looking at Figure 3.11, most of the reasoning done when using real transfers to deleverage still holds when using taxes. In this case taxes finance all expenditure and deleveraging and actually move much less in the Full Fiscal Union scenario compared to the other scenarios. In reality both taxes decrease rather than increase because the effect of the increase in GDP in country $F$ on the consolidated budget constraint is greater than the sum of the cost of deleveraging and of the effect of the decrease in GDP in country $H$ on the consolidated budget constraint. This shows how the main drivers of the dynamics are given by the variations in GDP in both countries, which are mainly given by the deflationary effect of the deleveraging shock on net exports through the terms of trade. Unfortunately this effect is amplified in the Full Fiscal union scenario, because taxes and thus prices decrease instead of increasing in country $H$ creating more deflation and bringing net exports lower, pushing GDP in country $F$ to persist longer away from its steady state. When using taxes to deleverage in the Full Fiscal Union scenario the only stabilization is on the fiscal instruments, while all other variables are more volatile and persistent. This brings us to think that it is not convenient to consolidate budget constraints across countries with this kind of fiscal policy coordination. This is true with incomplete markets and even more when deleveraging government debt. On the other hand some different form of fiscal policy coordination might provide more stabilization, along with the same consolidation of budget constraints.

### 3.4.5 Net Shocks from Deleveraging

Figures 3.8 and 3.9 show the dynamic simulations after a negative technology shock in country $H$ in the Pure Currency Union and Full Fiscal Union scenarios, respectively, when country $F$ is forced to reduce its government debt. The huge government debt deleveraging process that the fiscal authority is forced to follow makes the effects of other shocks marginal on the economy. For this reason, in this Section we try to isolate the effects of a productivity shock from those of the huge deleveraging shock hitting the economy.

In Figures 3.12 and 3.13 we show the impulse response functions to a negative technology shock in country $H$ when neither country $H$ nor country $F$ are deleveraging (PCU and FFU without deleveraging) and in the case of deleveraging (PCU and FFU with deleveraging). In the latter case, the impulse responses are presented in deviation from the path implied by the government debt reduction, by subtracting the effect of the pure deleveraging shock. This way we try to disentangle the effect of the deleveraging shock from the constraint it imposes on fiscal policy stabilization, while leaving the effects

Figure 3.12: Net Shock with Government Transfers - Technology Shock in Country H

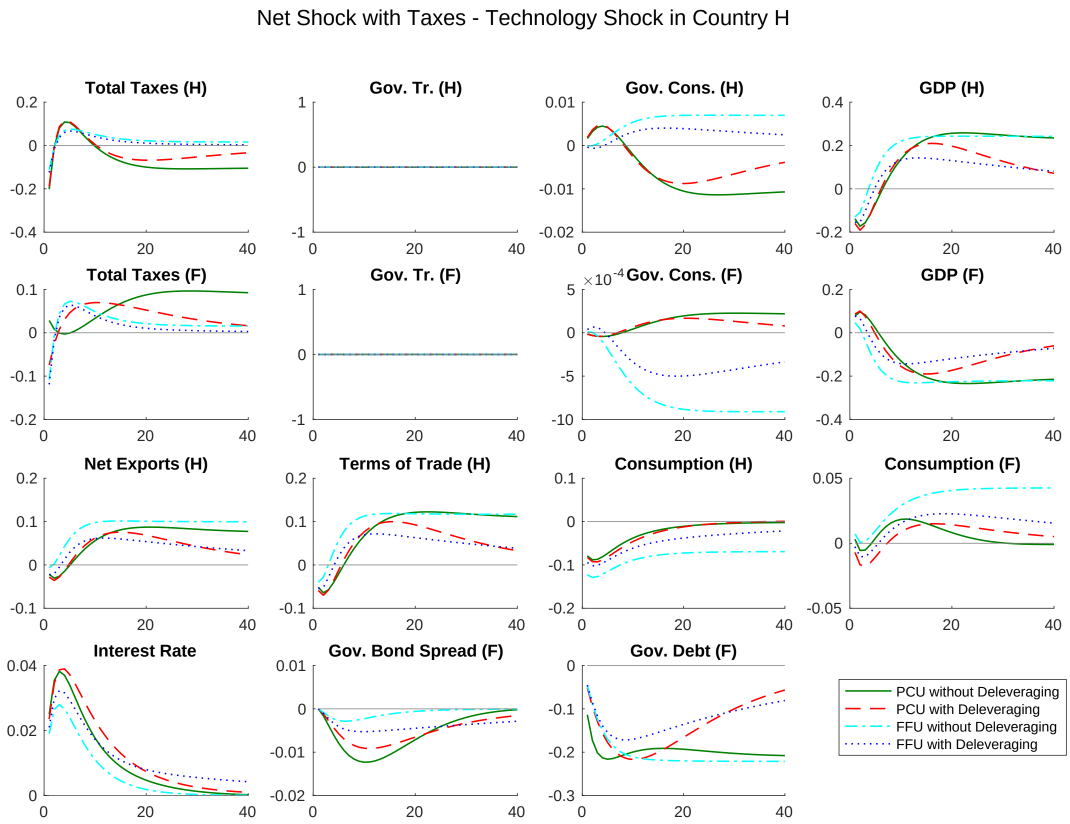



of the technology shock for all cases[14]. After a negative productivity shock in country $H$, the terms of trade deteriorate, reducing net exports and output in country $H$ on impact. Because of the assumption of incomplete markets, household consumption in the two countries is not synchronized and moves in different directions, decreasing in country $H$ and increasing in country $F$.

Figure 3.12 analyzes the case in which the government in country $F$ uses transfers to deleverage. The graph confirms that the Pure Currency Union scenario stabilizes most variables of the economy more than the Full Fiscal Union one, as documented in the previous Section. In particular, consumption and output are less volatile, while CPI inflation increases more on impact, as can be inferred from the dynamic response of the nominal interest rate. In the Pure Currency Union scenario, by subtracting the effect of the deleveraging shock, we obtain dynamics which are quantitatively close to those without deleveraging, while in the Full Fiscal Union scenario the distance between the two dynamics is still quantitatively big. When the government in country $F$ has to deleverage in the Full Fiscal Union scenario, government transfers and taxes move jointly in both countries, reducing the volatility of real foreign government debt. As a consequence all variables are more stabilized in the Full Fiscal Union scenario with deleveraging compared to other scenarios, except for household consumption in country $H$. On the other hand, when both budgets are balanced in the Pure Currency Union scenario, consumption

[14]A similar procedure is followed by Bodenstein, Guerrieri and Gust (2013) which presents impulse responses in deviation from the path implied by a severe recession.

Figure 3.13: Net Shock with Taxes - Technology Shock in Country H




paths are more persistently distant from their steady state levels reflecting the fact that markets are incomplete and country-specific fiscal instruments dampen the effects of the shock. In both scenarios (PCU and FFU) the overall volatility and persistence is lower under deleveraging. We interpret this finding in this way: when a productivity shock hits the economy, no matter if it originates in the other country, and one country is in a relevant deleveraging process, the shock contributes less in driving the dynamic response of the economy.

Figure 3.13 presents the same analysis when taxes are the instrument used to deleverage government debt. The dynamics do not vary significantly with respect to the case in which transfers are used. However, the volatility of the economy turns out to be higher than in the case in which government transfers are used to deleverage. As an example, the average volatility of foreign consumption under the Pure Currency Union scenario is about double when taxation varies to accomplish deleveraging, and it is even bigger (about 5 times bigger) in the Full Fiscal Union scenario. This phenomenon is amplified in the Full Fiscal Union scenario, making it worst in terms of stabilization even with deleveraging, compared to both Pure Currency Union scenarios. This result suggests that characterizing the best fiscal instrument to stabilize the economy is a difficult task while deleveraging occurs and that the conclusions can be radically different when we analyze only the effects of a shock. In the end, in the presence of a shock, the deleveraging process drives most variables back to steady state faster, overcoming partially the volatility and persistence induced by the shock.

## 3.5 Welfare Analysis based on an ad hoc Loss Function

As a further step of the analysis, we compare the stabilization properties of the fiscal policy scenarios and of the deleveraging instruments by means of an ad hoc Loss Function. Since the main focus of this work is to evaluate the role of fiscal policy as a stabilizer of the economy, we prefer to rely on a quadratic loss function rather than look at consumption equivalent variations (CEV). Specifically, fiscal policy has a stabilizing function for the real economy similar to the role played by monetary policy for prices and, hence, together they aim to reduce both the inflation gap and the output gap. As argued in Blanchard, Erceg and Lindé (2015), since our model and calibration assume a large resource slack (negative net exports) between the periphery (country $F$) and the core (country $H$), the gains in terms of consumption and unemployment related to closing the output gap are underestimated by utility-based measures.

Using a standard quadratic Loss Function, the policymakers are assumed to care only about minimizing the square of the output gap and of the inflation gap in both regions. Each region's Loss Function is, hence, simply the sum of the square of the inflation gap and the square of the output gap, with weights 3 and 1 respectively. The overall Loss Function is given by the weighted average of each region's Loss Function:

$$Loss = \sum_{j=0}^{\infty} \beta^j \left\{ h \left[ (\hat{\pi}_{t+j})^2 + \frac{1}{3} (\hat{Y}_{t+j})^2 \right] + (1-h) \left[ (\hat{\pi}^*_{t+j})^2 + \frac{1}{3} (\hat{Y}^*_{t+j})^2 \right] \right\} \tag{3.166}$$

The welfare costs for each scenario are computed as the difference in the conditional mean of the second order approximation of the Loss Function around the non-stochastic steady state with respect to the scenario with the lowest loss.

Table 3.2: Welfare Costs: Comparison of Fiscal Scenarios

| Welfare Costs based on ad hoc loss function | | | |
|---|---|---|---|
| Debt stabilization with taxes without deleveraging | | | |
| | Country H | Country F | Average |
| PCU* | 0% | 0% | 0% |
| CCU | 230% | 227% | 228% |
| FFU | 155% | 157% | 156% |

Welfare Costs are computed as $\frac{Loss_a - Loss_b}{Loss_b}$, with $b$ the scenario featuring the lowest loss for the selected fiscal instrument (indicated with *)

At first we compare the stabilizing properties of the the different fiscal policy scenarios in a model with incomplete markets in which nominal government debt in both countries is kept constant by using distortionary taxes. The welfare losses with this model specification are reported in Table 3.2 for each of the three fiscal policy scenarios taken into consideration. This allows us to support the results of Section 3.4.1, which compares our incomplete markets setting with a complete markets one. In particular, looking at Table 3.2, it is clear that stabilizing an international variable, such as net exports, by moving government consumption strongly reduces the stabilizing power of fiscal policy. This contrasts with the results in Cole, Guerello and Traficante (2016) because in the case of complete markets we find that targeting net exports is strongly welfare improving with respect to the case with exogenous fiscal policy, while targeting the output gap leads to a higher loss in terms of output and price stabilization with respect to both the case of targeting net exports and of exogenous fiscal policy. The main reason for the opposite ranking of the different degrees of fiscal policy coordination observed under incomplete rather than complete markets relies on the different drivers of consumption in the two models and, hence, on the ability of fiscal policy to effectively stabilize its target. With complete markets, thanks to full international risk sharing, household consumption in both countries moves jointly through the adjustment of net foreign assets, the terms of trade and, hence, net exports. Therefore, stabilizing net exports by counteracting variations in domestic aggregate demand with government consumption strongly reduces international spillovers, mainly through the substitution effect of a price change. On the contrary, if financial markets are incomplete, the response of consumption in the two countries typically diverges after a shock because consumption in both countries is mainly driven by movements in the interest rate, while consumption in country $F$ is also driven by the government bond spread. Therefore, the international variables, such as net exports and the terms of trade, depend mainly on the different price rigidity in the two countries. In this case, stabilizing net exports reduces international spillovers less because it does not counteract the substitution effect of a price change, while the movements in the tax rates, which impact household consumption in both countries, counteract the direct effect of increasing government consumption on net exports. With incomplete markets consumption follows closely the interest rate, net exports adjusts to relative price changes and create a gap between consumption and GDP in the same country, that is reduced by targeting the output gap, through the effect of government consumption on output and that of taxes on consumption.

Looking at Table 3.2 we observe that the Full Fiscal Union scenario is more stabilizing than the Coordinated Currency Union one, although the cost of targeting the net exports gap rather than the output gap is still large. This contrasts with the results under complete markets (See Cole, Guerello and Traficante (2016)), for which there are marginal differences between reducing international spillovers by means of targeting net exports, consolidating the budget constraints and both, because symmetric movements in taxes drive consumption in the same direction for both countries and, hence, reduce the substitution effect of a price change as well as stabilize net exports. If financial markets are incomplete, international spillovers are less relevant and movements in taxes reduce the fluctuations in household consumption in both countries. Although consolidating budget constraints amplifies the fluctuations in consumption because taxes are more stable, this does not affect the dynamics of output much, while it reduces price volatility in both countries.

Table 3.3 compares the welfare costs in terms of the Loss Function for the three fiscal policy scenarios in the case in which country $F$ has a level of debt-to-GDP above the target of 60% and, hence, follows a linear deleveraging scheme. The results are reported for the three alternative fiscal instruments for deleveraging and support the findings in Section 3.4.4, that targeting the output gap is slightly welfare improving with respect to targeting the net exports gap, for all instruments used for deleveraging. However, the results regarding the Full Fiscal Union scenario do not hold as with complete markets because, under deleveraging, consolidating budget constraints is much more destabilizing than targeting net exports. This is mainly due to the fact that the fiscal policy instruments move jointly, which implies a reduction in the tax rates for both countries under deleveraging with transfers and a reduction only in the medium-to-long run under deleveraging with taxes. The movement in the opposite direction for taxes in country $H$ in the Full Fiscal Union scenario, makes household consumption, inflation and output more persistent and more volatile.

Looking at Table 3.3, it is also possible to notice that the welfare costs from consolidating budget constraints is extremely amplified if deleveraging is achieved by using government consumption. To deeply investigate this phenomenon, Table 3.4 reports the welfare costs of using a specific fiscal instrument

Table 3.3: Welfare Costs: Comparison of Deleveraging Scenarios by Instrument

| Welfare Costs based on ad hoc loss function | | | |
|---|---|---|---|
| **Deleveraging Instrument: Government Consumption** | | | |
| | Country H | Country F | Average |
| PCU* | 0% | 0% | 0% |
| CCU | -1% | 1% | 0% |
| FFU | 243% | 115% | 181% |
| **Deleveraging Instrument: Government Transfers** | | | |
| | Country H | Country F | Average |
| PCU* | 0% | 0% | 0% |
| CCU | 2% | 1% | 1% |
| FFU | 1% | 82% | 47% |
| **Deleveraging Instrument: Taxes on Sales and on Wages** | | | |
| | Country H | Country F | Average |
| PCU* | 0% | 0% | 0% |
| CCU | 2% | 1% | 1% |
| FFU | 21% | 31% | 26% |

Welfare Costs are computed as $\frac{Loss_a - Loss_b}{Loss_b}$, with $b$ the scenario featuring the lowest loss for the selected fiscal instrument (indicated with *)

for deleveraging in each of the three scenarios for fiscal policy coordination. It is possible to notice that if the cost of deleveraging, along with the cost of government consumption, is financed by distortionary taxes, in all three scenarios we achieve the most stabilization. As discussed in Section 3.4.3, if deleveraging is achieved with a different instrument than taxes, this adds an additional source of fluctuation in the economy, because the large movements in this fiscal instrument drive large fluctuations in output in both countries and only marginally affect consumption. Therefore, if the deleveraging is performed by moving government consumption, it needs to fluctuate largely to achieve its goal and this destabilizes further output in both countries. In particular, given the direct impact of government consumption on output, its dynamics are amplified when government consumption is employed for deleveraging and government transfers target either the output gap or the net exports gap.

From the analysis of Table 3.3 we can see that under deleveraging, to reduce international spillovers, by targeting net exports, does not make the economy more stable, but marginally increases welfare costs. This result holds even if using taxes rather than government consumption or government transfers is welfare improving, as shown in Table 3.4. Consolidating budget constraints, especially by moving jointly the fiscal instruments, is highly destabilizing with any of the three fiscal instruments because it increases the persistence of consumption in both countries. Furthermore, these results are amplified if government consumption or government transfers are used to reduce government debt, because the large fluctuations of these instruments amplify movements in output.

## 3.6 Conclusions and Possible Extensions

In this paper we build a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union with *incomplete* financial markets, to study the effects of government debt deleveraging in one country and the stabilization properties of different fiscal instruments for deleveraging and of different scenarios for fiscal policy coordination. This allows us to study also the international transmission of shocks and

Table 3.4: Welfare Costs: Comparison of Deleveraging Instruments by Scenario

| Welfare Costs based on ad hoc loss function | | | |
|---|---|---|---|
| **Fiscal Scenario: Pure Currency Union** | | | |
| | Country H | Country F | Average |
| Gov. Cons. | 44% | 30% | 37% |
| Gov. Tr. | 1% | 22% | 12% |
| Taxes* | 0% | 0% | 0% |
| **Fiscal Scenario: Coordinated Currency Union** | | | |
| | Country H | Country F | Average |
| Gov. Cons. | 41% | 30% | 35% |
| Gov. Tr. | 2% | 23% | 12% |
| Taxes* | 0% | 0% | 0% |
| **Fiscal Scenario: Full Fiscal Union** | | | |
| | Country H | Country F | Average |
| Gov. Cons. | 308% | 113% | 204% |
| Gov. Tr. | -16% | 70% | 23% |
| Taxes* | 0% | 0% | 0% |

Welfare Costs are computed as $\frac{Loss_a - Loss_b}{Loss_b}$, with $b$ the instrument featuring the lowest loss for the selected fiscal scenario (indicated with *)

their stabilization in a setting with *incomplete* international financial markets.

In the simulations after a technology shock in incomplete markets without deleveraging, we find that stabilizing output by using government consumption or transfers produces more stabilization than without using a targeting rule, while stabilizing net exports actually amplifies most dynamics. Also, although consolidating budget constraints may provide more stabilization, moving fiscal instruments jointly does not, which implies that the Full Fiscal Union scenario might be more stabilizing than the Coordinated Currency Union scenario, but is surely not more stabilizing than the Pure Currency union scenario. The ranking by stabilization property of these scenarios is reversed compared to that obtained in a similar model with *complete* international financial markets, as in Cole, Guerello and Traficante (2016), which shows how important the assumption of *incomplete* financial markets is for the dynamics of the model in this paper.

By studying different deleveraging schemes, we find that the backloading scheme is more stabilizing than both the linear and frontloading schemes in all scenarios, while the linear deleveraging scheme produces more stabilization than the frontloading scheme.

The main drivers of the dynamics after a deleveraging shock are given by the variations in GDP in both countries, which are mainly given by the deflationary effect of the deleveraging shock on net exports through the terms of trade. This is because variations in GDP affect the tax base, which pushes tax rates to vary so as to satisfy the budget constraint. This effect is amplified in the Full Fiscal Union scenario, because taxes and thus prices decrease instead of increase in country $H$, creating more deflation and bringing net exports lower, pushing GDP in country $F$ to persist longer away from its steady state. This brings us to think that it is not convenient to consolidate budget constraints across countries with this kind of fiscal policy coordination. This is true with incomplete markets and even more when deleveraging government debt. On the other hand some different form of fiscal policy coordination might provide more stabilization, along with the same consolidation of budget constraints.

By comparing the dynamics after a technology shock with and without deleveraging, we can disentangle the effect of the limitations imposed on fiscal policy stabilization by the deleveraging shock from the effect of the deleveraging shock itself. In both the Pure Currency Union scenario and the Full Fiscal Union one the overall volatility and persistence is lower under deleveraging, compared to no deleveraging. This is because when a productivity shock hits the economy, no matter if it originates in the other country, and one country is in a relevant deleveraging process, the shock contributes less in driving the dynamic response of the economy. This phenomenon is amplified in the Full Fiscal Union scenario, making it worst in terms of stabilization even with deleveraging, compared to the two Pure Currency Union scenarios. This result suggests that characterizing the best fiscal instrument to stabilize the economy is a difficult task while deleveraging occurs and that the conclusions can be radically different when we analyze only the effects of a shock. In the end, in the presence of a shock, the deleveraging process drives most variables back to steady state faster, overcoming partially the volatility and persistence induced by the shock.

From the analysis of the welfare costs based on an ad hoc Loss Function, by fiscal scenario and by deleveraging instrument, we can see that by targeting net exports the economy is not more stable, but welfare costs increase marginally, except when using government consumption to deleverage, which decreases welfare costs marginally. This result holds even if using taxes to deleverage, rather than government consumption or government transfers, is welfare improving. Consolidating budget constraints and moving fiscal instruments jointly, instead, is highly destabilizing with any of the three fiscal instruments because it increases the persistence of output and inflation in both countries. Furthermore, these results are amplified if government consumption or government transfers are used to reduce government debt, because the large fluctuations of these instruments amplify movements in output. The results given by the welfare costs in terms of an ad hoc Loss Function confirm the findings from the analysis of the different simulations.

Our main results are the following. First, with incomplete markets targeting the output gap produces more stabilization than targeting the net exports gap, while under deleveraging it makes hardly a difference because the deleveraging shock drives the dynamics much more than other shocks. Second, consolidating budget constraints may provide more stabilization, but the joint movement of the fiscal instruments does not in an incomplete market setting, which is even more true under deleveraging. These are the two results which are overturned in a setting with *complete* international financial markets. Third, taxes are a better instrument for deleveraging compared to government consumption or transfers, because they counteract the deflationary effect of the deleveraging shock much more. Fourth, by backloading the deleveraging process one can achieve greater stabilization over time, because the fiscal instruments move less initially, as they need to deleverage less at the beginning and more over time.

Our model and results have nonetheless some shortcomings, which entail possible future avenues of research. First, our model focuses on a very specific design of fiscal policy, and one can consider coordination strategies which are different from ours, and which might bring to an increase in stabilization in the economy. Second, even only allowing the Central Bank to be more aggressive in targeting inflation, might reduce the deflationary effect induced by the deleveraging shock. Third, considering the Zero Lower Bound on the nominal interest rate might make a big difference, especially because the deleveraging shock pushes the interest rate down, possibly bringing the economy into a liquidity trap.

This research was conducted to assess the effects of deleveraging in the Eurozone, as requested by the European Commission, and the stabilization properties of different instruments for deleveraging and different scenarios for fiscal policy coordination, to bring to the proper government debt management in a Currency Union. The policy prescriptions for the Eurozone in this case are that government debt should be reduced more gradually, to avoid excessive volatility in the economy, and should be reduced by varying distortionary taxes, while countries should concentrate on reducing their output gap and avoid creating a fiscal union like the one we describe, until financial markets are integrated completely, allowing for perfect international risk-sharing, like in our previous complete markets model.

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# Appendix A

# Appendix to Chapter 2

As a tool for students and researchers studying models of this kind for the first time, we provide as many mathematical derivations of the model as possible here in the Appendix. This is meant for learning purposes, especially concerning complex models with cumbersome mathematical derivations.

## A.1 Derivation of the Consumption Index in the case of Unitary Substitutability between Domestic and Foreign Goods

Here the aim is to compute the limit of the Consumption Index for $\eta \to 1$:

$$\lim_{\eta\to 1}\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{A.1}$$

Substituting $\eta = 1$ yields an expression of the form $1^{\infty}$, which is indeterminate. To solve it, the expression must be transformed in one of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ to be able to apply l'Hôpital's rule.

$$= \lim_{\eta\to 1} e^{\ln\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}} \tag{A.2}$$

$$= \lim_{\eta\to 1} e^{\frac{\eta}{\eta-1}\ln\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]} \tag{A.3}$$

$$= \lim_{\eta\to 1} e^{\frac{\ln\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]}{\frac{\eta-1}{\eta}}} \tag{A.4}$$

This is an expression with a fraction of the form $\frac{0}{0}$ for which l'Hôpital's rule can be applied. The rule says that if

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{0}{0} \implies \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)} \tag{A.5}$$

So now, computing the two derivatives yields:

$$\frac{\partial \ln\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]}{\partial\eta} = \tag{A.6}$$

$$= \frac{-\frac{1}{\eta^2}(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}\ln(1-\alpha)+\frac{1}{\eta^2}(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}\ln C_{H,t}-\frac{1}{\eta^2}\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\ln\alpha+\frac{1}{\eta^2}\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\ln C_{F,t}}{\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]} \tag{A.7}$$

$$= \frac{(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}(\ln C_{H,t}-\ln(1-\alpha))+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}(\ln C_{F,t}-\ln\alpha)}{\eta^2\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]} \tag{A.8}$$

$$\frac{\partial\left[\frac{\eta-1}{\eta}\right]}{\partial\eta} = \frac{\eta-\eta+1}{\eta^2} = \frac{1}{\eta^2} \tag{A.9}$$

Now the two derivatives can be substituted in expression A.4 to compute the limit.

$$\lim_{\eta\to 1} e^{\frac{\frac{(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}\left[\ln C_{H,t}-\ln(1-\alpha)\right]+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\left[\ln C_{F,t}-\ln\alpha\right]}{\eta^2\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]}}{\frac{1}{\eta^2}}} = \tag{A.10}$$

$$= e^{\frac{(1-\alpha)\left[\ln C_{H,t}-\ln(1-\alpha)\right]+\alpha\left[\ln C_{F,t}-\ln\alpha\right]}{[(1-\alpha)+\alpha]}} = \tag{A.11}$$

$$= e^{(1-\alpha)\ln\left(\frac{C_{H,t}}{1-\alpha}\right)+\alpha\ln\left(\frac{C_{F,t}}{\alpha}\right)} = \tag{A.12}$$

$$= e^{\ln\left(\frac{C_{H,t}}{1-\alpha}\right)^{1-\alpha}} e^{\ln\left(\frac{C_{F,t}}{\alpha}\right)^{\alpha}} = \tag{A.13}$$

$$= \left(\frac{C_{H,t}}{1-\alpha}\right)^{1-\alpha}\left(\frac{C_{F,t}}{\alpha}\right)^{\alpha} \tag{A.14}$$

## A.2 Derivation of the Utility function in the case of Unitary Intertemporal Elasticity of Substitution

Here the aim is to compute the limit of the utility function $U(C_t)$ for $\sigma \to 1$:

$$\lim_{\sigma\to 1}\frac{C_t^{1-\sigma}-1}{1-\sigma} \tag{A.15}$$

Substituting $\sigma = 1$ yields an expression of the form $\frac{0}{0}$, so l'Hôpital's rule can be applied. The rule says that if

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{0}{0} \implies \lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)} \tag{A.16}$$

Applying the rule by taking the ratio of derivatives with respect to $\sigma$ yields:

$$\lim_{\sigma\to 1}\frac{C_t^{1-\sigma}-1}{1-\sigma} = \lim_{\sigma\to 1}\frac{-C_t^{1-\sigma}\ln C_t}{-1} = \ln C_t \tag{A.17}$$

## A.3 Derivation of the Elasticity of Intertemporal Substitution

The elasticity of intertemporal substitution is defined as the percent change in consumption growth divided by the percent change in the gross real interest rate:

$$\frac{\frac{\partial\left(\frac{C_{t+1}}{C_t}\right)}{\left(\frac{C_{t+1}}{C_t}\right)}}{\frac{\partial(1+r_t)}{(1+r_t)}} \equiv \frac{\partial\left(\frac{C_{t+1}}{C_t}\right)}{\partial(1+r_t)}\frac{(1+r_t)}{\left(\frac{C_{t+1}}{C_t}\right)} \tag{A.18}$$

Assume consumers maximize present-value utility $U \equiv \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$, where $U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$ subject to a simple budget constraint of the kind:

$$C_t + D_{t+1} \leq D_t(1 + r_{t-1}) + W_t N_t \tag{A.19}$$

where all variables are in real values and $D_t$ are risk-free assets held at the beginning of period $t$, $W_t$ is nominal wage in period $t$, and $N_t$ is labour supplied in period $t$. Then the first order conditions, with respect to $C_t$, $C_{t+1}$, and $D_{t+1}$, respectively, are:

$$\beta^t U'(C_t) = \lambda_t \tag{A.20}$$

$$\beta^{t+1} U'(C_{t+1}) = \lambda_{t+1} \tag{A.21}$$

$$\lambda_t = \lambda_{t+1}(1 + r_t) \tag{A.22}$$

Then, dividing the first by the second and substituting into the third yields:

$$\frac{U'(C_t)}{\beta U'(C_{t+1})} = (1 + r_t) \tag{A.23}$$

which is the standard Euler Equation. Computing marginal utilities yields:

$$\frac{C_t^{-\sigma}}{\beta C_{t+1}^{-\sigma}} = (1 + r_t) \tag{A.24}$$

Expressing consumption growth as a function of the interest rate, and making the simplifying assumption $\beta = 1$, to isolate the role of the interest rate, yields:

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = (1 + r_t) \implies \frac{C_{t+1}}{C_t} = (1 + r_t)^{\frac{1}{\sigma}} \tag{A.25}$$

Taking the derivative with respect to $(1 + r_t)$ yields:

$$\frac{\partial \left(\frac{C_{t+1}}{C_t}\right)}{\partial(1 + r_t)} = \frac{1}{\sigma}(1 + r_t)^{\frac{1}{\sigma}-1} \tag{A.26}$$

Then, substituting the derivative in expression A.18 yields the elasticity of intertemporal substitution:

$$\frac{\partial \left(\frac{C_{t+1}}{C_t}\right)}{\partial(1 + r_t)} \frac{(1 + r_t)}{\left(\frac{C_{t+1}}{C_t}\right)} = \frac{1}{\sigma}(1 + r_t)^{\frac{1}{\sigma}-1} \frac{1 + r_t}{(1 + r_t)^{\frac{1}{\sigma}}} = \frac{1}{\sigma}(1 + r_t)^{-1}(1 + r_t) = \frac{1}{\sigma} \tag{A.27}$$

## A.4 Derivation of the Frisch Elasticity of labour Supply

The Frisch elasticity of labour supply is defined as the percent change in the supply of labour divided by the percent change in the nominal wage:

$$\frac{\frac{\partial N_t}{N_t}}{\frac{\partial W_t}{W_t}} \equiv \frac{\partial N_t}{\partial W_t} \frac{W_t}{N_t} \tag{A.28}$$

Assume consumers maximize present-value utility $U \equiv \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$, where $U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$ subject to a simple budget constraint of the kind:

$$C_t + D_{t+1} \leq D_t(1 + r_{t-1}) + W_t N_t \tag{A.29}$$

where all variables are in real values and $D_t$ are risk-free assets held at the beginning of period $t$, $W_t$ is nominal wage in period $t$, and $N_t$ is labour supplied in period $t$. Then the first order conditions, with respect to $C_t$ and $N_t$, respectively, are:

$$\beta^t U'(C_t) = \lambda_t \tag{A.30}$$

$$\beta^t U'(N_t) = \lambda_t W_t \tag{A.31}$$

Then, dividing the second by the first yields:

$$\frac{U'(N_t)}{U'(C_t)} = W_t \tag{A.32}$$

Computing marginal utilities and expressing $N_t$ as a function of $W_t$ yields:

$$\frac{-N_t^{\varphi}}{C_t^{-\sigma}} = W_t \implies N_t^{\varphi} = -W_t C_t^{-\sigma} \implies N_t = (-W_t C_t^{-\sigma})^{\frac{1}{\varphi}} \tag{A.33}$$

Taking the derivative with respect to $W_t$ yields:

$$\frac{\partial N_t}{\partial W_t} = \frac{1}{\varphi}(-W_t C_t^{-\sigma})^{\frac{1}{\varphi}-1}(-C_t^{-\sigma}) \tag{A.34}$$

Then, substituting the derivative in expression A.28 yields the Frisch elasticity of labour supply:

$$\frac{\partial N_t}{\partial W_t}\frac{W_t}{N_t} = \frac{1}{\varphi}(-W_t C_t^{-\sigma})^{\frac{1}{\varphi}-1}(-C_t^{-\sigma})\frac{W_t}{(-W_t C_t^{-\sigma})^{\frac{1}{\varphi}}} = \frac{1}{\varphi}(-W_t C_t^{-\sigma})^{-1}(-C_t^{-\sigma} W_t) = \frac{1}{\varphi} \tag{A.35}$$

## A.5 Derivation of the Optimal Allocation of Consumption Expenditures on Domestic and Foreign Goods by Category of Goods

The demand functions for the different varieties of goods produced in countries $H$ and $F$ can be derived, both in the same way, by maximizing consumption $C_{H,t}$ (respectively $C_{F,t}$) for any given level of expenditure defined by:

$$\int_0^h P_{H,t}(j) C_{H,t}(j)\, dj \equiv Z_{H,t} \tag{A.36}$$

respectively,

$$\int_h^1 P_{F,t}(j) C_{F,t}(j)\, dj \equiv Z_{F,t} \tag{A.37}$$

for households in country $H$, and by maximizing consumption $C^*_{H,t}$ (respectively $C^*_{F,t}$) for any given level of expenditure defined by:

$$\int_h^1 P^*_{H,t}(j) C^*_{H,t}(j)\, dj \equiv Z^*_{H,t} \tag{A.38}$$

respectively,

$$\int_0^h P^*_{F,t}(j)C^*_{F,t}(j)\,dj \equiv Z^*_{F,t} \tag{A.39}$$

for households in country $F$, where $Z_{H,t}$ and $Z_{F,t}$ are consumption expenditures by households in country $H$ on *Home* and *Foreign* goods, respectively, while $Z^*_{H,t}$ and $Z^*_{F,t}$ are consumption expenditures by households in country $F$ on domestic and foreign goods, respectively.

I derive here the demand functions for households in country $H$. The demand functions for households in country $F$ are derived in a specular way.

Maximization can be formalized by means of the Lagrangeans:

$$\mathcal{L}_H = \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} - \lambda_H \left(\int_0^h P_{H,t}(j)C_{H,t}(j)\,dj - Z_{H,t}\right) \tag{A.40}$$

$$\mathcal{L}_F = \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} - \lambda_F \left(\int_h^1 P_{F,t}(j)C_{F,t}(j)\,dj - Z_{F,t}\right) \tag{A.41}$$

The derivatives of the Lagrangeans with respect to $C_{H,t}(j)$ and $C_{F,t}(j)$ yield:

$$\frac{\partial \mathcal{L}_H}{\partial C_{H,t}(j)} = \frac{\varepsilon}{\varepsilon-1}\frac{\varepsilon-1}{\varepsilon}\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon-1}} C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}-1}\left(\int_0^h C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}-1} - \lambda_H P_{H,t}(j) = 0 \tag{A.42}$$

$$\frac{\partial \mathcal{L}_F}{\partial C_{F,t}(j)} = \frac{\varepsilon}{\varepsilon-1}\frac{\varepsilon-1}{\varepsilon}\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon-1}} C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}-1}\left(\int_h^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}-1} - \lambda_F P_{F,t}(j) = 0 \tag{A.43}$$

which simplify to:

$$C_{H,t}(j)^{-\frac{1}{\varepsilon}}\left(\left(\frac{1}{h}\right)\int_0^h C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{1}{\varepsilon-1}} = \lambda_H P_{H,t}(j) \tag{A.44}$$

$$C_{F,t}(j)^{-\frac{1}{\varepsilon}}\left(\left(\frac{1}{1-h}\right)\int_h^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{1}{\varepsilon-1}} = \lambda_F P_{F,t}(j) \tag{A.45}$$

which, from the definitions of $C_{H,t}$ and $C_{F,t}$, equations 2.5 and 2.7, become:

$$C_{H,t}(j)^{-\frac{1}{\varepsilon}}\left(\frac{C_{H,t}}{h}\right)^{\frac{1}{\varepsilon}} = \lambda_H P_{H,t}(j) \tag{A.46}$$

$$C_{F,t}(j)^{-\frac{1}{\varepsilon}}\left(\frac{C_{F,t}}{1-h}\right)^{\frac{1}{\varepsilon}} = \lambda_F P_{F,t}(j) \tag{A.47}$$

This holds for all *Home* goods $j \in [0,h]$ and all *Foreign* Goods $j \in [h,1]$, so taking the expression for any two goods $j$ and $l$, and dividing one by the other yields:

$$\frac{C_{H,t}(j)^{-\frac{1}{\varepsilon}}}{C_{H,t}(l)^{-\frac{1}{\varepsilon}}} = \frac{P_{H,t}(j)}{P_{H,t}(l)} \tag{A.48}$$

$$\frac{C_{F,t}(j)^{-\frac{1}{\varepsilon}}}{C_{F,t}(l)^{-\frac{1}{\varepsilon}}} = \frac{P_{F,t}(j)}{P_{F,t}(l)} \tag{A.49}$$

from which expressions for $C_{H,t}(j)$ and $C_{F,t}(j)$ can be derived, given by:

$$C_{H,t}(j) = C_{H,t}(l)\left(\frac{P_{H,t}(j)}{P_{H,t}(l)}\right)^{-\varepsilon} \tag{A.50}$$

$$C_{F,t}(j) = C_{F,t}(l)\left(\frac{P_{F,t}(j)}{P_{F,t}(l)}\right)^{-\varepsilon} \tag{A.51}$$

which substituted into the expression for consumption expenditures on *Home* and *Foreign* goods, respectively, yield:

$$\int_0^h P_{H,t}(j)C_{H,t}(l)\left(\frac{P_{H,t}(j)}{P_{H,t}(l)}\right)^{-\varepsilon} dj = Z_{H,t} \tag{A.52}$$

$$\int_h^1 P_{F,t}(j)C_{F,t}(l)\left(\frac{P_{F,t}(j)}{P_{F,t}(l)}\right)^{-\varepsilon} dj = Z_{F,t} \tag{A.53}$$

which, from the definitions of the price indices $P_{H,t}$ and $P_{F,t}$, equations 2.33 and 2.34, become:

$$h(P_{H,t})^{1-\varepsilon}C_{H,t}(l) = P_{H,t}(l)^{-\varepsilon}Z_{H,t} \tag{A.54}$$

$$(1-h)(P_{F,t})^{1-\varepsilon}C_{F,t}(l) = P_{F,t}(l)^{-\varepsilon}Z_{F,t} \tag{A.55}$$

which simplify to:

$$C_{H,t}(l) = \left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\frac{Z_{H,t}}{hP_{H,t}} \tag{A.56}$$

$$C_{F,t}(l) = \left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\varepsilon}\frac{Z_{F,t}}{(1-h)P_{F,t}} \tag{A.57}$$

for all *Home* goods $l \in [0,h)$ and all *Foreign* goods $l \in [h,1]$. Then, substituting them into the definitions of $C_{H,t}$ and $C_{F,t}$, equations 2.5 and 2.7, yields:

$$C_{H,t} = \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}}\int_0^h\left[\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\frac{Z_{H,t}}{hP_{H,t}}\right]^{\frac{\varepsilon-1}{\varepsilon}} dl\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\frac{1}{h}\int_0^h P_{H,t}(l)^{1-\varepsilon}\,dl\right)^{\frac{\varepsilon}{\varepsilon-1}} Z_{H,t}(P_{H,t})^{\varepsilon-1} \tag{A.58}$$

$$C_{F,t} = \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}}\int_h^1\left[\left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\varepsilon}\frac{Z_{F,t}}{(1-h)P_{F,t}}\right]^{\frac{\varepsilon-1}{\varepsilon}} dl\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\frac{1}{1-h}\int_h^1 P_{F,t}(l)^{1-\varepsilon}\,dl\right)^{\frac{\varepsilon}{\varepsilon-1}} Z_{F,t}(P_{F,t})^{\varepsilon-1} \tag{A.59}$$

Substituting the definitions of the price indices $P_{H,t}$ and $P_{F,t}$, equations 2.33 and 2.34, in the previous expressions yields:

$$C_{H,t} = (P_{H,t})^{-\varepsilon}Z_{H,t}(P_{H,t})^{\varepsilon-1} \implies P_{H,t}C_{H,t} = Z_{H,t} = \int_0^h P_{H,t}(j)C_{H,t}(j)\,dj \tag{A.60}$$

$$C_{F,t} = (P_{F,t})^{-\varepsilon}Z_{F,t}(P_{F,t})^{\varepsilon-1} \implies P_{F,t}C_{F,t} = Z_{F,t} = \int_h^1 P_{F,t}(j)C_{F,t}(j)\,dj \tag{A.61}$$

In the end, combining the last equations with equations A.56 and A.57 yields the demand functions:

$$C_{H,t}(l) = \left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\frac{C_{H,t}}{h} \tag{A.62}$$

$$C_{F,t}(l) = \left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\varepsilon} \frac{C_{F,t}}{1-h} \tag{A.63}$$

for all $l \in [0, h)$ for country $H$ and all $l \in [h, 1]$ for country $F$. These equations for households in country $H$ are true for any $l, j \in [0, 1]$.
A specular derivation brings to similar results for households in country $F$, where consumption expenditures on domestic and foreign goods are given respectively by:

$$P^*_{H,t}C^*_{H,t} = Z^*_{H,t} = \int_h^1 P^*_{H,t}(j)C^*_{H,t}(j)\,dj \tag{A.64}$$

$$P^*_{F,t}C^*_{F,t} = Z^*_{F,t} = \int_0^h P^*_{F,t}(j)C^*_{F,t}(j)\,dj \tag{A.65}$$

while the demand functions for different varieties of domestic and foreign goods are given respectively by:

$$C^*_{H,t}(l) = \left(\frac{P^*_{H,t}(l)}{P^*_{H,t}}\right)^{-\varepsilon} \frac{C^*_{H,t}}{1-h} \tag{A.66}$$

$$C^*_{F,t}(l) = \left(\frac{P^*_{F,t}(l)}{P^*_{F,t}}\right)^{-\varepsilon} \frac{C^*_{F,t}}{h} \tag{A.67}$$

for all domestic goods $l \in [h, 1]$ and all foreign goods $l \in [0, h)$. These equations for households in country $F$ are true for any $l, j \in [0, 1]$.

## A.6 Derivation of the Optimal Allocation of Consumption Expenditures between Domestic and Foreign Goods by Country of Origin

The demand functions for domestic and foreign goods can be derived by maximizing consumption $C_t$ for any given level of expenditure defined by:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \equiv Z_t \tag{A.68}$$

for households in country $H$ and by maximizing consumption $C^*_t$ for any given level of expenditure defined by:

$$P^*_{H,t}C^*_{H,t} + P^*_{F,t}C^*_{F,t} \equiv Z^*_t \tag{A.69}$$

for households in country $F$, where $Z_t$ and $Z^*_t$ are total consumption expenditures on both domestic and foreign goods for households in countries $H$ and $F$, respectively.
I derive here the demand functions for households in country $H$. The demand functions for households in country $F$ are derived in a specular way.
Maximization can be formalized by means of the Lagrangean:

$$\mathcal{L} = \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} - \lambda(P_{H,t}C_{H,t} + P_{F,t}C_{F,t} - Z_t) \tag{A.70}$$

The derivative of the Lagrangean with respect to $C_{H,t}$ yields:

$$\frac{\partial\mathcal{L}}{\partial C_{H,t}} = \frac{\eta}{\eta-1}\frac{\eta-1}{\eta}(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{-\frac{1}{\eta}}\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} - \lambda P_{H,t} = 0 \tag{A.71}$$

which simplifies to:

$$(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{-\frac{1}{\eta}}C_t^{\frac{1}{\eta}} = \lambda P_{H,t} \tag{A.72}$$

The derivative of the Lagrangean with respect to $C_{F,t}$, instead, yields:

$$\frac{\partial \mathcal{L}}{\partial C_{F,t}} = \frac{\eta}{\eta-1}\frac{\eta-1}{\eta}\alpha^{\frac{1}{\eta}}(C_{F,t})^{-\frac{1}{\eta}}\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} - \lambda P_{F,t} = 0 \tag{A.73}$$

which simplifies to:

$$\alpha^{\frac{1}{\eta}}(C_{F,t})^{-\frac{1}{\eta}}C_t^{\frac{1}{\eta}} = \lambda P_{F,t} \tag{A.74}$$

Dividing one first order condition by the other yields:

$$\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\eta}}\left(\frac{C_{F,t}}{C_{H,t}}\right)^{\frac{1}{\eta}} = \frac{P_{H,t}}{P_{F,t}} \tag{A.75}$$

from which we can express foreign consumption as a function of domestic consumption and viceversa as:

$$C_{F,t} = \frac{\alpha}{1-\alpha}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{\eta}C_{H,t} \tag{A.76}$$

respectively,

$$C_{H,t} = \frac{1-\alpha}{\alpha}\left(\frac{P_{F,t}}{P_{H,t}}\right)^{\eta}C_{F,t} \tag{A.77}$$

Substituting the expression for foreign consumption in the expression for consumption expenditures, equation A.69, yields:

$$P_{H,t}C_{H,t} + P_{F,t}\frac{\alpha}{1-\alpha}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{\eta}C_{H,t} = Z_t \tag{A.78}$$

which becomes:

$$C_{H,t}\left(P_{H,t} + \frac{\alpha}{1-\alpha}(P_{F,t})^{1-\eta}(P_{H,t})^{\eta}\right) = Z_t \tag{A.79}$$

simplifying to:

$$C_{H,t} = \frac{(1-\alpha)(P_{H,t})^{-\eta}Z_t}{(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}} \tag{A.80}$$

Substituting the definition of the CPI, equation 2.31, in the previous equation yields:

$$C_{H,t} = \frac{(1-\alpha)(P_{H,t})^{-\eta}Z_t}{P_t^{1-\eta}} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\frac{Z_t}{P_t} \tag{A.81}$$

Substituting the expression for domestic consumption, instead, in the expression for consumption expenditures, equation A.69, yields:

$$P_{H,t}\frac{1-\alpha}{\alpha}\left(\frac{P_{F,t}}{P_{H,t}}\right)^{\eta}C_{F,t} + P_{F,t}C_{F,t} = Z_t \tag{A.82}$$

which becomes:

$$C_{F,t}\left(\frac{1-\alpha}{\alpha}(P_{H,t})^{1-\eta}(P_{F,t})^{\eta} + P_{F,t}\right) = Z_t \tag{A.83}$$

simplifying to:

$$C_{F,t} = \frac{\alpha(P_{F,t})^{-\eta}Z_t}{(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}} \tag{A.84}$$

Substituting the definition of the CPI, equation 2.31, in the previous equation yields:

$$C_{F,t} = \frac{\alpha(P_{F,t})^{-\eta} Z_t}{P_t^{1-\eta}} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \frac{Z_t}{P_t} \tag{A.85}$$

Now, substituting both equations A.81 and A.85 in the consumption index, equation 2.3, yields:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} (1-\alpha)\frac{Z_t}{P_t}\right]^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left[\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} \alpha\frac{Z_t}{P_t}\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{A.86}$$

which becomes:

$$C_t = \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta}\right]^{\frac{\eta}{\eta-1}} \frac{Z_t}{P_t} \tag{A.87}$$

simplifying to:

$$C_t = [(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{\eta}{\eta-1}} \frac{Z_t}{P_t^{1-\eta}} \tag{A.88}$$

Substituting the definition of the CPI, equation 2.31, in the previous equation yields:

$$C_t = P_t^{-\eta} \frac{Z_t}{P_t^{1-\eta}} = \frac{Z_t}{P_t} \implies P_t C_t = Z_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{A.89}$$

Substituting the new expression for $Z_t$ in the demand functions, equations A.81 and A.85, yields the final demand functions for domestic and foreign goods, respectively:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{A.90}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{A.91}$$

A specular derivation brings to similar results for households in country $F$, where consumption expenditures on both domestic and foreign goods are given by:

$$P_t^* C_t^* = Z_t^* = P_{H,t}^* C_{H,t}^* + P_{F,t}^* C_{F,t}^* \tag{A.92}$$

while the final demand functions for domestic and foreign goods are given respectively by:

$$C_{H,t}^* = (1-\alpha)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^* \tag{A.93}$$

$$C_{F,t}^* = \alpha\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* \tag{A.94}$$

## A.7 Derivation of the Consumer Price Index (CPI) in the case of Unitary Substitutability between Domestic and Foreign Goods

Here the aim is to compute the limit of the CPI for $\eta \to 1$:

$$\lim_{\eta \to 1}[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}} \tag{A.95}$$

Substituting $\eta = 1$ yields an expression of the form $1^{\infty}$, which is indeterminate. To solve it, the expression must be transformed in one of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ to be able to apply l'Hôpital's rule.

$$= \lim_{\eta \to 1} e^{\ln[(1-\alpha)(P_{H,t})^{1-\eta}+\alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}} \tag{A.96}$$

$$= \lim_{\eta \to 1} e^{\frac{1}{1-\eta}\ln[(1-\alpha)(P_{H,t})^{1-\eta}+\alpha(P_{F,t})^{1-\eta}]} \tag{A.97}$$

$$= \lim_{\eta \to 1} e^{\frac{\ln[(1-\alpha)(P_{H,t})^{1-\eta}+\alpha(P_{F,t})^{1-\eta}]}{1-\eta}} \tag{A.98}$$

This is an expression with a fraction of the form $\frac{0}{0}$ for which l'Hôpital's rule can be applied. The rule says that if

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0} \implies \lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)} \tag{A.99}$$

So now, computing the two derivatives yields:

$$\frac{\partial \ln[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]}{\partial \eta} = -\frac{(1-\alpha)(P_{H,t})^{1-\eta}\ln P_{H,t} + \alpha(P_{F,t})^{1-\eta}\ln P_{F,t}}{(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}} \tag{A.100}$$

$$\frac{\partial(1-\eta)}{\partial \eta} = -1 \tag{A.101}$$

Now the two derivatives can be substituted in expression A.98 to compute the limit.

$$= \lim_{\eta \to 1} e^{\frac{-\frac{(1-\alpha)(P_{H,t})^{1-\eta}\ln P_{H,t}+\alpha(P_{F,t})^{1-\eta}\ln P_{F,t}}{(1-\alpha)(P_{H,t})^{1-\eta}+\alpha(P_{F,t})^{1-\eta}}}{-1}} \tag{A.102}$$

$$= e^{\frac{(1-\alpha)\ln P_{H,t}+\alpha\ln P_{F,t}}{(1-\alpha)+\alpha}} = e^{(1-\alpha)\ln P_{H,t}+\alpha\ln P_{F,t}} \tag{A.103}$$

$$= e^{\ln(P_{H,t})^{1-\alpha}+\ln(P_{F,t})^{\alpha}} = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha} \tag{A.104}$$

## A.8 Derivations for Households

The optimal allocation of any given consumption expenditure on *Home* and *Foreign* goods, respectively, within each category of goods produced in each country yields the following demand functions[1]:

$$C^i_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \frac{C^i_{H,t}}{h} \tag{A.105}$$

[1]See Appendix A.5 for a derivation.

$$C^i_{F,t}(j) = \left(\frac{P_{H,t}(j)}{P_{F,t}}\right)^{-\varepsilon} \frac{C^i_{F,t}}{1-h} \tag{A.106}$$

for households in country $H$ and for all domestic goods $j \in [0,h)$ and all foreign goods $j \in [h,1]$, while the optimal allocation of any given consumption expenditure on domestic and foreign goods, respectively, within each category of goods produced in each country yields the following demand functions:

$$C^{*i}_{H,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} \frac{C^{*i}_{H,t}}{1-h} \tag{A.107}$$

$$C^{*i}_{F,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{F,t}}\right)^{-\varepsilon} \frac{C^{*i}_{F,t}}{h} \tag{A.108}$$

for households in country $F$ and for all domestic goods $j \in [h,1]$ and all foreign goods $j \in [0,h)$, where $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country $H$ and $P_{F,t}$ is a price index for goods imported from country $F$, respectively defined by[2]:

$$P_{H,t} \equiv \left(\frac{1}{h}\int_0^h P_{H,t}(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{A.111}$$

$$P_{F,t} \equiv \left(\frac{1}{1-h}\int_h^1 P_{F,t}(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{A.112}$$

while $P^*_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country $F$ and $P^*_{F,t}$ is a price index for goods imported from country $H$, respectively defined by:

$$P^*_{H,t} \equiv \left(\frac{1}{1-h}\int_h^1 P^*_{H,t}(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{A.113}$$

$$P^*_{F,t} \equiv \left(\frac{1}{h}\int_0^h P^*_{F,t}(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{A.114}$$

Consumption expenditures on *Home* and *Foreign* goods, respectively simplify to[3]:

$$\int_0^h P_{H,t}(j)C^i_{H,t}(j)\,dj = P_{H,t}C^i_{H,t} \tag{A.115}$$

$$\int_h^1 P_{F,t}(j)C^i_{F,t}(j)\,dj = P_{F,t}C^i_{F,t} \tag{A.116}$$

[2]Both domestic and foreign price indices for country $H$, $P_{H,t}$ and $P_{F,t}$ can be derived by minimizing a given expenditure on domestic and foreign goods under the constraint that equations 2.5 and 2.7 equal to one, respectively:

$$\min_{C^i_{H,t}(j)} \int_0^h P_{H,t}(j)C^i_{H,t}(j) \qquad s.t. \qquad \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}}\int_0^h C^i_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1 \tag{A.109}$$

$$\min_{C^i_{F,t}(j)} \int_h^1 P_{F,t}(j)C^i_{F,t}(j) \qquad s.t. \qquad \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}}\int_h^1 C^i_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1 \tag{A.110}$$

The same applies to the respective domestic and foreign price indices for country $F$, $P^*_{H,t}$ and $P^*_{F,t}$.

[3]See Appendix A.5 for a derivation.

for households in country $H$, while for households in country $F$ consumption expenditures on domestic and foreign goods respectively simplify to:

$$\int_h^1 P^*_{H,t}(j)C^{*i}_{H,t}(j)\,dj = P^*_{H,t}C^{*i}_{H,t} \tag{A.117}$$

$$\int_0^h P^*_{F,t}(j)C^{*i}_{F,t}(j)\,dj = P^*_{F,t}C^{*i}_{F,t} \tag{A.118}$$

Finally, the optimal allocation of consumption expenditures between domestic and imported goods is given respectively by[4]:

$$C^i_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C^i_t \tag{A.119}$$

$$C^i_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C^i_t \tag{A.120}$$

for households in country $H$, while for households in country $F$ the optimal allocation of consumption expenditures between domestic and imported goods is given respectively by:

$$C^{*i}_{H,t} = (1-\alpha^*)\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^{*i}_t \tag{A.121}$$

$$C^{*i}_{F,t} = \alpha^*\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^{*i}_t \tag{A.122}$$

where $P_t$ is the Consumer Price Index (CPI) for country $H$, given by[5]:

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{A.123}$$

while $P^*_t$ is the Consumer Price Index (CPI) for country $F$, given by:

$$P^*_t \equiv \left[(1-\alpha^*)(P^*_{H,t})^{1-\eta} + \alpha^*(P^*_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{A.124}$$

Total consumption expenditures by households in country $H$ are then given by[6]:

$$P_{H,t}C^i_{H,t} + P_{F,t}C^i_{F,t} = P_tC^i_t \tag{A.125}$$

while total consumption expenditures by households in country $F$ are given by:

$$P^*_{H,t}C^{*i}_{H,t} + P^*_{F,t}C^{*i}_{F,t} = P^*_tC^{*i}_t \tag{A.126}$$

[4] See Appendix A.6 for a derivation

[5] The Consumer Price Index (CPI) for country $H$, $P_t$, can be found by minimizing a given expenditure on both domestic and foreign goods under the constraint that equation 2.3 equals one:

$$\min_{C^i_{H,t},C^i_{F,t}} P_{H,t}C^i_{H,t} + P_{F,t}C^i_{F,t} \qquad s.t. \qquad \left[(1-\alpha)^{\frac{1}{\eta}}(C^i_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C^i_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} = 1$$

The same applies to the respective Consumer Price Index (CPI) for country $F$, $P^*_t$. In the particular case in which $\eta = 1$, the CPI in country $H$ takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$, while the CPI in country $F$ takes the form $P^*_t = (P^*_{H,t})^{1-\alpha^*}(P^*_{F,t})^{\alpha^*}$. For a derivation see Appendix A.7.

[6] See Appendix A.6 for a derivation

Finally, the period budget constraint for households in country $H$ can then be rewritten as:

$$P_t C_t^i + D_t^i + B_t^i \leq \frac{D_{t-1}^i}{\mathcal{Q}_{t-1,t}} + B_{t-1}^i(1+i_{t-1}) + (1-\tau_t^w)W_t N_t^i + T_t^i + \Gamma_t^i \tag{A.127}$$

while the period budget constraint for households in country $F$ can be rewritten as:

$$P_t^* C_t^{*i} + D_t^{*i} + B_t^{*i} \leq \frac{D_{t-1}^{*i}}{\mathcal{Q}_{t-1,t}} + B_{t-1}^{*i}(1+i_{t-1}) + (1-\tau_t^{*w})W_t^* N_t^{*i} + T_t^{*i} + \Gamma_t^{*i} \tag{A.128}$$

Aggregating the demand functions for domestic and imported goods, within each category of goods produced in each country, for households in country $H$, equations A.105 and A.106, yields:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \frac{C_{H,t}}{h} \tag{A.129}$$

respectively,

$$C_{F,t}(j) = \left(\frac{P_{H,t}(j)}{P_{F,t}}\right)^{-\varepsilon} \frac{C_{F,t}}{1-h} \tag{A.130}$$

where $C_{H,t}$ and $C_{F,t}$ are aggregate consumption of domestic and imported goods for households in country $H$, respectively defined by:

$$C_{H,t} \equiv \int_0^h C_{H,t}^i \, di = hC_{H,t}^i \qquad \text{and} \qquad C_{F,t} \equiv \int_0^h C_{F,t}^i \, di = hC_{F,t}^i \tag{A.131}$$

while, aggregating the demand functions for domestic and imported goods, within each category of goods, for households in country $F$, equations A.107 and A.108, yields:

$$C_{H,t}^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} \frac{C_{H,t}^*}{1-h} \tag{A.132}$$

respectively,

$$C_{F,t}^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{F,t}^*}\right)^{-\varepsilon} \frac{C_{F,t}^*}{h} \tag{A.133}$$

where $C_{H,t}^*$ and $C_{F,t}^*$ are aggregate consumption of domestic and imported goods for households in country $F$, respectively defined by:

$$C_{H,t}^* \equiv \int_h^1 C_{H,t}^{*i} \, di = (1-h)C_{H,t}^{*i} \qquad \text{and} \qquad C_{F,t}^* \equiv \int_h^1 C_{F,t}^{*i} \, di = (1-h)C_{F,t}^{*i} \tag{A.134}$$

Aggregating instead the demand functions for domestic and imported goods, by country of origin, for households in country $H$, equations A.119 and A.120, yields:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{A.135}$$

respectively,

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{A.136}$$

where $C_t$ is aggregate consumption for households in country $H$, defined by:

$$C_t \equiv \int_0^h C_t^i \, di = hC_t^i \tag{A.137}$$

and $C_{H,t}$ and $C_{F,t}$ are aggregate consumption of domestic and imported goods for households in country $H$, as previously defined. Aggregating the demand functions for domestic and imported goods, by country of origin, for households in country $F$, equations A.121 and A.122, yields:

$$C_{H,t}^* = (1-\alpha^*)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^* \tag{A.138}$$

respectively,

$$C_{F,t}^* = \alpha^*\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* \tag{A.139}$$

where $C_t^*$ is aggregate consumption for households in country $F$, defined by:

$$C_t^* \equiv \int_h^1 C_t^i \, di = (1-h)C_t^{*i} \tag{A.140}$$

and $C_{H,t}^*$ and $C_{F,t}^*$ are aggregate consumption of domestic and imported goods for households in country $F$, as previously defined.

The first order conditions for the household problem in country $H$ are derived by maximizing present-value utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\frac{(C_t^i)^{1-\sigma}-1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi}\right] \tag{A.141}$$

subject to the budget constraint given by:

$$P_t C_t^i + D_t^i + B_t^i \leq \frac{D_{t-1}^i}{\mathcal{Q}_{t-1,t}} + B_{t-1}^i(1+i_{t-1}) + (1-\tau_t^w)W_t N_t^i + T_t^i + \Gamma_t^i \tag{A.142}$$

I derive here the first order conditions for households in country $H$. The first order conditions for households in country $F$ are derived in a specular way.

The problem for households in country $H$ can be formalized by means of the following Lagrangean:

$$\begin{aligned}\mathcal{L} = &\sum_{t=0}^{\infty} \beta^t \xi_t \left[\frac{(C_t^i)^{1-\sigma}-1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi}\right] \\ &- \sum_{t=0}^{\infty} \lambda_t \beta^t \left[P_t C_t^i + D_t^i + B_t^i - \frac{D_{t-1}^i}{\mathcal{Q}_{t-1,t}} - B_{t-1}^i(1+i_{t-1}) - (1-\tau_t^w)W_t N_t^i - T_t^i - \Gamma_t^i\right]\end{aligned} \tag{A.143}$$

where $\lambda_t$ is the multiplier on the budget constraint.

Taking derivatives yields:

$$\frac{\partial \mathcal{L}}{\partial C_t^i} = \beta^t \xi_t (C_t^i)^{-\sigma} - \lambda_t \beta^t P_t = 0 \implies \xi_t (C_t^i)^{-\sigma} = \lambda_t P_t \tag{A.144}$$

$$\frac{\partial \mathcal{L}}{\partial N_t^i} = -\beta^t \xi_t (N_t^i)^{\varphi} + \lambda_t \beta^t (1-\tau_t^w) W_t = 0 \implies \xi_t (N_t^i)^{\varphi} = \lambda_t (1-\tau_t^w) W_t \tag{A.145}$$

$$\frac{\partial \mathcal{L}}{\partial D_t^i} = -\lambda_t \beta^t + \frac{\lambda_{t+1}\beta^{t+1}}{\mathcal{Q}_{t,t+1}} = 0 \implies \mathcal{Q}_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{A.146}$$

$$\frac{\partial \mathcal{L}}{\partial B_t^i} = -\lambda_t \beta^t + \lambda_{t+1}\beta^{t+1}(1+i_t) = 0 \implies \frac{1}{1+i_t} = \beta\frac{\lambda_{t+1}}{\lambda_t} \tag{A.147}$$

Combining the first two derivatives yields the intratemporal optimality condition:

$$\frac{(C_t^i)^{-\sigma}}{P_t} = \frac{(N_t^i)^{\varphi}}{(1-\tau_t^w)W_t} \implies (C_t^i)^{\sigma}(N_t^i)^{\varphi} = (1-\tau_t^w)\frac{W_t}{P_t} \tag{A.148}$$

Combining the first and the third derivative yields the intertemporal optimality condition:

$$\mathcal{Q}_{t,t+1} = \beta\frac{\xi_{t+1}(C_{t+1}^i)^{-\sigma}}{P_{t+1}}\frac{P_t}{\xi_t(C_t^i)^{-\sigma}} \implies \mathcal{Q}_{t,t+1} = \beta\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{P_t}{P_{t+1}} \tag{A.149}$$

which is the household's Euler Equation. Combining the third and fourth derivatives and using the fact that $i_t$ is known in period t, while $Q_{t,t+1}$ is not, yields the no-arbitrage condition:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} \tag{A.150}$$

## A.9 Derivations for International Risk-Sharing

Given the Euler Equations of households in countries $H$ and $F$, equations 2.14 and 2.20, respectively:

$$\mathcal{Q}_{t,t+1} = \beta\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{P_t}{P_{t+1}} \tag{A.151}$$

$$\mathcal{Q}_{t,t+1} = \beta\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^{*i}}{C_t^{*i}}\right)^{-\sigma}\frac{P_t^*}{P_{t+1}^*} \tag{A.152}$$

one can equate the two prices of the state-contingent bonds, $\mathcal{Q}_{t,t+1}$, which are the same, to yield:

$$\beta\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma}\frac{P_t}{P_{t+1}} = \beta\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^{*i}}{C_t^{*i}}\right)^{-\sigma}\frac{P_t^*}{P_{t+1}^*} \tag{A.153}$$

Simplifying and rearranging yields:

$$\left(\frac{C_{t+1}^i}{C_t^i}\frac{C_t^{*i}}{C_{t+1}^{*i}}\right)^{-\sigma} = \frac{\xi_t}{\xi_{t+1}}\frac{\xi_{t+1}^*}{\xi_t^*}\frac{P_{t+1}}{P_t}\frac{P_t^*}{P_{t+1}^*} \tag{A.154}$$

Substituting in the real exchange rate $\mathcal{Q}_t$ and rearranging the equation to express consumption in country $H$ as a function of consumption in country $F$ yields:

$$C_{t+1}^i = \left(\frac{\xi_{t+1}}{\xi_t}\frac{\xi_t^*}{\xi_{t+1}^*}\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\right)^{\frac{1}{\sigma}}\frac{C_t^i}{C_t^{*i}}C_{t+1}^{*i} \tag{A.155}$$

By repeated substitution of *Home* consumption backward in time and moving backwards one period, the previous equation reduces to one linking the consumption of savers in the two countries as a function of initial conditions, the real exchange rate and preference shocks:

$$C_t^i = \left(\frac{\xi_0^*}{\xi_0}\frac{1}{\mathcal{Q}_0}\right)^{\frac{1}{\sigma}}\left(\frac{C_0^i}{C_0^{*i}}\right)\left(\frac{\xi_t}{\xi_t^*}\mathcal{Q}_t\right)^{\frac{1}{\sigma}}C_t^{*i} \tag{A.156}$$

By assuming symmetric initial conditions for households in countries $H$ and $F$, the previous expression reduces to:

$$C_t^i = \left(\frac{\xi_t}{\xi_t^*}\mathcal{Q}_t\right)^{\frac{1}{\sigma}} C_t^{*i} \tag{A.157}$$

## A.10 Derivation of the Demand Constraint for Firms

The demand constraint that firms face in country $H$ can be derived by maximizing domestic output $Y_t$, defined as:

$$Y_t \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.158}$$

for any given level of expenditure on domestic goods, defined by:

$$\int_0^h P_{H,t}(j) Y_t(j)\, dj \equiv Z_t \tag{A.159}$$

where $Z_t$ is total expenditure on domestic goods in country $H$.

The demand constraint that firms face in country $F$ can be derived by maximizing domestic output $Y_t^*$, defined as:

$$Y_t^* \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.160}$$

for any given level of expenditure on domestic goods, defined by:

$$\int_h^1 P_{H,t}^*(j) Y_t^*(j)\, dj \equiv Z_t^* \tag{A.161}$$

where $Z_t^*$ is total expenditure on domestic goods in country $F$.

We derive here the demand constraint for country $H$. The demand constraint for country $F$ is derived in a specular way.

Maximization can be formalized by means of the Lagrangean:

$$\mathcal{L} = \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} - \lambda\left(\int_0^h P_{H,t}(j) Y_t(j)\, dj - Z_t\right) \tag{A.162}$$

The derivative of the Lagrangean with respect to $Y_t(j)$ yields:

$$\frac{\partial \mathcal{L}}{\partial Y_t(j)} = \frac{\varepsilon}{\varepsilon-1}\frac{\varepsilon-1}{\varepsilon}\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon-1}} Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}-1}\left(\int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}-1} - \lambda P_{H,t}(j) = 0 \tag{A.163}$$

which simplifies to:

$$Y_t(j)^{-\frac{1}{\varepsilon}}\left(\frac{1}{h}\int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{1}{\varepsilon-1}} = \lambda P_{H,t}(j) \tag{A.164}$$

which, from the definition of $Y_t$, equation A.158, becomes:

$$Y_t(j)^{-\frac{1}{\varepsilon}}\left(\frac{Y_t}{h}\right)^{\frac{1}{\varepsilon}} = \lambda P_{H,t}(j) \tag{A.165}$$

This holds for all $j \in [0, h]$, so taking the expression for any two goods $j$ and $l$, and dividing one by the other yields:

$$\frac{Y_t(j)^{-\frac{1}{\varepsilon}}}{Y_t(l)^{-\frac{1}{\varepsilon}}} = \frac{P_{H,t}(j)}{P_{H,t}(l)} \tag{A.166}$$

from which an expression for $Y_t(j)$ can be derived, given by:

$$Y_t(j) = Y_t(l) \left( \frac{P_{H,t}(j)}{P_{H,t}(l)} \right)^{-\varepsilon} \tag{A.167}$$

which substituted into the expression for domestic expenditure yields:

$$\int_0^h P_{H,t}(j) Y_t(l) \left( \frac{P_{H,t}(j)}{P_{H,t}(l)} \right)^{-\varepsilon} dj = Z_t \tag{A.168}$$

which, from the definition of the price index $P_{H,t}$, equation 2.33, becomes:

$$h(P_{H,t})^{1-\varepsilon} Y_t(l) = P_{H,t}(l)^{-\varepsilon} Z_t \tag{A.169}$$

which simplifies to:

$$Y_t(l) = \left( \frac{P_{H,t}(l)}{P_{H,t}} \right)^{-\varepsilon} \frac{Z_t}{h P_{H,t}} \tag{A.170}$$

for all $l \in [0, h)$. Then, substituting it into the definition of $Y_t$, equation A.158, yields:

$$Y_t = \left( \int_0^h \left[ \left( \frac{P_{H,t}(l)}{P_{H,t}} \right)^{-\varepsilon} \frac{Z_t}{h P_{H,t}} \right]^{\frac{\varepsilon-1}{\varepsilon}} dl \right)^{\frac{\varepsilon}{\varepsilon-1}} = \left( \frac{1}{h} \int_0^h P_{H,t}(l)^{1-\varepsilon} \, dl \right)^{\frac{\varepsilon}{\varepsilon-1}} Z_t (P_{H,t})^{\varepsilon-1} \tag{A.171}$$

Substituting the definition of the price index $P_{H,t}$, equation 2.33, in the previous expression yields:

$$Y_t = (P_{H,t})^{-\varepsilon} Z_t (P_{H,t})^{\varepsilon-1} \implies P_{H,t} Y_t = Z_t = \int_0^1 P_{H,t}(j) Y_t(j) \, dj \tag{A.172}$$

In the end, combining the last equation with equation A.170 yields the demand function:

$$Y_t(l) = \left( \frac{P_{H,t}(l)}{P_{H,t}} \right)^{-\varepsilon} \frac{Y_t}{h} \tag{A.173}$$

for all $l \in [0, h)$. This equation is true not only for $l$, but for any $l, j \in [0, h)$. At the same time it is true not only for time $t$, but for any time $t + k$, so that we can rewrite it as:

$$Y_{t+k}(j) = \left( \frac{P_{H,t+k}(j)}{P_{H,t+k}} \right)^{-\varepsilon} \frac{Y_{t+k}}{h} \tag{A.174}$$

but since $P_{H,t+k}(j)$ is the price chosen by firm $j$ in period $t$, denoted $\bar{P}_{H,t}(j)$, which lasts until period $t + k$ with probability $\theta^k$ for $k = 0, 1, 2, \ldots$, and since all firms resetting prices in a given period will choose the same price, we can drop the $j$ subscript on the price and rewrite the demand constraint as:

$$Y_{t+k|t}(j) = \left( \frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \frac{Y_{t+k}}{h} \tag{A.175}$$

where $Y_{t+k|t}(j)$ is the demand for firm $j$'s output in period $t+k$ conditional on the firm having last reset its price in period $t$.

The demand function for goods produced in country $F$ is instead given by:

$$Y_t^*(l) = \left(\frac{P_{H,t}^*(l)}{P_{H,t}^*}\right)^{-\varepsilon} \frac{Y_t^*}{1-h} \tag{A.176}$$

for all $l \in [h,1]$. This equation is true not only for $l$, but for any $l, j \in [h,1]$. Instead, the specific demand constraint faced by firms in country $F$ is given by:

$$Y_{t+k|t}^*(j) = \left(\frac{\bar{P}_{H,t}^*}{P_{H,t+k}^*}\right)^{-\varepsilon} \frac{Y_{t+k}^*}{1-h} \tag{A.177}$$

where $\bar{P}_{H,t}^*(j)$ is the price chosen by firm $j$ in period $t$ and $Y_{t+k|t}^*(j)$ is the demand for firm $j$'s output in period $t+k$ conditional on the firm having last reset its price in period $t$, all for firms in country $F$.

## A.11 Derivations for Firms

The firm's problem in country $H$ can be solved by first substituting the stochastic discount factor and the demand constraint into the objective function:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k \frac{\xi_{t+k}}{\xi_t} \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \frac{Y_{t+k}}{h} \left[(1-\tau_{t+k}^s)\bar{P}_{H,t} - MC_{t+k}^n\right] \right\} \tag{A.178}$$

Then, taking the derivative with respect to $\bar{P}_{H,t}$ and equating it to zero yields:

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}}{\xi_t} \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \frac{Y_{t+k}}{h} \left[(1-\tau_{t+k}^s)(1-\varepsilon) + \varepsilon \frac{MC_{t+k}^n}{\bar{P}_{H,t}}\right] \right\} = 0 \tag{A.179}$$

Multiplying all by $\frac{\xi_t C_t^{-\sigma} h}{(1-\varepsilon) P_t \bar{P}_{H,t}^{-\varepsilon}}$ and rearranging yields:

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} \left((1-\tau_{t+k}^s)\bar{P}_{H,t} - \frac{\varepsilon}{(\varepsilon-1)} MC_{t+k}^n\right) \right\} = 0 \tag{A.180}$$

which is the first order condition of the firm's problem. Rearranging the previous equation one can express the optimal price chosen by the firm as a function of only aggregate variables:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC_{t+k}^n \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1-\tau_{t+k}^s) \right\}} \tag{A.181}$$

Substituting nominal marginal cost with the expression for real marginal cost and dividing both sides by $P_{H,t}$ yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{(P_{H,t+k})^{\varepsilon+1}}{P_{H,t}} Y_{t+k} MC_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} (P_{H,t+k})^{\varepsilon} Y_{t+k} (1-\tau_{t+k}^s) \right\}} \tag{A.182}$$

Multiplying numerator and denominator on the right hand side of the previous equation by $\frac{P_t}{P^{\varepsilon}_{H,t}}$ yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon-1}\frac{\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\xi_{t+k}(C_{t+k})^{-\sigma}\frac{P_t}{P_{t+k}}\left(\frac{P_{H,t+k}}{P_{H,t}}\right)^{\varepsilon+1}Y_{t+k}MC_{t+k}\right\}}{\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\xi_{t+k}(C_{t+k})^{-\sigma}\frac{P_t}{P_{t+k}}\left(\frac{P_{H,t+k}}{P_{H,t}}\right)^{\varepsilon}Y_{t+k}(1-\tau^s_{t+k})\right\}} \tag{A.183}$$

Rewriting the previous equation in terms of inflation yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon-1}\frac{\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\xi_{t+k}(C_{t+k})^{-\sigma}Y_{t+k}MC_{t+k}\left[\prod_{s=1}^{k}\frac{(\Pi_{H,t+s})^{\varepsilon+1}}{\Pi_{t+s}}\right]\right\}}{\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\xi_{t+k}(C_{t+k})^{-\sigma}Y_{t+k}(1-\tau^s_{t+k})\left[\prod_{s=1}^{k}\frac{(\Pi_{H,t+s})^{\varepsilon}}{\Pi_{t+s}}\right]\right\}} \tag{A.184}$$

Rewriting the previous equation in recursive form yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon-1}\frac{K_t}{F_t} \tag{A.185}$$

where $K_t$ and $F_t$ are defined by:

$$K_t = \xi_t(C_t)^{-\sigma}Y_tMC_t + \beta\theta E_t\left\{\frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}}K_{t+1}\right\} \tag{A.186}$$

$$F_t = \xi_t(C_t)^{-\sigma}Y_t(1-\tau^s_t) + \beta\theta E_t\left\{\frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}}F_{t+1}\right\} \tag{A.187}$$

The optimal relative price in country $F$ is given by:

$$\frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\varepsilon}{\varepsilon-1}\frac{\sum_{k=0}^{\infty}(\beta\theta^*)^k E_t\left\{\frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}}\frac{(P^*_{H,t+k})^{\varepsilon+1}}{P^*_{H,t}}Y^*_{t+k}MC^*_{t+k}\right\}}{\sum_{k=0}^{\infty}(\beta\theta^*)^k E_t\left\{\frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}}(P^*_{H,t+k})^{\varepsilon}Y^*_{t+k}(1-\tau^{*s}_{t+k})\right\}} \tag{A.188}$$

Rewriting the previous equation in terms of inflation rates yields:

$$\frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\varepsilon}{\varepsilon-1}\frac{\sum_{k=0}^{\infty}(\beta\theta^*)^k E_t\left\{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}Y^*_{t+k}MC^*_{t+k}\left[\prod_{s=1}^{k}\frac{(\Pi^*_{H,t+s})^{\varepsilon+1}}{\Pi^*_{t+s}}\right]\right\}}{\sum_{k=0}^{\infty}(\beta\theta^*)^k E_t\left\{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}Y^*_{t+k}(1-\tau^{*s}_{t+k})\left[\prod_{s=1}^{k}\frac{(\Pi^*_{H,t+s})^{\varepsilon}}{\Pi^*_{t+s}}\right]\right\}} \tag{A.189}$$

Rewriting again the previous equation in recursive form yields:

$$\frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\varepsilon}{\varepsilon-1}\frac{K^*_t}{F^*_t} \tag{A.190}$$

for the optimal relative price in country $F$, where $K^*_t$ and $F^*_t$ are respectively defined recursively by:

$$K^*_t = \xi^*_t(C^*_t)^{-\sigma}Y^*_tMC^*_t + \beta\theta^* E_t\left\{\frac{(\Pi^*_{H,t+1})^{\varepsilon+1}}{\Pi^*_{t+1}}K^*_{t+1}\right\} \tag{A.191}$$

$$F^*_t = \xi^*_t(C^*_t)^{-\sigma}Y^*_t(1-\tau^{*s}_t) + \beta\theta^* E_t\left\{\frac{(\Pi^*_{H,t+1})^{\varepsilon}}{\Pi^*_{t+1}}F^*_{t+1}\right\} \tag{A.192}$$

## A.12 Aggregate Supply

Under the assumption of staggered prices following Calvo (1983), since in country $H$ each period only a fraction $1-\theta$ of firms change their prices, while a fraction $\theta$ keep their prices unchanged, from the definition of the domestic price index, equation 2.33:

$$P_{H,t} = \left[\frac{1}{\theta h}\int_0^{\theta h} P_{H,t-1}(j)^{1-\varepsilon}\, dj + \frac{1}{(1-\theta)h}\int_{\theta h}^{h} \bar{P}_{H,t}(j)^{1-\varepsilon}\, dj\right]^{\frac{1}{1-\varepsilon}} = \left[\theta(P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A.193}$$

which shows the evolution of prices over time. Dividing both sides by $P_{H,t}$ yields:

$$1 = \left[\theta\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{1-\varepsilon} + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A.194}$$

which can be rewritten to yield a relationship between gross PPI inflation and the optimal relative price:

$$\left(\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} = \frac{\bar{P}_{H,t}}{P_{H,t}} \tag{A.195}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ is gross PPI inflation in country $H$. Combining the last equation with the equation for the optimal relative price set by firms yields:

$$\left(\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{K_t}{F_t} \tag{A.196}$$

which is an equation linking the evolution of PPI inflation to real marginal cost dynamics in country $H$, through the recursive functions $K_t$ and $F_t$, defined above.
Specular price dynamics apply to country $F$, where each period only a fraction $1-\theta^*$ of firms change their prices, while a fraction $\theta^*$ keep their prices unchanged. From the definition of the domestic price index for country $F$, equation 2.35, the following expression can be derived:

$$P^*_{H,t} = \left[\theta^*(P^*_{H,t-1})^{1-\varepsilon} + (1-\theta^*)(\bar{P}^*_{H,t})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A.197}$$

Dividing both sides by $P^*_{H,t}$ yields:

$$1 = \left[\theta^*\left(\frac{P^*_{H,t-1}}{P^*_{H,t}}\right)^{1-\varepsilon} + (1-\theta^*)\left(\frac{\bar{P}^*_{H,t}}{P^*_{H,t}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{A.198}$$

which can be rewritten to yield a relationship between gross PPI inflation and the optimal relative price:

$$\left(\frac{1-\theta^*(\Pi^*_{H,t})^{\varepsilon-1}}{1-\theta^*}\right)^{\frac{1}{1-\varepsilon}} = \frac{\bar{P}^*_{H,t}}{P^*_{H,t}} \tag{A.199}$$

where $\Pi^*_{H,t} \equiv \frac{P^*_{H,t}}{P^*_{H,t-1}}$ is gross PPI inflation in country $F$. Combining the last equation with the equation for the optimal relative price set by firms yields:

$$\left(\frac{1-\theta^*(\Pi^*_{H,t})^{\varepsilon-1}}{1-\theta^*}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{K^*_t}{F^*_t} \tag{A.200}$$

which is an equation linking the evolution of PPI inflation to real marginal cost dynamics in country $F$, through the recursive functions $K_t^*$ and $F_t^*$, defined above.
Combining price dispersions, equations 2.66, with aggregate price dynamics, equations A.194 and A.198, yields two equations showing the evolution over time of price dispersion in countries $H$ and $F$ respectively:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} \tag{A.201}$$

Combining price dispersion, equation 2.66, with aggregate price dynamics, equations A.194, yields an equation showing the evolution over time of price dispersion in country $H$:

$$d_t = \frac{1}{h}\int_0^h \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj = \frac{1}{\theta h}\int_0^{\theta h} \left(\frac{P_{H,t-1}(j)}{P_{H,t}}\right)^{-\varepsilon} dj + \frac{1}{(1-\theta)h}\int_{\theta h}^h \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj \tag{A.202}$$

which multiplying and dividing the first term by $P_{H,t-1}^{-\varepsilon}$ yields:

$$d_t = \frac{1}{\theta h}\int_0^{\theta h} \left(\frac{P_{H,t-1}(j)}{P_{H,t-1}}\right)^{-\varepsilon}\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\varepsilon} dj + \frac{1}{(1-\theta)h}\int_{\theta h}^h \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj \tag{A.203}$$

$$d_t = \theta d_{t-1}\Pi_{H,t}^{\varepsilon} + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} \tag{A.204}$$

for country $H$ and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left(\frac{\bar{P}_{H,t}^*}{P_{H,t}^*}\right)^{-\varepsilon} \tag{A.205}$$

for country $F$, which, substituting the optimal relative price with the function of inflation derived above, yields two equations showing the evolution of price dispersion over time depending on PPI inflation in countries $H$ and $F$ respectively:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.206}$$

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left[\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.207}$$

Clearing of the labour market in country $H$ implies that aggregate labour supply, equation 2.16, must equal aggregate labour demand, equation 2.63, to yield:

$$h^{1+\frac{\sigma}{\varphi}} C_t^{-\frac{\sigma}{\varphi}}\left[(1-\tau_t^w)\frac{W_t}{P_t}\right]^{\frac{1}{\varphi}} = \frac{Y_t}{A_t}d_t \tag{A.208}$$

Expressing the real wage as a function of everything else yields:

$$\frac{W_t}{P_t} = \left[\frac{Y_t d_t C_t^{\frac{\sigma}{\varphi}}}{A_t(1-\tau_t^w)^{\frac{1}{\varphi}} h^{1+\frac{\sigma}{\varphi}}}\right]^{\varphi} \tag{A.209}$$

To derive a relationship between nominal marginal cost and output, one can start from deriving a relationship between real marginal cost and real wage:

$$MC_t = \frac{W_t}{A_t P_{H,t}} = \frac{W_t}{P_t}\frac{P_t}{P_{H,t}}\frac{1}{A_t} \tag{A.210}$$

Then, by substituting the real wage implied by labour market clearing and the price ratio as a function of the terms of trade, yields a relationship between real marginal cost and output through the terms of trade:

$$MC_t = \left[\frac{Y_t d_t C_t^{\frac{\sigma}{\varphi}}}{A_t(1-\tau_t^w)^{\frac{1}{\varphi}} h^{1+\frac{\sigma}{\varphi}}}\right]^{\varphi} \left[1-\alpha+\alpha\mathcal{S}_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \frac{1}{A_t} \tag{A.211}$$

$$MC_t = \frac{Y_t^{\varphi} d_t^{\varphi} C_t^{\sigma}}{(1-\tau_t^w)A_t^{1+\varphi} h^{\varphi+\sigma}} \left[1-\alpha+\alpha\mathcal{S}_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{A.212}$$

Following the same procedure for the *Foreign* country, real marginal cost in country $F$ will be given by:

$$MC_t^* = \frac{(Y_t^*)^{\varphi}(d_t^*)^{\varphi}(C_t^*)^{\sigma}}{(1-\tau_t^{*w})(A_t^*)^{1+\varphi}(1-h)^{\varphi+\sigma}} \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{A.213}$$

The previous equations together with the price setting equations constitute the aggregate supply equations for countries $H$ and $F$ linking inflation in each country to output and consumption in each country.

## A.13 Aggregate Demand

Clearing of the goods market implies the following condition for all goods $j \in [0, h)$ produced in country $H$:

$$Y_t(j) = \int_0^h C_{H,t}^i(j)\, di + \int_h^1 C_{F,t}^{*i}(j)\, di + G_t(j) \tag{A.214}$$

Substituting the derived aggregate demand functions for private domestic consumption, equation A.129, for foreign consumption of domestic goods, equation A.132, and for public consumption, equation 2.100, into the market clearing condition for good $j$, yields:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \frac{C_{H,t}}{h} + \left(\frac{P_{F,t}^*(j)}{P_{F,t}^*}\right)^{-\varepsilon} \frac{C_{F,t}^*}{h} + \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \frac{G_t}{h} \tag{A.215}$$

while substituting in equations A.135 and A.139, and using the law of one price ($P_{F,t}^*(j) = P_{H,t}(j)$ and $P_{F,t}^* = P_{H,t}$), yields:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[\frac{1-\alpha}{h}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{\alpha^*}{h}\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* + \frac{G_t}{h}\right] \tag{A.216}$$

Substituting the previous equation into the definition of aggregate domestic output, equation 2.65, yields:

$$Y_t = \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h \left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[\frac{1-\alpha}{h}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{\alpha^*}{h}\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* + \frac{G_t}{h}\right]\right]^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.217}$$

Since only the first fraction depends on $j$ the previous equation can be rewritten in the following way:

$$Y_t = \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h \left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \frac{1}{h}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^*\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^* + G_t\right] \tag{A.218}$$

Simplifying exponents yields:

$$Y_t = \left(\frac{1}{h}\int_0^h \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{1-\varepsilon} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^* \left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t + G_t\right] \tag{A.219}$$

Substituting in the definition of $P_{H,t}$, equation 2.33, simplifies the first term and yields:

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^* \left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t + G_t \tag{A.220}$$

Making use of the law of one price, equation 2.41, and the definition of the real exchange rate $Q_t$, defined by equation 2.51, yields:

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^* \left(\frac{P_{H,t}}{P_t Q_t}\right)^{-\eta} C^*_t + G_t \tag{A.221}$$

Collecting terms in the previous expression yields:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} [(1-\alpha)C_t + \alpha^*(Q_t)^\eta C^*_t] + G_t \tag{A.222}$$

By assuming symmetric initial conditions in countries $H$ and $F$, foreign consumption in the previous equation can be substituted through the international risk sharing condition, equation 2.56, to yield:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[1-\alpha+\alpha^* \frac{1-h}{h}\left(\frac{\xi^*_t}{\xi_t}\right)^{\frac{1}{\sigma}} (Q_t)^{\eta-\frac{1}{\sigma}}\right] C_t + G_t \tag{A.223}$$

Substituting the price ratio and the real exchange rate with expressions of the terms of trade yields:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[1-\alpha+\alpha^* \frac{1-h}{h}\left(\frac{\xi^*_t}{\xi_t}\right)^{\frac{1}{\sigma}} (\mathcal{S}_t)^{\eta-\frac{1}{\sigma}} \left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{1-\eta}+\frac{1}{\eta\sigma-\sigma}}\right] C_t + G_t \tag{A.224}$$

which rearranging, finally yields:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[1-\alpha+\alpha^* \frac{1-h}{h}\left(\frac{\xi^*_t}{\xi_t}\right)^{\frac{1}{\sigma}} (\mathcal{S}_t)^{\eta-\frac{1}{\sigma}} \left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(\eta-1)}}\right] C_t + G_t \tag{A.225}$$

The same procedure can be applied to the goods market clearing condition for all goods $j \in [0,1]$ produced in country $F$:

$$Y^*_t(j) = \int_h^1 C^{*i}_{H,t}(j)\, di + \int_0^h C^i_{F,t}(j)\, di + G^*_t(j) \tag{A.226}$$

Substituting the derived demand functions for private domestic consumption, equations A.132 and A.138, for foreign consumption of domestic goods, equations A.129 and A.136, and for public consumption, equation 2.112, yields:

$$Y^*_t(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} \left[\frac{1-\alpha^*}{1-h}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t + \frac{\alpha}{1-h}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + \frac{G^*_t}{1-h}\right] \tag{A.227}$$

Plugging the market clearing condition into the definition of aggregate domestic output in country $F$, equation 2.65, yields:

$$Y_t^* = (1-\alpha^*)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^* + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + G_t^* \tag{A.228}$$

Making use of the law of one price, equation 2.41, and the definition of the real exchange rate $Q_t$, defined by equation 2.51, yields:

$$Y_t^* = \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}\left[(1-\alpha^*)C_t^* + \alpha(Q_t)^{-\eta}C_t\right] + G_t^* \tag{A.229}$$

By assuming symmetric initial conditions in countries $H$ and $F$, foreign consumption in the previous equation can be substituted through the international risk sharing condition, equation 2.56, to yield:

$$Y_t^* = \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta}\left[1-\alpha^* + \alpha\frac{h}{1-h}\left(\frac{\xi_t}{\xi_t^*}\right)^{\frac{1}{\sigma}}(Q_t)^{\frac{1}{\sigma}-\eta}\right]C_t^* + G_t^* \tag{A.230}$$

Substituting the price ratio and the real exchange rate with expressions of the terms of trade and rearranging yields the aggregate demand equation for country $F$:

$$Y_t^* = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}}\left[1-\alpha^*+\alpha\frac{h}{1-h}\left(\frac{\xi_t}{\xi_t^*}\right)^{\frac{1}{\sigma}}(\mathcal{S}_t)^{\frac{1}{\sigma}-\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(1-\eta)}}\right]C_t^*+G_t^* \tag{A.231}$$

Demand for output in both countries depends on private consumption in the two countries, through the terms of trade, relative degree of openness and relative size of the two countries, other than relative preference shocks. Government consumption, instead, affects aggregate demand directly because of complete home bias in public consumption.

## A.14 Equilibrium Conditions

Here we collect all the equilibrium conditions of the full model, differentiating between a pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union and between different policy rules.
The equilibrium conditions of the model are grouped into the following blocks:

**Aggregate Demand Block**
The aggregate demand block is composed of aggregate demand in both countries $H$:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\left[1-\alpha+\alpha^*\frac{1-h}{h}\left(\frac{\xi_t^*}{\xi_t}\right)^{\frac{1}{\sigma}}(\mathcal{S}_t)^{\eta-\frac{1}{\sigma}}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(\eta-1)}}\right]C_t+G_t \tag{A.232}$$

and $F$:

$$Y_t^* = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}}\left[1-\alpha^*+\alpha\frac{h}{1-h}\left(\frac{\xi_t}{\xi_t^*}\right)^{\frac{1}{\sigma}}(\mathcal{S}_t)^{\frac{1}{\sigma}-\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(1-\eta)}}\right]C_t^*+G_t^* \tag{A.233}$$

while the evolution of private consumption is given by the households' Euler Equation in country $F$:

$$\frac{1}{1+i_t} = \beta E_t\left\{\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{1}{\Pi_{t+1}^*}\right\} \tag{A.234}$$

and by the international risk-sharing condition in country $H$:

$$C_t = \frac{h}{1-h}\left[\frac{\xi_t}{\xi_t^*}\mathcal{S}_t\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1}{1-\eta}}\right]^{\frac{1}{\sigma}} C_t^* \tag{A.235}$$

while the relationship between CPI inflation and PPI inflation is given by:

$$\Pi_t = \Pi_{H,t}\left[\frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{A.236}$$

in country $H$ and:

$$\Pi_t^* = \Pi_{H,t}^*\left[\frac{1-\alpha^*+\alpha^*\mathcal{S}_t^{\eta-1}}{1-\alpha^*+\alpha^*(\mathcal{S}_{t-1})^{\eta-1}}\right]^{\frac{1}{1-\eta}} \tag{A.237}$$

in country $F$, and the evolution of the terms of trade is given by:

$$\mathcal{S}_t = \frac{\Pi_{H,t}^*}{\Pi_{H,t}}\mathcal{S}_{t-1} \tag{A.238}$$

while the exogenous demand shocks evolve according to:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \tag{A.239}$$

$$\xi_t^* = (\xi_{t-1}^*)^{\rho_\xi^*} e^{\varepsilon_t} \tag{A.240}$$

**Aggregate Supply Block**

The aggregate supply block is composed of the aggregate supply equation for country $H$:

$$\left(\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{K_t}{F_t} \tag{A.241}$$

where:

$$K_t = \xi_t(C_t)^{-\sigma}Y_t MC_t + \beta\theta E_t\left\{\frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}}K_{t+1}\right\} \tag{A.242}$$

$$F_t = \xi_t(C_t)^{-\sigma}Y_t(1-\tau_t^s) + \beta\theta E_t\left\{\frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}}F_{t+1}\right\} \tag{A.243}$$

and marginal cost in country $H$ is given by:

$$MC_t = \frac{(Y_t)^\varphi(d_t)^\varphi(C_t)^\sigma}{(1-\tau_t^w)(A_t)^{1+\varphi}(h)^{\varphi+\sigma}}\left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{A.244}$$

and the aggregate supply equation for country $F$:

$$\left(\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{K_t^*}{F_t^*} \tag{A.245}$$

where:

$$K_t^* = \xi_t^*(C_t^*)^{-\sigma}Y_t^* MC_t^* + \beta\theta^* E_t\left\{\frac{(\Pi_{H,t+1}^*)^{\varepsilon+1}}{\Pi_{t+1}^*}K_{t+1}^*\right\} \tag{A.246}$$

$$F_t^* = \xi_t^*(C_t^*)^{-\sigma} Y_t^*(1-\tau_t^{*s}) + \beta\theta^* E_t \left\{ \frac{(\Pi_{H,t+1}^*)^{\varepsilon}}{\Pi_{t+1}^*} F_{t+1}^* \right\} \tag{A.247}$$

and marginal cost in country $F$ is given by:

$$MC_t^* = \frac{(Y_t^*)^{\varphi}(d_t^*)^{\varphi}(C_t^*)^{\sigma}}{(1-\tau_t^{*w})(A_t^*)^{1+\varphi}(1-h)^{\varphi+\sigma}} \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{A.248}$$

while the evolution of price dispersion is given by:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.249}$$

for country $H$, and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left[\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.250}$$

for country $F$, while the levels of technology evolve exogenously according to:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \tag{A.251}$$

$$A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \tag{A.252}$$

**Net Exports, Net Foreign Assets and the Balance of Payments**

Real Net Exports for country $H$ are given by:

$$\widetilde{NX}_t = Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{A.253}$$

Real Net Foreign Assets for country $H$ are given by:

$$\widetilde{NFA}_t = \tilde{D}_t + \tilde{B}_t - \tilde{B}_t^G \tag{A.254}$$

The real Balance of Payments for country $H$ is given by:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + \frac{i_{t-1}}{\Pi_{H,t}} \widetilde{NFA}_{t-1} \tag{A.255}$$

so that real Net Foreign Assets for country $H$ evolve according to:

$$\widetilde{NFA}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \tag{A.256}$$

**Monetary Policy**

Monetary policy sets the nominal interest rate following the rule:

$$\beta(1+i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} \left[\beta(1+i_{t-1})\right]^{\rho_i} \tag{A.257}$$

where union-wide CPI inflation is defined by:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{A.258}$$

**Fiscal Policy in a Pure Currency Union - Consumption Scenario**

Fiscal policy, in the consumption scenario of the Pure Currency Union setting, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.259}$$

for country $H$, and:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.260}$$

for country $F$, while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.261}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ ($\gamma^* \in [0,1]$ for country $F$) and $1-\gamma$ ($1-\gamma^*$ for country $F$) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{A.262}$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{A.263}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.264}$$

$$G_t^* + \tilde{T}_t^* + i_{t-1}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.265}$$

**Fiscal Policy in a Pure Currency Union - Transfer Scenario**

In the transfer scenario of the Pure Currency Union setting, fiscal policy sets real transfers following the rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{A.266}$$

for country $H$, and:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{A.267}$$

for country $F$, while keeping government consumption constant and balancing the budget:

$$G_t = G \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad G_t^* = G^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.268}$$

**Fiscal Policy in a Coordinated Currency Union - Consumption Scenario**

Fiscal policy, in the consumption scenario of the Coordinated Currency Union setting, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.269}$$

for country $H$, and:

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.270}$$

for country $F$, while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.271}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ ($\gamma^* \in [0,1]$ for country $F$) and $1-\gamma$ ($1-\gamma^*$ for country $F$) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{A.272}$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{A.273}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.274}$$

$$G_t^* + \tilde{T}_t^* + i_{t-1}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.275}$$

**Fiscal Policy in a Coordinated Currency Union - Transfer Scenario**

In the transfer scenario of the Coordinated Currency Union setting, fiscal policy sets real transfers following the rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{A.276}$$

for country $H$, and:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{A.277}$$

for country $F$, while keeping government consumption constant and balancing the budget:

$$G_t = G \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad G_t^* = G^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.278}$$

**Fiscal Policy in a Full Fiscal Union - Consumption Scenario**

Fiscal policy, in the consumption scenario of the Full Fiscal Union setting, sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.279}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.280}$$

while keeping real transfers constant and balancing the overall budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \text{and} \qquad \tilde{\mathcal{B}}_t^G = \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G + S_t\tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.281}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$, while distributing equally among the two countries the cost of fiscal policy, through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{A.282}$$

$$(\tau_t^s - \tau^s) = (\tau_t^{*s} - \tau^{*s}) \tag{A.283}$$

$$(\tau_t^w - \tau^w) = (\tau_t^{*w} - \tau^{*w}) \tag{A.284}$$

with the following consolidated budget constraint:

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1}\frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} = (\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* + \tilde{\mathcal{B}}_t^G - \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \tag{A.285}$$

**Fiscal Policy in a Full Fiscal Union - Transfer Scenario**

In the transfer scenario of the Full Fiscal Union setting, fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{A.286}$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{A.287}$$

while keeping government consumption constant and balancing the overall budget:

$$G_t = G \qquad G_t^* = G^* \qquad \text{and} \qquad \tilde{\mathcal{B}}_t^G = \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G + S_t\tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.288}$$

We can now define an equilibrium for the Currency Union.

**Definition 1** (Equilibrium)**.** *An Imperfectly competitive equilibrium is a sequence of stochastic processes:*

$$\mathcal{X}_t \equiv \{Y_t, Y_t^*, C_t, C_t^*, \Pi_{H,t}, \Pi_{H,t}^*, \Pi_t, \Pi_t^*, \Pi_t^U, S_t, K_t, K_t^*, F_t, F_t^*, MC_t, MC_t^*, d_t, d_t^*, \widetilde{NX}_t, \widetilde{NFA}_t, \widetilde{CA}_t\}$$

*and exogenous disturbances:*

$$\mathcal{Z}_t \equiv \{\xi_t, \xi_t^*, A_t, A_t^*\}$$

*satisfying equations A.232 through A.256 and the definition of union-wide inflation A.258, given initial* *conditions:*

$$\mathcal{I}_{-1} \equiv \{C_{-1}, C^*_{-1}, \Pi_{H,-1}, \Pi^*_{H,-1}, S_{-1}, d_{-1}, d^*_{-1}, \widetilde{NFA}_{-1}\}$$

*plus monetary and fiscal policies:*

$$\mathcal{P}_t \equiv \{i_t, G_t, G^*_t, \tilde{T}_t, \tilde{T}^*_t, \tau^s_t, \tau^{*s}_t, \tau^w_t, \tau^{*w}_t, \tilde{B}^G_t, \tilde{B}^{*G}_t\}$$

*specified in equation A.257 for monetary policy and in equations A.259 through A.288 for the various specifications of fiscal policy, for $t \geq 0$.*

## A.15 The Steady State

We describe the symmetric (in terms of per capita consumption and prices) non-stochastic steady state with constant government debt and zero inflation, which will be the starting point of our simulations. We focus on the perfect foresight steady state and equilibrium deviations from it, given by different shocks. *Perfect Foresight* is a viable assumption because, despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a large number (more accurately, a continuum) of firms, as explained in Calvo (1983).
The symmetric non-stochastic steady state with constant government debt and zero inflation, which will be the starting point of our simulations, is defined by the following assumptions and equations.
All shocks are constant at their long-run levels of 1:

$$\xi = \xi^* = A = A^* = 1 \tag{A.289}$$

There is no inflation and no price dispersion:

$$\Pi_H = \Pi^*_H = \Pi = \Pi^* = \Pi^U = 1 \implies d = d^* = 1 \tag{A.290}$$

The terms of trade and the real exchange rate are equal to 1:

$$S = 1 \implies Q = 1 \tag{A.291}$$

Per-capita consumption is equal across countries:

$$\frac{C}{h} = \frac{C^*}{1-h} \tag{A.292}$$

Aggregate demand in each country is given by:

$$Y = \left(1 - \alpha + \alpha^* \frac{1-h}{h}\right) C + G \tag{A.293}$$

$$Y^* = \left(1 - \alpha^* + \alpha \frac{h}{1-h}\right) C^* + G^* \tag{A.294}$$

Combining the previous equations we can derive per-capita consumption in each country as a function of output and government spending and equate the two to derive an equation linking output and government spending in the two countries:

$$Y = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} [Y^* - G^*] + G \tag{A.295}$$

From the Euler Equations:

$$\frac{1}{1+i} = \beta \implies i = \frac{1}{\beta} - 1 \tag{A.296}$$

Recalling marginal costs in steady state from price-setting:

$$MC = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^s) \tag{A.297}$$

$$MC^* = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^{*s}) \tag{A.298}$$

Marginal costs are also given by labour market equilibrium:

$$MC = \frac{(Y)^{\varphi}(C)^{\sigma}}{(1 - \tau^w)(h)^{\varphi+\sigma}} \tag{A.299}$$

$$MC^* = \frac{(Y^*)^{\varphi}(C^*)^{\sigma}}{(1 - \tau^{*w})(1-h)^{\varphi+\sigma}} \tag{A.300}$$

Equating the two marginal cost expressions for each country yields consumption in terms of output:

$$C = \left[\frac{\varepsilon - 1}{\varepsilon}\frac{(1 - \tau^s)(1 - \tau^w)(h)^{\varphi+\sigma}}{(Y)^{\varphi}}\right]^{\frac{1}{\sigma}} \tag{A.301}$$

$$C^* = \left[\frac{\varepsilon - 1}{\varepsilon}\frac{(1 - \tau^{*s})(1 - \tau^{*w})(1-h)^{\varphi+\sigma}}{(Y^*)^{\varphi}}\right]^{\frac{1}{\sigma}} \tag{A.302}$$

Deriving per-capita consumption in the two countries and equating the two yields an equation linking output in the two countries:

$$Y^* = \frac{1-h}{h}\left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)}\right]^{\frac{1}{\varphi}} Y \tag{A.303}$$

Substituting foreign output from the supply equation A.303 into the demand equation A.295 yields a relationship linking domestic output to government consumption in both countries[7]:

$$Y = \left[1 - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\frac{1-h}{h}\left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)}\right]^{\frac{1}{\varphi}}\right]^{-1}\left[G - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}G^*\right] \tag{A.304}$$

Fiscal policy multipliers and spillovers in steady state for country $H$ can be found from the previous equation by computing the effect on domestic output of variations of $G$ and $G^*$ respectively:

$$\frac{\partial Y}{\partial G} = \left[1 - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\frac{1-h}{h}\left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)}\right]^{\frac{1}{\varphi}}\right]^{-1} \tag{A.305}$$

$$\frac{\partial Y}{\partial G^*} = -\left[\frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\right]\left[1 - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\frac{1-h}{h}\left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)}\right]^{\frac{1}{\varphi}}\right]^{-1} \tag{A.306}$$

which shows how fiscal multipliers and spillovers depend on the parameters of the model, other than the tax rates in both countries.

We can see from the previous equations that a necessary and sufficient condition for fiscal multipliers to

[7] A specular equation can be derived linking foreign output to government consumption in both countries. This would allow to derive foreign fiscal multipliers and spillovers. The results and analysis would be symmetric with respect to the counterpart variables of the other country.

be positive and greater than 1 and at the same time for fiscal spillovers to be negative is:

$$\frac{(1-\alpha)h+\alpha^*(1-h)}{(1-\alpha^*)(1-h)+\alpha h}\frac{1-h}{h}\left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^{s})(1-\tau^{w})}\right]^{\frac{1}{\varphi}}<1 \tag{A.307}$$

From the previous condition, sufficient, but not necessary, conditions for fiscal multipliers to be positive and greater than 1 and at the same time for fiscal spillovers to be negative are:

$$\frac{\alpha^*}{\alpha}<\frac{h}{1-h} \tag{A.308}$$

$$\tau^{*s}+\tau^{*w}-\tau^{*s}\tau^{*w}>\tau^{s}+\tau^{w}-\tau^{s}\tau^{w} \tag{A.309}$$

while, by changing the sign of all the inequalities, we get alternative conditions for fiscal multipliers to be negative and at the same time for fiscal spillovers to be positive and greater than 1. This tells us that when fiscal multipliers are positive and greater than 1, fiscal spillovers are negative, and viceversa.
Under the condition given by Equation A.307, fiscal multipliers in country $H$ are positive and increasing with the openness to trade of the foreign country $\alpha^*$, with the relative size of the foreign country $\frac{1-h}{h}$ and with the steady state values of the domestic tax rates on firm sales $\tau^s$ and labour income $\tau^w$, while they are positive and decreasing with the inverse of the Frisch elasticity of labour supply $\varphi$, with the openness to trade of the domestic country $\alpha$ and with the steady state values of the foreign tax rates on firm sales $\tau^{*s}$ and labour income $\tau^{*w}$. Under the same conditions, fiscal spillovers in country $H$ are negative and increasing in absolute terms with the openness to trade of the foreign country $\alpha^*$, with the relative size of the foreign country $\frac{1-h}{h}$ and with the steady state values of the domestic tax rates on firm sales $\tau^s$ and labour income $\tau^w$, while they are negative and decreasing in absolute terms with the inverse of the Frisch elasticity of labour supply $\varphi$, with the openness to trade of the domestic country $\alpha$ and with the steady state values of the foreign tax rates on firm sales $\tau^{*s}$ and labour income $\tau^{*w}$. In steady state real net exports are given by:

$$\widetilde{NX}=Y-C-G \tag{A.310}$$

while real net foreign assets are:

$$\widetilde{NFA}=\tilde{D}+\tilde{B}-\tilde{B}^G \tag{A.311}$$

The real balance of payments is given by:

$$\widetilde{BP}=\widetilde{NX}+\left(\frac{1}{\beta}-1\right)\widetilde{NFA} \tag{A.312}$$

while from the budget constraints of households and governments, or equivalently from the evolution of net foreign assets:

$$\widetilde{NFA}=\widetilde{NFA}+\widetilde{BP} \tag{A.313}$$

which implies that in steady state the balance of payments must be zero and so net exports pin down net foreign assets:

$$\widetilde{BP}=0 \implies \widetilde{NX}=-\left(\frac{1}{\beta}-1\right)\widetilde{NFA} \tag{A.314}$$

These relations yield an equation linking output, government consumption and household consumption in the two countries in steady state, which mainly comes from the fact that net exports are in zero international net supply in steady state:

$$Y + Y^* = C + C^* + G + G^* \tag{A.315}$$

The household budget constraints in steady state for countries $H$ and $F$ are given by:

$$C = \left(\frac{1}{\beta} - 1\right)(\tilde{D} + \tilde{B}) + \tilde{T} + Y(1 - \tau^s - \tau^w MC) \tag{A.316}$$

$$C^* = \left(\frac{1}{\beta} - 1\right)(\tilde{D}^* + \tilde{B}^*) + \tilde{T}^* + Y^*(1 - \tau^{*s} - \tau^{*w} MC^*) \tag{A.317}$$

Instead the government budget constraints of the two countries in steady state read:

$$G + \tilde{T} + \left(\frac{1}{\beta} - 1\right)\tilde{B}^G = Y(\tau^s + \tau^w MC) \tag{A.318}$$

$$G^* + \tilde{T}^* + \left(\frac{1}{\beta} - 1\right)\tilde{B}^{*G} = Y^*(\tau^{*s} + \tau^{*w} MC^*) \tag{A.319}$$

## A.16 Derivations for Calibration

The relationship that links the two parameters of openness $\alpha$ and $\alpha^*$, equation 2.154, which is used to calibrate $\alpha^*$ is derived in the following way.
From the relationship linking demand for consumption, equation A.295, dividing all by $Y$ and rearranging yields:

$$1 - \frac{G}{Y} = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\left[1 - \frac{G^*}{Y^*}\right]\frac{Y^*}{Y} \tag{A.320}$$

From the relationship linking supply of consumption goods, equation A.303, dividing all by $Y$ yields:

$$\frac{Y^*}{Y} = \frac{1-h}{h}\left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)}\right]^{\frac{1}{\varphi}} \tag{A.321}$$

Substituting the second equation into the first yields:

$$1 - \frac{G}{Y} = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\left[1 - \frac{G^*}{Y^*}\right]\frac{1-h}{h}\left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)}\right]^{\frac{1}{\varphi}} \tag{A.322}$$

$$\frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} = \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^s)(1-\tau^w)}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} \tag{A.323}$$

$$(1-\alpha)h + \alpha^*(1-h) = [(1-\alpha^*)(1-h) + \alpha h]\frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^s)(1-\tau^w)}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} \tag{A.324}$$

dividing all by $1-h$ and rearranging to explicit $\alpha^*$ yields:

$$\begin{aligned}
&\alpha^*\left[1 + \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^s)(1-\tau^w)}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}}\right] = \\
&\alpha\frac{h}{1-h}\left[1 + \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^s)(1-\tau^w)}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}}\right] + \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^s)(1-\tau^w)}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} - \frac{h}{1-h}
\end{aligned} \tag{A.325}$$

then finally dividing all by what multiplies $\alpha^*$ yields:

$$\alpha^* = \alpha\frac{h}{1-h} + \frac{h}{1-h}\frac{\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^s)(1-\tau^w)}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} - 1}{1+\frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^s)(1-\tau^w)}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}}} \tag{A.326}$$

which is the final equation linking the two parameters of openness through the shares government consumption in output and the tax rates.

# Appendix B

# Appendix to Chapter 3

As a tool for students and researchers studying models of this kind for the first time, I provide as many mathematical derivations of the model as possible here in the Appendix. This is meant for learning purposes, especially concerning complex models with cumbersome mathematical derivations.

## B.1 Derivations for Households

The optimal allocation of any given consumption expenditure on *Home* and *Foreign* goods, respectively, within each category of goods produced in each country yields the following demand functions:

$$C^i_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \frac{C^i_{H,t}}{h} \tag{B.1}$$

$$C^i_{F,t}(j) = \left(\frac{P_{H,t}(j)}{P_{F,t}}\right)^{-\varepsilon} \frac{C^i_{F,t}}{1-h} \tag{B.2}$$

for households in country $H$ and for all domestic goods $j \in [0, h)$ and all foreign goods $j \in [h, 1]$, while the optimal allocation of any given consumption expenditure on domestic and foreign goods, respectively, within each category of goods produced in each country yields the following demand functions:

$$C^{*i}_{H,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{H,t}}\right)^{-\varepsilon} \frac{C^{*i}_{H,t}}{1-h} \tag{B.3}$$

$$C^{*i}_{F,t}(j) = \left(\frac{P^*_{H,t}(j)}{P^*_{F,t}}\right)^{-\varepsilon} \frac{C^{*i}_{F,t}}{h} \tag{B.4}$$

for households in country $F$ and for all domestic goods $j \in [h, 1]$ and all foreign goods $j \in [0, h)$, where $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country $H$ and $P_{F,t}$ is a price index for goods imported from country $F$, respectively defined by:

$$P_{H,t} \equiv \left(\frac{1}{h}\int_0^h P_{H,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \tag{B.5}$$

$$P_{F,t} \equiv \left(\frac{1}{1-h}\int_h^1 P_{F,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \tag{B.6}$$

while $P^*_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country $F$ and $P^*_{F,t}$ is a price index for goods imported from country $H$, respectively defined by:

$$P^*_{H,t} \equiv \left( \frac{1}{1-h} \int_h^1 P^*_{H,t}(j)^{1-\varepsilon}\, dj \right)^{\frac{1}{1-\varepsilon}} \tag{B.7}$$

$$P^*_{F,t} \equiv \left( \frac{1}{h} \int_0^h P^*_{F,t}(j)^{1-\varepsilon}\, dj \right)^{\frac{1}{1-\varepsilon}} \tag{B.8}$$

Consumption expenditures on *Home* and *Foreign* goods, respectively simplify to:

$$\int_0^h P_{H,t}(j) C^i_{H,t}(j)\, dj = P_{H,t} C^i_{H,t} \tag{B.9}$$

$$\int_h^1 P_{F,t}(j) C^i_{F,t}(j)\, dj = P_{F,t} C^i_{F,t} \tag{B.10}$$

for households in country $H$, while for households in country $F$ consumption expenditures on domestic and foreign goods respectively simplify to:

$$\int_h^1 P^*_{H,t}(j) C^{*i}_{H,t}(j)\, dj = P^*_{H,t} C^{*i}_{H,t} \tag{B.11}$$

$$\int_0^h P^*_{F,t}(j) C^{*i}_{F,t}(j)\, dj = P^*_{F,t} C^{*i}_{F,t} \tag{B.12}$$

Finally, the optimal allocation of consumption expenditures between domestic and imported goods is given respectively by:

$$C^i_{H,t} = (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C^i_t \tag{B.13}$$

$$C^i_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C^i_t \tag{B.14}$$

for households in country $H$, while for households in country $F$ the optimal allocation of consumption expenditures between domestic and imported goods is given respectively by:

$$C^{*i}_{H,t} = (1-\alpha^*) \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} C^{*i}_t \tag{B.15}$$

$$C^{*i}_{F,t} = \alpha^* \left( \frac{P^*_{F,t}}{P^*_t} \right)^{-\eta} C^{*i}_t \tag{B.16}$$

where $P_t$ is the Consumer Price Index (CPI) for country $H$, given by:

$$P_t \equiv \left[ (1-\alpha)(P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{B.17}$$

while $P^*_t$ is the Consumer Price Index (CPI) for country $F$, given by:

$$P^*_t \equiv \left[ (1-\alpha^*)(P^*_{H,t})^{1-\eta} + \alpha^* (P^*_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{B.18}$$

Total consumption expenditures by households in country $H$ are then given by:

$$P_{H,t} C^i_{H,t} + P_{F,t} C^i_{F,t} = P_t C^i_t \tag{B.19}$$

while total consumption expenditures by households in country $F$ are given by:

$$P_{H,t}^{*}C_{H,t}^{*i} + P_{F,t}^{*}C_{F,t}^{*i} = P_t^{*}C_t^{*i} \tag{B.20}$$

Finally, the period budget constraint for households in country $H$ can then be rewritten as:

$$\begin{aligned} &P_tC_t^i + D_t^i + B_{H,t}^i + B_{F,t}^i \\ &\quad \leq \frac{D_{t-1}^i}{\mathcal{Q}_{t-1,t}} + B_{H,t-1}^i(1+i_{t-1}) + B_{F,t-1}^i(1+i_{t-1}^{*})(1-\delta_{t-1}) + (1-\tau_t^w)W_tN_t^i + T_t^i + \Gamma_t^i + \mathcal{I}_t \end{aligned} \tag{B.21}$$

while the period budget constraint for households in country $F$ can be rewritten as:

$$P_t^{*}C_t^{*i} + D_t^{*i} + B_{F,t}^{*i} \leq \frac{D_{t-1}^{*i}}{\mathcal{Q}_{t-1,t}} + B_{F,t-1}^{*i}(1+i_{t-1}^{*}) + (1-\tau_t^{*w})W_t^{*}N_t^{*i} + T_t^{*i} + \Gamma_t^{*i} \tag{B.22}$$

Aggregating the demand functions for domestic and imported goods, within each category of goods produced in each country, for households in country $H$, equations B.1 and B.2, yields:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\frac{C_{H,t}}{h} \tag{B.23}$$

respectively,

$$C_{F,t}(j) = \left(\frac{P_{H,t}(j)}{P_{F,t}}\right)^{-\varepsilon}\frac{C_{F,t}}{1-h} \tag{B.24}$$

where $C_{H,t}$ and $C_{F,t}$ are aggregate consumption of domestic and imported goods for households in country $H$, respectively defined by:

$$C_{H,t} \equiv \int_0^h C_{H,t}^i \, di = hC_{H,t}^i \qquad \text{and} \qquad C_{F,t} \equiv \int_0^h C_{F,t}^i \, di = hC_{F,t}^i \tag{B.25}$$

while, aggregating the demand functions for domestic and imported goods, within each category of goods, for households in country $F$, equations B.3 and B.4, yields:

$$C_{H,t}^{*}(j) = \left(\frac{P_{H,t}^{*}(j)}{P_{H,t}^{*}}\right)^{-\varepsilon}\frac{C_{H,t}^{*}}{1-h} \tag{B.26}$$

respectively,

$$C_{F,t}^{*}(j) = \left(\frac{P_{H,t}^{*}(j)}{P_{F,t}^{*}}\right)^{-\varepsilon}\frac{C_{F,t}^{*}}{h} \tag{B.27}$$

where $C_{H,t}^{*}$ and $C_{F,t}^{*}$ are aggregate consumption of domestic and imported goods for households in country $F$, respectively defined by:

$$C_{H,t}^{*} \equiv \int_h^1 C_{H,t}^{*i} \, di = (1-h)C_{H,t}^{*i} \qquad \text{and} \qquad C_{F,t}^{*} \equiv \int_h^1 C_{F,t}^{*i} \, di = (1-h)C_{F,t}^{*i} \tag{B.28}$$

Aggregating instead the demand functions for domestic and imported goods, by country of origin, for households in country $H$, equations B.13 and B.14, yields:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{B.29}$$

respectively,

$$C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \tag{B.30}$$

where $C_t$ is aggregate consumption for households in country $H$, defined by:

$$C_t \equiv \int_0^h C_t^i \, di = hC_t^i \tag{B.31}$$

and $C_{H,t}$ and $C_{F,t}$ are aggregate consumption of domestic and imported goods for households in country $H$, as previously defined. Aggregating the demand functions for domestic and imported goods, by country of origin, for households in country $F$, equations B.15 and B.16, yields:

$$C_{H,t}^* = (1 - \alpha^*) \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \tag{B.32}$$

respectively,

$$C_{F,t}^* = \alpha^* \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} C_t^* \tag{B.33}$$

where $C_t^*$ is aggregate consumption for households in country $F$, defined by:

$$C_t^* \equiv \int_h^1 C_t^i \, di = (1-h) C_t^{*i} \tag{B.34}$$

and $C_{H,t}^*$ and $C_{F,t}^*$ are aggregate consumption of domestic and imported goods for households in country $F$, as previously defined.
The first order conditions for the household problem in country $H$ are derived by maximizing present-value utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[ \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \right] \tag{B.35}$$

subject to the budget constraint given by:

$$\begin{aligned} & P_t C_t^i + D_t^i + B_{H,t}^i + B_{F,t}^i \\ & \quad \le \frac{D_{t-1}^i}{\mathcal{Q}_{t-1,t}} + B_{H,t-1}^i (1 + i_{t-1}) + B_{F,t-1}^i (1 + i_{t-1}^*)(1 - \delta_{t-1}) + (1 - \tau_t^w) W_t N_t^i + T_t^i + \Gamma_t^i + \mathcal{I}_t \end{aligned} \tag{B.36}$$

I derive here the first order conditions for households in country $H$. The first order conditions for households in country $F$ are derived in a specular way.
The problem for households in country $H$ can be formalized by means of the following Lagrangean:

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \xi_t \left[ \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \right] - \sum_{t=0}^{\infty} \lambda_t \beta^t \left[ P_t C_t^i + D_t^i + B_{H,t}^i + B_{F,t}^i \right] \\ - & \sum_{t=0}^{\infty} \lambda_t \beta^t \left[ - \frac{D_{t-1}^i}{\mathcal{Q}_{t-1,t}} - B_{H,t-1}^i (1 + i_{t-1}) - B_{F,t-1}^i (1 + i_{t-1}^*)(1 - \delta_{t-1}) - (1 - \tau_t^w) W_t N_t^i - T_t^i - \Gamma_t^i - \mathcal{I}_t \right] \end{aligned} \tag{B.37}$$

where $\lambda_t$ is the multiplier on the budget constraint.
Taking derivatives yields:

$$\frac{\partial \mathcal{L}}{\partial C_t^i} = \beta^t \xi_t (C_t^i)^{-\sigma} - \lambda_t \beta^t P_t = 0 \implies \xi_t (C_t^i)^{-\sigma} = \lambda_t P_t \tag{B.38}$$

$$\frac{\partial \mathcal{L}}{\partial N_t^i} = -\beta^t \xi_t (N_t^i)^{\varphi} + \lambda_t \beta^t (1-\tau_t^w) W_t = 0 \implies \xi_t (N_t^i)^{\varphi} = \lambda_t (1-\tau_t^w) W_t \tag{B.39}$$

$$\frac{\partial \mathcal{L}}{\partial D_t^i} = -\lambda_t \beta^t + \frac{\lambda_{t+1}\beta^{t+1}}{\mathcal{Q}_{t,t+1}} = 0 \implies \mathcal{Q}_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{B.40}$$

$$\frac{\partial \mathcal{L}}{\partial B_{H,t}^i} = -\lambda_t \beta^t + \lambda_{t+1}\beta^{t+1}(1+i_t) = 0 \implies \frac{1}{1+i_t} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{B.41}$$

$$\frac{\partial \mathcal{L}}{\partial B_{F,t}^i} = -\lambda_t \beta^t + \lambda_{t+1}\beta^{t+1}(1+i_t^*)(1-\delta_t) = 0 \implies \frac{1}{(1+i_t^*)(1-\delta_t)} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{B.42}$$

Combining the first two derivatives yields the intratemporal optimality condition:

$$\frac{(C_t^i)^{-\sigma}}{P_t} = \frac{(N_t^i)^{\varphi}}{(1-\tau_t^w)W_t} \implies (C_t^i)^{\sigma}(N_t^i)^{\varphi} = (1-\tau_t^w)\frac{W_t}{P_t} \tag{B.43}$$

Combining the first and the third derivative yields the intertemporal optimality condition:

$$\mathcal{Q}_{t,t+1} = \beta \frac{\xi_{t+1}(C_{t+1}^i)^{-\sigma}}{P_{t+1}} \frac{P_t}{\xi_t (C_t^i)^{-\sigma}} \implies \mathcal{Q}_{t,t+1} = \beta \frac{\xi_{t+1}}{\xi_t} \left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{B.44}$$

which is the household's Euler Equation. Combining the third, fourth and fifth derivatives and using the fact that $i_t$, $i_t^*$ and $\delta_t$ are known in period t, while $Q_{t,t+1}$ is not, yields the no-arbitrage condition:

$$\frac{1}{(1+i_t^*)(1-\delta_t)} = \frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} \tag{B.45}$$

## B.2 Derivations for Firms

The firm's problem in country $H$ can be solved by first substituting the stochastic discount factor and the demand constraint into the objective function:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k \frac{\xi_{t+k}}{\xi_t} \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \frac{Y_{t+k}}{h} \left[(1-\tau_{t+k}^s)\bar{P}_{H,t} - MC_{t+k}^n\right] \right\} \tag{B.46}$$

Then, taking the derivative with respect to $\bar{P}_{H,t}$ and equating it to zero yields:

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}}{\xi_t} \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \frac{Y_{t+k}}{h} \left[(1-\tau_{t+k}^s)(1-\varepsilon) + \varepsilon \frac{MC_{t+k}^n}{\bar{P}_{H,t}}\right] \right\} = 0 \tag{B.47}$$

Multiplying all by $\frac{\xi_t C_t^{-\sigma} h}{(1-\varepsilon) P_t \bar{P}_{H,t}^{-\varepsilon}}$ and rearranging yields:

$$\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} \left((1-\tau_{t+k}^s)\bar{P}_{H,t} - \frac{\varepsilon}{(\varepsilon-1)} MC_{t+k}^n\right) \right\} = 0 \tag{B.48}$$

which is the first order condition of the firm's problem. Rearranging the previous equation one can express the optimal price chosen by the firm as a function of only aggregate variables:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC^n_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1 - \tau^s_{t+k}) \right\}} \tag{B.49}$$

Substituting nominal marginal cost with the expression for real marginal cost and dividing both sides by $P_{H,t}$ yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{(P_{H,t+k})^{\varepsilon+1}}{P_{H,t}} Y_{t+k} MC_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} (P_{H,t+k})^{\varepsilon} Y_{t+k} (1 - \tau^s_{t+k}) \right\}} \tag{B.50}$$

Multiplying numerator and denominator on the right hand side of the previous equation by $\frac{P_t}{P^{\varepsilon}_{H,t}}$ yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \xi_{t+k}(C_{t+k})^{-\sigma} \frac{P_t}{P_{t+k}} \left( \frac{P_{H,t+k}}{P_{H,t}} \right)^{\varepsilon+1} Y_{t+k} MC_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \xi_{t+k}(C_{t+k})^{-\sigma} \frac{P_t}{P_{t+k}} \left( \frac{P_{H,t+k}}{P_{H,t}} \right)^{\varepsilon} Y_{t+k} (1 - \tau^s_{t+k}) \right\}} \tag{B.51}$$

Rewriting the previous equation in terms of inflation yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \xi_{t+k}(C_{t+k})^{-\sigma} Y_{t+k} MC_{t+k} \left[ \prod_{s=1}^{k} \frac{(\Pi_{H,t+s})^{\varepsilon+1}}{\Pi_{t+s}} \right] \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \xi_{t+k}(C_{t+k})^{-\sigma} Y_{t+k} (1 - \tau^s_{t+k}) \left[ \prod_{s=1}^{k} \frac{(\Pi_{H,t+s})^{\varepsilon}}{\Pi_{t+s}} \right] \right\}} \tag{B.52}$$

Rewriting the previous equation in recursive form yields:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_t}{F_t} \tag{B.53}$$

where $K_t$ and $F_t$ are defined by:

$$K_t = \xi_t (C_t)^{-\sigma} Y_t MC_t + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}} K_{t+1} \right\} \tag{B.54}$$

$$F_t = \xi_t (C_t)^{-\sigma} Y_t (1 - \tau^s_t) + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}} F_{t+1} \right\} \tag{B.55}$$

The optimal relative price in country $F$ is given by:

$$\frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}} \frac{(P^*_{H,t+k})^{\varepsilon+1}}{P^*_{H,t}} Y^*_{t+k} MC^*_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi^*_{t+k}(C^*_{t+k})^{-\sigma}}{P^*_{t+k}} (P^*_{H,t+k})^{\varepsilon} Y^*_{t+k} (1 - \tau^{*s}_{t+k}) \right\}} \tag{B.56}$$

Rewriting the previous equation in terms of inflation rates yields:

$$\frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \xi^*_{t+k}(C^*_{t+k})^{-\sigma} Y^*_{t+k} MC^*_{t+k} \left[ \prod_{s=1}^{k} \frac{(\Pi^*_{H,t+s})^{\varepsilon+1}}{\Pi^*_{t+s}} \right] \right\}}{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \xi^*_{t+k}(C^*_{t+k})^{-\sigma} Y^*_{t+k} (1 - \tau^{*s}_{t+k}) \left[ \prod_{s=1}^{k} \frac{(\Pi^*_{H,t+s})^{\varepsilon}}{\Pi^*_{t+s}} \right] \right\}} \tag{B.57}$$

Rewriting again the previous equation in recursive form yields:

$$\frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K^*_t}{F^*_t} \tag{B.58}$$

for the optimal relative price in country $F$, where $K^*_t$ and $F^*_t$ are respectively defined recursively by:

$$K^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t MC^*_t + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon+1}}{\Pi^*_{t+1}} K^*_{t+1} \right\} \tag{B.59}$$

$$F^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t (1 - \tau^{*s}_t) + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon}}{\Pi^*_{t+1}} F^*_{t+1} \right\} \tag{B.60}$$

## B.3 Aggregate Supply

Under the assumption of staggered prices following Calvo (1983), since in country $H$ each period only a fraction $1 - \theta$ of firms change their prices, while a fraction $\theta$ keep their prices unchanged, from the definition of the domestic price index, equation 3.37:

$$P_{H,t} = \left[ \frac{1}{\theta h} \int_0^{\theta h} P_{H,t-1}(j)^{1-\varepsilon} \, dj + \frac{1}{(1-\theta)h} \int_{\theta h}^{h} \bar{P}_{H,t}(j)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}} = \left[ \theta (P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{B.61}$$

which shows the evolution of prices over time. Dividing both sides by $P_{H,t}$ yields:

$$1 = \left[ \theta \left( \frac{P_{H,t-1}}{P_{H,t}} \right)^{1-\varepsilon} + (1-\theta) \left( \frac{\bar{P}_{H,t}}{P_{H,t}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{B.62}$$

which can be rewritten to yield a relationship between gross PPI inflation and the optimal relative price:

$$\left( \frac{1 - \theta (\Pi_{H,t})^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\bar{P}_{H,t}}{P_{H,t}} \tag{B.63}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$ is gross PPI inflation in country $H$. Combining the last equation with the equation for the optimal relative price set by firms yields:

$$\left( \frac{1 - \theta (\Pi_{H,t})^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_t}{F_t} \tag{B.64}$$

which is an equation linking the evolution of PPI inflation to real marginal cost dynamics in country $H$, through the recursive functions $K_t$ and $F_t$, defined above.

Specular price dynamics apply to country $F$, where each period only a fraction $1 - \theta^*$ of firms change their prices, while a fraction $\theta^*$ keep their prices unchanged. From the definition of the domestic price index for country $F$, equation 3.39, the following expression can be derived:

$$P^*_{H,t} = \left[ \theta^* (P^*_{H,t-1})^{1-\varepsilon} + (1-\theta^*)(\bar{P}^*_{H,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{B.65}$$

Dividing both sides by $P^*_{H,t}$ yields:

$$1 = \left[\theta^* \left(\frac{P^*_{H,t-1}}{P^*_{H,t}}\right)^{1-\varepsilon} + (1-\theta^*)\left(\frac{\bar{P}^*_{H,t}}{P^*_{H,t}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{B.66}$$

which can be rewritten to yield a relationship between gross PPI inflation and the optimal relative price:

$$\left(\frac{1-\theta^*(\Pi^*_{H,t})^{\varepsilon-1}}{1-\theta^*}\right)^{\frac{1}{1-\varepsilon}} = \frac{\bar{P}^*_{H,t}}{P^*_{H,t}} \tag{B.67}$$

where $\Pi^*_{H,t} \equiv \frac{P^*_{H,t}}{P^*_{H,t-1}}$ is gross PPI inflation in country $F$. Combining the last equation with the equation for the optimal relative price set by firms yields:

$$\left(\frac{1-\theta^*(\Pi^*_{H,t})^{\varepsilon-1}}{1-\theta^*}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{K^*_t}{F^*_t} \tag{B.68}$$

which is an equation linking the evolution of PPI inflation to real marginal cost dynamics in country $F$, through the recursive functions $K^*_t$ and $F^*_t$, defined above.
Combining price dispersions, equations 3.66, with aggregate price dynamics, equations B.62 and B.66, yields two equations showing the evolution over time of price dispersion in countries $H$ and $F$ respectively:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} \tag{B.69}$$

Combining price dispersion, equation 3.66, with aggregate price dynamics, equations B.62, yields an equation showing the evolution over time of price dispersion in country $H$:

$$d_t = \frac{1}{h}\int_0^h \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj = \frac{1}{\theta h}\int_0^{\theta h}\left(\frac{P_{H,t-1}(j)}{P_{H,t}}\right)^{-\varepsilon} dj + \frac{1}{(1-\theta)h}\int_{\theta h}^h \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj \tag{B.70}$$

which multiplying and dividing the first term by $P^{-\varepsilon}_{H,t-1}$ yields:

$$d_t = \frac{1}{\theta h}\int_0^{\theta h}\left(\frac{P_{H,t-1}(j)}{P_{H,t-1}}\right)^{-\varepsilon}\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\varepsilon} dj + \frac{1}{(1-\theta)h}\int_{\theta h}^h \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj \tag{B.71}$$

$$d_t = \theta d_{t-1}\Pi^{\varepsilon}_{H,t} + (1-\theta)\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon} \tag{B.72}$$

for country $H$ and:

$$d^*_t = \theta^* d^*_{t-1}(\Pi^*_{H,t})^{\varepsilon} + (1-\theta^*)\left(\frac{\bar{P}^*_{H,t}}{P^*_{H,t}}\right)^{-\varepsilon} \tag{B.73}$$

for country $F$, which, substituting the optimal relative price with the function of inflation derived above, yields two equations showing the evolution of price dispersion over time depending on PPI inflation in countries $H$ and $F$ respectively:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.74}$$

$$d_t^* = \theta^* d_{t-1}^* (\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*) \left[ \frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.75}$$

Clearing of the labour market in country $H$ implies that aggregate labour supply, equation 3.18, must equal aggregate labour demand, equation 3.63, to yield:

$$h^{1+\frac{\sigma}{\varphi}} C_t^{-\frac{\sigma}{\varphi}} \left[ (1-\tau_t^w) \frac{W_t}{P_t} \right]^{\frac{1}{\varphi}} = \frac{Y_t}{A_t} d_t \tag{B.76}$$

Expressing the real wage as a function of everything else yields:

$$\frac{W_t}{P_t} = \left[ \frac{Y_t d_t C_t^{\frac{\sigma}{\varphi}}}{A_t (1-\tau_t^w)^{\frac{1}{\varphi}} h^{1+\frac{\sigma}{\varphi}}} \right]^{\varphi} \tag{B.77}$$

To derive a relationship between nominal marginal cost and output, one can start from deriving a relationship between real marginal cost and real wage:

$$MC_t = \frac{W_t}{A_t P_{H,t}} = \frac{W_t}{P_t} \frac{P_t}{P_{H,t}} \frac{1}{A_t} \tag{B.78}$$

Then, by substituting the real wage implied by labour market clearing and the price ratio as a function of the terms of trade, yields a relationship between real marginal cost and output through the terms of trade:

$$MC_t = \left[ \frac{Y_t d_t C_t^{\frac{\sigma}{\varphi}}}{A_t (1-\tau_t^w)^{\frac{1}{\varphi}} h^{1+\frac{\sigma}{\varphi}}} \right]^{\varphi} \left[ 1-\alpha+\alpha \mathcal{S}_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \frac{1}{A_t} \tag{B.79}$$

$$MC_t = \frac{Y_t^{\varphi} d_t^{\varphi} C_t^{\sigma}}{(1-\tau_t^w) A_t^{1+\varphi} h^{\varphi+\sigma}} \left[ 1-\alpha+\alpha \mathcal{S}_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{B.80}$$

Following the same procedure for the *Foreign* country, real marginal cost in country $F$ will be given by:

$$MC_t^* = \frac{(Y_t^*)^{\varphi} (d_t^*)^{\varphi} (C_t^*)^{\sigma}}{(1-\tau_t^{*w})(A_t^*)^{1+\varphi}(1-h)^{\varphi+\sigma}} \left[ 1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1} \right]^{\frac{1}{1-\eta}} \tag{B.81}$$

The previous equations together with the price setting equations constitute the aggregate supply equations for countries $H$ and $F$ linking inflation in each country to output and consumption in each country.

## B.4 Aggregate Demand

Clearing of the goods market implies the following condition for all goods $j \in [0,h)$ produced in country $H$:

$$Y_t(j) = \int_0^h C_{H,t}^i(j)\, di + \int_h^1 C_{F,t}^{*i}(j)\, di + G_t(j) \tag{B.82}$$

Substituting the derived aggregate demand functions for private domestic consumption, equation B.23, for foreign consumption of domestic goods, equation B.26, and for public consumption, equation 3.101, into the market clearing condition for good $j$, yields:

$$Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \frac{C_{H,t}}{h} + \left( \frac{P_{F,t}^*(j)}{P_{F,t}^*} \right)^{-\varepsilon} \frac{C_{F,t}^*}{h} + \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \frac{G_t}{h} \tag{B.83}$$

while substituting in equations B.29 and B.33, and using the law of one price ($P^*_{F,t}(j) = P_{H,t}(j)$ and $P^*_{F,t} = P_{H,t}$), yields:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[\frac{1-\alpha}{h}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{\alpha^*}{h}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t + \frac{G_t}{h}\right] \tag{B.84}$$

Substituting the previous equation into the definition of aggregate domestic output, equation 3.65, yields:

$$Y_t = \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h \left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[\frac{1-\alpha}{h}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \frac{\alpha^*}{h}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t + \frac{G_t}{h}\right]\right]^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.85}$$

Since only the first fraction depends on $j$ the previous equation can be rewritten in the following way:

$$Y_t = \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h \left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \frac{1}{h}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^*\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t + G_t\right] \tag{B.86}$$

Simplifying exponents yields:

$$Y_t = \left(\frac{1}{h}\int_0^h \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{1-\varepsilon} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^*\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t + G_t\right] \tag{B.87}$$

Substituting in the definition of $P_{H,t}$, equation 3.37, simplifies the first term and yields:

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^*\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t + G_t \tag{B.88}$$

Making use of the law of one price, equation 3.46, and the definition of the real exchange rate $Q_t$, defined by equation 3.56, yields:

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^*\left(\frac{P_{H,t}}{P_t Q_t}\right)^{-\eta} C^*_t + G_t \tag{B.89}$$

Collecting terms in the previous expression yields:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)C_t + \alpha^*(Q_t)^\eta C^*_t\right] + G_t \tag{B.90}$$

Substituting the price ratio and the real exchange rate with expressions of the terms of trade yields:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha)C_t + \alpha^*(\mathcal{S}_t)^\eta \left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{1-\eta}} C^*_t\right] + G_t \tag{B.91}$$

The same procedure can be applied to the goods market clearing condition for all goods $j \in [0,1]$ produced in country $F$:

$$Y^*_t(j) = \int_h^1 C^{*i}_{H,t}(j)\,di + \int_0^h C^i_{F,t}(j)\,di + G^*_t(j) \tag{B.92}$$

Substituting the derived demand functions for private domestic consumption, equations B.26 and B.32, for foreign consumption of domestic goods, equations B.23 and B.30, and for public consumption, equation 3.116, yields:

$$Y_t^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} \left[\frac{1-\alpha^*}{1-h}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^* + \frac{\alpha}{1-h}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + \frac{G_t^*}{1-h}\right] \tag{B.93}$$

Plugging the market clearing condition into the definition of aggregate domestic output in country $F$, equation 3.65, yields:

$$Y_t^* = (1-\alpha^*)\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^* + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + G_t^* \tag{B.94}$$

Making use of the law of one price, equation 3.46, and the definition of the real exchange rate $Q_t$, defined by equation 3.56, yields:

$$Y_t^* = \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} \left[(1-\alpha^*)C_t^* + \alpha(Q_t)^{-\eta} C_t\right] + G_t^* \tag{B.95}$$

Substituting the price ratio and the real exchange rate with expressions of the terms of trade and rearranging yields the aggregate demand equation for country $F$:

$$Y_t^* = \left[1-\alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha^*)C_t^* + \alpha(\mathcal{S}_t)^{-\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{\eta-1}} C_t\right] + G_t^* \tag{B.96}$$

Demand for output in both countries depends on private consumption in the two countries, through the terms of trade and relative degree of openness. Government consumption, instead, affects aggregate demand directly because of complete home bias in public consumption.

## B.5 Equilibrium Conditions

Here we collect all the equilibrium conditions of the full model, differentiating between a pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union and between different policy rules.
The equilibrium conditions of the model are grouped into the following blocks:

**Aggregate Demand Block**
The aggregate demand block is composed of aggregate demand in both countries $H$:

$$Y_t = \left[1-\alpha + \alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha)C_t + \alpha^*(\mathcal{S}_t)^{\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{1-\eta}} C_t^*\right] + G_t \tag{B.97}$$

and $F$:

$$Y_t^* = \left[1-\alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha^*)C_t^* + \alpha(\mathcal{S}_t)^{-\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{\eta-1}} C_t\right] + G_t^* \tag{B.98}$$

while the evolution of private consumption is given by the households' Euler Equation in countries $H$:

$$\frac{1}{1+i_t} = \beta E_t\left\{\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\Pi_{t+1}}\right\} \tag{B.99}$$

and $F$:

$$\frac{1-\delta_t}{1+i_t} = \beta E_t \left\{ \frac{\xi^*_{t+1}}{\xi^*_t} \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{1}{\Pi^*_{t+1}} \right\} \tag{B.100}$$

while the relationship between CPI inflation and PPI inflation is given by:

$$\Pi_t = \Pi_{H,t} \left[ \frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}} \right]^{\frac{1}{1-\eta}} \tag{B.101}$$

in country $H$ and:

$$\Pi^*_t = \Pi^*_{H,t} \left[ \frac{1-\alpha^*+\alpha^*\mathcal{S}_t^{\eta-1}}{1-\alpha^*+\alpha^*(\mathcal{S}_{t-1})^{\eta-1}} \right]^{\frac{1}{1-\eta}} \tag{B.102}$$

in country $F$, and the evolution of the terms of trade is given by:

$$\mathcal{S}_t = \frac{\Pi^*_{H,t}}{\Pi_{H,t}} \mathcal{S}_{t-1} \tag{B.103}$$

while the exogenous demand shocks evolve according to:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \tag{B.104}$$

$$\xi^*_t = (\xi^*_{t-1})^{\rho^*_\xi} e^{\varepsilon_t} \tag{B.105}$$

**Aggregate Supply Block**

The aggregate supply block is composed of the aggregate supply equation for country $H$:

$$\left( \frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1} \frac{K_t}{F_t} \tag{B.106}$$

where:

$$K_t = \xi_t (C_t)^{-\sigma} Y_t MC_t + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}} K_{t+1} \right\} \tag{B.107}$$

$$F_t = \xi_t (C_t)^{-\sigma} Y_t (1-\tau^s_t) + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}} F_{t+1} \right\} \tag{B.108}$$

and marginal cost in country $H$ is given by:

$$MC_t = \frac{(Y_t)^\varphi (d_t)^\varphi (C_t)^\sigma}{(1-\tau^w_t)(A_t)^{1+\varphi}(h)^{\varphi+\sigma}} \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{B.109}$$

and the aggregate supply equation for country $F$:

$$\left( \frac{1-\theta^*(\Pi^*_{H,t})^{\varepsilon-1}}{1-\theta^*} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1} \frac{K^*_t}{F^*_t} \tag{B.110}$$

where:

$$K^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t MC^*_t + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon+1}}{\Pi^*_{t+1}} K^*_{t+1} \right\} \tag{B.111}$$

$$F^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t (1-\tau^{*s}_t) + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon}}{\Pi^*_{t+1}} F^*_{t+1} \right\} \tag{B.112}$$

and marginal cost in country $F$ is given by:

$$MC_t^* = \frac{(Y_t^*)^{\varphi}(d_t^*)^{\varphi}(C_t^*)^{\sigma}}{(1-\tau_t^{*w})(A_t^*)^{1+\varphi}(1-h)^{\varphi+\sigma}} \left[1-\alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{B.113}$$

while the evolution of price dispersion is given by:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.114}$$

for country $H$, and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left[\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.115}$$

for country $F$, while the levels of technology evolve exogenously according to:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \tag{B.116}$$

$$A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \tag{B.117}$$

**Net Exports, Net Foreign Assets and the Balance of Payments**

Real Net Exports for country $H$ are given by:

$$\widetilde{NX}_t = Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{B.118}$$

Real Net Foreign Assets for country $H$ are given by:

$$\widetilde{NFA}_t = \tilde{B}_{F,t} \tag{B.119}$$

The real Balance of Payments for country $H$ is given by:

$$\widetilde{BP}_t = \widetilde{NX}_t + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}} \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \tag{B.120}$$

so that real Net Foreign Assets for country $H$ evolve according to:

$$\widetilde{NFA}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \tag{B.121}$$

while the transaction cost is given by:

$$\delta_t \equiv (1-\rho_\delta)\delta^B \left(\frac{B_{t-1}^{*G}}{P_{H,t-1}^* Y_{t-1}^*} - \frac{B^{*G}}{P_H^* Y^*}\right) + \rho_\delta \delta_{t-1} + \varepsilon_t \tag{B.122}$$

**Monetary Policy**

Monetary policy sets the nominal interest rate following the rule:

$$\beta(1+i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} \left[\beta(1+i_{t-1})\right]^{\rho_i} \tag{B.123}$$

where union-wide CPI inflation is defined by:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{B.124}$$

**Fiscal Policy in a Pure Currency Union - Transfer Scenario**

Fiscal policy, in the transfer scenario of the Pure Currency Union setting, sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{B.125}$$

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{B.126}$$

while using real transfers to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}_t^G = \gamma_t \left(\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}^G\right) \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{B.127}$$

and keeping the tax rates on labour income and firm sales constant:

$$\tau_t^w = \tau^w \qquad \tau_t^s = \tau^s \qquad \tau_t^{*w} = \tau^{*w} \qquad \tau_t^{*s} = \tau^{*s} \tag{B.128}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{B.129}$$

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1} + \delta_{t-1}}{1-\delta_{t-1}} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{B.130}$$

**Fiscal Policy in a Pure Currency Union - Consumption Scenario**

In the consumption scenario of the Pure Currency Union setting, fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{B.131}$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{B.132}$$

while using government consumption to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}_t^G = \gamma_t \left(\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}^G\right) \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{B.133}$$

and keeping the tax rates on labour income and firm sales constant:

$$\tau_t^w = \tau^w \qquad \tau_t^s = \tau^s \qquad \tau_t^{*w} = \tau^{*w} \qquad \tau_t^{*s} = \tau^{*s} \tag{B.134}$$

**Fiscal Policy in a Pure Currency Union - Distortionary Tax Scenario**

In the distortionary tax scenario of the Pure Currency Union setting, fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{B.135}$$

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{B.136}$$

while using equally taxes on labour income and on firm sales to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}_t^G = \gamma_t \left(\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}^G\right) \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{B.137}$$

and keeping real transfers constant:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tilde{T}_t = \tilde{T} \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad \tilde{T}_t^* = \tilde{T}^* \tag{B.138}$$

**Fiscal Policy in a Coordinated Currency Union - Transfer Scenario**

Fiscal policy, in the transfer scenario of the Coordinated Currency Union setting, sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{B.139}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{B.140}$$

while using real transfers to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}_t^G = \gamma_t \left(\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} - \tilde{B}^G\right) \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{B.141}$$

and keeping the tax rates on labour income and firm sales constant:

$$\tau_t^w = \tau^w \qquad \tau_t^s = \tau^s \qquad \tau_t^{*w} = \tau^{*w} \qquad \tau_t^{*s} = \tau^{*s} \tag{B.142}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{B.143}$$

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1} + \delta_{t-1}}{1 - \delta_{t-1}}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{B.144}$$

**Fiscal Policy in a Coordinated Currency Union - Consumption Scenario**

In the consumption scenario of the Coordinated Currency Union setting, fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{B.145}$$

$$\frac{\tilde{T}^*_t}{\tilde{T}^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_t)} \left(\frac{\tilde{T}^*_{t-1}}{\tilde{T}^*}\right)^{\rho^*_t} e^{\varepsilon_t} \tag{B.146}$$

while using government consumption to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G_t = \gamma_t \left(\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G\right) \qquad \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}\right) \tag{B.147}$$

and keeping the tax rates on labour income and firm sales constant:

$$\tau^w_t = \tau^w \qquad \tau^s_t = \tau^s \qquad \tau^{*w}_t = \tau^{*w} \qquad \tau^{*s}_t = \tau^{*s} \tag{B.148}$$

**Fiscal Policy in a Coordinated Currency Union - Distortionary Tax Scenario**

In the distortionary tax scenario of the Coordinated Currency Union setting, fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{B.149}$$

$$\frac{G^*_t}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} \tag{B.150}$$

while using equally taxes on labour income and on firm sales to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G_t = \gamma_t \left(\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G\right) \qquad \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}\right) \tag{B.151}$$

and keeping real transfers constant:

$$\tau^w_t - \tau^w = \tau^s_t - \tau^s \qquad \tilde{T}_t = \tilde{T} \qquad \tau^{*w}_t - \tau^{*w} = \tau^{*s}_t - \tau^{*s} \qquad \tilde{T}^*_t = \tilde{T}^* \tag{B.152}$$

**Fiscal Policy in a Full Fiscal Union - Transfer Scenario**

Fiscal policy, in the transfer scenario of the Full Fiscal Union setting, sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{B.153}$$

$$\frac{G^*_t}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} \tag{B.154}$$

while using real transfers equally in both countries to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G_t = \gamma_t \left( \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G \right) \qquad \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left( \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G} \right) \tag{B.155}$$

and keeping the tax rates on labour income and firm sales constant:

$$\tilde{T}_t - \tilde{T} = \tilde{T}^*_t - \tilde{T}^* \qquad \tau^w_t = \tau^w \qquad \tau^{*w}_t = \tau^{*w} \qquad \tau^s_t = \tau^s \qquad \tau^{*s}_t = \tau^{*s} \tag{B.156}$$

with the following consolidated budget constraint:

$$\begin{aligned} G_t + \tilde{T}_t + S_t(G^*_t + \tilde{T}^*_t) + i_{t-1}\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} + \frac{i_{t-1} + \delta_{t-1}}{1 - \delta_{t-1}} \frac{S_{t-1}\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} = \\ (\tau^s_t + \tau^w_t MC_t d_t)Y_t + (\tau^{*s}_t + \tau^{*w}_t MC^*_t d^*_t)S_t Y^*_t + \tilde{B}^G_t - \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} + S_t \tilde{B}_t^{*G} - \frac{S_{t-1}\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} \end{aligned} \tag{B.157}$$

**Fiscal Policy in a Full Fiscal Union - Consumption Scenario**

In the consumption scenario of the Full Fiscal Union setting, fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left( \frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_t)} \left( \frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \tag{B.158}$$

$$\frac{\tilde{T}^*_t}{\tilde{T}^*} = \left( \frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}} \right)^{-\psi^*_{nx}(1-\rho^*_t)} \left( \frac{\tilde{T}^*_{t-1}}{\tilde{T}^*} \right)^{\rho^*_t} e^{\varepsilon_t} \tag{B.159}$$

while using government consumption equally in both countries to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G_t = \gamma_t \left( \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G \right) \qquad \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left( \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G} \right) \tag{B.160}$$

and keeping the tax rates on labour income and firm sales constant:

$$G_t - G = G^*_t - G^* \qquad \tau^w_t = \tau^w \qquad \tau^{*w}_t = \tau^{*w} \qquad \tau^s_t = \tau^s \qquad \tau^{*s}_t = \tau^{*s} \tag{B.161}$$

**Fiscal Policy in a Full Fiscal Union - Distortionary Tax Scenario**

In the tax on income scenario of the Full Fiscal Union setting, fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left( \frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left( \frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \tag{B.162}$$

$$\frac{G^*_t}{G^*} = \left( \frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}} \right)^{-\psi^*_{nx}(1-\rho^*_g)} \left( \frac{G^*_{t-1}}{G^*} \right)^{\rho^*_g} e^{\varepsilon_t} \tag{B.163}$$

while using the tax rates on labour income and firm sales equally across tax rates and across countries to deleverage government debt, through the debt rules:

$$\frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G_t = \gamma_t \left( \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} - \tilde{B}^G \right) \qquad \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left( \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G} \right) \tag{B.164}$$

and keeping real transfers constant:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \qquad \tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \qquad \tilde{T}_t = \tilde{T} \qquad \tilde{T}_t^* = \tilde{T}^* \quad \text{(B.165)}$$

We can now define an equilibrium for the Currency Union.

**Definition 2** (Equilibrium)**.** *An Imperfectly competitive equilibrium is a sequence of stochastic processes:*

$$\mathcal{X}_t \equiv \{Y_t, Y_t^*, C_t, C_t^*, \Pi_{H,t}, \Pi_{H,t}^*, \Pi_t, \Pi_t^*, \Pi_t^U, S_t, K_t, K_t^*, F_t, F_t^*, MC_t, MC_t^*, d_t, d_t^*, \widetilde{NX}_t, \widetilde{NFA}_t, \widetilde{CA}_t\}$$

*and exogenous disturbances:*

$$\mathcal{Z}_t \equiv \{\xi_t, \xi_t^*, A_t, A_t^*\}$$

*satisfying equations B.97 through B.121 and the definition of union-wide inflation B.124, given initial conditions:*

$$\mathcal{I}_{-1} \equiv \{C_{-1}, C_{-1}^*, \Pi_{H,-1}, \Pi_{H,-1}^*, S_{-1}, d_{-1}, d_{-1}^*, \widetilde{NFA}_{-1}\}$$

*plus monetary and fiscal policies:*

$$\mathcal{P}_t \equiv \{i_t, G_t, G_t^*, \tilde{T}_t, \tilde{T}_t^*, \tau_t^s, \tau_t^{*s}, \tau_t^w, \tau_t^{*w}, \tilde{B}_t^G, \tilde{B}_t^{*G}\}$$

*specified in equation B.123 for monetary policy and in equations B.125 through B.165 for the various specifications of fiscal policy, for $t \geq 0$.*

## B.6 The Steady State

We describe its symmetric (in terms of per capita consumption and prices) non-stochastic steady state with constant government debt and zero inflation. We focus on the perfect foresight steady state and equilibrium deviations from it, given by different shocks. *Perfect Foresight* is a viable assumption because, despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a large number (more accurately, a continuum) of firms, as explained in Calvo (1983).

The symmetric non-stochastic steady state with constant government debt and zero inflation is defined by the following assumptions and equations.

All shocks are constant at their long-run levels of 1:

$$\xi = \xi^* = A = A^* = 1 \tag{B.166}$$

There is no inflation and no price dispersion:

$$\Pi_H = \Pi_H^* = \Pi = \Pi^* = \Pi^U = 1 \implies d = d^* = 1 \tag{B.167}$$

The terms of trade and the real exchange rate are equal to 1:

$$S = 1 \implies Q = 1 \tag{B.168}$$

Per-capita consumption is equal across countries:

$$\frac{C}{h} = \frac{C^*}{1-h} \tag{B.169}$$

Aggregate demand in each country is given by:

$$Y = (1-\alpha)\, C + \alpha^* C^* + G \tag{B.170}$$

$$Y^* = (1-\alpha^*)\, C^* + \alpha C + G^* \tag{B.171}$$

Combining the previous equations we can derive per-capita consumption in each country as a function of output and government spending and equate the two to derive an equation linking output and government spending in the two countries:

$$Y = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\left[Y^* - G^*\right] + G \tag{B.172}$$

From the Euler Equations:

$$\frac{1}{1+i} = \beta \implies i = \frac{1}{\beta} - 1 \tag{B.173}$$

Recalling marginal costs in steady state from price-setting:

$$MC = \frac{\varepsilon - 1}{\varepsilon}(1-\tau^s) \tag{B.174}$$

$$MC^* = \frac{\varepsilon - 1}{\varepsilon}(1-\tau^{*s}) \tag{B.175}$$

Marginal costs are also given by labour market equilibrium:

$$MC = \frac{(Y)^{\varphi}(C)^{\sigma}}{(1-\tau^w)(h)^{\varphi+\sigma}} \tag{B.176}$$

$$MC^* = \frac{(Y^*)^{\varphi}(C^*)^{\sigma}}{(1-\tau^{*w})(1-h)^{\varphi+\sigma}} \tag{B.177}$$

Equating the two marginal cost expressions for each country yields consumption in terms of output:

$$C = \left[\frac{\varepsilon - 1}{\varepsilon}\frac{(1-\tau^s)(1-\tau^w)(h)^{\varphi+\sigma}}{(Y)^{\varphi}}\right]^{\frac{1}{\sigma}} \tag{B.178}$$

$$C^* = \left[\frac{\varepsilon - 1}{\varepsilon}\frac{(1-\tau^{*s})(1-\tau^{*w})(1-h)^{\varphi+\sigma}}{(Y^*)^{\varphi}}\right]^{\frac{1}{\sigma}} \tag{B.179}$$

Deriving per-capita consumption in the two countries and equating the two yields an equation linking output in the two countries:

$$Y^* = \frac{1-h}{h}\left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)}\right]^{\frac{1}{\varphi}} Y \tag{B.180}$$

In steady state real net exports are given by:

$$\widetilde{NX} = Y - C - G \tag{B.181}$$

while real net foreign assets are:

$$\widetilde{NFA} = \tilde{B}_F \tag{B.182}$$

The real balance of payments is given by:

$$\widetilde{BP} = \widetilde{NX} + \left(\frac{1}{\beta} - 1\right)\widetilde{NFA} \tag{B.183}$$

while from the budget constraints of households and governments, or equivalently from the evolution of net foreign assets:

$$\widetilde{NFA} = \widetilde{NFA} + \widetilde{BP} \tag{B.184}$$

which implies that in steady state the balance of payments must be zero and so net exports pin down net foreign assets:

$$\widetilde{BP} = 0 \implies \widetilde{NX} = -\left(\frac{1}{\beta} - 1\right)\widetilde{NFA} \tag{B.185}$$

while the transaction cost in steady state is zero, because debt is constant at steady state level and there are no shocks:

$$\delta = 0 \tag{B.186}$$

These relations yield an equation linking output, government consumption and household consumption in the two countries in steady state, which mainly comes from the fact that net exports are in zero international net supply in steady state:

$$Y + Y^* = C + C^* + G + G^* \tag{B.187}$$

The household budget constraints in steady state for countries $H$ and $F$ are given by:

$$C = \left(\frac{1}{\beta} - 1\right)(\tilde{B}_H + \tilde{B}_F) + \tilde{T} + Y(1 - \tau^s - \tau^w MC) \tag{B.188}$$

$$C^* = \left(\frac{1}{\beta} - 1\right)\tilde{B}_F^* + \tilde{T}^* + Y^*(1 - \tau^{*s} - \tau^{*w} MC^*) \tag{B.189}$$

Instead the government budget constraints of the two countries in steady state read:

$$G + \tilde{T} + \left(\frac{1}{\beta} - 1\right)\tilde{B}^G = Y(\tau^s + \tau^w MC) \tag{B.190}$$

$$G^* + \tilde{T}^* + \left(\frac{1}{\beta} - 1\right)\tilde{B}^{*G} = Y^*(\tau^{*s} + \tau^{*w} MC^*) \tag{B.191}$$

## B.7 Derivations for Calibration

The relationship that links the two parameters of openness $\alpha$ and $\alpha^*$, equation 3.165, which is used to calibrate $\alpha^*$ is derived in the following way.
From the relationship linking demand for consumption, equation B.172, dividing all by $Y$ and rearranging yields:

$$1 - \frac{G}{Y} = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\left[1 - \frac{G^*}{Y^*}\right]\frac{Y^*}{Y} \tag{B.192}$$

From the relationship linking supply of consumption goods, equation B.180, dividing all by $Y$ yields:

$$\frac{Y^*}{Y} = \frac{1-h}{h}\left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)}\right]^{\frac{1}{\varphi}} \tag{B.193}$$

Substituting the second equation into the first yields:

$$1 - \frac{G}{Y} = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\left[1 - \frac{G^*}{Y^*}\right]\frac{1-h}{h}\left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^{s})(1-\tau^{w})}\right]^{\frac{1}{\varphi}} \tag{B.194}$$

$$\frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} = \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^{s})(1-\tau^{w})}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} \tag{B.195}$$

$$(1-\alpha)h + \alpha^*(1-h) = [(1-\alpha^*)(1-h) + \alpha h]\frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^{s})(1-\tau^{w})}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} \tag{B.196}$$

dividing all by $1-h$ and rearranging to explicit $\alpha^*$ yields:

$$\begin{aligned}&\alpha^*\left[1 + \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^{s})(1-\tau^{w})}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}}\right] = \\ &\alpha\frac{h}{1-h}\left[1 + \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^{s})(1-\tau^{w})}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}}\right] + \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^{s})(1-\tau^{w})}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} - \frac{h}{1-h}\end{aligned} \tag{B.197}$$

then finally dividing all by what multiplies $\alpha^*$ yields:

$$\alpha^* = \alpha\frac{h}{1-h} + \frac{h}{1-h}\frac{\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^{s})(1-\tau^{w})}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}} - 1}{1 + \frac{h}{1-h}\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\left[\frac{(1-\tau^{s})(1-\tau^{w})}{(1-\tau^{*s})(1-\tau^{*w})}\right]^{\frac{1}{\varphi}}} \tag{B.198}$$

which is the final equation linking the two parameters of openness through the shares government consumption in output and the tax rates.

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