Phd Program in Economics (XXV Cycle)
Department of Economics and Finance

Essays on Optimal Contracts with Costly State Verification

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Chapter 1

Introduction

The present dissertation collects two essays on the theory of optimal contracts in presence of a Costly State Verification Technology.

In the first essay (Chapter 2), we extend the standard Costly State Verification framework in such a way to study how the presence of pre-contractual, non-contractible investment affects the terms of the contract between a Principal and an Agent. In particular, we will present a model in which, before the contracting stage, an agent can invest an amount of effort to generate a future surplus in a bilateral relationship with a principal who is endowed with a costly auditing technology. To induce the Principal not to walk away from this relationship, both parties sign a contract with which payments from the agent to the principal are specified. We assume that parties are asymmetrically informed: the agent is privately informed about the state of nature and tries to exploit his informational advantage in order to give the principal lower payments. On the other hand, the principal tries to avoid agent's fraudulent behaviour by using (or threatening to use) the costly audit technology.

The results that we will present share some key features of both the Costly State Verification (CSV) and the hold-up literature. In particular, we will show how our optimal

contract inherits almost all the essential characteristics of the typical optimal contract in the CSV framework. However, there is a fundamental difference. Due to our assumption of relationship-specific and non-contractible investment, if the principal is not able to credibly commit to pre-specified auditing policies, then investment can no longer be used as a commitment device. This will lead to underinvestment in effort with respect to the full commitment case. Furthermore, we will show how different allocations of bargaining power between the contracting parties may lead to the hold-up problem. In particular, we will show that if the principal has all the bargaining power, then hold-up arises in its most severe form, that is, no effort at all will be exerted at the pre-contractual stage. Finally, we will briefly discuss how the introduction of bonus payments in case of successful project's outcome may mitigate the hold-up problem.

In the second essay (Chapter 3) we depart from the canonical model of costly auditing and study the optimal contract between a lender a borrower in the case in which the verification and reporting decisions are taken simultaneously (monitoring) and not sequentially (auditing). We consider two different scenarios: one in which the Principal is able to fully commit to the verification policy announced at the contracting stage and another one where such an ability to commit is assumed away.

We will show that the optimal contract under full commitment prescribes a null expected rent to the Agent if the low state of the world realizes along with the so-called maximum punishment principle, that is, the Agent must give his entire wealth to the Principal if caught cheating. Furthermore, the Agent will be left with a positive rent (bonus) if he truthfully reports the higher state of the world. In the case without commitment, we will show that the optimal contract will inherit all the main features of the contract under full commitment but with two relevant differences. First, the expected rent for the Agent in the high state is lower. Second, the loan size acts as a commitment device and therefore turns out to be

larger in the case in which full commitment to verification policies is assumed away.

Chapter 2

Costly auditing with non-contractible investment

2.1 Introduction

The literature on Costly State Verification (CSV) that originated from the seminal contributions of [Tow79] and [GH85] has always treated loan size or investment as fully contractible variables. In the typical scenario, a cashless entrepreneur borrows funds from a lender which is endowed with a (perfect) costly verification (or auditing) technology. The amount of the loan to be invested in the contractual relationship between the two agents is usually assumed to be fully contractible.

However, we can think of many situations in the real world in which the assumption of full contractibility happens to be too restrictive. For instance, consider[1] a situation in which an agent (e.g., a start-up inventor) can make an investment in basic research activity today in order to generate a possible surplus tomorrow. Once the investment has been made, suppose the agent asks a principal to participate in the project (for instance, in order to develop

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[1]This example was inspired by [Sch12].

a marketable innovation or final product). Given this sequence of events[2], the agent's investment turns out to be a sunk cost at the contracting stage. The literature[3] has usually interpreted a situation of this kind as an *incomplete contract* scenario in which parties cannot agree upon a pre-specified investment level.

The aim of this chapter is to extend the standard CSV framework in such a way to study how the presence of pre-contractual, non-contractible investment affects the terms of the contract between the two parties. In particular, we will present a model in which, before the contracting stage, an agent can invest an amount of effort to generate a future surplus in a bilateral relationship with a principal who is endowed with a costly verification technology. To induce the Principal not to walk away from this relationship, both parties sign a contract with which payments from the agent to the principal are specified. We assume that parties are asymmetrically informed: the agent is privately informed about the state of nature and tries to exploit his informational advantage in order to give the principal lower payments. On the other hand, the principal tries to avoid agent's fraudulent behavior by using (or threatening to use) the costly audit technology.

The results that we will present share some key features of both the CSV and the hold-up literature. In particular, we will show how our optimal contract inherits almost all the essential characteristics of the typical optimal contract in the CSV framework. However, there is a fundamental difference. Due to our assumption of relationship-specific and non-contractible investment, if the principal is not able to credibly commit to pre-specified auditing policies, then investment can no longer be used as a commitment device[4]. This will lead to under-investment in effort with respect to the full commitment case, which is in sharp contrast to [KP98]. Furthermore, we will show how different allocations of bargaining power between the contracting parties may lead to the hold-up problem. In particular, we will show that

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[2]This scenario is typical in the hold-up literature. See, for instance, Hart and Moore (1990).
[3]See [HM88] and (1999), Aghion and Tirole (1994).
[4]On the role of investment as a commitment device, see [KP98].

if the principal has all the bargaining power, then hold-up arises in its most severe form, that is, no effort at all will be exerted at the pre-contractual stage. Finally, we will briefly discuss how the introduction of bonus payments in case of successful project's outcome may mitigate the hold-up problem.

The chapter is organized as follows. In Section 2.2 we will describe the basic model. Then we proceed to study the optimal contract design problem both under the assumption of full commitment to auditing policies (Section 2.3) and under the assumption that the principal cannot credibly commit to auditing policies in Section 2.4. The effort choice problem is studied in Section 2.5 while Section 2.6 discusses the effects of different allocations of bargaining power between the two parties. Then conclusions follow.

2.2   The model

Let us consider a simple environment with two risk-neutral, expected utility maximizing players: A (the Agent) and P (the Principal). The Agent is endowed with a risky project for the functioning of which an investment in effort is required. A can invest an amount of effort $e \in [0, 1]$ that would affect the project's outcome distribution over the set of states $S = \{h, l\}$. In particular, the project's outcome is represented by a random variable y with support $\{y_h, y_l\}$, where $y_h > y_l \geq 0$. The high state realization y_h occurs with probability e while the low state y_l occurs with probability $1 - e$.

Exerting effort is costly and these costs are represented by the twice continuously differentiable function $\psi : [0, 1] \to \mathbb{R}_+$ that satisfies $\psi(0) = 0$, $\lim_{e \to 1^-} \psi(e) = +\infty$, $\psi'(0) = 0$, $\psi'(e) > 0$ for $e > 0$, $\lim_{e \to 1^-} \psi'(e) = +\infty$, and $\psi''(\cdot) > 0$.

The investment in effort is assumed to be relationship-specific and non-contractible. Furthermore, in order for the project returns to realize, agent P participation is assumed to be necessary. Players are asymmetrically informed about the true state of the world and

they write a contract with which payments from A to P are specified. In particular, after investing effort e, player A privately observes the realization y_s of y, where $s \in S$. A makes a report $\hat{y}_m$ to player P, where $m \in M = \{h, l\}$. Contingent on the report, a payment specified by the contract is made to player P. The latter can either accept the payment or use a perfect audit technology with which he can discover the true state of the world at cost $c > 0$. We also assume $c < y_h - y_l$. After auditing, payments contingent both on the reported and the true state of the world take place. In particular, the payment functions specified by the contract between the two parties are $R(m|s)$ and $R(m|\cdot)$, where $R(m|s)$ is the payment when $\hat{y}_m$ is reported and y_s is the true state while $R(m|\cdot)$ is the payment when $\hat{y}_m$ is reported and no auditing occurs.

The Agent is protected by limited liability, so that $R(m|s) \leq y_s$ and $R(m|\cdot) \leq \hat{y}_m$ for $m, s = \{l, h\}$.

The sequence of events is the following:

- **Stage 1**: The Agent exerts effort $e \in [0, 1]$

- **Stage 2**: A contract specifying payments from A to P is signed

- **Stage 3**: A privately observes the true state of the world and sends a reports to P

- **Stage 4**: P decides whether to audit or not and final payments take place
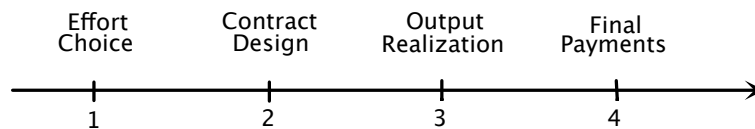
The sequence of events is

Figure 2.1: Sequence of events with pre-contractual effort

2.3 Optimal contract with full commitment

In this section we characterize the optimal contract under the assumption that, at the contracting stage, the Principal can fully commit to a pre-specified (and possibly random) auditing policy. We denote by $\widehat{\mathcal{C}}$ the resulting contract and we will use throughout the superscript $\widehat{}$ to indicate relevant variables in the full commitment case.

A contract is a list of payment functions $\widehat{R}(m|s)$ and $\widehat{R}(m|\cdot)$ and an auditing schedule $\widehat{\alpha}(\hat{y}_m)$ contingent on the reported state $\hat{y}_m$. $\widehat{R}(m|s)$ is the payment from party A to party P if the state $m \in M$ is reported and the true state $s \in S$ is discovered by the Principal using the (perfect) auditing technology he is endowed with. $\widehat{R}(m|\cdot)$ is the payment to P in the case in which m is reported and no auditing occurs, while $\widehat{\alpha}(\hat{y}_m)$ is the probability with which P will audit the reported state m.

By standard arguments[5], we can reduce the number of relevant components of the optimal contract by noticing that the agent A's incentive to lie arises only in the case in which a high state h realizes and, as a consequence, agent P has an incentive to audit only if a low state l is reported, the auditing technology being costly to use.

Therefore, a contract is a list $\widehat{\mathcal{C}} = \left\{ \widehat{R}(l|\cdot), \widehat{R}(l|l), \widehat{R}(l|h), \widehat{R}(h|\cdot), \widehat{\alpha} \right\}$, where $\widehat{\alpha}$ is the probability with which a low state report will be audited.

Since we are working under the full commitment assumption, we can make use of the Revelation Principle and then confine our attention to direct truthful mechanisms. In other words, without loss of generality we can consider contracts for which the Agent is asked to report his private information, i.e. the state s, and truthful reporting is optimal at equilibrium. This implies that, given an arbitrary contract $\mathcal{C} = \{R(l|\cdot), R(l|l), R(l|h), R(h|\cdot), \alpha\}$, agent A expected payoff is

$$W = e\left[y_h - R(h|\cdot)\right] + (1-e)\left[y_l - (1-\alpha)R(l|\cdot) - \alpha R(l|l)\right] \tag{2.1}$$

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[5]See, for instance, [KP98] and [Cho98].

while the Principal's expected payoff is

$$Z = eR\left(h|\cdot\right) + \left(1 - e\right)\left[\alpha R\left(l|l\right) + \left(1 - \alpha\right)R\left(l|\cdot\right) - \alpha c\right]. \tag{2.2}$$

The optimal contract $\widehat{\mathcal{C}}$ is the solution to the following program

Problem 1.

$$\max_{\{R(m|s),R(m|\cdot),\alpha\}} W$$

subject to

$$Z \geq 0 \tag{2.3}$$

$$R\left(h|\cdot\right) \leq \alpha R\left(l|h\right) + \left(1 - \alpha\right)R\left(l|\cdot\right) \tag{2.4}$$

$$R\left(m|s\right) \leq y_s \quad for \quad m, s \in \{l, h\} \tag{2.5}$$

$$R\left(m|\cdot\right) \leq y_m \quad for \quad m \in \{l, h\}, \tag{2.6}$$

where (2.3) is P's participation constraint, (2.4) is A's incentive compatibility (or truth-telling) constraint, (2.5) and (2.6) are the limited liability constraints.

The solution of the maximization problem above is described in the following

Proposition 1. *The contract* $\widehat{\mathcal{C}} = \left\{\widehat{R}\left(l|\cdot\right), \widehat{R}\left(l|l\right), \widehat{R}\left(l|h\right), \widehat{R}\left(h|\cdot\right), \widehat{\alpha}\right\}$ *which solves Problem 1 is such that*

- $\widehat{R}\left(l|\cdot\right) = \widehat{R}\left(l|l\right) = y_l,$

- $\widehat{R}\left(l|h\right) = y_h,$

- $y_l < \widehat{R}\left(h|\cdot\right) < y_h,$

- $\widehat{\alpha} = \frac{\widehat{R}(h|\cdot) - y_l}{y_h - y_l} < 1.$

The results of Proposition 1 are standard[6]. It is apparent that, in order to induce truthful reporting, the Agent is left with an expected payoff equal to zero in the low state l while he receives a positive expected rent in the high state h. Moreover, the punishment in the case of fraudulent report is maximal, that is, $\widehat{R}(l|h) = y_h$. Finally, A's incentive compatibility constraint is binding and $\widehat{\alpha}$ is less than 1. We will see in Section 2.6 how the latter result crucially depends on the allocation of bargaining power between the two contracting parties.

2.4   Optimal contract without commitment

In this section we provide the full characterization of the optimal contract $\widetilde{\mathcal{C}}$ between parties A and P under the assumption that, at the contracting stage, party P cannot commit himself not to renegotiate the auditing policy announced in stage 1. We use the superscript $\sim$ to denote all relevant variables under no commitment.

The reason why we also consider the scenario without commitment rests upon the inherent dynamic inconsistency of the optimal contract under full commitment. In fact, let us consider the scenario outlined in the previous section. Once the terms of the optimal contract have been agreed upon by both parties, the Agent will truthfully report the observed state of the world and then the Principal will audit according to the equilibrium auditing policy $\widehat{\alpha}$. Clearly, the Principal would not have any incentive *ex post* to audit because he knows for sure that A's reports are truthful and because auditing is costly. Furthermore, the Agent may anticipate P's incentive not to audit and then he may try to take advantage of the Principal by sending fraudulent reports. In other words, the P's promise to commit to the announced policy $\widehat{\alpha}$ is not credible. Without the full commitment assumption, we thus consider a scenario with *strategic* auditing and we will try to determine an optimal contract which is also renegotiation-proof.

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[6]See, for instance, [BS87], [MP89], and [KP98]

Before going through the contract design problem, we study and solve the reporting game that takes place once a given contract has been signed.

Let an arbitrary contract $\mathcal{C} = (R(m|s), R(m|\cdot))$ be given. After signing such a contract, the Agent privately observes the true state of the world $s \in S$ and then sends a message $m \in M$ to the Principal. The latter may then enforce the payment $R(m|\cdot)$ without any auditing. Alternatively, he may audit the report $m \in M$ at cost $c > 0$ to discover with certainty the true state $s \in S$ and then enforce the payment $R(m|s)$. We refer to this sequence of events as the 'reporting game' induced by contract $\mathcal{C}$.

The reporting game described above falls in the category of signalling (or sender-receiver) games. Player A is the sender whose private information is given by the state $s \in S$, that is, A's type. A signals his type to player P by means of one out of the two messages available, that is, $m \in M$.

We use behavioral strategies to describe each player's plan of action. In particular, we can define a behavioral strategy σ_A for player A as a probability distribution $(\sigma_A(y_s), 1 - \sigma_A(y_s))$, where $\sigma_A(y_s)$ is the probability of truthfully reporting the state of the world. In other words, it is the probability of reporting $\hat{y}_s$ when y_s is privately observed. A behavioral strategy σ_P for player P is a probability distribution $(\sigma_P(\hat{y}_m), 1 - \sigma_P(\hat{y}_m))$, where $\sigma_P(\hat{y}_m)$ is the probability of auditing the report $\hat{y}_m$.

Furthermore, player P has beliefs about the true type of player A. We define those beliefs as $\mu(s|m) = \mathsf{Prob}[y = y_s|\hat{y}_m]$. In words, $\mu(s|m)$ is the posterior probability that player P assigns to type s of player A after having received the message $m \in M$.

To solve the reporting game we use the standard solution concept found in the literature on sender-receiver games, that is, the Perfect Bayesian Equilibrium (PBE)[7]. A PBE is a behavioral strategy profile (σ_A^*, σ_P^*) and a belief system $\mu^*(s|m)$ such that beliefs $\mu^*(s|m)$

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[7]In our simple scenario, the Perfect Bayesian Equilibrium we are after will also satisfy the requirements of the Sequential Equilibrium proposed by Kreps and Wilson (1982). On the equivalence between PBE and Sequential Equilibrium in our simple scenario, see Fudenberg and Tirole (1991).

are determined by Bayes' rule whenever possible and, given those beliefs, the strategy profile (σ_A^*, σ_P^*) is sequentially rational.

As we already did in the full commitment scenario, we can simplify the structure of the game by noticing that at every equilibrium both $\sigma_A(y_l) = 1$ and $\sigma_P(\hat{y}_h) = 0$ will hold. Intuitively, the Agent will never send a high-state report $\hat{y}_h$ when a low state y_l realizes because in that case he would expect a lower payment from truthful reporting. In other words, the incentive lo lie arises only in the case in which a high state y_h realizes. On the other hand, the Principal anticipates agent A's behavior and, since auditing is costly, he will never audit a report $\hat{y}_h$.

Thus a contract $\widetilde{\mathcal{C}}$ has to specify four payments: $\left\{ \widetilde{R}(l|l), \widetilde{R}(l|h), \widetilde{R}(l|\cdot), \widetilde{R}(h|\cdot) \right\}$. Clearly, the no commitment assumption implies that any auditing policy announced by the Principal in the contracting stage cannot be contracted upon.

To save on notation, we introduce $\alpha := \sigma(\hat{y}_l)$, that is, the probability of auditing a low-income report, and $\gamma := \sigma_A(y_h)$, that is, the probability of truthfully reporting a high-income realization.

Given an arbitrary contract $\mathcal{C} = (R(m|s), R(m|\cdot))$, player A's expected payoff from $\mathcal{C}$ is

$$
\begin{aligned}
W &= e\left\{ \gamma\left(y_h - R(h|\cdot)\right) + (1-\gamma)\left[\alpha\left(y_h - R(l|h)\right) + (1-\alpha)\left(y_h - R(l|\cdot)\right)\right]\right\} + \\
&\quad + (1-e)\left\{\alpha\left(y_l - R(l|l)\right) + (1-\alpha)\left(y_l - R(l|\cdot)\right)\right\},
\end{aligned}
\tag{2.7}
$$

while agent P's expected payoff is

$$
\begin{aligned}
Z &= e\gamma R(h|\cdot) + \\
&\quad + (1-e\gamma)\left\{(1-\alpha)R(l|\cdot) + \alpha\left[\mu(h|l)R(l|h) + (1-\mu(h|l))R(l|l) - c\right]\right\}.
\end{aligned}
\tag{2.8}
$$

As shown, among others, by [KP98] and [Cho98], the PBE we are after is a mixed strategy equilibrium. This implies that $\alpha \in (0, 1)$ and $\gamma \in (0, 1)$. This implies that every player should be indifferent between the pure actions that will be played with positive probability in the corresponding equilibrium mixed strategy. Formally, the condition

$$\mu(h|l) R(l|h) + (1 - \mu(h|l)) R(l|l) - c = R(l|\cdot) \tag{2.9}$$

and the condition

$$\alpha R(l|h) + (1 - \alpha) R(l|\cdot) = R(h|\cdot) \tag{2.10}$$

must simultaneously hold. Note that equality (2.9) means that agent P must be indifferent between auditing and not auditing in the case in which a low state is reported. We also know that, if the high state is reported, then P will play the pure action 'not audit' with probability 1. Equality (2.10) instead refers to player A and it means that he must be indifferent between telling a lie and telling the truth when the high state of world is observed. If the low state is observed, then we already know that A will send the report $\hat{y}_l$ with probability 1.

As for beliefs, we can use Bayes' rule to easily obtain

$$\mu(h|l) = \frac{e(1 - \gamma)}{1 - e\gamma} \tag{2.11}$$

and

$$\mu(l|l) = 1 - \mu(h|l) = \frac{1 - e}{1 - e\gamma}. \tag{2.12}$$

Using (2.11) and (2.12) in (2.9), we obtain

$$\widetilde{\gamma} = \frac{eR(l|h) + (1 - e) R(l|l) - R(l|\cdot) - c}{e\left[R(l|h) - R(l|\cdot) - c\right]}. \tag{2.13}$$

From (2.10), we get

$$\widetilde{\alpha} = \frac{R\left(h|\cdot\right) - R\left(l|\cdot\right)}{R\left(l|h\right) - R\left(l|\cdot\right)}. \tag{2.14}$$

Thus we can conclude that in every PBE of our reporting game, player A and player P will randomize according to the above probabilities $\widetilde{\gamma}$ and $\widetilde{\alpha}$, respectively.

Now we are ready to set up and solve the optimal contract design problem.

As we already did in the full commitment case, we assume that the Agent has all the bargaining power and act as the contract designer. In Section 2.6 we will discuss the effect of different allocations of bargaining power between the two parties.

The optimal contract design problem reads as follows:

Problem 2.

$$\max_{\{R(m|s), R(m|\cdot)\}} W$$

subject to

$$Z \geq 0 \tag{2.15}$$

$$\alpha = \widetilde{\alpha} \tag{2.16}$$

$$\gamma = \widetilde{\gamma} \tag{2.17}$$

$$R\left(m|s\right) \leq y_s \quad for \quad m, s \in \{l, h\} \tag{2.18}$$

$$R\left(m|\cdot\right) \leq y_m \quad for \quad m \in \{l, h\}. \tag{2.19}$$

Condition (2.15) is player P's participation constraint, (2.16) and (2.17) ensure equilibrium play in the reporting game that will start out after signing the contract, (2.18) and (2.19) are the limited liability constraints.

The solution to Problem 2 is reported in the following

Proposition 2. *The contract* $\widetilde{\mathcal{C}} = \left\{ \widetilde{R}\left(l|\cdot\right), \widetilde{R}\left(l|l\right), \widetilde{R}\left(l|h\right), \widetilde{R}\left(h|\cdot\right) \right\}$ *which solves Problem 2 is such that:*

- $\widetilde{R}\left(l|\cdot\right) = \widetilde{R}\left(l|l\right) = y_l,$

- $\widetilde{R}\left(l|h\right) = y_h,$

- $y_l < \widetilde{R}\left(h|\cdot\right) < y_h.$

Furthermore, the following inequalities hold:

- $\widehat{\alpha} \leq \widetilde{\alpha} = \frac{\widetilde{R}(h|\cdot) - y_l}{y_h - y_l} < 1,$

- $\widetilde{R}\left(h|\cdot\right) \geq \widehat{R}\left(h|\cdot\right).$

The results of the Proposition above are standard[8]. Nonetheless, some remarks are in order. In particular, we can argue that the contract $\widetilde{\mathcal{C}}$ share some features of $\widehat{\mathcal{C}}$ except for a couple of remarkable differences. First, since the sender-receiver game induced by the contract $\widetilde{\mathcal{C}}$ has a mixed strategy equilibrium in which $\widetilde{\gamma} < 1$, this implies that *strategic default* occurs with positive probability at equilibrium. Clearly, this is in sharp contrast to the results of the previous section, the reason being that removing the full commitment assumption does not allow us to use the revelation principle. Second, even though the Agent is left with zero rent in the low state also in the no commitment case, now he is called upon to give the Principal a greater payment $\widetilde{R}\left(h|\cdot\right)$. The reason is that, in order for the P's auditing threat to be credible, the difference between payments $\widetilde{R}\left(l|\cdot\right)$ and $\widetilde{R}\left(h|\cdot\right)$ must be increased in order to avoid pooling at the low state. Via (2.14), also the auditing probability $\widetilde{\alpha}$ will be higher at equilibrium. Finally, we can conclude that in the case of no commitment the Agent is able to extract a smaller rent from the principal due to the higher payment $\widetilde{R}\left(h|\cdot\right)$ in the high state.

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[8]In particular, see [Kha97], [KP98], and [Cho98].

2.5   Investment in effort

Due to our assumption of relationship-specific, non contractible effort, player A chooses the optimal level of effort at stage 1, that is, before the contract with player P is signed. The Agent clearly makes his decision by anticipating his expected payoff from the contract that will be signed at stage 2. In this section we will study the effort choice problem in the case in which parties will agree on the contract $\widehat{\mathcal{C}}$ and also in the alternative scenario in which, under no commitment, $\widetilde{\mathcal{C}}$ will be signed.

As a benchmark, let us consider the first-best level of effort e^{FB}, that is, the level of effort that would be chosen under complete and perfect information. The Agent would solve

Problem 3.

$$\max_{e \in [0,1]} \quad ey_h + (1 - e) y_l - \psi(e)$$

and e^{FB} would be implicitly determined by the first-order condition

$$\psi'\left(e^{FB}\right) = (y_h - y_l). \tag{2.20}$$

Clearly, $e^{FB} > 0$ because of the assumptions $y_h > y_l$ and $\psi'(\cdot) > 0$ for $e > 0$.

Let us consider now the case of full commitment. The effort choice at stage 1 is determined by the following program

Problem 4.

$$\max_{e \in [0,1]} \widehat{W} - \psi(e),$$

where $\widehat{W}$ is A's expected payoff from contract $\widehat{\mathcal{C}}$.

Analogously, in the case of no commitment, agent A solves

Problem 5.

$$\max_{e \in [0,1]} \widetilde{W} - \psi(e),$$

where $\widetilde{W}$ is A's expected payoff from contract $\widetilde{\mathcal{C}}$.

The solutions to Problems 4 and 5 are summarized in the following

Proposition 3. *The Agent's effort choice at stage 1 is such that*

$$0 < \widetilde{e} \le \widehat{e} < e^{FB}.$$

Clearly, the presence of asymmetric information between the parties gives rise to underinvestment in effort with respect to the firs-best scenario. Furthermore, the inability to commit cause a further decrease in the optimal level of effort with respect to the scenario under full commitment. The reason for such an underinvestment is due to the fact that the absence of commitment allow the Agent to extract a smaller rent from the Principal at the contracting stage. Anticipating the resulting lower expected payoff, the Agent will exert lower effort in the no commitment case. The result is quite intuitive and is in contrast to the findings contained in [KP98]. In fact, they show that, when investment is fully contractible, the absence of commitment to audit policies implies a higher level of investment. This is due to the fact that the amount of investment (in [KP98], the loan size) is used to make the P's auditing threat credible. That is, the higher is the investment at stake in the contractual relationship with A, the higher is the probability with which the Principal will audit fraudulent reports. In our case, investment cannot be used as a commitment device because it is assumed to be non-contractible. The only commitment device the Principal can use is (an increase in) the payment $\widetilde{R}(h|\cdot)$. But increasing $\widetilde{R}(h|\cdot)$ would cause the Agent to gain a lower expected payoff from $\widetilde{\mathcal{C}}$, hence the underinvestment with respect to the full commitment case.

2.6 Bargaining power and audit efficiency

In this section we provide some extensions of the results in Proposition 3 to different assumptions about the allocation of bargaining power between the contracting parties and about the audit efficiency of the optimal contract. More precisely, we will first characterize the optimal contract under no commitment in the case in which player P is the contract designer and then we will see how the allocation of bargaining power between A and P affects the effort investment choice at stage 1. Finally, we will discuss how bargaining power affects the audit efficiency of the optimal contract.

Let us consider first the optimal contract design problem in the case in which the Principal is assumed to have all the bargaining power. The optimal contract $\widetilde{C}_P$ now solves the following program:

Problem 6.

$$\max_{\{R(m|s),R(m|\cdot)\}} Z$$

subject to

$$W \geq 0 \tag{2.21}$$

$$\alpha = \widetilde{\alpha} \tag{2.22}$$

$$\gamma = \widetilde{\gamma} \tag{2.23}$$

$$R\left(m|s\right) \leq y_s \quad for \quad m, s \in \{l, h\} \tag{2.24}$$

$$R\left(m|\cdot\right) \leq y_m \quad for \quad m \in \{l, h\}. \tag{2.25}$$

Clearly, (2.21) is the Agent's participation constraint while (2.22), (2.23), (2.24), and (2.25) are equivalent to the corresponding constraints in Problem 2. It is apparent that the only

difference between the problem above and Problem 2 is given by the objective function to maximize.

The solution to Problem 6 is reported in the following

Proposition 4. *The contract* $\widetilde{\mathcal{C}}_P = \left\{ \widetilde{R}_P (l|\cdot), \widetilde{R}_P (l|l), \widetilde{R}_P (l|h), \widetilde{R}_P (h|\cdot) \right\}$ *which solves Problem 6 is such that*

- $\widetilde{R}_P (l|\cdot) = \widetilde{R}_P (l|l) = y_l,$

- $\widetilde{R}_P (l|h) = \widetilde{R}_P (h|\cdot) = y_h,$

- $\widetilde{\alpha}_P = 1.$

The results of Proposition 4 are quite intuitive. Since the Principal operates as the contract designer, he is able to push down the Agent's expected payoff to its reservation value, i.e. zero. More precisely, the Agent is left with an expected payoff equal to zero in both states l and h (and not only in state h as in Proposition 2). In other words, the Principal is now able to maximize his expected payoff by setting $\widetilde{R}_P (h|\cdot) = y_h$, so auditing with probability 1 any low-state report by player A.

As for the effort choice, it is evident that if P is the contract designer, no effort at all will be exerted by A at stage 1. To consider a more interesting scenario, we follow [Sch12] and assume that at stage 1 there is a probability ϕ that the contract will be designed by player A at stage 2 and a probability $1-\phi$ that the contract designer will be player P. The probability ϕ can be interpreted as a measure of A's bargaining power[9].

Then the effort investment problem becomes

Problem 7.

$$\max_{e \in [0,1]} \quad \phi \widetilde{W} + (1 - \phi) \widetilde{W}_P - \psi(e), \tag{2.26}$$

---

[9]To use [RW86]'s words, our simple scenario implies that 'the expected surplus at stage 2 is split according to the generalized Nash bargaining solution, where ϕ is party A's bargaining power'.

where $\widetilde{W}$ is agent A's expected payoff from contract $\widetilde{\mathcal{C}}$ while $\widetilde{W}_P$ is A's expected payoff from contract $\widetilde{\mathcal{C}}_P$, which is equal to zero.

The solution is reported in the following

Proposition 5. *The effort level $\widetilde{e}_\phi$ which solves Problem 7 is increasing with respect to ϕ. Moreover, if $\phi = 0$, then $\widetilde{e}_\phi = 0$ and if $\phi = 1$, then $\widetilde{e}_\phi = \widetilde{e}$.*

The results of the above proposition identifies the so-called hold-up problem. Because of contract incompleteness at stage 1 and relationship-specific investment, the Agent's incentive to invest is increasing with his bargaining power. In the extreme case in which $\phi = 0$, then A is totally held-up by the Principal and no effort will be exerted.

A further aspect to consider is the relationship between the hold-up problem and the audit efficiency of the optimal contract. The issue can be put in the following terms. When A has no bargaining power at all, the hold-up problem arises in its most severe form because of P's inability to commit and the auditing inefficiency[10] of the resulting optimal contract. In fact, expected auditing costs are equal to $\widetilde{\alpha} c (1 - e\gamma)$ under the contract $\widetilde{\mathcal{C}}$ whereas they are equal to $c (1 - e\gamma)$ under $\widetilde{\mathcal{C}}_P$. That is, the fact that the Principal sets $\widetilde{R}_P (h|\cdot) = y_h$ to extract as much surplus as possible from A implies that the resulting optimal contract gives rise to inefficiently high expected auditing costs. Therefore, in order to reduce this efficiency loss and possibly mitigate the resulting hold-up problem, a benevolent planner may think of changing the contracting terms between the parties and introduce an upper bound $\overline{R} < y_h$ on the payment $R (h|\cdot)$ so that it has to satisfy $R (h|\cdot) \leq \overline{R}$. The introduction of this payment cap can be interpreted as a bonus that the Agent can retain in case of successful outcome of the risky project he is endowed with.

Introducing the payment cap $\overline{R}$, the contract design problem in the case in which P has all the bargaining power becomes

---

[10]For a detailed analysis of auditing efficiency, see [GS06]. In addition, from a normative perspective [Cho98] shows that auditing efficiency implies that the informed party should be the contract designer.

Problem 8.

$$\max_{\{R(m|s),R(m|\cdot)\}} Z$$

subject to

$$W \geq 0 \tag{2.27}$$

$$\alpha = \widetilde{\alpha} \tag{2.28}$$

$$\gamma = \widetilde{\gamma} \tag{2.29}$$

$$R\left(m|s\right) \leq y_s \quad for \quad m,s \in \{l,h\} \tag{2.30}$$

$$R\left(l|\cdot\right) \leq y_l \tag{2.31}$$

$$R\left(h|\cdot\right) \leq \overline{R} < y_h. \tag{2.32}$$

We denote as $\overline{\mathcal{C}}_P$ the solution to this program. Furthermore, the effort choice problem would become

Problem 9.

$$\max_{e \in [0,1]} \phi \widetilde{W} + (1 - \phi) \overline{W}_P - \psi\left(e\right),$$

where $\overline{W}_P$ is the Agent's expected payoff from the contract $\overline{\mathcal{C}}_P$.

The results are summarized in the following

Proposition 6. *Suppose an upper bound $\overline{R}$ on the payment $R\left(h|\cdot\right)$ is introduced. Then Problem 9 is solved by the contract $\overline{\mathcal{C}}_P$ which satisfies*

- $\overline{R}_P\left(l|\cdot\right) = \overline{R}_P\left(l|l\right) = y_l,$

- $\overline{R}_P\left(|h\right) = y_h,$

- $\overline{R}_P\left(h|\cdot\right) = \overline{R}.$

Furthermore, the effort level $\overline{e}_\phi$ that solves Problem 9 is increasing with respect to ϕ. Moreover, if $\phi = 0$, then $\overline{e}_\phi > 0$. If $\phi = 1$, then $\overline{e}_\phi = \widetilde{e}_\phi = \widetilde{e}$.

Clearly, the payment cap $\overline{R}$ is a binding constraint. Since $\overline{R} < y_h$, we also obtain that A's expected payoff from $\overline{\mathcal{C}}_P$ is positive, even though the Principal has all the bargaining power. This fact would mitigate the hold-up problem. Indeed now we have that A's incentive to exert effort is increasing with his bargaining power, but even in the case in which $\phi = 0$ a positive level of effort $\overline{e}_\phi$ would be chosen. The latter result is clearly in contrast to the most severe form of hold-up characterized in Proposition 5.

2.7 Conclusion

In this chapter we have provided the full characterization of the optimal contract between a Principal and an Agent under the assumption that the latter can invest a non-contractible, relationship-specific amount of effort before the contracting stage. The resulting optimal contract shares most of the main features of the optimal mechanisms usually found in the CSV literature. In particular, we have that the Agent is left with zero expected rent in the low state realization and the so-called maximum punishment principle applies. Moreover, in the case in which the Principal is not able to fully commit to the announced auditing policy, strategic default will appear at equilibrium. However, the amount of investment cannot be used as a commitment device and this leads to underinvestment with respect to the case with full commitment. We have also seen how different allocations of bargaining power directly affect the amount of effort invested.

2.8 Appendix

This Appendix contains all the proofs omitted in the Chapter text.

2.8.1 Proof of Proposition 1

The proof technique is the same as in [KP98].

The Lagrangean associated with Problem 1 is

$$
\begin{aligned}
\mathfrak{L} = \quad & e \quad [y_h - R(h|\cdot)] + (1-e)[y_l - (1-\alpha)R(l|\cdot) - \alpha R(l|l)] \\
& - \quad \lambda[R(h|\cdot) - \alpha R(l|h) - (1-\alpha)R(l|\cdot)] \\
& + \quad \theta\{eR(h|\cdot) + (1-e)[\alpha R(l|l) + (1-\alpha)R(l|\cdot) - \alpha c]\},
\end{aligned}
\tag{2.33}
$$

where $\lambda \geq 0$ and $\theta \geq 0$ are the Lagrange multipliers associated with the constraints (2.4) and (2.3), respectively.

A solution $\left\{\widehat{R}(l|\cdot), \widehat{R}(l|l), \widehat{R}(l|h), \widehat{R}(h|\cdot), \widehat{\alpha}, \widehat{\lambda}, \widehat{\theta}\right\}$ must satisfy the following set of first-order conditions

$$
\frac{\partial \mathfrak{L}}{\partial R(l|h)} = \lambda\alpha \geq 0 \quad \text{and} \quad [y_h - R(l|h)] \cdot \frac{\partial \mathfrak{L}}{\partial R(l|h)} = 0
\tag{2.34}
$$

$$
\frac{\partial \mathfrak{L}}{\partial R(l|\cdot)} = (1-\alpha)[\lambda + \theta(1-e) - (1-e)] \geq 0 \quad \text{and} \quad [y_l - R(l|\cdot)] \cdot \frac{\partial \mathfrak{L}}{\partial R(l|\cdot)} = 0
\tag{2.35}
$$

$$
\frac{\partial \mathfrak{L}}{\partial R(l|l)} = -\alpha[(1-e) - \theta(1-e)] \geq 0 \quad \text{and} \quad [y_l - R(l|l)] \cdot \frac{\partial \mathfrak{L}}{\partial R(l|l)} = 0
\tag{2.36}
$$

$$
\frac{\partial \mathfrak{L}}{\partial R(h|\cdot)} = \theta e - e - \lambda \geq 0 \quad \text{and} \quad [y_h - R(h|\cdot)] \cdot \frac{\partial \mathfrak{L}}{\partial R(h|\cdot)} = 0
\tag{2.37}
$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = (1-e)\left[R\left(l|\cdot\right) - R\left(l|l\right)\right] + \lambda\left[R\left(l|h\right) - R\left(l|\cdot\right)\right] + \theta\left(1-e\right)\left[R\left(l|l\right) - R\left(l|\cdot\right) - c\right] \geq 0$$

$$\text{and} \quad (1-\alpha) \cdot \frac{\partial \mathcal{L}}{\partial \alpha} = 0. \tag{2.38}$$

The proof continues along the following steps.

- $\widehat{\theta} > 1$. Condition (2.37) implies $\widehat{\theta} \geq 1 + \frac{\widehat{\lambda}}{e}$. Clearly, if $\widehat{\theta} = 1$, then $\widehat{\lambda} = 0$. By condition (2.38), $\widehat{\theta} = 1$ and $\widehat{\lambda} = 0$ imply $c \leq 0$, which contradicts the assumption $c > 0$. $\widehat{\theta} > 1$ clearly implies that P's participation constraint (2.3) is binding, i.e. P's expected profit is lowered until it reaches its reservation value, which is assumed to be zero.

- $\widehat{R}\left(l|l\right) \leq \widehat{R}\left(l|\cdot\right)$. Suppose not. Then we could lower $\widehat{R}\left(l|l\right)$ and increase $\widehat{R}\left(l|\cdot\right)$ so that expected payoffs remain unchanged. Due to the increase in $\widehat{R}\left(l|\cdot\right)$, the constraint (2.4) is slack. Consequently, we can lower both $\widehat{\alpha}$ and $\widehat{R}\left(h|\cdot\right)$ so that both (2.4) and (2.3) are satisfied. Contradiction.

- $\widehat{R}\left(l|l\right) = \widehat{R}\left(l|\cdot\right) = y_l$. The result $\widehat{R}\left(l|l\right) = y_l$ follows from $\widehat{\theta} > 1$, $\widehat{\alpha} > 0$, and condition (2.36). Furthermore, since we already know that $\widehat{R}\left(l|l\right) \leq \widehat{R}\left(l|\cdot\right)$, by limited liability we also obtain $\widehat{R}\left(l|\cdot\right) = y_l$.

- $\widehat{\alpha} < 1$, $\widehat{\lambda} > 0$, and $\widehat{R}\left(l|h\right) = y_h$. The previous result $\widehat{R}\left(l|l\right) = \widehat{R}\left(l|\cdot\right) = y_l$ and (2.38) imply $\widehat{\lambda} > 0$, which means that agent A's incentive compatibility constraint is binding (as it usually happens in this class of problems). Consequently, if $\widehat{\alpha} = 1$, then $\widehat{R}\left(h|\cdot\right) = y_h$. But then the agent A receives an expected payoff equal to zero while P receives a positive expected payoff, which contradicts the fact that constraint (2.3) must be binding.

- $y_l < \widehat{R}\left(h|\cdot\right) < y_h$ and $\widehat{\alpha} = \frac{\widehat{R}(h|\cdot) - y_l}{y_h - y_l}$. From the previous results and the conditions (2.4)

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and (2.3), which are binding at optimum, we get

$$\widehat{R}(h|\cdot) = \widehat{\alpha} y_h + (1 - \widehat{\alpha}) y_l \tag{2.39}$$

and

$$e\widehat{R}(h|\cdot) + (1 - e)(y_l - \widehat{\alpha}c) = 0. \tag{2.40}$$

Since $\widehat{\alpha} \in (0, 1)$, (2.39) implies that $y_l < \widehat{R}(h|\cdot) < y_h$. Furthermore, solving the two equations (2.39) and (2.40) in the two unknowns $\widehat{\alpha}$ and $\widehat{R}(h|\cdot)$ we easily obtain

$$\widehat{\alpha} = \frac{\widehat{R}(h|\cdot) - y_l}{y_h - y_l}$$

and

$$\widehat{R}(h|\cdot) = \frac{(1 - e) y_l (y_l - y_h - c)}{e(y_h - y_l) - (1 - e)c}.$$

2.8.2 Proof of Proposition 2

Again, the proof technique is the same as in [KP98].

- First we prove $\widetilde{R}(l|l) = y_l$. Since A has all the bargaining power, P's participation constraint must be binding, that is

$$
\begin{aligned}
Z &= e\gamma R(h|\cdot) + \tag{2.41}\\
&+ (1 - e\gamma)\{(1 - \alpha)R(l|\cdot) + \alpha[\mu(h|l)R(l|h) + (1 - \mu(h|l))R(l|l) - c]\}\\
&= 0.
\end{aligned}
$$

We already know that agent A mixes according to the equilibrium probability

$$\widetilde{\gamma} = \frac{eR(l|h) + (1 - e)R(l|l) - R(l|\cdot) - c}{e[R(l|h) - R(l|\cdot) - c]} \tag{2.42}$$

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whereas player P forms beliefs according to

$$\mu\left(h|l\right) = \frac{e\left(1 - \gamma\right)}{1 - e\gamma}. \tag{2.43}$$

Substituting $\mu\left(h|l\right)$ from (2.43) and γ from (2.42) to (2.41) we have

$$Z = R\left(h|\cdot\right) - \frac{\left(1 - e\right) R\left(l|\cdot\right)\left[R\left(l|l\right) - R\left(l|h\right)\right]}{eR\left(l|h\right) + \left(1 - e\right) R\left(l|l\right) - R\left(l|\cdot\right) - c} = 0. \tag{2.44}$$

Now we can use condition (2.10) to rewrite player A's expected payoff as

$$W = \left[y_h - R\left(h|\cdot\right)\right] + \left(1 - e\right)\left[y_l - \alpha R\left(l|l\right) - \left(1 - \alpha\right) R\left(l|\cdot\right)\right]. \tag{2.45}$$

Substituting α from (2.14) and $R\left(h|\cdot\right)$ from (2.44) in (2.45) and then differentiating (2.45) with respect to $R\left(l|l\right)$, we get

$$\frac{\partial W}{\partial R\left(l|l\right)} = \frac{c\left(1 - e\right) R\left(l|\cdot\right)\left[R\left(l|h\right) - R\left(l|\cdot\right) - c\right]}{\left[c + R\left(l|\cdot\right) + R\left(l|l\right)\left(e - 1\right) - R\left(l|h\right) e\right]^2\left[R\left(l|h\right) - R\left(l|\cdot\right)\right]} > 0, \tag{2.46}$$

where the inequality follows from the fact that $\widetilde{\gamma} > 0$.

Inequality (2.46) and limited liability imply $\widetilde{R}\left(l|l\right) = y_l$.

- Now we prove $\widetilde{R}\left(l|h\right) = y_h$ and $y_l < \widetilde{R}\left(h|\cdot\right) < y_h$. Again, we can substitute α from (2.14) and $R\left(h|\cdot\right)$ from (2.44) in (2.45) and then differentiate (2.45) with respect to $R\left(l|h\right)$ so as to obtain

$$\frac{\partial W}{\partial R\left(l|h\right)} = \frac{c\left(1 - e\right) R\left(l|\cdot\right)\left\{\left[y_l - R\left(l|\cdot\right)\right]\left[c + R\left(l|\cdot\right) - y_l\right] + e\left[R\left(l|h\right) - y_l\right]\right\}^2}{\left[R\left(l|h\right) - R\left(l|\cdot\right)\right]^2\left[eR\left(l|h\right) + \left(1 - e\right) y_l - R\left(l|\cdot\right) - c\right]^2} > 0. \tag{2.47}$$

The latter inequality along with limited liability imply $\widetilde{R}\left(l|h\right) = y_h$. Furthermore, from (2.10) and the fact that $\widetilde{\alpha} \in \left(0, 1\right)$, it follows that $y_l < \widetilde{R}\left(h|\cdot\right) < y_h$.

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- To prove $\widetilde{R}(l|\cdot) = y_l$ we can compare the slopes of the player A's and player P's isoprofit functions. Using $\widetilde{R}(l|l) = y_l$ and $\widetilde{R}(l|h) = y_h$, the slope of agent A's isoprofit function in the $(R(l|\cdot), R(h|\cdot))$ space is

$$\frac{\mathsf{d}R(h|\cdot)}{\mathsf{d}R(l|\cdot)} = \frac{(e-1)[y_h - y_l][y_h - R(h|\cdot)]}{[y_h - R(l|\cdot)][ey_h + (1-e)y_l - R(l|\cdot)]} < 0 \qquad (2.48)$$

Moreover,

$$\frac{\mathsf{d}^2 R(h|\cdot)}{\mathsf{d}R(l|\cdot)^2} < 0. \qquad (2.49)$$

As for player P's isoprofit function, its slope is

$$\frac{\mathsf{d}R(h|\cdot)}{\mathsf{d}R(l|\cdot)} = -\frac{(1-e)[y_h - y_l][y_h - R(h|\cdot) - c]}{[y_h - R(l|\cdot) - c][ey_h + (1-e)y_l - R(l|\cdot) - c]}. \qquad (2.50)$$

To sign (2.50), we can observe that from P's zero-profit condition and (2.9) we have

$$y_h - R(h|\cdot) - c = \frac{[y_h - R(l|\cdot) - c][ey_h + (1-e)y_l - c]}{[ey_h + (1-e)y_l - R(l|\cdot) - c]} > 0. \qquad (2.51)$$

Thus, (2.50) < 0.

Now we can observe that, due to limited liability, the $(R(l|\cdot), R(h|\cdot))$ space is bounded by y_l and y_h. Since $y_h > y_l$ and $y_h > \widetilde{R}(h|\cdot)$, at the optimal solution the two isoprofit curves can either have a tangency point or cross at the corner solution $R(l|\cdot) = y_l$. Since a tangency can occur only if $c = 0$, we must have $\widetilde{R}(l|\cdot) = y_l$.

- Finally, we have to show $\widetilde{R}(h|\cdot) \geq \widehat{R}(h|\cdot)$ and $\widetilde{\alpha} \geq \widehat{\alpha}$. Let $\widehat{W}$ and $\widetilde{W}$ denote agent A's expected payoff from contract $\widehat{\mathcal{C}}$ and $\widetilde{\mathcal{C}}$, respectively. From previous results, it is immediate to obtain

$$\widehat{W} = e\left[y_h - \widehat{R}(h|\cdot)\right]$$

and

$$\widetilde{W} = e\left[y_h - \widetilde{R}\left(h|\cdot\right)\right].$$

Now, the lack of commitment in Problem 2 implies $\widehat{W} \geq \widetilde{W}$, that is

$$e\left[y_h - \widehat{R}\left(h|\cdot\right)\right] \geq e\left[y_h - \widetilde{R}\left(h|\cdot\right)\right],$$

from which it is immediate to get $\widehat{R}\left(h|\cdot\right) \leq \widetilde{R}\left(h|\cdot\right)$.

Substituting the previous results of this proposition in (2.14), it is immediate to get

$$\widetilde{\alpha} = \frac{\widetilde{R}\left(h|\cdot\right) - y_l}{y_h - y_l}.$$

Using $\widehat{R}\left(h|\cdot\right) \leq \widetilde{R}\left(h|\cdot\right)$, we easily obtain

$$\widehat{\alpha} \leq \widetilde{\alpha}.$$

2.8.3   Proof of Proposition 3

First we prove $\widehat{e}$, $\widetilde{e} > 0$.

From Propositions 1 and 2 we easily obtain

$$\widehat{W} = e\left[y_h - \widehat{R}\left(h|\cdot\right)\right]$$

and

$$\widetilde{W} = e\left[y_h - \widetilde{R}\left(h|\cdot\right)\right],$$

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respectively.

The optimal effort levels $\widehat{e}$ and $\widetilde{e}$ are implicitly determined by the first-order conditions

$$\psi'(\widehat{e}) = y_h - \widehat{R}(h|\cdot)$$

and

$$\psi'(\widetilde{e}) = y_h - \widetilde{R}(h|\cdot),$$

respectively. Note that these conditions are not only necessary but also sufficient due to the concavity of Problems 4 and 5.

Now, recalling $\widehat{R}(h|\cdot) < y_h$ from Proposition 1, $\widetilde{R}(h|\cdot) < y_h$ from Proposition 2, and $\psi'(e) > 0$ for $e > 0$ by assumption, it follows immediately that $\widehat{e}, \widetilde{e} > 0$.

To complete the proof, note that from Proposition 1 and 2 we can write

$$y_h - y_l > y_h - \widehat{R}(h|\cdot) \geq y_h - \widetilde{R}(h|\cdot).$$

Since $\psi(\cdot)$ is strictly convex by assumption, then

$$e^{FB} > \widehat{e} \geq \widetilde{e}.$$

2.8.4   Proof of Proposition 4

Since P has all the bargaining power, agent A's participation constraint is binding. That is,

$$
\begin{aligned}
W &= e\left\{\gamma\left(y_h - R(h|\cdot)\right) + (1-\gamma)\left[\alpha\left(y_h - R(l|h)\right) + (1-\alpha)\left(y_h - R(l|\cdot)\right)\right]\right\} + \\
&\quad + (1-e)\left\{\alpha\left(y_l - R(l|l)\right) + (1-\alpha)\left(y_l - R(l|\cdot)\right)\right\} \\
&= 0.
\end{aligned}
\tag{2.52}
$$

Using condition (2.10), the equation above can be rewritten as

$$W = e\left[y_h - R\left(h|\cdot\right)\right] + (1-e)\left[y_l - \alpha R\left(l|l\right) - (1-\alpha) R\left(l|\cdot\right)\right] = 0, \tag{2.53}$$

which is equivalent to

$$e\left[y_h - R\left(h|\cdot\right)\right] = -(1-e)\left[y_l - \alpha R\left(l|l\right) - (1-\alpha) R\left(l|\cdot\right)\right]. \tag{2.54}$$

By limited liability, the left-hand side of (2.54) is non-negative while the right-hand side is non-positive. Thus, in order for (2.54) to hold, both sides must be equal to zero. Consequently,

$$y_h = \widetilde{R}_P\left(h|\cdot\right) \tag{2.55}$$

and

$$y_l = \alpha R\left(l|l\right) + (1-\alpha) R\left(l|\cdot\right). \tag{2.56}$$

The last equality, along with limited liability, implies

$$y_l = \widetilde{R}_P\left(l|l\right) = \widetilde{R}_P\left(l|\cdot\right). \tag{2.57}$$

Finally, we need to prove $\widetilde{R}_P\left(l|h\right) = y_h$. Using $\widetilde{R}_P\left(h|\cdot\right) = y_h$ and $\widetilde{R}_P\left(l|\cdot\right) = y_l$ we easily obtain

$$\widetilde{\alpha}_P = \frac{y_h - y_l}{\widetilde{R}_P\left(l|h\right) - y_l}. \tag{2.58}$$

By limited liability and by the fact that, by definition, $\widetilde{\alpha}_P \in [0,1]$, we clearly obtain

$$\widetilde{R}_P\left(l|h\right) = y_h, \tag{2.59}$$

which in turn implies

$$\widetilde{\alpha}_P = 1.$$

2.8.5   Proof of Proposition 5

From Proposition 4, we easily obtain $\widetilde{W}_P = 0$. Thus all the results follows immediately from the first-order condition

$$\psi'(e) = \phi \left[y_h - \widetilde{R}(h|\cdot) \right] \tag{2.60}$$

to the effort choice problem and from the fact that $\psi'(\cdot)$ satisfies $\psi'(0) = 0$, $\psi'(e) > 0$ for $e > 0$, and $\psi''(\cdot) > 0$ by assumption.

2.8.6   Proof of Proposition 6

- First we prove the characterization of the optimal contract of Problem 8.

  Note that Problem 8 is nothing other than Problem 6 expect for the introduction of the payment cap $\overline{R}$. It is immediate to see that payments $\overline{R}_P(l|l)$, $\overline{R}_P(l|h)$, and $\overline{R}_P(l|\cdot)$ are equal to the corresponding payments in Problem 6 and they are not affected by the introduction of $\overline{R}$. That is,

$$\overline{R}_P(l|l) = R_P(l|l) = y_l, \quad \overline{R}_P(l|\cdot) = R_P(l|\cdot) = y_l, \quad \overline{R}_P(l|h) = R_P(l|h) = y_h. \tag{2.61}$$

To find out $\overline{R}_P(h|\cdot)$, we can first use the results in (2.61) to obtain

$$\overline{\alpha}_P = \frac{\overline{R}_P(h|\cdot) - y_l}{y_h - y_l}, \tag{2.62}$$

$$\overline{\gamma}_P = \frac{ey_h + (1-e)y_l - y_l - c}{e(y_h - y_l - c)} \tag{2.63}$$

and beliefs

$$\mu\left(h|l\right) = \frac{c}{y_h - y_l}. \tag{2.64}$$

Substituting in player P's profit function yields

$$Z = \left[\frac{e\left(y_h - y_l\right)}{y_h - y_l - c}\right] \overline{R}_P\left(h|\cdot\right) + \left[\frac{1-e}{y_h - y_l - c}\right] \left[y_l\left(y_h - y_l\right)\right]. \tag{2.65}$$

Since $y_h - y_l > c > 0$ by assumption, (2.65) is increasing with respect to $\overline{R}_P\left(h|\cdot\right)$. Thus the constraint on payment $\overline{R}_P\left(h|\cdot\right)$ is binding and we clearly have $\overline{R}_P\left(h|\cdot\right) = \overline{R}$.

- As for the effort level $\overline{e}_P$, the proof is analogous to that of Proposition 5 and it follows from the first-order condition

$$\psi'\left(e\right) = \phi\left[y_h - \widetilde{R}\left(h|\cdot\right)\right] + \left(1-\phi\right)\left[y_h - \overline{R}\right] \tag{2.66}$$

and from the convexity of $\psi\left(\cdot\right)$ together with the assumption $\overline{R} < y_h$.

Chapter 3

Timing of verification, commitment, and loan size

3.1 Introduction

The literature on Costly State Verification that originated from the seminal contributions of [Tow79] and [GH85] has always considered verification in terms of auditing. That is, the Principal/Financier's verification decision is commonly assumed to take place after observing a report sent by the Agent/Borrower. This implies that the game induced by the optimal contract is a *sequential* game with incomplete information. However, we may think of many real life examples in which the verification policy is devised *before* observing any report, so implying that the reporting/verification game is a *simultaneous-move* game of incomplete information.

For example, following [Str05], consider an organization in which a Supervisor/Principal delegates the functioning of a risky project to an Employee/Agent who is privately informed about the realization of the project itself. Since exerting effort in the project is costly, the Agent clearly has an incentive to shirk and then attribute the resulting poor performance

to bad luck. To induce the Agent not to shirk, the Principal can choose one out of two verification procedures. At the end of the project, she can require the Agent to report on his activity and then, on the basis of that report, perform a costly verification. Alternatively, she can verify the Agent's effort intensity while he is working on the project, that is, before final outcomes realize. The two verification procedures only differ in the *timing* with which they are employed. In the first case, the verification decision takes place *after* observing the Agent's report and from now on we will call this procedure *auditing*. In the second case, the verification decision is taken *before* observing any report from the Employee. We will use the term *monitoring* to refer to the latter scenario.

Given the two verification procedures, one obvious question is: Why using *monitoring* instead of *auditing*? The answer crucially depends upon the Principal's ability to commit to the verification policy announced at the contracting stage. Suppose that Principal is able to fully commit to the verification procedure announced at the contracting stage. Then the optimal contract with auditing would be weakly superior to the contract with monitoring in that the Principal can commit herself to replicate the strategy that would have been used under monitoring. Nonetheless, this means that the Principal commit herself to disregard the additional information that she will receive by observing the Agent's report, which is a dynamic inconsistent choice or, in other words, a non-credible threat. On the other hand, by removing the full commitment assumption [Str05] shows that, under some circumstances, the optimal contract with monitoring can be a Pareto improving with respect to the contract under auditing. In particular, [Str05] shows that without commitment the double moral hazard[1] problem with auditing is more severe than with monitoring. Namely, providing incentive for the Principal to audit requires "steeper incentives", i.e. larger payments to the Agent, than under monitoring.

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[1]That is, moral hazard for the Agent that would try to shirk instead of working and moral hazard for the Principal that would save on verification costs by not monitoring/auditing.

In this chapter we put aside the comparison[2] between *monitoring* and *auditing* and consider a lender/borrower relationship in which the lender can only use monitoring as a verification procedure. Furthermore, we will consider two different scenarios: one in which the Principal is able to fully commit to the verification policy announced at the contracting stage and another scenario where such an ability to commit is assumed away.

We will show that the optimal contract under full commitment prescribes a null expected rent to the Agent if the low state of the world realizes along with the so-called maximum punishment principle[3], that is, the Agent must give his entire wealth to the Principal if caught cheating. Furthermore, the Agent will be left with a positive rent (bonus) if he truthfully reports the higher state of the world. In the case without commitment, we will show that the optimal contract will inherit all the main features of the contract under full commitment but with two relevant differences. First, the expected rent for the Agent in the high state is lower. Second, the loan size acts as a commitment device and therefore turns out to be larger in the case in which full commitment to verification policies is assumed away.

To the best of our knowledge, the closest paper to this chapter are [KP98] and [Str05].

The modelling framework that we will use is essentially the same as that of [KP98]. However, the key difference is that we assume that the only verification procedure available is monitoring whereas [KP98] assume that only auditing is available. As we will discuss later, many results of [KP98] carry over to our scenario. In particular, we will show how their result on the loan size as a commitment device is still valid under monitoring.

Our definition of monitoring and auditing is taken from [Str05]. In particular, he considers a framework with (*ex ante*) moral hazard and finds conditions under which monitoring is

---

[2]The reason why we only consider monitoring can be justified in terms of higher costs that the Principal has to sustain in order to use auditing. For example, we may think that collecting reports before auditing is costly due to the necessity of hiring additional employees. This become even more relevant if we extend our analysis in such a way to consider a financial intermediary that lends its funds to a larger number of borrowers.

[3]See, for instance, [BS87].

more efficient than auditing. However, we consider a framework with *ex post* moral hazard and confine our attention to monitoring procedures.

As for the different assumptions about commitment and the use of stochastic contracts, at least since [MP89] the literature on Costly State Verification has established that stochastic contracts usually outperform deterministic mechanisms, namely standard debt contracts. However, [KV00] have shown that deterministic mechanisms turn out to be optimal even under no commitment if one introduces enforcement as a decision variable. Nonetheless, since our analysis will not focus on (costly) enforcement, we consider a static environment *à la* [MP89] in which stochastic contracts turn out to be optimal if the assumption of full commitment is removed.

The chapter is organized as follows. In Section 3.2 we set up the model and provide a full characterization of the monitoring game induced by a given contract. Section 3.3 characterizes the optimal contract between the lender and the borrower in the case in which the former is able to fully commit to the monitoring policy announced at the contracting stage. The optimal contract under the hypothesis of no commitment is characterized in Section 3.4. Then conclusions follow.

3.2   The model

Let us consider an economy with one cashless Agent or Entrepreneur A (he) and a (possibly) infinite number of Principals of Financiers (she) that operate in a perfectly competitive market for lending funds. The Agent is endowed with a risky production technology which transforms input capital $I \in [0, \bar{I}]$ into output $y_h(I)$ with probability $p \in (0, 1)$ or $y_l(I)$ with probability $(1 - p)$. The production function $y(\cdot)$ satisfies the standard assumptions of monotonicity and concavity, i.e. $y'(\cdot) > 0$ and $y''(\cdot) < 0$. Furthermore, following [KP98],

for every $I \in [0, \bar{I}]$ the production technology satisfies the following conditions:

$$y_h(I) > I > y_l(I), \qquad y'_h(I) > 1 > y'_l(I), \qquad \text{and} \quad py_h(I) + (1-p) y_l(I) > I,$$

which ensure that financing the Agent's project has a positive net present value. To finance her project, the Agent tries to collect funds by making a take-it-or-leave-it offer to one Principal or Financier[4].

Information is asymmetrically distributed across market participants. Once a contract has been signed, the Agent privately observes the true state of the world $s \in \{l, h\}$ and prepares a report $\hat{y}_m \in \{y_l(I), y_h(I)\}$ to be send to the Principal. On the other hand, the principal simultaneously chooses a monitoring intensity α with which the report received from the borrower will be verified. This means that the Principal is endowed with a perfect verification technology that allows him to discover the true state of the world at a cost $0 < c < y_h(I) - y_l(I)$.

A contract is thus a list of repayment functions $R(m|s)$ and $R(m|\cdot)$ along with the loan size I, where $R(m|s)$ is the repayment in the case in which the state $m \in \{l, h\}$ is reported and the true state $s \in \{l, h\}$ is discovered with verification while $R(m|\cdot)$ is the repayment when $m \in \{l, h\}$ is reported and no verification occurs. The Agent is protected by limited liability, so that $R(m|\cdot) \leq y_m$ and $R(m|s) \leq y_s$, for $m, s \in \{l, h\}$. Clearly, if verification were not possible, the Agent would always claim a low output realization. Consequently, the contract should be designed in such a way to induce the borrower to truthfully report not only the low state but also the high state output realization.

The sequence of events is the following:

- **Stage 1**: A makes a take-it-or-leave-it contract offer to P specifying loan size I and payments $R(m|\cdot)$ and $R(m|s)$

---

[4]We work under the assumption of *exclusive* financial contracts, that is, the borrower is allowed to sign financial contracts with only one lender.

- **Stage 2**: If the contract offer is refuted, then the game end and both players are left with zero utility. Otherwise, A privately observes the true state of the world and sends a reports to P; at the same time P decides whether to monitor or not the report that will be sent by A

- **Stage 3**: Final payments take place

The sequence of events is also reported in Figure 3.1.
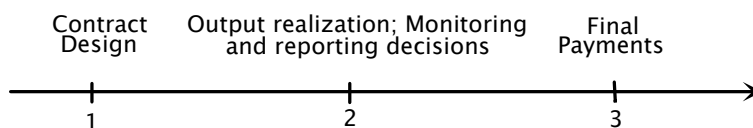
Figure 3.1: Sequence of events with monitoring

As a benchmark, notice that under complete and symmetric information the states of the world become common knowledge and the first-best loan size I^* will be achieved. In particular, I^* is the loan size that maximizes the social surplus

$$py_h(I) + (1-p) y_l(I) - I$$

and is implicitly determined by the first-order condition

$$py_h'(I) + (1-p) y_l'(I) = 1.$$

3.2.1 The monitoring game

Let us consider an arbitrary contract $\mathcal{C} = \{I, R(l|\cdot), R(l|l), R(h|\cdot), R(h|h), R(h|l), R(l|h)\}$. This contract induces a simultaneous-move game of incomplete information. To simplify

things and avoid uninteresting cases, we assume that the incentive for the Agent to lie arises only if a high output realization occurs. In other words, when a low state is observed, then A always sends a truthful report. But when a state h realizes, then he tries to take advantage of the Principal by claiming that the project produced a low output.

The monitoring game induced by a given contract $\mathcal{C}$ is a simultaneous-move game of incomplete information. Players, Actions, beliefs, payoffs.

The set of players is $\mathcal{N} = \{A, P\}$, where A is the Agent and P is the Principal.

The type space for player A is $\Theta_A = \{l, h\}$ while his action set is $S_A = \{\hat{y}_l, \hat{y}_h\}$. Consequently, a (possibly mixed) strategy for the Agent is a mapping $\sigma_A : \Theta_A \to \Delta(S_A)$.

The action set for P is $S_P = \{\mathsf{Monitor}, \mathsf{Don't\ Monitor}\}$ and a (possibly mixed) strategy is $\sigma_P \in \Delta(S_P)$.

Due to the simultaneous occurrence of moves, the Principal's prior beliefs about A's type are given by the probability distribution over the set of states. That is, $\mu(h) = p$ and $\mu(l) = 1 - p$, where $\mu(s)$ denote the Principal's belief of facing type s of player A.

Abusing notation, let $\sigma_A = (\sigma_A(h), 1 - \sigma_A(h), \sigma_A(l), 1 - \sigma_A(l))$ be a mixed strategy for player A, where $\sigma_A(s)$ is the probability of sending the truthful report $\hat{y}_s$. Similarly, let $\sigma_P = (\alpha, 1 - \alpha)$ be a mixed strategy for player P, where α indicates the probability of monitoring the Agent's report.

To simplify notation, introduce $\gamma := \sigma_A(h)$.

Given $\mathcal{C}$, expected payoffs are

$$
\begin{aligned}
W \;=\; & (1-p)\left[y_l(I) - \alpha R(l|l) - (1-\alpha) R(l|\cdot)\right] \\
& + p\{\gamma\left[\alpha\left(y_h(I) - R(h|h)\right) + (1-\alpha)\left(y_h(I) - R(h|\cdot)\right)\right] \\
& + (1-\gamma)\left[\alpha\left(y_h(I) - R(l|h)\right) + (1-\alpha)\left(y_h(I) - R(l|\cdot)\right)\right]\}
\end{aligned}
\tag{3.1}
$$

for player A and

$$
\begin{aligned}
Z \;=\; & \alpha \left\{ (1-p)\,R\,(l|l) + p\,[\gamma R\,(h|h) + (1-\gamma)\,R\,(l|h)] - c \right\} \qquad (3.2) \\
+ \; & (1-\alpha)\left\{ p\gamma R\,(h|\cdot) + (1-p\gamma)\,R\,(l|\cdot) \right\},
\end{aligned}
$$

for player P.

The natural solution concept to use is the Bayesian Nash Equilibrium (BNE). Recall that a BNE is a *strategy profile* and a *belief system* that maximize every player's expected payoff given his or her beliefs about other players' types and given other player's strategies.

Lemma 1. *The (unique) Bayesian Nash Equilibrium of the monitoring game is the strategy profile* $(\sigma_A^*, \sigma_P^*) = ((\gamma^*, 1-\gamma^*, 1, 0), (\alpha^*, 1-\alpha^*))$ *such that*

$$
\gamma^* = \frac{(1-p)\,R\,(l|l) + pR\,(l|h) - R\,(l|\cdot) - c}{p\,[R\,(h|\cdot) + R\,(l|h) - R\,(h|h) - R\,(l|\cdot)]} \qquad (3.3)
$$

and

$$
\alpha^* = \frac{R\,(h|\cdot) - R\,(l|\cdot)}{R\,(h|\cdot) + R\,(l|h) - R\,(h|h) - R\,(l|\cdot)}. \qquad (3.4)
$$

The associated belief system is given by $(\mu\,(h), \mu\,(l)) = (p, 1-p)$.

Notice that the monitoring probability is independent of the verification cost c while the probability of truthfully reporting the high state h is decreasing with respect to c. Intuitively, if the verification cost increases, even though the monitoring intensity is unaffected, the Principal will have to sustain a higher cost to monitor. Anticipating this, the Agent will react at equilibrium by decreasing γ. Notice that, independently of c, at every equilibrium *strategic default* will appear with positive probability.

3.3 Optimal contract with full commitment to monitoring

In this section we characterize the optimal contract under the assumption that, at the contracting stage, the Principal can fully commit to a pre-specified (and possibly random) monitoring policy. We denote by $\widehat{\mathcal{C}}$ the resulting contract and we will use throughout the superscript $\,\widehat{}\,$ to indicate relevant variables in the full commitment case.

In this case, a contract is a list of repayment functions $\widehat{R}(m|s)$ and $\widehat{R}(m|\cdot)$, a monitoring schedule $\widehat{\alpha}$, and the loan size $\widehat{I}$. $\widehat{R}(m|s)$ is the repayment from party A to party P if the state $m \in M$ is reported and the true state $s \in S$ is discovered by the Principal using the (perfect) verification technology he is endowed with. $\widehat{R}(m|\cdot)$ is the repayment to P in the case in which m is reported and no auditing occurs, while $\widehat{\alpha}$ is the probability with which P will monitor the reported state m.

Thus a contract is a list $\widehat{\mathcal{C}} = \left\{\widehat{I}, \widehat{R}(l|\cdot), \widehat{R}(l|l), \widehat{R}(l|h), \widehat{R}(h|\cdot), \widehat{R}(h|h), \widehat{\alpha}\right\}$. Notice that, due to the hypothesis of simultaneous monitoring and reporting decisions, both low state and high state reports can possibly be verified.

Since we are working under the full commitment assumption, we can make use of the Revelation Principle and then confine our attention to direct truthful mechanisms. In other words, without loss of generality we can consider contracts for which the Agent is asked to report his private information, i.e. the state s, and truthful reporting is optimal at equilibrium. This implies that, given an arbitrary contract $\mathcal{C} = \{I, R(l|\cdot), R(l|l), R(l|h), R(h|\cdot), R(h|h), \alpha\}$, by the revelation principle, player A's expected payoff from $\mathcal{C}$ is

$$W = p\left[y_h(I) - \alpha R(h|h) - (1-\alpha)R(h|\cdot)\right] + (1-p)\left[y_l(I) - \alpha R(l|l) - (1-\alpha)R(l|\cdot)\right]$$

$$(3.5)$$

while player P's expected (gross) payoff is

$$Z = p\left[\alpha R\left(h|h\right) + \left(1 - \alpha\right) R\left(h|\cdot\right)\right] + \left(1 - p\right)\left[\alpha R\left(l|l\right) + \left(1 - \alpha\right) R\left(l|\cdot\right)\right] - \alpha c. \tag{3.6}$$

Without loss of generality, the Principal's reservation utility is set equal to I. Furthermore, we assume that player A has all the bargaining power and acts as the contract designer. The optimal contract $\widehat{\mathcal{C}}$ is the solution to the following program

Problem 10.

$$\max_{\{I, R(m|s), R(m|\cdot), \alpha\}} W$$

subject to

$$Z \geq I \tag{3.7}$$

$$\alpha R\left(h|h\right) + \left(1 - \alpha\right) R\left(h|\cdot\right) \leq \alpha R\left(l|h\right) + \left(1 - \alpha\right) R\left(l|\cdot\right) \tag{3.8}$$

$$R\left(m|s\right) \leq y_s\left(I\right) \quad for \quad m, s \in \{l, h\} \tag{3.9}$$

$$R\left(m|\cdot\right) \leq y_m\left(I\right) \quad for \quad m \in \{l, h\}, \tag{3.10}$$

where (3.7) is the Principal's participation constraint, (3.8) is the Agent's incentive compatibility (or truth-telling) constraint, (3.9) and (3.10) are the limited liability constraints. The solution to the above problem is reported in the following

Proposition 7. *The contract* $\widehat{\mathcal{C}} = \left\{\widehat{I}, \widehat{R}\left(l|\cdot\right), \widehat{R}\left(l|l\right), \widehat{R}\left(l|h\right), \widehat{R}\left(h|h\right), \widehat{R}\left(h|\cdot\right), \widehat{\alpha}\right\}$ *which solves Problem 10 is such that:*

- $\widehat{R}\left(l|\cdot\right) = \widehat{R}\left(l|l\right) = y_l\left(I\right),$

- $\widehat{R}\left(l|h\right) = y_h\left(I\right),$

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- $y_l\left(I\right) < \widehat{R}\left(h|\cdot\right), \widehat{R}\left(h|h\right) < y_h\left(I\right),$

- $\widehat{\alpha} = \frac{\widehat{R}(h|\cdot) - y_l(I)}{y_h(I) - y_l(I) + \widehat{R}(h|\cdot) - \widehat{R}(h|h)} < 1,$

- $\widehat{I} < I^*.$

Three properties of the above proposition are worth noticing. First, the Agent is left with an expected rent equal to zero in the low state l. Second, the so-called *maximum punishment* principles applies. That is, if the Agent is caught cheating by sending a fraudulent report on the state h, he is called upon to repay the highest possible level of wealth $y_h\left(I\right)$. These two properties are also shared by the optimal contract with auditing[5]. Finally, the fact that $\widehat{R}\left(h|h\right) < y_h\left(I\right)$ means that, in case of truthful report of the high state h the Agent is left with a positive expected rent which can be interpreted as a bonus left to A to reward his honest behaviour. The latter property is in sharp contrast with the properties of the optimal contract with auditing and is a direct consequence of our assumption of simultaneous monitoring and reporting.

3.4 Optimal contract without commitment to monitoring

In this section we provide the full characterization of the optimal contract $\widetilde{\mathcal{C}}$ between parties A and P under the assumption that, at the contracting stage, party P cannot commit herself not to renegotiate the monitoring policy announced in stage 1. We use the superscript $\widetilde{}$ to denote all relevant variables under no commitment.

The reason why we also consider the scenario without commitment rests upon the inherent dynamic inconsistency of the optimal contract under full commitment. In fact, let us consider the scenario outlined in the previous section. Once the terms of the optimal contract

---

[5]See, among others, [KP98], [Boy01], and [Cho98].

have been agreed upon by both parties, the Agent will truthfully report the observed state of the world and then the Principal will monitor according to the pre-announced, equilibrium monitoring policy $\widehat{\alpha}$. Clearly, the Principal would not have any incentive *ex post* to monitor because he knows for sure that A's reports are truthful and because verification is costly. Furthermore, the Agent may anticipate P's incentive not to monitor and then she may try to take advantage of the Principal by sending fraudulent reports. In other words, the P's promise to commit to the announced policy $\widehat{\alpha}$ is not credible. Without the full commitment assumption, we thus consider the same scenario with *strategic* monitoring outlined in section 3.2.1 and we will try to determine an optimal contract which is also renegotiation-proof.

Having in mind the reporting game described before, we have that a contract $\widetilde{\mathcal{C}}$ has to specify five payments: $\left\{ \widetilde{R}\left(l|l\right), \widetilde{R}\left(l|h\right), \widetilde{R}\left(l|\cdot\right), \widetilde{R}\left(h|\cdot\right), \widetilde{R}\left(h|h\right) \right\}$ along with the loan size $\widetilde{I}$. Clearly, the no commitment assumption implies that any monitoring policy announced by the Principal in the contracting stage cannot be contracted upon.

Given an arbitrary contract $\widetilde{\mathcal{C}} = \left\{ \widetilde{I}, \widetilde{R}\left(l|\cdot\right), \widetilde{R}\left(l|l\right), \widetilde{R}\left(l|h\right), \widetilde{R}\left(h|\cdot\right), \widetilde{R}\left(h|h\right) \right\}$, player A's expected payoff from $\mathcal{C}$ is

$$
\begin{aligned}
W \;=\; & (1-p)\left[y_l\left(I\right) - \alpha R\left(l|l\right) - (1-\alpha) R\left(l|\cdot\right)\right] \\
+ \; & p\{\gamma\left[\alpha\left(y_h\left(I\right) - R\left(h|h\right)\right) + (1-\alpha)\left(y_h\left(I\right) - R\left(h|\cdot\right)\right)\right] \qquad (3.11) \\
+ \; & (1-\gamma)\left[\alpha\left(y_h\left(I\right) - R\left(l|h\right)\right) + (1-\alpha)\left(y_h\left(I\right) - R\left(l|\cdot\right)\right)\right]\}
\end{aligned}
$$

while agent P's expected payoff is

$$
\begin{aligned}
Z \;=\; & \alpha\left\{(1-p) R\left(l|l\right) + p\left[\gamma R\left(h|h\right) + (1-\gamma) R\left(l|h\right)\right] - c\right\} \qquad (3.12) \\
+ \; & (1-\alpha)\left\{p\gamma R\left(h|\cdot\right) + (1-p\gamma) R\left(l|\cdot\right)\right\}.
\end{aligned}
$$

Clearly, removing the full commitment assumption entails that the revelation principle can no longer be used[6].

As we already did in the full commitment case, we assume that the Agent has all the bargaining power and act as the contract designer.

The optimal contract $\widetilde{\mathcal{C}}$ is the solution to the following program

Problem 11.

$$\max_{\{I,R(m|s),R(m|\cdot)\}} W$$

subject to

$$Z \geq I \tag{3.13}$$

$$\alpha = \alpha^* \tag{3.14}$$

$$\gamma = \gamma^* \tag{3.15}$$

$$R(m|s) \leq y_s(I) \quad for \quad m, s \in \{l, h\} \tag{3.16}$$

$$R(m|\cdot) \leq y_m(I) \quad for \quad m \in \{l, h\}, \tag{3.17}$$

where (3.13) is the Principal's participation constraint while (3.16) and (3.17) are the limited liability constraints. (3.14) and (3.15) ensure equilibrium play in the monitoring game induced by the contract $\widetilde{\mathcal{C}}$. The solution $\widetilde{\mathcal{C}}$ to the program above is reported in the following

Proposition 8. *The contract* $\widetilde{\mathcal{C}} = \left\{ \widetilde{I}, \widetilde{R}(l|\cdot), \widetilde{R}(l|l), \widetilde{R}(l|h), \widetilde{R}(h|\cdot), \widetilde{R}(h|h) \right\}$ *which solves Problem 11 is such that:*

- $\widetilde{R}(l|\cdot) = \widetilde{R}(l|l) = y_l(I),$

---

[6]To be fully rigorous, we can show that a modified version of the Revelation Principle applies to the case without commitment (See for instance [BS01] and [BS07]). In particular, [BS07] show that the revelation principle continues to hold provided that a *noisy* communication device is used. Given the simultaneous-move assumption of our model and to simplify things, we follow [Kha97] and [KP98] and work without the revelation principle.

- $\widetilde{R}\left(l|h\right) = y_h\left(I\right)$,

- $y_l\left(I\right) < \widetilde{R}\left(h|\cdot\right), \widetilde{R}\left(h|h\right) < y_h\left(I\right)$.

Furthermore, the following inequalities hold:

- $\widehat{I} < \widetilde{I} < I^*$,

- $\widehat{\alpha}\widehat{R}\left(h|h\right) + \left(1 - \widehat{\alpha}\right)\widehat{R}\left(h|\cdot\right) < \widetilde{\alpha}\widetilde{R}\left(h|h\right) + \left(1 - \widetilde{\alpha}\right)\widetilde{R}\left(h|\cdot\right)$.

The results of the above proposition show that the optimal contract without commitment inherits all the main features of the contract under full commitment characterized in the previous section. That is, also in Proposition 8 the maximum punishment principle applies and the Agent is left with a null expected rent in the low state l. Moreover, the Agent is rewarded with a positive rent for truthfully reporting the high state h. However, the striking difference between Proposition 7 and Proposition 8 is contained in the last two items of the latter. In particular, the absence of commitment implies that to induce the Principal to perform monitoring, she must receive higher payments in the high state h. Clearly, this corresponds to a lower expected payoff from the optimal contract for the Agent. Furthermore, the Principal's inability to commit affects the loan size. In particular, the fact that $\widetilde{I}$ is greater than $\widehat{I}$ suggests that the loan size acts as a commitment device. In other words, the higher the amount of funds at stake, the more credible is the monitoring threat of the Principle because the higher would be the loss she sustains in case of fraudulent report. This results resembles that of [KP98] and shows that the key point of their analysis is still valid in a framework with monitoring.

3.5   Conclusion

In this chapter we have provided the full characterization of the optimal contract between a lender a borrower under monitoring, i.e. under the hypothesis that verification and reporting

decisions are taken simultaneously and not sequentially. We have shown that the resulting mechanisms inherits many features of the optimal contracts with auditing. Nonetheless, contrary to what is usually found with sequential decisions, we have seen how the presence of monitoring entails a positive bonus to the Agent in the case in which he truthfully reports a high state of the world. Moreover, then standard result of [KP98] on the loan size as commitment device also applies in our scenario. It would be interesting to extend our analysis and make a comparison between monitoring and auditing similar to that of [Str05] and see how the corresponding loans sizes would behave.

3.6　Appendix

This Appendix contains all the proofs omitted in the Chapter text.

3.6.1　Proof of Lemma 1

For completeness, the following two tables report all possible payoffs of the monitoring game in the case in which the true state of the world is $s = h$ and $s = l$, respectively.

	$s = h$		
A's action	P's action	A's payoff	P's payoff
$\hat{y}_l$	Monitor	$y_h(I) - R(l\|h)$	$R(l\|h) - c$
$\hat{y}_l$	Don't Monitor	$y_h(I) - R(l\|\cdot)$	$R(l\|\cdot)$
$\hat{y}_h$	Monitor	$y_h(I) - R(h\|h)$	$R(h\|h) - c$
$\hat{y}_h$	Don't Monitor	$y_h(I) - R(h\|\cdot)$	$R(h\|\cdot)$

	$s = l$		
A's action	P's action	A's payoff	P's payoff
$\hat{y}_l$	Monitor	$y_l(I) - R(l\|l)$	$R(l\|l) - c$
$\hat{y}_l$	Don't Monitor	$y_l(I) - R(l\|\cdot)$	$R(l\|\cdot)$
$\hat{y}_h$	Monitor	$y_l(I) - R(h\|l)$	$R(h\|l) - c$
$\hat{y}_h$	Don't Monitor	$y_l(I) - R(h\|\cdot)$	$R(h\|\cdot)$

Note that assuming the Agent always truthfully reports the low state $s = l$ amounts to assuming $\alpha R(h|l) + (1 - \alpha) R(h|\cdot) \geq R(l|l), R(l|\cdot), \quad \forall \alpha \in [0, 1]$.

It is obvious that the BNE of our monitoring game is a mixed-strategy equilibrium[7]. The intuition why we cannot have a pure strategy equilibrium is straightforward. Roughly speaking, suppose that (all types of) player A always reports truthfully, so that P's best response

---

[7]To be fully rigorous, we have a hybrid equilibrium in which the low type of player A always plays $\hat{y}_l$ while both the high type of player A and player P randomize according to γ and α, respectively. See on this [Str05] and the references therein.

is not to monitor. But then truthfully reporting is no longer optimal and A would have an incentive to deviate. On the other hand, suppose that P always monitors so that A's best response is to truthfully report. But then P would have an incentive to deviate, the monitoring technology being costly to use. The remaining cases are similar.

The equilibrium characterization follows directly from the property that, in a mixed strategy equilibrium, each (type of each) player must be indifferent between his or her pure strategies that will be played with positive probability at equilibrium[8]. This means that, at equilibrium, the (high type of the) Agent must be indifferent between truthfully reporting the high state h and sending a fraudulent report while the Principal must be indifferent between monitoring and not monitoring. Formally,

$$\alpha \left[y_h \left(I \right) - R \left(h|h \right) \right] + \left(1 - \alpha \right) \left[y_h \left(I \right) - R \left(h|\cdot \right) \right] = \alpha \left[y_h \left(I \right) - R \left(l|h \right) \right] + \left[y_h \left(I \right) - R \left(l|\cdot \right) \right] \quad (3.18)$$

and

$$\left(1 - p \right) \left[R \left(l|l \right) - c \right] + p \left\{ \gamma \left[R \left(h|h \right) - c \right] + \left(1 - \gamma \right) \left[R \left(l|h \right) - c \right] \right\} =$$
$$= \left(1 - p \right) R \left(l|\cdot \right) + p \left\{ \gamma R \left(h|\cdot \right) + \left(1 - \gamma \right) R \left(l|\cdot \right) \right\}. \quad (3.19)$$

The Left-Hand Side of (3.18) is the expected payoff from truthfully reporting the state $s = h$ while the Right-Hand Side is the expected payoff from sending a fraudulent report. The LHS of (3.19) is the expected payoff from monitoring while the RHS is the expected payoff from not monitoring.

Solving (3.18) for α and (3.19) for γ yields

$$\alpha^* = \frac{R \left(h|\cdot \right) - R \left(l|\cdot \right)}{R \left(h|\cdot \right) + R \left(l|h \right) - R \left(h|h \right) - R \left(l|\cdot \right)} \quad (3.20)$$

---

[8]For a formal proof, see for instance Lemma 33.2 in Osborne and Rubinstein (1994).

and

$$\gamma^* = \frac{(1-p)R(l|l) + pR(l|h) - R(l|\cdot) - c}{p[R(h|\cdot) + R(l|h) - R(h|h) - R(l|\cdot)]}, \tag{3.21}$$

respectively.

3.6.2 Proof of Proposition 7

Recalling that

$$W = p[y_h(I) - \alpha R(h|h) - (1-\alpha)R(h|\cdot)] + (1-p)[y_l(I) - \alpha R(l|l) - (1-\alpha)R(l|\cdot)] \tag{3.22}$$

and

$$Z = p[\alpha R(h|h) + (1-\alpha)R(h|\cdot)] + (1-p)[\alpha R(l|l) + (1-\alpha)R(l|\cdot)] - \alpha c, \tag{3.23}$$

the Lagrangian of problem (10) is

$$
\begin{aligned}
\mathfrak{L} &= p[y_h(I) - \alpha R(h|h) - (1-\alpha)R(h|\cdot)] + (1-p)[y_l(I) - \alpha R(l|l) - (1-\alpha)R(l|\cdot)] \\
&\quad - \lambda[\alpha R(h|h) + (1-\alpha)R(h|\cdot) - \alpha R(l|h) - (1-\alpha)R(l|\cdot)] \\
&\quad + \theta\{p[\alpha R(h|h) + (1-\alpha)R(h|\cdot)] + (1-p)[\alpha R(l|l) + (1-\alpha)R(l|\cdot)] - \alpha c - I\},
\end{aligned}
\tag{3.24}
$$

where λ and θ are the Lagrange multipliers associated with the Agent's incentive compatibility (or truth-telling) constraint and the Principal's participation constraint, respectively. The corresponding first-order conditions are

$$\frac{\partial \mathfrak{L}}{\partial I} = py_h'(I) + (1-p)y_l'(I) - \theta = 0 \tag{3.25}$$

$$\frac{\partial \mathfrak{L}}{\partial R(l|h)} = \lambda\alpha \geq 0 \quad \text{and} \quad [y_h(I) - R(l|h)] \cdot \frac{\partial \mathfrak{L}}{\partial R(l|h)} = 0 \tag{3.26}$$

$$\frac{\partial \mathcal{L}}{\partial R(l|\cdot)} = (1-\alpha)\left[\lambda + \theta(1-p) - (1-p)\right] \geq 0 \quad \text{and} \quad \left[y_l(I) - R(l|l)\right] \cdot \frac{\partial \mathcal{L}}{\partial R(l|\cdot)} = 0 \quad (3.27)$$

$$\frac{\partial \mathcal{L}}{\partial R(l|l)} = -\alpha\left[(1-p) - \theta(1-p)\right] \geq 0 \quad \text{and} \quad \left[y_l(I) - R(l|l)\right] \cdot \frac{\partial \mathcal{L}}{\partial R(l|l)} = 0 \quad (3.28)$$

$$\frac{\partial \mathcal{L}}{\partial R(h|\cdot)} = (1-\alpha)\left[\theta p - p - \lambda\right] \geq 0 \quad \text{and} \quad \left[y_h(I) - R(h|\cdot)\right] \cdot \frac{\partial \mathcal{L}}{\partial R(h|\cdot)} = 0 \quad (3.29)$$

$$\frac{\partial \mathcal{L}}{\partial R(h|h)} = \alpha\left[\theta p - p - \lambda\right] \geq 0 \quad \text{and} \quad \left[y_h(I) - R(h|h)\right] \cdot \frac{\partial \mathcal{L}}{\partial R(h|h)} = 0 \quad (3.30)$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \alpha} =\ & p\left[R(h|\cdot) - R(h|h)\right] + (1-p)\left[R(l|\cdot) - R(l|l)\right] \quad (3.31) \\
+\ & \lambda\left[R(h|\cdot) - R(h|h) + R(l|h) - R(l|\cdot)\right] \\
+\ & \theta\left\{p\left[R(h|h) - R(h|\cdot)\right] + (1-p)\left[R(l|l) - R(l|\cdot)\right] - c\right\} \geq 0 \\
& \text{and} \quad (1-\alpha) \cdot \frac{\partial \mathcal{L}}{\partial \alpha} = 0.
\end{aligned}
$$

The proof continues along the following steps.

- $\widehat{\theta} > 0$. Since $p \in [0,1]$ and $y_l'(I), y_h'(I) > 0$ by assumption, from (3.25) we easily obtain $\widehat{\theta} > 0$. As expected, this means that P's participation constraint is binding.

- $\widehat{\theta} > 1$ and $\widehat{\lambda} > 0$. Adding up constraints (3.29) and (3.30) we obtain $\widehat{\theta}p - p - \widehat{\lambda} \geq 0$, which is equivalent to $\widehat{\theta} \geq 1 + \frac{\widehat{\lambda}}{p}$, hence $\widehat{\theta} \geq 1$. Clearly, if $\widehat{\theta} = 1$, then $\widehat{\lambda} = 0$. By condition (3.31), $\widehat{\theta} = 1$ along with $\widehat{\lambda} = 0$ imply $c \leq 0$, which is a contradiction. Therefore, $\widehat{\theta} > 1$ and consequently $\widehat{\lambda} > 0$. The latter inequality means that the Agent's incentive compatibility constraint is binding, hence $\widehat{\alpha}\widehat{R}(h|h) + (1-\widehat{\alpha})\widehat{R}(h|\cdot) = \widehat{\alpha}\widehat{R}(l|h) + (1-\widehat{\alpha})\widehat{R}(l|\cdot)$.

- $\widehat{R}(l|l) = y_l(I)$. Using $\widehat{\alpha} > 0$ and $\widehat{\theta} > 1$ in (3.28) we have $\frac{\partial \mathcal{L}}{\partial R(l|l)} > 0$ and by the corresponding complementary slackness condition we can conclude that $\widehat{R}(l|l) = y_l(I)$.

- $\widehat{R}(l|h) = y_h(I)$. Using $\widehat{\lambda} > 0$ and $\widehat{\alpha} > 0$ in (3.26) we easily obtain $\frac{\partial \mathcal{L}}{\partial R(l|h)} > 0$, hence

$\widehat{R}(l|h) = y_h(I)$.

- $\widehat{\alpha} < 1$. Suppose by way of contradiction that $\widehat{\alpha} = 1$. Then by previous results and by the Agent's participation constraint (which is binding) we easily obtain $\widehat{R}(h|h) = \widehat{R}(l|h) = y_h(I)$. But then, using $\widehat{R}(l|l) = y_l(I)$ and $\widehat{R}(l|h) = y_h(I)$, A's expected payoff turns out to be equal to zero, i.e. $W = 0$, while the Principal would get a net positive payoff. Indeed we have $Z = py_h(I) + (1-p)y_l(I) - c > L$. Recall that the net social surplus $py_h(I) + (1-p)y_l(I) - c - L$ is positive by assumption. Now, $Z > L$ contradicts $\widehat{\theta} > 0$. Then we conclude that $\widehat{\alpha} < 1$.

- $\widehat{R}(l|\cdot) = y_l(I)$. Using $\widehat{\alpha} \in (0,1)$, $\widehat{\lambda} > 0$ and $\widehat{\theta} > 1$, by condition (3.27) it follows that $\frac{\partial \mathfrak{L}}{\partial R(l|\cdot)} > 0$. By the associated complementary slackness condition, we easily obtain $\widehat{R}(l|\cdot) = y_l(I)$.

- $\widehat{\alpha} = \frac{\widehat{R}(h|\cdot) - y_l(I)}{y_h(I) - y_l(I) + \widehat{R}(h|\cdot) - \widehat{R}(h|h)}$. It suffices to take the binding incentive compatibility constraint

$$\widehat{\alpha}\widehat{R}(h|h) + (1-\widehat{\alpha})\widehat{R}(h|\cdot) = \widehat{\alpha}y_h(I) + (1-\widehat{\alpha})y_l(I),$$

then solving for $\widehat{\alpha}$ yields

$$\widehat{\alpha} = \frac{\widehat{R}(h|\cdot) - y_l(I)}{y_h(I) - y_l(I) + \widehat{R}(h|\cdot) - \widehat{R}(h|h)}.$$

- $y_l(I) < \widehat{R}(h|h), \widehat{R}(h|\cdot) < y_h(I)$. Given $\widehat{\alpha} \in (0,1)$, it follows that $\widehat{R}(h|h) < y_h(I)$ and $y_l(I) < \widehat{R}(h|\cdot)$. Then, using $\widehat{\alpha} < 1$, $\widehat{R}(h|h) < y_h(I)$ and $y_l(I) < \widehat{R}(h|\cdot)$ in the incentive compatibility constraint

$$\widehat{\alpha}\widehat{R}(h|h) + (1-\widehat{\alpha})\widehat{R}(h|\cdot) = \widehat{\alpha}y_h(I) + (1-\widehat{\alpha})y_l(I)$$

we finally get $y_l(I) < \widehat{R}(h|\cdot) < y_h(I)$ and $y_l(I) < \widehat{R}(h|h) < y_h(I)$.

- $\widehat{I} < I^*$. Notice that I^* maximizes the social welfare

$$py_h(I) + (1-p)\, y_l(I) - I$$

and is implicitly determined by the first-order condition

$$py_h'(I^*) + (1-p)\, y_l'(I^*) - 1 = 0, \tag{3.32}$$

whereas $\widehat{I}$ is implicitly determined by (3.25). Comparing (3.32) with (3.25) and using $\widehat{\theta} > 1$ along with the assumptions of strict monotonicity and strict concavity of the technology yields the result.

3.6.3   Proof of Proposition 8

We organize the proof in several steps. First of all, the players' expected payoffs under no commitment are

$$
\begin{aligned}
W &=& (1-p)\left[y_l(I) - \alpha R(l|l) - (1-\alpha) R(l|\cdot)\right] \\
&+& p\{\gamma\left[\alpha\left(y_h(I) - R(h|h)\right) + (1-\alpha)\left(y_h(I) - R(h|\cdot)\right)\right] \\
&+& (1-\gamma)\left[\alpha\left(y_h(I) - R(l|h)\right) + (1-\alpha)\left(y_h(I) - R(l|\cdot)\right)\right]\}
\end{aligned} \tag{3.33}
$$

and

$$
\begin{aligned}
Z &=& \alpha\left\{(1-p) R(l|l) + p\left[\gamma R(h|h) + (1-\gamma) R(l|h)\right] - c\right\} \\
&+& (1-\alpha)\left\{p\gamma R(h|\cdot) + (1-p\gamma) R(l|\cdot)\right\},
\end{aligned} \tag{3.34}
$$

respectively.

- As it is common in this class of problems, we know that the Agent will be left with zero rent in the low state, hence $\widetilde{R}(l|\cdot) = \widetilde{R}(l|l) = y_l(I)$ must hold. Then expected payoffs become

$$
\begin{aligned}
W \;=\; & p\{\gamma\left[\alpha\left(y_h(I) - R(h|h)\right) + (1-\alpha)\left(y_h(I) - R(h|\cdot)\right)\right] && (3.35)\\
& +\; (1-\gamma)\left[\alpha\left(y_h(I) - R(l|h)\right) + (1-\alpha)\left(y_h(I) - R(l|\cdot)\right)\right]\}
\end{aligned}
$$

for A and

$$
\begin{aligned}
Z \;=\; & \alpha\left\{(1-p)\,y_l(I) + p\left[\gamma R(h|h) + (1-\gamma)R(l|h)\right] - c\right\} && (3.36)\\
& +\; (1-\alpha)\left\{p\gamma R(h|\cdot) + (1-p\gamma)R(l|\cdot)\right\}
\end{aligned}
$$

for P.

Moreover, since A has all the bargaining power, the Principal's Participation constraint must be binding, that is

$$
\begin{aligned}
Z \;=\; & \alpha\left\{(1-p)\,y_l(I) + p\left[\gamma R(h|h) + (1-\gamma)R(l|h)\right] - c\right\} && (3.37)\\
& +\; (1-\alpha)\left\{p\gamma R(h|\cdot) + (1-p\gamma)R(l|\cdot)\right\} = I.
\end{aligned}
$$

Notice that, using condition (3.18), the expected payoff (3.35) can be rewritten as

$$
W = p\left[y_h(I) - \alpha R(h|h) - (1-\alpha)R(h|\cdot)\right] \tag{3.38}
$$

while using condition (3.19), the expected payoff (3.37) can be rewritten as

$$
Z = \left[p\gamma R(h|\cdot) + (1-p\gamma)\,y_l(I)\right] - I = 0. \tag{3.39}
$$

Solving (3.39) for $R(h|\cdot)$, plugging in (3.38) and using (3.21) yields

$$
\begin{aligned}
W \;=\; p\Bigg\{ & y_h(I) - R(h|h) \cdot \frac{R(h|\cdot) - y_l(I)}{R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)} \\
& - \frac{R(l|h) - R(h|h)}{R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)} \times \\
& \times \left\{ \frac{I\left[R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)\right]}{(1-p)\,y_l(I) + pR(l|h) - y_l(I) - c} \right. \\
& \left. - y_l(I) \cdot \frac{R(h|\cdot) - R(h|h) + (1-p)\left[R(l|h) - y_l(I)\right]}{(1-p)\,y_l(I) + pR(l|h) - y_l(I) - c} \right\}\Bigg\}.
\end{aligned}
\tag{3.40}
$$

The first derivative of (3.40) with respect to $R(l|h)$ is

$$
\begin{aligned}
\frac{\partial W}{\partial R(l|h)} \;=\; p\Bigg\{ & \frac{R(h|h)\left[R(h|\cdot) - y_l(I)\right]\left[R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)\right]}{\left[R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)\right]^2} \\
& - \frac{R(h|\cdot) + R(l|h) - R(h|h) - y_l(I) - \left[R(l|h) - R(h|h)\right]}{\left[R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)\right]^2} \times \\
& \times \left\{ \frac{I\left[R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)\right]}{(1-p)\,y_l(I) + pR(l|h) - y_l(I) - c} \right. \\
& \left. - y_l(I) \cdot \frac{R(h|\cdot) - R(h|h) + (1-p)\left[R(l|h) - y_l(I)\right]}{(1-p)\,y_l(I) + pR(l|h) - y_l(I) - c} \right\} \\
& - \frac{R(l|h) - R(h|h)}{R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)} \times \\
& \times \left\{ \frac{I\left[(1-p)\,y_l(I) + pR(l|h) - y_l(I) - c\right] - pI\left[R(h|\cdot) + R(l|h) - R(h|h) - y_l(I)\right]}{\left[(1-p)\,y_l(I) + pR(l|h) - y_l(I) - c\right]^2} \right. \\
& \left. - y_l(I) \cdot \frac{(1-p)\,R(l|h) - p\left\{R(h|\cdot) - R(h|h) + (1-p)\left[R(l|h) - y_l(I)\right]\right\}}{\left[(1-p)\,y_l(I) + pR(l|h) - y_l(I) - c\right]^2} \right\} > 0,
\end{aligned}
\tag{3.41}
$$

where the inequality follows from $p \in (0,1)$, $R(h|\cdot) > y_l(I)$, and $R(l|h) > R(h|h)$.
The sign of this derivative along with limited liability yields $\widetilde{R}(l|h) = y_h(I)$.

- The proof of $y_l(I) < \widetilde{R}(h|\cdot)$, $\widetilde{R}(h|h) < y_h(I)$ is identical to that provided for Proposition 7.

- To prove $\widetilde{I} < I^*$, see [KP98].

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- To prove $\widetilde{I} > \widehat{I}$, we proceed as in [KP98].

First notice that the first-order condition (3.25) can be rewritten as

$$p\frac{1}{\widehat{\theta}}y'_h\left(\widehat{I}\right) + (1-p)\frac{1}{\widehat{\theta}}y'_l\left(\widehat{I}\right) = 1. \tag{3.42}$$

From constraint (3.31) we have

$$p\left[\widehat{R}\left(h|\cdot\right) - \widehat{R}\left(h|h\right)\right] + \widehat{\lambda}\left[\widehat{R}\left(h|\cdot\right) - \widehat{R}\left(h|h\right) + y_h\left(\widehat{I}\right) - y_l\left(\widehat{I}\right)\right] + \tag{3.43}$$

$$+\widehat{\theta}\left\{p\left[\widehat{R}\left(h|h\right) - \widehat{R}\left(h|\cdot\right)\right] - c\right\} = 0$$

From conditions (3.29) and (3.30) we get

$$\widehat{\lambda} = \widehat{\theta}p - p. \tag{3.44}$$

Substituting (3.44) in (3.43) and solving for $\widehat{\theta}$ yields

$$\widehat{\theta} = \frac{p\left(y_h\left(\widehat{I}\right) - y_l\left(\widehat{I}\right)\right)}{p\left(y_h\left(\widehat{I}\right) - y_l\left(\widehat{I}\right)\right) - c}. \tag{3.45}$$

As for the problem under no commitment, we can rewrite Problem 11 as

$$\max_{\{I\}} \quad p\gamma y_h\left(I\right) + (1 - p\gamma) y_l\left(I\right) - I$$

subject to

$$\gamma = \gamma^*$$

so that the corresponding first-order condition is

$$p\gamma^* y'_h\left(\widetilde{I}\right) + (1 - p\gamma^*) y'_l\left(\widetilde{I}\right) + p\frac{\partial\gamma^*}{\partial I}y_h\left(\widetilde{I}\right) - p\frac{\partial\gamma^*}{\partial I}y_l\left(\widetilde{I}\right) = 1. \tag{3.46}$$

Subtracting the LHS of (3.46) from the LHS of (3.43) and evaluating the difference at $I = \widetilde{I}$, we obtain

$$p\frac{1}{\theta}y_h'(I) + (1-p)\frac{1}{\theta}y_l'(I) - [p\gamma y_h'(I) + (1-p\gamma)y_l'(I)] - p\frac{\partial\gamma}{\partial I}[y_h(I) - y_l(I)], \quad (3.47)$$

where we dropped the superscript $\sim$ to simplify notation. It follows that if $(3.47) < 0$, then $\widetilde{I} > \widehat{I}$.

Let us rewrite (3.47) as

$$\left[\frac{1}{\theta} - \gamma\right]py_h'(I) + \left[\frac{1}{\theta}(1-p) - (1-p\gamma)\right]y_l'(I) - p\frac{\partial\gamma}{\partial I}[y_h(I) - y_l(I)]. \quad (3.48)$$

To save on notation let us define

$$H := R(h|\cdot) - R(h|h)$$

and

$$\Delta y := y_h(I) - y_l(I).$$

Using (3.45) and (3.21) we get

$$\frac{1}{\theta} - \gamma = \frac{H(p\Delta y - c)}{p\Delta y(H + \Delta y)}, \quad (3.49)$$

$$\begin{aligned}\frac{1}{\theta}(1-p) - (1-p\gamma) &= \frac{1}{\theta} - 1 - p\left[\frac{1}{\theta} - \gamma\right] \quad (3.50)\\ &= \frac{pH(c - \Delta y) - c(H + \Delta y)}{p\Delta y(H + \Delta y)},\end{aligned}$$

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and

$$\frac{\partial \gamma}{\partial I} = \frac{\partial}{\partial I} \cdot \frac{p\Delta y - c}{p\left(\Delta y + H\right)} \tag{3.51}$$

$$= \frac{\left[y_h'\left(I\right) - y_l'\left(I\right)\right]\left(pH + c\right)}{p\left(H + \Delta y\right)^2}. \tag{3.52}$$

Substituting into (3.48) we obtain

$$y_h'\left(I\right)\frac{H\left(p\Delta y - c\right)}{\Delta y\left(H + \Delta y\right)} + y_l'\left(I\right)\frac{pH\left(c - \Delta y\right) - c\left(H + \Delta y\right)}{p\Delta y\left(H + \Delta y\right)}$$
$$-\Delta y\frac{\left[y_h'\left(I\right) - y_l'\left(I\right)\right]\left(pH + c\right)}{\left(H + \Delta y\right)^2} \tag{3.53}$$

which is equivalent to

$$\frac{py_h'\left(I\right)\left(H + \Delta y\right)H\left(p\Delta y - c\right)}{p\Delta y\left(H + \Delta y\right)^2} \tag{3.54}$$
$$+\frac{y_l'\left(I\right)\left(H + \Delta y\right)\left[pH\left(c - \Delta y\right) - c\left(H + \Delta y\right)\right]}{p\Delta y\left(H + \Delta y\right)^2}$$
$$-\frac{p\left(\Delta y\right)^2\left[y_h'\left(I\right) - y_l'\left(I\right)\right]\left(pH + c\right)}{p\Delta y\left(H + \Delta y\right)^2}.$$

Now, if $c = 0$, then (3.54) becomes

$$\frac{y_h'\left(I\right)p\left(H + \Delta y\right)Hp\Delta y - y_l'\left(I\right)\left(H + \Delta y\right)\left(p\Delta y H\right) - p\left(\Delta y\right)^2\left(y_h'\left(I\right) - y_l'\left(I\right)\right)pH}{p\Delta y\left(H + \Delta y\right)^2} < 0, \tag{3.55}$$

where the inequality follows from the assumptions $y_h'\left(I\right) > y_l'\left(I\right)$, $\Delta y > 0$, and $p \in (0, 1)$.

If $c = \Delta y$, then (3.54) becomes

$$
\begin{aligned}
&- \frac{y'_h(I) \, p \, (H + \Delta y) \, H \, [\Delta y (1 - p)] + y'_l \, (H + \Delta y) \, [\Delta y \, (H + \Delta y)]}{p \Delta y \, (H + \Delta y)^2} \quad (3.56) \\
&- \frac{p \, (\Delta y)^2 \, [y'_h \, (I) - y'_l \, (I)] \, (pH + \Delta y)}{p \Delta y \, (H + \Delta y)^2} < 0.
\end{aligned}
$$

Furthermore, the derivative of (3.54) with respect to c is

$$
-y'_h(I) \, p \, (H + \Delta y) \, H - p \, (\Delta y)^2 \, [y'_h(I) - y'_l(I)] + pHy'_l(H + \Delta y) - y'_l(I)(H + \Delta y)^2 < 0.
$$

$$(3.57)$$

Recalling that $0 < c < \Delta y$ by assumption, the latter results means that there is no value of c such that (3.47) is non-negative, so establishing the result

- Finally, we have to prove that

$$
\widetilde{\alpha} \widetilde{R}(h|h) + (1 - \widetilde{\alpha}) \, \widetilde{R}(h|\cdot) > \widehat{\alpha} \widehat{R}(h|h) + (1 - \widehat{\alpha}) \, \widehat{R}(h|\cdot). \quad (3.58)
$$

Notice that the Agent's expected profit under full commitment is

$$
\widehat{W} = p \left[y_h \left(\widehat{I} \right) - \widehat{\alpha} \widehat{R}(h|h) - (1 - \widehat{\alpha}) \, \widehat{R}(h|\cdot) \right] > 0 \quad (3.59)
$$

whereas he gets

$$
\widetilde{W} = p \left[y_h \left(\widetilde{I} \right) - \widetilde{\alpha} \widetilde{R}(h|h) - (1 - \widetilde{\alpha}) \, \widetilde{R}(h|\cdot) \right] > 0 \quad (3.60)
$$

without commitment.

The definition of commitment implies

$$p\left[y_h\left(\widehat{I}\right) - \widehat{\alpha}\widehat{R}\left(h|h\right) - (1-\widehat{\alpha})\,\widehat{R}\left(h|\cdot\right)\right] \geq p\left[y_h\left(\widetilde{I}\right) - \widetilde{\alpha}\widetilde{R}\left(h|h\right) - (1-\widetilde{\alpha})\,\widetilde{R}\left(h|\cdot\right)\right],$$
$$(3.61)$$

which can be rewritten as

$$\widetilde{\alpha}\widetilde{R}\left(h|h\right) + (1-\widetilde{\alpha})\,\widetilde{R}\left(h|\cdot\right) - \widehat{\alpha}\widehat{R}\left(h|h\right) - (1-\widehat{\alpha})\,\widehat{R}\left(h|\cdot\right) \geq y_h\left(\widetilde{I}\right) - y_h\left(\widehat{I}\right) > 0, \quad (3.62)$$

where the last equality comes from $\widehat{I} < \widetilde{I}$ and the assumption $y'\left(\cdot\right) > 0$.

Thus we easily obtain both

$$\widetilde{\alpha}\widetilde{R}\left(h|h\right) + (1-\widetilde{\alpha})\,\widetilde{R}\left(h|\cdot\right) > \widehat{\alpha}\widehat{R}\left(h|h\right) + (1-\widehat{\alpha})\,\widehat{R}\left(h|\cdot\right) \qquad (3.63)$$

and

$$\widehat{W} > \widetilde{W}. \qquad (3.64)$$

Chapter 4

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