HETEROGENEITY, COMMONALITY AND INTERDEPENDENCE IN THE EURO AREA:

Size and Dynamics of Fiscal Spillover Effects in macroeconomic-financial linkages

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Abstract

The paper develops empirical measures to estimate the strength and dymanic of fiscal spillover effects in the Euro Area. It moves for estimating a Bayesian VAR model of real and financial variables in order to examine in depth economic policy coordination and policy making, with a strong attention on the current financial crisis. Spillovers are estimated recursively with weakly-exogenous common factors. The aim of the project accounts for interdependencies across countries within the Euro Area and derives impulse response functions and conditional forecasts with the output of an Monte Carlo Marco Chain (MCMC) routine. However, the paper attempts to estimate the systemic contribution and cross-country transmission of unexpected shocks on the productivity in the EA between June 1995 and March 2014. Overall, the positive impact on outputs in the financial dimension indicates the importance of coordinated fiscal actions in the EA. Shocks overflow in a heterogeneous way across countries. Moreover, financial variables show higher amplification of spillover effects which can be seen as a result of increased interdependence between variables. Finally, the analysis is consistent and robust with the more recent literature on business cycles, which recognizes the importance of both group-specific and global factors in evaluating cross-country spillovers and responses to an unexpected shocks.

Keywords: Fiscal Policy, Cross-country Spillovers, Impulse Responses, Conditional Forecasts, Bayesian VAR, Financial Crisis, Common Features, Causality, Catching-up, Competitiveness.

Contents

Non-Technical Summary

In the debate on global imbalances, the euro area countries did not receive much attention so far. While the current account has been close to balance over the past decades at the aggregate level, divergences between individual member states have increased since the introduction of the common currency. Disparities across the member states are striking, for example persistent current account deficits of Portugal, Ireland, Greece, and Spain (PIGS) are accompanied by huge surpluses in Germany, Austria, and Netherlands.

Since the Euro started[1], the most research findings focused on debt dynamics, current account balances, contagion effects, and fiscal rules. However, there are many channels and factors through which macroeconomic and financial linkages can be analyzed. For istance, they can arise by a deterioration of financial conditions affecting the economy through a negative wealth effect on consumption and investment decisions. Additionally, business cycles, demographic developments, and fiscal policy are important determinants of empirical realisations of inward growth spillover effects. Furthermore, the European integration process certainly made stronger interdependecies across countries to a fiscal shock. On the other hand, in the meantime tight institutional and economic interdependencies may have made euro area countries more alike, the recent recession has shown that there may still be a substantial degree of heterogeneity, with some common behavior, in real and financial linkages across countries. In addition, those linkages may have changed over time because of economic/institutional implications. Up to now, research on these issues have still not been analysed in the necessary depth. The aim of the poject is to try to fill this gap, accounting for break-time effects, time-varying variables, and macroeconomic-financial linkages.

This paper addresses the topic of commonality and heterogeneity across countries and over time within the euro area. It analyzes sign, dimension, and transmission of fiscal spillover effects across countries, with a particular emphasis on the recent recession and fiscal consolidations. Finally, it quantifies the prominent role of transmission channels and economic/institutional implications in driving height difference and spreading of shocks and *cause-effect* relationships. The project accounts for a BVAR model where real, financial, and

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[1]The euro area consists of those European Union (EU) Member States which have adopted the euro as their single currency. The euro area were introduced on 1 January 1999, as stage III of economic and monetary union began, in 11 countries and expanded through a series of enlargements to 18 countries, so far.

selected *latent* factors[2] are jointly modelled for a total of 12 countries of the eurozone for the period from 1999 to 2014. The project runs out evaluating a Seemengly Unrelated Model (SUR) in order to analyze the evolution (and, hence, strength and dynamic), commonality, and heterogeneity of fiscal spillover effects in macroeconomic-financial linkages. The selected eurozone countries are: Italy (IT), Spain (ES), France (FR), Belgium (BE), Netherlands (NL), Austria (AT), Germany (DE), Finland (FI), Luxembourg (LU), Ireland (IE), Portugal (PT), and Greece(GR). The first 11 countries are the founding euro-area Member States. On 1 January 2001, Greece joined the euro area.

The analysis confirms the need to allow for cross-country and cross-variable interdependence and the importance of economic/institutional implications when studying real-financial linkages. The empirical model including real and financial variables for the countries of the EA12 identifies significant spillovers. A shock to a variable in a given country affects all other countries and the transmission is more intense among financial variables, mainly during the recent fiscal consolidations. This result seems to prove higher interdependencies among financial variables and consolidations occurred simultaneously behind more coordinated fiscal actions across members states. During the financial crisis, the imbalances have been reduced and the evidence found seems to believe that the global imbalances will decline in the period ahead.

However, growth shocks spill over in a heterogenous way across countries, with some common behavior. This latter increased during financial crisis in its financial dimension and even more in its real dimension. The common component is larger during recent fiscal consolidations.

In addition, country-specific factors remain very important in explaining the presence of a heterogeneous pattern in real-financial linkages. Moreover, they are potentially strongly sensitive to trade and capital flows (selected to measure transmission channels) and to common and idionsyncratic factors (selected to measure economic and institutional implications) across countries of the eurozone. The analysis accounts for national policy factors (final consumption, unemployment rate), competitiveness factors (unit labour costs, international investment positions), and private factors (household's private consumptions). The result is consistent with the more recent literature on business cycles and effects of fiscal policy, which recognizes the importance of existing transmission channels of spillovers and the determinants generating inward and outward growth spillovers.

These findings cast a new perspective for theoretical models of idiosyncratic business cycles and policy making.

From a modelling perspective, the analysis appears to favour models that assign an important role to *catching-up* and competitiveness factors in explaining current account imbalances and debt dynamics. Moreover, transmission channels suggest that trade channels matter relatively less than financial channels. Growth shocks appear to be predominantly transmitted via financial linkages. The interdependence is stronger in financial dimension, while real component shows higher degree of heterogeneity and it is mainly affected by latent

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[2]Latent or hidden factors are variables that are not directly observed but are rather inferred from other variables that are observed and, hence, directly measured.

confounding effects. The results are consistent with the recent literature which recognizes the importance of accounting for both country-specific and global factors when studying real and financial interactions. Moreover, the analysis is consistent with the premise that for countries to be an important source of growth spillovers, growth should rely on a greater extent of autonomous domestic sources.

From a policy perspective, several considerations can be displayed. First, despite high degree of heterogeneity, countries of the eurozone share common financial shocks and, hence, the analysis is in line with rapidly increasing cross-border trade and financial linkages. Second, despite a common monetary policy, national policies of fiscal policy, investments, and structural reforms in labour and complementary markets remain heterogenous across the euro area. Thus, national authorities may be tempted to design domestic policies so as to counteract world conditions, but those policies may be ineffective and counter-productive for the domestic economy. Third, structural differences among national policy may also be driven by idiosyncratic business cycles and, hence, the importance of accounting for transmission channels and latent confounding effects. Fourth and probably most importantly, divergence across countries were driven by different degrees of productivity growth.

Introduction

Since the Euro started, the most research findings focused on debt dynamics, contagion effects, global financial markets, and asset market linkages. (see e.g., Sala-I-Martin *et al.*, 2004, Imbs *et al.*, 2005, and Fatas and Mihov, 2006). This paper addresses three main questions. (i) What was the impact of cross-country fiscal spillover effects in the Euro Area and how it changed during the great recession. (ii) What are possible transmission channels allowing shocks to spill over. (iii) What is the importance of economic and institutional implications in driving the transmission of a shock?

The recent financial crisis that started in mid-2007 and affected the whole world by September 2008 is one of the most challenge episodes for policy makers both at governments and central banks since the introduction of the euro. It developed through two main perspectives. (i) The credit squeeze affected borrowing conditions for firms and households with subsequent adverse effects on domestic investment. (ii) The consumption demand and the downturn in the global economy affected export demand severely.

In a worldwide context, the effects of this disruption was not limited to the financial sector. Global real output and trade declined dramatically, and central banks took unprecedented coordinated action, in part, to alleviate the adverse impacts of the financial markets shocks on real activity. These findings show the deep interdependence between the financial and real sectors. Against such as background, the more recent literature disclosed a large body of empirical evidence based on pre-EMU data points to the presence of significant differences across countries or heterogeneity in the transmission mechanism of shocks in Europe (see e.g., Guiso *et al.*, 2000, and Angeloni*et al.*, 2002).

The recent recession has shown some common behaviors. Thus, institutional and economic interdependencies may have made Euro Area (EA) countries more alike. Nevertheless, there may still be a substantial degree of heterogeneity in economic-financial linkages across countries within the EA and the European Union, and that those linkages may have changed over time (see e.g., Canova and Marrinan, 1998, Canova and De Nicolo', 2000, Del Negro and Obiols, 2001, Lane and Milesi-Ferretti, 2007, and Hirata *et al.*, 2011).

Regarding cross-country spillovers, there is a vast literature reporting their intensification in the last decades (see e.g., Hirata *et al.*, 2011). In addition, a variety of approaches and methods on how shocks spill over across countries and between real and financial variables

have been proposed. Hirata *et al.*, 2001, and Lane and Milesi-Ferri, 2007 argumented on an intensification of the processes of economic unification in different regions including an explosion in the number of regional trade agreements. Recent empirical works were employed in modelling for role of financial factors in driving real outcomes in theory, procyclical nature of real and financial variables, and implications of financial crises for the real economy (see e.g., Kiyotaki and Moore, 1997, Reinhart and Rogoff, 2009, and Hirata, 2012). The evidence of this paper would confirm the need to allow for cross-country and cross-variable interdependencies when studying real and financial linkages. However, country-specific factors remain very important explaining the heterogeneous behaviour across countries observed over time and the presence of a heterogeneous pattern in macroeconomic-financial linkages.

When dealing with muticountry data, the empirical literature took a number of short cuts and neglected some problems. For example, it is typical to assume that slope coefficients are common across units (see e.g., Fatas and Mihov, 2006); there are no lagged interdependencies across units (see e.g., Dees *et al.*, 2005); the structural relationships are stable over arbitrary samples (see e.g., Imbs *et al.*, 2005); no time variation is allowed in the parameters, and there are no interdependencies either among different variables within units or among the same variables across units. In this study, heterogeneity, interdependence, and fiscal spillover effects in macroeconomic-financial linkages are analyzed in a unified framework. Real and financial variables are jointly modelled for a set of a total of 12 countries of the EA.

The specification of the econometric model is the same for all countries considered. Bayesian methods and Maximum Likelihood Estimates (MLE) are used to reduce the dimensionality of the model, put structure on the time variations, and simultaneously evaluate omitted variable bias and issues of endogeneity. In the case of fully hierarchical priors, a MCMC method (or alternatives) can be employed to calculate posterior distributions. To be more precise, MCMC methods are used to model for bayesian inference and numerical integration, to compute impulse responses and conditional forecasting experiments to unexpected perturbations in the innovations of either the VAR or the factors. A BVAR model is used for the following reasons. First, it provides a flexible coefficient factorization that renders estimation easy. Second, the econometric approach makes model selection and inference tractable measuring the evolution of heterogeneity and spillovers in an unified framework. Third, possible commonalities can be analyzed jointly for all variables and countries. The evidence would confirm the need to allow for cross-country and cross-variable interdependencies when studying real and financial linkages. However, country-specific factors remain very important explaining the presence of a heterogeneous pattern across members and of co-movements in economic activity. The specification model used in this study is consistent with the recent literature which recognizes the importance to separate common shocks from propagation of country-specific shocks through different channels.

The paper is structured as follows. Chapter 1, provides theoretical background of spillovers and potential spillover effects. Fiscal policy actions, contagion effects, and fiscal consolidations will be discussed. A brief review will be also done about dynamic growth

and recent crisis. Chapter 2, describes the empirical model and further specification. Chapter 3, illustrates the data. Chapter 4, discusses related literature. Chapter 5, explains the structure of multicountry VAR model. Discussion and relationship will be illustrated. Chapter 6, accurately describes model estimation, prior assumptions, and posterior distributions. MCMC methods and Bayesian factor will be discussed. Chapter 7, describes dynamic analyses of the model. Chapter 8, provides summary statistics of data. Chapter 9, provides estimates of fiscal spillover effects and systemic contributions. Chapter 10, discusses the systemic contribution and contagion index in real and financial dimension before crisis period. Chapter 11 examines in depth common and country-specific factors during the recent crisis. Chapter 12 discusses the role of commonalities and heterogeneity across countries over time. Finally, theoretical and empirical findings are discussed. Appendix A reports spillover net matrices for real and financial dimension. Appendix B, provides useful Bayesian inference completing the model. Appendix C, provides additional computations.

Chapter 1

Theoretical Background: Euro Area, Fiscal Policy, and Policy Coordination under EMU

The euro area is a unique form of a monetary union without historical precedence. The member states of the euro area have assigned the framing of monetary policy to a common monetary authority, the European Central Bank (ECB), set up as a highly independent central bank to insure that it will be able to carry out a policy of price stability. Fiscal policy within the European Union (EU) remains the task of the national governments under a set of rules given in the Maastricht Treaty and the Stability and Growth Pact (SGP). These rules, pertaining to the Economic and Monetary Union (EMU), cover euro area member states as well as member states that have not adopted the euro. They are monitored the existing fiscal policy framework of the euro area that complements the monetary union and its single currency, the euro.

The adoption of a common monetary policy in Europe has eliminated the possibility to use monetary policy for the stabilization of country-specific shocks. This is generally considered as the main cost of forming a monetary union. How large this cost actually is depends on what alternative mecchanisms are available to ensure economic adjustment to idiosyncratic shocks. With perfectly flexible factor markets, stabilization policy is irrelevant as production factors move instantaneously to that part of the union where under-capacity prevails. This reflects Mundell's (!961) argument that labour mobility is a desirable feature of a common currency area subject to cuntry-specific disturbances. In reality, labour mobility is notoriously low, both within and across countries. Hence, not much can be expected from this channel of adjustment. Despite the huge capital flows observed nowadays within and outside the EMU, cross-border asset holdings still seem to be much smaller than predicted by standard theoretical models (see e.g., Gordon and Bovenberg, 1996). This means that instead of shifting savings to places where the risk-return trade-off is most favorable, agents

invest most of their savings locally (see Obstfeld and Rogoff, 2000). The findings by Yosha and Sorensen (1998) confirm the negligible role of capital income flows in absorbing the effects of country-specific shocks in Europe.

As monetary policy can no longer address country-specific shocks and factor mobility does not solve the problem either, other solutions need to be found.One possibility would be a centralization at the European level of the tax-transfer systems that now mainly operate at the national level. Another possibility, discussed for example by von Hagen and Hammond (1995) and Beetsma and Bovenberg (2001), would be the adoption of a system of cross-border fiscal transfers to countries hit by exceptionally bad shocks. Both options, especially the first one, are politically sensitive and cannot be expected to materialize in the foreseeacble future.

The only remaining instrument in the hands of national authorities and capable to stabilize local macroeconomic conditions is fiscal policy. However, fiscal flexibility is hampered by large public debts and formal institutional constraints: the Maastricht rules and the SGP, which forbid public deficits exceeding 3 percent of GDP. It has nevertheless been argued that if countries adhere to a medium-term objective of budget balance or budget surplus, these restrictions are unlikely to be binding in the event of a recession (see e.g., Buti, Franco and Ongena, 1998, Eichengreen and Wyplosz, 1998, and Pina, 2001.

In the context of the EU, the issue of policy coordination is often addressed in institutional terms, the question being whether decisions about a given policy instrument should be taken at the central level (the union level) pr be decentralized (At the national, regional or local levels). As emphasized by Alesina dn Wacziarg (1999), the optimal degree of decentralization of policy prerogatives generally depends on a trade-off between the specific needs of individual decision-making entities[1] and the extent to which the decentralized manipulation of the policy instrument generates spillovers in areas under the jurusdiction of other decision units. Hence, everything else equal, the larger the cross-border externalities associated with decentralized policy actions, the stronger the case for shifting decision-making powers to a higher level of government, possibly even to a supranational instituion able to internaliza all externalities and to deliver more efficient policies.

In principle, all national policies generating cross-border spillovers could be subject to some degree of policy coordination or centralization at the supranational level. Potentially important areas for EMU-wide coordination are structural policies, such as labour market regulations, the tax system, goods market liberalizations, etc.) and various dimensions of fiscal policy, such as capital income taxation, infrastructure expenditure, and tax exemptions for non-resident investors).

For example, in the area of fiscal policy, tax competion has received a lot of attention from policymakers and researchers alike. The problem is that national governments have an incentive to reduce taxes on mobile factors will be inefficiently low, at the expense of inefficiently high taxes on less mobile factors like labour.

A second area of fiscal coordination that is attracting more and more attention since the inception of the EMU is the need for national governments to closely coordinate decisions

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[1]For example, it is because of heterogeneous preferences or constraints

on the overall fiscal stance. As European policymakers become more vocal on the necessity for this type of coordination, it is important to assess whether there is an economic rationale for coordination efforts that go beyond what already exists in the context of the Excessive Deficit Procedure[2] and the Multilateral Surveillance Porcedure[3].

The general debate about the merits and costs of coordination is enriched bya aseries of issues that are specific to monetary unions and that either reinforce or weaken the overall case for fiscal coordination. The first issue dates back to the optimum currency area literature initiated by Mundell (1961) and concerns the stabilization of asymmetric demand shocks. Since monetary unification prevents nominal exchange rate to country-specific demand disturbances, aggregate-demand management through fiscal means becomes more important and can be made easier,and globally more efficient, if countries agree to internalize demand externalities so as to adequately share the burden of adjustment. The argument is reinforced by the fact that monetary integration should foster further trade integration and increase demand-side externalities associated with national fiscal policies.

A second issue specific to monetary unions is that the prevailing policy mix now results from interactions among a large number of players, one central bank and many governments. The risk of a poorly coordinated policy mix is ths potentially greater than in the usual situation in which there is one central bank and only one government. However, even if it reduces the dimension of the fiscal-monetary coordination problem, 'horizontal' coordination limited to fiscal authorities only does not necessarily yield better outcomes. Given the relatively narrow mandate of the ECB, which is primary focus on price stability, it is conceivable that fiscal coordination amplifies the inconsistency between what fiscal authorities jointly perceive as the appropriate policies in the various individual countries and the broader assessment made by the ECB for the aggreagate level. A related concern is that fiscal coordination increases the strategic weight of the fiscal authorities vis-à-vis the central bank, with potentially adverse consequences on the expansionary bias characterizing time-consistent macroeconomic policies (see e.g., Beetsma and Bovenberg, 1998, and Debrun, 2000). These two elements point towards the risk of counter-productive fiscal coordination.

Even though the interaction with the ECB is a key aspect to determine whether coordination is desirable, the debate often remains focused on the magnitude and the signs of the fiscal spillovers that could justify a more cooperative approach to demand-side fiscal policies (see von Hagen, 1998). The sign of these spillovers is particularly important as it helps to determine whether coordination should lead to a more expansionary or more restrictive fiscal stance in the member states. Should the fiscal authorities perceive negative externalities, they would interpret non-cooperative policies in response to bad economic shocks as too expansionary and agree on a more restrictive stance in all countries. Conversely, if governments perceive positive spillovers, coordination should eliminate free-riding behavior and promote more expansionary policies in response to bad shocks.

The theoretical literature does not provide a clear-cut answer about the sign of fiscal pol-

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[2]Article 104 of the Amsterdam Treaty

[3]Article 99 of the Treaty

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icy spillovers. In classic analyses of policy coordination (see e.g., Mundell, 1968, and Hamada, 1985), ad-hoc fixed-price models generally assume direct, positive demand spillovers. By contrast, micro-founded models of EMU tend to conclude in favor of negative spillovers. Andersen and Sorensen (1995) and Jensen (1996) emphasize the adverse terms-of-trade effect of a balanced-budget foreign fiscal expansion on the domestic economy (see e.g., Pina, 1999, and Tirelli, 2000).

The possibility to accumulate public debt adds other sources of negative spillovers through the common real interest rate and the credibility of monetary policy. For istance. Levine and Brociner (1994) propose a model that combines terms-of-trade (negative), real interest rate (negative) and external demand (positive) spillovers and argue that negative spillovers probably dominate. Dixon and Santoni (1997) demonstrated the possibility of positive demand spillovers in a micro-founded model of EMU with monopolistic competition and unionized labor markets leading to excessive unemployment. Important for their result is the assumption that a 'specie-flow' mechanism is at work to balance intra-EMU trade. Hence, a domestic fiscal expansion entails a trade deficit fiannced by a decrease in the net foreign assets of the economy.

Overall, the validity of the argument in favor of negative spillovers primarily depends on the empirical importance of intra-EMU terms-of-trade effects and on the reaction of the common interest rate to changes in fiscal policy. In most of the theoretical models reviewed above, terms-of-trade effects are significant because they implicitly assume strategic interaction within a group of large countries making up the world economy. However, Europe is better described as a club of small economies open to the rest of the world. More specifically, the goods exchanged among EMU member states are also traded outside of the EMU and at the world level, at which individual EMU economies can be assumed to be small in the trade-theoretic sense. Iti si therefore unclear whther a domestic fiscal impulse in a EU member state could have a significant impact on that country's terms of trade since prices are mostly determined at the world level.

Every since the plans for a single European currency were launched about twenty years ago, the institutional system for framing fiscal policies and for preserving the fiscal sustainability of the monetary union has been the subject of a heated debate, among economists as well as among policy-makers (see e.g., Buti and Franco, 2005, Korkman, 2005, and Wierts, 2006). The recent global financial crisis and mainly the European debt crisis have added new impulses to the debate about the proper fiscal policy arrangements within the European Union.

Moreover, the recent crisis has highlighted deficiencies in both the fiscal framework and the financial regulatory framework of the euro area.

The current sovereign debt crisis with its epicenter in the euro area has forcefully revived the academic and policy debate on the economic impact of public debt. Market concerns with respect to fiscal sustainability in vulnerable euro area countries have grown and spread to other countries. Against this background, empirical research has started to focus on

estimates of the impact of public debt on economic activity, inter alia by attempting to unveil possible non-linearities.

Nevertheless, the empirical literature on this topic remains scarce (see e.g., Schclarek, 2004, and Reinhart and Rogoff, 2010) and only few studies employ a non-linear impact analysis and are of particulat interest for this project.

Over the last two years, the euro zone has been going through an agonizing debate over the handling of its own home grown crisis. Starting from Greece, Ireland, Portugal, Spain and more recently Italy, these euro zone economies have witnessed a downgrade of the rating of their sovereign debt, fears of default and a dramatic rise in borrowing costs. These developments threaten other Euro zone economies and even the future of the Euro.

Such a situation is a far cry from the optimism and grand vision that marked the launch of the Euro in 1999 and the relatively smooth passage it enjoyed thereafter. While the Euro zone may be forced to do what it takes, it is unlikely that the situation will soon return to business as usual on its own. Yet, this crisis is not a currency crisis in a classic sense. Rather, it is about managing economies in a currency zone and the economic and political tensions that arise from the fact that its constituents are moving at varying speeds, have dramatically different fiscal capacities and debt profiles but their feet are tied together with a single currency.

Given the large economic weight of the euro zone in the globe, and regularity with which the crisis is spreading from one euro zone economy to the next, the stage for 'palliatives' is over. The manner, in which the euro zone crisis is dealt this point onwards, is likely to be of far reaching significance to the world.

In Chapter 4, more recent literature and discussion on theoretical studies and empirical evidence will be explained.

Chapter 2

Econometric and Specification Model

The paper estimates two BVAR models.

(i) The first accounts for all variables of the system and group-specific factors[1]. Empirical evidence about the size and height difference are displayed in Chapter 9. It has the following form:

$$Y_{it} = A_{it,j}(L)Y_{i,t-1} + B_{it,j}(L)W_{i,t-1} + \varepsilon_{it} \tag{2.1}$$

where Y_{it} is a $M \cdot 1$ vector of variables for each i, the subscript $i=1,2,\ldots,N$ is a country index, $t=1,2,\ldots,T$ denotes time, $A_{it,j}$ are $NM \cdot NM$ matrices of coefficients $Y_{i,t-1}$ is a $M \cdot 1$ vector of variables lagged, W_t is a $q \cdot 1$ vector that include trade and capital factors, and ε_{it} is a $M \cdot 1$ vector of random disturbances. Here, there are p_1 lags for each of the M endogenous variables and p_2 lags for the q variables in W_t. The error terms are assumed to be i.i.d. N(0, Σ)[2]. Here, $p = p_1 = p_2 = 1$.

(ii) The second accounts for all variables of the system, group-specific factors, and latent factors in order to evaluate economic and institutional implications across countries and over time. Empirical evidence about the size and height difference are displayed in Section 12.1. It has the following form:

$$Y_{it} = A_{it,j}(L)Y_{i,t-1} + B_{it,j}(L)W_{i,t-1} + C_{it,j}(L)Z_{i,t-1} + \varepsilon_{it} \tag{2.2}$$

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[1]The variables are output growth and real and financial factors. Group-specific factors accounts for trade and capital flows for real and financial variables, respectively. The factors are weakly-exogenous variables and would analyze transmission channels of spillovers across countries.They are described in Chapter 3 and heavily discussed in Section 8.1.

[2]The equation 2.1 can be written in matrix form in different ways. Some of the literature expresses results in terms of the multivariate Normal and others in terms of the matric-variate Normal distribution [see e.g., Canova (2007) and Kadiyala and Karlsson (1997)].

where $C_{it,j}$ is a $\xi \cdot 1$ vector that include common and idiosyncratic factors[3]. Here, there are p_3 lags for each of the NM endogenous variables. Thus, $p = p_1 = p_2 = p_3 = 1$.

The models hold three important features. (i) The coefficients of the specification are allowed to vary over time. (ii) Dynamic relationship are allowed to be country-specific. In this way, heterogeneity biases are minimized. (iii) Cross-unit lagged interdependencies exist whenever the matrix $A_t(L) = [A_{1t}(L), A_{2t}(L), \ldots A_{Nt}(L)]'$ is not block diagonal for some L. To be more precise, stacking the elements of $A_{it,j}$ over i, a matrix that is not block diagonal for at least one j can be obtained. Thus, dynamic feedback across countries is possible.

This feature adds flexibility to the specification but it is costly. In fact, the number of coefficients is increased by factor N ($k = NM \cdot p_1 + q \cdot p_2$ coefficients in equation 2.1 and $k = NM \cdot p_1 + q \cdot p_2 + \xi \cdot p_3$ in equation 2.2). However, in 2.1 the dynamic relationships are allowed to be unit specific and the coefficients could vary over time. Following the framework in Canova and Ciccarelli [8], the models 2.1 and 2.2 can be re-written in a simultaneous-equation form in order to avoid the matter of dimensionality. Let δ_{it}^m be $k \cdot 1$ vectors, with $\delta_{it} = (\delta_{it}^{1'}, \delta_{it}^{2'}, \ldots, \delta_{it}^{M'})'$, which contains, stacked, the M rows of the matrices $A_{it,j}$ and $B_{it,j}$, a $NMk \cdot 1$ vector $\delta_t = (\delta_{1t}', \delta_{2t}', \ldots, \delta_{Nt}')'$ can be defined. The specification of models assumes the form:

$$Y_t = X_t \cdot \delta_t + E_t \tag{2.3}$$

where, accounting for equation 2.1, $X_t = I_{NG} \otimes X_t'$, with $X_t = (Y_{t-1}', W_t', W_{t-1}')'$, Y_t and E_t are $NM \cdot 1$ vectors containing the endogenous variables and the random disturbances of the model. Here, Y_{it} is expressed in terms of X_t. The crucial aspect of equation 2.3 is that there is no subscript i since variables of all countries in the system are stacked in X_t. However, in equation 2.2, the vector $X_t = (Y_{t-1}', W_t', W_{t-1}', Z_t', Z_{t-1}')'$ contains endogenous and exogenous variables of the system.

Now, since δ_t varies in different time periods for each country-variable pair, whenever δ_{it} is unrestricted, it is impossible to estimate it. Moreover, its sheer dimension (k=NMp parameters in each equation) could prevent any meaningful unconstrained estimation. There are more coefficients than data points. To solve it, a flexible structure where δ_t is factored can be assumed:

$$\delta_t = \Xi \cdot \theta_t + u_t \qquad u_t \sim N(0, \Sigma \otimes V) \tag{2.4}$$

where Ξ is a matrix of coefficients, $\dim(\theta_t) \ll \dim(\delta_t)$, and u_t captures unmodelled and idiosyncratic variations present in δ_t.

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[3]The variables are unemployment rate, final consumption, and unit labour costs for all selected period. During financial crisis and fiscal consolidations, other private and public factors are also added, such as private consumptions and international investment positions. All variables are described in Chapter 3 and heavily discussed in Section 8.1.

The selection of the type of factors is often a matter of choice, that is typically dictated by the needs of the investigation. In a cross-country study of business cycle transmissions, for example, common and country-specific factors are probably sufficient although, when constructing indicators of GDP, one way want to specify, at least, a common, a country-specific, and a variable-specific factor. In equation 2.4, all factors are permitted to be time-varying and, hence, time invariant structures can be obtained via restrictions on their law of motion, as explained below.

Empirical evidence are discussed in Section 12.1 in order to estimate economic and structural implications in driving the transmission of a shock in maceroeconomic-financial linkages. Running equations 2.3 and 2.4 for both equations 2.1 and 2.2, the SUR model is:

$$\Xi \cdot \theta_t = \Xi_1 \cdot \theta_{1t} + \Xi_2 \cdot \theta_{2t} + \Xi_3 \cdot \theta_{3t} + \Xi_4 \cdot \theta_{4t} + \Xi_5 \cdot \theta_{5t} + \Xi_6 \cdot \theta_{6t} \qquad (2.5)$$

where Ξ_1 and Ξ_3 are matrices of dimensions $NMk \cdot N$, with $k = NM \cdot p_1 + q \cdot p_2$. θ_{1t} and θ_{3t} are mutually orthogonal $NM \cdot 1$ factors capturing, respectively, movements in the coefficient vector which are country specific. They account, respectively, for real and financial variables plus common-specific factors, so that $\Xi_{1it} = \sum_{m=1}^{3} \cdot \sum_j y_{timp-j}$ and $\Xi_{3it} = \sum_{m=4}^{6} \cdot \sum_j y_{timp-j}$, $i = 1, \ldots, 12$, $p = p_1 = p_2 = 1$.

Ξ_2 and Ξ_4 are matrices of dimensions $NMk \cdot N$, with $k = NM \cdot p_1 + q \cdot p_2 + \xi \cdot p_3$. θ_{2t} and θ_{4t} are mutually orthogonal $NM \cdot 1$ factors capturing movements in the coefficient vector across all countries. They account, respectively, for real and financial variables plus common factors and exogenous variables, so that $\Xi_{2it} = \sum_{m=1}^{3} \cdot \sum_j y_{timp-j}$ and $\Xi_{4it} = \sum_{m=4}^{6} \cdot \sum_j y_{timp-j}$, $i = 1, \ldots, 12$, $p = p_1 = p_2 = p_3 = 1$.

Ξ_5 is a matrix of dimension $NMk \cdot M_1$. θ_{5t} is mututally orthogonal $NM_1 \cdot 1$ factor capturing movements in the coefficient vector which are variable (or group-variable) specific, where $M_1 \leq M$ denotes the number of variable groups. It corresponds to two groups accounting for trade and capital flows and six groups accounting for common factors and exogenous variables, so that $\Xi_{5it} = \sum_{m=1}^{6} \cdot \sum_j y_{tim_1p-j}$, $p = p_1 = p_2 = p_3 = 1$.

Ξ_6 is a matrix of dimensions $NMk \cdot 1$, θ_{6t} is mutually orthogonal $N \cdot 1$ factor capturing movements in the coefficient vector which are common across all countries and variables. It accounts for all real and financial variables plus common factors and, respectively, with and without exogenous variables, so that $\Xi_{6it} = \sum_{m=1}^{6} \cdot \sum_j y_{timp-j}$, $p = p_1 = p_2 = p_3 = 1$.

The idea behind the first four effects is to provide a possible reason about different reactions or co-movements across countries to a common shock (e.g., fiscal consolidations to improve the economic growth against increasing local and economic imbalances). The fifth and sixth effect would highlight the importance of economic and structural factors and transmission channels in driving the spreading of a shock when studying macroeconomic-financial linkages.

Hence, $\theta_t = (\theta_{1t}', \theta_{2t}', \theta_{3t}', \theta_{4t}', \theta_{5t}', \theta_{6t}')'$ is a $NMk \cdot 1$ vector and the estimated model is:

$$y_t = \chi_{1t}\theta_1 t + \chi_{2t}\theta_2 t + \chi_{3t}\theta_3 t + \chi_{4t}\theta_4 t + \chi_{5t}\theta_5 t + \chi_{6t}\theta_6 t + \eta_t \qquad with \qquad \theta_t = \theta_{t-1} + \nu_t \quad (2.6)$$

where $\chi_t \equiv X_t \cdot \Xi$, with $\Xi = [\Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_5, \Xi_6]$.

Factoring δ_t as in equation 2.4 reduce the problem of estimating NMk coefficients into the one of estimating for example, $NM + M_1 + 1$ factors characterizing their dynamics. Moreover, this finding is able to transform the overparametrized multicountry VAR into a parsimonious Seemingly Unrelated Regression (SUR) model. By equations 2.4 and 2.3, it can be written as:

$$Y_t = \chi_t \theta_t + \eta_t \tag{2.7}$$

By construction, χ_{it} are linear combinations of right-hand side variables of the multi-country VAR and correlated among each other. The correlation decreases as M or N or $p = max[p_1, p_2, p_3]$ increase, and comovements are emphasized across lagged variables.

The vector of endogenous variables depends on a small number of observable indices, χ_{it}, and the factors θ_{it} load on the indices. They are vector time-varying loadings to be estimated. In fact, they are smooth linear functions of the lagged endogenous variables. Thus, in equation 2.6, $\chi_{1t}\theta_{1t}$, $\chi_{2t}\theta_{2t}$, $\chi_{3t}\theta_{3t}$, $\chi_{4t}\theta_{4t}$ are observable country indicators for Y_t, $\chi_{5t}\theta_{5t}$ is observable country-specific component for Y_t, and $\chi_{6t}\theta_{6t}$ is observable common indicator for Y_t.

Chapter 3

Data Description

The last 15 years have observed an increased globalization of world economies. The model is estimated for 12 economies of the Euro Area: Italy, Spain, France, Belgium, Netherlands, Austria, Germany, Finland, Luxembourg, Ireland, Greece, Portugal. The sample period is 1998q4 - 2014q2. This span of data includes a large number of quarters before and after the financial crisis. Hence, the model is able to capture not only possible time variation around business cycle phases, but also time variation caused by possible structural changes (see e.g., Canova *et al.* [29]).

For each of the EA12 countries, the real variables included are general government spending, real GDP growth rate, and gross fixed capital formation in order to capture business cycles and main spillover channels in real dimension. To be more precise, the general government spending (*gov*) denotes all financial accounts in percentage of GDP. Real GDP growth rate (*gdpg*) is computed respect to the same quarter of the previous year (q/q-4) . Gross fixed capital formation (*cap*), also known as Investments, consists of resident producers' acquisitions, less disposals, of fixed assets plus certain additions to the value of non-produced assets. These assets acquired are intended for use in processes of production. GFCF includes acquisition less disposals of, e.g. buildings, structures, machinery and equipment, mineral exploration, computer software, literary or artistic originals and major improvements to land such as the clearance of forests.

The financial variables included are interest rate, general government debt, and general government deficit, which are most suitable to capture business cycles and spillover channels in financial dimension. To be more precise, the interest rate (*int*) denotes EMU convergence criterion series relates to interest rates for long-term government bonds denominated in national currencies. Selection guidelines require data to be based on central government bond yields on the secondary market, gross of tax, with a residual maturity of around 10 years. The General government debt (*debt*) corresponds to quarterly non-financial accounts for the general government sector which are conceptually consistent with the corresponding annual data compiled on a national accounts basis. The general government sector comprises

central government, state government, local government, and social security funds and is observed in percentage of GDP. General government surplus/deficit (*curr*), also known as current account balance, is defined in the Maastricht Treaty as general government net borrowing/lending according to the European System of Accounts and observed in percentage of GDP.

The (directly) observable variable to measure the effects from fiscal shocks in real and financial components, respectively, is the productivity as proxy for economic growth (*prod*). It is defined as $prod_{it} = \ln(\frac{Y_{it}}{Y_{it-1}})$, by considering the computations of Sala-i-Martin [39].

Bilateral flows of trade (*real*) and bilateral flows of capital (*fin*) are computed to capture interactions between real and financial variables across countries, respectively. To be more precise, the variable *real* denotes exports and imports by Member Stated of the Euro Area at the current prices and weighted for the GDP. The variable *fin* denotes financial transactions computed on the total economy in million units of national currency and weighted for the GDP. The values are expressed at the net on the total transactions.

There are five indicators which describe macroeconomic imbalances and, hence, economic/structural implications (called *imbalances*). The project considers one indicator monitoring external positions, one indicators capturing competitiveness developments and *catching-up* factors, three indicators reflecting internal imbalances. The specification model described in equation 2.7 is able to observe interdependence and time-varying effects across countries and over time. For the all selected period, net investment position, nominal unit labour cost, general government consumption, private sector consumption, unemployment rate.

To be more precise, international investment positions (*inv*) are observed in million euro and weighted for the GDP of eurozone countries.

Unit labour costs (*lab*) measure the average cost of labour per unit of output and are calculated as the ratio of total labour costs to real output. In broad terms, unit labour costs show how much output an economy receives relative to wages, or labour cost per unit of output. Generally, it represents a direct link between productivity and the cost of labour used in generating output. A rise in an economy‚Äôs unit labour costs represents an increased reward for labour‚Äôs contribution to output. However, a rise in labour costs higher than the rise in labour productivity may be a threat to an economy's cost competitiveness, if other costs are not adjusted in compensation.

The variable (*cons*) denotes the final consumption aggregates at the current prices and weighted for the GDP. The variable (*priv*) consists of expenditure incurred by resident institutional units on goods or services that are used for the direct satisfaction of individual needs or wants or the collective needs of members of the community. The consumption expenditure may take place on the domestic territory or abroad. Private final consumption expenditure includes households' and Non Profit Institutions Serving Households' (NPISH) final consumption expenditure. NPISH are separate legal entities serving households and account, for example, for trade unions, professional societies, political parties, sports clubs and so on.

Finally, unemployment rate (*unem*) denotes the growth rates of the unemployment by sex and age groups in percentage of GDP.

Running the BVAR model in equation 2.2, the above mentioned variables are able to evaluate bilateral spillover effects and systemic contribution between real and financial sectors[1]. Running the SUR model in equation 2.7, they can estimate possible co-movements, heterogeneity, and dispersion indices. Emprical evidence are heavily discussed in Chapter 12, where *catching-up* and competitiveness factors and *cause-effect* relationship due to economic/institutional implications can be explained[2].

All endogenous and exogenous variables in the system are expressed in quarters. Since the analysis does not consider all Euro Area Countries, adjusted weights by the EA12 GDP are used to compute posterior of IRFs (except for the interest rate which is taken on the basis of EMU convergence criterion bond yields). However, all variables in the system are observed in annual growth rates with respect to the same quarter of the previous year (q/q-4). Mostly data come from Eurostat, OECD, and IMF databases.

In equation 2.1, all real and financial variables are treated as endogenous, while group-variable specific are treated as weakly-exogenous factors. To be more precise, trade flows *real* corresponds to bilateral net exports and capital flows *fin* are determined as bilateral net financial transactions. Thus, they correspond to weak-exogenous variables in order to capture co-movements, spreading of spillover effects, heterogeneity, and some economic/structural implications when studying macroeconomic-financial linkages. In Section **??**, the variables are significant and potentially strongly correlated with real and financial dimension. Moreover, the variable specific *fin* is more significant than transmission channels in real dimension. This result has implications for the extent to which some economies can be considered engines of global or regional growth or alternatively can be considered transmitters of growth shocks that originate elsewhere.

The advantage of this approach is that it is easier to model the endogenous variables conditional on the exogenous variables if these show some kind of irregular behaviour, which would be difficult to model within a VAR framework. It is very tempting to draw inference from the conditional or partial model rather than modelling the exogenous variables less carefully or not at all. Thus, one would work with smaller systems int erms of the parameters to be estimated with a gain in efficiency. The above-mentioned approach is valid if and only if the assumption of weak exogeneity is statisfied (see e.g., Engle, Hendry and Richard [20] and Barassi [4]). To be more precise, the basis for this discussion is provided by the analysis of joint and conditional densities and sequential factorisation.

For example, let $D_x(Y_t, W_t | X_{t-1}, \delta)$ be the sequential density at time t of the random vector $X_t = (Y_t : W_t)'$ conditional on $X_{t-1} = (X_0, X_1, \ldots, X_{t-1}$, where $\delta = (\delta_1, \ldots, \delta_n)' \in \Delta$ which is compact subset of $\mathfrak{R}^n$. Generally speaking, W_t is endogenous in the framework of

---

[1]Bilateral net spillover effects and systemic contribution are discussed in Chapters 9 and 10, respectively.
[2]In Section **??**, preliminary tests are carried out in order to highlight the presence of those relationships.

the joint density function, but if W_t is weakly exogenous it is possible to factorise the joint density such that knowledge of how the process W_t is determined is not necessary in order to investigate the properties of the process Y_t. Thus, allowing for the existence of many one-to-one transformations from the original n parameters $\delta = (\delta_1, \ldots, \delta_n)' \in \Delta$ to any new set of parameters $\alpha \in \dot{A}$, with $\alpha = (\alpha_1, \alpha_2)'$, the factorization of the joint density function is: $D_x(Y_t, W_t | X_{t-1}, \delta) = D_{y|w}(Y_t | W_t, X_{t-1}, \alpha_1) D_w(W_t | X_{t-1}, \alpha_2)$. This latter involves a subset ϕ of the parameters δ, where ϕ is a vector of parameters of interest.

However, the first requirement for a variable W_t to be regarded as weakly exogenous for a set of parameters of interest ϕ is that the marginal process for W_t should add no useful information about ϕ, that is one must be able to learn about ϕ from α_1 alone. The second condition one needs to justify taking W_t as given is that α_1 should not depend on α_2. If this were the case one could learn indirectly about ϕ from α_2. Thus, W_t would be weakly-exogenous for ϕ if and only if: ϕ is function of α_1 and does not depend on α_2; α_1 and α_2 are variation-free.

The exogenous variables observed in equation 2.2 and in the specification model 2.7 account for common and idiosyncratic component, which describes latent factors that both change over time and across countries affecting either real or financial dimension.

The analysis of cross-country growth spillovers and, more generally, multi-country estimations is generally hampered by dimensionality constrints. Foru different VAR-based approaches have been suggested to tackle this issue: Bayesian VARs, factor model VARs, global VARs, and VARs based on regional groupings. All four techniques require additionally an approach for resolving the identification issue. The Bayesian VAR approach tackles the problem with the use of priors about the cross-country correlation patterns, which are subsequently updated with the data (see e.g., Banbura *et al.*, 2007, and Canova and Ciccarelli, 2006). Factor models, instead, collpase cross-country co-movements of several variables into common factors which are then allowed to affect the dynamics of the individual countries (see e.g., Bénassy-Quéré and Cimadomo, 2006). Global VARs reduce the individual countries' spillovers to their share in a weighted average for the variable of interest, which then affects the individual countries' dynamics. The spillover in the global VAR has thus a direct interpretation, unlike the spillover in the factor VAR (see e.g., Bussiére *et al.*, 2009, Galesi and Sgherri, 2009, and Dees *et al.*, 2007). A fourth approach focuses on a small set of countries or regions and then use the traditional structural VAR (SVAR) approach (see e.g., Bayoumi and Swiston, 2009, and Danninger, 2008). The degrees of freedom are preserved by reducing the number of regressors, for example by reducing either the number of countries involved or the number of variables considered, or a combination of both. Bayesian VARs and SVARs are more general than global VARs or factors VARs since they impose less structure on the inter-linkages. Compared to SVARs, Bayesian VARs require making more assumptions on the data generating process in return for more degrees of freedom, which makes the estimation feasible, if the number of regressors is high relative to the size of the available data sample. The SVAR approach proposed by Bayoumi ans Swiston (2009) requires an extensive dataset, but has the advantage that it imposes no structure on the inter-linkages, and thus

the coefficient estimates are purely data driven.

In this analysis, to study interdependence across countries, one may estimate a large VAR model that includes variables of all countries in the vector Y_{it}. In such model, all variables are treated as endogenous. However, for the large number of variables and of coefficients to be estimated and the relatively small number of observations, a large VAR may be intractable. A BVAR model offers an alternative approach by treating all variables as endogenous and bilateral flows of trade and capital as weakly exogenous. They highlight the possible existence of cross-country linkages. Due to the limited length of the time series, the model is estimated with only one lag of the endogenous variables ($p_1 = 1$), a constant, one lag of the group-variables ($p_2 = 1$), and one lag of the exogenous factors ($p_3 = 1$).

Each equation of the VAR has $k = [(12 \cdot 6) \cdot 1] + (2 \cdot 1) + (5 \cdot 1) = 79$ coefficients and there are 72 equations in the system. Thus, without restrictions, there would be a total of $79 \cdot 72$ regression parameters. The total number of draws is $5000 + 1000 = 6000$, which corresponds to the sum of final number of draws to save and draws to discard, respectively. The study checked convergence recursively calculating the first two moments of the posterior of the parameters using 1000, 2000, 3000, 4000, 5000 draws and found that convergence is easily obtained with about 1000 draws. The analysis has been experimented with different combinations of runs and priors keeping the total number of iterations fixed. Thus, results would be robust to this choice. Finally, with this routine, unexpected shocks are computed in order to estimate macroeconomic-financial linkages and fiscal spillover effects on the productivity in the Euro Area. For the specification model described in equation 2.7 are based on chains with 150000 draws. In particular, it corresponds to 3000 blocks of 50 draws and retained the last draw for each block. Finally, 2000 draws were used to conduct posterior inference at each t.

Chapter 4

Discussion and Relationship with the literature

Macroeconomic coordination of fiscal policies is a recurring theme both in the policy discussions and in the economic literature. Recent developments have given a new impulse to the study of this issue using multicountry VAR models and panel VAR for applied macroeconomic analysis. For example, Bénassy-Quéré and Cimadomo [3] estimate the impact of fiscal spillovers from Germany to the remaining G7 countries using an augmented SVAR model. Blanchard and Perotti [6], Perotti [35], and Pappa [7] estimate fiscal shocks in the U.S. economy and selected G7 economies using an augmented country-specific VAR model. Beetsma *et al.* [5] consider fiscal spillover effects through trade proceeding in two steps. First, they obtain estimates on the effects of a fiscal shock on output using an European panel VAR. Second, they impose homogeneity restrictions and plug the panel VAR estimates into a trade-gravity type of model. Pesaran *et al.* [34] and Pesaran [33] develop a multi-country Global Vector Autoregression (GVAR) approach in which estimate the spillover effects of a domestic budget balance shock on the members of the EA by combining all country-specific VAR models in one multi-country model and treating all variables as endogenous. In a recent paper, Ciccarelli, Ortega and Valderrama [9] investigate heterogeneity and spillovers in macro-financial linkages across developed economies adopting a panel Bayesian VAR model.

By the same token, empirical measures for the strenght and dynamic of spillover effects have attracted widespread attention by academia, policy makers and market participants, specially during the recent financial crisis. After the collapse of Lehman Brothers in autumn 2008, the fear of contagion has been one of the most prominent issues on the agenda both for financial research and policy making. A variey of approaches and methods on how to measure contagion has been proposed. Allen and Gale [1] define contagion as a consequence of excess spillover. More generally, Hartmann *et al.* (2005) summarize five main criteria to identify contagion. First, an idiosyncratic negative shock that affects a financial institution and spreads to other parts of the financial system. Second, the interdependencies between assets are different than in tranquil times. Third, the excess dependencies cannot be ex-

plianed by common shock. Fourth, events associated with extreme left tail returns. Fifth, interdependencies evolve sequentially. Costancio [16] extends the identification of contagion in the abnormal speed and strength of potential spillovers as a consequence of a trigger-event (e.g., financial instability).

One advantage of the flexible coefficient factorization in equation 2.4 is that the over-parametrization of the original multicountry TVC-VAR is dramatically reduced. In fact, in the resulting SUR model , estimation and specification searches are constrained only by the dimensionality of θ_t (δ_t is integrated out). A second advantage is that, given the MA nature of many chi_{it}, the regressors of equation 2.7 capture low-frequency comovements present in the lags of the VAR. Since the model averages out not only cross section, but also time-series noise, reliable and stable estimates of θ_t can potentially be obtained, making the framework useful for a variety of medium-term policy analyses exercises. A third advantage is that the SUR model in equation 2.7 has some economic content. For example, if θ_{1t} and θ_{2t} capture information that is common to all lags and the coefficients of the VAR, $\chi_{1t}\theta_{1t}$ and œá$\chi_{2t}\theta_{2t}$ are indicators for Y_t based on common information. By the same token, indicators containing other types of information can also be easily constructed. Finally, since χ_{it} are predetermined, leading versions of these indicators can be obtained projecting θ_t on the information available at $t - \tau$, whit $\tau = 1, 2, \ldots$.

Some commentators have argued that the equal and exogenous weights that equation 2.4 imposes on the regressors of 2.7 are restrictive and suggested the possibility to estimate the Ξ,Äôs. The structure of this framework is no more restrictive than the one used in related literature. Clearly, the equal weighting scheme is appropriate if all variables are measured in the same units (e.g., growth rates) and their variability is comparable; otherwise, preliminary transformations need to be used or the vector of Ξ_i appropriately scaled. For example, if the variability of the variables of country 1 is considerably larger than the variability of the variables of country 2, then one could specify $Xi_1 = (\sigma_1^{-1}, \ldots, \sigma_1^{-1}, \sigma_2^{-1}, \ldots, \sigma_2^{-1}, \ldots)$ where σ_1 and σ_2 measure the average standard deviation of the variables in country 1 and 2. The idea of estimating the Ξ,Äôs is not considered in this framework; the weights are a priori determined by the flexible factorization used. This latter is feasible if one directly starts from equation 2.7, treats Ξ_i as unknown, and employs the factor models techniques described in Chapter 6. Further, this approach has two types of advantages over single-country or two-country VARs. (i) If the information is weak or the sample short, cross sectional information may help to get better estimates and smaller standard errors. (ii) If the momentum that shocks induce across countries is the result of lagged interdependencies, the model described in equation 2.7 will be able to capture it.

In addition, the SUR model has also some similarities with the models used by Pesaran [34] and Pesaran *et al.* [33] to model global interdependencies, even though the starting point, the underlying specification, and the estimation technique differ. In fact, in these papers, the baseline specification is a traditional micro-panel structure with unobservable common components in the error term, instead of a VAR; no time variations are allowed in

the coefficients and no lagged interdependences are present; N is assumed to be large. In this setup, it is possible to obtain a consistent estimate of the common unobservable component by arithmetically averaging the dependent and the independent variables of the unit-specific regressions. Therefore, the estimated specification looks like a set of unrelated single-country VARs, where common factors are proxied by averages of the variables across countries. The specification model described in equation 2.7 shares the idea of using arithmetic averages as regressors. It can be interpreted as an F-factor generalization of these authors‚Äô approach, where each factor spans a difference space; instead, the model described in equation 2.7 allows for lagged interdependecies in the error term and for time-varying loading. Finally, this specification does not need a large N to work.

The recent recession has shown two main matters. First, there are institutional and economic interdependencies across countries, specially between Eurozone countries having relinquished independent monetary and exchange rate policies. Second, there may still be a substancial degree of heterogeneity with some common behaviours in macroeconomic-financial linkages across countries and that those linkages may have changed over time. There exists a variety amount of empirical work on spillovers effects from fiscal policy shock. A possible reason for this is that, while the theoretical literature suggests a variety of possible channels through which fiscal policy may cause cross-border spillovers, empirically, it has proved difficult to find significant spillover effects. Neverthless, there are some exceptions. In a study that is closest to the current one, Canova *et al.* [8] confirm the need to allow for cross-country and cross-variable interdependencies when studying real and financial linkages. Moreover, country-specific factors remain very important, which explains the presence of a heterogeneous pattern in macroeconomic-financial linkages. They extend recent empirical work that assesses the macro-economic effects of impulses in the economy by using time-varying multicountry VAR models to study interdependence and time variation simultaneously across a panel of countries.

According to the above-mentioned literature, the aim of the project is to understand common and heterogeneous patterns between financial and real variables, with a strong attention on the recent recession. This study presents a method to estimate the strength and dynamic of fiscal spillover effects in the EA using a Bayesian VAR approach to study cross-unit interdependencies, unit-specific dynamics, group- and variable- specific effects, and time variations in the coefficients. The framework of analysis is Bayesian in order to reduce the dimensionality of the model and put structure on the time variations. Posterior of impulse response functions and conditional forecasts are obtained with the output of an MCMC simulations. This study contributes to the literature for the effects of fiscal shocks on the economies in the EA. Bilateral trade and capital for real and financial variables are respectively computed in order to account for cross-country linkages. The paper finds that spillovers of fiscal coordination in the real dimension are no more larger than financial dimension. Moreover, cross-border spillovers have excarbated the negative effects of consolidations, with a substantial degree of heterogeneity in real dimension and a deeper interdependence in financial dimension. From a policy perspective, optimal policy coordination in the EA

would have required a differentiation of consolidation efforts depending on the fiscal space to minimise the negative spillovers.

Chapter 5

Multicountry VAR Setup and Related literature

To illustrate the structure of the matrices $\Xi's$ and of X_{it} suppose there are M=2 variables for each of n=2 countries and that the BVAR has p=1 lags and no intercept:

$$
\begin{bmatrix} y_t^1 \\ x_t^1 \\ y_t^2 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} b_{1,1,t}^{1,y} & b_{2,1,t}^{1,y} & b_{1,2,t}^{1,y} & b_{2,2,t}^{1,y} \\ b_{1,1,t}^{1,x} & b_{2,1,t}^{1,x} & b_{1,2,t}^{1,x} & b_{2,2,t}^{1,x} \\ b_{1,1,t}^{2,y} & b_{2,1,t}^{2,y} & b_{1,2,t}^{2,y} & b_{2,2,t}^{2,y} \\ b_{1,1,t}^{2,x} & b_{2,1,t}^{2,x} & b_{1,2,t}^{2,x} & b_{2,2,t}^{2,x} \end{bmatrix} \cdot \begin{bmatrix} y_{t-1}^1 \\ x_{t-1}^1 \\ y_{t-1}^2 \\ x_{t-1}^2 \end{bmatrix} + \eta_t
\tag{5.1}
$$

Here, $\delta_t = [b_{1,1,t}^{1,y}, b_{1,1,t}^{1,x}, b_{1,1,t}^{2,y}, b_{1,1,t}^{2,x}, b_{2,1,t}^{1,y}, b_{2,1,t}^{1,x}, b_{2,1,t}^{2,y}, b_{2,1,t}^{2,x}, b_{1,2,t}^{1,y}, b_{1,2,t}^{1,x}, b_{1,2,t}^{2,y}, b_{1,2,t}^{2,x}, b_{2,2,t}^{1,y}, b_{2,2,t}^{1,x}, b_{2,2,t}^{2,y}, b_{2,2,t}^{2,x}]'$ is a $(16 \cdot 1)$ vector containing the time varying coefficients of the model. Note that the typical element of δ_t, $b_{l,s,t}^{i,j}$, is indexed by the country i, the variable j, the variable in an equation l (independent of the country) and the country in an equation s (independent of the variable). Given the factorization described in equation 2.4, the VAR() can be rewritten as:

$$
\begin{bmatrix} y_t^1 \\ x_t^1 \\ y_t^2 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} \chi_{1,t} \\ \chi_{1,t} \\ \chi_{1,t} \\ \chi_{1,t} \end{bmatrix} \theta_{1t} + \begin{bmatrix} \chi_{2,t}^1 & 0 \\ \chi_{2,t}^1 & 0 \\ 0 & \chi_{2,t}^1 \\ 0 & \chi_{2,t}^1 \end{bmatrix} \theta_{2t} + \begin{bmatrix} \chi_{3,t}^1 & 0 \\ \chi_{3,t}^1 & 0 \\ 0 & \chi_{3,t}^1 \\ 0 & \chi_{3,t}^1 \end{bmatrix} \theta_{3t} + \eta_t
\tag{5.2}
$$

where $\chi_{1t} = y_{t-1}^1 + x_{t-1}^1 + y_{t-1}^2 + x_{t-1}^2$, $\chi_{2,t}^1 = y_{t-1}^1 + x_{t-1}^1$, $\chi_{2,t}^2 = y_{t-1}^2 + x_{t-1}^2, \chi_{3,t}^1 = y_{t-1}^1 + y_{t-1}^2, \chi_{3,t}^2 = x_{t-1}^1 + x_{t-1}^2$. In In the empirical application, all variables are measured in standardized and demeaned growth rates and therefore this type of averaging will indeed be approriate. Note that if θ_{1t} is large relative to θ_{2t}, y_t^1 and x_t^1 comove with y_t^2 and x_t^2. On the other hand, if θ_{1t} is zero, y_t^1 and x_t^1 may drift apart from y_t^2 and x_t^2. In the general case when p>1, lags could be weighted using a decay factor in the same spirita s Doan *et al.* (1984). The regressors in equation 2.7 are combinations of lags of the right hand side variables of the VAR, while θ_{it} play the role of time varying loadings. Using averages as

regressors is common in the signal extraction literature (see e.g., Sargent, 1989) and in the factor model literature (see e.g., Forni and Reichlin [21]). However, there are three several important differences between regressors in equation 2.7 and standard factor models. (i) The indices are used weighting equally the information in all variables, while in factor models the weights generally depend on the variablity of the components. (ii) The indices dynamically span lagged interdependencies across units and variables, while in standard factor models they statistically span the space of the variables of the system. (iii) The indices are directly observable, while in factor models they are estimated. However, these indices are correlated by construction since the factorization is applied on the coefficient vector rather than on the variables. Finally, this averaging approach creates moving average terms of order p in the regressors of equation 2.7, even when y_{it} are serially independent. Hence, contrary to what occurs in factor models, the indicators implicitly filter out from the right hand side variables of the VAR high frequency variability.

Exploiting SUR model, the regressors emphasize the low frequencies movements in the variables of the VAR. This finding is important in forecasting in the medium run and in detecting turning points of GDP growth (Ciccarelli and Rebucci [12]). The SUR model in Chapter 2 has some similarities with the Global VAR (GVAR) model (see e.g., Pesaran *et al.*, 2005), even though the starting point. Nevertheless, the underlying specification and the estimation technique differ. To be more precise, in the GVAR models the estimated specification looks like a set of unrelated single country VARs where common factors are proxied by averages of the variables across countries. The approach illustrated in Chapter 2 would share the idea of using arithmetic averages as regressors and can be interpreted as an F-factor generalization of these author's approach, where each factor spans a difference space allowing for lagged interdependencies in the error term and for time-varying loading.

Chapter 6

Model Estimation

As stated in Chapter 2, Bayesian VAR model is a feasible solution to the overfitting problem. To be more precise, there are three statistical regularities of time-series data. (i) Trending behaviour. (ii) More recent values contain more information than past values. (iii) Past values of the variable contain more information than past values of other variables. Here, regularities are transformed in prior assumptions. Bayesian estimation requires the specification of these prior assumptions.

6.1 Prior Information

In hierarchical models, many problems involve multiple parameters which can be regarded as related in some way by the structure of the problem. A joint probability model for those parameters should reflect their mutual dependence. Typically, the dependence can be summarized by viewing these parameters as a sample from a common population distribution. Hence, the problem can be modelled hierarchically, with observable outcomes (Y_i) modeled conditionally on certain parameters (θ_i), which themselves are assigned a distribution in terms of further (possibly common) parameters, hyperparameters (α). This hierarchical thinking may help solve the trade-off between inaccurate fit and overfitting, and also plays an important role in developing computational strategies. Given equation described in equation 2.7, the prior $p(\theta)$ typically depends on hyperparameters. Collecting the latter in a vector α, it leads that:

$$p(\theta|Y,\alpha) = \frac{p(\theta,\alpha,Y)}{p(Y|\alpha)} = \frac{p(Y|\theta,\alpha)p(\theta|\alpha)}{p(Y|\alpha)} \tag{6.1}$$

If η is unknown, the second stage prior distribution (hyperprior), $p(\theta)$, is:

$$p(\theta) = \int p(\theta,\alpha)d\alpha = \int p(\theta|\alpha)p(\alpha)d\alpha \tag{6.2}$$

The posterior will be:

$$p(\theta, \alpha|Y) = p(\theta|\alpha, Y)p(\alpha|Y) \propto p(Y|\theta, \alpha) \cdot p(\theta|\alpha)p(\alpha) \tag{6.3}$$

Then,

$$p(\theta|Y) = \int p(\theta, \alpha|Y)d\eta = \int p(\theta|\alpha, Y)p(\alpha|Y)d\alpha \tag{6.4}$$

Furthermore, equation 2.7 can be alternatively written in the following manner, accounting for indices[1]:

$$Y_{ij} = \chi_{ij}\theta_i + \eta_{ij} \tag{6.5}$$

where, $i=1,2, \ldots, n$ and $j=1,2, \ldots, J$. Stacking, this latter become:

$$Y_j = \chi_j\theta + \eta_j \tag{6.6}$$

where, Y_j is a $(n \cdot 1)$ vector, χ_j is a $(n \cdot k)$ matrix, θ is a $(k \cdot 1)$ vector, and η_j is a $(n \cdot 1)$ vector, with $k = \sum_{i=1}^{n} k_i$. Stacking further:

$$Y = \chi\theta + \eta \tag{6.7}$$

If $\eta_j \sim N(0, \Sigma)$, then $\eta \sim N(0, \Omega)$, where $\Omega = (\Sigma \otimes I)$.

The non-zero covariances imply that equation 6.5 is related and individual regressions are tied into a system of equations that can be analyzed together. Variances can also differ across j, while η_i are independent across i. Generally speaking, equation 6.7 is a linear regression model, where:

$Y_j = (Y_{1j}, Y_{2j}, \ldots, Y_{nj})'$; $\theta = (\theta_1, \theta_2, \ldots, \theta_n)'$; $\eta_j = (\eta_{1j}, \eta_{2j}, \ldots, \eta_{nj})'$; $Y = (Y_1, Y_2, \ldots, Y_J)'$; $\chi = (\chi_1, \chi_2, \ldots, \chi_J)'$; $\eta = (\eta_1, \eta_2, \ldots, \eta_J)'$. χ_j and Ω are matrix having the following form:

$$\chi_j = \begin{pmatrix} \chi'_{1j} & 0 & \ldots & 0 \\ 0 & \chi'_{2j} & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \ldots & 0 & \chi'_{nj} \end{pmatrix} \quad \Omega = \begin{pmatrix} \Sigma & 0 & \ldots & 0 \\ 0 & \Sigma & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \ldots & 0 & \Sigma \end{pmatrix}$$

.

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[1]Remember that Y_{ij} represents the effect on the productivity given the impulse response of the variable i for a shock in the variable j.

The specification allows the disturbance to productivity across fiscal shocks on real and financial dimension for a particular country to be correlated, buti t assumes zero correlation across countries. Finally, equation 6.7 has the following state-space structure:

$$Y_t = (X_t \cdot \Xi_t)\theta_t + \eta_t \qquad \eta_t = X_t u_t + E_t \tag{6.8}$$

$$\theta_t = (I - C)\bar{\theta} + C\theta_{t-1} + \upsilon_t \qquad \upsilon_t \sim N(0, B) \tag{6.9}$$

$$\bar{\theta} = P\mu + \varepsilon \qquad \varepsilon \sim N(0, \Psi) \tag{6.10}$$

where, $E_t \sim N(0, \Omega)$ and $\upsilon_t \sim N(0, \Sigma \otimes V)$. Moreover, $\bar{\theta}$ is the unconditional mean of θ_t; P, C are known matrices; η_t and ε are mutually independent and independent of E_t and u_t; and B is a block diagonal matrix, with $B = diag(\bar{B}_1, \ldots, \bar{B}_F)$. Let $\Omega = \Sigma \otimes I$ and $V = \sigma^2 \cdot I_k$, where V is a $(k \cdot k)$ matrix and Ω is a $(NM \cdot NM)$ matrix. Here, σ^2 is known and $B_f = b_f \cdot I$, where $f = 1, \ldots, F$ and b_f controls the tightness of factor i in the coefficients.

The intuition behind this specification is simple. The factors obey the stochastic restrictions implied by equation 6.9 permitting time variations. In the model 6.8, it is assumed a general AR() structure. Since the matrix C is arbitrary, many posterior are allowed in the specification. Although, C is treated as fixed, hence it is possible to make it function of a small set of hyperparameters whose posterior can be jointly obtained with one of the other parameters. This approach is not followed here since that a choice joins the computational costs and that a near random walk specification for θ_t is for all purposes satisfactory. Moreover, the spherical assumption on V reflects the fact that factors are measured in common units. The block diagonality of B is needed to guarantee the orthogonality of the factors, which is preserved a-posterior and, hence, their identifiability. The assumption of $\Omega = \Sigma \otimes I$ is standard (see e.g., Kadiyala and Karlsson, 1997). In this way, prior assumptions can be specified and, hence, Bayesian computations are feasibles. Further, the factors θ_{it} drive the coefficients vector δ_t. The idea is to shrink δ_t into θ_{it} obtaining a much smaller dimensional vector. Ξ_j are matrices with elements equal to zero or one. Finally, independence among the errors is standard. To be more precise, E_t, u_t, η_t, and ε are assumed to have normal distribution, but it easy to allow for fat tails if non-normal observations are presumed to be present.

For example, let $(u_t | X_t \Xi) \sim N(0, (X_t \Xi)(\Omega \otimes V))$, where $X_t^{-1} \sim \chi^2(\nu, 1)$. This latter holds since, unconditionally, $u_t \sim t_\varepsilon(0, \Omega \otimes V)$. Nevertheless, by construction, the forecast errors of specified SUR model already display fat tail distributions, even when all disturbances are normal. Hence, this extension will not be considered here. Further complication allowing, for example, for skewness in the errors or for time variations in the variance of shocks to the factors are easy to introduce (see e.g., Canova, 1993, and Fernandez and Steel, 1998). All of these additions go in the directions of capturing non-normal patterns in Y_t, if this is needed. Numerous specifications are nested in the model 6.8. For example, a factor is time invariant when $B_{it} = 0$ and the appropriate elements of C are set to zero. No exchangeability obtains

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when Ψ is large; whereas, exact pooling obtains when $\Psi = 0$ and the factorization becomes exact when $\sigma^2 = 0$.

6.2   Bayes Factor and Model Selection

According to the factorization in equation 2.4, the type of factors δ_t depends on the nature of the problem. Nevertheless, one may be interested in having a method to statistically determine the number of indices needed to capture the heterogeneities present across time, units, and variables in the multicountry VAR, or to verify general hypotheses on the type of indices to be included. In order to discriminate across models with different indices, the (conditional) marginal likelihood for a h generic index can be defined as:

$$L(Y^T|M_h) = \int \mathscr{F}(Y^T|\phi_h, M_h)p(\phi_h|M_h)d\phi_h \tag{6.11}$$

where, Y_T denotes the data, $P(\phi_h|M_h)$ is the prior density for ϕ in model M_h, and $\mathscr{F}(Y_T|\phi_h, M_h)$ is density of the data under the parameterization produced by M_h. Equation 6.11 is conceptually simple, but can be evaluated analytically only in few elementary cases. More often, it is intractable and must be computed by numerical methods, using the output of the MCMC sampler, as suggested by Raftery *et al.* [37], Chib [15], and Chib [13].

Given, the complexity of the model, these numerical computation are not entirely straindht-forward. As an alternative, one can rely on asymptotic (normal) approximations to 6.11, for example Laplace's method, which takes a second-order expansion of 6.11 around the model (or the Schwarz criterion) which expands 6.11 around the maximum-likelihood estimator. Since in hierarchical models, asymptotic normality might not be a sensible approximation. Thus, a good idea is to compute alternative measures of marginal likelihood before taking decisions about the size of h. Once the marginal likelihood is obtained for any model h, the Bayes factor is:

$$B_{hh} = \frac{L(Y_T|M_h)}{L(Y_T|M_h')} \tag{6.12}$$

It can be used to decide whether M_h or M_h' fits the data better. Since marginal likelihoods can be decomposed into the product of one-step ahead predictive record. Moreover, since the ML implicitly discounts the performance of models with a larger number of indices, the equation 6.13 directly trades off the predictive record with the dimensionality of the model. By equation 6.11, it is also possible to conduct useful specification searches. For example, it is possible to examine whether the fatorization in equation 2.4 is exact, letting ϕ_h unrestricted and $\phi_h' = (\dots, \sigma^2 = 0, \dots)$; or whether there are time variations in θ_t, letting ϕ_h be unrestricted and $\phi_h' = (\dots, b_f = 0, \dots)$ for some f. Finally, support for the presence

of interdependencies is obtained by comparing the marginal likelihoods of the unrestricted model and that of a vector of country-specific time-varying VARs.

Instead of examining hypotheses on the structure of the model, one may want to incorporate model uncertainty directly into posterior estimates. Let M_1 be the model with one index, M_h the model with h indices, with $h=2,\ldots,H$, and that the Bayes factor B_{h1} for each M_h is computed. The posterior probability of model h is:

$$p(M_h|Y_T) = \frac{\beta_{it}B_{h1}}{\sum_{h=2}^{H}\beta_h B_{h1}} \qquad (6.13)$$

where, β_h are the prior odds for M_h, and model uncertainty can be accounted for weighting $G(\phi_h)$ by $p(M_h|Y^T)$, with G() denotes the Gamma distribution.

Given the SUR model in equation 6.8, prior densities are assumed for $\phi_0 = (\Omega^{-1}, \mathcal{B}, \theta_0)$, the factorization is exact (for example, $\delta_t = \Xi_t\theta_t$), $C = I$, $\Psi = 0$, so that hierarchical prior with exchangeability are allowed, and b_f controls the tightness of each factor in the coefficients (e.g., $X_t\Xi$). In this way, the equation 6.9 becomes:

$$\theta_t = \theta_{t-1} + \upsilon_t \qquad \upsilon_t \sim N(0,\mathcal{B}) \qquad (6.14)$$

The random-walk assumption is very common in the time-varying VAR literature and has the advantage of focusing on permanent shifts and reducing the number of parameters in the estimation procedure[2].

6.3   Prior Assumptions

Let $\phi_0 = (\Phi_{-1}, b_f, \theta_0)$ to be the prior densities, three tentative beliefs (assumptions) can be defined accounting for the model described in equation 6.7. (i) Conditional Normality: $p(\eta|\phi_0) = N(0,\Phi)$. This is a hierarchical prior for η. (ii) Conditional Independence: $p(\eta|\phi_0) = p(\eta|\phi_0)p(\chi|\phi_0)$. (iii) Exogeneity: $p(\chi|\phi_0) = p(\chi)$. With these assumptions, the likelihood function is:

$$p(Y,\chi|\phi_0) = p(Y|\phi_0)p(\chi) \propto p(Y|\phi_0) \propto (\Omega)^{-\frac{n}{2}} \cdot exp[-\frac{1}{2}(Y-\chi\theta)^{'}(\Omega)^{-1}(Y-\chi\theta)] \qquad (6.15)$$

Running the likelihood function, the estimated model in 6.8 is:

$$Y = \chi\hat{\theta} + \eta \qquad (6.16)$$

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[2]See e.g., Primiceri (2005) for a discussion on alternative specifications.

By equation 6.16, the likelihood in equation 6.15 can be developed as:

$$(\Omega)^{-\frac{n}{2}}exp[-\frac{1}{2}(Y-\chi\hat{\theta})'(\Omega)^{-1}(Y-\chi\hat{\theta})] = (\Omega)^{-\frac{n}{2}}exp\{-\frac{1}{2}[(\chi\theta+\eta)-\chi\hat{\theta}]'\Omega^{-1}\cdot$$
$$\cdot[(\chi\theta+\eta)-\chi\hat{\theta}]\} \tag{6.17}$$

By completing the square, equation 6.18 can be re-written for convenience:

$$(\Omega)^{-\frac{n}{2}}exp[-\frac{1}{2}\chi'(\theta-\hat{\theta})'+\eta'\Omega^{-1}\eta+\chi(\theta-\hat{\theta})]$$
$$= (\Omega)^{-\frac{v}{2}}(\Omega)^{-\frac{k}{2}}exp\{-\frac{1}{2}[(\theta-\hat{\theta})'\chi'\Omega^{-1}\chi(\theta-\hat{\theta})+vS]\}$$
$$= (\Omega)^{-\frac{k}{2}}exp[-\frac{1}{2}[(\theta-\hat{\theta})'\chi'(\Omega)^{-1}\chi(\theta-\hat{\theta})]\}(\Omega)^{-\frac{v}{2}}exp\{-\frac{vS}{2}(\Omega)^{-1}\} \tag{6.18}$$

where, $\hat{\theta}=(\chi'\chi)^{-1}\chi'Y$, $S=(\frac{1}{v})(Y-\chi\hat{\theta})'(Y-\chi\hat{\theta})$, and $v=n-k$. In the following manner and perspective, it is easy to notice that: $(\Omega)^{-\frac{k}{2}}exp\{-\frac{1}{2}[(\theta-\hat{\theta})'\chi'(\Omega)^{-1}\chi(\theta-\hat{\theta})]\}$ is the kernel of a $N(\hat{\theta},\Omega(\chi'\chi)^{-1})$ and $(\Omega)^{-\frac{v}{2}}exp\{-\frac{vS}{2}(\Omega)^{-1}\}$ is the kernel of an $IG(\frac{v-2}{2},\frac{vS}{2})$.

A hierarchical prior for η has been already specified. Thus, in order to complete the model, a prior moments on (χ_0,Ω^{-1},b_f) need to be defined. It is just viewed that the likelihood function can be derived from the sampling density $p(Y|\phi_0)$, thus it is considered as a function of the parameters. To be more precise, it can be shown to be of a form that breaks into three parts. (i) A distribution for factors θ given Ω. (ii) A distribution where Ω^{-1} has a Wishart distribution. (iii) A distribution for b_f, where $b_f=vec(\mathcal{B})$ has a Inverse Gamma distribution[3]. That is:

$$\theta|\Omega,Y \sim N(\hat{\theta},\Omega\otimes(\chi'\chi)^{-1}) \tag{6.19}$$

$$\Omega^{-1}|Y \sim W(S^{-1},T-K-M-1) \tag{6.20}$$

$$b_f|Y \sim IG(\frac{\bar{\omega}}{2},\frac{vS}{2}) \tag{6.21}$$

Furthermore, such prior assumptions will generally be influenced, for example, by common or subjective beliefs about marginal effects of economic variables. Hence, Independent Normal Wishart Prior is used in this analysis, since it assumes that tentative beliefs on $(\theta_0,\Omega^{.1},b_f)$ derive from separate considerations[4].

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[3]See Appendix C for more details
[4]See Appendix B for discussion and relantionship with the literature

6.4 Inference

Rearranging equations 6.8 and 6.14, the SUR model described in equation 6.7 can be easily re-written as:

$$Y_t = (X_t \cdot \Xi_t)\theta_t + \eta_t \qquad \eta_t = X_t u_t + E_t \qquad (6.22)$$

$$\theta_t = \theta_{t-1} + v_t \qquad v_t \sim N(0, \mathcal{B}) \qquad (6.23)$$

In order to conduct inference, letting $\theta_t = \theta \; \forall t$, the estimation is easy since it only requires regressing each element of Y_t on appropriate averages, adjusting estimates of the standard errors for the presence of heteroschedasticity. With a prior for θ, posterior estimates would be straightforward to construct. However, when the θ_t‚Äôs are time-varying, with $\mathcal{B}_{it} \neq 0$, MCMC methods can be employed to construct their exact posterior distributions. Let data run from $(-\tau, T)$, where $(-\tau, 0)$ is a training sample used to estimate features of the prior. When such a sample is unavailable, it is sufficient to modify the expressions for the prior moments in equations 6.19, 6.20, and 6.21 as:

$$p(\Omega^{-1}, b_f, \theta_0) = p(\Omega^{-1})\Pi_f p(b_f) p(\theta_0) \qquad (6.24)$$

where, $p(\Omega^{-1}) = W(\theta_1, z_1)$, $p(b_f) = IG(\frac{\bar{\omega}_0}{2}, \frac{S_0}{2})$, and $p(\theta_0|\mathscr{F}_{-1}) = N(\bar{\theta}_0, \bar{R}_0)$. Here, $N()$ stands for Normal, $W()$ for Wishart, and $IG()$ for Inverse Gamma distributions; while $\mathscr{F}_{-1}$ denotes the information available at time *-1*. The prior for θ_0 and the law of motion for the factors imply that $p(\theta_t|\mathscr{F}_{-1}) = N(\bar{\theta}_{t-1|t-1}, \bar{R}_{t-1|t-1} + \mathcal{B}_t)$, where $\bar{\theta}_{t-1|t-1}$ and $\bar{R}_{t-1|t-1}$ are, respectively, the mean and the (variance)-covariance matrix of the conditional distribution of $\bar{\theta}_{t-1|t-1}$. The hyperparameters are all known[5]. To be more precise, collecting them in a vector α, where $\alpha = (z_1, Q_1, \bar{\omega}_0, S_0, \bar{\theta}_0, \bar{R}_0)$, they are treated as fixed and are either obtained from the data (this is the case for $\bar{\theta}_0$ and Q_1) to tune the prior to the specific applications or selected a-priori to produce relatively loose priors (this is the case for z_1, $\bar{\omega}_0$, S_0, $\bar{R}_0$). The value used are: $z_1 = N \cdot M + 7$, $Q_1 = \hat{Q}_1$, $\bar{\omega}_0 = 10^7$, $S_0 = 1.0$, $\bar{\theta}_0 = \hat{\theta}_0$, and $\bar{R}_0 = I_f$. Here, $\hat{Q}_1$ is a block diagonal matrix, where $\hat{\theta}_1 = diag(Q_{11}, \ldots, Q_{1N})$ and Q_{1i} is the estimated covariance matrix of the time invariant version for each country VAR, and $\hat{\theta}_0$ is obtained with the OLS on a time invariant version of equation 6.7.

The posterior distributions for $\phi = (\Omega^{-1}, b_f, \{\theta_t\}_{t=1}^T)$ are calculated combining the prior with the (conditional) likelihood on initial conditions of the data, which is proportional to:

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[5]For instance, prior hyper-parameters are own computations. See Appendix B for detailed empirical evidence.

$$L(Y^T|\phi) \propto (\Omega)^{-\frac{T}{2}} exp\{-\frac{1}{2}[\Sigma_t(Y_t - (X_t\Xi)\theta_t)^{'}\Omega^{-1}\Sigma_t(Y_t - (X_t\Xi)\theta_t)]\} \qquad (6.25)$$

where, $Y^T = (Y_1, \ldots, Y_T)$ denotes the data, $\phi = (\Omega^{-1}, b_f, \{\theta_t\})$ the unknowns whose joint distribution needs to be found, with ϕ_{-k} standing the vector ϕ excluding the parameter k.

6.5   Posterior Distributions and MCMC Methods

Given a prior $p(\phi)$, according to the Bayes rule, the conditional posterior $p(\phi|Y^T)$ is proportional to:

$$p(\phi|Y^T) = \frac{p(\phi)L(Y^T|\phi)}{p(Y^T)} \propto p(\phi)L(Y^T|\phi) \qquad (6.26)$$

Given $p(\phi|Y^T)$, the posterior distribution for the elements of ϕ can be obtained by integrating out nuisance parameters from $p(\phi|Y^T)$. Once these distributions are found, location and dispersion measures can be obtained for ϕ or for any interesting continuous function of these parameters. Despite the dramatic parameter reduction obtained with equation 6.7, analytical computation of posterior distributions $p(\phi|Y^T)$ is unfeasible. However, through Monte Carlo techniques, a variant of the Gibbs sampler approach can be used in this framework it only requires knowledge of the conditional posterior distribution of ϕ. Thus, the posterior distributions for ϕ are:

$$\theta_t|Y^T, \phi_{-\theta_t} \sim N(\tilde{\theta}_{t|T}, \tilde{R}_{t|T}) \qquad with \qquad t \leq T \qquad (6.27)$$

$$\Omega|Y^T, \phi_{-\Omega} \sim iW(\hat{z}_1, \hat{\theta}_1) \qquad (6.28)$$

$$b_f|Y^T, \phi_{-b_f} \sim IG(\frac{\bar{\omega}_f}{2}, \frac{\bar{S}}{2}) \qquad (6.29)$$

where, $\tilde{\theta}_{t|T} = \tilde{R}_{t|T}(\bar{R}_0^{-1}\bar{\theta}_0 + \Sigma_t(X_t\Xi)^{'}\Omega^{-1}Y_t)$ and $\tilde{R}_{t|T} = (\bar{R}_0^{-1} + (X_t\Xi)^{'}\Omega^{-1}(X_t\Xi))^{-1}$, with $\tilde{\theta}_{t|T}$ and $\tilde{R}_{t|T}$ denoting the smoothed one-period-ahead forecasts of θ_t and of the variance-covariance matrix of the forecast error, respectively; $\hat{z}_1 = z_1 + T$ and $\hat{\theta}_1 = [\theta_1 + (Y_t - (X_t\Xi)\theta_t)^{'}\Omega^{-1}(Y_t - (X_t\Xi)\theta)]^{-1}$; $\bar{\omega}_f = K + \bar{\omega}_0$ and $\bar{S} = S_0 + \Sigma_t(\theta_t^f - \theta_{t-1}^f)^{'}(\theta_t^f - \theta_{t-1}^f)$, with θ_t^f denoting the fth subvector of θ_t, $K = NM$, and f the factors described in equation 2.5.

The conditional posterior of $(\theta_1, \ldots, \theta_T|Y^T, \phi_{-\theta_t})$ can be obtained with a run of the Kalman filter and of a simulation smoother as in Chib and Greenberg [25]. To be more precise, the Kalman (1960, 1963) filter technique is adopted to estimate linear regression

models with time-varying coefficients . This class of models consists of two equation. (i) The transition equation, describing the evolution of the state variables. (ii) The measurement equation, describing how the observed data are generated form the state variables. This approach is extremely useful for investigating the issue of parameters constancy, because it is an updating method producing estimates for each time period based on the observations available up to the current period. It is important to realise that recursive OLS estimation is not a suitable technique to use here. Recursive estimation is essentially a test of structural stability. For example, given H_0 be the null hypothesis that the parameters are constant and H_1 be that alternative that the parameters are estimated through recursive estimation. But as the underlying assumption of OLS is always that the parameters are constant, recursive estimation does not provide a consistent estimate of a time-varying parameters. In particular, given $\theta_{0|0}$ and $R_{0|0}$, the Kalman filter gives the recursions[6]:

$$\theta_{t|t} = \tilde{\theta}_{t-1|t-1} + [\tilde{R}_{t|t-1}(X_t\Xi)F_{t|t-1}^{-1}][Y_t - (X_t\Xi)\theta_t] \tag{6.30}$$

$$R_{t|t} = [I - \tilde{R}_{t|t-1}(X_t\Xi)'F_{t|t-1}^{-1}(X_t\Xi)](\tilde{R}_{t-1|t-1} + \mathcal{B}) \tag{6.31}$$

$$F_{t|t-1} = (X_t\Xi)'\tilde{R}_{t|t-1}(X_t\Xi) + \Omega_t \tag{6.32}$$

Hence, in order to obtain a sample $\{\theta_t\}$ from the joint posterior distribution $(\theta_1, \ldots, \theta_T | Y^T, \phi_{-\theta_t})$, the output of the Kalman filter is used to simulate θ_T from $N(\theta_{T|T}, R_{T|T})$, θ_{T-1} from $N(\theta_{T-1}, R_{T-1})$, and θ_1 from $N(\theta_1, R_1)$, where:

$$\theta_t = \theta_{t|t} + R_{t|t} \cdot R_{t+1|t}^{-1} \cdot (\theta_{t+1} - \theta_{t|t}) \tag{6.33}$$

$$R_t = R_{t|t} - R_{t|t} \cdot R_{t+1|t}^{-1} \cdot R_{t|t} \tag{6.34}$$

The recursion can be started choosing $R_{0|0}$ to be diagonal with elements equal to small values, whereas $\theta_{0|0}$ can be estimated in the training sample or initialized using a constant coefficient version of the model.

Under regularity conditions (see e.g., Geweke [23]), cycling through the conditional distributions in equation 6.27 in the limit it produces draws from the joint posterior of interest. In fact, convergence only requires the algorithm to be able to visit all partitions of the parameter-space in a finite number of iterations. Thus, the marginal distributions of θ_t can be computed averaging over draws in the nuisance dimensions and the posterior distributions of indicators can be obtained. A credible 95% interval for every indicator described in equation 2.5 is obtained ordering the draws of $\chi_{it}\theta_{it}$ for each t and taking the *5th* and the *95th* percentile of the distribution.

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[6]For instance, see the dynamic analysis described in Chapter 7

However, the regressors of the SUR model in equation 6.22 are correlated, but the presence of correlation, even of extreme form, does not create problems in identifying the loading as long as the priors are proper (see e.g., Ciccarelli and Rebucci [12]). In addition, the choice of making E_t and u_t correlated allows conjugation between the prior and the likelihood, avoids identification issues, and greatly simplifies the computation of the posterior. This latter is also used in Minnesota prior (see e.g., Doan *et al.*, 1984). Hence, as stated in Section 6.3, the forecast error $\eta = Y_t - (X_t\Xi)\theta_t$ has the form $(\eta|\sigma^2) \sim N(0, \sigma_t\Omega)$. Therefore, unconditionally, η_t has a multivariate t-distribution centered at *0*, with a scale matrix proportional to Ω and ν_n degree of freedom. Thus, the innovations of the model described in equation 6.22 are endogenously allowed to have fat tails. Finally, since the fit improves when $\sigma^2 \to 0$, the model in equation 6.22 presents an exact factorization of δ_t.

In order to compute conditional heteroschedasticity in Y_t, Cogley and Sargent [40] specify Ω to be a function of a set of stochastic volatility processes. The above discussion shows that a similar result can be equivalently obtained with a simpler set of assumptions. The selection model in equation 6.22 appeals on another count. To be more precise, since shocks to the model may alter its dynamics, by construction, it has built-in an endogenous adaptive scheme that allows coefficients to adjust when breaks in the relantionship occur. Posterior distributions for any continuous function $G(\phi)$ can be obtained using the output of the MCMC algorithm and the ergodic theorem. For example, $E[G(\phi)] = \int G(\phi)p(\phi|Y_T)d\phi$ can be approximated using $\frac{1}{\bar{L}}[\Sigma_{l=\bar{L}+1}^{\bar{L}+L}G(\phi^l)]$, where the first $\bar{L}$ observations represent a burn-out sample discarded in the calculation. Predictive distributions for future Y_{it}'s can be estimated using the recursive nature of the model and the conditional structure of the posterior. Let $Y^{t+\tau} = (Y_{t+1}, \ldots, Y_{t+\tau})$, consider the conditional density of $Y^{t+\tau}$, given the data up to t, and a function $G(Y^{t+\tau})$, then:

$$F[G(Y^{t+\tau})|Y_t] = \int F[G(Y^{t+\tau})|Y^t, \phi] \; p(\phi|Y^t) \; d\phi \tag{6.35}$$

Here, forecasts for $Y^{t+\tau}$ can be obtained drawing $\phi^{(l)}$ from the posterior distribution and simulating the vector $Y^{t+\tau}$ from the density $F(Y^{t+\tau}|Y_t, \phi^{(l)})$. Turning point distributions can also be constructed by appropriately choosing G. Impulse responses and conditional forecasts can be obtained with the same approach as detailed in Chapter 7.

6.6   Variance Component Model

Considering the model with the following state-space structure:

$$Y_{it} = \gamma_{it} + T_t \qquad (1 - \rho_t L)T_t = e_t \tag{6.36}$$

$$\gamma_{it} = \gamma_i + \vartheta_{it} \qquad (1 - w_i L)\vartheta_{it} = (X\Xi)_{it} \tag{6.37}$$

$$\gamma_i = \gamma_0 + \epsilon \tag{6.38}$$

where, e_t is i.i.d. across t, ϑ_{it} is i.i.d. across t, and Y_{it} is a $NM \cdot 1$ vector for each $i=1,2,\ldots,$ N. This model has the following VAR representation[7]:

$$Y_t = \gamma_{0t}^* + A_t Y_{t-1} + B_t W_{t-1} + \eta_t = \gamma_{0t}^* + \delta_t(X_t\Xi) + \eta_t \tag{6.39}$$

where, $X_t = (Y_{t-1}', W_t', W_{t-1}', C_t', C_{t-1}')'$, Y_t is a $NM \cdot 1$ vector each t, $\gamma_{0t}^* = diag\{(1 - 1omega_i)\}(1 - \rho_t)\gamma_0$, $\eta_{it} = (1 - \omega_i L)e_t + (1 - \omega_i L)(1 - \rho_t L)\varepsilon + (1 - \rho_t L)(X\Xi)_{it}$, $A_{it} = \rho_t + \omega_i$, and $B_{it} = \rho_t + \omega_i$. Therefore, an error component model generates a particular error structure in the VAR. Note that γ_{0t}^* are time trends common to all the M variables for unit i. Thus, according to equation (2.6), $\delta_t = [vec(A_t), vec(B_t), vec(C_t)]$ is factorized as:

$$\delta_{timp} = \Xi_1\theta_{1t} + \Xi_2\theta_{2t} + \Xi_3\theta_{3t} + \Xi_4\theta_{4t} + \Xi_5\theta_{5t} + u_{timp}^\delta \tag{6.40}$$

where, θ_{1t}, θ_{2t}, θ_{3t}, and θ_{4t} are $NM_1 \cdot 1$ vectors of country-specific factors common to all lags p; $theta_{5t}$ is $M_1 \cdot 1$ vector of group-variable specific common to all variables m and lags p. Ξ_1, Ξ_2, Ξ_3, and Ξ_4 are matrices of dimensions $NMk \cdot 1$; Ξ_5 is matrix of dimension $NMk \cdot M_1$. Therefore, γ_{0t}^* is assumed to be:

$$\gamma_{0t}^* = \Xi_6\theta_{6t} + u_{pit}^\gamma \tag{6.41}$$

where, θ_{6t} is $NM{\cdot}1$ vector, and Ξ_6 is matrix of dimension $NMk{\cdot}1$. Equations6.40 and 6.41 represent a version of the model described in equation 2.6. Here, the number of parameters to be estimated is $NM + NM_1 + M_1$, which is still relatively large. To further reduce the dimensionality of the parameter vector one could make θ_{6t} time- or unit- independent and exploit averages in the remaining dimensions to construct the appropriate regressors. Disregarding how γ_{0t}^* is parametrized, the SUR model is:

$$(Y_t - \gamma_{0t}^*) = \theta_{1NM_1}\chi_{1t} + \theta_{2NM_1}\chi_{2t} + \theta_{3NM_1}\chi_{3t} + \theta_{4NM_1}\chi_{4t} + \theta_{5M_1}\chi_{5t} + \eta_t \tag{6.42}$$

where, χ_{1t}, χ_{2t}, χ_{3t}, and χ_{4t} are country-specific indices, χ_{5t} is a group-variable specific factors, $\chi_{6t} = (Y_t - \gamma_{0t}^*)$ denotes common and idiosyncratic components across all countries and variables, and η_t is composite error whose variance depends on group-specific factors, on a common index, on variable-specific effects, on the lags p, and on a time- or unit-independence index. Hence, the reparametrization maintains the original error component structure, but somewhat reduces the dimensionality of the parameters space.

---

[7]In this case, the multicountry selected VAR model corresponds to equation 2.2.

Chapter 7

Dynamic Analyses

Dynamic Analysis is non-standard in the SUR model as described in equation 6.22, because of the specification of the error term and the time variations potentially present in the coefficients. Hence, in the following sections, impulse responses and conditional forecasts are described providing statistics useful for academics and policymakers.

7.1 Impulse Responses

Impulse responses are generally computed as the difference between two realizations of $y_{t+\tau}$, with $\tau = 1, 2, \ldots$, which are identical up to time t. Thus, between $t+1$ and $t+\tau$, one can assume two time impulses in the *jth* component of $e_{t+\tau}$. (i) One that occurs only at time $t+1$. (ii) The other that no shocks take place at all dates between $t+1$ and $t+\tau$. In a model with time-varying coefficients, the approach is inadequate since it overlooks that between $t+1$ and $t+\tau$, structural coefficients may also change. Therefore, impulse responses are obtained as the difference between two conditional expectations of $y_{t+\tau}$. In both cases, they are conditioned on the history of the data Y_t and of the factors θ^t, the parameters of the law of motion of the coefficients, and all future shocks. However, impulse responses are conditioned on a random draw for the current shocks, whereas in the other the current shocks is set to its unconditional value. Hence, they are worked out on future shocks instead of integrating them out because, computationally, such a choice gives more stable responses, even though this makes standard error bands larger than in the case where future shocks are integrated out. Given the equation 6.22, one has two potential types of impulses. (i) One to the variables of the system. (ii) One to the factors.

Here, the reparametrized SUR is:

$$y_t = \chi_t \theta_t + (E_t + X_t u_t) \qquad with \qquad \theta_t = \theta_{t-1} + \upsilon_t \tag{7.1}$$

where $\theta_t = [\theta'_{1t}, \theta'_{2t}, \ldots, \theta'_{Ft}]'$, $\chi_t = [\chi_{1t}, \ldots, \chi Ft]$, $\chi_{it} = \Xi X_t$, $X_t = [Y_{t-1}, W_t, W_{t-1}]$. Let $\mho = [(E_t + X_t u_t)', v']'$ be the vector of reduced-form shocks and $Z_t = [H_t^{-1}(E_t + X_t u_t)', H_t^{-1} v'_t]'$ be the vector of structural shocks where $E_t = H_t v_t$, $H_t H'_t = \Omega$ so that $var(v_t) = \sigma^2 I_k$ and $H_t = J \cdot K_t$ where $K_t K'_t = I$ and J is a matrix that orthogonalizes the VAR shocks.

Here, a Choleski system is obtained setting $K_t = I$, $\forall t$, and choosing J to be lower triangular whereas more structural identification schemes are obtained letting J be an arbitrary square root matrix and K_t a matrix implementing certain theoretical restrictions. The identification matrix K_t is allowed to be time-varying since, when recursive estimations are used, estimates of Ω depends on t.

Let $\mathbb{Z}_t = (\Omega, \sigma^2, B_t, \Phi)$, let $\bar{Z}_{j,t}$ be a particular realization of $Z_{j,t}$ and $Z_{-j,t}$ indicate the structural shocks, excluding the one in the jth component. Let $F_t^1 = \{Y^{t-1}, \theta^t, \mathbb{Z}_t, H_t, Z_{j,t}\}$, with $Z_{j,t} = \{\bar{Z}_{j,t}, Z_{-j,t}\mho_{t+1}^{t+\tau}\}$, and $F_t^2 = \{Y^{t-1}, \theta^t, \mathbb{Z}_t, H_t, Z_{j,t}\}$, with $Z_{j,t} = \{EZ_{j,t}, Z_{-j,t}\mho_{t+1}^{t+\tau}\}$ be two conditioning sets. Thus, responses to a shock at t in the jth component of Z_t are obtained as:

$$IR(t, t+\tau) = E(Y_{t+\tau}|F_t^1 - E(Y_{t+\tau}|F_t^2) \qquad t = 1, 2, \ldots \qquad (7.2)$$

In order to see what definition equation 7.2 involves, rewrite the original VAR model 2.1 in a companion form[1]:

$$Y_{t+\tau} = A_{t+\tau} Y_{t+\tau-1} + B_{t+\tau} W_{t+\tau-1} + E_{t+\tau} \qquad (7.3)$$

and let

$$\delta_{t+\tau} = \Xi[\theta_{t+\tau-1} + v_{t+\tau}] + u_{t+\tau} \qquad (7.4)$$

where $\delta_{t+\tau} = [vec(A_{1t+\tau}), vec(B_{t+\tau})]$ and $A_{1t+\tau}$ is the first row of $A_{t+\tau}$. Taking $Y^{t-1} = (Y_{t-1}, Y_{t-2}, \ldots, W_{t-1}, W_{t-2}, \ldots,)$, $A^t = (A_t, A_{t-1}, \ldots)$, $B^t = (B_t, B_{t-1}, \ldots)$, and $H_{t+\tau} = H_t$ for $\forall \tau$ as given.

Solving backward, equations 7.3 and 7.4 can be rewritten as:

$$Y_{t+\tau} = (\prod_{k=0}^{\tau} A_{t+\tau-k})Y_{t-1} + B_{t+\tau}W_{t+\tau-1} + \sum_{h=1}^{\tau}(\prod_{k=0}^{h-1} A_{t+\tau-k})B_{t-\tau-h}W_{t+\tau-h-1} +$$

$$+ H_{t-\tau}\eta_{t+\tau} + \sum_{h=1}^{\tau}(\prod_{k=0}^{h-1} A_{t+\tau-k})H_{t+\tau-h}\eta t + \tau - h \qquad (7.5)$$

---

[1]The same computations are done for the model 2.2 accounting for exogenous variables Z_t

and as

$$\delta_{t+\tau} = \Xi\theta_{t-1} + \Xi\sum_{k=0}^{\tau} v_{t+\tau-k} + u_{t+\tau} \tag{7.6}$$

Consider first the case of a *(m+1)*-period impulse in the *jth* component of v. For example: $v_{j,t+k} = \bar{v}_{j,t+k}$; $v_{-j,t+k}$, *k = 0, 1, 2, ..., m* and $v_{t+m'}$, with $\forall m' > m$, are restricted. Then,

$$
\begin{aligned}
IR_{t,t+\tau} &= E_t[Y_{t+\tau}|Y_{t-1}, A^t, B^t, \mathbb{Z}_t, H_t, \{\bar{\eta}_{jt+m}\}_{k=o}^m, \{\eta_{-jt+k}\}_{k=0}^m, \{\eta_{t+k}\}_{k=m+1}^\tau] - \\
&\quad - E_t[Y_{t+\tau}|Y^{t-1}, A^t, B^t, \mathbb{Z}_t, H_t, \{\eta_{t+k}\}_{k=0}^\tau] \\
&= E_t[(\prod_{k=0}^{\tau-1})^j H_t^j(\bar{\eta}_{jt} - E\eta_{jt}) + (\prod_{k=0}^{\tau-2} A_{t+\tau-k})^j \cdot H_{t+1}^j(e\bar{t}a_{jt+1} - E\eta_{jt+1}) + \dots \\
&\quad \dots + (\prod_{k=0}^{\tau-m-1} A_{t+\tau-k})^j \cdot H_{t+m}^j(e\bar{t}a_{jt+m} - E\eta_{jt+m})] \tag{7.7}
\end{aligned}
$$

where the superscript j refers to the jth column of the matrix. It is easy to see that, when $A_t = A$ and $B_t = B$, $\forall t$, equation 7.7 reduces to standard impulse responses and, when E_t and v_t are correlated (that is both the sign and the size of the shocks matter a shock in v_t), may induce changes in A_t or B_t. Given 7.2, responses in the SUR model can be computed as follows:

1. Choosing t, τ, and J_t. Draw $\Omega^l = H_t^l(H_t^l)', (\sigma^2)'$ from their posterior distribution and u_t^l from $N(0, (\sigma^2)^2 I \otimes H_t^l(H_t^l)')$. Computing $y_t^l = \chi_t\theta_t + H_t\bar{\eta}_t + X_t u_t^l$.

2. Drawing $\Omega = H_{t+1}^l(H_{t+1}^l)', (\sigma^2)^l, \mathcal{B}_{t+1}^l, \phi^l$. Drawing η_{t+1}^l from their posterior distribution. Using the law of motion of the factors to compute θ_{t+1}^l, $l = 1, 2, ..., L$, and the definition of Ξ to compute χ_{t+1}. Drawing u_{t+1}^l from $N(0, (\sigma^2)^l I \otimes H_{t+1}^l(H_{t+1}^l)')$ and computing $y_{t+1}^l = \chi_{t+1}\theta_{t+1} + H_{t+1}\bar{\eta}_{t+1} + X_{t+1}u_{t+1}, l = 1, 2, ..., L$.

3. Repeating Step 2 and computing θ_{t+k}^l, θ_{t+k}^l, *k = 1, 2, ..., τ*.

4. Repeating Steps 1 - 3 by setting $\eta_{t+K} = E(\eta_{t+1})$, *k = 1, 2, ..., m* and using the draws for the shocks in 1 - 3.

Responses to structural shocks to the law of motion of the factors can be computed in the same way. An impulse in $v_t = \bar{v}$ lasting *(m+1)* periods implies from equation 7.6 that:

$$E(\bar{\delta}_{t+\tau} - \delta_{t+\tau}) = \Xi \sum_{k=0}^{m} H_{t+k}(\bar{\eta}_{t+\tau-k} - E\eta_{t+\tau} - k) \qquad (7.8)$$

and

$$IR_{t,t+\tau} = E_t[\prod_{k=0}^{\tau}(\bar{A}_{t+1,\tau-k} - A_{t+\tau-k})Y_{t+1} + \sum_{h=1}^{\tau}\prod_{k=0}^{h-1}(\bar{A}_{t+1,\tau-k} - A_{t+\tau-k}) \cdot B_{t+\tau-h-1} +$$

$$+ \sum_{h=1}^{\tau}\prod_{k=0}^{h-1}(\bar{A}_{t+1,\tau-k} - A_{t+\tau-k})H_{t+\tau-h}\eta_{t+\tau-h} \qquad (7.9)$$

7.2 Conditional Forecasts

There are two types of conditional forecasts one can compute in this framework. Those involving displacement of the exogenous variables W_t from their unconditional path, and those involving a particular path for a subset of the endogenous variables. Both types of conditional forecasts can be constructed using the output of the Gibbs sampler routine. Consider first displacing the exogenous variables from their expected future path for $m+1$ periods. Calling the new path $\bar{W}_{t+k}$, $k = 0, 1, \ldots, m$. Then, the response of $Y_{t+\tau}$ is:

$$IR_{t,t+\tau} = E_t[(\prod_{k=0}^{\tau-2} A_{t+\tau-k})B_{t+1}(\bar{W}_{jt} - W_{jt}) + (\prod_{k=0}^{\tau-3} A_{t+\tau-k})B_{t+2}(\bar{W}_{jt+1} - W_{jt+1}) + \ldots$$

$$\ldots + (\prod_{k=0}^{\tau-2-m} A_{t+\tau-k})B_{t+m+1}(\bar{W}_{jt+m} - W_{jt+m})] \qquad (7.10)$$

Thus, to compute conditional forecasts of this type in the SUR model, one need to:

1. Choosing t, τ, and a path $\{\bar{W}_{t+k}\}_{k=0}^{m}$. Drawing Ω^l, $(\sigma^2)^l$ from their posterior, drawing $E_t^l + X_t u_t^l$ and computing y_t^l.

2. Drawing $(B_t)^l$, Ψ^l from their posterior distribution; drawing v_{t+1}^l and using the law of motion of the factors to draw θ_{t+1}^l, $1, 2, \ldots, L$ and the definition of Ξ to compute χ_{t+1}. Then, $E_{t+1}^l + X_{t+1}U_{t+1}^l$ are drawn to compute $y_{t+1}^l = \chi_{t+1}\theta_{t+1}^l + (E_{t+1}^l + X_{t+1}u_{t+1}^l)$, $l = 1, 2, \ldots, L$.

3. Repeating Step 2 in order to compute θ_{t+k}^l, y_{t+k}^l, $k = 1, 2, \ldots, \tau$.

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4. Repeating Steps 1 - 3. In this way, it sets $W_{t+k} = E(W_{t+k})$, $k = 0, 1, \ldots, m$, using the draws for the shocks in 1 - 3.

Finally, considering the case in which the future path of a subset of Y_t's is fixed. For example, in a system with output growth, inflation, and the nominal rate, one would like to work out on a given path for the future interest rate. Hence, partioning $Y_t = A_t Y_{t-1} + BW_{t-1} + E_t$ in two blocks, let $Y_{2t+k} = \bar{Y}_{2t+k}$ be the fixed variables and Y_{1t+k} those allowed to adjust, the Impulse Responses are:

$$
\begin{aligned}
IR_{t,t+\tau} \quad = \quad & E[H_t^1 (\prod_{k=0}^{t-1} A_{t+\tau-k})^1 (\bar{\eta}_{2t} - \eta_{2t}) + H_{t+1}^1 (\prod_{k=0}^{t-2} A_{t+\tau-k})^1 (\bar{\eta}_{2t+1} - \eta_{2t+1}) + \ldots \\
& \ldots + H_{t+m}^1 (\prod_{k=0}^{t-1-m} A_{t+\tau-k})^1 (\bar{\eta}_{2t+m} - \eta_{2t+m})]
\end{aligned}
\tag{7.11}
$$

where $\bar{\eta}_{2t+k} = \bar{Y}_{2t+k} - A_{21,t+k} Y_{1t-k-1} - A_{22,t+k} Y_{2t-k-1} - B_{2t+k} W_{t+k-1}$ and the super-script *1* refers to the first row of the matrix. Hence, to compute this type of conditional forecasts one need to:

1. Partitioning $y_t = (y_{1t}, y_{2t})$, choosing t and a path $\{y_{2t+k}\}_{k=0}^{\tau}$. Using the model to solve for the $\bar{\eta}_{2t}$ that gives $y_{2t} = \bar{y}_{2t}$, backing out the implied y_{1t}^l once draws for E_{1t}^l, and computing u_t^l from their posterior distribution. Thus, *upsilon*$_{t+1}^l$ can de drawn using the law of motion of the factors to obtain θ_{t+1}^l, with $l = 1, 2, \ldots, L$, and the definition of Ξ to compute χ_{t+1}.

2. Using the model to solve for $\bar{\eta}_{2t}$ that gives $y_{2t+1} = \bar{y}_{2t+1}$, backing out the implied y_{1t+1}^l once draws for E_{1t+1}^l, and computing u_{t+1}^l as above. Hence, once can draw v_{t+2}^l, using the law of motion of the factors to compute θ_{t+2}^l, with $l = 1, 2, \ldots, L$, and the definition of Ξ to compute χ_{t+2}.

3. Repeating Step 2 and computing θ_{t+k}^l, y_{t+k}^l, $k = 2, 3, \ldots$.

4. Repeating Steps 1 - 3, once can set $\eta_{2t+k}^l = E(\eta_{t+k})$, $\forall k$ using the draws for the shocks in 1 - 3.

In Step 2 of all algorithms, it has implicitly assumed that selecting a path for the shocks does not alter neither the law of motion of the factors nor the beliefs about the true structural shocks . If this were not the case, an intermediate Step, where a run of the Kalman filter updates the information about the factors, needs to be used (for istance, see Section 6.5).

7.3 Recursive Unconditional Forecasts

Given the information at time t, unconditional forecasting exercises only require the computation of the predictive distribution of future observations. In some cases, recursive unconditional forecasts are needed, in which case the predictive density of future observations has to be constructed for every $t = \bar{t}, \ldots, T$ once recursive estimates of $p(\phi_h | Y^T)$ are computed[2]. These recursive distributions are straightforward to obtain (e.g., a MCMC routine need to be run for every t) and, although computationally demanding, they are feasible on available machines.

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[2]See for istance Section 6.2 according to the Bayes factor

Chapter 8

Summary Statistics

Before proceeding to the empirical analysis, it is useful to identify possible festures and some stylized facts about the variables in the system. In particular, the project is interested in the degree of commonality and heterogeneity across countries and over time, focusing on the current financial crisis and fiscal consolidation effects. The analysis would test for co-movements and interdependence in order to highlights causality, catching-up, and competitiveness effects when studying real and financial linkages inter-countries within a common-currency area.

8.1   Evaluating trends and changes of the data

A first approach for identifying the degree of heterogeneity would be by just looking at the first and second moments of the data. Thus, mean and standard deviation are shown below for each variable in the system, including the observable variable, trade and capital flows, for the periods 'before crisis' and 'crisis period' and, hence, from 1998 to 2006 and from 2007 to 2014, respectively. It is interesting to divide the 'crisis period' in two subperiods: from 2007 to 2011 and from 2011 to 2014. The aim of this analysis is one to observe the effects of fiscal consolidation on real and financial sectors; what componet, between heterogeneity and commonality, seems to prevail during the two subperiods; in which sector, each of components shows the biggest size; and why.

In the Section 8.1.1, real and financial variables are observed during the two periods: from 1999 to 2006 and from 2007 to 2014. The Section 8.1.2 shows the other two selected periods spinning from 2007 to 2011 and from 2011 to 2014. Mean and standard deviation are computed in order to study how average and dispersion changed over time.

8.1.1 Before Crisis vs. Crisis Period

In Table 8.1, real variables are taken into account (general government spending [*gov*], GDP growth rate [*gdpg*], and gross fixed capital [*cap*]). All variables decrease during the crisis period, excepting Germany for the variables *gov* and *gdpg*. France, Belgium, and Finland show a small increase in the variable *cap*. The dispersion around the mean show a small increase, except for the variable *gdpg* remaining almost equal. It could reflect the inherent catching-up effect and, hence, the possibility to neglect important economic and institutional factors in driving the transmission of a shock.

Accounting for the coefficient of variation[1] (CV), mostly countries show an increase of index, except for the variable *gov*, proving the presence of omitted factors and a strong common component in government spending across countries. At the same time, the dispersion index highlights the presence of deeper heterogeneity during the crisis period, except in variable *gov* for the presence of commonality after-specified.

For example, paying attention to the variable *gdpg*, Spain, Portugal, Ireland, Italy, and Greece (PIIGS) show the highest values of CV during crisis period. In the variable *cap*, Ireland, Spain, Greece, and Portugal have values bigger than the average, following other smaller open economies as Netherlands, Finland, and Belgium in order of size. In *gov*, Germany show the highest value of the CV than the rest of the EA12. This results seems to be caused by the trade deep exposures that Germany has with respect to the other countries. In general, the dispersion index is smaller than one in the 'before crisis' period and, hence, highlighting the strong interdependence between real and financial dimension[2].

In Table 8.1, the above-mentioned results are shown.

---

[1]The coefficient of variation is a normalized measure of dispersion of a probability distribution, also known as the variation coefficient. It is defined as the ratio of the standard deviation to the mean.

[2]The variable *gov* is computed accounting for Financial accounts for general government spending weithed for the GDP

Table 8.1: Real Variables

Country	GOV				GDPG				CAP			
	1998 - 2006		2007 - 2014		1998 - 2006		2007 - 2014		1998 - 2006		2007 - 2014	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
IT	-0.029	0.033	-0.033	0.027	0.010	0.006	0.001	0.008	0.207	0.005	0.194	0.014
ES	0.000	0.023	-0.072	0.064	0.019	0.003	0.000	0.009	0.272	0.019	0.233	0.046
FR	-0.027	0.039	-0.050	0.035	0.010	0.003	0.004	0.007	0.188	0.006	0.205	0.005
BE	-0.007	0.058	-0.032	0.055	0.011	0.005	0.006	0.009	0.203	0.009	0.208	0.009
NL	-0.006	0.026	-0.030	0.034	0.013	0.006	0.003	0.010	0.203	0.015	0.183	0.016
AT	-0.019	0.039	-0.026	0.035	0.010	0.005	0.006	0.008	0.227	0.010	0.211	0.005
DE	-0.025	0.026	-0.011	0.023	0.006	0.006	0.006	0.012	0.190	0.017	0.178	0.006
FI	0.036	0.039	-0.003	0.069	0.011	0.008	0.005	0.016	0.196	0.006	0.199	0.011
LU	0.023	0.035	0.009	0.031	0.022	0.024	0.010	0.021	0.216	0.019	0.192	0.018
IE	0.020	0.050	-0.115	0.106	0.025	0.023	-0.003	0.028	0.240	0.022	0.154	0.059
GR	-0.061	0.105	-0.100	0.054	0.015	0.011	0.002	0.011	0.217	0.011	0.159	0.031
PT	-0.042	0.032	-0.061	0.039	0.011	0.007	0.001	0.011	0.250	0.021	0.191	0.028
Average	**-0.011**	**0.042**	**-0.044**	**0.048**	**0.014**	**0.009**	**0.003**	**0.012**	**0.217**	**0.013**	**0.192**	**0.021**

Mean and standard deviation for each real variable in the system for the period 1998 - 2006 and 2007 - 2014. The bold row shows the average for each column. The government spending (*gov*) denotes all financial accounts weighted for GDP, the GDP growth rate (*gdpg*) is weighted for the population for each country, and the gross fixed capital formation (*cap*) consists of resident producers' acquisitions, less disposals, of fixed assets during a selected period plus certain additions to the value of non-produced assets. This latter, also known as Investment, includes acquisition less disposals of buildings, structures, machinery and equipment, computer software, and so on.

Graphically, containing the analysis for the whole EA, the variables show the following trends and changes over time.


Figure 8.1: All variables are computed accountig for the Euro Area. Productivity in the first difference denotes the output growth not autocorrelated in order to deal with the problem of omitted variables.

In the period 2007 - 2014, all variables show a worsening of their trends after the monetary union period (1999 - 2001). The productivity in the first difference (*Dprod*) shows larger values during financial crisis proving the presence of potential latent variables affecting the transmission of a shock on the productivity growth. Today, the level tends to be almost equal to the period 1999 - 2001.

In Table 8.2, financial variables are shown (interest rate [*int*], general government debt [*debt*], and general government surplus/deficit [*curr*]). The variable *int*, following restrictive fiscal actions during the post-2008 the economic recovery, decreased in order to bring the economy to sustainable growth in a context of price stability. Italy, Spain, Ireland, Greece, and Portugal show larger values. The dispersion increse twice over proving the presence of heterogeneity and the important of economic and institutional implications between countries. The variable *debt* deeply increase during financial crisis accounting for a general *cause-effect* relationship after-specified.

The variability of coefficient increases during 'crisis period' in all variables. In *int*, Germany, Finland and other peripherical countries (as Irelan, Greece, and Portugal) show values bigger than the average. Italy, France and other smaller open economies (as Belgium and Austria) show smaller variation of coefficients during crisis. The result would prove the presence of more accommodating recovery actions in the first four years of the current crisis. In variable *debt*, the variation in CV bigger than two percentage points is observed in Greece, Italy, Finland, Ireland, and Netherlands. Belgium, Germany, Austria, and France shows the smallest increasing in CV accounting for size. The variable *curr* shows smaller CV during 'crisis period'. Germany, Netherlands, France, and Portugal show the smallest dispersion in the current crisis. Moreover, some countries as Greece, Italy, Belgium, Portugal, and Ireland even if observing a decreasing in CV during 'crisis period' show the biggest debt in percentage of GDP with respect to the resto of EA12. These results would highlight the high degree of heterogeneity and interdependence across countries emphasized by financial crisis and fiscal actions. For example, a direct recovery-action or shock affecting the countries' current price get effects in real and financial dimension, contemporaneously, across to transmission channels. Thus, spillovers are caused in both sectors (interdependence), but in different ways and with different outcomes on the variables of the system.

Table 8.2: Financial Variables

	INT				DEBT				CURR			
	1998 - 2006		2007 - 2014		1998 - 2006		2007 - 2014		1998 - 2006		2007 - 2014	
Country	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
IT	4.580	0.687	4.678	0.642	108.546	2.254	117.746	9.389	-0.356	1.181	-1.707	2.065
ES	4.466	0.743	4.679	0.709	50.543	6.528	60.296	19.817	-4.909	2.169	-4.732	4.087
FR	4.403	0.678	3.338	0.789	61.989	3.813	80.107	10.368	0.981	1.266	-1.514	0.968
BE	4.504	0.743	3.679	0.768	102.621	7.483	96.871	6.582	2.752	2.601	-0.482	4.409
NL	4.406	0.696	3.153	0.953	52.700	2.806	61.046	9.376	5.141	3.104	7.582	2.818
AT	4.467	0.740	3.360	0.905	68.611	2.511	70.418	4.892	1.066	2.761	3.068	2.363
DE	4.319	0.649	2.834	1.045	63.779	3.694	74.750	6.310	2.103	3.012	6.811	1.244
FI	4.452	0.735	3.155	0.995	42.182	2.401	44.304	8.290	6.031	3.113	0.861	3.198
LU	3.952	1.111	3.295	1.157	6.100	0.291	16.511	6.329	10.422	6.505	6.904	5.280
IE	4.443	0.751	5.624	1.960	32.557	4.941	77.443	36.416	-1.163	1.825	-0.057	4.822
GR	4.875	0.966	10.267	6.616	103.357	4.234	137.721	23.433	-7.197	4.327	-9.075	7.317
PT	4.526	0.739	6.519	2.811	58.911	6.366	94.218	22.980	-9.175	2.260	-7.543	5.118
Average	**4.450**	**0.770**	**4.548**	**1.612**	**62.658**	**3.943**	**77.619**	**13.682**	**0.475**	**2.844**	**0.010**	**3.641**

Mean and standard deviation for each financial variable in the system for the period 1998 - 2006 and 2007 - 2014. The bold row shows the average for each column. The interest rate (*int*) denotes EMU convergence criterion series related to interest rates for long-term government bonds denominated in national currencies, the general government debt (*debt*) are measured in euro and presented as a percentage of GDP, and the general government surplus/deficit (*curr*) surplus is defined in the Maastricht Treaty as general government net borrowing/lending. The series are measured in euro and presented as a percentage of GDP.

The Picture 8.2 shows trends and changes occurred in the euro area. The unemployment rate (*unem*), matching it with the vairable *debt* would betray a first approach to identify common features and *cause-effect* relationships across countries in the system.

Financial Variables
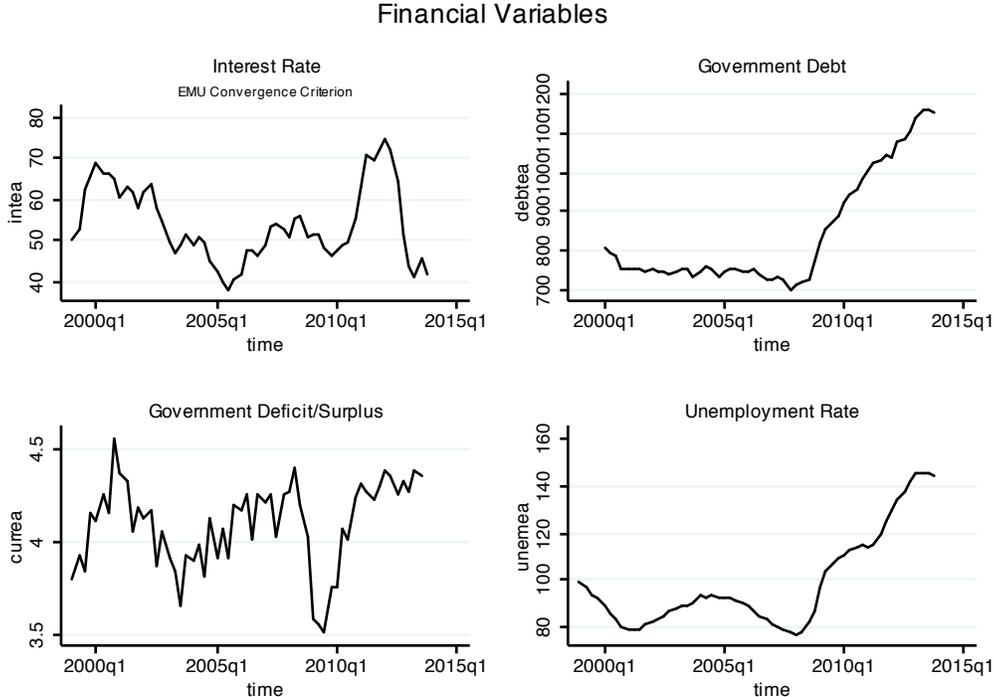
Figure 8.2: All variables are computed accountig for the Euro Area and presented as a percentage of GDP, except the interest rate which is taken in levels. The variable (*unem*) is computed accounting for sex and age groups.

In Table 8.3, shows trade and capital flows (*real* and *fin*, respectively) and the (directly) observable variable (*prod*). The output gap for the periods 1998-2006 and 2007-2014 increase in all countries, except Germany showing a slight improvement by previous (restrictive) deflazionary fiscal actions. On average, the variable *real* increased in mostly countries proving the strong interdependence across countries within a common-currency area. On the contrary, capital flows decreased due to strong losses in competitiveness inter-countries. This findings will be heavily discussed in the next section.

Accounting for the CV, the variable *fin* shows higher values than the variable *real*. This result seems to prove a deeper common component in driving shocks through trade flows. In Germany, Luxembourg, and Greece, the CV in trade flows decreases during the current crisis. It shows the presence of large trade exposures across countries and, hence, strong interdependence and commonality affecting the transmission of a shock. In fact, the variables *prod* and *fin* show higher CV than one in the variable *real*.

Spain, Greece, Portugal, Italy, and other smaller economies (as Finland and Netherlands) show the biggest dispersion in productivity. Moreover, the dispersion index spills over in different ways across countries, particularly during the current crisis highlighting the importance of potential latent spillovers occurring over time. This analysis will be following deepened. The Tabel 8.3 summarizes this findings.

Table 8.3: Dependent Variable and Common Factors

	PROD				REAL				FIN			
	1998 - 2006		2007 - 2014		1998 - 2006		2007 - 2014		1998 - 2006		2007 - 2014	
Countries	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
IT	0.010	0.006	0.001	0.008	0.258	0.013	0.281	0.023	-0.010	0.029	-0.014	0.052
ES	0.019	0.003	0.000	0.009	0.270	0.013	0.289	0.036	-0.042	0.022	-0.042	0.042
FR	0.010	0.003	0.004	0.007	0.271	0.012	0.267	0.014	0.006	0.055	-0.018	0.056
BE	0.011	0.005	0.006	0.009	0.765	0.034	0.825	0.046	0.044	0.025	0.011	0.040
NL	0.013	0.006	0.003	0.010	0.670	0.037	0.797	0.071	0.066	0.018	0.060	0.029
AT	0.010	0.005	0.006	0.008	0.493	0.043	0.564	0.030	0.022	0.023	0.026	0.037
DE	0.006	0.006	0.006	0.012	0.368	0.048	0.483	0.031	0.040	0.056	0.073	0.060
FI	0.011	0.008	0.005	0.016	0.413	0.024	0.416	0.036	0.056	0.096	-0.018	0.065
LU	0.022	0.024	0.009	0.021	1.483	0.118	1.745	0.074	0.406	0.032	0.321	0.042
IE	0.025	0.023	-0.004	0.028	0.884	0.079	0.959	0.108	-0.009	0.041	0.001	0.183
GR	0.015	0.011	0.001	0.011	0.224	0.016	0.216	0.015	-0.103	0.138	-0.119	0.080
PT	0.011	0.007	0.001	0.011	0.282	0.013	0.341	0.042	-0.078	0.026	-0.060	0.058
Average	**0.014**	**0.009**	**0.003**	**0.013**	**0.532**	**0.037**	**0.599**	**0.044**	**0.033**	**0.047**	**0.018**	**0.062**

Mean and standard deviation for each financial variable in the system for the period 1998 - 2006 and 2007 - 2014. The bold row shows the average for each column. The productivity (*prod*) denotes the observable variable in the system and, hence, the GDP growth rate. It is computed following Sala-i-Martin and Barro (1986). Common factors for real (*real*) and financial (*fin*) dimension denote trade and capital flows, respectively, across Member Stated of the EA12. The former accounts for exports and imports at the current prices and weighted with respect to GDP; the factor *fin* are computed on the total economy in million units of national currency and weighted with respect to GDP. The values are expressed at net on the total transactions.

Graphically, the variable *real* falls during the crisis period. However, in the period 1999-2001 and 2009-2013, it increases following restrictive fiscal actions mirroring deeper austerity on real dimension. Capital flows shows more trends and, hence, serial correlation over time. In the next section, tests on common features and break-time effects will be estimated. The Picture also shows trends and changes of public and private sectors (general government consumption [*cons*], unit labour costs [*lab*]). The above-mentioned variables will be heavily analyzed in the SUR model accounting for economic and institutional implications.
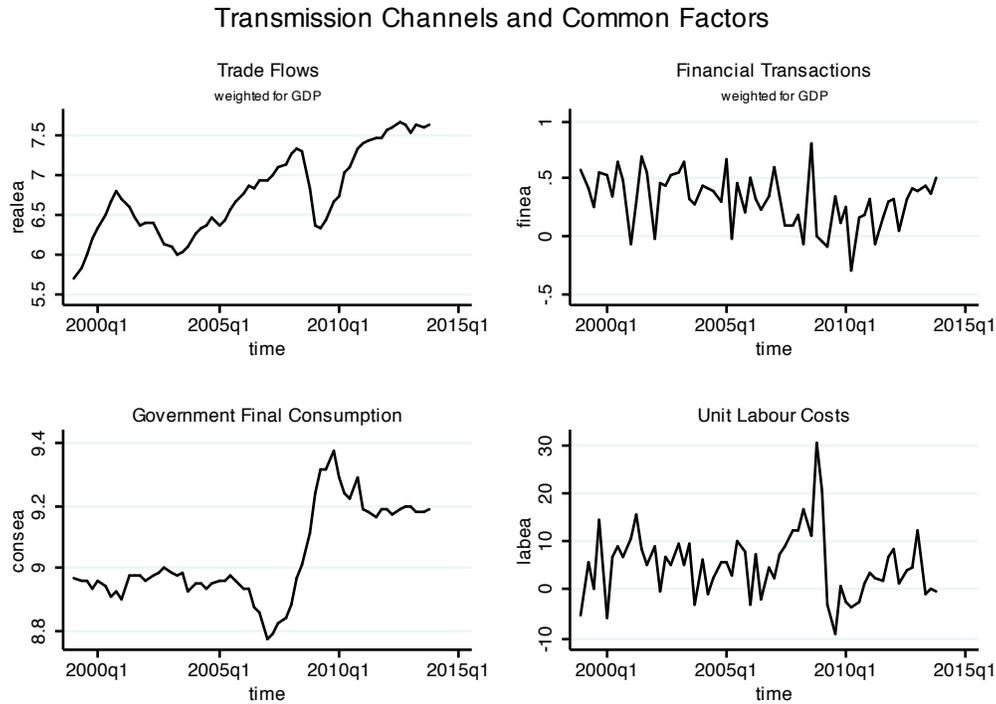
Figure 8.3: All variables are computed accountig for the Euro Area. Trade flows (*real*), financial transactions (*fin*), and general government consumption (*cons*) are presented as a percentage of GDP. Unit labour costs (*lab*) denotes a public and private variable in order to deep economic/instituional implications.

8.1.2 Crisis Period vs. Fiscal Consolidation

In the following subsection, the entire dataset will be analyzed in two different periods: 'crisis period' (2007 - 2011) and 'fiscal consolidation' (2011 - 2014). The aim of this analysis is to highlight how the presence of heterogeneity, interdependence, and common factors across countries changed recently. Mean and standard deviation are shown for each real and financial variables adding two instrumental variables in the system (private consumption [*priv*] and international investment position [*inv*])[3].

In Table 8.4, the first and second moments for each real variable of the system is shown. The variable *gov* further decreases in terms of all financial accounts in mostly countries as Spain, Belgium, Netherlands, and Finland. Italy, France, Austria, Germany, Luxembourg, Ireland, Greece, and Portugal show light improvements. This result seems to be strictly due to restrictive measures on competitiveness price in export trade. However, there is heterogeneity across countries; in fact, the variation of coefficient increases during fiscal consolidation period due to latent variables which need to be analyzed when studying real and financial linkages. The variable *gdpg* observes a light improvement, except in Italy, Spain, and France.

Belgium, Netherlands, Austria, Finland, and some large economies (as Italy and Spain) which show a CV bigger than the average. Over all, the CV decreased during fiscal consolidations proving cross-country interdependency and co-movements following fiscal actions; but, however, they spill over in heterogenous way. The variable *cap* decreased in the last years. The dispersion index tends to decrease during fiscal consolidation, but with less intensity with respect to the decreasing observed between before 1999 - 2007 and 2007 - 2014. It proves the presence of commonalities across countries within a common-currency area; however, there is a high degree of heterogeneity due to omitted factors affecting interdependencies between Members.

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[3]These variables will be important instrumental factors in order to study the transmission of a shock following economic and institutional effects

Table 8.4: Real Variables

	GOV				GDPG				CAP			
	2007 - 2011		2011 - 2014		2007 - 2011		2011 - 2014		2007 - 2011		2011 - 2014	
Countries	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
IT	-0.037	0.027	-0.031	0.026	0.002	0.010	0.000	0.004	0.202	0.010	0.181	0.008
ES	-0.060	0.065	-0.091	0.057	0.002	0.010	-0.002	0.004	0.257	0.039	0.192	0.013
FR	-0.053	0.039	-0.048	0.028	0.004	0.009	0.004	0.004	0.205	0.006	0.203	0.002
BE	-0.030	0.059	-0.037	0.050	0.007	0.011	0.005	0.004	0.212	0.009	0.203	0.004
NL	-0.027	0.037	-0.036	0.027	0.005	0.011	0.002	0.005	0.191	0.013	0.170	0.009
AT	-0.031	0.035	-0.021	0.032	0.007	0.010	0.006	0.003	0.210	0.005	0.212	0.002
DE	-0.016	0.027	-0.002	0.011	0.005	0.014	0.008	0.005	0.179	0.007	0.177	0.004
FI	0.012	0.074	-0.019	0.056	0.006	0.019	0.005	0.007	0.202	0.013	0.194	0.005
LU	0.015	0.034	0.001	0.026	0.008	0.026	0.011	0.009	0.195	0.020	0.185	0.011
IE	-0.127	0.123	-0.094	0.061	-0.005	0.034	0.008	0.019	0.181	0.058	0.107	0.009
GR	-0.106	0.041	-0.090	0.067	0.002	0.014	0.003	0.003	0.174	0.028	0.130	0.000
PT	-0.069	0.040	-0.052	0.034	0.003	0.010	-0.003	0.012	0.209	0.015	0.163	0.015
Average	**-0.044**	**0.050**	**-0.043**	**0.040**	**0.004**	**0.015**	**0.004**	**0.007**	**0.201**	**0.018**	**0.176**	**0.007**

Mean and standard deviation for each real variable in the system for the period 2007 - 2011 and 2011 - 2014. The bold row shows the average for each column. The government spending (*gov*) denotes all financial accounts weighted for GDP, the GDP growth rate (*gdpg*) is weighted for the population for each country, and the gross fixed capital formation (*cap*) consists of resident producers' acquisitions, less disposals, of fixed assets during a selected period plus certain additions to the value of non-produced assets. This latter, also known as Investment, includes acquisition less disposals of buildings, structures, machinery and equipment, computer software, and so on.

Graphically, containing the analysis for the whole EA, the variables show the following trends and changes over time.

Figure 8.4: All real variables are computed accountig for the Euro Area. Productivity in the first difference denotes the output growth not autocorrelated in order to deal with the problem of omitted variables.

In Table 8.5, the first and second moments for each financial variables are shown accounting for crisis and fiscal consolidation period. Despite restrictive fiscal actions, the interest rates for long-term government bonds (*int*) decreased over time due to deeper deflationary programs inter-countries[4]as France, Belgium, Netherlands, Austria, Germany, and Finland.

The variation of coefficient also decreased across countries proving the high degree of interdependence and coordinated fiscal actions. The variable *debt* increased in all countries keeping heavily high levels with respect to own country-wealth. The variation of coefficient shows values smaller than the 'crisis period' due to strong interdependencie across countries in financial dimension. The variable *curr* observes a further deficit in current account, but with lower dispersion due to the monitoring of the Growth Stability Pact (GSP). In financial sector, on average, the dispersion seems to be similar across countries. This result proves a stronger interdependence in financial dimension. Thus, a shock in the former deeply affects real sector through the common component but in an heterogenous way given a high degree of divergence in economic and institutional factors. This findings will be heavily discussed in Chapter 10

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[4]ECB(2013)

Table 8.5: Financial Variables

	INT				DEBT				CURR			
	2007 - 2011		2011 - 2014		2007 - 2011		2011 - 2014		2007 - 2011		2011 - 2014	
Countries	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
IT	4.425	0.311	5.078	0.765	112.467	6.702	126.217	5.015	-2.617	1.548	-0.070	1.905
ES	4.349	0.437	5.283	0.665	47.983	11.077	79.792	10.892	-6.939	2.961	-0.760	2.504
FR	3.795	0.514	2.688	0.569	74.267	8.061	89.767	3.353	-1.417	1.043	-1.690	0.839
BE	4.046	0.416	3.214	0.879	93.528	5.675	102.308	2.700	0.533	4.873	-2.310	2.773
NL	3.753	0.560	2.293	0.600	56.083	7.776	68.967	3.886	6.189	2.241	10.090	1.859
AT	3.915	0.498	2.568	0.670	68.000	4.420	74.442	1.536	3.461	2.528	2.360	1.952
DE	3.494	0.627	1.892	0.637	71.700	5.916	80.392	1.070	6.483	1.308	7.400	0.899
FI	3.783	0.567	2.250	0.634	39.594	6.124	51.717	3.910	1.950	3.401	-1.100	1.479
LU	4.034	0.659	2.199	0.643	13.356	5.551	21.717	2.510	7.578	5.871	5.690	4.001
IE	5.499	1.812	6.522	2.620	56.283	27.078	112.783	10.565	-2.678	3.488	4.660	2.883
GR	6.757	3.310	16.100	5.997	124.972	17.727	160.158	10.916	-12.800	4.564	-2.370	6.599
PT	5.082	1.492	9.028	2.609	79.344	12.041	117.667	11.188	-10.922	2.038	-1.460	2.604
Average	**4.411**	**0.934**	**4.926**	**1.441**	**69.798**	**9.846**	**90.494**	**5.628**	**-0.931**	**2.989**	**1.703**	**2.525**

Mean and standard deviation for each financial variable in the system for the period 2007 - 2011 and 2011 - 2014. The interest rate (*int*) denotes EMU convergence criterion series related to interest rates for long-term government bonds denominated in national currencies, the general government debt (*debt*) are measured in euro and presented as a percentage of GDP, and the general government surplus/deficit (*curr*) surplus is defined in the Maastricht Treaty as general government net borrowing/lending. The series are measured in euro and presented as a percentage of GDP.

In Picture 8.5, financial variables and unemployment rate are plotted for crisis and fiscal consolidation periods.

Financial Variables
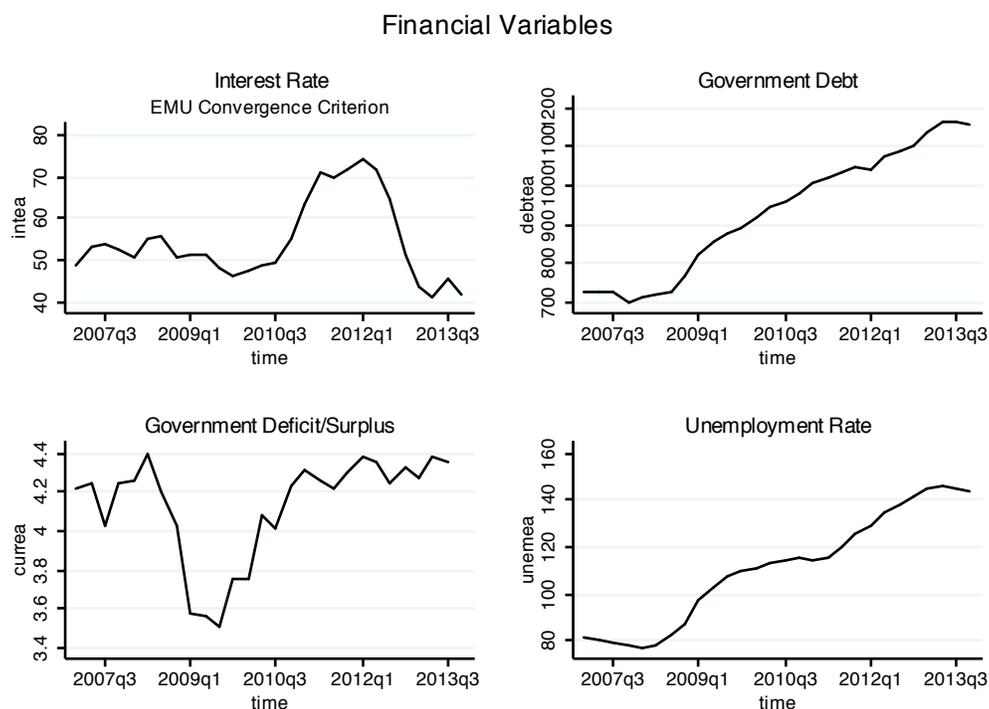
Figure 8.5: All variables are computed accountig for the Euro Area and presented as a percentage of GDP, except the interest rate which is taken in levels. The variable (*unem*) is computed accounting for sex and age groups.

Finally, in Table 8.6, mean and standard deviation for real and financial transmission channels and productivity are displayed for the two selected periods. The variable *prod* shows a undeniable improvement during fiscal consolidation.

Observing the only standard deviation, the index is almost equal; but, accounting for the CV, the dispersion index only improves for some countries, while it increases in mostly Members as Italy, France, Belgium, Netherlands, Ireland, Greece, and Portugal. Thus, the high degree of heterogeneity cannot be explained by the only catchin-up effect and proves the need to focus on the competitiveness level in each country and to estimate the variables of the system in a time-varying model. The decreasing in variables *real* and *fin* show the high degree of heterogeneity across countries and, thus, strong economic/instituional implications following restrictive fiscal actions. Moreover, this result is proved by computing the CV which increase during consolidation period. The paper will highlight on how heterogeneity is increased by strong cross-country interdependence and co-movements of some real and financial factors. Italy, Greece, Portugal, and Ireland shows careless improvements in *fin*.

Table 8.6: Dependent Variable and Common Factors

	PROD				REAL				FIN			
	2007 - 2014		2011 - 2014		2007 - 2014		2011 - 2014		2007 - 2014		2011 - 2014	
Countries	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
IT	0.002	0.010	0.298	0.008	0.270	0.022	0.002	0.041	-0.028	0.056	0.000	0.004
ES	0.002	0.010	0.325	0.017	0.266	0.020	-0.009	0.031	-0.064	0.030	-0.002	0.004
FR	0.004	0.009	0.278	0.003	0.260	0.014	-0.023	0.064	-0.019	0.059	0.004	0.004
BE	0.007	0.011	0.857	0.009	0.807	0.048	0.001	0.029	0.019	0.046	0.005	0.004
NL	0.005	0.011	0.867	0.023	0.753	0.049	0.073	0.024	0.053	0.031	0.002	0.005
AT	0.007	0.010	0.574	0.003	0.557	0.036	0.019	0.036	0.030	0.039	0.006	0.003
DE	0.005	0.014	0.511	0.007	0.467	0.027	0.070	0.067	0.081	0.060	0.008	0.006
FI	0.006	0.019	0.403	0.008	0.424	0.043	-0.024	0.038	0.081	0.060	0.005	0.007
LU	0.008	0.026	1.772	0.022	1.734	0.090	0.293	0.024	0.340	0.038	0.011	0.009
IE	-0.005	0.034	1.058	0.028	0.900	0.088	0.005	0.084	-0.007	0.222	0.005	0.022
GR	0.001	0.014	0.218	0.001	0.215	0.019	-0.109	0.016	-0.127	0.099	-0.001	0.002
PT	0.003	0.010	0.382	0.022	0.314	0.023	-0.008	0.048	-0.098	0.021	-0.002	0.012
Average	**0.004**	**0.015**	**0.628**	**0.013**	**0.581**	**0.040**	**0.024**	**0.042**	**0.022**	**0.063**	**0.003**	**0.007**

Mean and standard deviation for the observable variable and trade and capital flows for the period 2007 - 2014 and 2011 - 2014. The productivity (*prod*) denotes the observable variable in the system and, hence, the GDP growth rate. It is computed following Sala-i-Martin and Barro (1986). Common factors for real (*real*) and financial (*fin*) dimension denote trade and capital flows, respectively, across Member Stated of the EA12. The former accounts for exports and imports at the current prices and weighted with respect to GDP; the factor *fin* are computed on the total economy in million units of national currency and weighted with respect to GDP. The values are expressed at net on the total transactions.

In Picture 8.6, transmission channels and the remaining observed latent factors are drawn accounting for the two selected subperiods (2007 - 2011 and 2011 - 2014).

Transmission Channels and Common Factors
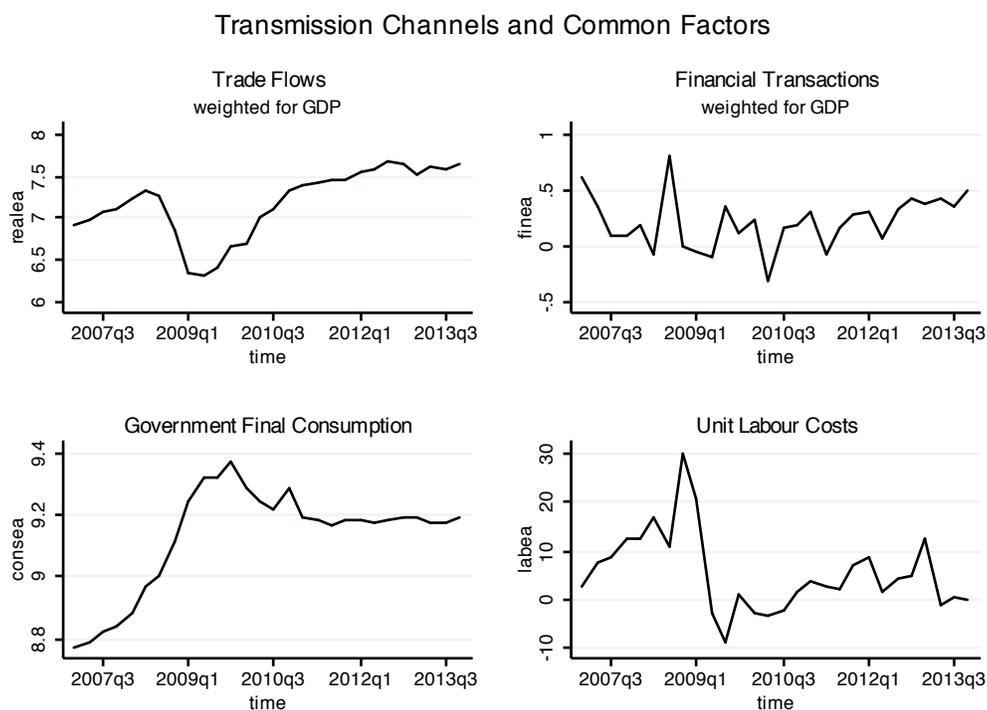
Figure 8.6: All variables are computed accountig for the Euro Area. Trade flows (*real*), financial transactions (*fin*), and general government consumption (*cons*) are presented as a percentage of GDP. Unit labour costs (*lab*) denotes a public and private variable in order to deep economic/instituional implications.

In summary, simple descriptive statistics reveal differences in the evolution of the real and financial variables in this analysis. The above description of trends and changes for each variable in the system shows that while the speed of growth of almost all variables, especially real variables, in the mostly of countries was slower after 1999 and even more recently, the volatility of many variables and countries in the selected sample increased. Thus, the degree of heterogeneity is high and strongly affects co-movements and interdependencies due to fiscal actions. Moreover, trends and changes in the majority variables seem to be mostly driven by the large fall and increased uncertainty in almost all variables during financial crisis and recent fiscal measures. Thus, the need to account for potential omitted variables engaging with real and financial sectors.

Two additional latent variables will be discussed in the SUR model (international investment position [*inv*] and private consumption [*priv*]) in order to estimate the importance of transmission channels and economic/institutional implications when studying real and

financial linkages across countries within an optimum currency area. The Picture 8.7 confirms that heterogeneity and common features derive by factors much more problematic than cause-effect and catching-up relationship. The variable *inv* shows an improvement following first fiscal consolidation measures. Then, it falls until to observe an almost similar value of one in 2011. The variable *priv*, after the first wave of consolidations, in the majority countries decreased affecting the mean of the whole area. This decreasing persists today and once more this result seems to be mostly driven by deep cross-country economic and institutional implications.

Private Factors
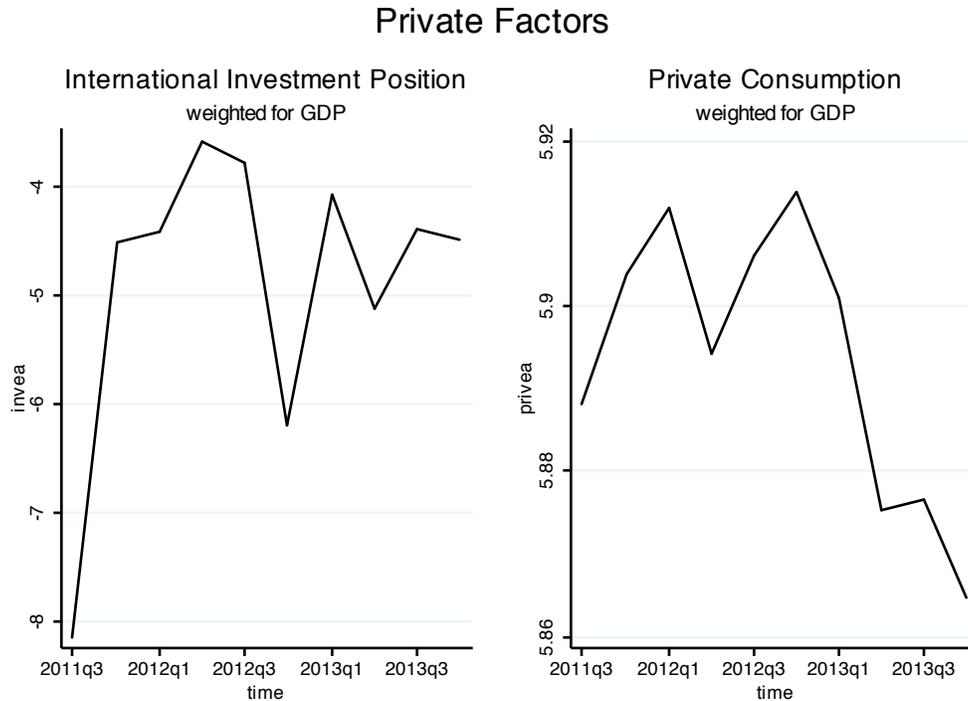
Figure 8.7: All variables are computed accountig for the Euro Area and presented as percentage in GDP. International Investment Position (*inv*) denotes the total of investments, for both real and financial sectors, with foreign partners. The private consumption (*priv*) consists of expenditure incurred by resident institutional units on goods or services that are used for the direct satisfaction of individual needs. They include for example trade unions, professional societies, political parties, sport clubs, and so on.

8.2   Time-series, Features of the data, and Testing

Time series analysis comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data. In this section, principal methods will be employed and discussed in order to analyze all variables in the system and features of the data.

Given the specification model described in equation 6.22. A first approach would be estimating the significance and correlation between real and financial variables before and during crisis period in order to highlight the presence of potential serial correlation in the system. The result would prove the importance of transmission channels and economic/structural implications when studying macroeconomic-financial linkages, even more so countries within a common currency area.

Table 8.7 shows a robust regression between real and financial variables accounting for cross-country and common factors[5]. As illustrated in Chapter 3, trade and capital flows are observed in order to analyze economic-financial transmission channels (which will be indicated as *weights*). Some private and public variables are observed in order to highlight recent empirical studies and empirical evidence on growth spillover effects, *catching-up* and competitiveness factors, *cause-effect* relationship, eurozone member states. They are indicated as *outliers*.

All estimates are significant and show output growth is potentially strongly sensitive in changes and features of real and financial dimension. Moreover, accounting for components *weights* and *outliers*, the analysis would be significant in its coutry-specific component and even more in its common component.

Focusing on cross-country specific factors, component *weights* is significant and robust in its real dimension and even more in its financial dimension. Accounting for both components *weights* and *outliers*, real dimension is stronger sensitive than financial component. These findings would prove the presence of potential economic-institutional factors affecting the transmission of a growth spillover to a shock in real economic and financial variables.

Interdependence across countries is stronger in financial component than real component proving the presence of coordinated fiscal actions. Those spillover swiftly affect real economy through transmission channels. Nevertheless, potential latent factors would durably hampered the convergence across countries and the degree of homogeneity given a growth shocks originating within the EMU.

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[5]In Chapters 9 and 12 empirical evidence are heavily employed and discussed.

Table 8.7: Cross-Country Factors vs. Common Factors

	Cross-Country Factors			
	Real Component		Financial Component	
	weights	weights & outliers	weights	weights & outliers
Number of obs.	63	63	63	63
F(12 , 50)	4.82	9.96	5.55	6.19
Prob > F	0.00	0.00	0.00	0.00
	Common Component			
	weights		weights & outliers	
Number of obs	63		63	
F(2 , 60)	13.45		24.67	
Prob > F	0.00		0.00	

In the Table are shown the F-statistic test and *p-value* for real and financial dimension accounting fro cross-country and common factors. The analysis accounts for the coefficient vectors χ_{1t}, χ_{2t}, χ_{3t}, χ_{4t}, and χ_{6t} in equation 6.22.

Information criterion is a useful analysis to understand the optimal lag working with stochastic processes. When the three Information Criteria are disagree, in the context of VAR models, Akaike Information Criterion (AIC) tends to be more accurate with monthly data, Hannan and Quinn Information Criterion (HQIC) works better for quarterly data on samples over 120, and Schwarz-Bayesian Information Criterion (SBIC) works fine with any sample size for quarterly data. The Table 8.8 shows the above-mention information criteria. The analysis would observe the SBIC index working wiht quarterly data and large sample. The optimal lag is $p = 1$.

Table 8.8: Information Criteria

Sample	1998q4 - 2014q2		
Number of obs	63		
Lag	AIC	HQIC	SBIC
0	2.85	2.98	3.19
1	-8.36	-7.07	-4.96∗
2	-8.82	-6.34	-2.34
3	-9.94	-6.29	-0.39
4	-15.16∗	-10.34∗	-2.55

In the Table are shown Akaike Information Criterion (AIC), Hannan and Quinn Information Criterion (HQIC), and Bayesian Information Criterion, respectively. The analysis accounts for the coefficient vector χ_{5t} described in equation 6.22.

A second approach to study features of the data is a unit root analysis. Here, if the model is not significant in the considered period, then the series have a unit root and, hence, not stationary. It means that there is more than one trend in the series. In Table 8.9, the data are obseved in three different periods: from 1998q4 to 2006q4 selected as 'before crisis' period, from 2007q1 - 2011q2 selected as 'crisis period', and from 2011q2 - 2014q2 selected as 'fiscal consolidation' period. All variables in the system accounting for *weights* and *outliers* are observed. Empirical evidence shows the presence of one or more trends in the selected periods and, hence, the necessity to test for stationarity.

Table 8.9: Unit Root Analysis

1998q4 - 2006q4		2007q1 - 2014q1		2009q3 - 2014q1	
Number of obs.	33	Number of obs	18	Number of obs	12
F(2,30)	4.31	F(2,15)	3.16	F(2,9)	1.71
Prob > F	0.05	Prob > F	0.06	Prob > F	0.07
R^2	0.23	R^2	0.20	R^2	0.19
Adj R^2	0.18	Adj R^2	0.13	Adj R^2	0.08
Root MSE	0.04	Root MSE	0.10	Root MSE	0.05

In the Table is shown unit root analysis accounting for the coefficient vector χ_{5t} of the equation 6.22.

One of the most commonly use tests for stationarity is the Dickey-Fuller test. The null hypothesis is that the series has a unit root (or stochastic trends). Accounting for real and financial dimension, the empirical evidence seems to highlight the presence of potential latent factors affecting real and financial variables. Those factors could evaluate the important of economic and institutional implications across eurozone countries in analyzing size and height of spillover effects.

The Table 8.10 shows the presence of stochastic trends. One way to deal with unit roots is by taking a difference between the time trend and unit root processes. It is a useful transformation that the data required to generate a stationary time series.

Table 8.10: Unit Root Test

Augmented Dickey-Fuller Test for unit root		
	Government Spending	GDP Growth Rate	Gross Fixed Capital
Number of obs.	63	63	63
Test Statistic Z(t)	-2.681	-3.177	-1.313
p-value for Z(t)	0.24	0.09	0.88
	Interest Rate	Public Debt	Public Deficit
Number of obs.	63	63	63
Test Statistic Z(t)	-2.63	-1.32	-2.11
p-value for Z(t)	0.26	0.88	0.54
Critical Value (5%)		-3.492

In the Table is shown Augmented Dickey-Fuller test for unit root accounting
for the coefficient vectors χ_{1t}, χ_{2t}, χ_{3t}, χ_{4t} of the equation 6.22.

Since the data are have a unit root and, hence, stochastic trends, a test in order to estimate serial correlation is appropriate. Breush-Godfrey and Durbin-Watson are used. The null in both tests is that there is no serial correlation. The Table 8.11 proves that there is serial correlation between variables; thus, the necessity to permit coefficent vectors to vary over time and to account for transmission channels and other potentially strongly correlated with real and financial variables.

Table 8.11: Serial Correlation

Cross-Country Factors		Common Factors	
Number of obs	63	Number of obs	63
F(12, 50)	11.30	F(2, 60)	12.65
Prob > F	0.00	Prob > F	0.00
R-squared	0.28	R-squared	0.99
Adj R-squared	0.26	Adj R-squared	0.99
Root MSE	0.09	Root MSE	0.00
D-statistic(3.61)	0.87	D-statistic(51.61)	2.72
Durbin's alternative Test		Durbin's alternative Test	
Chi^2	28.88	Chi^2	10.50
Prob>Chi^2	**0.00**	Prob>Chi^2	**0.00**
Breusch-Godfrey LM Test		Breusch-Godfrey LM Test	
Chi^2	20.42	Chi^2	34.37
Prob>Chi^2	**0.00**	Prob>Chi^2	**0.00**

In the Table is shown Breush-Godfrey and Durbin-Watson tests for serial correlation accounting for the coefficient vectors χ_{1t}, χ_{2t}, χ_{3t}, χ_{4t}, and χ_{6t} of the equation 6.22.

Finally, Granger-causality test is runned. It is a statistical hypothesis test for determining whether one time series is useful in forecasting another. However, it is considered as *predictive analysis* since does not account for latent confounding effects and does not capture instantaneous and non-linear causal relationships. Nevertheless, it is a useful test to highlight the presence of *cause-effect* relationships. It will be deepened in Chapter 12 accounting for latent factors and common components across countries. Accounting for cross-country factors, the Table 8.12 shows that the strong linkage between real economy and financial variables. Moreover, trade channels suggests that trade channels matter relatively less than financial and other non-trade channels. Thus, the importance of competitiveness factors when studying real-financial linkages. Private and public variables are also observed. Each variable Granger-cause the other affecting both real and financial sectors, except the variable *investment* and, hence, the importance of economic and institutional factors within the EMU. It highlights findings for future researches: growth shocks from outside the EMU are

to a relatively larger extent transmitted via trade compared to growth shocks originating within the EMU. This latter appear to be predominantly transmitted via monetary and financial linkages.

Table 8.12: Granger-Causality Test

Granger Causality		
Impulse Variable	Chi^2	Prob>Chi^2
government spending	134.64	0.00
real gdp growth rate	30.47	0.01
gross fixed capital	59.88	0.00
interest rate	63.22	0.00
public debt	154.01	0.00
public deficit	131.74	0.00
real factor	9.86	0.13
financial factor	50.54	0.00
consumption	51.65	0.00
real unemployment rate	167.93	0.00
labour cost	51.88	0.00
private cons	30.09	0.01
investment	16.63	0.28

In the Table is shown Granger-causality effect accounting for the coefficient vectors χ_{1t}, χ_{2t}, χ_{3t}, and χ_{4t} of the equation 6.22.

Chapter 9

Fiscal spillover effects and shock transmission

In this Chapter, the aim of the analysis is to compute sign and dimension of fiscal spillovers in real and financial dimension accounting for cross-country and common factors and to answer the following questions: Is heterogeneity across countries the component mainly affecting output gap? What is the role of co-movements and interdependence across countries in driving spillovers between real and financial dimension? Are these components[1] larger in financial or real dimension? And why?

A central issue in economics and economic policy guidance is the effect of a change in fiscal policy on the domestic economy. Moreover, in an integrated world, domestic fiscal actions can also affect foreign economies. In the context of a currency union where the exchange rate between member countries is fixed, individual countries need some protection from shocks of uncoordinated fiscal policies (see e.g., fiscal agreements like the Stability and Growth Pact and successive measures adopted by EA). In this paper, spillovers are defined as the transmission of an unexpected but identified shock from one variable to responding variables in the system. Aggregation of net spillover effects at each point in time yields then a contagion index. The article addresses two main spillover channels of an expansionary fiscal policy in one member country into the rest of the Eurozone. (i) Spillover effects through trade. A fiscal expansion stimulates domestic activities driving the exchange rate to appreciate and the domestic interest rate to increase. In a currency union, the exchange rate between members is fixed and the interest rate is ultimately determined at the union level, hence domestic money under circulation increases. (ii) Spillover effects through capital flows. The increase in the domestic interest rate attracts capital flows into domestic economy out of the rest of the union. Overall, economic theory provides reasoning to expect positive and negative spillover effects.

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[1]Heterogeneity, co-movements, and interdependence.

The BVAR model is able to determine how trends and changes in a particular variable in a given country affect other countries, using generalized impulse response functions. Moreover, this analysis can assess whether negative financial shock in one country affects other countries and how affect real sector. In addition, inderdependecies and transmission channels can be evaluated in this framework. Fiscal spillover effects are computed by generalized impulse response functions (GIRFs)[2] as the difference between a conditional and an unconditional projection of a variable for each country in a particular period. The analysis considers conditional impulse response functions (IRFs) for each variable in the system obtained over the same period conditionally on the actual path of another variable, that is an unexpected shock sent, for that period. The output deriving from the model absorbs conditional forecasts computed on the time frame of 10 quarters (2 years and half). The aim of this choose is to compute potential fiscal spillover effects, absorbing each single draw obtained from the posterior of regression coefficients. The prediction would reach December 2016 until the conclusion of actual *fiscal measure-path*.

In order to capture potential spillovers that could trigger financial contagion across the EA, a multicountry econometric framework is used to derive impulse responses from each variable to all other variables in the system. A matrix of potential spillover effects from each variable in the system has been constructed in order to define (individual) bilateral spillover effects. They describe the dynamics of impulse responses from a shock in real and financial variables within the Euro Area as weighted average of responses of each variables. Bilateral spillover effects can either be negative or positive. Here, two components can be defined, with N=1,2,...,12.

1. The average sum of the impulse responses to others defines (individual) bilateral OUT spillover effects:

$$SE_{OUT,y_i \to *} = \sum_{j=1}^{N} IR_{y_i \to y_j} \qquad (9.1)$$

2. The average sum of the impulse responses from others defines (individual) bilateral IN spillover effects:

$$SE_{IN,* \to y_i} = \sum_{j=1}^{N} IR_{y_j \to y_i} \qquad (9.2)$$

They account for time-varying impact in real and financial variables within the EA. They incorporate feedback effects from the impulse variables and temporary or persistent long-run effect of a potential shock. By the same token, bilateral net spillover effects is defined as the

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[2]See e.g., Pesaran and Shin, 1998, for a definition of generalized impulse responses.

difference between the impulse responses sent and received from/to another variable. When the bilateral net spillover effect is positive, the variable (country) is a Net Sender of the system, and vice-versa. The following two equations are used in the framework:

$$SE_{NET, y_i \to y_j} = IR_{y_i \to y_j} - IR_{y_j \to y_i} \tag{9.3}$$

and

$$SE_{NET, y_j \to y_i} = IR_{y_j \to y_i} - IR_{y_i \to y_j} \tag{9.4}$$

They represent the amplification contribution of the impulse variable to the response variable and is able to capture sequential feature associated with systemic events.

$$SE_{NET, y_i \to y_j} + SE_{NET, y_j \to y_i} = 0 \tag{9.5}$$

Using $SE_{NET, y_i \to y_j}$ for each variable, bilateral net spillover effects and its main components will be described and analyzed.

Finally, total bilateral net spillover effect can be computed by (9.1) and (9.2). It correspond to the sum of its bilateral net effects:

$$TSE_{NET, y_i} = \sum_{j=1}^{N} (IR_{y_i \to y_j} - IR_{y_j \to y_i}) \tag{9.6}$$

9.1 Summary statistics and Net Spillover matrices

Running the selected BVAR model, *beta* posteriors drawn by bayesian computations are significant at the 95% of the confidence interval. The below Table shows the principal measures of fit in order to prove the robustness of the analysis. In Appendix A, impulse responses are shown for each variables in real and financial component.

The goodness of fit of the model is:

Table 9.1: Dependent Variable and Common Factors

1999 - 2014	Real Component	Financial Component
SSE	21.67	10.07
R^2	0.6073	0.6357
Adj. R^2	0.6047	0.6333
RMSE	0.2659	0.2557

In the Table are shown the principal measures of fit as the Sum of Squared Error (*SSE*), R^2 and Adj. R^2, and Root Mean Squared Error (*RMSE*)

Thus, mostly variability of the observable variable is explained by regression coefficients. It is closed to zero either for real or financial dimension. Moreover, financial sector shows a small improvement in measuring fitting objects. This results would highlights the bigger interdependencies across countries in financial dimension as after proved.

Nevertheless, financial components show the presence of greater omitted variables proving the importance to consider economic and institutional factors when studying real and financial linkages.
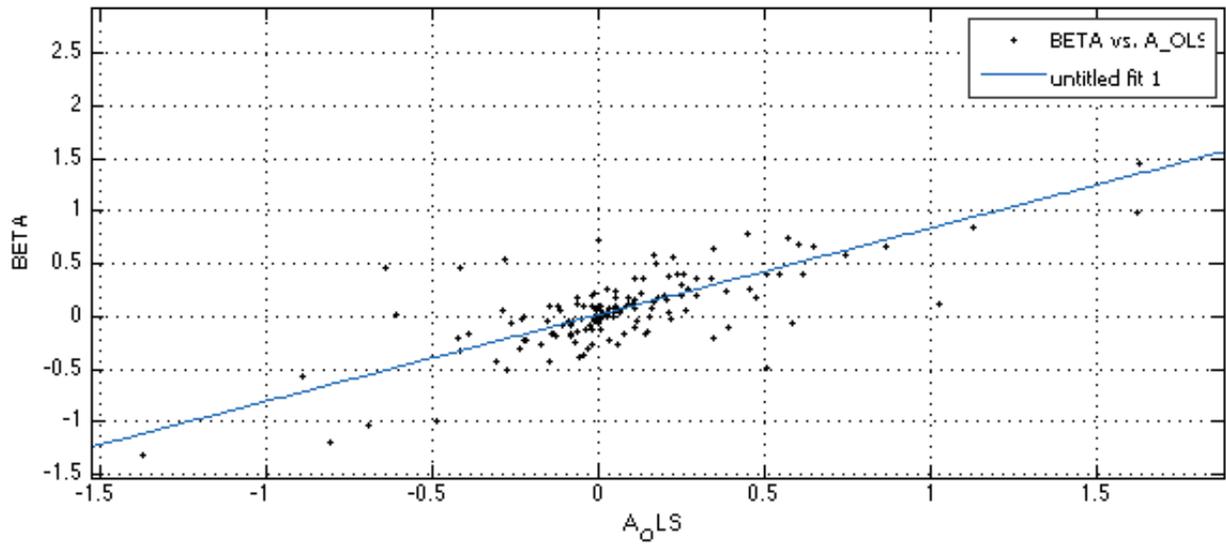
Figure 9.1: The Picture shows the robustness of the model drawing a scatter plot between the matrix of regression coefficients and the single draw from the posterior of *beta* for real dimension accounting for weights (which corresponds to the variable *real*)
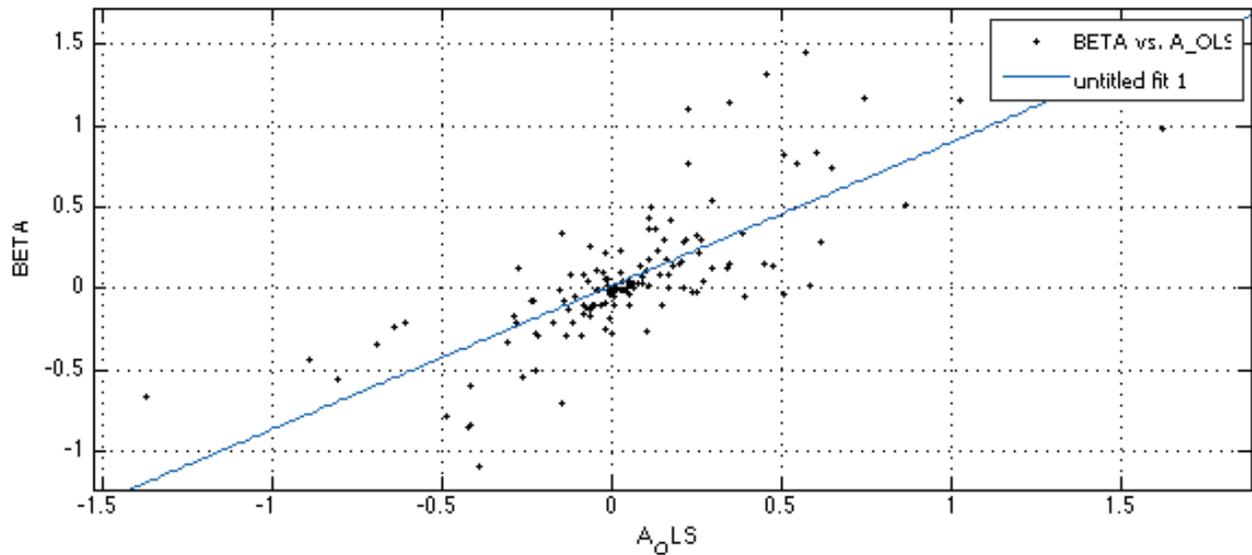
Figure 9.2: The Picture shows the robustness of the model drawing a scatter plot between the matrix of regression coefficients and the single draw from the posterior of *beta* for financial dimension accounting for weights (which corresponds to the variable *fin*)

Using $SE_{NET, y_i \to y_j}$ and focusing on the only *respondents* in Table 9.2, net spillover effects can be computed.

The results would find partial but more significant spillovers to the same financial variables in mostly countries than to their real economy. Moreover, the transmission across countries trough a variety of episodes seems to be stronger between trade exposures than between capital flows. A proof of this findings is heavily discussed in Section 9.2.

Real Dimension

Accounting for each component in real dimension, the mean and standard deviation of estimated regression coefficients are shown for the before crisis period: 1999 - 2006. The Table 9.2 summarizes the impulse responses of productivity[3] to 1% shock to real variables:

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[3]Productivity is the current observable variable used in the model.

Table 9.2: Responses of GDP to 1% shock to real variables

	SPILLOVER MATRIX IN REAL DIMENSION					
	GOV		GDPG		CAP	
	1999-2006		1999-2006		1999-2006	
Country	Out	In	Out	In	Out	In
Italy	0.074	0.061	0.260	-0.008	0.147	0.045
Spain	0.037	0.026	0.502	0.006	0.086	0.056
France	0.721	0.011	0.292	-0.018	0.552	0.023
Belgium	0.136	0.142	-1.151	0.065	-1.288	0.034
Netherlands	-0.083	0.067	-0.024	-0.035	-0.161	-0.006
Austria	0.158	0.125	0.157	0.050	0.399	0.069
Germany	-0.176	0.130	-0.263	0.040	0.238	0.024
Finland	0.091	0.051	0.102	-0.031	-0.242	0.026
Luxembourg	0.007	0.169	0.339	-0.041	0.576	0.060
Ireland	0.143	0.004	0.198	0.221	0.062	-0.077
Greece	0.014	0.153	0.088	0.106	0.141	0.065
Portugal	-0.118	0.065	-0.074	0.072	-0.145	0.050
Average	**0.084**		**0.036**		**0.031**	

Outward and inward growth spillovers are shown for each country in real dimension for the period 1999 - 2006. They are computed running the selected BVAR model (in Appendix A are drawn the corresponding impulse responses to a shock to real variables).

Estimates for the sample 1999 - 2006 suggest a limited Germany's role in generating outward spillovers despite its large size. This result, in part, reflects Germany's own dependence on growth in the rest of the Eurozone[4].

Accounting for the variable *gov*, Netherlands and Portugal also are net receiver of the system. Germany is particularly sensitive to shocks in the other two large EA countries (France and Spain) and in the other two smaller open economies (Finland and Belgium). France is much less sensitive to shocks in Germany than Germany is to shocks in France. Spain's growth is particularly affected by shocks in Italy and in other three smaller european countries (Belgium, Austria, and Netherlands). This result seems to hold for Italy's growth with lesser extent, but it is a net sender of unexpected shocks in Spain. Germany responds to a shocks in France more strongly than Italy. Overlooking responses of Luxembourg's growth, Germany responds to a growth shock in Eurozone countries more strongly than any of the other large EA countries and exhibits the second largest response (after Belgium) to shocks in smaller european countries. This result seems to be consistent with Germany's large trade to the rest of EA. About GIPS countries (Greece, Ireland, Portugal, and Spain), they are sensitive to shocks in France (Portugal, Greece, and Ireland according to size). Greece and Portugal are sensitive to shocks in Spain and, for this latter, in Italy. On average, mostly countries are net receiver of shocks.

The Picture 9.3 draws inward growth spillovers to 1% shock to variable *gov*. Mostly countries show positive values, hence, they are net receiver of the system. In particular, Austria shows greater responses following by Finland, Germany, Greece, Belgium, and Spain. Ireland, Netherlands, and Italy tend to be net sender and, hence, affected from other economies with trade exposures. This result seems to comply with reforms following monetary union in 2001 - 2002 and the recovery processes in order to keep the stability of price strongly affected by real component in the majority of countries (commonalities).

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[4]IMF, 2011
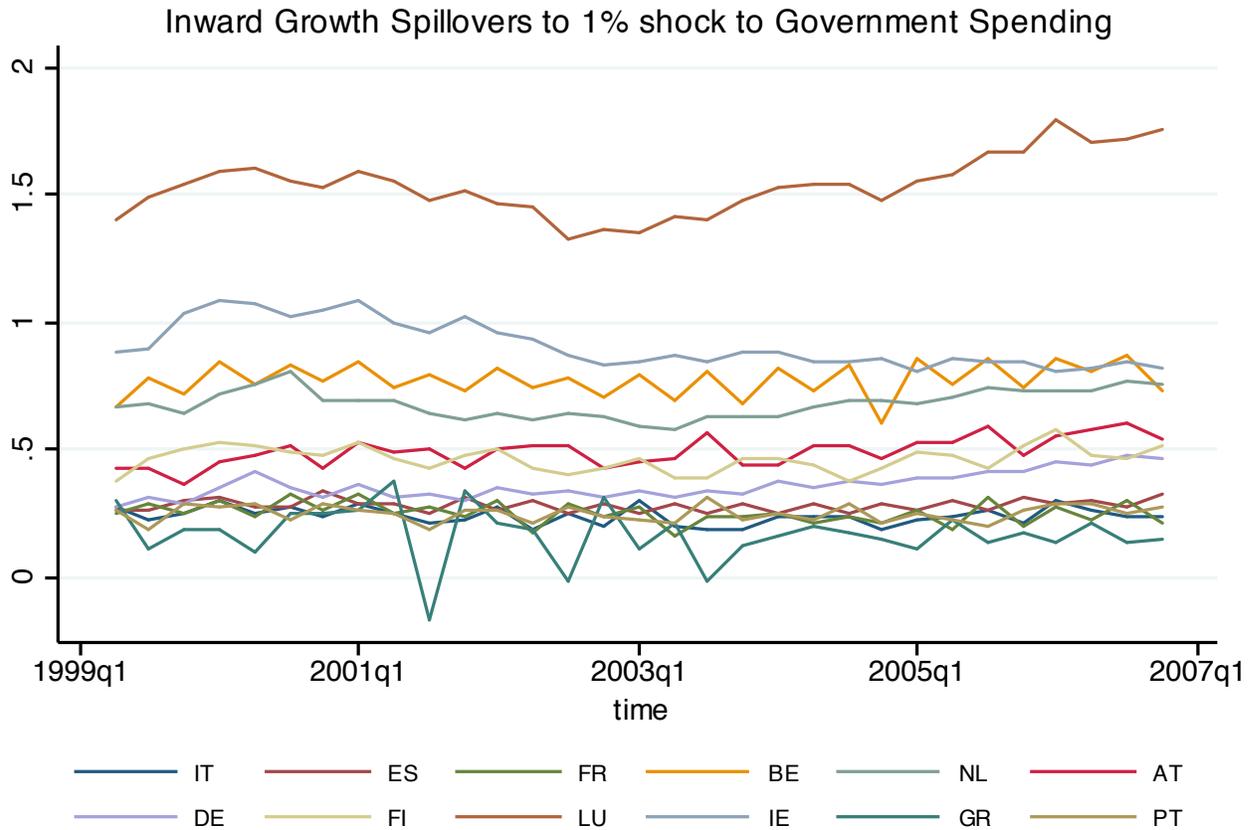
Figure 9.3: The Picture shows inward growth spillovers to a 1% shock to government spending for the period 1999 - 2007.

In Table 9.2, growth's responses to shocks in the variable *gdpg* follow a similar path of the variable *gov*. Germany is a net receiver. According to size, Ireland, Greece, Belgium, Austria, and Portugal are net receiver of the system.

Accounting for outward growth spillovers, Spain shows higher values with respect to other large countries (France and Italy) and smaller european economies (Austria and Finland). Italy's growth is sensitive to Germany but less than Spain. Growth in Ireland, Greece, and Portugal are sensitive to other EA countries.

The Picture 9.4 shows inward growth spillovers to 1% shock to variable *gdpg*. Mostly countries show positive values, hence, they are net receiver of the system. As in the variable *gov*, Austria shows greater responses following by Finland, Germany, Greece, Belgium, and Spain. Ireland, Netherlands, and Italy tend to be net sender affecting other economies with trade exposures.
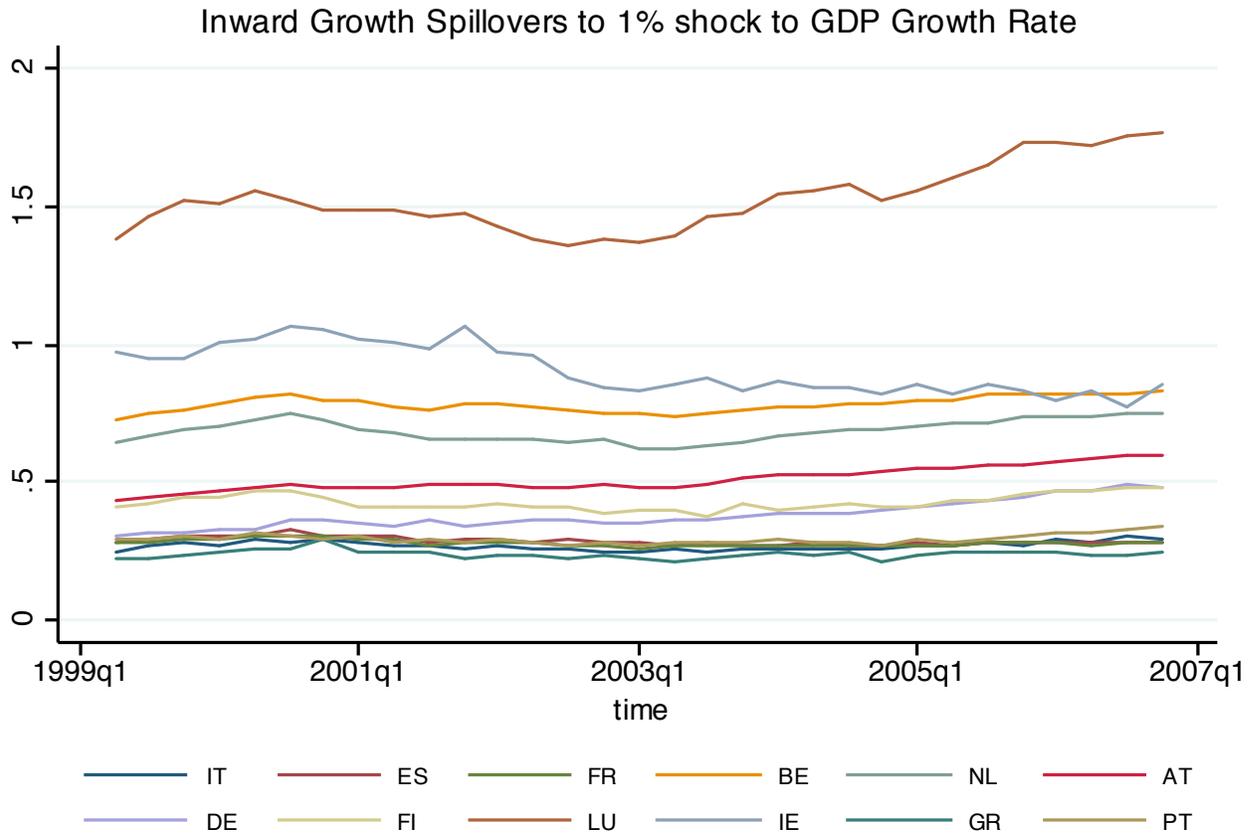
Figure 9.4: The Picture shows inward growth spillovers to a 1% shock to real GDP growth rate for the period 1999 - 2007.

In Table 9.2, accounting for the variable *cap*, Germany's growth is a net sender proving large trade exposures with other european countries and, possibly, large dimension in international trade accounts. Belgium, Netherlands, Finland, and Portugal are net receiver and, hence, inward growth spillovers are sensitive to shocks sent by the rest of Europe. For example, Portugal's inward growths are less sensitive to Austria, Greece, Netherlands, and Belgium; like so Netherlands did not much affect by shocks to France, Finland, and Belgium.

In Picture 9.5, inward spillovers to 1% shock to variable *cap*, mostly countries show positive values, hence, they are net receiver of the system. As in the variable *gdpg*, Austria shows greater responses following by Finland, Germany, Greece, Belgium, and Spain. Ireland, and Italy tend to be net sender affecting other economies with trade exposures. The responses are bigger than *gov* and *gdpg* showing importance of trade transmission channels in driving spillover effects.
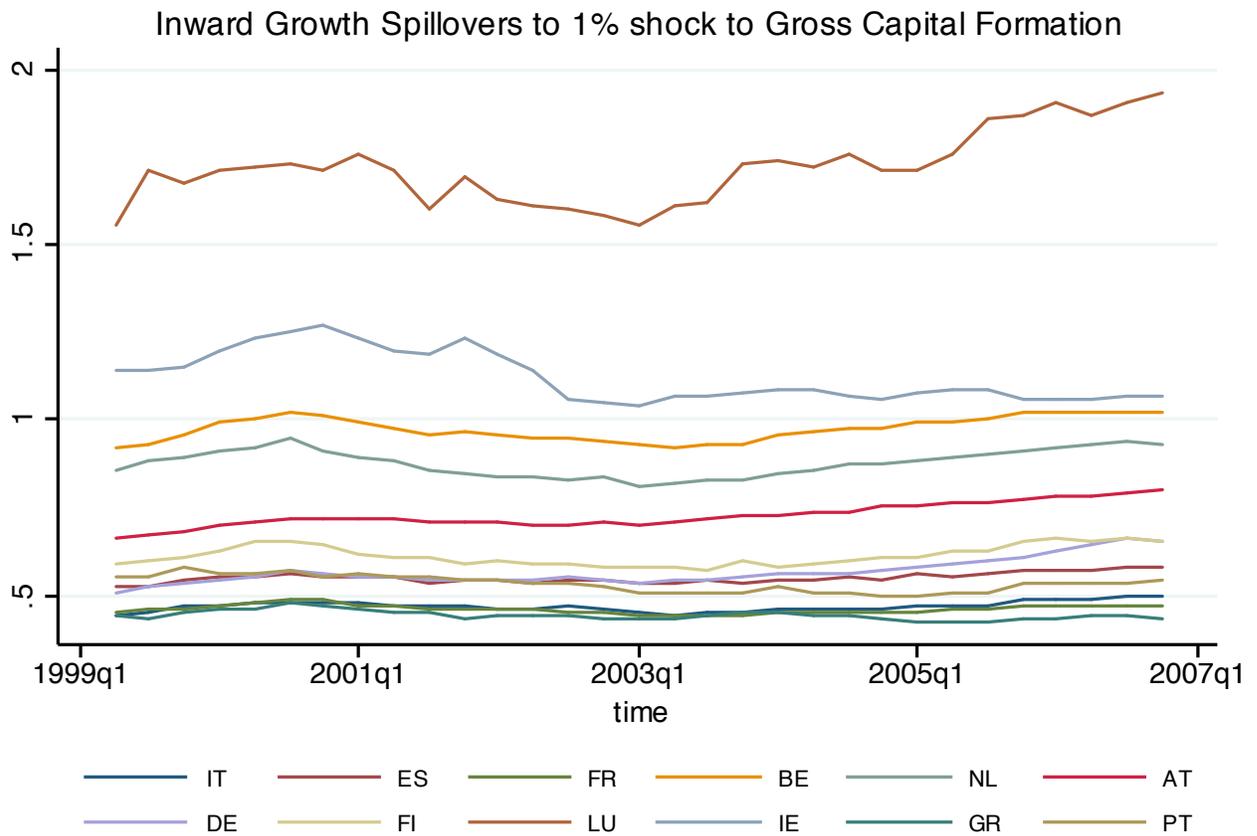
Figure 9.5: The Picture shows inward growth spillovers to a 1% shock to gross capital formation for the period 1999 - 2007.

Financial Dimension

Accounting for for each component in financial dimension, the mean and standard deviation of estimated regression coefficients are shown for the 'before crisis' period: 1999 - 2006. The Table 9.3 summarizes the impulse responses of productivity to 1% shock to financial variables:

Table 9.3: Responses of GDP to 1% shock to financial variables

SPILLOVER MATRIX IN FINANCIAL DIMENSION						
	INT		DEBT		CURR	
	1999-2006		1999-2006		1999-2006	
Country	Out	In	Out	In	Out	In
Italy	1.836	0.063	50.016	6.477	3.089	0.027
Spain	0.657	0.098	-0.174	2.489	0.083	0.066
France	1.828	0.092	0.145	4.904	0.555	0.177
Belgium	-1.979	0.144	0.245	13.594	-0.107	0.465
Netherlands	-2.109	0.108	0.032	4.177	-0.008	0.895
Austria	1.012	0.119	-0.143	12.190	0.079	0.338
Germany	0.263	0.120	0.000	4.571	-0.108	0.233
Finland	0.397	0.075	0.119	-2.410	0.194	0.915
Luxembourg	-3.077	0.194	-0.107	-0.393	-0.113	0.630
Ireland	0.162	0.144	-0.711	3.161	0.007	0.463
Greece	1.875	0.190	0.208	-2.234	-0.063	-0.343
Portugal	0.568	0.085	0.114	3.217	-0.021	-0.281
Average	**0.120**		**4.145**		**0.299**	

Outward and inward growth spillovers are shown for each country in financial dimension for the period 1999 - 2006. They are computed running the selected BVAR model (in Appendix A are drawn the corresponding impulse responses to a shock to real variables).

Accounting for the variable *int*, inward growth spillovers are positive and, hence, Stated Members are net receiver of the system. It is evident since the variable *int* referred to the EMU convergence criterior choosen by policy. Given a shock in the variable, outward growth spillovers are sensitive except to some smaller open economies (as Netherlands and Belgium) which are particularly sensitive to growth shocks on other large Euro area countries (Italy, Spain, France, and Germany). Germany's growth is particularly sensitive to shock in Italy, Spain, and France and some smaller economies (Austria and Finland). France's growth is sensitive to a shock in Italy and Spain, but to a lower extent than Italy and Spain are to growth shocks in France.

The Picture 9.6 draws inward growth spillovers to 1% shock to variable *int*. The impulse responses are smoother with respect to real dimension. In fact, the variable is stricly correlated with fiscal actions and, hence, there have been coordinated recovery actions in the period 1998 - 2007 during the last EMU period (2001 - 2002). The inward growth spillovers was bigger during the period from 2000 to 2002. A significant increasing has been starting from 2006 with the worsening in world economy. This result would prove the need to account for economic and institutional implications when studying real and financial linkages.
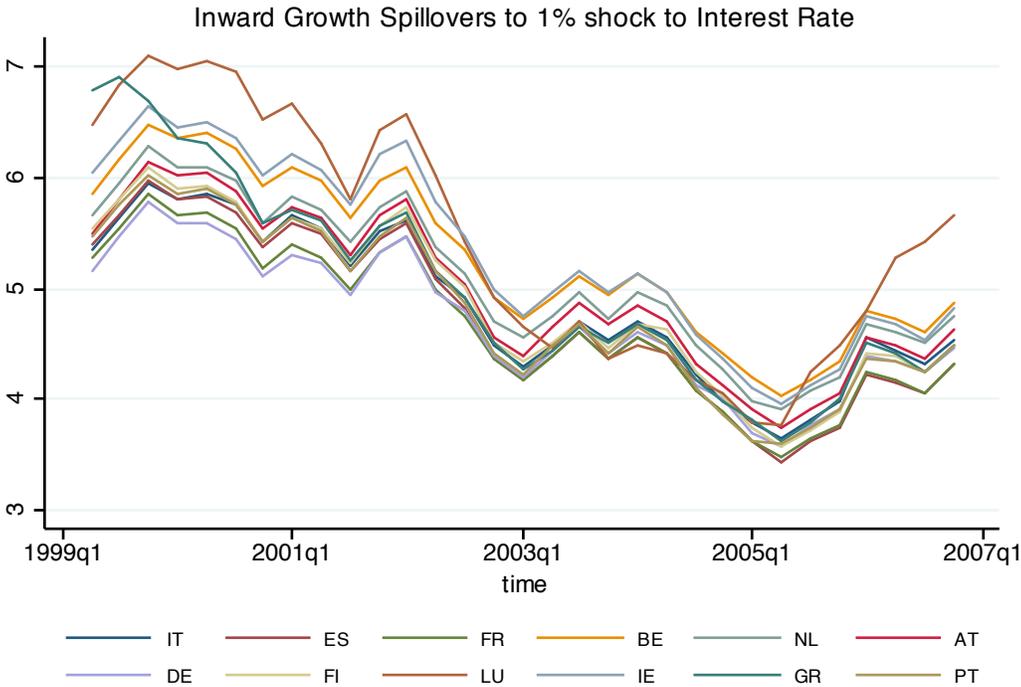
Figure 9.6: The Picture shows inward growth spillovers to a 1% shock to interest rate for the period 1999 - 2007.

In Table 9.3, mostly countries are net receiver of the system. Belgium, Austria, Italy, and France show higher inward growth spillovers than the rest of the EA12. An unexpected growth shock increases divergence and heterogeneity across countriess due to the inflexibility of converge criterions. In addition, the strong interdependence in a common-currency area affects real dimension and other correlated components.

Accounting for outward growth spillovers, mostly countries are net sender of the system; thus, an increasing in the variable *debt* negatively affects the countries' growth.

The Picture 9.7 shows inward growth spillovers to 1% shock to variable *debt*. Italy, Belgium, and Portugal shows higher inward spillover effects; in fact, the same countries have greater debt than others. There is heterogeneity across countries accentuated by divergence in public and private sectors.
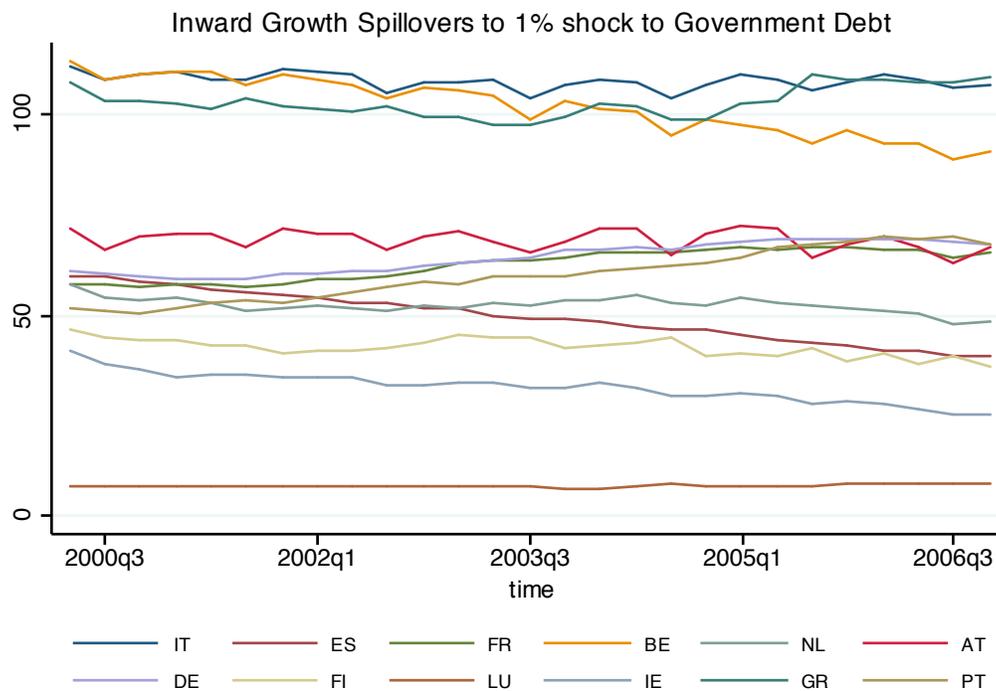
Figure 9.7: The Picture shows inward growth spillovers to a 1% shock to real GDP growth rate for the period 1999 - 2007.

In Table 9.3, accounting for the variable *curr*, mostly countries are net receiver of the system and, hence, potentially sensitive to growth shocks in other countries. Germany'growth is sensitive to Italy, Spain and other smaller economies (Austria and Finland). Spain's growth is much more sensitive to growth shocks in Germany than Germany is to growth shocks in Spain.

Accounting for outward growth spillovers, mostly countries are net sender of the system (e.g., Germany and other smaller open economies as Netherlands and Belgium) and, hence, affect gowths in other countries because of large trade and capital exposures.

This findings would prove greater convergence in the variable *curr* with respect to others in either financial or real dimension because of GSP's monitoring.

In Picture 9.8, inward spillovers to 1% shock to variable *curr* are drawn. During 'before crisis' period, the inward growth spillovers develop in different way showing more trends over time. This result seems to prove the presence of latent factors increasing divergence across countries and worsening economic growth to an unexpected shock.
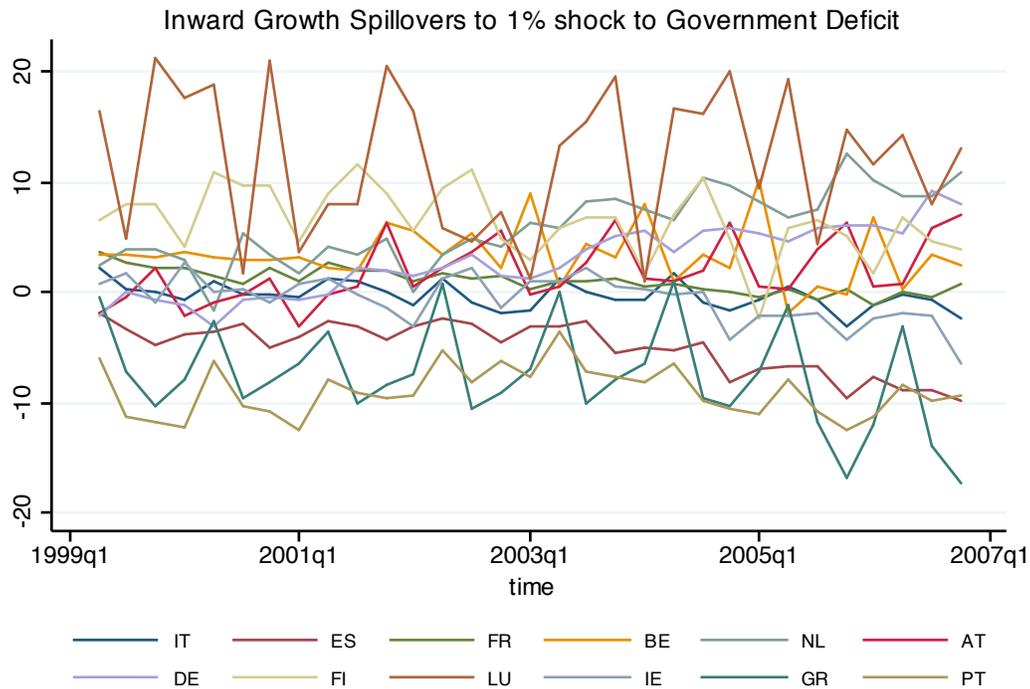
Figure 9.8: The Picture shows inward growth spillovers to a 1% shock to government surplus/deficit for the period 1999 - 2007.

9.2 Bilateral Spillover Effects for common and cross-country factors

Bilateral net spillover effects are computed in real and financial dimension for the period 1999 - 2006. They represent the amplification contribution of the first two lags of the impulse variable to the response variable in order to capture possible sequential features associated with systemic events.

Real Dimension

According to real dimension, selected features of responses associated with unexpected shocks are shown:
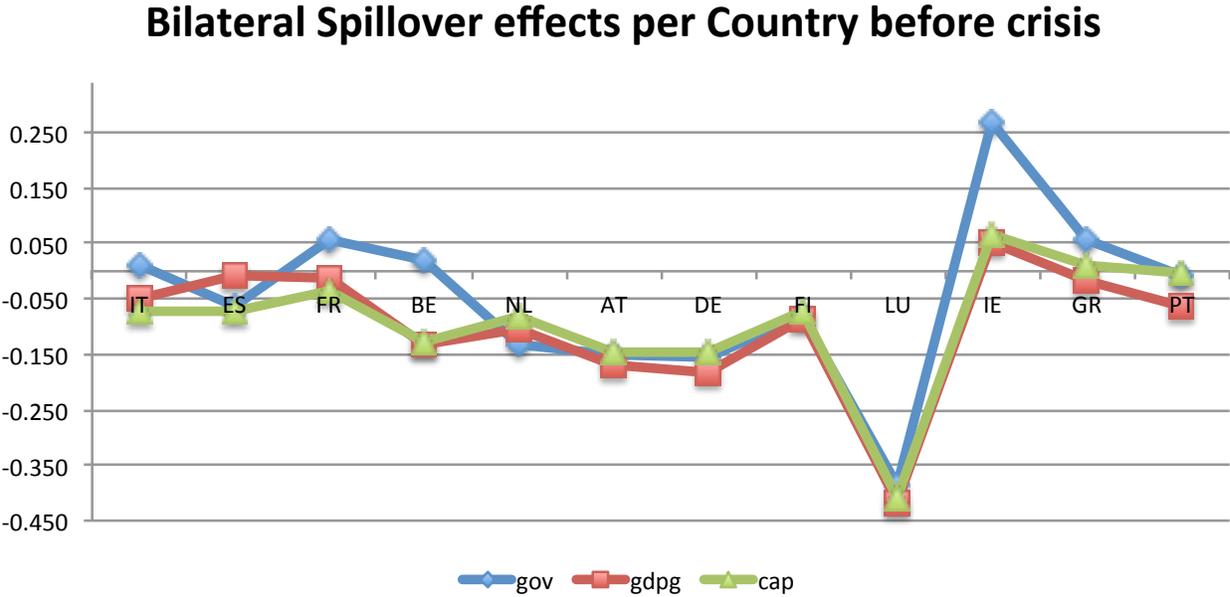
Figure 9.9: The Picture shows bilateral net spillover effects for real dimension for the period 1999 - 2006. The index is computed for each real variable in the system by equation (9.3).

The variable *gov* shows higher values for the strong interdependence with directly policy guidance. About the half of countries are net sender of the system (Italy, France, Belgium, Irelan, and Greece); while, the rest of EA12 countries are net receiver as proved in Section 9.1.

The variable *gdpg* and *cap* show similar path in Netherlands, Finland and some large economies as Germany and France due to the presence of co-movements in real dimension.

Bilateral net spillover effects are also computed for the period 1999 - 2006 ('before crisis'). In Picture 9.10, the variable *cap* is strongly affected by the presence of economic and institutional factors interacting over time. For example, in 2000 - 2002 according to the replacement of the eurozone currencies by the Euro. In the beginning of 2007, the variable seems to increase approaching with the current crisis.

The variable *gdpg* draws lower trends with respect to *gov* adn *cap*. This result holds important cause-effect relationship affecting growth's path in an approximate future.

The variable *gov* seems to be simply affected by direct and/or indirect fiscal actions in Eurozone's governments.
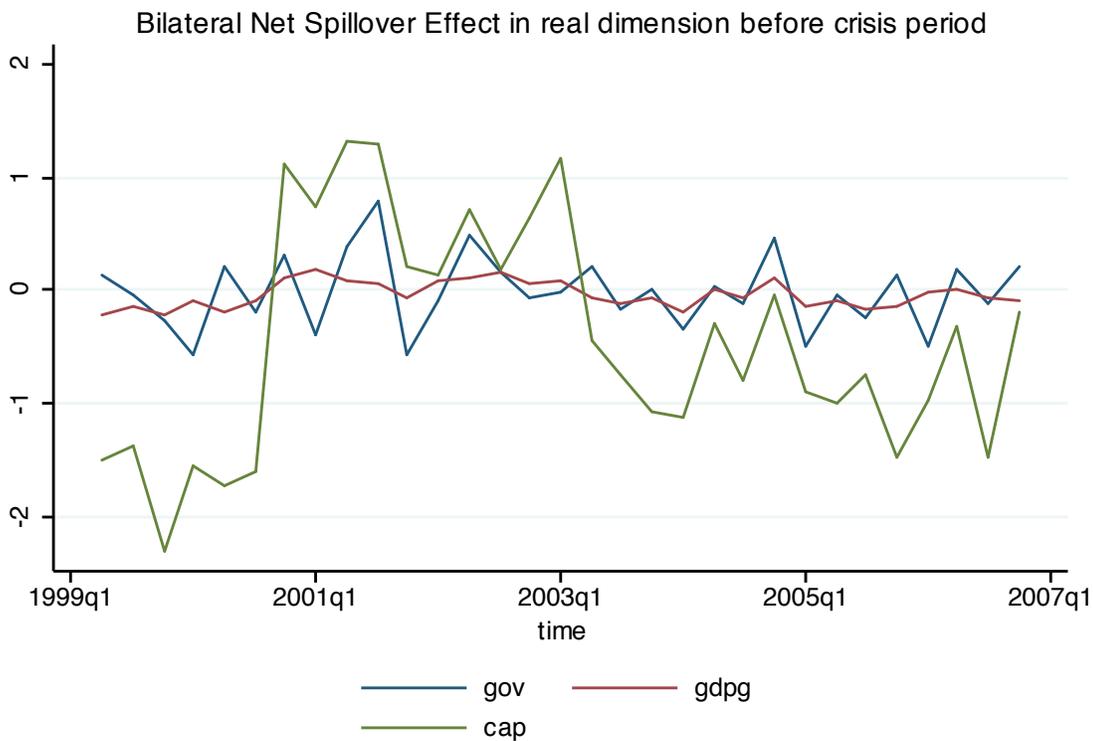
Figure 9.10: The Picture shows bilateral net spillover effects accounting for the selected period 1999 - 2006. The index is computed for each real variable in the system by equation (9.3).

Financial Dimension

According to financial dimension, Picture 9.11 shows bilateral spillover effects for the period 1999 - 2006 per country. Mostly countries are net sender of the system and, hence, unexpected shocks directly affect own output growth and real economy because of interdependencies.

Bilateral Spillover effects per Country before crisis
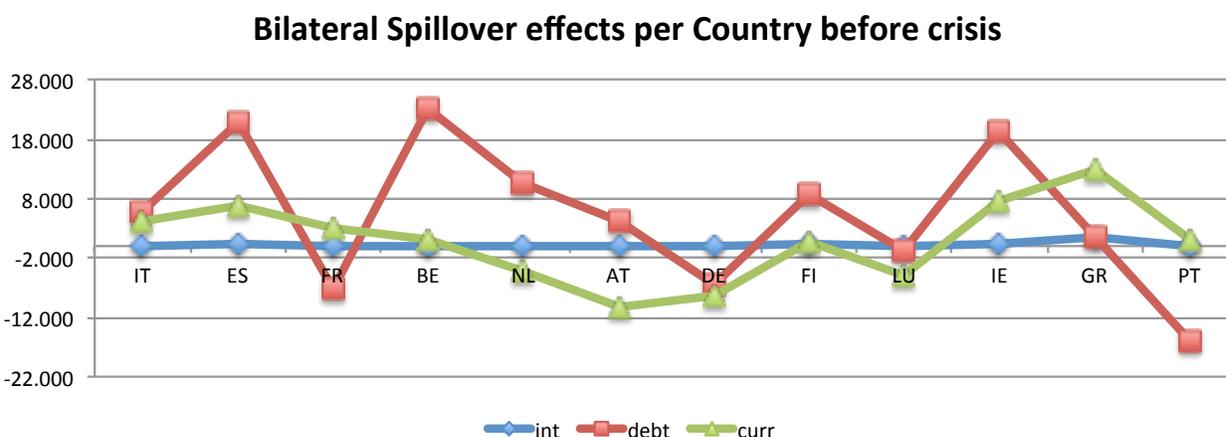
Figure 9.11: ThePicture shows bilateral net spillover effects for real dimension for the period 1999 - 2006. The index is computed for each real variable in the system by equation (9.3).

France, Germany, Luxembourg and Portugal are net receiver of the system in the variable *debt* since sensitive to growth shocks in other countries (see Table 9.3).

Germany and Austria are net receiver of the system in the variable *curr*. Mostly countries are net sender of the system due to *austerity* measures in order to keep under own current account. In the variable *int*, Germany and Luxembourg show negative values. This result would confirm the replacement of the eurozone currencies by the Euro.

Bilateral net spillover effects are also computed for the period 1999 - 2006. In Picture 9.12, countries are either net receiver or sender over time relative to euro convergence criteria and the official launch of the euro (on 1 january 1999 until 2002 with the total replacement of all national currencies). The variables *debt* and *curr* show higher values since potentially strongly affected by policy commitment.

Positive output effects are larger in financial dimension proving that consolidations occurred simultaneously.
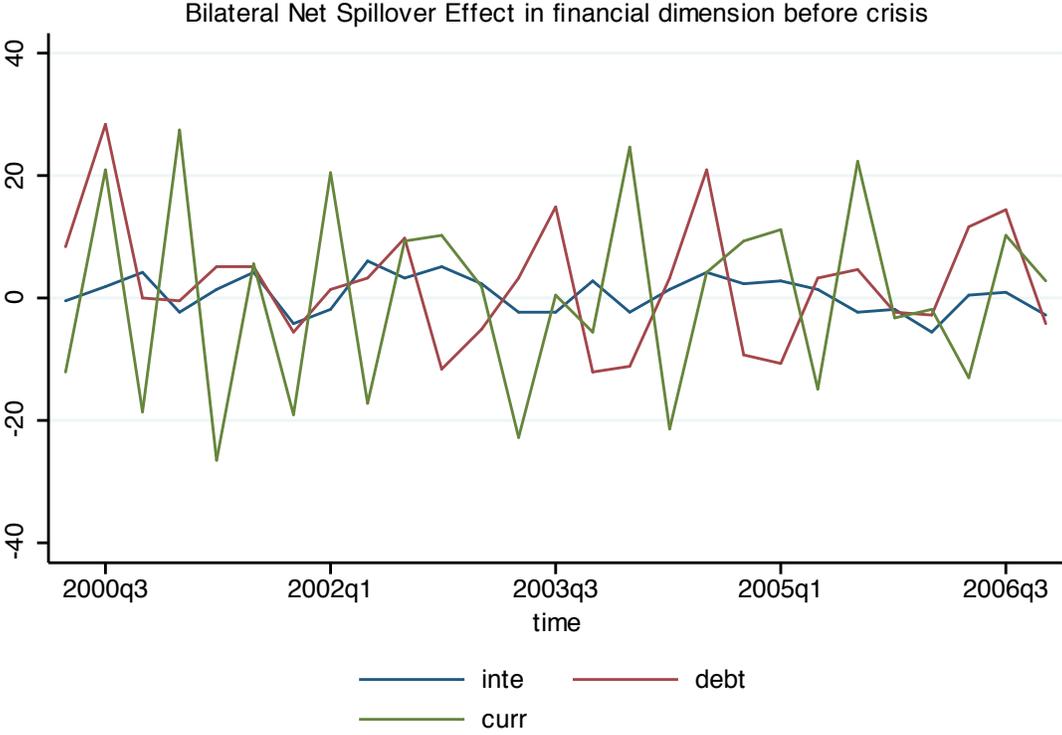
Figure 9.12: The Picture shows bilateral net spillover effects accounting for the selected period 1999 - 2006. The index is computed for each real variable in the system by equation (9.3).

Chapter 10

Systemic Contribution and Contagion Index

In recent years, successive consolidations have depressed growth in the Euro Area. Output effects are significantly larger as consolidations occurred simultaneously, which led to significant spillovers across the Euro Area. Shocks spill over in a heterogeneous way across countries. Moreover, financial variables show higher amplification of spillover effects which can be seen as a result of increased interdependence between variables. The transmission is faster and deeper between financial than real variables. Positive output effects are larger in financial dimension proving that consolidations occurred simultaneously (see Section **??**). A first approach in order to study size and dimension of spreading of spillovers' effects , two index are discussed in this chapter: the systemic contribution and the contagion index.

The systemic contribution is defined as the ratio between the total net contagion effects and the total net positive spillover of the system:

$$SC_{y_i} = \frac{TEC_{NET,y_i}}{TNPspillover} \tag{10.1}$$

The total contagion index of the system is introduced as the average potential spillover effects in the system. There, the cumulative impulse responses are restricted in the interval [0, 1] and the (individual) spillover effects are restricted in the interval [-1, +1] so that the index will be bound between 0 and 100 (or between -100 and 0 if negative effects occur). There are several computational forms. In this analysis, the below formula is used in order to account each variable and its *contagion effect* in both real and financial dimension.

$$CI_{fin} = \frac{100}{N(N-1)} \cdot IR_{y_i \to y_j} \tag{10.2}$$

where, $IR_{y_i \to y_j}$ denotes individual (out) spillover effects. The equation 10.2 is used for real and financial dimension.

The above-mentioned indices are able to observe and, hence, answer to the following questions: how do economic and institutional events affect real economy and financial dimension? How was effect changed over time? What component is more sensitive to unexpected shocks? In which component have unexpected changes had larger impact?

Real Dimension

The Picture 10.1 shows the systemic contribution index for each real variables during 'before crisis' period. The variables *gdpg* and *cap* observe similar path over time because of strict existing correlation between real GDP growth rate and gross capital formation (or Investments). They show negative values until the quarter 2001q1 keeping similar trends for all period in relation to euro convergence criteria. Later, the variables showed positive impact for the following two years. In the previous years to the current crisis, the variables have again observed a negative impact; but, nevertheless, they showed smaller values than oen in the quarter 2001q1.

The variable *gov* show a different trend with respect to *gdpg* and *cap*. This result would confirm the strong impact of pressures of fiscal policy (*austerity*) in relation to financial measures in order to keep a sustainable growth and to guarantee the respect of stablished euro convergence criteria.

The non smooth responses of the real variables, given an unexpected change in real economy, would confirm the strong divergence across countries yet. This divergence become more stringent accounting for public and private factors (see Chapter 12). The Picture 10.1 shows this findings.
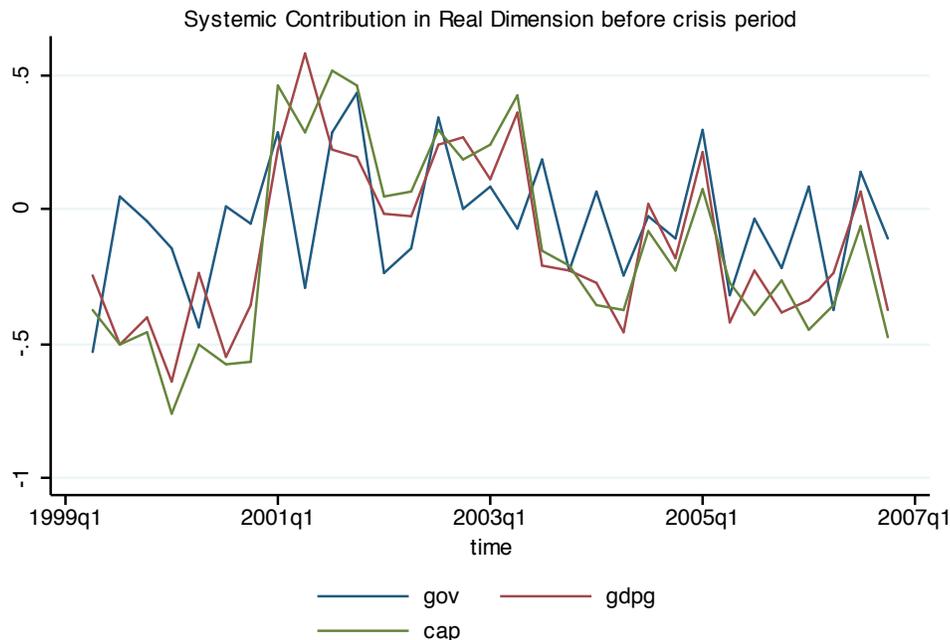
Figure 10.1: The Picture shows the systemic contribution index for the period 1999 - 2006 in real dimension. The index is computed for each real variable in the system by equation (10.1).

The Contagion Index (CI) would confirm the presence of deeper co-movements in real dimension. In Picture 10.2, in the top left box, the CI is observed for each real variable. The variable *cap* show greater value than variables *gov* and *gdpg*. This result seems to prove the strict relationship of the variable *cap* with financial sector. The top right box draws the index per countries in the variable *gov*. There are co-movements across countries like so in the variable *gdpg* and *cap*. Luxembourg shows higher contagion because of small size and little trade and capital flows with the rest of the Stated Memebers. It attended by Ireland and other smaller open economies (Netherlands, Belgium, Austria, and Finland). In the two bottom boxes, the variables *gdpg* and *cap* show the same trend over time. Nevertheless, the variable *gdpg* shows larger impact of the index than the variable *gov*. These findings would confirm the low independence in responding to shocks to real economy (*wide-ranging austerity measures*). In addition, the variable *cap* shows larger index than variables *gov* and *gdpg* due to the strong relationship between investments and trade flows. This result would confirm Euro Area imbalances can be traced back to competitiveness factors rather than *catching-up* relationship.
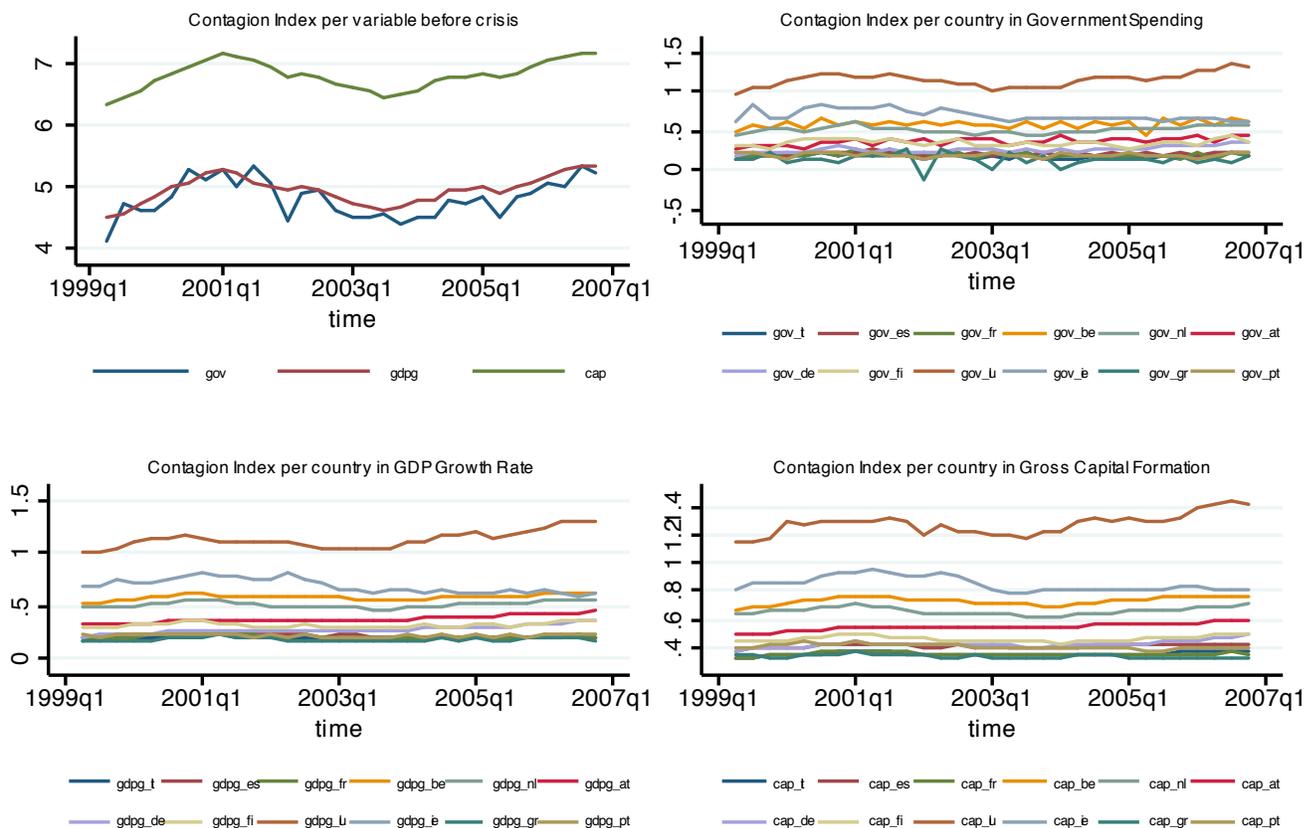
Figure 10.2: The Picture shows the contagion index for the period 1999 - 2006 in real dimension. The index is computed for each real variable in the system by equation 10.2.

Financial Dimension

Accounting for financial dimension, Picture 10.3 shows financial variables differently react to shocks. For example, the variable *int* observe the maximum negative value in the quarter 1999q1 soon after the strong depreciation happened in participant countries. The variables *debt* and *curr* show different trends over time observing positive and negative values during all period in which Euro (€) adopted as common currency.

The responses, given an unexpected change in macroeconomic-financial dimension, are larger than one observed in real economy and, hence, the presence of deeper interdependencies across countries.

Financial component has greater weight than real component; but there is deeper heterogeneity in responding to unexpected shocks in economy. This latter is because of potentially strong relationship with public and private factors. The same relationship turn out to be inverse following a high degree of divergence across countries.

Larger systemic contributions in financial dimension would confirm the prominent role of coordinated fiscal actions across Members; but, at the same time, deeper and faster consolidations depressed growth across countries.
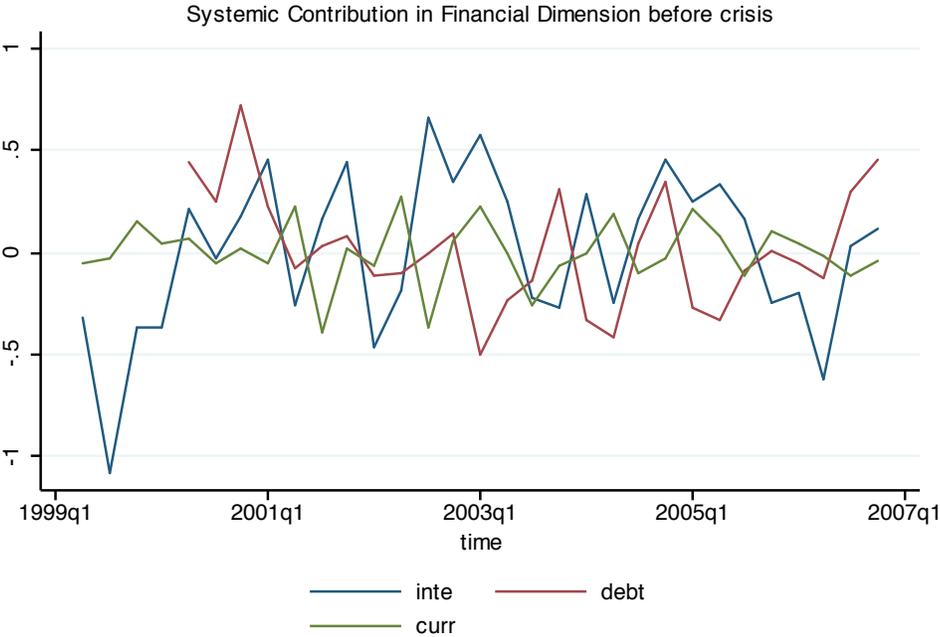
Figure 10.3: The Picture shows the systemic contribution index for the period 1999 - 2006 in financial dimension. The index is computed for each financial variable in the system by equation (10.1).

In financial dimension, the CI shows larger values than one in real dimension. This

latter confirms consolidations occurred simultaneously behind more coordinated fiscal actions across members. A proof of this result is the plot observed in the top right box. Here, the contagion index per countries in the variable *int* is drawn. The index is homogeneously distributed across countries confirming the prominent role of coordinated fiscal actions for the presence of deeper interdependencies. Nevertheless, in the bottom boxes, the contagion index for the variables *debt* and *curr* is plotted. The variables show larger heterogeneity in their trend over time and, hence, lower co-movements than one in real economy. In the variable *debt*, Italy shows greater effect following Greece and Belgium. Moreover, it shows larger values than variables *int* and *curr*. This result would confirm imbalances in the EA are traced back to competitiveness factors and divergence in respecting euro convergence criteria. In the variable *cap*, trends are non smooth than the others because of latent factors behind economic and institutional implications. To be more precise, the financial measurements in keeping the government surplus and deficit at the imposed level combine to bring about deeper divergence across countries given a shock to real and financial economy. In the top left box, the variable *debt* is badly larger than *int* and *curr* because of more accomodating tolerance allowance.
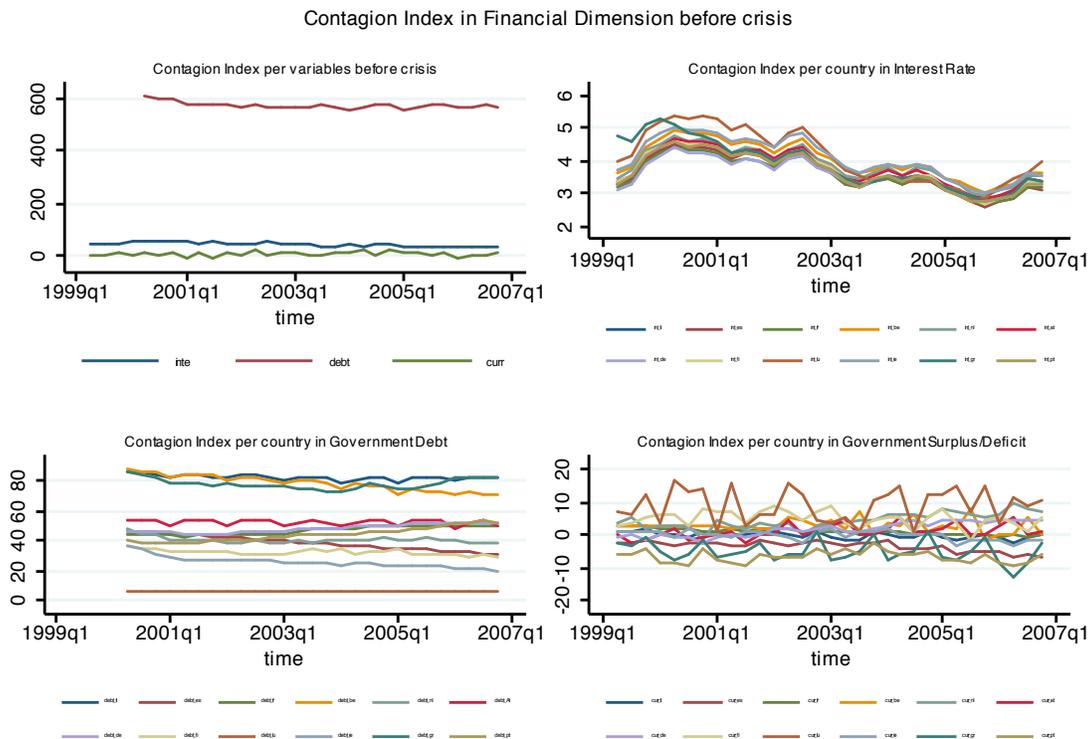
Figure 10.4: The Picture shows the contagion index for the period 1999 - 2006 in financial dimension. The index is computed for each financial variable in the system by equation 10.2.

Chapter 11

How did common and cross-country factors change during the last recession?

11.1 Net Spillover matrix during crisis period and fiscal consolidation

In this Section, real and financial dimension have shown the largest shocks for the crisis period. At the same time, inward growth spillovers across countries seem to have been as sizeable as in recent fiscal consolidations (2011 - 2014), but with mainly synchronized feedback given a shocks. To be more precise, the analysis is consistent with the possibility that larger co-movements or macroeconomic-financial linkages observed in the last recession could be more related to the size of the shocks than to the intensification of their transmission to previous recessions[1]. In Section 12.1, these findings are examined in detail.

Real Dimension

Accounting for the 'crisis period' (2007 - 2011) and 'fiscal consolidation' (2011 - 2014), the Table 11.1 shows outward and inward growth spillovers to 1% shocks to real dimension.

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[1] See e.g., Stock and Watson, 2012

Table 11.1: Responses of GDP to 1% shock to real variables

| | GOV | | | | GDPG | | | | CAP | | | |
| | 2007 - 2011 | | 2011 - 2014 | | 2007 - 2011 | | 2011 - 2014 | | 2007 - 2011 | | 2011 - 2014 | |
Country	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In
Italy	0.67	-0.01	0.01	0.05	-0.10	-0.03	0.41	-0.06	0.39	-0.16	0.20	0.05
Spain	-0.82	0.16	-0.19	0.09	0.58	-0.06	2.81	0.17	0.15	-0.11	-0.02	-0.16
France	0.54	-0.01	0.17	0.02	1.55	-0.13	-0.45	0.11	0.12	-0.16	-0.46	0.08
Belgium	1.25	0.03	-0.34	0.11	-1.79	-0.26	2.42	0.82	-5.61	-0.51	1.01	0.32
Netherlands	0.30	0.07	0.33	0.04	-0.62	-0.23	0.65	-0.20	-0.87	-0.54	0.35	0.19
Austria	0.41	0.11	0.30	0.10	-1.04	0.01	0.26	0.28	2.05	-0.15	-0.19	0.12
Germany	-0.95	0.05	0.11	-0.17	1.41	-0.21	-0.74	0.25	2.89	-0.20	0.34	0.20
Finland	-0.65	0.14	0.26	-0.17	0.25	-0.38	-0.70	-0.23	-2.90	-0.45	0.40	0.09
Luxembourg	-0.09	0.13	-0.04	-0.09	-0.04	-0.69	-0.06	-0.07	0.55	-1.28	-0.13	1.15
Ireland	-0.17	0.16	0.01	0.07	0.72	0.00	-0.32	1.48	-0.11	0.03	-0.18	-0.31
Greece	0.09	0.11	-0.12	0.19	-0.02	0.00	-0.11	-0.13	0.19	-0.07	0.03	0.02
Portugal	0.46	0.08	0.07	0.31	-2.97	-0.09	-1.12	0.64	-0.67	-0.23	0.52	0.14
Average	**0.09**		**0.05**		**-0.17**		**0.25**		**-0.32**		**0.16**	

The heading above the table spans: SPILLOVER MATRIX IN REAL DIMENSION

Outward and inward growth spillovers are shown for each country in real dimension for the period 2007 - 2011 and 2011 - 2014. They are computed running the selected BVAR model.

In the variable *gov*, mostly countries are net senders of the system due to unexpected shocks deriving from financial crisis, as Italy, France, Belgium, Netherlands, Austria, Greece, and Portugal. Germany follows by showing negative outward growth spillovers and, hence, less sensisitivity to shocks in other Eurozone countries. Germany is particularly sensitive to shocks in Belgium, France, and Italy according to size. France becomes a net sender of unexpected shocks on growth in Germany and in Spain (possibly due to large trade exposures). Positive inward growth spillovers proves the presence of co-movements across countries for a given shock affecting real component.

Focusing on the last period (2011 - 2014), all inward growth spillovers are negative and, hence, net sender of the system. According to size, higher component is observed in Greece, France, Spain, Italy, and Portugal. Germany and Austria show relatively less extent with respect to the other EA countries. Thus, restrictive fiscal measures on financial dimension heavily affect real component for the presence of common features and inter-linkages between two selected sectors.This result holds observing negative outward growth spillovers and, hence, the mostly countries are net receiver of unexpected shocks, except Germany and an another smaller economy as Belgium and peripheral countries as Ireland and Portugal.

Grahically,
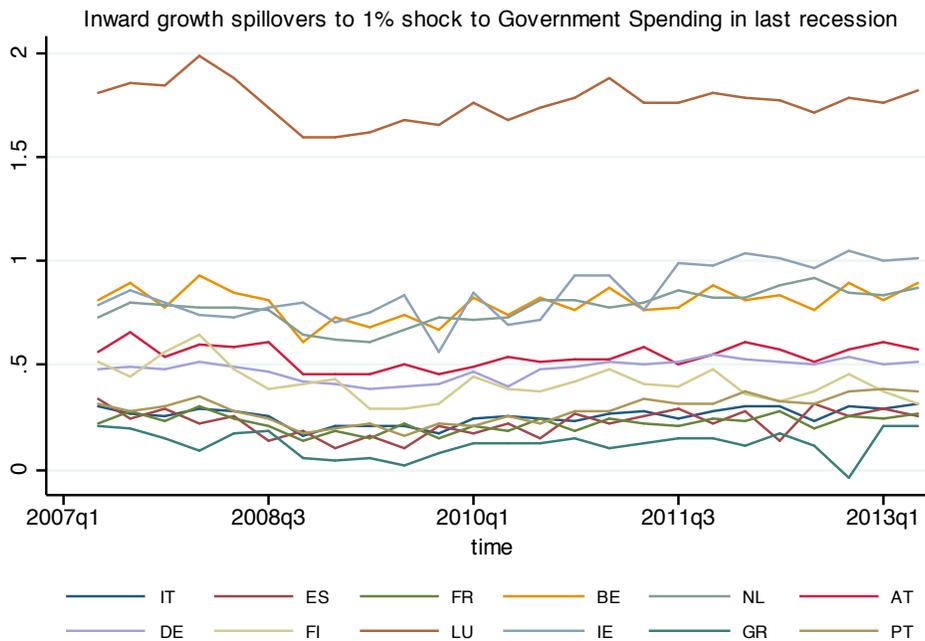
Figure 11.1: The Picture shows inward growth spillover effects to 1% shock to government spending accounting for the crisis period 2007 - 2014.

Accounting for the variable *gdpg*, during crisis period, inward growth spillovers are neg-

ative in majority of countries and, hence, net receiver of the system. Germany's growth is sensitive to other large countries as France and Spain, but less sensitive than Spain is to Germany. This latter is also sensitive to Ireland, Greece, and France.

France is less sensitive to Spain than Spain is to France. Greece, Portugal, Italy and other smaller economies (Netherlands, Austria, and Belgium) are sensitive to unexpected shocks negatively affecting outward growth spillovers.

In Picture 11.2, the responses to an unexpected shocks in Germany, France and Spain, and other smaller open economies (Netherlnads, Austria, and Finland) show higher size. Nevertheless, the increasing in *gdpg* is smaller than one observed in *gov* for the presence of omitted factors affecting the GDP growth rate.
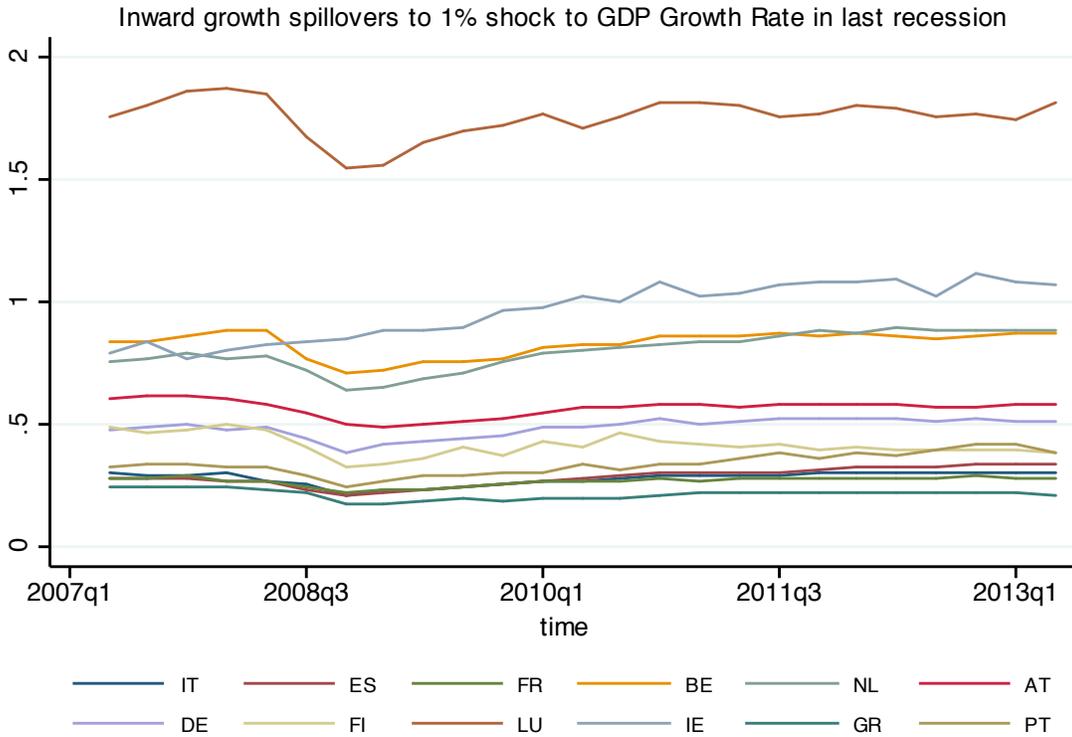
Figure 11.2: The Picture shows inward growth spillover effects to 1% shock to GDP growth rate accounting for the crisis period 2007 - 2014.

In variable *cap* of Table 11.1, inward growth spillovers are negative in the majority of countries. Nevertheless, Germany, attended by Italy, Spain and France, is affected by unexpected shocks to other countries (e.g., crashing into trade exposures). Germany's growth is sensitive to large economies as France and Spain, and to smaller open economies as Austria. However, the former is less sensitive than France and Spain are to Germany.

Greece, Ireland, and Portugal (GIP) are net receiver with respect to the rest of Eurozone countries.

In Picture 11.3, impulse responses are smoother than *gov* and *gdpg* for the presence of large trade exposures between countries, as Germany, France and Spain, and other smaller open economies (Netherlands, Austria, and Finland). The same trade channels will be important future researches for international and non-EA trade flows.
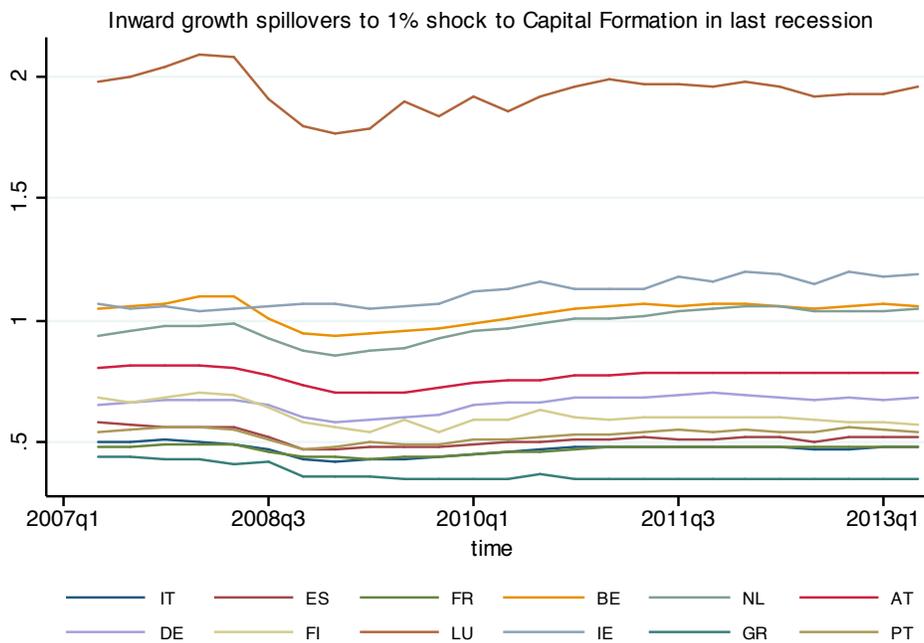
Figure 11.3: The plots show inward growth spillover effects to 1% shock to gross capital formation accounting for the crisis period 2006 - 2014.

Financial Dimension

Accounting for the 'crisis period' (2007 - 2011) and 'fiscal consolidation' (2011 - 2014), the Table 11.2 shows outward and inward growth spillovers to 1% shocks to financial dimension.

Table 11.2: Responses of GDP to 1% shock to financial variables

SPILLOVER MATRIX IN FINANCIAL DIMENSION											
INT				DEBT				CURR			
2007 - 2011		2011 - 2014		2007 - 2011		2011 - 2014		2007 - 2011		2011 - 2014	
Country	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In
Italy	7.46	0.51	2.08	0.60	62.17	5.93	8.43	8.47	0.92	0.60	9.53	1.77
Spain	0.73	0.70	-1.85	0.25	0.36	1.19	0.35	5.84	0.11	-0.14	-1.44	1.54
France	-1.31	0.64	1.37	-0.04	-0.21	1.39	0.76	26.17	-0.05	-0.03	0.16	1.00
Belgium	5.91	0.67	-5.37	0.26	1.76	6.00	-1.90	30.89	0.59	0.67	-0.83	0.15
Netherlands	1.31	0.65	2.68	-0.12	-1.81	12.75	0.73	-2.74	0.08	-0.38	0.12	-0.43
Austria	-13.41	0.61	0.86	0.04	0.87	6.79	0.26	26.83	0.25	-0.81	-0.75	-1.26
Germany	1.59	0.74	6.96	-0.22	0.43	4.27	0.88	21.42	-0.13	0.61	0.66	-0.41
Finland	1.84	0.61	-1.07	-0.03	-0.87	5.75	0.44	5.42	-0.23	0.95	-0.45	-1.65
Luxembourg	3.05	0.73	-3.54	-0.06	0.02	4.43	-0.54	19.48	-0.03	1.22	-0.21	-5.72
Ireland	-0.04	0.37	-0.60	-0.23	-0.76	5.18	-0.73	-15.07	-0.08	0.20	-0.20	1.64
Greece	0.03	0.56	0.09	0.70	0.28	6.57	-0.21	-94.10	0.19	-0.40	0.69	7.93
Portugal	-0.06	0.33	-0.10	0.37	0.36	2.37	-0.06	-24.22	-0.07	-0.95	0.02	2.77
Average	**0.59**		**0.13**		**5.22**		**0.70**		**0.13**		**0.61**	

Outward and inward growth spillovers are shown for each country in financial dimension for the period 2007 - 2011 and 2011 - 2014. They are computed as the impulse responses drawn in selected BVAR model (see Appendix A).

In the variable *int*, impulse responses increased during financial crisis (2007 - 2011). To be more precise, either inward or outward growth spillovers are higher than the 'before crisis' period. Inward gorwth spillovers are always positive proving the strong impact of fiscal actions are on Eurozone countries.

According to outward growth spillovers, some countries (Belgium and Netherlands) becomes net sender of the system affecting other countries through spillovers' transmission channels. Germany's growth is sensitive to growth shocks in Italy and other smaller economies (Netherlands and Finland). Nevertheless, the former affects Spain, France, Austria, and Portugal. Italy's growth is sensitive to growth shocks in Germany, Finland, and Ireland. France's growth is sensitive to a shock in Italy, Germany and other smaller economies (Netherlands, Belgium, and Finland). Moreover, Germany is potentially strongly affected by growth shocks in other countries by own growth independence.

Focusing on the last period (2011 - 2014), Italy, Greece, and Portugal are bigger than one in 'crisis period' due to pressing fiscal-recovery actions. During last years, mostly countries become net sender of the system (Germany, France, Netherlands, and Ireland). This findings would highlight the high degree of heterogeneity across countries given a fiscal spillover effect and the presence of stronger accomodating policy (*austerity*).

In Picture 11.4, Portugal, Greece, Ireland show higher inward growth spillovers attended by Italy, Spain, and France. The other countries seem to show coordinated responses to a fiscal shocks; thus, the degree of heterogeneity could be affected by potential latent factors in public and private sectors.
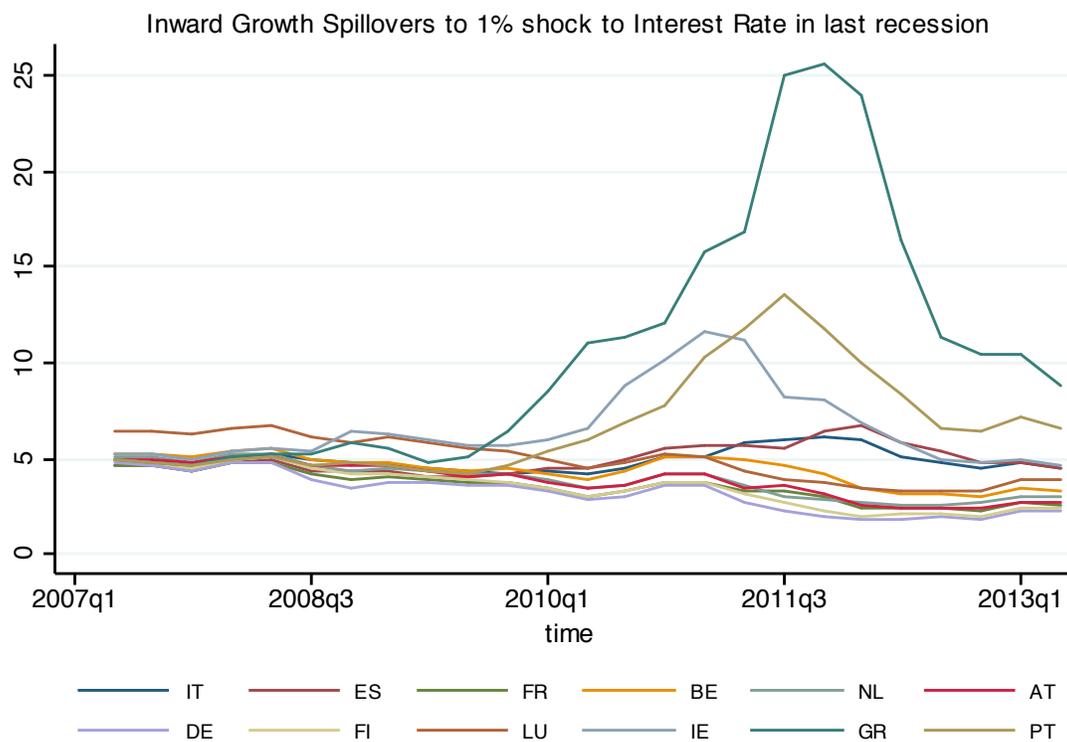
Figure 11.4: The Picture shows inward growth spillover effects to 1% shock to interest rate accounting for the crisis period 2007 - 2014.

Accounting for the variable *debt*, during crisis period, inward growth spillovers are positive in all countries and, hence, they are net receiver of the system. This result seems to two important findings proved in the described SUR model. Accounting for size, Italy, Belgium, and Austria show greater outward growth spillovers in respect to debt load. France, Ireland, and other smaller economies (Netherlands and Finland) are net receiver of the system and, hence, affected by growth shocks in other countries.

During fiscal consolidation (2011 - 2014), Greece, Portugal, and Ireland, in order of size, are net sender of the system affecting other countries given a shock in own current account. Mostly countries are net receiver and, hence, potentially strongly sensitive to growth shocks due to recovery fiscal actions (*austerity*). Accounting for size, Belgium, Austria, and France show greater inward growth spillovers than others. Mostly countries are sensitive to unexpected growth shocks (negative spillovers).

According to outward growth spillovers, Belgium, Ireland, Greece, and Portugal are net receiver of the system and, hence, potentially sensitive to unexpected growth shocks.

In Picture 11.5, Greece, Italy, Portugal, and Ireland show bigger inward shocks spillovers than others.
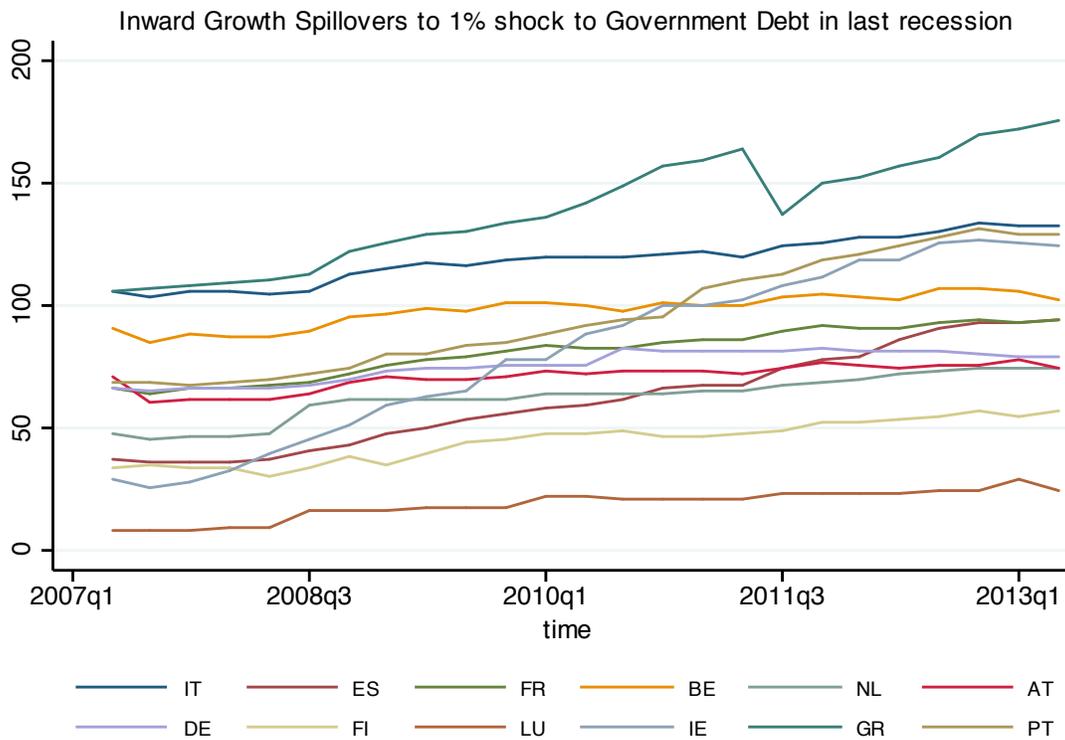
Figure 11.5: The Picture shows inward growth spillover effects to 1% shock to government surplus/deficit accounting for the crisis period 2007 - 2014.

By Table 11.2, inward growth spillovers given a shock in the variable *curr* are bigger than one during 'before crisis' period. Moreover, some countries (Spain, France, Greece, Portugal ans other smaller economies as Netherlands and Austria) become from net receiver to net sender of the system. Therefore, these countries are sensitive to growth shocks and affect other countries trough trade and capital transmission channels. These findings increase interdependencies across countries and, hence, the size and magnitude on how shocks spill over.

In Picture 11.6, during fiscal consolidation, heterogeneity in inward growth spillovers across countries shrink. The maximum value is observed during 'crisis period' due to economic and institutional implications arisen from divergence in public and private sectors.
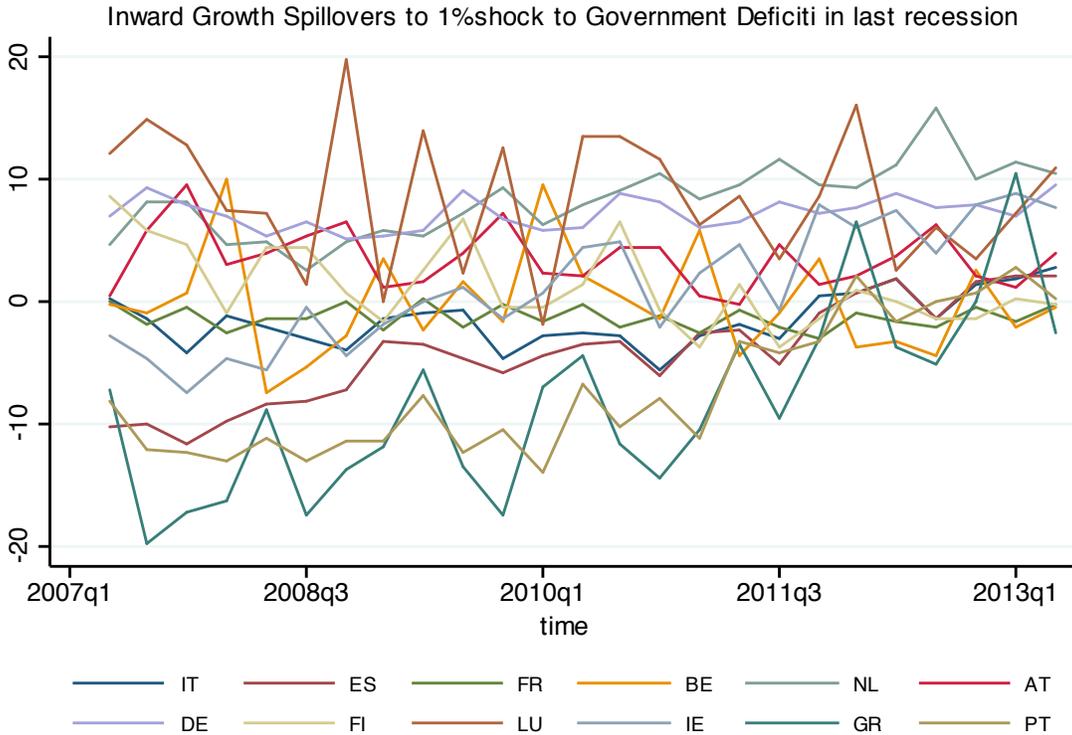
Figure 11.6: The plots show inward growth spillover effects to 1% shock to government surplus/deficit accounting for the crisis period 2007 - 2014.

11.2 Fiscal spillover effects and their main components

Bilateral Net spillover effects are drawn for 'crisis period' and 'fiscal consolidation' accounting for real and financial dimension.

Real Dimension

The Picture 11.7 shows the variable *gdpg* is negative for mostly countries and, hence, are net receiver of the system. PIG shows greater values than the rest of the EA12, following Spain, France, and Italy . The variable *cap* is almost constant and a net sender in the majority of countries. It seems to be due to strong interdependencies with external transmission channels. The variable *gov* is rather uneven across countries proving the existence of strong degree of heterogeneity between public and private sectors.
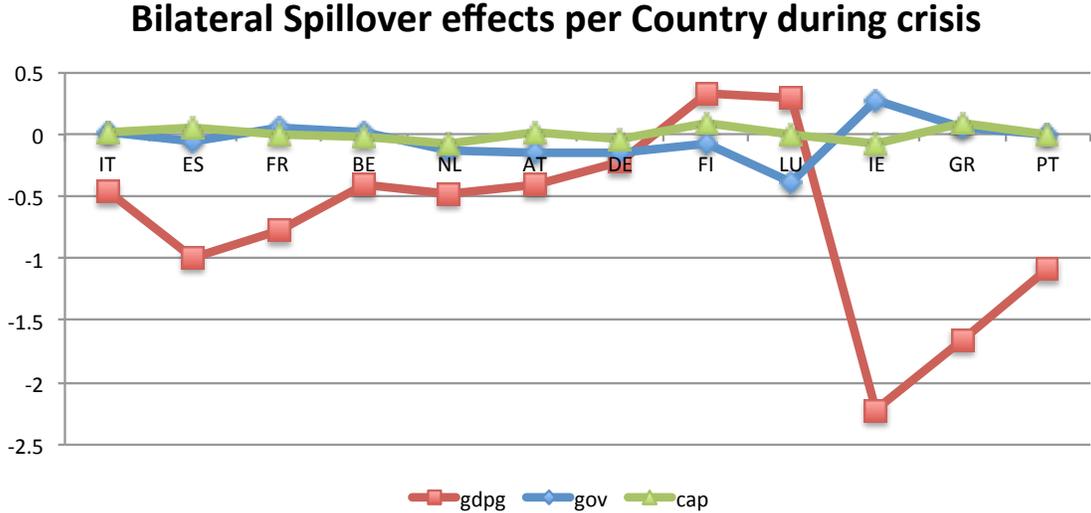
Figure 11.7: The plots show bilateral spillover effects per country during crisis period. The index is computed by equation 9.6

During fiscal sonsolidations, there is a sharp improvement on extent of transmission shocks across countries showing a smoother responses. Nevertheless, the degree of heterogeneity holds over time.

The variable *gov* and *gdpg* are again net receiver of the system in mostly countries as Italy, Spain, Belgium, Germany, Austria for the large trade exposures to the rest of Europe.

The variable *cap* is otherwise a net sender in more than half countries just as it is an important component for transmission trade channels.

## Bilateral Spillover effects per Country during fiscal consolidations
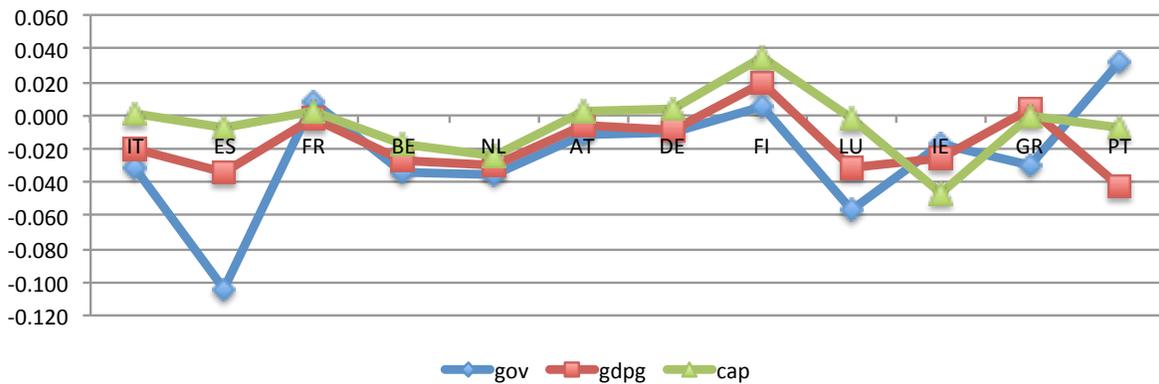
Figure 11.8: The plots show bilateral spillover effects per country during fiscal consolidation. The index is computed by equation 9.6

During crisis period, total bilateral spillover effects are negative (net receiver) and, hence, countries are sensitive to growth shocks given. To be more precise, an unexpected shock (given an economic/institutional occurrence) on real economy affects own output growth. The maximum value is observed during financial crisis for the period from 2009 to 2011. The variable *gdpg* show bigger values than others the above-mentioned period.

Accounting for size, it is attended by *gov* and *cap*. Unlike everyone else, the former is negative and reach positive value in the quarters 2011q4, 2012q3, and 2013q1 during fiscal consolidations.

Therefore, there is large interdependence between real and financial sectors and economic/institutional events affect spillovers' trend over time.
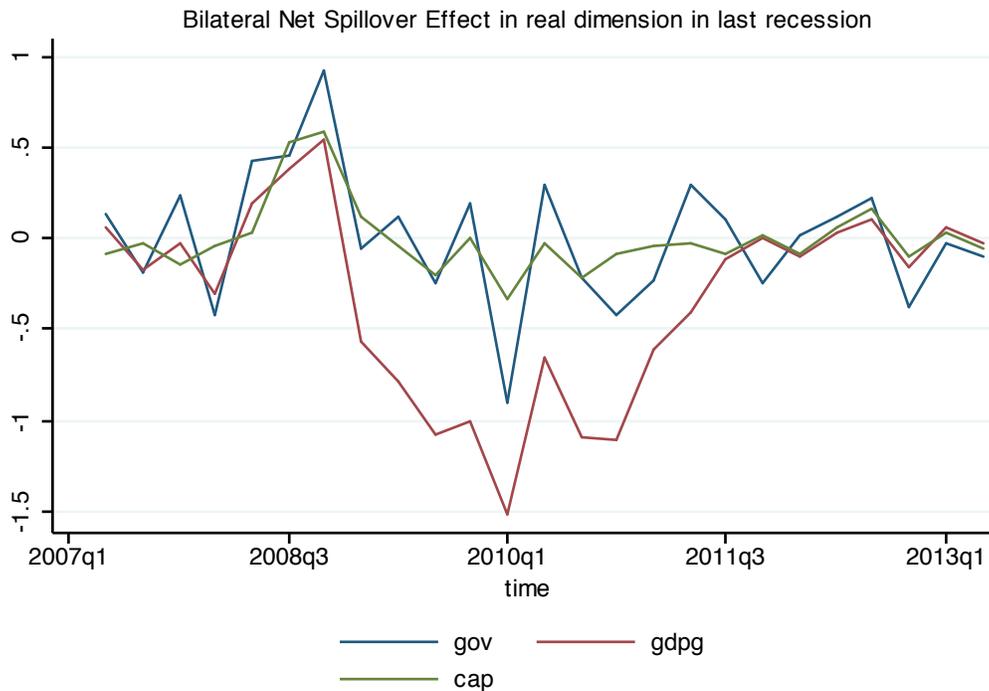
Figure 11.9: The plots show bilateral spillover effects for the period from 2007 to 2014. The index is computed by equation 9.6

Financial Dimension

The Picture 11.10 shows the variable *int* is negative in Portugal, Ireland, Italy, Greece, and Spain (PIIGS) and, hence, are net receiver of the system being potentially sensistive to growth shocks in other countries. The remaining countries are net sender of the system because of financial measures. Germany observed the greater total bilateral net spillover effects (as *leader country*) and attended by Luxembourg, Belgium, France, and other smaller economies (Austria and Netherlands). The variable *debt* is negative in all countries and, hence, negatively affected by financial measures in order to keep a sustainable growth. The variable *curr* observe smoother responses to economic/institutional events than others. Spain, France, and other smaller open economies (Finland, Ireland, Greece, and Portugal) are net receivers of the system due to a greater worsening in own current account.
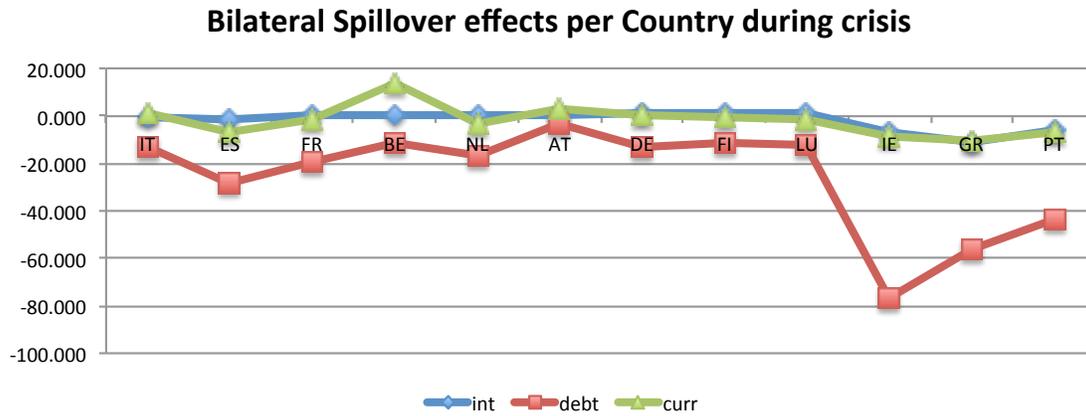
Figure 11.10: The plots show bilateral spillover effects per country during crisis period. The index is computed by equation 9.6

During fiscal sonsolidations, there is a sharp improvement in the variable *debt* observing smaller values than one during 'crisis period'. However, countries remain net receiver of the system, except Germany, because of strong worsening in debt accounts given recovery fiscal actions (*austerity*).

The variable *curr* seems to be more monitored than *debt* by GSP's commitment.

Finally, the variable *int* is positive in mostly countries sending impulses to other countries trough inter-linkages across countries and to own output growth since tied under fiscal control actions.
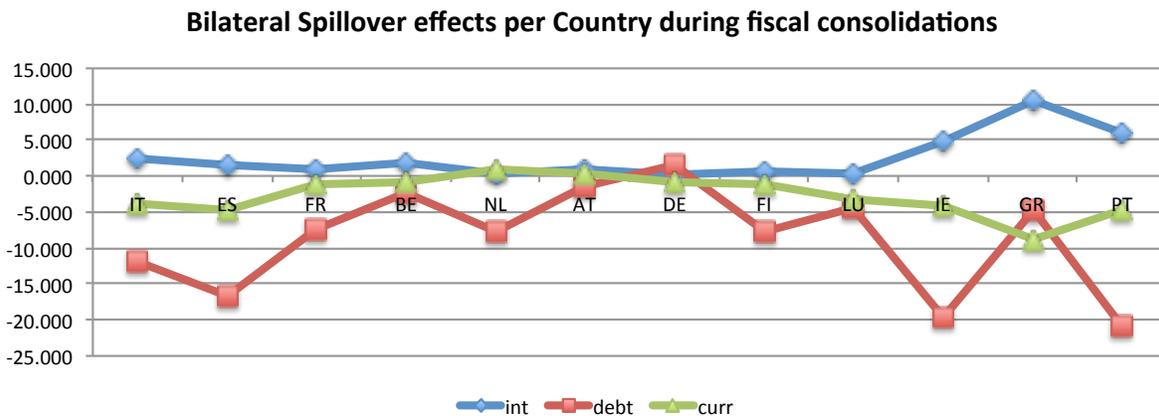
Figure 11.11: The plots show bilateral spillover effects per country during fiscal consolidation. The index is computed by equation 9.6

Accounting for the entire period from 2007 to 2014, total bilateral net spillover effects show greater size and magnitude than one in real dimension. However, there is heterogeneity on how shocks spill over across countries showing non smooth trends over time. In fact, BSEs are either positive or negative with relation to financial shocks and adopted fiscal measures.

During fiscal consolidations, BSEs show unchanged trends, but they become net sender of the system affecting growth and real economy of Stated Members. This result seems to confirm the financial dimension has a common and an idiosyncratic component, but the former was larger during the more recent crisis in its financial dimension and even more in its real dimension.

Positive output effects are larger in financial dimension proving that consolidations occurred simultaneously.
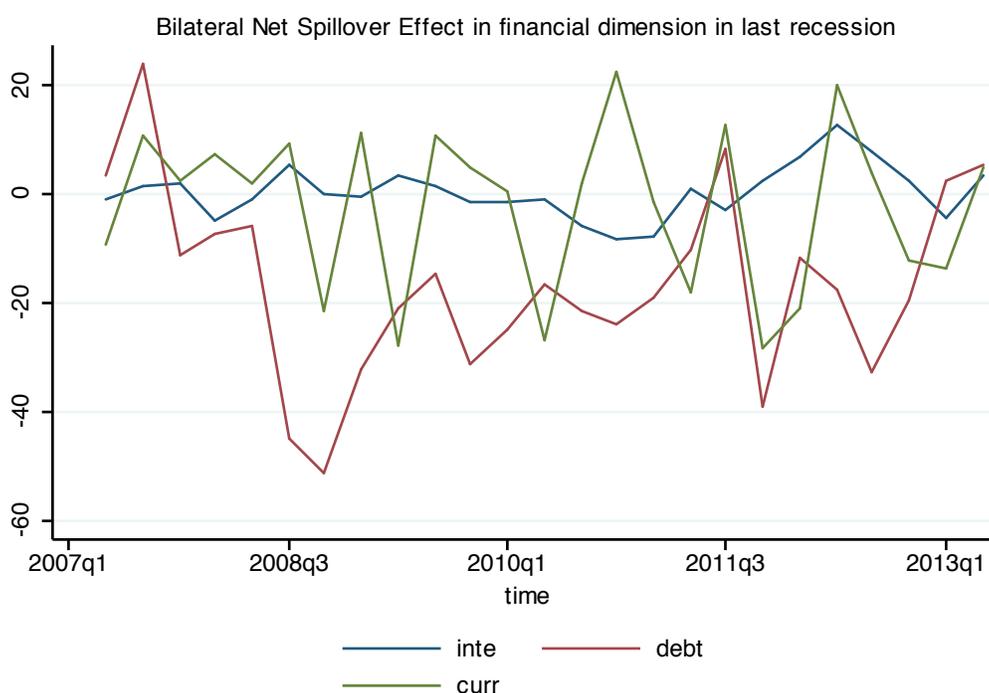
Figure 11.12: The plots show bilateral spillover effects for the period from 2007 to 2014. The index is computed by equation 9.6

The Section 11.2 show several important findings. In real dimension, the most countries are net receiver of the system for the entire selected *time-series* (1999 - 2014). This result would confirm the strong interdependence between real and financial dimension and real dimension has a greater common component highlighted by large trade exposures across countries and by the importance of *austerity* policies in the last period.

Given an unexpected shock following extreme economic/institutional changes, financial dimension show higher size and magnitude in BSEs than one in real dimension. The find-

ings prove the greater incidence of financial sectors and the stronger interdependence across countries in financial component due to *austerity*'s fiscal policy. Later, these changes affect real dimension trough trade flows, which has a larger significance than capital transactions.

The responses are larger in financial component proving the importance of the size of shocks than to the intensification of their transmission. Moreover, countries show greater heterogeneity in own financial accounts and larger co-movements in their real dimension. However, in this latter, countries typically are net receiver of the system and, hence, more sensitive to growth shocks in other countries than into their financial dimension.

These inter-linkages across countries result into a worsening in output gap becuase of strong divergence in latent factors (public and private sectors) and the importance of the competitiveness in supporting their current account and unexpected changes in real economy to the detriment of *catching-up* and *causality* relationships.

Systemic Contribution and Contagion Index

The Picture 11.13 shows the systemic contribution index for each real variables during crisis period and the recent fiscal consolidations. The variables show positive responses to shocks to real economy. Positive values confirm deeper pressure on the real variables, especially for the quarters from 2008q1 to 2010q2.

During fiscal consolidations, there have been lower effects on real economy. Nevertheless, the variables show positive values with respect to one during 'before crisis' period in Picture 11.13.

In the previous quarters to fiscal consolidations, real economy showed negative value because of no significant responses to shocks. Finally, the trend of the systemic contribution over time would confirm deeper common component in real dimension.
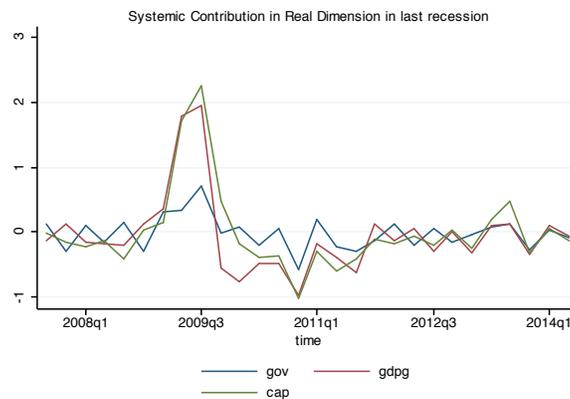

Figure 11.13: The Picture shows the systemic contribution index for the period 2007 - 2014 in real dimension. The index is computed for each real variable in the system by equation (10.1).

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Accounting for financial dimension, Picture 11.14 shows financial variables differently react to shocks. For example, the variable *int* observe the maximum value in the quarter 2013q4 soon after the strong depreciation happened in participant countries by fiscal measurements (*austerity*). Different trends confirm lower common component in financial dimension. Moreover, this result would confirm the prominent role of latent factors affecting financial variables responses and, hence, the impact on real economy. In fact, observing Picture 11.14, the variable *debt* show negative impact during crisis period because of more accomodating tolerance allowances. Conversely, the variable *cap* is less sensitive than other variables due to more stringent control on its tolerance level.

Focusing on the current crisis, financial variables mostly react negatively to systemic events with respect to real dimension due to extreme fiscal-agreement measures (or deeper *wide-ranging austerity measures*). Therefore, the increasing of divergence across countries and deeper imbalances in the last recession can be traced back to important economic and institutional implications and to competitiveness factors affecting real economy trough large treade exposures across countries.

Larger systemic contributions in financial dimension would confirm the prominent role of coordinated fiscal actions across Stated Members; but, at the same time, deeper and faster consolidations depressed growth across countries.
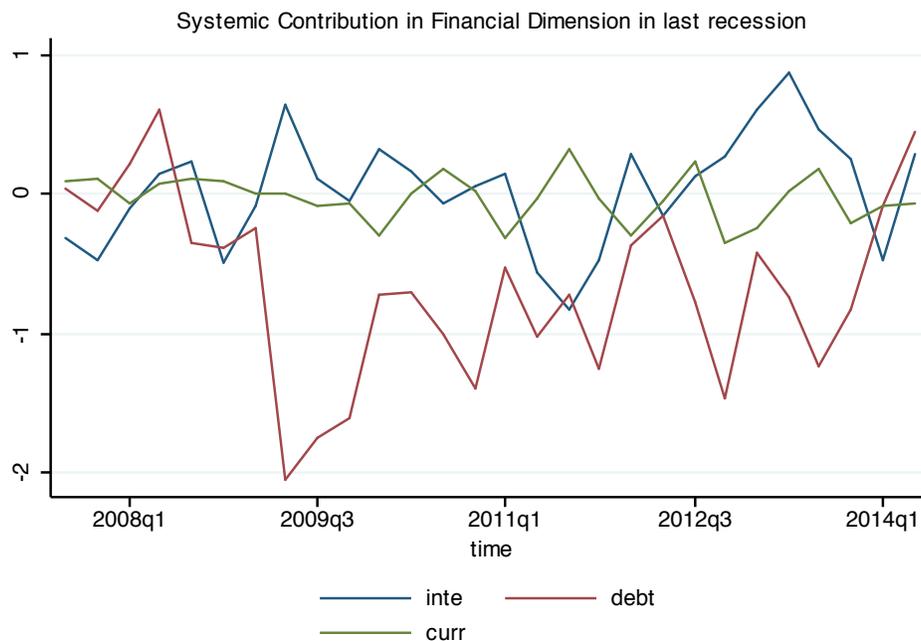
Figure 11.14: The Picture shows the systemic contribution index for the period 2007 - 2014 in financial dimension. The index is computed for each financial variable in the system by equation (10.1).

During crisis period, the Contagion Index (CI) in real dimension is greater than one for the period from 1998 to 2014. The variable *gdpg* observes larger values during fiscal consolidation. This result would confirm the increasing of imbalances in responding to shocks to real economy across countries. The variables *gov* and *cap* increased during fiscal consolidations, but observing on average values lower than one during crisis period. These findings are shown in top left box of Picture 10.2.

Generally, the index shows the common component has a strong impact over time. This result would confirm real dimension is affected by common factors. These latters differently spill over across countries for a large degree of heterogeneity in generating inward spillover effects. In addtion, the same heterogeneity is increased due to a strong divergence existing across countries' public and private sectors.
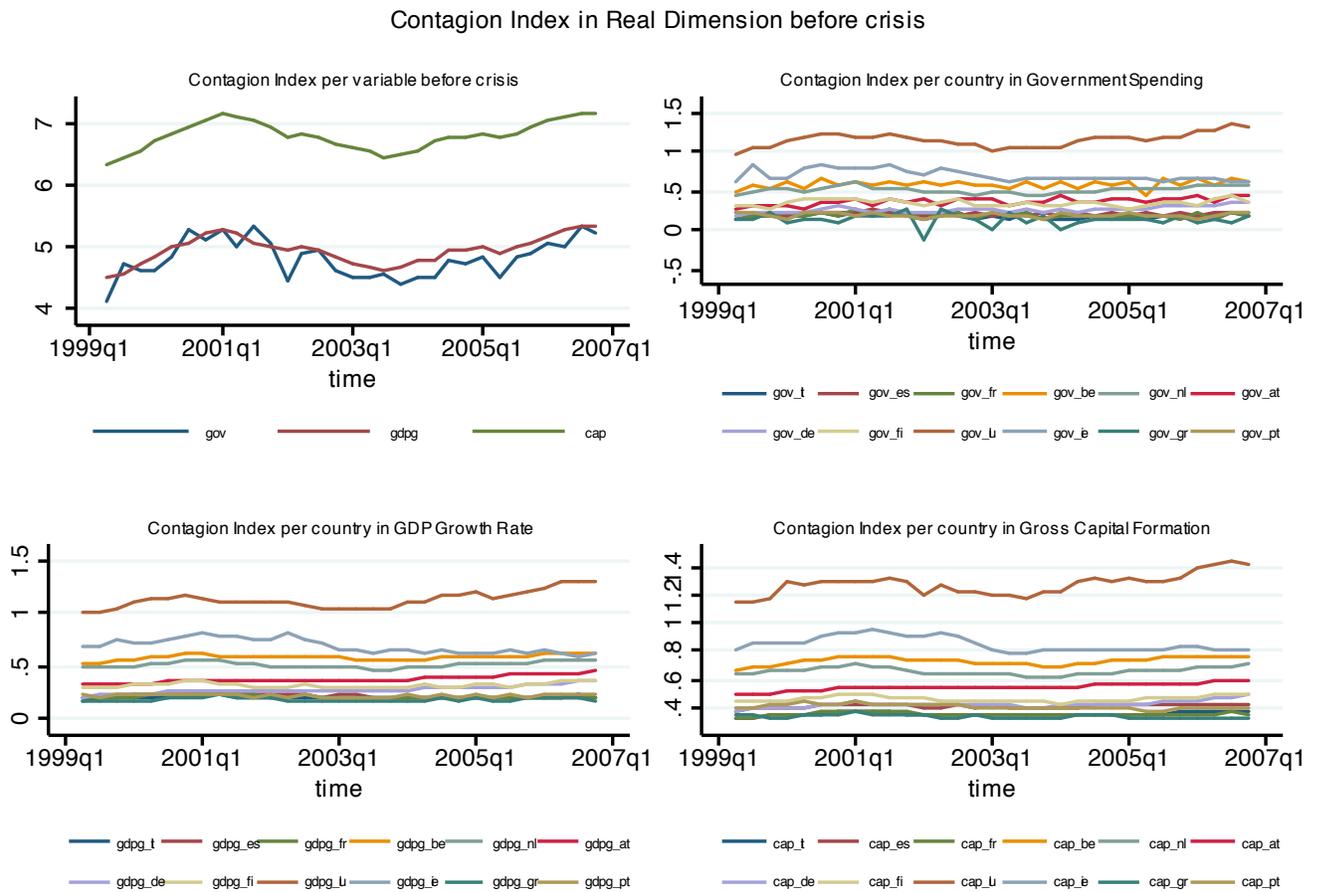

Figure 11.15: The Picture shows the contagion index for the period 1999 - 2006 in real dimension. The index is computed for each real variable in the system by equation 10.2.

In financial dimension, the CI shows the importance of economic and institutional factors in affecting inward growth spillovers. During fiscal consolidations, the variable *int* shows different indices. For example, Greece, Ireland, and Portugal observed greater values than other countries. Following Spain, France, and Italy in order of size. This result is drawn in the top right box of Picture 10.4. The variable *debt* observe higher value than *int* and *curr*. This result seems to derive on too much deeper tolerance allowances. The variable *curr* shows non smooth trends in the contagion index. These findings would confirm increasing of imbalances during last recession and even now in observing inward growth spillovers can be traced back to competitiveness and other economic/institutional factors triggering a *cause-effect* relationships rather than *catching-up* events (see Section **??**).
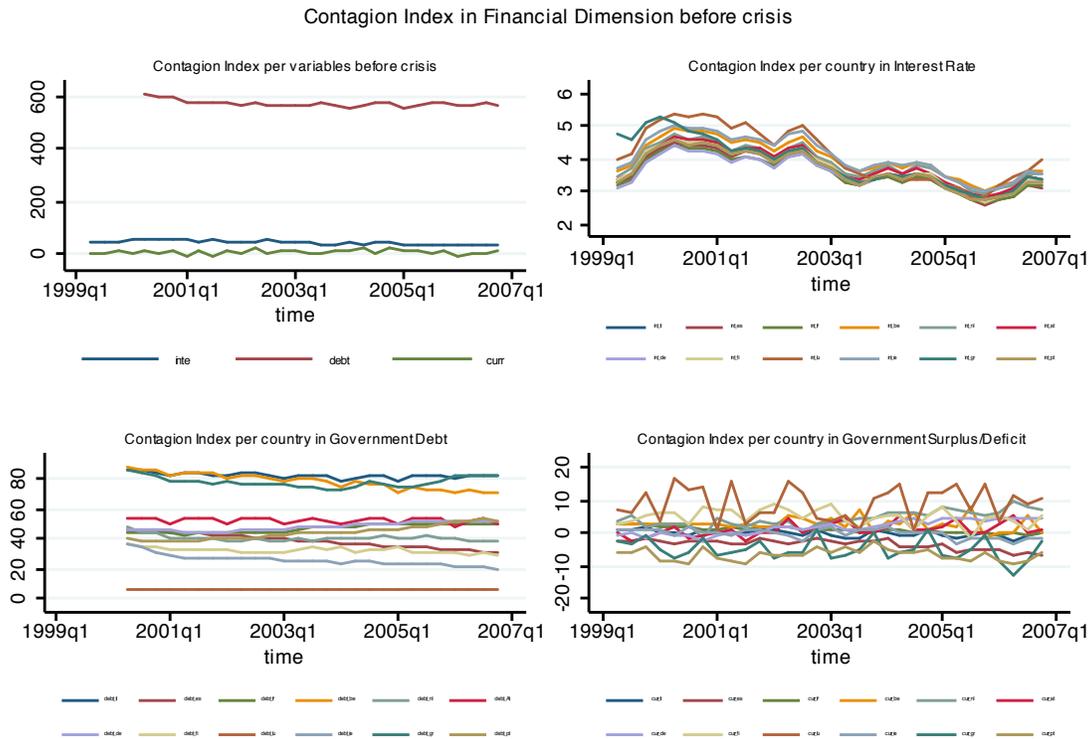
Figure 11.16: The Picture shows the contagion index for the period 1999 - 2006 in financial dimension. The index is computed for each financial variable in the system by equation 10.2.

Chapter 12

Commonality vs. Heterogeneity

12.1   Evolution of group-specific and common factors

Accounting for common factors, systemic contribution and contagion index are shown for real and financial dimension. Transmission channels and selected latent factors are observed.

In Picture 12.1, in bottom boxes, financial variables are observed. They keep the same feature of the data with low hight difference. This result seems to confirm that growth shocks within the EMU are to a relatively larger extent transmitted via monetary and financial linkages.Focusing on financial component, the results confirm the prominent role of coordinated fiscal actions for the presence of deeper interdependencies. In fact, the contagion index decreased more than half during fiscal during spillover consolidations.

Upper boxes draw real dimension showing different trend with respect to financial component. For example, during 'crisis period', the systemic contribution index keep positive and greater values than one in financial dimension. These findings would prove that trade channels and economic/institutional implications are very important in evaluationg growth shocks. The latter appear to be relatively larger with respect to trade channels. In fact, in the top-right box, the systemic contribution observes lower values. Nevertheless, the index in real dimension follows to observe higher values during 'crisis period' than one in financial dimension. This result would prove the high degree of heterogeneity in spreading of spillover effects in real dimension and the presence of potential outliers within and outside the EMU. The latter appear to be predominantly transmitted via trade channels.

Systemic Contribution
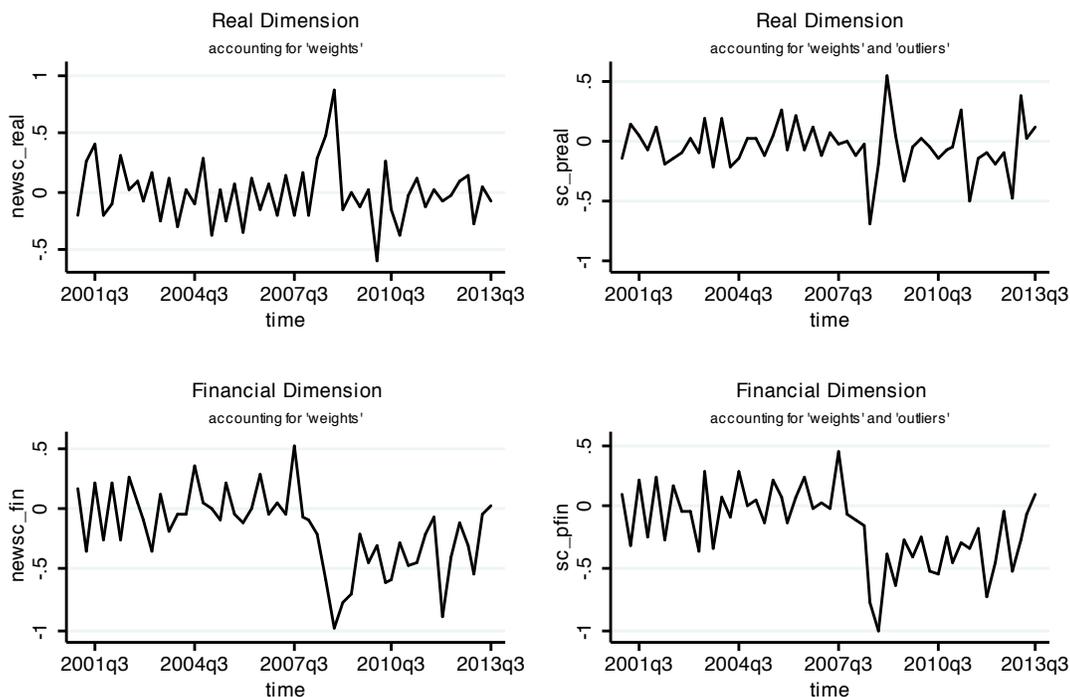
Figure 12.1: The Picture shows systemic contribution for real and financial dimension accounting for *weights* and *outliers*. The coefficient vectors observed in the analysis correspond to χ_{1t}, χ_{2t}, χ_{3t}, and χ_{4t} of equation 6.22.

Thus, the project aims at measuring whether there are significant co-movements among these countries and variables that simple summary statistics and bilateral spillover effects cannot identify in depth. The model estimated is described in equation 6.22.

After estimating different specification of this model, the highest marginal likelihood was found for the model including four country-specific components for each economy, four variable-type components, and two common components for all series. The first accounts for the coefficient vectors χ_{1t}, χ_{2t}, χ_{3t}, and χ_{4t}: χ_{1t} and χ_{3t} shared by real and financial variables, respectively, across countries accounting for *weights* and χ_{2t} and χ_{4t} shared by real and financial variables, respectively, across countries accounting for *weights* and *outliers*. The four variable-type components correspond to coefficient vector χ_{5t}: one shared by all real variables with *weights*, another shared by all financial variables with *weights*, another shared by all real variables with *weights* and *outliers*, and another shared by all financial variables with *weights* and *outliers*. The two common components account for the coefficient vector χ_{6t} shared by all series with *weights* and *outliers*, respectively.

These common, country-specific and variable-type components quantify the relative contribution of common and heterogenous factors in macroeconomic-financial linkages and help

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to address the following questions: Is there a significant common component in the real and financial interactions across eurozone members or do cuntry-specific heterogeneities matter more? How did *weights* and *outliers* factors affect real economy and financial variables over time? What is the importance of transmission channels and latent confounding effects when studying growth shocks across countries within a common currency area?

Despite the heterogenous behaviour showed in Chapter 9, there is indeed a significant common component, especially in the last recession, in its financial dimension and even more in its real dimension. The result seems to confirm the existence of a statistically significant common factor linking these seemingly heterogenous real and financial series across all countries and throughout several cycles.

For example, focusing on Pictures 12.2 and 12.3, evolution of the first and third country-specific factors over time is drawn. Real and financial dimension accounting for *weights* are estimated, respectively. Real variables, in Picture 12.2, show higher degree of heterogeneity than financial component (Picture 12.3). In fact, the box plot is comparatively large and it suggests that overall countries have a low level of *agreement* with each other. These findings would confirm the increasingly importance of capital flows in driving the spreading of spillover effects. The result is consistent with the more recent literature and empirical evidence of IMF (2014) and ECB (2013) recognizing growth shocks are predominantly transmitted via financial linkages. Moreover, box plotes in Picture 12.2 box large difference between them and, hence, there is high degree of divergence across countries to fiscal shocks originating within the EMU.

Higher values in Picture 12.3 confirm more coordinated fiscal actions over time in financial dimension with respect to real dimension. Moreover, the median (the line that divides the box into two parts) observe negative values. These findings would confirm that country are by turns net receivers and net sender of the system over time, absorbing and generating, respectively, growth shocks (see Chapter 9 for more details). Stronger effects are observed in some smaller open economies (Netherlands, Austria, Belgium, and Finland) and, with the debt crisis, also to larger countries such as Spain, Italy, and France).
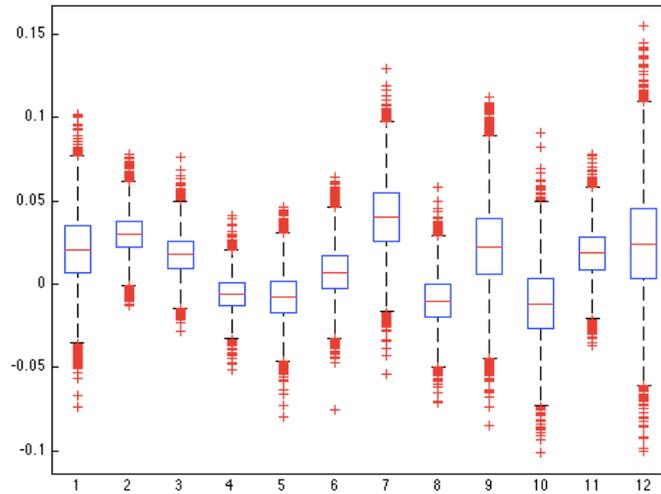
Figure 12.2: The Picture draws the country factors of all real variables with *weights*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to χ_{1t} in the model described in 6.22. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.
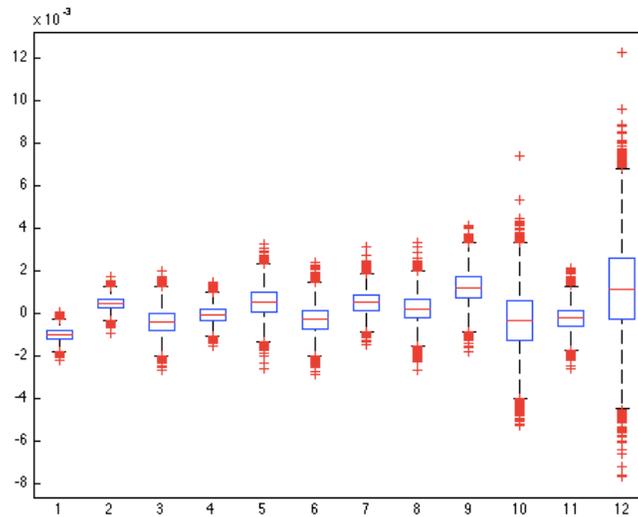

Figure 12.3: The Picture draws the country factors of all financial variables with *weights*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to χ_{3t} in the model described in 6.22. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

Pictures 12.4 and 12.5 show real and financial dimension, respectively, accounting for both *weights* and *outliers*. These findings would confirm the importance of accounting for other sectors such as labour and household's market and other *latent* factors such as competitiveness, evolution of consumption, investments, productivity across countries. Variability decreases in real dimension proving the strong divergence across countries. Box plots in Picture 12.4 observe larger different distributions. Moreover, median is mostly positive and, hence, real economy is a net receiver of the system. This results would confirm potential unobserved variables strongly affect real economy. To be more precise, countries more strongly affected by outward growth shocks becuase of large trade exposures with other member states

In Picture 12.5, box plots observe higher value than ones in Picture 12.3 and, hence, more coordinated fiscal action in financial dimension. However, those severe adjustment pressures have depressed output for the presence of a persistent divergence across countries in their real component.
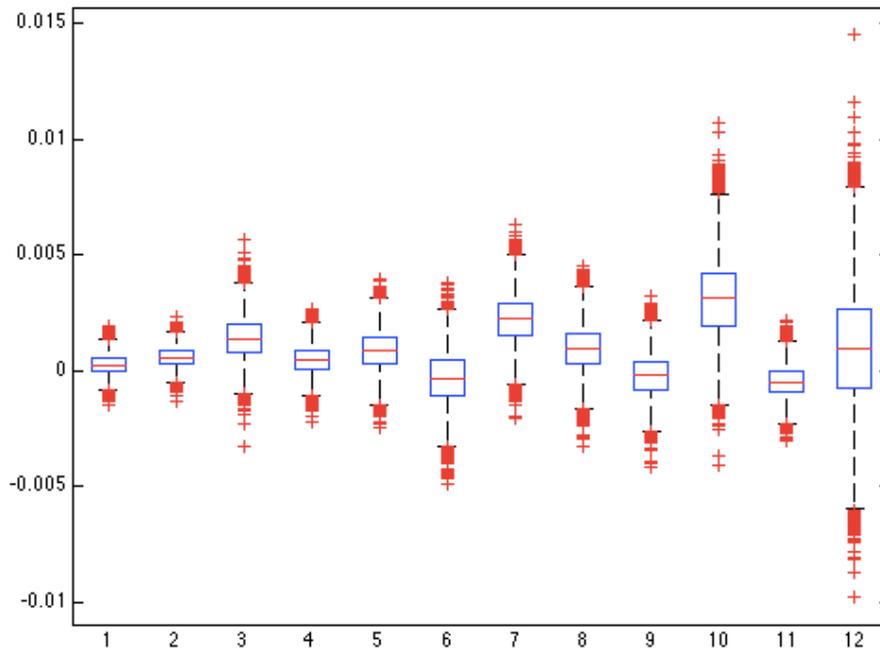

Figure 12.4: The Picture draws the country factors of all real variables with *weights* and *outliers*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to χ_{2t} in the model described in 6.22. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.
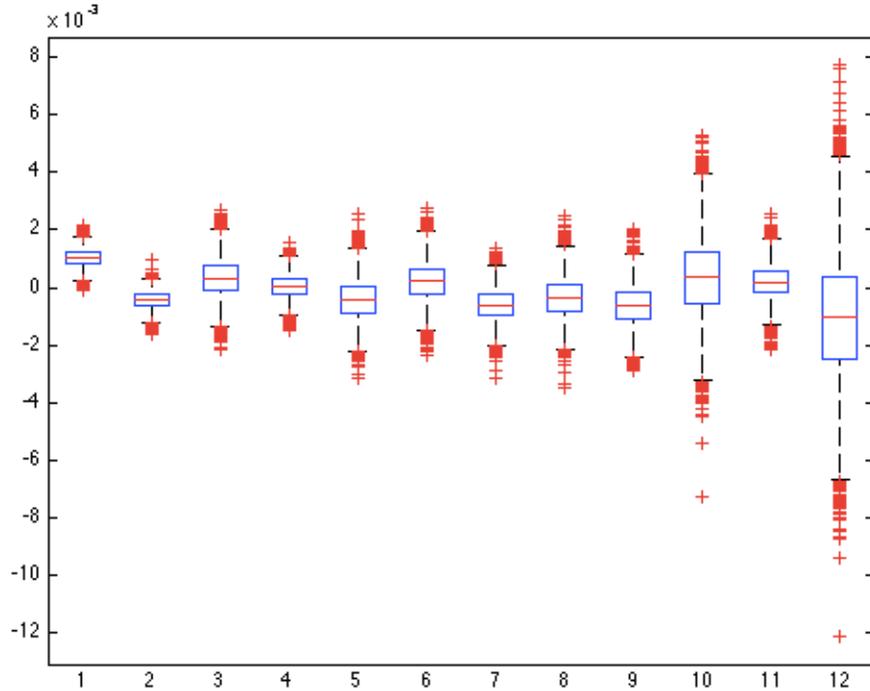
Figure 12.5: The Picture draws the country factors of all financial variables with *weights* and *outliers*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to χ_{4t} in the model described in 6.22. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

Accounting for group-variable specific factors, Picture 12.6. draws real and financial dimension for all sample with and without *outliers*, respectively. Financial dimension without *outliers*(second box-plot) shows larger variability and, hence, more sensitive to inward growth shocks originating within the EMU. In the third and fourth box-plots, latent factors are accounted for other potential inter-linkages across countries. Real dimension shows higher variability than financial dimension. This result seems to confirm that real economy is potentially strongly sensitive to outward growth shocks originating by changes and responses in other sectors.
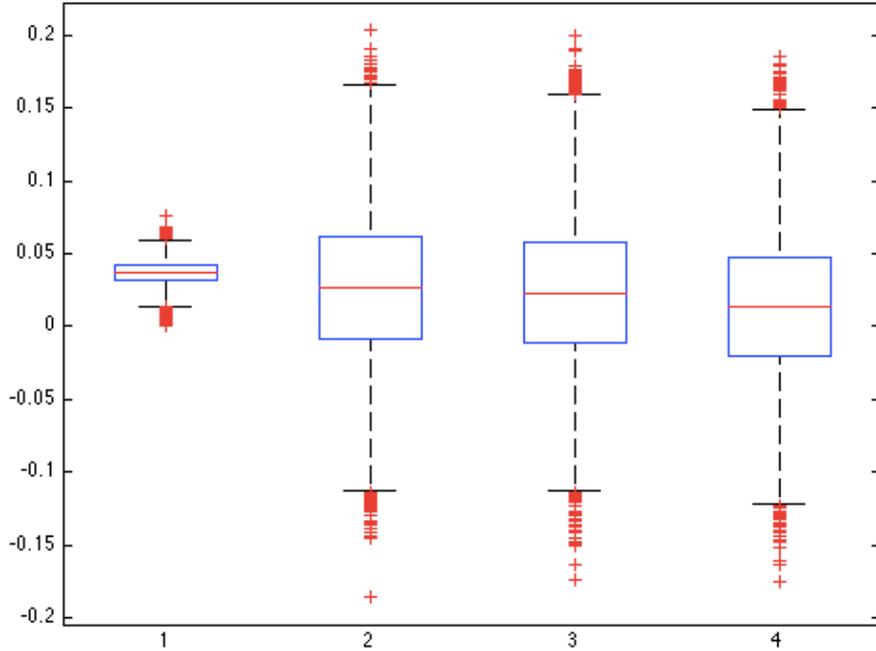
Figure 12.6: The Picture draws the variable factors across all countries for real and financial dimension, respectively, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to χ_{5t} in the model described in 6.22. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

Finally, common componet accounts for all real and financial variables in an only coefficient vector observed with and without *outliers*. The result seems to prove the importance of accounting for both common and idiosyncratic components, which is large in its financial dimension and even more in its real dimension. Thus, real variables are strongly sensitive to outward growth shocks due to large interactions with financial and no-financial sectors.
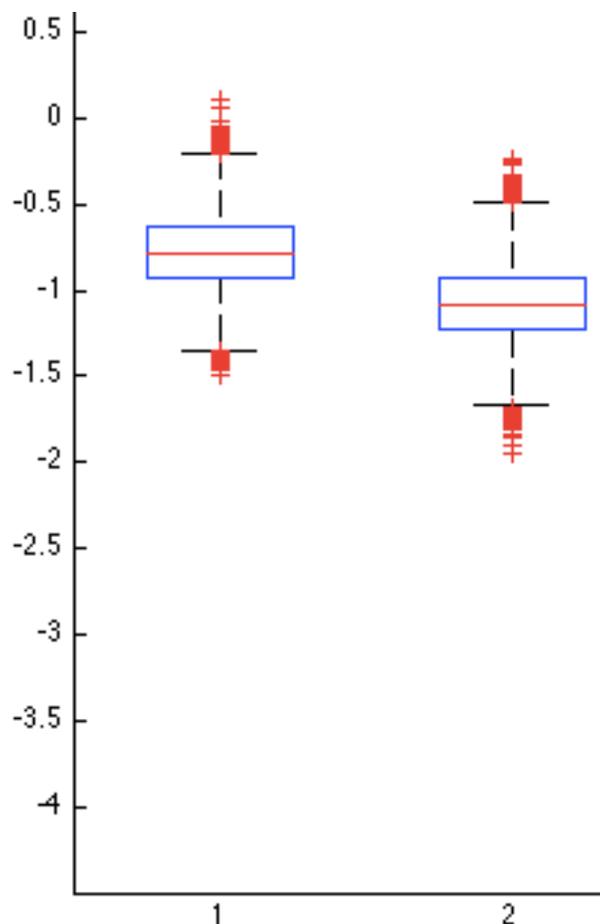
Figure 12.7: The Picture draws the country factors of all macroeconomic and financial variables with and without *outliers*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to χ_{6t} in the model described in 6.22. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

12.2 Sign and dimension of cross-country spillovers from real and financial shocks

In order to highlight commonality and heterogeneity accounting for omitted economic/institutional implications, a 3D-dimensional graph will be plotted for real and financial dimension. The aim of the analysis is to observe magnitude and effect size of potential spillover effects over

time in the Eurozone. For this latter, the following order of magnitude and size[1] will be used:

- *yellow* : empty sample corresponding to no significant impulse response,

- *orange* and *light green* : *small size* with $0.1 \leq x \leq 0.3$,

- *sky blue* : *small & medium size* with $0.4 \leq x \leq 0.5$

- *navy blue* : *high size* with $0.6 \leq x \leq 0.7$

- *red* : *extreme size* with $0.8 \leq x \leq 1$

The analysis draws surface plots in all EA12, accounting for real and financial dimension, and for transmission channels (as driven extent). The selected periods are: 1999 - 2007 and 2007 - 2014.

The below surface of fitting objects complies with previous analysis. The effect size and magnitude of inward spillover effects appear limited over period, with values lower than 0.2 (*light green*). High values are shown when crisis is getting close with a magnitude of 0.8 (*navy blue*). Nevertheless, there is a considerable increasing in terms of heights reaching the minimum and maximum point in -1.5 and +1.5 respectively. Negative (net receiver country) and positive (net sender country) values confirm heterogeneity in transmission of growth spillovers across countries. In addition, the increasing of potential spillover although with low magnitude proves the presence of latent factors strictly correlated to real and financial linkages.

[1]The analysis restricts the cumulative impulse responses in the interval [0,1]
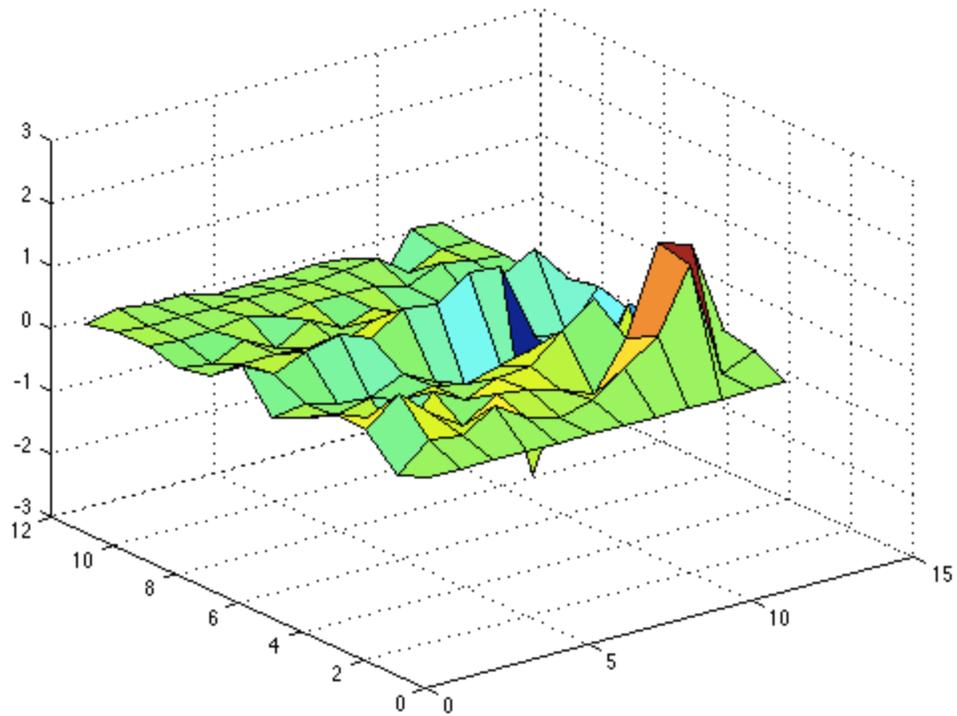
Figure 12.8: The Picture plots height and magnitude of potential growth spillovers given a 1% shock in real dimension for the period from 1999 to 2007. The plot is obtained creating a three-dimensional shaded surface from the z components in matrix Z (fitting objects), using x = 1:n and y = 1:m, where [m,n] = size(Z). The height, Z, is a single-valued function defined over a geometrically rectangular grid. Z specifies the color data, as well as surface height, so color is proportional to surface height. The coefficient vector analyzed in this study is χ_{6t} of the equation 6.22.

Proceeding with the analysis, surface plot is also conducted in the last recession account-ing for fiscal consolidation. The Picture 12.2 would prove the existence of higher values. Mostly magnitude keep values bigger than 0.8 with an improvement on the spillover heights. Financial crisis and fiscal consolidation have, hence, affected almost contemporaneously real and financial dimension. This results refers to the presence of strong interlinkages between sectors, which can be analyzed by considering trade and capital flows across countries. More-over, higher values in terms of divergence (or not-smoother surface of fitting objects) would highlights the importance of economic and institutional factors[2] indirectly affecting the usual relationship between real and financial variables. Given high heterogeneity across countries in public and private sectors, spillovers differently affect countries creating divergence and no-coordinated impulse responses.
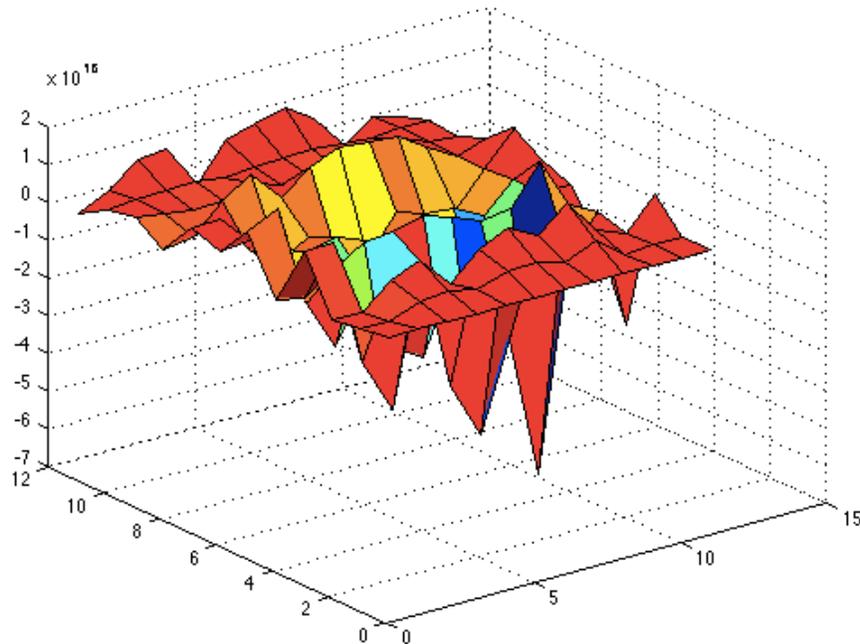

Figure 12.9: The Picture plots height and magnitude of potential growth spillovers given a 1% shock in real dimension for the period from 2007 - 2014. The plot is obtained creating a three-dimensional shaded surface from the z components in matrix Z (fitting objects), using x = 1:n and y = 1:m, where [m,n] = size(Z). The height, Z, is a single-valued function defined over a geometrically rectangular grid. Z specifies the color data, as well as surface height, so color is proportional to surface height.χ_{6t} of the equation 6.22.

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[2]In this analysis, the selected variables *unem*, *lab*, *cons*, *priv*, and *inv* are considered in data frame.

Conclusion

The paper develops an approach to conduct inference in time-varying coefficients using a Bayesian multicountry VAR models with lagged cross-unit interdependencies and unit-specific dynamics. Bayesian computations are used to estimate and restrict the coefficients to have a low-dimensional time-varying factor structure. The specification model uses a hierarchical prior for the vector of factors in order to permit exchangeability, time variations, and heretoskedasticity in the innovations in the factors. An overparametrized VAR is transformed into a parsimonius SUR model where the regressor are observable linear combinations of the right-hand side variables of the VAR, and the loadings are the time-varying coefficient factors. Generalized impulse response fuctions and conditional forecasts are obtained with the output of an MCMC routine.

The evidence would confirm the need to allow fro cross-country and cross-factos interdependencies when analyzing macroeconomic-financial linkages. Net spillover matrices including real and financial variables for the EA12 are constructed to define total bilateral net spillover effects. They incorporate feedback effects from the impulse variables and temporary or persistent long-run effects of potential shocks that may lead to contagion. Analyzing the entire time-series period, shocks spill over in a heterogeneous way across countries, more intensive among financial variables. This finding accounts for higher amplification of spillover effects which can be seen as a result of increased interdependences between variables.

In this paper, spillovers are defined as the transmission of an unexpected but identified shock from one variable to receiving variables in the system. Accounting for cross-country and cross-variable interdependencies, conditional forecasts for bilateral trade and capital are computed. In this way, the model is able to investigate interactions between real and financial variables and to capture changes of interdependencies over time. Following the definition by Allen and Gale (2000), the contagion index proposed in this paper is defined as a consequence of excess spillover. Thus, extreme amplification of spillover effects can be seen as alarming levels which could lead to contagion. Optimal policy coordination in the Euro Area would have required a differentiation of consolidation efforts depending on the fiscal space to minimise the negative spillovers. Spillovers of fiscal consolidations are larger in financial dimension. Larger output effects prove that consolidations occurred simultaneously. The positive impact on outputs of most members in the financial dimension indicates the importance of coordinated fiscal actions in the Euro Area.

After estimating different specification of this model, the highest marginal likelihood was found for the model including four country-specific component for each economy, four variable-type components, and two common components for all series. These common, country-specific and variable-type components quantify the relative contribution of common and heterogenous factors in macroeconomic-financial linkages and help to address the following questions: Is there a significant common component in the real and financial interactions across eurozone members or do cuntry-specific heterogeneities matter more? How did *weights* and *outliers* factors affect real economy and financial variables over time? What is the importance of transmission channels and latent confounding effects when studying growth shocks across countries within a common currency area?

Here, some considerations are in order. Country-specific factors remain very important explaining the presence of a heterogeneous pattern across members. However, interactions between real and financial dimension are important to understand co-movements in economic activity. Thus, bilateral trade and capital conduct a prominent role when analyzing foreign and domestic policies. Higly indebted countries were forced into taking wide-ranging austerity measures, having lost access to the financial markets. This has led to call for stronger cross-country differentiation and for temporary stimulus measures in countries not facing financial market pressure. Therefore, cross-border spillovers have exacerbated the negative effects of consolidations. This finding accounts for a substantial degree of heterogeneity in real dimension and a deeper interdependence in financial dynamic.

These findings cast a new perspective for theoretical models of idiosyncratic business cycles and policy making.

From a modelling perspective, the analysis appears to favour models that assign an important role to *catching-up* and competitiveness factors in explaining current account imbalances and debt dynamics. Moreover, transmission channels suggest that trade channels matter relatively less than financial channels. Growth shocks appear to be predominantly transmitted via financial linkages. The interdependence is stronger in financial dimension, while real component shows higher degree of heterogeneity and it is mainly affected by latent confounding effects. The results are consistent with the recent literature which recognizes the importance of accounting for both country-specific and global factors when studying real and financial interactions. Moreover, the analysis is consistent with the premise that for countries to be an important source of growth spillovers, growth should rely to a greater extent on autonomous domestic sources. Nevertheless, testing for commonality and heterogeneity, the idiosyncratic components in driving fiscal shock transmissions is high, suggesting the necessity of accounting also for growth shocks outside the EMU that are to a relatively larger extent transmistted via trade. Finally the analysis is consistent with the possibility that larger co-movements or macroeconomic-financial linkages observed, mainly in the last recession, could be more related to the size and height difference of the shocks than to the intensification of their transmission.

From a policy perspective, several considerations can be displayed. First, despite high

degree of heterogeneity, countries of the eurozone share common financial shocks and, hence, the analysis is in line with rapidly increasing cross-border trade and financial linkages. Although early indications suggest that the imbalances have been reduced and the eurozone countries are weathering the current *storm* during current recession, without the appropriate adjustment of the private and public sector, euro area imbalances could pick up again if the macroeconomic conditions normalize. Second, despite a common monetary policy, national policies of fiscal policy, investments, and structural reforms in labour and complementary markets remain heterogenous across the euro area. This might have contributed to the emergence of different country-specific developments of competitiveness, consumption, investment, and production structures affecting national economy. Thus, national authorities may be tempted to design domestic policies so as to counteract world conditions, but those policies may be ineffective and counter-productive for the domestic economy. Third, structural differences among national policy may also be driven by idiosyncratic business cycles and, hence, the importance of accounting for transmission channels and latent confounding effects. Fourth and probably most importantly, divergence across countries were driven by different degrees of productivity growth. Thus, in the euro area, structural reforms without coordinated national fiscal actions affect the adjustment capacity of the currency union as a whole because of high degree of divergence.

These considerations raise interesting questions that could be addressed in future research. (i) The importance of fiscal and monetary policy interactions in a currency Union when analyzing macroeconomic-financial linkages. (ii) International business cycles play a prominent role with countries endogenously reacting to foreign impulses.

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Appendix A

Impulse responses functions accounting for real and financial weights

In this Appendix, generalised impulse response functions are drawn. They are computed as the difference between a conditional and an unconditional projection of output growth for each country in a given period.

The unconditional projection is the one the model would have obtained for output growth for that period based only on historical information, and consistent with a model-based forecast path for the ohter variables.

The conditional projection for output growth is the one the model would have obtained over the same period conditionally on the actual path of unexpected shock for that period. The below panels show the responses of productivity growth in all countries to a 1% shock to each real and financial variable accounting for selected time-series (1999 - 2014).

Overall, impulse responses are large and closed to zero; thus, they are significant and able in explaining co-movements and heterogeneity across countries with a relatively small biases.

Real Dimension

The Picture A.1 draws impulse responses of output growth to a 1% shock in the variable *gov*. The responses die out at a similar rate but display different magnitudes (see e.g., Section 9.1 for more details).
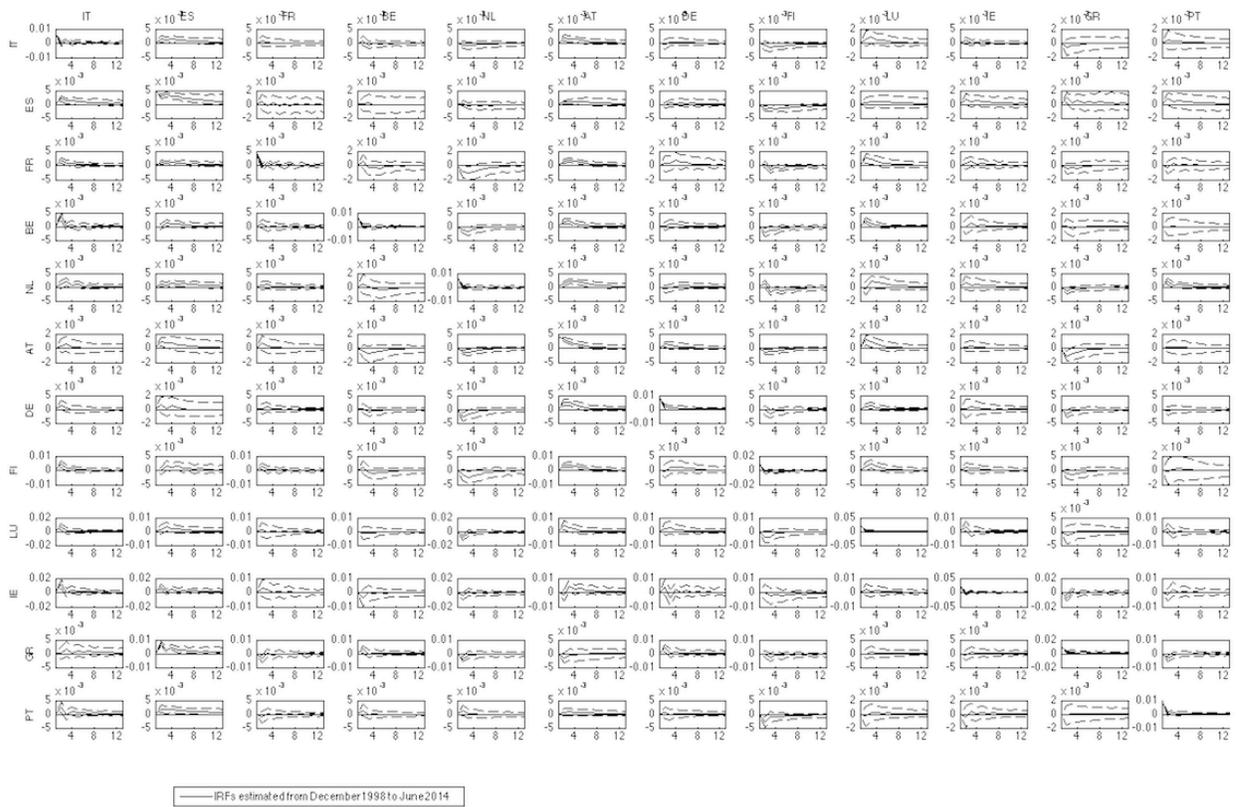
Figure A.1: Responses of output growth a 1% shock to the variable *gov* (*solid lines*) and 60% confidence bands (*dashed lines*).

Accounting for the variable *gdpg*, the responses are significant and show a more similar growth path with respect to *gov*. This result seems to prove the presence of co-movements in GDP due to catiching-up and competitiveness relationship inter-countries.
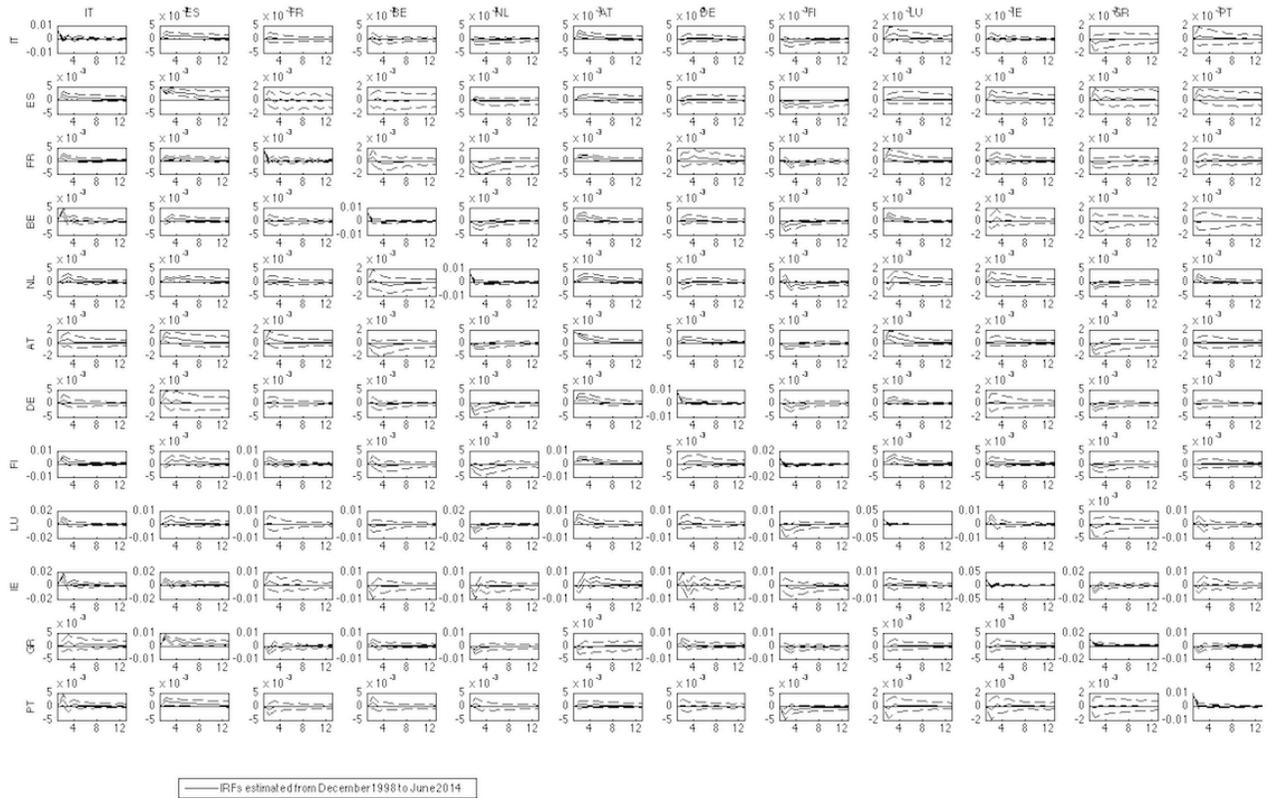

Figure A.2: Responses of output growth a 1% shock to the variable *gdpg* (*solid lines*) and 60% confidence bands (*dashed lines*).

In the variable *cap*, the responses are large and, hence, significant wit 60% confidence bands. Mostly countries show bigger values with respect to *gov* and *gdpg*.

Figure A.3: Responses of output growth a 1% shock to the variable *cap* (*solid lines*) and 60% confidence bands (*dashed lines*).

Financial Dimension

The Picture A.4 draws impulse responses of output growth to a 1% shock in the variable *int*. The responses die out at a similar rate but display different magnitudes. Moreover, growth spillovers seem to have higher size than one in real dimension.
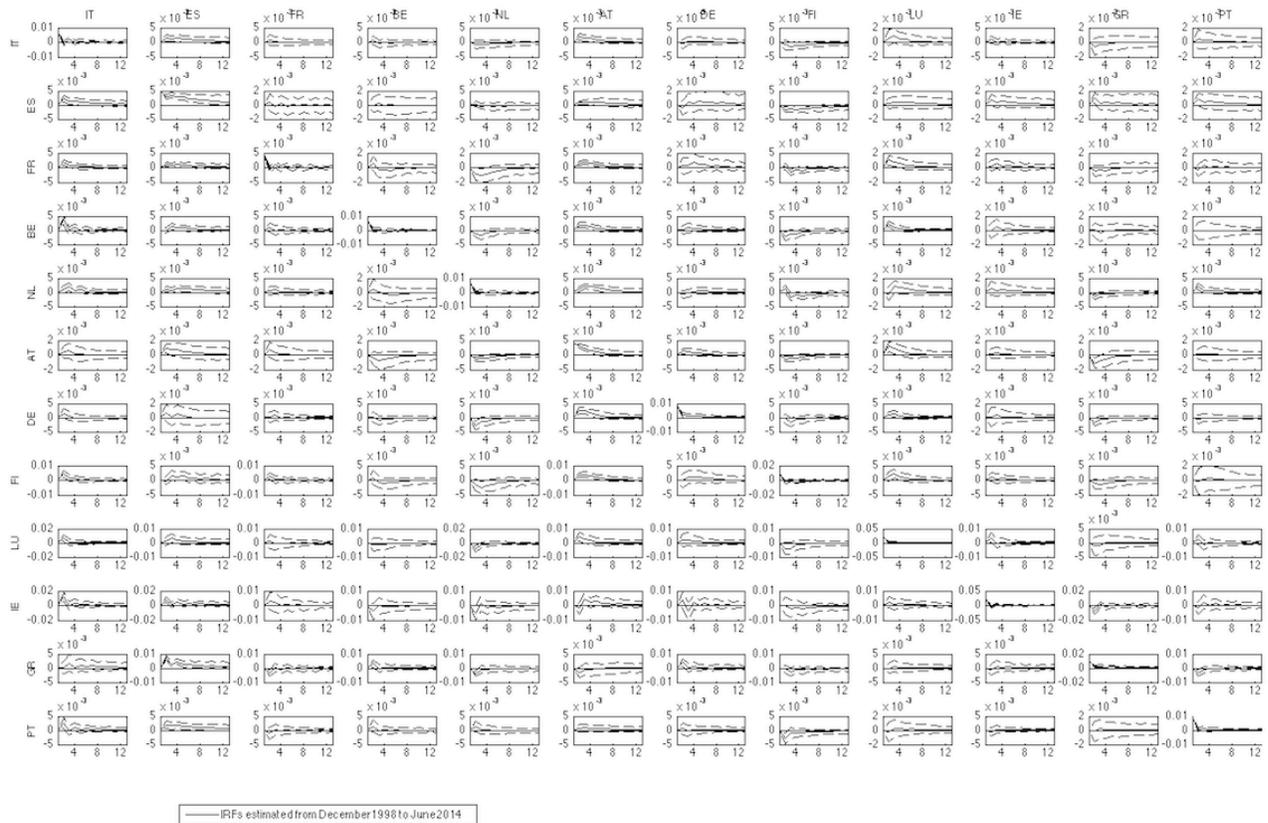

Figure A.4: Responses of output growth a 1% shock to the variable *int* (*solid lines*) and 60% confidence bands (*dashed lines*).

Accounting for the variable *debt*, the responses are significant and show a higher size than one in *int*. This result seems to prove the presence of co-movements in GDP due to catiching-up and competitiveness relationship inter-countries.
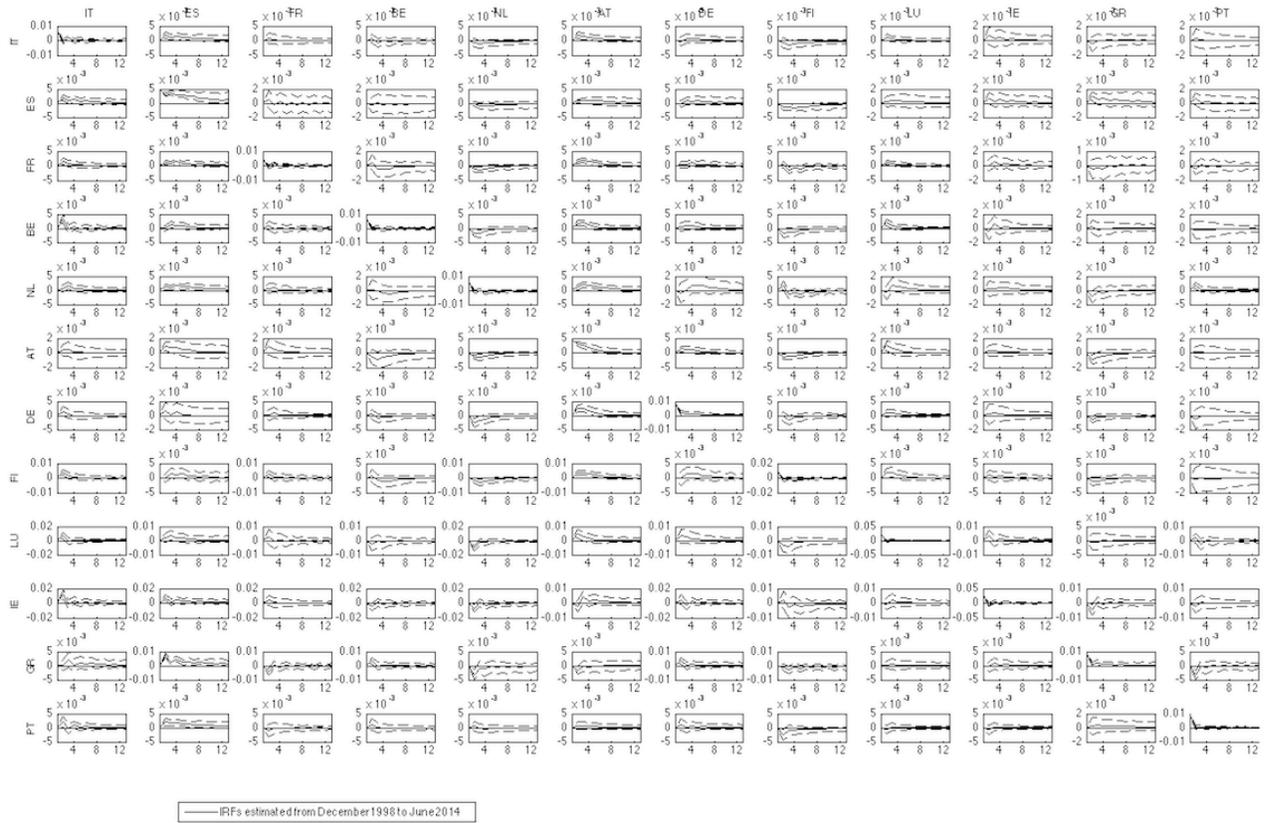

Figure A.5: Responses of output growth a 1% shock to the variable *debt* (*solid lines*) and 60% confidence bands (*dashed lines*).

In the variable *curr*, the responses are large and, hence, significant wit 60% confidence bands. The impulse responses are smaller than one in variable *debt*. This results could be due to GSP's austerity.
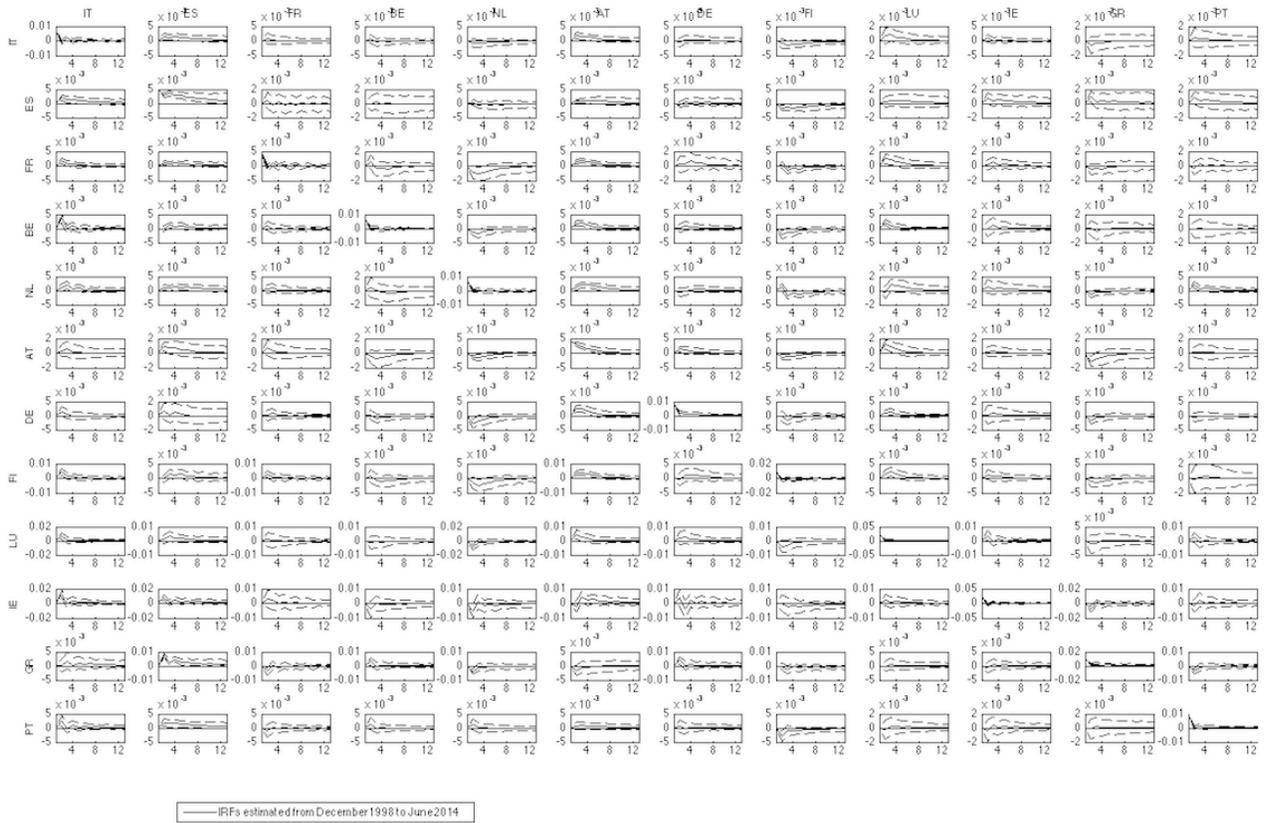

Figure A.6: Responses of output growth a 1% shock to the variable *curr* (*solid lines*) and 60% confidence bands (*dashed lines*).

Appendix B

Bayesian Inference: Notation and Related Literature

Estimating VAR models, a variety of priors can be used. However, there are three principal issues in which they differ. (i) VAR models are not parsimonious models. They have a great many coefficients and, with quarterly macroeconomic data, the number of observations on each variables might be at most a few hundred. Thus, without prior informations, it is hard to obtain significant estimates of so many coefficients and impulse response functions and forecasts will tend to be imprecisely estimated. (ii) The priors used with VARs differ in whether they lead to analytical results for the posterior and predictive densities or whether MCMC methods are required to carry out Bayesian inference. Moreover, with the VARs, natural conjugate priors lead to analytical results, which can greatly reduce the computational burden. For example, if one is carrying out a recursive forecasting exercise, which requires repeated calculation of osterior and predictive distributions, non-conjugate priors which require MCMC methods can be very computationally demanding. (iii) The priors differ in how easily they can handle departures from the unrestricted VAR allowing for different equations to have different explanatory variables, allowing for VAR coefficients to change over time, allowing for heteroskedastic structures for the errors of various sorts, and so forth.

Minnesota Prior

Early work with Bayesian VARs with shrinkage priors was done by researchers at the University of Minnesota and the Federal Reserve Bank of Minneapolis (see e.g., Doan, Litterman and Sims, 1984, and Litterman, 1986). The priors they used have come to be known as Minnesota priors. They are based on an approximation which leads to great simplifications in prior elicitation and computation. This approximation involves replacing Ω with an estimate, $\hat{\Omega}$. The original Minnesota prior simplifies even further by assuming Ω to be a diagonal matrix. For example, according to the model described in equation 6.22, each equation of the TVC-VAR can be estimated one at a time so that $\hat{\sigma}_{ii} = S_i$, where S_i is the

standard OLS estimate of the error variance in the ith equation and $\hat{\sigma}_{ii}$ is the ii^{th} element of $\hat{\Omega}$. When Ω is not assumed to be diagonal, a simple estimate such as $\hat{Omega} = \frac{S}{T}$ can be used. A disadvantage of this approach is that it involves replacing an unknown matrix of parameters by an estimate rather than integrating it out in a Bayesian fashion. Furthermore, replacing Ω by an estimate simplifies computation since analytical posterior and predictive results are available. Finally, it strategies allows for a great range of flexibility in the choice of prior. If Ω is not replaced by an estimate, the only fully Bayesian approach which leads to analytical results involves the use of a natural conjugate prior. This prior is often not very appealing because it assumes that prior beliefs about θ_m imply that something about Ω must be known. Nevertheless, a conjugate prior joined with a hierarchical structure with independent prior assumptions can be applied in this setup. Exploiting SUR model in equation 6.22, the estimation of θ_m indicators containing the parameters of the mth equation, with $m = 1, \ldots, M$, is:

$$p(\theta_m) = N(\bar{\theta}_{mn}, \bar{R}_{mn}) \qquad or \qquad \theta_m \sim N(\bar{\theta}_{mn}, \bar{R}_{mn}) \tag{B.1}$$

In this context, without loss of generality, time-invariant factors are assumed. The Minnesota prior can be thought of as a way of automatically choosing θ_{mn} and R_{mn} is a manner which is sensible in many empirical contexts. By the same token, the (conditional) likelihood on initial conditions is:

$$L(Y^T|\phi_0) \propto (\Omega_{mn})^{-\frac{T}{2}} exp\{-\frac{1}{2}[(Y_m - (X_m\Xi)\theta_m)'\Omega_{mn}^{-1}(Y_m - (X_m\Xi)\theta_m)]\} \tag{B.2}$$

where, Y^T denotes the data and ϕ_0 denotes the prior for $\theta_0, \Omega^{-1})$. To explain the Minnesota prior, note first that the explanatory variables in the VAR in any equation can be divided into the own lags of the dependent variable, the lags of the other dependent variables and exogenous variables. For the prior mean, $\bar{\theta}_{mn}$, the Minnesota prior involves setting most or all of its elements to zero. Thus, ensuring of the BVAR coefficients towards zero and lessening the risk of over-fitting. When using growth rates data, which are typically found to be stationary and exhibit little persistence, it is sensible to simply set $\bar{\theta}_{mn} = 0_{KM}$ except for the elements corresponding to the first own lag of the dependent variable in each equation. These elements are set to one. These are the tradional choices for $\bar{\theta}_{mn}$, but anything is possible. However, the Minnesota prior assumes the prior covariance matrix, $\bar{R}_{mn}$, to be diagonal. For example, denoting $\bar{R}_i$ the block of $\bar{R}_{mn}$ associated with the K coefficients in equation i and $\bar{R}_{i,jj}$ its diagonal elements, a common implementation of the Minnesota prior is to set $\bar{R}_{i,jj}$ equal to: $\frac{\bar{\theta}_1}{p^2}$ for coefficients on own lag p, $\frac{\bar{\theta}_2\sigma_{ii}}{p^2\sigma_{jj}}$ for coefficients on lag p of variable $j \neq i$, and $\bar{\theta}_3\sigma_{ii}$ for coefficients on exogenous variables, whit $p = p_1, p_2$. This prior simplifies the complicated choice of fully specifying all the elements of $\bar{R}_{mn}$ to choosing three scalars, $\bar{\theta}_1$, $\bar{\theta}_2$, and $\bar{\theta}_3$. This form captures the sensible properties that, as lag length increases, coefficients are increasingly shrunk towards zero and that, by setting $\bar{\theta}_1 > \bar{\theta}_2$, own lags are more likely to be important predictors than lags of other variables. The exact choice

of values for $\bar{\theta}_1$, $\bar{\theta}_2$, and $\bar{\theta}_3$ depends on the empirical application and the researcher may wish to experiment with different values for them. Typically, $\sigma_{ii} = S_i$, but Litterman (1986) provides additional motivation and discussion of these choices . Furthermore, many variants of the Minnesota prior have been used in practice. For istance, Kadiyala and Karlsson (1997)] divide prior variances by p instead of the p^2. Then, Banbura, Giannone and Reichlin (2010) use a slight modification of the Minnesota prior in a large VAR with over 100 dependent variables. Typically, factor methods are used with such large panels of data, but they find that the Minnesota prior leads to even better forecasting performance than factor methods. A big advantage of the Minnesota prior is that it leads to simple posterior inference involving only the Normal distribution. To be more precise, the posterior is going to be:

$$p(\theta_m|Y^T, \phi_{-\theta_m}) = N(\tilde{\theta}_{mn}, \tilde{R}_{mn}) \qquad or \qquad \theta_m|Y^T, \phi_{-\theta_m} \sim N(\tilde{\theta}_{mn}, \tilde{R}_{mn}) \tag{B.3}$$

The conditional likelihood on posterior distributions is proportional to:

$$p(\theta_m|\phi) = |(\Omega_{mn})^{-\frac{T}{2}}| \; |(\bar{R}_{mn})^{-\frac{T}{2}}| \; exp\{-\frac{1}{2}[(\theta_m - \bar{\theta}_m)'\bar{R}_{mn}^{-1}(\theta_m - \bar{\theta}_m) +$$

$$+ (Y_m - (X_m\Xi)\theta_m)'\Omega_{mn}^{-1}(Y_m - (X_m\Xi)\theta_m)]\} \tag{B.4}$$

Thus,

$$p(\theta_m|Y^T, \phi_{-\theta_m}) \propto exp\{-\frac{1}{2}[(\theta_m - \tilde{\theta}_m)'\tilde{R}_{mn}^{-1}(\theta_m - \tilde{\theta}_m)]\} \tag{B.5}$$

where, $\tilde{\theta}_m = \tilde{R}_{mn}\{\bar{R}_{mn}^{-1}\bar{\theta}_m + [\hat{\Omega}_{mn}^{-1} \otimes (X_m\Xi)]\bar{Y}_m\}$, with $\bar{Y}_m = (X_m\Xi)\hat{\theta}_m + \hat{\Omega}_{mn}$, and

$\tilde{R} = \{\bar{R}_{mn}^{-1} + [\hat{\Omega}_{mn}(X_m\Xi)'(X_m\Xi)]\}^{-1}$. Here, prior and posterior independence lie between equations. Elements of Ω are obtained from univariate $AR(p)$. $\bar{\theta}_m$ and $\bar{R}_{mn}$ are unknown and specified in terms of few known parameters. Assuming time-variant factors, with $\mathscr{B} \neq 0$, the posterior distribution can be calculated involving the Inverse Gamma distribution, that is the conjugate prior of a normal distribution with unknown mean and variance . Finally, if $\bar{R}_{mn}^{-1} = 0$, $\tilde{\theta}_m = [(X_m\Xi)'(X_m\Xi)]^{-1}(X_m\Xi)'Y_m$, hence the OLS estimator of θ_m. Nevertheless, as stressed above, a disadvantage of the Minnesota prior is that it does not provide a full Bayesian treatment of Ω as an unknown parameter. This latter means that analytical methods are not available and MCMC methods are required. Instead, the Minnesota prior simply plugs in $\Omega = \hat{\Omega}$, ignoring any uncertainty in this parameter.

Conjugate Prior Distribution

In the natural conjugate prior, the prior, likelihood and posterior come from the same family of distributions. According to the likelihood funtion described in Section 6.3 (see for istance equations 6.19, 6.20, and 6.21) and letting time-invariant factors, the natural conjugate prior has the form:

$$\theta|\Omega \sim N(\bar{\theta}, \Omega \propto \bar{R}) \qquad or \qquad p(\theta|\Omega) = N(\bar{\Omega}, \Omega \propto \bar{R}) \tag{B.6}$$

$$\Omega^{-1} \sim W(\theta_1^{-1}, z_1) \qquad or \qquad p(\Omega^{-1}) = W(\theta_1^{-1}, z_1) \tag{B.7}$$

where, $\bar{\theta}$, $\bar{R}$, z_1, θ_1 are hyperparameters chosen by the researcher. With this prior, the posterior is:

$$\theta|Y, \Omega^{-1} \sim N(\tilde{\theta}, \tilde{R}) \qquad or \qquad p(\theta|Y, \Omega^{-1}) = N(\tilde{\theta}, \Omega \propto \tilde{R}) \tag{B.8}$$

$$\Omega^{-1}|Y, \theta \sim W(\hat{\theta}_1^{-1}, \hat{z}_1) \qquad or \qquad p(\Omega^{-1}|Y, \theta) = W(\hat{\theta}_1^{-1}, \hat{z}_1) \tag{B.9}$$

where, $\tilde{\theta} = \tilde{R}[\bar{R}^{-1}\bar{\theta} + (X\Xi)'(X\Xi)\hat{\theta}]$, $\tilde{R} = [\bar{R}^{-1} + (X\Xi)'(X\Xi)]^{-1}$,

$\hat{\theta}_1 = S + \theta_1 + \hat{\theta}'(X\Xi)'(X\Xi)\hat{\theta} + \bar{\theta}'\bar{R}\bar{\theta} - \tilde{\theta}'(\bar{R} + (X\Xi)'(X\Xi))\tilde{\theta}$, and $\hat{z}_1 = T + z_1$. Thus, any

values for the hyperparameters $\bar{\theta}$, $\bar{R}$, z_1, θ_1 can be chosen[1].

For the natural conjugate prior, analytical results exist which allow for Bayesian estimation and prediction. Hence, the posterior distribution of, for example, impulse responses can be obtained by Monte Carlo integration. That is, draws of Ω^{-1} can be obtained from equation 6.9 and, conditional on these, draws of θ can be taken from equation 6.8, and draws of $\mathscr{B}$ derive from equation 6.10. Then, draws of impulse responses can be computed using these drawn values of Ω^{-1}, θ. If $\mathscr{B} = 0$ allowing for time-variant factors, draws of b_f can be taken from a Normal-Inverse Gamma distribution (as well described in Section 6.3 and discussed in Appendix C). The natural conjugate prior has the large advantage that analytical results are available for posterior inference and prediction. However, it assumes each equation to have the same explanatory variables and it restricts the prior covariance of the coefficients in any two equations to be proportional to one another. Thus, there is no need to use posterior simulation algorithms unless interest centers on non-linear functions of the parameters (e.g., impulse response analysis such as those which arise in structural VARs) . Moreover, these properties do not allow to use the Minnesota prior. For example, such as showed previously, the Minnesota prior covariance matrix is written in terms of blocks which vary across equations and it is not allowed for in the natural conjugate prior. There are generalizations of this prior, such as the extended natural conjugate prior of Kadiyala and Karlsson (1997),

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[1]The non-informative prior is obtained by setting $\bar{R}^{-1} = cI$ and letting $c \to 0$

which surmount these problems. In practice, they proposed analytical and simulation strategy about unknown parameters $\bar{\theta}$, $\bar{R}$, z_1, and θ_1. A Minnesota-type of specification for these matrices could be adapted and used here. For example, $\bar{\theta}$, is specified as dependent upon only one hyperparameter that controls the mean of the first lag of the endogenous variable. Then, $\bar{R}$ is specified as a diagonal matrix according to an implementation scheme and the diagonal elements of z_1 are set as $z_{1(m,m)} = (\theta - n - 1)\hat{\theta}'_{mm}$, with $\hat{\theta}'_{mm}$ is estimated from an univariate $AR(p)$ model, and θ_1 are obtained from the data. With time-invariant factors, hyper-parameters of a Normal-Inverse Gamma distribution can be selected a-priori to produce relatively loose priors. Nevertheless, such as noted previously, this latter is not very appealing since it requires that unknown parameters are specified in terms of few known parameters and are either estimated or assumed to be known based on the Minnesota rules of thumb. Therefore, a hierarchical Bayes estimation with independent beliefs is required (see e.g., Chib and Greenberg [14]). The important difference between Chib-Greenberg and Minnesota prior is that hyper-parameters $(\Omega^{-1}, b_f, \{\theta_t\})$ are given proper prior, hence they can be collected in an only vector and easily estimated.

Hierarchical Bayes Estimation

A more general framework for VAR modelling can be introduced. In these models, Bayesian inference requires posterior simulation algorithms such as the Gibbs sampler. According to the natural conjugate prior, $\theta|\Omega$ and Ω^{-1} have Normal and Wishart distributions, respectively. Note that now time-invariant factors are assumed. Then, the fact that the prior for θ depends on Ω implies that θ and Ω are not independent of one another. To be more precise, the estimation works with a prior which has VAR coefficients and error covariance being independent of one another. Hence, it is often called independent Normal-Wishart prior. To allow for different equations in the VAR to have different explanatory variables, previous notation have to be modified. Such as discussed in Section 6.1, the restricted VAR can be written as a normal linear regression model, with an error covariance matrix of a particular form. Given the model in equation 6.22, a general prior which does not involve the restrictions inherent in the natural conjugate prior is the independent Normal-Wishart prior:

$$p(\theta, \Omega^{-1}) = p(\theta)p(\Omega^{-1}) \tag{B.10}$$

where,

$$\theta \sim N(\bar{\theta}, \bar{R}_\theta) \qquad or \qquad p(\theta) = N(\bar{\theta}, \bar{R}_\theta) \tag{B.11}$$

$$\Omega^{-1} \sim W(\theta_1^{-1}, z_1) \qquad or \qquad p(\Omega^{-1} = W(\theta_1^{-1}, z_1) \tag{B.12}$$

Here, the prior allows for the prior covariance matrix, $\bar{R}_\theta$, to be anything the researcher chooses, rather the restrictive $\Omega \otimes \bar{R}$ form of the natural conjugate prior. For istance, the researcher can set θ and $\bar{R}_\theta$ exactly as in the Minnesota prior . But, the joint posterior $p(\theta, \Omega^{-1}|Y)$ does not have a convenient form that would allow easy Bayesian analysis (e.g., posterior means and variances do not have analytical forms). However, the (conditional) likelihood function is proportional to:

$$p(\phi|Y^T) \propto p(\Omega)exp\{-\frac{1}{2}[\Sigma_t(Y_t - (X_t\Xi)\theta_t)'\Omega^{-1}\Sigma_t(Y_t - (X_t\Xi)\theta_t)]\} \tag{B.13}$$

where, $p(\phi|Y^T) \propto p(\phi)L(\phi|Y^T)$ and ϕ stands for priors densities $(\Omega^{-1}, \{\theta_t\})$. Thus, posterior distributions $p(\theta|Y, \Omega^{-1})$ and $p(\Omega^{-1}|Y, \theta)$ have the following forms:

$$\theta|Y, \Omega^{-1} \sim N(\tilde{\theta}, \tilde{R}_\theta) \qquad or \qquad p(\theta|Y, \Omega^{-1}) = N(\tilde{\theta}, \tilde{R}_\theta) \tag{B.14}$$

$$\Omega^{-1}|Y, W \sim W(\hat{\theta}_1^{-1}, \hat{z}_1) \qquad or \qquad p(\Omega^{-1}|Y, \theta = W(\hat{\theta}_1^{-1}, \hat{z}_1) \tag{B.15}$$

where,

$$\tilde{\theta} = \tilde{R}_\theta[\bar{R}_\theta^{-1}\bar{\theta} + \Sigma_{t=1}^T(X_t\Xi)'\Omega^{-1}(X_t\Xi)\hat{\theta}] \tag{B.16}$$

$$\tilde{R} = [\bar{R}_\theta^{-1} + \Sigma_{t=1}^{-1}(X_t\Xi)'\Omega^{-1}(X_t\Xi)]^{-1} \tag{B.17}$$

$$\hat{\theta}_1 = \theta_1 + \Sigma_{t=1}^T(Y_t - (X_T\Xi)\theta_t)'(Y_t - (X_t\Xi)\theta_t) \tag{B.18}$$

$$\hat{z}_1 = T + z_1 \tag{B.19}$$

In equation B.16, $\hat{\theta}$ is the GLS estimator, with $\hat{\theta} = [(X_t\Xi)'\Omega^{-1}(X_t\Xi)]^{-1} \cdot (X_t\Xi)'\Omega^{-1}Y_t$. Rearranging terms, the equation B.16 can be written as:

$$\tilde{\theta} = \tilde{R}_\theta[\bar{R}_\theta^{-1}\bar{\theta} + \Sigma_{t=1}^T(X_t\Xi)'\Omega^{-1}Y_t] \tag{B.20}$$

Assuming time-variant factors, the (conditional) likelihood function in equation B.13 is going to be:

$$p(\phi|Y^T) = p(\Omega)exp\{-\frac{1}{2}[\Sigma_t(Y_t - (X_t\Xi)\theta_t)'\Omega^{-1}\Sigma_t(Y_t - (X_t\Xi)\theta_t) +$$

$$+ (\theta_t - \bar{\theta})'\bar{R}_\theta^{-1}(\theta_t - \bar{\theta})]\} \cdot \Omega^{-\frac{1}{2}(\bar{\omega}_0 + T - 1)}exp[-\frac{1}{2}S_0Q_1^{-1}] \tag{B.21}$$

where, $\bar{\omega}$ and S_0 are hyper-parameters of an Inverse Gamma distribution selected a-priori and $Q_1 = \hat{Q}_1$ denotes a block diagonal matrix[2].

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[2]See Section 6.4 for more details.

Appendix C

Additional Computations

C.1 Matrix Algebra

In this Section, additional matrix algebra used in the framework are discussed.

Kronecker Product

Let A and B be matrices with dimension $(m \cdot n)$ and $(p \cdot q)$, respectively, a matrix product $(A \cdot B)$ exists if and only if the number of columns in matrix (A) equals the number of rows in matrix (B), or (A) and (B) are scalar. The Kronecker product, $A \otimes B$, is defined for any pairs of matrices (A) and (B):

$$A \otimes B = \begin{pmatrix} a_{11}B & \ldots & a_{1n}B \\ a_{m1}B & \ldots & a_{mn}B \end{pmatrix} = C \tag{C.1}$$

where C has dimension $(m \cdot p) \cdot (n \cdot q)$.

Vec Operator

The vec operator transforms a matrix into a vector by stacking the columns of the matrix one underneath the other. For example, let a matrix (A), with dimension $(m \cdot n)$, and a_i which denotes the *j-th* column. The *vec* operator of the matrix (A) is:

$$vec = (m \cdot n) \cdot 1 \tag{C.2}$$

with $vec(A) = (a_1, a_2, \ldots, a_n)'$. Finally, the *vec*(A) is defined for any matrix (A) and not just for square matrices.

C.2 Multivariate Distributions

In this Section, additional multivariate distributions discussed in Chapter 6 are expounded.

Multivariate Normal

Let $x \; \forall \; \mathfrak{R}$ be a random variable so that:

$$x \sim N(\mu, \sigma^2) \qquad if \qquad p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \; exp \; \{-\frac{(x-\mu)^2}{2\sigma^2}\} \qquad (C.3)$$

where, $p(x)$ denotes the probability density function of x and $\mu = E(X)$ is the mean of the variable x, with $E(x) = \Sigma_x x \cdot Pr(x)$ whether x is a discrete variable and $E(x) = \int x \cdot p(x) \, dx$ whether x is a continuos variable. Thus, the vector x is normally distributed if:

$$p(x) \quad = \quad \frac{1}{|2\pi|^{\frac{N}{2}}} \cdot |\Sigma|^{\frac{1}{2}} \cdot \; exp \; \{-\frac{(X-\mu)' \cdot \Sigma^{-1} \cdot (X-\mu)}{2}\}$$

$$= \quad |2\pi|^{\frac{N}{2}} \cdot |\Sigma|^{\frac{1}{2}} \cdot \; exp \; \{-\frac{1}{2}(X-\mu)' \cdot \Sigma^{-1} \cdot (X-\mu)\} \qquad (C.4)$$

where, $\mu = E(x)$, with $E(x) = [E(x_1), E(x_2), \dots, E(x_N)]' \; \forall \; \mathcal{R}$ denotes the mean of random vector x.

Multivariate t Distribution

Given a n-dimensional random vector $x = (x_1, x_2, \dots, x_n)'$, its n-variate t distribution with υ degrees of freedom can be stated as:

$$x \sim t_\upsilon(\mu, \Sigma) \qquad (C.5)$$

where, μ and Σ denote mean and covariance matrix of x, respectively. Here, the joint probability density function of x corresponds to:

$$f(x) \quad = \quad \frac{\Gamma[\frac{(\upsilon+n)}{2}]}{|\pi \cdot \upsilon|^{\frac{n}{2}} \cdot \Gamma \cdot (\frac{\upsilon}{2}) \cdot |\Sigma|^{\frac{1}{2}}} \cdot [1 + \frac{1}{\upsilon}(x-\mu)' \cdot \Sigma^{-1} \cdot (x-\mu)]^{-\frac{(\upsilon+n)}{2}}$$

$$= \quad \frac{\Gamma[\frac{(\upsilon+n)}{2}]}{|\pi \cdot \upsilon|^{\frac{n}{2}} \cdot \Gamma \cdot (\frac{\upsilon}{2})} \cdot |\Sigma|^{\frac{1}{2}} \cdot [1 + \frac{1}{\upsilon}(x-\mu)' \cdot \Sigma^{-1} \cdot (x-\mu)]^{-\frac{(\upsilon+n)}{2}} \qquad (C.6)$$

However, supposing $n = 0$, $\mu = 0$, and $\Sigma = 1$, $f(x)$ reflects an univariate student's t distribution with υ degrees of freedom.

Wishart Distribution

The Wishart distribution $W(\Sigma, n, \upsilon)$ is a probability distribution of random non-negative-definite $n \cdot n$ matrices. It is used to model random covariance matrices:

$$W \approx W(\Sigma, n, \upsilon) \approx \Sigma_i x_i x_i^{'} \tag{C.7}$$

where, $x_i \approx N(0, \Sigma)$, with N reflecting a normal independent distribution; υ denotes the number of degrees of freedom; Σ is a non-negative definite symmetric $n \cdot n$ matrix, which called scale matrix; and $E(W) = nE(x_i x_i^{'}) = nCov(x_i) = n\Sigma$. Assuming that $\upsilon > n$ and Σ has full rank, so that $\exists \Sigma^{-1}$, the density of the random $n \cdot n$ matrix W is:

$$f(W, \upsilon, \Sigma) = \frac{|W|^{\frac{(\upsilon-n-1)}{2}} \cdot exp\left\{-\left[\frac{1}{2} \ tr \ (W\Sigma^{-1})\right]\right\}}{2^{\frac{n \cdot \upsilon}{2}} \cdot \pi^{\frac{n(n-1)}{4}} \cdot |\Sigma|^{\frac{\upsilon}{2}} \cdot \prod_{i=1}^{n} \Gamma(\frac{\upsilon+1-i}{2})} \tag{C.8}$$

where, $|W|$ and $|\Sigma|$ are the determinants of W and Σ matrices, respectively. However, the density $f()$ tends to zero unless the W matrix is symmetric and positive-definite.

Inverse Wishart Distribution

The density of the random $n \cdot n$ matrix W can be alternatively written as:

$$W \approx iW(\Sigma^{-1}, n, \upsilon) \tag{C.9}$$

Thus,

$$f(W, \upsilon, \Sigma) = \frac{|W|^{-\frac{(\upsilon-n-1)}{2}} \cdot exp\left\{-\left[\frac{1}{2} \ tr \ (W^{-1}\Sigma)\right]\right\} \cdot |\Sigma|^{\frac{\upsilon}{2}}}{2^{\frac{n \cdot \upsilon}{2}} \cdot \pi^{\frac{n(n-1)}{4}} \cdot \prod_{i=1}^{n} \Gamma(\frac{\upsilon+1-i}{2})} \tag{C.10}$$

Gamma and Inverse Gamma Distribution

The Gamma and Inverse Gamma distributions are widely used in Bayesian analysis. In probability theory and statistics, they correspond with a two-parameter family of continuous probability distributions. There are three different parametric notions in common use. (i)

With a *shape parameter* k and a *scale parameter* θ. (ii) With a *shape parameter* $\alpha = k$ and an inverse scale parameter $\beta = \frac{1}{\theta}$, called *rate parameter*. (iii) With a *shape parameter* k and a *mean parameter* $\mu = \frac{k}{\beta}$. In each of these three forms, both parameters are positive real numbers. In addition, the parameterization with k and θ appears to be more common in econometrics and other applied fields. The parameterization with α and β are more common in Bayesian statistics, where the Gamma distribution is used as a conjugate prior distribution for various types of inverse scale parameters (such as the λ of an exponential distribution or a poisson distribution). The closely related inverse gamma distribution is used as a conjugate prior for scale parameters (such as the variance of a normal distribution). Let the pdf of the gamma distribution be $f(x) = x^{k-1} \cdot \frac{exp[-\frac{x}{\theta}]}{\theta^k \cdot \Gamma(k)}$ and defining the transformation $y = g(x) = \frac{1}{x}$, the resulting transformation is:

$$
\begin{aligned}
f_y(y) &= f_x[g^{-1}(y)] \mid \frac{d}{dy} g^{-1}(y) \\[2mm]
&= \frac{1}{\theta^k \Gamma(k)} \cdot \left(\frac{1}{y}\right)^{k-1} \cdot exp\left[\frac{-1}{\theta_y} \cdot \frac{1}{y^2}\right] \\[2mm]
&= \frac{1}{\theta^k \Gamma(k)} \cdot \left(\frac{1}{y}\right)^{k+1} \cdot exp\left[\frac{-1}{\theta_y}\right] \\[2mm]
&= \frac{1}{\theta^k \Gamma(k)} \cdot y^{-k-1} \cdot exp\left[\frac{-1}{\theta_y}\right] \quad\quad\quad\quad\quad\quad \text{(C.11)}
\end{aligned}
$$

Replacing k with α; θ^{-1} with β; and y with x, equation C.11 can be rewritten:

$$
f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot x^{-\alpha-1} \cdot exp\left(\frac{-\beta}{x}\right) \quad\quad\quad\quad\quad\quad \text{(C.12)}
$$

This latter denotes the inverse Gamma distribution's pdf over the support $x > 0$.

Normal-Inverse Gamma Distribution

The normal-inverse Gamma distribution is a four-parameter family of multivariate continous probability distributions. It is the conjugate prior of a normal distribution with unknown mean and variance. Suppose that the variable x has the following normal (or gaussian) distribution:

$$
x|\sigma^2, \mu, \lambda \sim N(\mu, \frac{\sigma^2}{\lambda}) \quad\quad\quad\quad\quad\quad \text{(C.13)}
$$

where, μ denotes the mean and $\frac{\sigma^2}{\lambda}$ denotes the variance, with $\sigma^2|\alpha, \beta \sim IG(\alpha, \beta)$. Thus, the parameters (x, σ^2) have a normal-inverse gamma distribution denoted as:

$$(x, \sigma^2) \sim NIG(\mu, \lambda, \alpha, \beta) \tag{C.14}$$

In a multivariate form of equation C.14, the conditional distribution of x and σ^2 on the parameters is:

$$x|\sigma^2, \mu, \lambda, R^{-1} \sim N(\mu, \sigma^2 R) \tag{C.15}$$

where, x is a $k \cdot 1$ random vector following the multivariate normal distribution with mean μ and covariance $\sigma^2 R$.

In the univariate case of equation C.14, the conditional distribution of σ^2 on the parameters is:

$$\sigma^2|\alpha, \beta \sim IG(\alpha, \beta) \tag{C.16}$$

In multivariate models, with time-varying parameters (or factors), further specifications need to be considered. Hence, let the **pdf** be:

$$f(x, \sigma^2|\mu, \lambda, \alpha, \beta) = \frac{\sqrt{\lambda}}{\sigma\sqrt{2\mu}} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot (\frac{1}{\sigma^2})^{\alpha+1} \cdot exp\left[-\frac{2\beta + \lambda(x-\mu)^2}{2\sigma^2}\right] \tag{C.17}$$

For the multivariate form, where x is a $k \cdot 1$ random vector, equation C.17 becomes:

$$f(x, \sigma^2|\mu, R^{-1}, \alpha, \beta) = |R|^{-\frac{1}{2}} \cdot (2\pi)^{-\frac{k}{2}} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot (\frac{1}{\sigma^2})^{\frac{k}{(2+\alpha+1)}} \cdot$$

$$\cdot exp\left[-\frac{2\beta + (x-\mu)' \cdot R^{-1} \cdot (x-\mu)}{2\sigma^2}\right] \tag{C.18}$$

where, $|R|$ is the determinant of the $K \cdot K$ matrix R. This latter equation reduces to the first form if $k = 1$ so that x, R, and μ are scalars. Finally, it is also possibile to let $\gamma = \frac{1}{\lambda}$ so that the pdf in equation C.18 becomes:

$$f(x, \sigma^2|\mu, \gamma, \alpha, \beta) = \frac{1}{\sigma\sqrt{2\pi\gamma}} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot (\frac{1}{\sigma^2})^{\alpha+1} \cdot exp\left[-\frac{2\gamma\beta + (x-\mu)^2}{2\gamma\sigma^2}\right. \tag{C.19}$$

In the multivariate form, the corresponding change is to regard the covariance matrix R, instead of its inverse (R^{-1}), as a parameter. Thus, the marginal distributions are $(x, \sigma^2) \sim NIG(\mu, \lambda, \alpha, \beta)$, $\sigma^2 \sim IG(\alpha, \beta)$, and $x \sim t_{2\alpha}(\mu, \frac{\beta}{\alpha}R)$.