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Optimal Taxation with Home Ownership and Wealth Inequality *

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Abstract

We consider optimal taxation in a model with wealth-poor and wealth-rich households, where wealth derives from business capital and home ownership, and investigate the consequences of a rising wealth inequality at steady state on these tax rates. The optimal tax structure includes some taxation of labor, zero taxation of financial and business capital, a housing wealth tax on the wealth-rich households and a housing subsidy on the wealth-poor households. When wealth inequality increases, the optimal balance between labor and housing wealth taxes depends on the source of the increasing wealth.

Keywords: housing wealth, wealth inequality, optimal taxation.

JEL Codes: E21, E62, H2, H21, G1.

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1 Introduction

There is mounting consensus, among scholars and commentators, that shifting taxation from labor to capital may be an optimal response to the increase in wealth–to–income ratios and wealth inequality that has been documented for many advanced economies over the last decades (Piketty and Saez, 2003; Piketty and Zucman, 2014; Saez and Zucman, 2016; Fagereng et al., 2016; Piketty et al., 2017). This policy is mostly motivated by distributional objectives and it is sometime credited as having small efficiency costs (Piketty et al., 2015; Saez and Zucman, 2019). In this paper we investigate the long-run social welfare effects of such tax reform under full commitment by considering a simple model where households accumulate different levels of wealth; the latter consisting in business capital, housing, and financial assets; and the government has access to a limited set of tax rates (a flat tax on wages, housing rents and wealth, the latter being possibly contingent on the types of wealth and on the households' net asset position). We show that an optimal tax structure implies heterogeneous tax rates/subsidies on housing wealth and no tax on financial and business capital.

In our model labor supply is inelastic and households can be lenders or borrowers, homeowners or renters. Wealth heterogeneity is based on the assumption that households have different time discount rates and face borrowing constraints, so that some households end up having zero net wealth. In this set up, the steady state distribution of wealth is perfectly polarized between a set of *wealth-rich* and *wealth-poor* households, although all of them may work and own some housing in different quantities. The only relevant difference between the two sets of households is that, due to an indivisibility, the wealth-poor sort themselves between renters, with zero home ownership, and homeowners, with the value of their home perfectly matched by mortgages. The supply side of the economy includes two produced goods; a perishable consumption good (also called *consumption*); and residential construction. The latter generates an evolving stock of housing subject to physical depreciation. Technologies employ labor and capital, although the housing sector also needs some flow of new land available for construction every period. All the revenues from the sale of land permits go to the government, either because it is the only land owner or because, despite land being privately owned, these revenues are fully taxed by the government.

Within this set up, we study the optimal tax problem assuming that the planner can chose among a limited set of linear tax rates: an income tax, a tax on business capital, a tax (or subsidy) on housing capital and a tax (or subsidy) on rental rates. Importantly, we assume that income taxes are non individual contingent whereas wealth taxes may be contingent on the households' net asset position, *i.e.*, on whether the household has positive or negative financial wealth net of mortgage debt. Within this limited menu of taxes, we show that the Chamley-Judd zero steady state tax on financial and business capital survives (Chamley, 1986; Judd, 1985), whereas housing wealth is taxed at a non zero rate. In particular, we show that it is optimal to impose a positive tax on the rich households' housing wealth, and a subsidy on the user cost of housing (or rent) faced by poor households. For poor homeowners, this can be implemented as a negative tax on housing wealth or imputed rents.

Using a sufficient statistics approach we provide a characterization of the optimal tax rates (and subsidies) on housing in terms of price elasticities in two distinctive cases: the case of quasilinear and CES utility. In the former case, income effects disappear and a standard result obtains: the housing wealth tax on the rich and the housing subsidy to the poor depend inversely on the housing price elasticity. For the case of a CES utility, instead, price elasticities are not the only determinants. Both housing taxes and subsidies are decreasing in the elasticity of substitution between consumption and housing services and, most importantly, the size of the rich households' capital income as a share of their consumption affects negatively the housing tax and positively the housing subsidy. More generally, the behavior of the housing tax rates is related to the "general equilibrium elasticities" of consumption and housing services[1]: the larger is the former relative to the latter, the larger are the efficiency gains from shifting taxation to housing wealth. In our model, the elasticity of consumption is decreasing, and the elasticity of housing increasing, in the households' capital income as a share of consumption. Hence, other things equal, the higher is the share of income coming from wealth, the lower is the optimal housing wealth tax. Since the poor have zero wealth, a rising aggregate wealth-to-income ratio has no or little effects on housing subsidies but it

[1] The term "general equilibrium elasticities" is from Atkeson et al. (1999). These are expressions capturing the efficiency cost of taxing the corresponding good.

may have a strong effect on the rich households' housing tax rates.

Based on this finding, we use a Cobb-Douglas representation of preferences and technology to evaluate numerically the impact on optimal tax rates of a rising aggregate wealth-to-income ratio generated by two alternative mechanisms: an increase in public debt or a fall in the real interest rate. We show that the behavior of the optimal tax rates changes dramatically according to which of the two mechanisms is in place. If wealth rises because of a rising public debt, then the optimal income tax rises; the housing subsidy is flat at around 2%; and the housing tax on the rich households falls substantially. When, instead, aggregate wealth rises as a consequence of a falling real rate, then the optimal income tax falls (by a small amount); the housing tax on the rich households rises strongly; and the housing subsidy falls by approximately one percentage point. In both scenarios, the housing subsidies are small, while the housing tax rates are large (between 30 to 60%).

We additionally find that the tax on income falls with wealth when the interest rate drops, compared to the first scenario, because the gross wage rises substantially, thereby generating a larger tax base. Note that, under our parametrization of preferences and technologies, the second mechanism (a drop in the interest rate) goes along with a stronger re-adjustment of all equilibrium variables, and, in particular, it generates a higher stock of capital (to compensate for the lower marginal productivity), a higher housing wealth (and prices), and a higher level of the poor households' mortgage debt[2]. These different patterns are consistent with the role of capital income in affecting the general equilibrium elasticities that come out from the planning optimum. Namely, if a rising wealth-to-income ratio is obtained through a larger public debt that leaves the real interest rate unaltered, then rich households' capital income rises, so that the general equilibrium elasticity of housing grows relative to consumption and, then, it is optimal to decrease the housing wealth tax. If, on the other hand, a rising wealth-to-income is obtained through a falling real rate, then it is possible (as it happens in our simulations) that capital income falls relative to wages, so that the optimal housing wealth tax rises. This suggests that the way wealth taxes should respond to rising wealth and wealth inequality is far from obvious.

[2]These effects are broadly consistent with the experience of many advanced economies in the last decades (see, for example, La Cava (2016)).

Finally, we provide an estimate of how the economy responds to an *ad hoc* shift of taxation from labor to wealth. In particular, using again the Cobb-Douglas specification of preferences and technologies, we simulate the impact of introducing of a 1% tax on net wealth starting from zero (*i.e.*, from the case in which only labor is taxed) and we show that this increases net wages modestly (by 1-3%), but has a strong positive effect on the user cost of housing faced by poor households (*i.e.*, the effective price of housing services). We estimate that an income-equivalent welfare loss of this policy for poor households is around 5%, and these numbers are not substantially affected when the aggregate wealth-to-income ratio increases. The basic intuition is that, at steady states, the net of tax interest rate is given by the rich households' rate of time preference (*i.e.*, it is invariant to the capital tax rate), so that the burden of the capital tax is shifted on the poor households, who face a higher user cost of housing and a higher cost of debt.

Our results depend on some strong assumptions. First, an inelastic labor supply makes the model biased towards the idea that wealth should not be taxed, so that a positive taxation on housing should be fairly robust[3]. Second, deriving the wealth distribution from different subjective discount factors and debt limits has some limitations, although it is a very standard practice in neoclassical growth theory and, in some way, necessary to produce the stronger observed polarization in wealth than in income which is not easily reproducible in models with homogeneous preferences (Jones, 2015). Third, by concentrating the analysis on steady states we miss the analysis of the transition from low to higher tax rates, which is motivated by the need to focus on long-run phenomena.

Following the seminal contributions by Chamley (1986) and Judd (1985), the literature on optimal taxation has provided various arguments why wealth should be taxed, even in the long-run and under commitment, ranging from life-cycle considerations, precautionary savings and imperfect information. More recently, Piketty and Saez (2013) and Saez and Stantcheva (2018) have advanced the idea that the optimality of positive capital tax rates may emerge due to the *non-infinite elasticity* of the long-run supply of capital[4]. In turn,

[3] Saez et al. (2009) argue that the estimated compensated elasticity of labor is small (close to zero for prime-age males).

[4] The reason why capital taxation is not optimal is that it implies exponentially growing distortions of investment over time, so that there are large benefits on long-run capital, or wealth, from shifting the tax burden from capital to labor. According to Saez and Stantcheva (2018) these growing distortions arise because

finite values for the elasticity of long-run wealth are obtained by assuming that the latter (or the services it generates) enters the individuals' utility function. In particular, Piketty and Saez (2013) consider a life-cycle model where households derive utility from bequests and Saez and Stantcheva (2018) assume that wealth enters the households utility function directly for various reasons, among which are "social status", "power", "philanthropy". In our model housing is both a store of value and an asset that generates utility services, whereas the supply of financial and business capital retains the property of being infinitely elastic in the long run. This explains why, in our model, taxing housing wealth may be optimal, while taxing financial wealth is not. In fact, housing taxation has been advocated in several studies, especially as a way to avoid a sub-optimal tax discrimination between factor inputs and sources of wealth, and many authors have highlighted the existence of substantial welfare gains from increased housing taxation, due to the failure to tax implicit rental income and because of mortgage interest deductibility characterizing existing tax codes in most advanced economies (see Poterba (1984), Gahvari (1984), Berkovec and Fullerton (1992), Auerbach and Hines (2002), Gervais (2002) and Mirrlees et al. (2011)). These distortions imply that housing investment crowds out business capital and generates excessive levels of home ownership. Furthermore, a heavier taxation of housing wealth may reduce inequality in economies where, because of capital market imperfections and indivisibilities, rental housing is concentrated among poor households (although Gervais (2002) finds that the distributional effects of eliminating housing tax incentives are quantitatively small). Our contribution differs from this literature because we are specifically interested in (differentiated) wealth taxation and the way it should evolve in response to increasing wealth inequality, instead of examining the welfare gains from reducing fiscal incentives on housing. Whereas the case for housing taxation is usually based on the unavailability of non distorting taxes, in our model housing taxes (and subsidies) survive despite the fact that labor taxes are non distortionary. The papers most related to ours are Alpanda and Zubairy (2016) and Bonnet et al. (2020). Alpanda and Zubairy (2016) consider a model with patient and impatient households, borrowers and lenders, homeowners and renters, and build a dynamic general-equilibrium model to study the transitional and steady-state effects of a large menu of exogenous taxes (mortgage interest

long-run capital supply is infinitely elastic and taxing infinitely elastic bases is not desirable.

deductions, taxation of imputed rental income, property tax rates and a reduction in depreciation allowance). Our model shares a similar environment and studies the optimal taxation with a smaller menu of taxes. Bonnet et al. (2020) consider an economy with heterogeneous wealth composition (business capital, housing and land) and heterogeneous households (capitalists/landlords and workers/tenants). Differently from our model, they assume that poor households have no wealth (in particular, no land and housing wealth) and obtain housing services by renting from rich households. In their model, capital should not be taxed and the first best allocation can be implemented by levying a tax on land. The optimality of a land tax follows from the planner's preference for redistribution and the fact that land is a fixed factor (*i.e.*, a land tax is non-distortionary).

The remainder of this paper is organized as follows: section 2 presents the model; section 3 considers the optimal taxation problem; section 4 presents quantitative results for the optima tax structure and for the introduction of an exogenous general wealth tax; finally, section 5 presents our conclusions.

2 The Model

In this section we present a model with two sectors: manufacturing and housing construction; different households, with preferences over consumption of a perishable manufacturing good and a durable good, which we call housing; and a government that uses a set of taxes to finance public spending. We first present a general characterization of the model and, then, in the next section, a reduced form with two types of households only, that we will be using in the main text to present the main results. Some comments on how these results may be affected when one turns to the more general setting are presented in section B of the Appendix.

2.1 General Set-Up

The economy has two sectors, *manufacturing* and *(housing) construction*; and a finite set $\mathcal{I}$ of households types indexed by i with preferences over consumption of the manufacturing good and housing services. The manufacturing good is a proxy for all non-construction consumption and the housing stock is a proxy for housing services. Household types have mass $m_i \in (0, 1)$ per total population, with $\sum_{i\in\mathcal{I}} m_i = 1$, and belong to infinitely lived dynasties. Life time utilities are represented by

$$\mathcal{U}^i = \sum_{t=0}^{\infty} \beta_i^t U(c_t^i, z_t^i), \tag{1}$$

where $U(.)$ is the per period strictly increasing and strictly concave utility function (identical across types); $\beta_i \in (0, 1)$ are the type-specific time discount rates; and c^i, z^i denote, respectively, household i's consumption of manufacturing goods and housing services. $U(.)$ is assumed to be twice continuously differentiable and both c and z are normal goods. All households supply one unit of labor inelastically and have different labor productivities. In particular, we let $\epsilon^i \in (0, 1)$ be the household i-specific contribution to production of a unit of labor and assume

$$\sum_{i\in\mathcal{I}} m_i \epsilon^i = 1.$$

Production takes place in the manufacturing (m) and housing (h) sector with heterogeneous neoclassical technologies. While the technology in manufacturing employs labor and capital only, production of new housing requires also land. In particular, technologies in the two sectors are defined by

$$y_t^m = f^m(k_t^m, l_t^m), \qquad y_t^h = f^h(k_t^h, l_t^h, x_t),$$

where k^j is the capital stock and l^j the amount of labor employed in sector j in efficiency units; x_t is the flow of new land available for housing construction. We think of the flow of new available land as "land permits" provided by the government on the basis of some physical constraint or environmental concern (Favilukis et al., 2017; Borri and Reichlin, 2018a). Both $f^m(.)$ and $f^h(.)$ are assumed to be increasing, strictly concave, to exhibit constant returns to scale, to be continuously differentiable and to verify Inada conditions. Then, for some given initial allocation of capital, k_0; and housing stock, h_0; a *feasible allocation* of individuals' consumption and sector specific capital and employment is a sequence

$\mathcal{A} = \{c_t^i, z_t^i, h_{t+1}^i, k_t^j, l_t^j, k_{t+1};\ i \in \mathcal{I},\ j = h, m\}_{t=0}^{\infty}$, satisfying, for all $t \geq 0$,

$$c_t + g_t + k_{t+1} \leq f^m(k_t^m, l_t^m) + (1 - \delta^k)k_t, \tag{2}$$

$$h_{t+1} \leq f^h(k_t^h, l_t^h, x_t) + (1 - \delta^h)h_t, \tag{3}$$

$$z_t \leq h_t \tag{4}$$

$$l_t^m + l_t^h \leq 1, \tag{5}$$

$$k_t^h + k_t^m \leq k_t, \tag{6}$$

where $c = \sum_{i \in \mathcal{I}} m_i c^i$ and $z = \sum_{i \in \mathcal{I}} m_i z^i$ are the aggregate demands of manufacturing and housing services, $k = k^h + k^m$ is the capital stock, g_t the total amount of public spending, $\delta^k \in (0, 1]$, $\delta^h \in (0, 1)$ are the capital and housing depreciation rates, and $\{x_t\}_{t=0}^{\infty}$ is the given sequence of government provided flow of new land permits. Assuming perfect competition in both sectors, profit maximization, and perfect labor mobility imply

$$\delta^k + r_t = f_k^m(k_t^m, l_t^m) = q_t f_k^h(k_t^h, l_t^h, x_t), \tag{7}$$

$$w_t = f_l^m(k_t^m, l_t^m) = q_t f_l^h(k_t^h, l_t^h, x_t). \tag{8}$$

where r is the real interest rate, f_k^j, f_l^j, f_x^j, for $j = h, m$, are the marginal products of capital, labor and land. Firms in the construction sector rebate any remaining profits to the government as a compensation for the use of land permits, and the government uses these resources to finance public spending. Then, the government revenue from land permits in units of labor efficiency is

$$\tau_t^L = q_t f_x^h(k_t^h, l_t^h, x_t)x_t. \tag{9}$$

We let manufacturing be the *numeraire* good; q_t the unit price of housing; s_t the unit price of housing rent; R_t the real gross interest rate; w_t the average real wage rate, with the i-specific wage rate being set at $\epsilon^i w_t$. Any household i, at all time $t \geq 0$, has access to some units, b_{t+1}^i, of a 1-period bond and some units, h_{t+1}^i, of residential property. Housing services enjoyed at time t, z_t^i, come from rental housing or home ownership. We denote with $z_t^{r,i}$ the housing services from renting and the units of housing rented; and with $z^{o,i}$ the housing services from owner occupied housing. Hence, one unit of housing capital generates one unit of housing services and home ownership must be at least as large as the housing services generated by home ownership, *i.e.*,

$$h_t^i \geq z_t^{o,i} \tag{10}$$

for all $t \geq 0$. These two type of housing services are assumed to be perfect substitutes, so that

$$z_t^i = z_t^{r,i} + z_t^{o,i}.$$

To generate a meaningful distinction between renting and owner occupied housing, we assume that housing capital is not perfectly divisible as in Gervais (2002). In particular, there exists a minimum size of owner occupied housing, $\bar{z}$, which also represents the smallest amount of housing services a homeowner (but not a renter) can consume. Hence, all households face the constraint:

$$z_t^{o,i} \geq \bar{z}. \tag{11}$$

This assumption will be used to generate polarization in the housing market between relatively rich homeowners (possibly landlords) and relatively poor renters, but it will play no significant role in our analysis because we will concentrate on equilibrium allocations such that (11) is non binding (see section B of the Appendix for further details on the role of this assumption). At all $t \geq 0$, the government can select from a menu of linear taxes or subsidies, $(\tau_t^y, \tau_t^s, \tau_t^{k,i}, \tau^{h,i})$, possibly contingent on the household's asset position. In particular, $\tau_t^y \in [0,1]$ is the income tax rate, $\tau_t^s \in [0,1]$ is a subsidy on rental housing paid by renters, $\tau_t^{k,i} \in [0,1]$ is the tax rate on financial assets, and $\tau_t^{h,i} \in [0,1]$ is the tax rate on housing wealth (*i.e.*, the value of the house property). Note that, as it is standard in the optimal taxation literature, income taxes and rent subsidies are not individual contingent. However, we allow financial and housing tax rates to be related to the household's net asset position. In particular, we impose, as common in most tax codes, that debt is untaxed and home owners may get exemptions (for their property taxes) due to mortgage payments. The fact that income taxes are non individual contingent plays an important role for the optimal tax design, but we believe this is a valid restriction in a second best framework close enough to actual tax systems.

Note that the $\tau^{h,i}$ can be considered a tax on housing wealth or, equivalently, a sale tax on housing transactions, and we are excluding subsidies or taxes on imputed rents (rarely implemented in reality), although these can be mimicked by a specific choice of the existing tax rates. Including taxes on imputed rents would have little consequences on our results: since we concentrate on steady state equilibria, any distinction between housing wealth taxes and indirect taxes on housing services is somewhat artificial, and the user cost of housing (a proxy for imputed rents) is proportional to the value of housing property. More generally, it is often noted that a tax on imputed rental income could be approximated through an annual recurrent tax on property since imputed rents are typically proportional to property values. However, taxing imputed rents for owner-occupied housing is difficult in practice, and in fact it is rarely implemented, as it involves some practical difficulties such as properly evaluating depreciation and capital gains[5]. The Mirrlees' Review suggests that a tax related to the consumption value of a property bears some resemblance with the British *council tax*, which is essentially a locally collected property tax based on a limited set of brackets (*bands*) for the property values (Mirrlees et al., 2011). Similar tax systems for housing wealth are applied in almost all advanced economies.

Now let $\hat{s}_t = (1-\tau_t^s)s_t$ be the after tax housing rent on the tenant; $\hat{\rho}_t = (1-\tau_t^y)s_t$ the after tax housing rent on the landlord; $\hat{w}_t = (1-\tau_t^y)w_t$ the after tax wage rate per units of efficiency; $\hat{b}_t^i = (1-\tau_t^{k,i})b_t^i$ the after tax net financial claims; and $\hat{q}_t^i = q_t(1-\tau_t^{h,i})$ the after tax housing price. Then, the per-period budget constraint of household i is

$$b_{t+1}^i/(1+r_{t+1}) + c_t^i + q_t h_{t+1}^i + \hat{s}_t z_t^{r,i} = \epsilon^i \hat{w}_t + \hat{\rho}_t(h_t^i - z_t^{o,i}) + \hat{b}_t^i + (1-\delta^h)\hat{q}_t^i h_t^i.$$

It is convenient to define households' before tax net assets as

$$a_{t+1}^i/(1+r_{t+1}) = b_{t+1}^i/(1+r_{t+1}) + q_t h_{t+1}^i \tag{12}$$

and the i-specific after tax net assets, $\hat{a}^i = (1-\tau^{k,i})a^i$, net interest rates, $\hat{R}^i = (1+r)(1-\tau^{k,i})$.

[5]In a recent report, Fatica and Prammer (2017) claim that, "while imputed rents are generally not taxed, all the euro area countries in the HFCS survey - except Malta - levy recurrent taxes on real estate property".

Then, using (12), the t-period budget constraint becomes

$$\hat{a}^i_{t+1}/\hat{R}^i_{t+1} + c^i_t + \hat{\pi}^i_t h^i_t + \hat{s}_t z^{r,i}_t = \epsilon^i \hat{w}_t + \hat{\rho}_t (h^i_t - z^{o,i}_t) + \hat{a}^i_t, \tag{13}$$

where

$$\hat{\pi}^i_t = \hat{R}^i_t q_{t-1} - (1-\delta^h)\hat{q}^i_t$$

is the after tax *user cost of housing*. The latter is a measure of the net of tax market price of housing services and it is equivalent to the present value of next period *imputed rents* from owner occupied housing. For simplicity we assume that households cannot be renters and home owners at the same time and that they choose to be home owners if the two alternative choices (owner occupying or renting) are utility equivalent. Finally, we assume that net assets must be non-negative at all periods,

$$\hat{a}^i_{t+1} \geq 0, \tag{14}$$

i.e., households debt must be fully collateralized by the housing wealth.

For a given set of prices, $\{\hat{R}^i_t, \hat{s}_t, \hat{\rho}_t, \hat{\pi}^i_t, \hat{w}_t\}_{t=0}^{\infty}$, any household $i \in \mathcal{I}$ selects a sequence of consumption levels, housing services, housing stocks and assets, $\{c^i_t, z^{r,i}_t, z^{o,i}_t, h^i_{t+1}, \hat{a}^i_{t+1}\}_{t=0}^{\infty}$, maximizing U^i subject to (10), the minimum home size restriction, (11), the budget constraints, (13), and the collateral constraint, (14), for all $t \geq 0$, and for given initial assets, $(\hat{a}^i_0, h^i_0)$. A sequence $\{c^i_t, z^{r,i}_t, z^{o,i}_t, h^i_{t+1}, \hat{a}^i_{t+1}\}_{t=0}^{\infty}$ solving the household i's utility maximization subject to the relevant constraints is said to be U^i-optimal at the given prices. A more detailed characterization of U^i-optimality is in section B of the Appendix. Here we give the key insights. First, denoting with $U^i_{c,t}$ and $U^i_{z,t}$ the marginal utilities of consumption and housing services of the i household at time t, the Euler equations take the following form:

$$U^i_{c,t}/U^i_{c,t+1} \geq \beta_i \hat{R}_{t+1}, \tag{15}$$

for all $i \in \mathcal{I}$, where the above holds with equality if the collateral constraint (14) is non binding. Furthermore, at interior solutions, *i.e.*, solutions at which the minimum home size restriction is non binding and home ownership is not a dominated choice, the optimal allocation of consumption and housing services implies

$$U^i_{c,t}\hat{\pi}^i_t = U^i_{z,t}, \tag{16}$$

$$U^i_{c,t}\hat{s}_t \geq U^i_{z,t}, \tag{17}$$

$$\hat{s}_t \geq \hat{\pi}^i_t \geq \hat{\rho}_t, \tag{18}$$

and the complementary slackness conditions

$$(\hat{s}^i_t U^i_{c,t} - U^i_{z,t}) z^{r,i}_t = (\hat{\pi}^i_t - \hat{\rho}_t)(h^i_t - z^{o,i}_t) = 0. \tag{19}$$

The above first order conditions are complemented by the transversality condition

$$\lim_{t\to\infty} \hat{a}^i_t / \prod_{j=0}^{t} \hat{R}^i_j = 0. \tag{20}$$

Equation (18) imposes two restriction on the range of prices that are implied by individual optimality. On the one hand, the rental price of housing cannot be smaller than the user cost of housing, otherwise no household would chose to be a home owner. On the other hand, the after tax rent, $\hat{\rho}$, cannot be greater than the user cost of housing, $\hat{\pi}^i$, otherwise, it would pay for the rich households to offer an unlimited amount of housing in the market. This eliminates any arbitrage opportunity in asset markets.

To close the model, we assume that the government, at all time t, issues one-period bonds in some amount b^g_{t+1} at the market interest rate, r_{t+1}. Then, the government budget constraint is

$$b^g_{t+1}/(1 + r_{t+1}) \geq g_t + b^g_t - T_t, \tag{21}$$

where

$$T_t = \sum_i m_i \left(\tau^{k,i}_t b^i_t + q_t(1 - \delta^h)\tau^{h,i}_t h^i_t + \tau^s_t s_t (h^i_t - z^{o,i}_t) \right) + \tau^y_t w_t + \tau^L_t$$

is the time-t revenue from taxation. Finally, the asset markets equilibrium conditions can be stated as

$$\sum_i m_i a^i_t/(1 + r_t) = k_t + b^g_t/(1 + r_t) + q_{t-1} h_t \tag{22}$$

for all $t \geq 0$ and some given initial stocks of capital, housing and public debt (k_0, h_0, b_0^g).

Given a sequence of public spending and land permits, $\{g_t, x_t\}_{t=0}^{\infty}$, and some initial stocks of capital, housing and public debt (k_0, h_0, b_0^g), we say that an allocation, $\mathcal{A}$, is *implementable* with tax rates $\mathcal{T} = \{\tau_t^y, , \tau_t^s, \tau_t^{h,i}, \tau_t^{k,i}; j = s, w, k, h, i \in \mathcal{I}\}_{t=0}^{\infty}$, if there exists a sequence of prices, $\{q_t, w_t, r_{t+1}\}_{t=0}^{\infty}$ and asset holdings, $\{a_{t+1}^i; i \in \mathcal{I}\}_{t=0}^{\infty}$, verifying (i) resource feasibility (equations (2)–(6) at all $t \geq 0$); (ii) profit maximization (equations (7)–(8)); (iii) U^i-optimality (maximization of U^i subject to (10), (11), (13), (14)); the government budget constraint (equation (21)); and the asset markets equilibrium condition (22).

2.2 The Polarized Model

For ease of exposition, from now on we focus on a simplified version of the economy, to be called the *(wealth) polarized model.* The interested reader can look at appendix B for a discussion of how the main results should be amended in a more general version of the model. The polarized model follows from a partition of the set of households into two sets of identical types. The first type are *wealth-rich* households, *i.e.*, households with positive net wealth at all periods. The second type are *wealth-poor*, *i.e.*, households with zero net wealth. In addition, we impose that the wealth-rich are all homeowners, and possibly landlords, whereas the wealth-poor may be homeowners or renters. For this this assumptions to be consistent with the model defined in the previous section, the discount rates, β_i, and the labor productivities, ϵ^i, are implicitly adjusted in such a way as to make this partition of households an outcome of households' utility maximization, as we explain below. The equilibrium allocation obtained in this way is called a *polarized allocation.* The key feature of this specific partition of the set of households is that our model boils down to a multi-goods, multi-assets, version of the Judd (1985)'s capitalist-worker model. Consistently with this simple partition, we reduce the set of households to $\{p, r\}$, where $i = p$ means that the household i is the wealth-poor and $i = r$ that it is the wealth-rich.

The procedure to derive this reduced form model from the more general economy described in the last section is based on two key assumptions. First, we assume that $\beta_r > \beta_p$. Because the wealth-rich are more patient than the wealth-poor, the former type of household ends up owning the total net wealth in the economy at or near steady states, whereas the latter type ends up with zero net wealth. In fact, the steady state equilibrium real interest rate equals the rate of time preference of the most patient households, so that the impatient households' debt limit is binding at all equilibria close enough to the steady state, *i.e.*, the collateral constraint (14) is binding for the poor households only. The second important assumption is that we discard equilibrium configurations at which the minimum home size constraint is binding for some household. To guarantee that all rich households are also home owners, the labor productivity, ϵ^r, and the initial wealth, a_0^r, must be large enough to overcome the minimum home size, $\bar{z}$, at all existing market prices. On the other hand, a polarized allocation is consistent with the possibility that the poor households may be home owners or renters. By (19), the former possibility arises when the poor household is sufficiently productive. Since the minimum home size constraint is assumed to be non binding at equilibrium, the first order conditions (16)–(19) imply that, at any polarized equilibrium with a positive mass of renters (and landlords), as well as poor homeowners,

$$\hat{\rho}_t = \hat{\pi}_t^r = U_{z,t}^r / U_{c,t}^r, \qquad \hat{s}_t = \hat{\pi}_t^p = U_{z,t}^p / U_{c,t}^p.$$

In this case households would be indifferent between renting and owning the occupied housing. In the more general model discussed in appendix B we allow for heterogeneous choices and labor productivities among the poor households. The assumption that renting is concentrated among the poor is consistent with the empirical evidence. Note, however, that, in our model, the poor are more impatient and renting is an optimal choice if and only if the optimal home size of the home owner is below the minimum $\bar{z}$. This mechanism generates a correlation between time discount rates and labor productivities in a polarized allocation: for the renters both of them must be low. However, note that individuals' capital income is also contributing to the ability to overcome the minimum home size constraint $\bar{z}$. Hence, there are parameter configurations for which we do not need to correlate labor productivity with time discount rates. We are not providing specific conditions to contain the length and complexity of the paper. In the general version of the model, we allow for sufficient heterogeneity among wealth poor households, so that these households may have low or high labor productivities. In other words, a strict positive correlation between labor productivity and degree of impatience can be relaxed.

To characterize a competitive equilibrium of the polarized model, we take an implementability approach (Lucas and Stokey, 1983; Atkeson et al., 1999; Chari and Kehoe, 1999), *i.e.*, we use first order conditions from utility maximization to replace market prices and exploit the transversality condition from households' optimal plans to derive a lifetime present value representation of the households' budget constraints. Note that, by the first order conditions (15)–(17), the no arbitrage condition (18), and the complementary slackness condition (19), multiplication of the left and right hand side of the household i's period-t budget constraint (13) delivers

$$\beta_i U^i_{c,t+1}\hat{a}^i_{t+1} + H^i(c^i_t, z^i_t, \hat{w}_t) = U^i_{c,t}\hat{a}^i_t, \tag{23}$$

for all $i \in \mathcal{I}$ independently of the type of households, *i.e.*, whether it is wealth-rich or wealth-poor, where

$$H^i(c^i_t, z^i_t, \hat{w}_t) \equiv U^i_{c,t}(c^i_t - \epsilon^i\hat{w}_t) + U^i_{z,t}z^i_t \tag{24}$$

denotes the individual i's optimal current net spending. The only condition to obtain the above representation of the budget constraint is that the minimum home size is non binding and the only difference between the two types of households is that $\hat{a}^r_t \geq 0$ and $\hat{a}^p_t = 0$ for all $t \geq 0$. Note, also, that, for $i = r$, the transversality condition, (20) allows for the following representation of the lifetime present value budget constraint:

$$\sum_{t=0}^{\infty} \beta^t_r H^r(c^r_t, z^r_t, \hat{w}_t) = U^r_{c,0}\hat{a}^i_0, \tag{25}$$

whereas, for $i = p$, the zero net wealth assumption implies the more restrictive constraint on consumption choices:

$$H^p(c^p_t, z^p_t, \hat{w}_t) = 0. \tag{26}$$

Conditions (25), (26) are called the *implementability constraints*. Note that, if we use the market clearing conditions in the good, housing, and asset markets (equations (2)–(6) and (22)) and profit maximization (equations (7)–(8)), the rich households' lifetime present value budget constraint (25) implies that the government budget constraints (21) are verified at every period $t \geq 0$. Then, we say that a *polarized allocation*, $\mathcal{A}$, is implementable as a competitive equilibrium with proportional taxes if and only if it satisfies the implementability constraints (25), (26) and the resource feasibility constraints, *i.e.*, equations (2)–(6) at all $t \geq 0$. Note that the implicit tax rates at a polarized allocation are

$$\begin{aligned}(1-\tau_t^{k,i})\frac{f_{k,t-1}^m}{f_{k,t-1}^h} - (1-\delta^h)(1-\tau_t^{h,i})\frac{f_{k,t}^m}{f_{k,t}^h} &= \frac{U_{z,t}^i}{U_{c,t}^i},\\ (1-\tau_t^{k,i})(f_{k,t}^m + 1 - \delta^k) &= \frac{U_{c,t-1}^r}{\beta_r U_{c,t}^r},\\ (1-\tau_t^y)f_{l,t}^m &= \hat{w}_t.\end{aligned}$$

If there is a positive mass of landlords, the subsidy on the tenants' rental housing is implicitly defined by

$$(1-\tau_t^s)f_{l,t}^m = \hat{w}_t \frac{U_{z,t}^p/U_{c,t}^p}{U_{z,t}^r/U_{c,t}^r}.$$

Note that, at a steady state equilibrium of the polarized model, profit maximization and the Euler equation (15) imply

$$f_k^m(k^m, l^m) = q f_k^h(k^h, l^h, x) = (\delta^k + r) = \frac{1}{\beta_r(1-\tau^k)} - (1-\delta^k), \tag{27}$$

i.e.,

$$\hat{R}^r = 1/\beta_r. \tag{28}$$

Since we are assuming that debt is untaxed, the above equation implies that, at steady state,

$$\hat{R}^p = 1 + r = 1/(\beta_r(1-\tau^k)),$$

i.e., the capital tax is fully shifted on the poor households, who face a higher gross interest rate on their debt relative to the case of a zero capital tax. This fact has the further implication that, at steady state, a positive financial tax raises the user cost of housing faced by the poor households on their owner occupied housing. Specifically, we have

$$\hat{\pi}^r = q\left(\left(\frac{1-\beta_r}{\beta_r}\right) + \delta^h + (1-\delta^h)\tau^{h,r}\right), \tag{29}$$

$$\hat{\pi}^p = q\left(\left(\frac{1-\beta_r}{\beta_r}\right) + \delta^h + (1-\delta^h)\tau^{h,p} + \frac{\tau^k}{\beta_r(1-\tau^k)}\right), \tag{30}$$

so that, in order to implement a tax structure guaranteeing a cheaper effective housing cost for the poor households than for the rich ones, *i.e.*, $\hat{\pi}^r \geq \hat{\pi}^p$, we need

$$\tau^k \leq \frac{(1-\delta^h)\beta_r(\tau^{h,r}-\tau^{h,p})}{1+(1-\delta^h)\beta_r(\tau^{h,r}-\tau^{h,p})}.$$

3 Optimal Tax Structure

In this section we consider the optimal taxation problem under commitment for the polarized model. While in section B of the Appendix we provide the extension to the more general model, the main message that we can derive from this example remains unaltered.

3.1 Framework

The planner maximizes a weighted average of per period utilities across households types at competitive equilibrium allocations by choosing appropriate values of the available tax rates. In order to obtain the steady state allocation as a possible solution to the optimal policy we assume that per period utilities are discounted at the same rate, β_r, *i.e.*, the discount rate of the most patient households. Note that this type of social welfare function implies that the impatient households will be saving more than they would if the planner was discounting utilities at the (heterogeneous) subjective discount rates. However, since we only consider equilibria with binding debt limits for the impatient households (at or near steady states), replacing their subjective discount rate with the higher value, β_r, has no consequences on these households' net wealth. Given these premises and the analysis in the previous section, the planning problem for the polarized economy is

$$\max_{\mathcal{A}} \sum_{t=0}^{\infty} \beta_r^t \left(\eta m_r U(c_t^r, z_t^r) + m_p U(c_t^p, z_t^p)\right) \qquad \text{s.t.: (2)–(6), (26) and} \tag{PP}$$

$$\sum_{t=0}^{\infty} \beta_r^t H^r(c_t^r, z_t^r, \hat{w}_t) \geq U_{c,0}^r \hat{a}_0^i, \tag{31}$$

where m_r and m_p are the fractions of rich and poor households, $\eta \geq 0$ represents the welfare weight attached to the rich households' utility and (31) defines the government's (as well as the rich households') budget constraint[6].

3.2 Characterization

The characterization of the optimal tax structure is based on the *general equilibrium elasticities* related, respectively, to the tax rates on capital and housing (Chari and Kehoe, 1999; Atkeson et al., 1999), defined, respectively, as

$$g_j^i = \frac{U_{c,j}^i c^i + U_{z,j}^i z^i}{U_j} - \frac{U_{c,j}^i c^i}{U_j^i} \left(\frac{\epsilon^i \hat{w}}{c^i} \right) \tag{32}$$

for $i = p, r$, $j = c, z$. These elasticities capture the extent to which a fall in the corresponding tax rates is reducing distortions. In particular, the higher is g_z relative to g_c, the higher are the efficiency costs from taxing housing. It is immediate to verify that for poor households, because c and z are normal goods and they have no wealth, their own general equilibrium elasticity of consumption is positive while the one related to housing is negative, *i.e.*,

$$g_c^p > 0 > g_z^p.$$

We summarize the main result of this section in the following proposition.

Proposition. *Let $\hat{\pi}_t^i$ and π_t be, respectively, the after tax and the before tax user costs of housing for household type $i = p, r$. Clearly, $\hat{\pi}_t^i > \pi_t$ ($\hat{\pi}_t^i < \pi_t$) implies that the housing of household type i is taxed (subsidized). Then, at any interior optimal allocation such that the government's lifetime budget constraint (31) is binding, the poor households' housing must be subsidized and the rich households' housing must be taxed if and only if $g_{c,t}^r > g_{z,t}^r$.*

Note that the motivation of rich households' tax on housing follows a standard comparison between demand elasticities. How should we instead understand the housing subsidy on the poor households? The basic intuition is the following. Since poor households are "hand to mouth", the private benefit of increasing their consumption ($U_{c,t}^p$) has an extra cost (over and above the standard resource cost) due to some additional distortions. In fact, in order to increase these households' consumption, the planner should raise net wages, and, since labor

[6] Whose equivalence is established by Walras' Law.

taxes are not individual specific, this can only be accomplished by shifting taxation from labor (which bears no distortions) to capital. Hence, since U_c^p must be relatively large, the poor households' marginal rate of substitution between housing and consumption (U_z^p/U_c^p) must be relatively small, *i.e.*, smaller than the before tax user cost of housing.

In the rest of this section we provide a more rigorous account of these insights. In particular, by recalling the definition of the individuals' optimal current net spending $H^i(.)$ defined in (24) for $i = p, r$, we first note that the partial derivatives of this expression with respect to consumption, housing and net wage (to be denoted, respectively, H_c^i, H_z^i, H_w^i) play an important role in the characterization of the optimal tax structure. In particular, note that

$$\begin{aligned} H_j^i(c^i, z^i, \hat{w}) &= U_j(c^i, z^i, \hat{w})\,(1 + g_j)\,, \qquad \text{for } j = c, z, \\ H_w^i(c^i, z^i, \hat{w}) &= -U_c(c^i, z^i)\epsilon^i. \end{aligned}$$

Since the poor households' current net spending holds with equality at all periods, the poor households' consumption, c^p, can be expressed as a function of housing demand and the net wage, *i.e.*,

$$c^i = \psi(z_t^i, \hat{w}_t), \tag{33}$$

where $\psi(.)$ is a differentiable function with partial derivatives with respect to housing services and net wage, ψ_z, ψ_w, such that

$$\psi_z = -\frac{H_z^p}{H_c^p} = -\frac{U_z^p(1 + g_z^p)}{U_c^p(1 + g_c^p)}, \qquad \psi_w = -\frac{H_w^p}{H_c^p} = \frac{\epsilon^p}{(1 + g_c^p)}. \tag{34}$$

Now we solve the planner's problem using a lagrangean approach. Following the existing literature we define the *pseudo welfare function*

$$\tilde{U}_t = \eta m_r U(c_t^r, z_t^r) + m_p U(c_t^p, z_t^p) + \mu m_r (H^r(c_t^r, z_t^r, \hat{w}_t) - U_{c,0}^r \hat{a}_0^i),$$

where the Lagrange multiplier μ takes care of the government's budget constraint (31). Then, we set up the lagrangean function

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta_r^t \Big(\tilde{U}_t + \lambda_t^m (f^m(k_t^m, l_t^m) + (1-\delta^k)k_t - c_t - g_t - k_{t+1}) +$$

$$+\lambda_t^h (f^h(k_t^h, l_t^h, x_t) - (1-\delta^h)h_t - h_{t+1}) + \nu_t(h_t - z_t) + \xi_t^l (1 - l_t^m - l_t^h) + \xi_t^k (k_t - k_t^m - k_t^h) \Big) .$$

We split the first order characterization of the optimal taxation problem into two sets of conditions. The first concerns the optimal allocation of capital, labor and land across sectors, consumption of manufacturing, and housing:

$$\lambda_t^h / \lambda_t^m = f_{l,t}^m / f_{l,t}^h = f_{k,t}^m / f_{k,t}^h, \tag{35}$$

$$\lambda_t^m / \lambda_{t+1}^m = \beta_r (f_{k,t+1}^m + 1 - \delta^k) = (\lambda_t^h / \lambda_t^m) f_{k,t+1}^h. \tag{36}$$

Note that, by the profit maximization conditions, (7), (8), the above imply

$$\lambda_t^h / \lambda_t^m = q_t, \qquad \lambda_t^m / \lambda_{t+1}^m = \beta_r (1 + r_{t+1}).$$

The second set of conditions concerns the optimal allocation of consumption, housing and labor across households. Letting

$$\pi_t = q_{t-1}(1 + r_t) - (1 - \delta^h) q_t,$$

the optimal allocation of consumption and housing services across rich households, *i.e.*,

$$\lambda_t^m = U_{c,t}^r (\eta + \mu (1 + g_{c,t}^r)), \tag{37}$$

$$\lambda_t^m \pi_t = U_{z,t}^r (\eta + \mu (1 + g_{z,t}^r)), \tag{38}$$

which provide an interpretation of λ^m and $\lambda^m \pi_t$ as the shadow prices of consumption and housing services, respectively. Note that, by (36), equation (37) implies that, at steady state,

$$f_k^m + 1 - \delta^k = 1 + r = 1/\beta_r, \tag{39}$$

which establishes the Chamley-Judd zero capital tax rate result at steady state.

Now consider the first order conditions related to the rich households' optimal allocation of consumption and housing services. Solving for the lagrange multipliers, we obtain

$$\frac{U^r_{z,t}}{U^r_{c,t}} \equiv \hat{\pi}^r_t = \pi_t \left(\frac{\eta + \mu(1 + g^r_{c,t})}{\eta + \mu(1 + g^r_{z,t})} \right), \tag{40}$$

which parallels a standard condition for the optimal tax structure in multi-good economies. Note that $g^r_c > g^r_z$ implies $U^r_z / U^r_c > \pi$, *i.e.*, the after tax user cost of housing for the rich households should be larger than before tax cost, a condition that calls for a housing tax.

Consider now the optimal allocation of the poor households' consumption and housing services. By equating the marginal cost of increasing the amount of housing services to the marginal benefit, we derive

$$\lambda^m_t (\pi_t + \psi_{z,t}) = U^p_{z,t} + U^p_{c,t} \psi_{z,t},$$

where the left hand side represents the net loss of resources due to an extra unit of housing services to the poor and the right hand side the net marginal utility benefit. From the above we obtain

$$\lambda^m_t = U^p_{c,t} \left(\frac{U^p_{z,t}/U^p_{c,t} + \psi_{z,t}}{\pi_t + \psi_{z,t}} \right),$$

implying that

$$U^p_{z,t}/U^p_{c,t} > \pi_t \qquad \Leftrightarrow \qquad U^p_{c,t} < \lambda^m. \tag{41}$$

In other words, a housing tax (subsidy) on the poor household is optimal if and only if the the poor households' marginal utility of consumption is smaller (larger) than the shadow price of consumption. In order to understand how these two variables are related, we have to consider how the planner sets the net wage, $\hat{w}$. In particular, note that an extra unit of net wage increases the poor households' marginal utility of consumption, but it also reduces the available resources, and it implies some extra cost in terms of distortions, since labor supply is inelastic and, then, a labor tax minimizes distortions. Formally, the first order conditions imply that the optimal net wage is set according to:

$$\underbrace{\lambda_t^m \psi_{w,t} - \mu H_{w,t}^r}_{\text{total cost of raising } \hat{w}_t} = \underbrace{U_{c,t}^p \psi_{w,t}}_{\text{gain of raising } \hat{w}} , \tag{42}$$

where the left hand side represents the total cost and the right hand side the marginal utility benefit of a higher net wage. Note that the total cost has two components: the first is the standard resource cost due to some additional consumption and the second, represented by $-\mu H_{w,t}^r = U_{c,t}^r \epsilon^r > 0$, is the extra tax distortions that we need to impose to compensate for the loss in the government revenue. Note that the latter shows up in the rich households budget constraint, or, equivalently, on the government budget constraint. In other words, the term $-\mu H_w^r$ represents the efficiency cost of lifting (non distortionary) income taxation, a sort of externality on the rich households' tax burden. Then, by (34), equation (42) can be written as

$$\lambda_t^m = U_{c,t}^p - \mu U_{c,t}^r (1 + g_{c,t}^p) \frac{\epsilon^r}{\epsilon^p}, \tag{43}$$

implying that the poor households' marginal utility of consumption is greater than the shadow price of consumption. By (41) we obtain

$$U_{z,t}^p / U_{c,t}^p < \pi_t, \tag{44}$$

i.e., the implicit after tax user cost of housing must be lower than the before tax cost and, then, the poor households' housing services must be subsidized. Note that the optimal housing subsidy is zero if $\epsilon^r = 0$, *i.e.*, when the rich households are "pure capitalists" as in Judd (1985). In our model this case corresponds to a situation in which the rich are landlords deriving their income from housing rents only. By using (43) into (42) and defining

$$\gamma = U_c^r \epsilon^r / U_c^p \epsilon^p,$$

we derive the following optimal tax formula for the poor households

$$\frac{U_{z,t}^p}{U_{c,t}^p} \equiv \hat{\pi}_t^p = \pi_t \left(\frac{1 - \mu\gamma(1 + g_{c,t}^p)}{1 - \mu\gamma(1 + g_{z,t}^p)} \right). \tag{45}$$

Note that, by (37) and (43), the value of μ, a proxy for how tight is the government budget constraint, is determined as

$$\gamma\mu = \frac{\epsilon^r - \eta\gamma\epsilon^p}{(1+g_c^r)\epsilon^p + (1+g_c^p)\epsilon^r}, \tag{46}$$

which is positive for $U_c^p > \eta U_c^r$.

3.3 Characterization in Terms of Price Elasticities

We now provide an estimate of the individual specific housing tax and subsidies in terms of price elasticities at steady states in the spirit of the sufficient statistics approach. In particular, we provide two leading examples. The first is based on the assumption of a quasi-linear utility (in consumption), that rules out income effects, and the other is based on a CES utility, *i.e.*, a preference specification allowing for income effects but characterized by a constant elasticity of substitution between consumption and housing services.

Let the price elasticities of consumption and housing services and the cross elasticity as

$$e_c^i = -U_c^i / c^i U_{cc}^i, \qquad e_z^i = -U_z^i / z^i U_{zz}^i, \qquad e_{cz}^i = U_z^i / c^i U_{zc}^i.$$

Define, also, the *housing-to-consumption expenditure ratio*,

$$\phi^i = U_z^i z^i / U_c^i c^i,$$

and note that, by the normality of consumption and housing services, $1/e_c^i + 1/e_{cz}^i > 0$ and $1/e_z^i + \phi^i / e_{cz}^i > 0$, and by the steady state budget constraints,

$$\frac{\hat{w}\epsilon^i - c^i}{c^i} = \phi^i - \omega^i,$$

where

$$\omega^i = \left(\frac{r}{1+r}\right)\frac{\hat{a}^i}{c^i}$$

is the *wealth income-to-consumption ratio*. In our (polarized) equilibrium, $\omega^p = 0$ and, by asset market clearing,

$$\omega^r = r\frac{k + qh + b^g/(1+r)}{m_r c^r}. \tag{47}$$

The term ω^r plays an important role in our analysis as it represents a sort of income effect on the optimal tax rates. In fact, the general equilibrium elasticities for the rich households are

$$g_c^r = \frac{\phi^r}{e_{cz}^r} + \frac{\phi^r - \omega^r}{e_c^r}, \qquad g_z^r = -\left(\frac{1}{e_z^r} + \frac{\phi^r - \omega^r}{e_{cz}^r}\right). \tag{48}$$

Then, a larger ω^r lowers the general equilibrium elasticity of consumption, g_c^r, and, provided that c and z are complements, it increases the general equilibrium elasticity of housing services, g_z^r, implying less scope for housing taxation. In our characterization we assume that the planner assigns zero weight to the rich households' welfare, *i.e.*, $\eta = 0$ and define t^r and t^p as the implicit tax and subsidy on the rich and poor households' housing services, respectively, so that

$$\hat{\pi}^r = \pi(1 + t^r), \qquad \hat{\pi}^p = \pi(1 - t^p),$$

where the variables t^p and t^r are, respectively, the *implicit housing subsidy* and the *housing tax* rates. By (29) and (30), the implicit tax rates defined above are implemented through the tax instruments considered in section 2 by setting

$$\tau^{h,r} = \left(\frac{r + \delta^h}{1 - \delta^h}\right) t^r, \qquad \tau^{h,p} = -\left(\frac{r + \delta^h}{1 - \delta^h}\right) t^p.$$

In our first example we assume that utility is linear in consumption, so that there are no income effects, a popular simplification in the optimal tax literature (cf. Saez and Stantcheva (2018)). In this case, $1/e_c^i = 1/e_{cz}^i = 0$. Then, by rearranging the optimal tax conditions (40), (45), and by (46), we obtain

$$t^r = \frac{1}{e_z^r - 1}, \qquad t^p = \frac{1}{1 + (\epsilon^p/\epsilon^r)e_z^p}.$$

We should remark that $e_z^r > 1$ is required to guarantee the existence of a steady state and that the above imply no upper bound on the housing tax, whereas the housing subsidy is always smaller than one and decreasing in the poor's relative labor productivity, ϵ^p/ϵ^r

(a result that extends to the case of CES utility considered below). Now consider a CES specification of $U(c, z)$ with elasticity of substitution equal to σ and a constant expenditure ratio between housing services and consumption equal to ϕ. Note that, for $\sigma = 1$, we obtain the Cobb-Douglas utility, where $\phi = \theta/(1-\theta)$. Under the CES specification, we get

$$e_c = \sigma(1 + 1/\phi), \qquad e_z = e_{cz} = \sigma(1 + \phi)$$

and

$$t^r = (1+\phi)\left(\frac{(1+\phi) - \omega^r}{(\sigma-1)(1+\phi) + \omega^r}\right), \qquad t^p = \frac{1}{1 + \frac{\epsilon^p/\epsilon^r}{1+\phi}\left(\sigma + \phi - \frac{\phi}{1+\phi}\omega^r\right)}. \tag{49}$$

Note that, for given ω^r, both the tax and the subsidy on housing services fall with the elasticity of substitution σ. Furthermore, t^r is increasing in the housing share of expenditure, ϕ, whereas this parameter has ambiguous effects on the housing subsidy (it has a negative effect for $\sigma \leq 1$). Finally, differently from the case of quasi-linear utility, the CES case shows that a higher wealth income-to-consumption ratio, ω^r, goes along with a lower housing tax and a higher housing subsidy. Since wealth and consumption are endogenous variables, these remarks are only suggestive of the effective consequences of a rising wealth, which we provide in the next section for a simulation of the model with plausible parameters. It turns out that, for the CES specification, the term ω^r is uniquely determined by the net wage to aggregate wealth ratio. The basic insight from the above discussion is that housing taxes are a useful and efficient tool to compensate the inequality that arises in this model due to the debt limits. Since capital taxes are sub-optimal, the planner may mitigate the effect on inequality of switching taxation from capital to labor by selecting a combination of taxes and subsidies on housing services. However, the size of these taxes should reflect the elasticities of housing demand, which, are turn, is affected by the wealth distribution. Somewhat counterintuitively, a higher wealth inequality calls for a lighter housing taxation (but possibly more housing subsidies), since higher wealth makes housing demand more elastic. As back of the envelope calculation, we take the wealth income-to-consumption ratio to be around 40% in the U.S., given that the capital-to-income ratio is approximately equal to 25% (Jones, 2014) and the personal consumption expenditure to income ratio to 65%. In addition, for the U.S., the parameter ϕ is around 25% (using data from BEA NIPA Table 2.3.5), and the parameter σ is close to 1 (Piazzesi et al., 2007). Then, the implicit housing tax rate on rich households (t^r) is positive and approximately equal to 2.5. In fact, when σ is close to 1, t^r is positive as long as $\omega^r < 1 + \phi$, condition which is verified using U.S. data. The value of the implicit housing subsidy on poor households additionally depends on poor's relative labor productivity (ϵ^p/ϵ^r). If the latter is close to 1, then t^p is approximately equal to 0.5, and smaller for higher values of poor's relative labor productivity.

4 Quantitative Analysis

In this section we presents quantitative results for the optimal tax structure at steady state when aggregate wealth-to-income and wealth inequality increase due to two alternative mechanisms: a rise in the government debt or a fall in the real interest rate (which, at steady state, corresponds to the rate of time preference of the wealth-rich households). Additionally, following recent proposals that have been circulating in the academic and political debates, we consider the effects of the introduction of an exogenous general flat tax on financial and housing net wealth to see how this would affect households' welfare for different values of the wealth-to-income ratios in our model.

4.1 Simulating the Optimal Tax Structure

We first consider the quantitative results of the optimal taxation problem. Here and in the following numerical exercises we use a very parsimonious parametrization of the model based on Cobb-Douglas preferences and technologies

$$U(c, z) = c^{1-\theta} z^{\theta}, \tag{50}$$

$$f^m(k^m, l^m) = (k^m)^{\alpha_k^m} (l^m)^{\alpha_l^m}, \tag{51}$$

$$f^h(k^h, l^h, x) = (k^h)^{\alpha_k^h} (l^h)^{\alpha_l^h} x^{\alpha_x^h} \tag{52}$$

where $\sum_{j=k,l} \alpha_j^m = \sum_{j=k,l,x} \alpha_j^h = 1$. We calibrate the model by borrowing some of the parameter values from existing literature and setting the others in order to match some moments of the data. We solve for the steady state of the model by solving the system described in detail in the separate online appendix and setting $\tau^k = 0$ and $\sigma = 1$. All the details about this calibration exercise and the specific parameter values are reported in section A of the Appendix along some robustness checks. We consider two scenarios that generate a path of increasing wealth inequality. In the first, we generate different levels of the wealth-to-income ratio by exogenously changing the level of government debt (b^g). In the second, we generate different levels of the wealth-to-income ratio by changing the level of the real interest rate. Specifically, we pick the level of interest rates to match the exogenous wealth ratios obtained under the fiscal scenario. Intuitively, wealth is decreasing in the level of the real interest rate.

It is worth noticing that the Cobb-Douglas representation of the production functions implies that the flow of land permits per unit of labor used in the housing sector and provided by the government is not affecting the equilibrium values of business capital, k, housing capital, qh, total wealth, $v = k + qh + b^g/(1 + r)$ and net wage, $\hat{w}$, although it affects the equilibrium price of the housing stock, q. More specifically, k^m and k^h are positively and linearly correlated. On the other hand, the flow of land permits affects q negatively, but this effect is compensated by a higher housing stock, h. Furthermore, the relation between business capital, k, and housing capital, qh, depends on the relative capital intensities in the two sectors. In particular, if we assume (as we do in our simulations) that the manufacturing sector is more capital intensive (*i.e.*, $k^m > k^h$), then, for given taxes and interest rate, business and housing capital are inversely correlated.

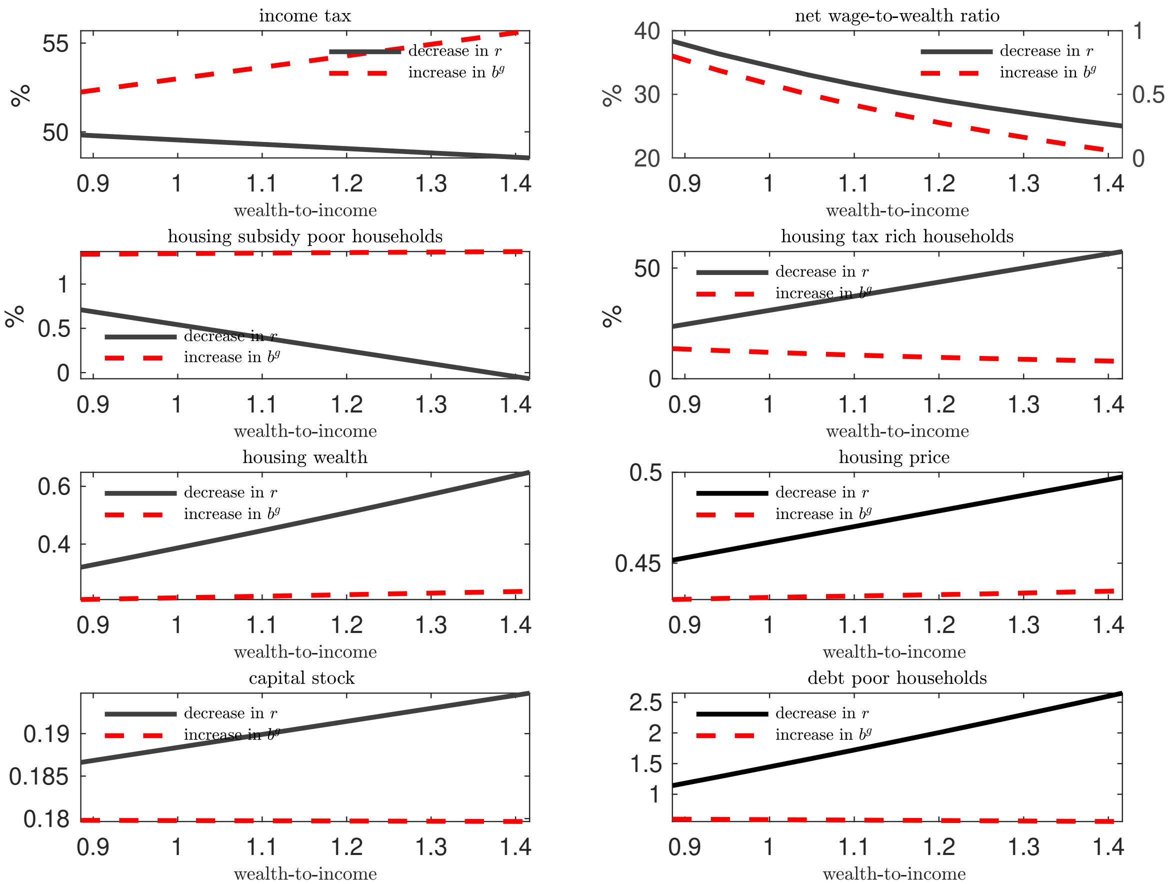
Figure 1 plots the steady state values for the income tax (τ^y); the net wage-to-wealth ratio ($\hat{w}/(k + qh + b^g/(1 + r))$); the housing subsidy to poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the housing wealth (qh); the housing price (q); the capital stock (k); the debt of poor households (qh^p); for different levels of the wealth-to-income ratio. We denote with a red dashed-line the first scenario, in which a higher wealth ratio is associated to higher public debt, and with the black solid line the second scenario, in which a higher wealth ratio is associated to a lower real interest rate. We start by describing the results for the first scenario. First, the income tax increases from approximately 52% to 55%, and wealth inequality, proxied by the net wage-to-wealth ratio, decreases with the level of the wealth ratio. Second, when the wealth ratio increases, the housing subsidy to poor households is approximately flat at 1.3%, while the housing tax on rich households decreases from 13% to 8%. Finally, the effect of an increase in the wealth ratio is neutral with respect to the housing wealth, the housing price, the capital stock and the debt of poor households. Intuitively, in the first scenario, the increase in public debt drains resources which are mostly financed with the increase in income tax. We now describe the second scenario, in which a declining real interest rate drives the increase in the wealth ratio. First, when the wealth ratio increases, the income tax declines by approximately 50% to 48%, and wealth inequality, proxied by the net wage-to-wealth ratio, decreases with the level of the wealth ratio. Second, the housing subsidy to poor households decreases from 0.7% to zero. Third, the housing tax on rich households increases from 23% to 57%. Fourth, the housing wealth, housing price, and capital stock are all increasing in wealth, and decreasing in the real interest rate. Note that the second scenario is "less neutral" relative to the first scenario, but more appealing, in terms of some important stylized facts that have characterized the experience of most advanced economies in the past thirty years. In particular, the rise in aggregate wealth went along with rising housing prices and falling real interest rates (Bonnet et al., 2020), and the latter may have been responsible for the increase in the value of housing property through a rise in the demand of housing (due to the fall in the cost of housing services) and a rise in the demand of housing mortgages (La Cava, 2016). Finally, we note that while in all simulations the wealth income-to-consumption ratio is in the range 20 to 40 percent, and thus in line with the back of the envelope estimate discussed in section 3, our model cannot generate wealth-to-income ratios as large as the current values in many advanced economies[7].

4.2 Effects of Introducing a General Wealth Tax

In this section we evaluate the quantitative effects of introducing a general (flat) tax on net wealth for two scenarios: a benchmark scenario where (housing and financial) wealth is untaxed, and an alternative scenario characterized by a flat 1% tax rate, τ^k, on total net

[7]De Nardi (2015) argues that most quantitative models of wealth inequality cannot generate wealth-to-income ratios as large as the current values in many advanced economies. For example, in the U.S. the net wealth-to-income ratio has been hovering around 4 to 5. In our simulations, for values of the wealth-to-income ratios as large as in the data, the housing tax rate is greater than 100 percent.

Figure 1: Steady State: Main Variables



Notes: This figure plots the steady state values for the income tax (τ^y); the net wage-to-wealth ratio ($\hat{w}/v$); the housing subsidy to poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the housing wealth ($v^h = qh$); the housing price (q); the capital stock (k); and the debt of poor households (qh^p) for different values of the total wealth-to-income ratio ($v/y = 0.8 \ldots 1.8$). The wage and housing tax rates, the housing subsidy, and the net wage-to-wealth ratio, are reported in percentage. We consider two scenarios that generate a path of increasing wealth inequality. In the first (dashed-red line), we exogenously change the level of govern debt (b^g). In the second (solid black line), we change the level of the real interest rate. The latter ranges from 0.5% to 5%. Parameters are from Table A1. Refer to Appendix A for details on the numerical solution of the model.

wealth, which is comparable to the rates we observe in existing tax codes (Jacobsen et al., 2017; Seim, 2017; Brülhart et al., 2019). The parameters are the same are discussed in section A of the appendix.

In our model, this implies $\tau^k = \tau^{h,i}$ for $i = p, r$, and, then, the following steady state user costs of housing

$$\hat{\pi}^p = q\left(\left(\frac{1-\beta_r}{\beta_r}\right) + \delta^h + \left(1 - \delta^h + \frac{1}{\beta_r(1-\tau^k)}\right)\tau^k\right), \tag{53}$$

$$\hat{\pi}^r = q\left(\left(\frac{1-\beta_r}{\beta_r}\right) + \delta^h + (1-\delta^h)\tau^k\right). \tag{54}$$

To reduce the dimensionality of the problem (from the point of view of the effects on the distribution of income and wealth), we only consider the case where the wealth-poor face the same cost of housing services, *i.e.*, we assume that the rent tax is such that $\hat{\pi}^p = \hat{s}$. Therefore, we are limiting the degree of inequality across households below the level that could be otherwise achieved (*i.e.*, for $\hat{s} \geq \hat{\pi}^p$). This is a necessary restriction to keep the model within the boundaries of the polarized economy with two types of households only.

We are especially interested in evaluating the effects of the introduction of a flat wealth tax for different levels of wealth and wealth inequality. In the model, we generate different levels of the wealth-to-income ratio, by assuming two scenarios. In the first, we exogenously generate an increasing level of the wealth-to-income ratio by increasing the level of government debt (b^g), while keeping all the other parameters unchanged. In the second scenario, we endogenously generate an increasing level of the wealth-to-income ratio by reducing the level of the real interest rate (r). To guarantee comparability between the two scenarios, we pick values for the real interest rates to match the exogenous wealth ratios obtained under the first scenario.

Table 1 presents the results of a comparison at the steady state of the benchmark model with zero wealth tax and the model with the 1% wealth tax for different levels of the wealth-to-income ratios (i.e., low = 0.8; medium = 1; and high = 1.5). Panel A refers to the first scenario, in which higher wealth is associated to higher public debt (b^g); while panel B to the second scenario, in which higher wealth is associated to a lower real interest rate (r). In order for the two simulations to be comparable, we change r in order to exactly match the wealth-to-income ratios obtained under the "increase in public debt" scenario (specifically, the interest rates is approximately equal to 5%, 2.6% and 1%, respectively). Public debt-to-income increases from 19% to 70%. Although they have zero net wealth, poor households are affected by the wealth tax because of the general equilibrium effect on prices. Specifically, poor and rich households face different net user costs of housing services (equations (29) and (30)). We summarize the results as follows. First, in both scenarios, the introduction of the wealth tax increases the user cost of housing services for poor households. Specifically, the user cost of housing services for poor households, after the introduction of the 1% flat wealth tax and for a medium wealth-to-income ratio, increases by approximately 30% in the first scenario, and by 20% in the second scenario. On the contrary, for rich households, the user cost of housing services increases in the first scenario, and it decreases in the second scenario. In particular, in the first scenario and for a medium wealth ratio, it increases by approximately 13%; in the second scenario, instead, it drops by approximately 2.3%. The latter result depends on the fact that, in the second scenario, the housing price is lower after the introduction of the wealth tax. Second, the introduction of the wealth tax, under both scenarios, increases the net wage and, conversely, decreases the income tax. Specifically, under the first scenario and for a medium wealth-to-income ratio, the net wage increases by approximately 0.5% and the income tax decreases by approximately 0.5%. The effects on the net wage and on the income tax are higher under the second scenario: the net wage increases by approximately 2.6% and the income tax decreases by approximately 3.8%. Third, the equivalent income loss for poor households of introducing the wealth tax is large and approximately equal to 5% for both scenarios. Fourth, while the effects on the net wage, income tax, and user costs of housing, are similar for the different levels of the wealth-to-income ratio under the first scenario, they are increasing with the wealth-to-income ratio under the second scenario. For example, the net user cost of housing for poor households, after the introduction of a 1% wealth tax, increases by approximately 13% for a low level of the wealth-to-income ratio, and by 30% for a high level of the wealth-to-income ratio.

5 Conclusions

This paper studies optimal tax rates when households accumulate different levels of wealth, the latter consisting in business capital, housing, and financial assets. The Chamley-Judd zero steady state tax on financial and business capital survives, whereas housing wealth is taxed at a non zero rate. In particular, it is optimal to impose a positive tax on rich

Table 1: Introducing a Flat 1% Wealth Tax

Panel A: increase in public debt (Δb^g)			
wealth-to-income	low	medium	high
public debt (%) (b^g/y)	19.00	39.75	70.88
Δ% net wage ($\hat{w}$)	0.53	0.55	0.60
Δ income tax (τ^y)	-0.49	-0.49	-0.49
Δ% net user cost of housing poor ($\hat{\pi}^p$)	29.76	29.94	30.01
Δ% net user cost of housing rich ($\hat{\pi}^r$)	13.11	13.26	13.33
Δ public debt (b^g/y)	19.00	39.75	70.88
Δ% equivalent income loss poor households	-5.57	-5.50	-5.47
Panel B: decrease in real interest rate (Δr)			
wealth-to-income (v/y)	low	medium	high
real interest rate (%) (r)	5.00	2.60	1.00
Δ% net wage ($\hat{w}$)	3.16	3.81	4.64
Δ income tax (τ^y)	-1.51	-1.85	-2.28
Δ% net user cost of housing poor ($\hat{\pi}^p$)	12.97	19.67	30.06
Δ% net user cost of housing rich ($\hat{\pi}^r$)	-1.90	-2.33	-2.95
Δ% equivalent income loss poor households	-5.36	-5.46	-5.54

Notes: This table reports the change in the net wage; income tax; net user costs of housing for poor and rich households, between the scenario with a flat 1% wealth tax and the scenario with a zero wealth tax, for different levels of total wealth-to-income ratio. For the income tax we report the difference in percentage points. For all other variables we report percentage changes. In addition, the table reports the equivalent income loss of poor households, in percentage, determined by the introduction of the flat 1% wealth tax. The equivalent income loss is equal to $1 - (\hat{\pi}_0^p/\hat{\pi}_1^p)^\theta$, where we denote with "0" the scenario with zero wealth tax and with "1" the scenario with the flat 1% wealth tax. Panel A corresponds to a "increase in public debt" scenario, in which the change in wealth is determined exogenously by changing government debt (b^g); panel B corresponds to a "decrease in real interest rate" scenario, in which the change in wealth is determined endogenously by changing the level of the real interest rate (r). We change r in order to exactly match the wealth-to-income ratios obtained under the "increase in public debt" scenario. Parameters are from Table A1. The values for the wealth-to-income ratios are equal to 0.8 (low), 1 (medium), and 1.5 (high); the corresponding values for the real interest rates are 5%, 2.3% and 0.5%. Additionally, in Panel A we also report the values for the public debt-to-income ratios. Refer to Appendix A for details on the numerical solution of the model.

households' housing wealth and a subsidy on poor's households user cost of housing (or rent). Finally, using Cobb-Douglas preferences and technology, we evaluate numerically the impact on optimal tax rates of a rising aggregate wealth-to-income ratio. We find that the mechanism used to generate different wealth levels matters for the optimal tax rates. While the results of a positive housing subsidy on poor households, and housing tax on rich households, are very robust, the evolution of the other optimal tax rates changes depending on the source of the increasing wealth.

Our results depend on several strong assumptions that can be relaxed in future research. Most importantly, we assume that income taxes cannot be contingent on types, or increase progressively with households' income. Although this is possibly an interesting extension, restricting the attention to flat tax rates on labor income and types of wealth (or sources of capital income) has the advantage of providing results that are more comparable with the existing literature on optimal taxation and to generate simpler (and more easily implementable) policy proposals. For example, it could be difficult to implement income tax rates contingent on households' wealth size and composition due to practical implementation or institutional constraints.

It is well known that housing capital receives preferential tax treatment, relative to other types of capital, in most advanced countries, due to the existence of mortgage interest deduction and limited or no taxation of imputed rental income from owner-occupied housing. This implies a distortion in individuals' asset portfolios against business capital, and potential efficiency gains from reducing favorable tax treatment of housing services. Using an overlapping generations model and a limited set of tax rates (linear tax rates on income, imputed rents and mortgage interest deductions) Gervais (2002) find that individuals at all income levels would rather live in a world where imputed rents are taxed, or one where mortgage interest payments are not deductible. Somewhat consistently with these findings, Alpanda and Zubairy (2016) examine a calibrated model with heterogeneous infinitely lived individuals (where renters are hand-to-mouth) and find significant gains (in terms of output losses for given tax revenues) from reducing mortgage interest rate deductions and taxing imputed rents. The model contains a rich set of tax rates and allows income tax rates to be contingent on whether a household is a renter or a home owner. Our results suggest that the second best effects of taxing or subsidizing housing services do not necessarily lend support for a uniform tax treatment across households' types and that these effects are very much dependent on the available menu of taxes. For instance, the Mirrlees' Review claims that most housing wealth taxes (like the council tax) are generally regressive relative to its base and should be replaced by a *housing service tax*, *i.e.*, a flat percentage of the rental value of property, whether it is rented or owner-occupied. Our findings suggest that, with an inequality averse planner, this tax should not be flat, but contingent on the size of individuals' net wealth.

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Appendix

A Calibration

All the parameters used in the quantitative analysis are reported in Table A1. We set the baseline consumption preference parameter $\theta = 0.2$ in order to match the U.S. households expenditure on housing services (approximately 15% of 2015 GDP according to the BEA NIPA Table 2.3.5), and assume Cobb-Douglas preferences and set the elasticity of substitution σ to 1. The latter is similar to the value used, for example, in Piazzesi et al. (2007), who set it to 1.05. The time discount parameter of patient households (*i.e.*, rich households) is set to $\beta^H = 0.96$, implying a steady state real interest rate of 4%, similar to the post-WWII U.S. average (for example, see Campbell and Cochrane (1999)). Impatient households (*i.e.*, poor households) have a lower value for the time discount parameter, which we set to $\beta^L = 0.90$. The annual depreciation of the housing stock is set equal to $\delta^h = 2\%$ as in Iacoviello and Neri (2010), while we assume full capital depreciation $\delta^k = 1$. We use O'Mahony and Timmer (2009)'s KLEMS data to have rough estimates of the capital factor shares in construction and manufacturing in the US over the 1970–2010 period and, accordingly, set $\alpha_k^m = 1/3$ and $\alpha_k^h = 1/5$. These numbers are in line with those in Valentinyi and Herrendorf (2008) who set the capital share in manufacturing and construction respectively to 0.4 and 0.2. We set the weight attached to the land input to $\alpha_x^h = 1/10$, which is in line with the value used by Davis and Heathcote (2005). Finally, we set the government expenditure g to 0.22 to match the U.S. Federal expenditure as fraction of GDP net of government transfers; and the share of patient (*i.e.*, rich) households to 75%, and we consider a constant flow of new land permits. Note that, in order to set the share of poor households, we refer to data from the U.S. Census relative to the net wealth of U.S. households. The latter is defined as the value of assets owned minus liabilities, and it does not include equities in pension plans and the value of home furnishings. We observe that in 2016, the last available data, households below the 10th percentile have negative net wealth (i.e., -3,500) and those below the 25th percentile have a small but positive net wealth (i.e., 4,134). For this reason, we set the share of poor households at the value of 25%. Robustness of our results with respect to changes in these parameter values is verified in the online appendix to this paper[8].

B A More General Model

In this section we give some insights on how the main results presented in the text should be amended in a more general version of the model with a larger degree of heterogeneity.

[8]We do not directly calibrate the housing wealth as a fraction of total wealth. In the simulations, this share is approximately equal to 70% and higher then in the data. For example, Iacoviello (2010) reports a value of approximately 50% for the U.S., where a large fraction of housing wealth (80 percent) is made up by the stock of owner-occupied homes).

Table A1: Model Parameters

Preferences		
consumption expenditure share (baseline):	$1-\theta$	0.80
housing expenditure share (baseline):	θ	0.20
discount rate rich households:	β^H	0.96
discount rate poor households:	β^L	0.90
elasticity of substitution:	σ	1.00
Technology		
housing depreciation:	δ^h	0.02
capital depreciation:	δ^k	0.00
capital share manufacturing:	α_k^m	0.33
capital share construction:	α_k^h	0.10
housing share construction:	α_x^h	0.10
Economy structure		
government expenditure:	g	0.22
share rich households:	m_r	0.75
share poor households:	m_p	0.25

Notes: This table reports all the parameters used in the simulation. The model is simulated for different values of total wealth under two scenarios. In the first scenario, which we label "fiscal contraction" scenario, we exogenously generate an increasing level of wealth by increasing the level of government debt (b^g) while keeping all the other parameters unchanged. In the second, which we label "preference shock" scenario, we endogenously generate an increasing level of wealth by reducing the level of the real interest rate (r). To guarantee comparability between the two scenarios, we pick values for the real interest rates to match the exogenous wealth levels obtained under the "fiscal contraction" scenario. The utility function u is Cobb-Douglas and described in equation (50). The production functions are Cobb-Douglas.

B.I Individuals' Optimization

The complete first order characterization of the households' utility maximization subject to the budget constraints (13), the debt limits (14) and

$$z_t^{r,i} \geq 0, \qquad z_t^{o,i} \geq \bar{z}, \qquad h_t \geq z_t^{o,i} \tag{A1}$$

is as follows:

$$U_{c,t}^i \hat{\pi}_t^i - U_{z,t}^i \geq 0, \quad U_{c,t}^i \hat{s}_t - U_{z,t}^i \geq 0, \quad U_{c,t}^i (\hat{\pi}_t^i - \hat{\rho}_t) \geq 0,$$

together with (15) and the complementary slackness conditions

$$(U_{c,t}^i \hat{\pi}_t^i - U_{z,t}^i)(z_t^{o,i} - \bar{z}) = U_{c,t}^i (\hat{\pi}_t^i - \hat{\rho}_t)(h_t^i - z_t^{o,i}) = (U_{c,t}^i \hat{s}_t - U_{z,t}^i) z_t^{r,i} = 0.$$

Note that the budget set is non convex, but solutions are unique under the assumption that, when indifferent between renting and owner occupying, the household selects the latter possibility. Intuitively, we have to impose $\hat{s} \geq \pi^i$ for all i in order for home ownership not to be sub-optimal. Then, the household is a renter if the interior solution such that $U_{c,t}^i \hat{\pi}_t^i = U_{z,t}^i$ implies $z_t^{o,i} = z_t^i > \bar{z}$. In this case, we have $z^{o,i} = 0$, $z^{r,i} = z_t^i$ and $U_{c,t}^i \hat{s}_t = U_{z,t}^i$. Using the above into the budget constraints (13), we derive

$$\beta_i U^i_{c,t+1}\hat{a}^i_{t+1} + U^i_{c,t}(c^i_t - \epsilon^i\hat{w}_t) + U^i_{z,t}z^i_t + \xi^i_t = U^i_{c,t}\hat{a}^i_t,$$

where $\xi^i_t = 0$ whenever $z^{o,i}_t = 0$, or the minimum home size constraint (11) is non binding. In this case the above corresponds to (23).

B.II Planning Optimum

To set up the planner's problem in the general model, we start by exploiting the market clearing conditions in the good, housing and asset markets, as well as profit maximization, to restate the t-period government budget constraint (21) as follows

$$\sum_i m_i \left(\frac{\hat{a}^i_{t+1}}{\hat{R}^i_{t+1}} + c^i_t + \hat{\pi}^i_t h^i_t + (s_t - \hat{\rho}_t)(h^i_t - z^i_t) - \hat{w}_t - \hat{a}^i_t \right) \geq 0. \tag{A2}$$

Now let $\mathcal{I} = \mathcal{R} \cup \mathcal{P}$ where $\mathcal{R}$ denotes the set of rich patient home owners and $\mathcal{P}$ the set of (impatient) poor households. Furthermore, let $\mathcal{S} \subset \mathcal{P}$ be the set of poor-renters. Equation (A2) can be simplified under the assumed household's partition. Specifically, we set $\beta_i = \beta_r$ for all $i \in \mathcal{R}$ and $\beta_i = \beta_l$ for all $i \in \mathcal{P}$, with $\beta_r > \beta_l$, and consider an equilibrium allocation at which $\hat{a}^i_t = 0$ for all $i \in \mathcal{P}$ and $t \geq 0$. Note that

$$\sum_{i \in \mathcal{P} \backslash \mathcal{S}} (c^i_t + \hat{\pi}^p h^i - \epsilon^i \hat{w}_t) = \sum_{i \in \mathcal{S}} (c^i_t + s_t z^i_t - \epsilon^i \hat{w}_t) = 0, \tag{A3}$$

and

$$\sum_{i \in \mathcal{R}} m_i (h^i_t - z^i_t) = \sum_{i \in \mathcal{S}} m_i z^i_t.$$

Then, using the above into equation (A2) and exploiting the no arbitrage condition (19), the latter is equivalent to

$$\sum_{i \in \mathcal{R}} m_i \left(\frac{\hat{a}^i_{t+1}}{\hat{R}^i_{t+1}} + c^i_t + \hat{\pi}^r z^i_t - \epsilon^i \hat{w}_t - \hat{a}^i_t \right) \geq 0. \tag{A4}$$

Using the first order conditions and the assumption that the minimum home size constraint is non binding; we can rewrite equations (A3), (A4) as

$$\sum_{i \in \mathcal{R}} m_i \left(\beta_r U^i_{1,t+1}\hat{a}^i_{t+1} + H^i(c^i_t, z^i_t, \hat{w}_t) - U^i_{c,t}\hat{a}^i_t \right) \quad \geq \quad 0, \tag{A5}$$

$$H^i(c^i_t, z^i_t, \hat{w}_t) \quad = \quad 0 \qquad i \in \mathcal{P} \tag{A6}$$

where

$$H^i(c^i, z^i, \hat{w}) \equiv U^i_c(c^i - \epsilon^i \hat{w}) + U^i_z z^i.$$

Equations (A5), (A6) are the *implementability conditions* and define the households' budget constraints in terms of first order conditions, instead of prices. Finally, using the transversality condition, equation (A5) can be iterated forward from period zero to provide the following present value representation of the government budget constraint

$$\sum_{t=0}^{\infty} \beta_r^t \sum_{i\in\mathcal{R}} m_i H^i(c_t^i, z_t^i, \hat{w}_t) \geq \sum_{i\in\mathcal{R}} m_i U_c(c_0^i, z_0^i)\hat{a}_0^i, \qquad \text{for all } i \in \mathcal{R}. \tag{A7}$$

Any sequence $\{c_t^i, z_t^i, \hat{w}_t; i = \mathcal{I}\}_{t=0}^{\infty}$ satisfying conditions (A6), (A7) together with the resource feasibility constraints (equations (2)–(6)), for all $t \geq 0$, and for some initial aggregate wealth (verifying (22)),

$$\sum_{i\in\mathcal{R}} m_i \hat{a}_0^i = \hat{R}_0(k_0 + b_0^g/R_0 + q_{-1}h_0),$$

is a competitive equilibrium implemented by some set of implicit individual specific tax rates. Now define the *pseudo welfare function*

$$\tilde{U}_t = \eta \sum_{i\in\mathcal{R}} m_i U(c_t^i, z_t^i) + \sum_{i\in\mathcal{P}} m_i U(c_t^i, z_t^i) + \mu \sum_{i\in\mathcal{R}} m_i H^i(c^i, z^i, \hat{w}), \tag{A8}$$

where the multiplier μ is positive if the Planner needs distortionary taxation to finance public spending. For a given policy, $\{g_t, x_t\}_{t=0}^{\infty}$, the Planner's decision variables are defined by

$$\mathcal{A} = \{c_t^i, z_t^i, k_t^j, h_{t+1}, l_t^j, k_t; i = \mathcal{I},\ j = h, m\}_{t=0}^{\infty},$$

and we define the optimal taxation problem as

$$\max_{(\mathcal{A},\mu)\geq 0} \sum_{t=1}^{\infty} \beta_r^t \tilde{U}_t - \mu W_0 \qquad \text{s.t. equations (2)–(6) at all } t \geq 0, \tag{PP}$$

where

$$W_0 = \sum_{i\in\mathcal{R}} m_i U_c(c_0^i, z_0^i)\hat{a}_0^i, \qquad \sum_{i\in\mathcal{R}} m_i a_0^i = (1 + r_0)k_0 + b_0^g + q_{-1}h_0.$$

Note that, since we only consider the case of full commitment, the planner is unable to revise the given initial tax rates, so that W_0 is a predetermined initial condition in the planning problem. To characterize the planning optimum, we follow the same notation for the Lagrange multipliers associated to the supply side of the economy, and for the general equilibrium elasticities that we have used in section 3. In particular, productive optimality is characterized by equations (35), (36), and then the optimal allocation of consumption and housing services across rich households, *i.e.*, for all $i \in \mathcal{R}$, is defined as in (37) and (38), which establish the Chamley-Judd zero capital tax rate result at steady state and define the implicit tax on housing for all $i \in \mathcal{R}$ from

$$\frac{U^i_{z,t}}{U^i_{c,t}} \equiv \hat{\pi}^i_t = \pi_t \left(\frac{\eta + \mu(1 + g^i_{c,t})}{\eta + \mu(1 + g^i_{z,t})} \right) \qquad \text{for all } i \in \mathcal{R}. \tag{A9}$$

Turning to the first order conditions for the optimal allocation of poor households' housing services, we get

$$\lambda^m_t \pi_t = U^i_{z,t} \left(1 - \left(\frac{1 + g^i_{z,t}}{1 + g^i_{c,t}} \right) \left(1 - \frac{\lambda^m_t}{U^i_{c,t}} \right) \right) \qquad \text{for all } i \in \mathcal{P}. \tag{A10}$$

Finally, the first order condition related to the optimal allocation of the net wages, $\hat{w}_t$, can be stated as follows

$$\underbrace{\left(\sum_{i \in \mathcal{P}} m_i \frac{U^i_{c,t} \epsilon^i}{1 + g^i_{c,t}} \right)}_{\text{extra consumption}} = \mu \underbrace{\left(\sum_{i \in \mathcal{R}} m_i U^i_{c,t} \epsilon^i \right)}_{\text{extra distortions}} + \lambda^m_t \underbrace{\left(\sum_{i \in \mathcal{P}} m_i \frac{\epsilon^i}{1 + g^i_{c,t}} \right)}_{\text{reduced resources}}. \tag{A11}$$

Note that (A11) equates the gain from any extra unit of net wage due to poor households' extra consumption to the sum of two different costs: the cost of the additional distortions following from the fall in the tax revenue plus the cost of the fall in the available resources. The last two costs are weighted, respectively, by the Lagrange multiplier μ (representing the gain from a fall in distortionary taxation) and the shadow price of consumption, λ^m_t. Now observe that, by (A9), $\hat{\pi}^i_t > \pi_t$ if and only if $g^i_{c,t} > g^i_{z,t}$ for all $i \in \mathcal{R}$, *i.e.*, the cost of housing services for rich households must be taxed if the gain in efficiency from a fall in the (implicit) tax on consumption exceeds the gain from a fall in the tax on housing. A second important observation is that, by (A10), and since $g^i_{c,t} \geq g^i_{z,t}$ for all $i \in \mathcal{P}$,

$$\frac{U^i_{z,t}}{U^i_{c,t}} \equiv \hat{\pi}^i_t = \pi_t \left(\frac{(1 + g^i_{c,t})\lambda^m_t}{U^i_{c,t}(g^i_{c,t} - g^i_{z,t}) + (1 + g^i_{z,t})\lambda^m_t} \right) \qquad \forall i \in \mathcal{P}. \tag{A12}$$

implying that, for all $i \in \mathcal{P}$,

$$U^i_{z,t}/U^i_{c,t} < \pi_t \qquad \Leftrightarrow \qquad \lambda^m_t < U^i_{c,t} \qquad \forall i \in \mathcal{P}. \tag{A13}$$

In other words, the cost of housing services for the worker must be subsidized if her marginal utility of consumption exceeds the shadow price of consumption. To understand the circumstances under which this condition holds, define the "weights"

$$\xi^i_t = \frac{m_i \epsilon^i/(1 + g^i_{c,t})}{\sum_{j \in \mathcal{P}} m_j \epsilon^i/(1 + g^j_{c,t})},$$

and notice that, by rearranging the terms in equation (A11), we obtain

$$\lambda_t^m = \sum_{i \in \mathcal{P}} \xi_t^i U_{c,t}^i - \mu \left(\frac{\sum_{i \in \mathcal{R}} m_i \epsilon^i U_{c,t}^i}{\sum_{j \in \mathcal{P}} m_j \epsilon^i / (1 + g_{c,t}^j)} \right).$$

Since the weights, ξ_t^i, are positive and they sum up to one and $\mu > 0$, then λ_t^m is strictly smaller than a convex linear combination of the poor households' marginal utilities of consumption. Namely, the shadow price of consumption falls short of an average of the poor households' marginal utility of consumption because of the extra-distortions implied by shifting taxation from labor to housing. This implies that $\lambda_t^m < \max_{i \in \mathcal{P}} U_{c,t}^i$, *i.e.*, by (A13), the user cost of housing faced by poor households whose marginal utility of consumption is relatively large must be subsidized.

Online Appendix (not for publication)

This online appendix contains additional robustness results and derivations for Borri, N. and P. Reichlin (2020): "Optimal Taxation with Homeownership and Wealth Inequality."

This online appendix is organized as follows:

- Appendix OA1: The model used in the numerical simulations
- Appendix OA2: The calibration
- Appendix OA3: Additional robustness results

OA1 Model

Here we provide a full specification of the model that we use in the numerical simulation for the parameters described in Table A1. The utility and production functions are specified as in (50), (51), (52), so that the parameter θ denotes the expenditure share on housing, α_k^j the capital shares in sector $j = h, m$ and α_x^h the land share in sector h. Given these premises, it is convenient to define the rich households' rate of time preference,

$$r^r = \frac{1-\beta_r}{\beta_r},$$

and the aggregate net wealth,

$$v \equiv qh + k + b^g/(1+r). \tag{OA1}$$

Then, the set of equations defining the equilibrium steady state is

$$\begin{aligned} c^p &= (1-\theta)\epsilon^p \hat{w}, && \text{(OA2)} \\ c^r &= (1-\theta)\left(\epsilon^r \hat{w} + r^r \frac{v}{m_r}\right), && \text{(OA3)} \\ z^p &= \left(\frac{\theta}{1-\theta}\right) c^p / \hat{\pi}^p, && \text{(OA4)} \\ z^r &= \left(\frac{\theta}{1-\theta}\right) c^r / \hat{\pi}^r. && \text{(OA5)} \end{aligned}$$

Note that, since b^g can be set arbitrarily, we can as well consider v as an arbitrary value. By the homogeneity of the utility function, we can aggregate the demand of consumption and housing services in the market clearing conditions (2), (4) at steady state to get:

$$\begin{aligned} (1-\theta)(\hat{w} + r^r v) &= f^m(k^m, l^m) - \delta^k k - g, && \text{(OA6)} \\ \theta\left(\frac{m^p \epsilon^p \hat{w}}{\hat{\pi}^p} + \frac{1}{\hat{\pi}^r}(m^r \epsilon^r \hat{w} + r^r v)\right) &= h, && \text{(OA7)} \\ 1 + r &= \frac{1+r^r}{1-\tau^k}. && \text{(OA8)} \end{aligned}$$

Using the steady state versions of (7), (8), (3), (5), (6), we obtain

$$
\begin{aligned}
\frac{k^m}{l^m} &= \left(\frac{\alpha_k^m}{r+\delta^k}\right)^{1/(1-\alpha_k^m)}, && \text{(OA9)}\\
\frac{k^h}{l^h} &= \left(\frac{\alpha_k^h}{\alpha_k^m}\right)\left(\frac{1-\alpha_k^m}{1-\alpha_k^h-\alpha_x^h}\right)\frac{k^m}{l^m}, && \text{(OA10)}\\
w &= (1-\alpha_k^m)\left(\frac{k^m}{l^m}\right)^{\alpha_k^m}, && \text{(OA11)}\\
q &= \left(\frac{\alpha_k^m}{\alpha_k^h}\right)^{\alpha_k^h}\left(\frac{1-\alpha_k^m}{1-\alpha_k^h-\alpha_x^h}\right)^{1-\alpha_k^h}\left(\frac{k^m}{l^m}\right)^{\alpha_k^m-\alpha_k^h}\left(\frac{x}{l^h}\right)^{-\alpha_x^h}, && \text{(OA12)}\\
1 &= l^m + l^h && \text{(OA13)}\\
k &= k^h + k^m && \text{(OA14)}\\
\delta^h h &= (k^h)^{\alpha_k^h}(l^h)^{1-\alpha_k^h-\alpha_x^h}x^{\alpha_x^h} && \text{(OA15)}
\end{aligned}
$$

Together with (OA6) and (OA7), equations (OA9)–(OA15) form a system of 9 equations in the 9 unknowns

$$(k^m, l^m, k^h, l^h, w, q, k, h, \hat{w}),$$

as functions of $(\tau^k, x, \hat{\pi}^p, \hat{\pi}^r, v)$. From this set of variables we can derive the individual specific consumptions of manufacturing and housing services, (c^i, c^r, z^i, z^r), for all $i \in \mathcal{P}$, from equations (OA2)–(OA5).

Exogenous Uniform Wealth Tax on the Rich. For the simulation in which we impose a uniform wealth tax, τ^k, on the rich households and no wealth tax on the poor households' deb, the user costs of housing are

$$\hat{\pi}^p = q\left(r^r + \delta^h + (1-\delta^h)\tau^k + (1+r^r)\frac{\tau^k}{1-\tau^k}\right), \qquad \hat{\pi}^r = q\left(r^r + \delta^h + (1-\delta^h)\tau^k\right).$$

Optimal Tax Rates. For the simulation of the optimal tax structure derived in section 4, we set $\tau^k = 0$, *i.e.*, $r^r = r$ and t^r and t^p as defined in equation (49) with $\sigma = 1$, obtaining

$$\hat{\pi}^r = \pi\left(1 + \frac{\xi}{1-\theta}\right), \qquad \hat{\pi}^p = \pi\left(1 - \frac{\epsilon^r}{\epsilon^r + \epsilon^p\left(1 - \frac{\theta}{1+\xi}\right)}\right),$$

where

$$\xi = \frac{m^r \epsilon^r \hat{w}}{rv}, \qquad \pi = q\left(r + \delta^h\right).$$

In the quantitative analysis, we consider two scenarios. In the first scenario, we generate different levels of the wealth-to-income ratio by exogenously changing b^g in (OA1). In the second scenario, we endogenously obtain different levels of the wealth-to-income ratio by changing the levels of the real interest rate. Note that wealth is decreasing in the real interest rate. To guarantee comparability of the quantitative results under the two scenarios, in the second scenario we choose levels of the real interest rate to match the levels of the wealth-to-income ratio of the first scenario.

OA2 Calibration

In this section we present additional data in support of the parameters used in the calibration and reported in Table A1. Specifically, Table OA1 reports data on household net wealth holdings by percentile. We refer to this data to set the share of rich households to 0.75. In fact, we observe that households below the 10th percentile have negative net wealth (i.e., -3.500 dollars in 2016), and those below the 25th percentile have a small but positive net wealth (i.e., 5,556 dollars in 201).

Table OA1: Household Net Wealth Holdings by Percentile

percentile	2013	2014	2015	2016
10th	-6,573	-4,113	-3,113	-3,500
25th	4,134	4,565	5,163	5,556
50th	82,920	84,160	89,870	94,670
75th	330,300	332,300	354,400	359,400
90th	879,700	820,000	897,600	952,300

Notes: This table reports the value of households net worth by percentile for the years 2013 to 2016. Data are in 2016 U.S. dollars and are from the U.S. Census Bureau. Survey of Income and Program Participation, 2014 Panel Waves 1 through 4.

Figure OA1 plots the evolution of the U.S. federal government expenditure as fraction of GDP. The blue line corresponds to the total expenditure, while the red line to the expenditure net of transfers payments (i.e., government social benefits to persons). The sample averages are, respectively, equal to 33% and 22%. In our baseline results, we set the value of $g = 0.22$, because the government expenditure net of transfer payments is the definition which is closer to that used in the model. If we, instead, set $g = 0.33$, to match the average expenditure including transfer payments, then we obtain larger values for the wage tax (around 80%), while all the remaining variables are qualitatively and quantitatively very similar.

OA3 Robustness

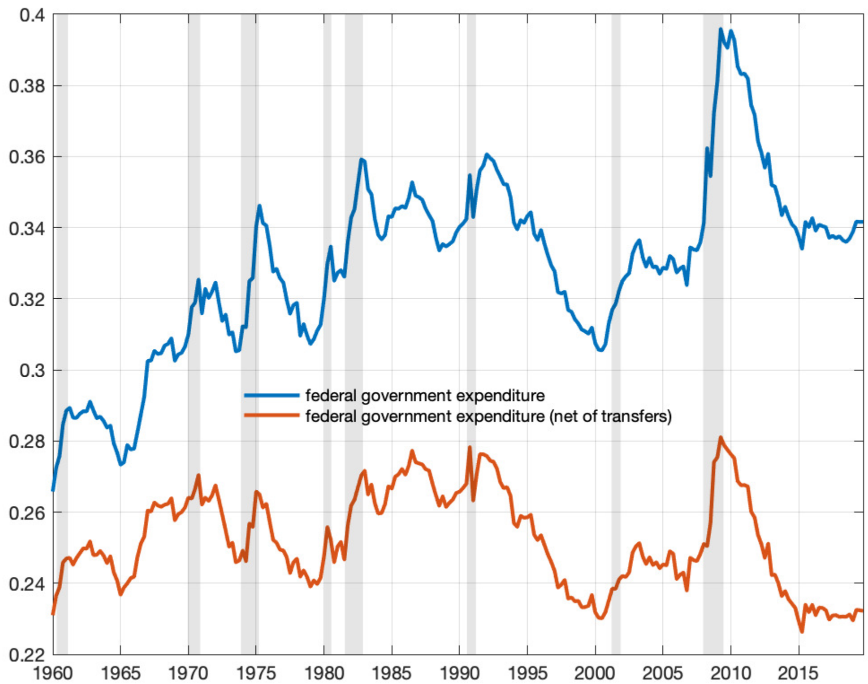
In this section we provide additional robustness results.

Steady States for Different Real Interest Rates

Figure OA2 plots the steady state values of the main variables for different levels of the real interest rate. Note that the wealth-to-income ratio is decreasing in the level of the real interest rate, we set equal to 1 to 5. We mark with a blue dashed-line the baseline interest rate used in the simulations (i.e., the value pinned down by β^H).

Figure OA3 and Figure OA4 plot the steady state values of the wage tax, the housing subsidy on poor households and housing tax on rich households, and the government revenues from the sale of land permits, for different values of the land share (α_x^h) and of the share of rich households (m_r), for a given level of aggregate wealth. Recall that in the model poor households have zero net wealth. Therefore, in the baseline calibration we set the fraction

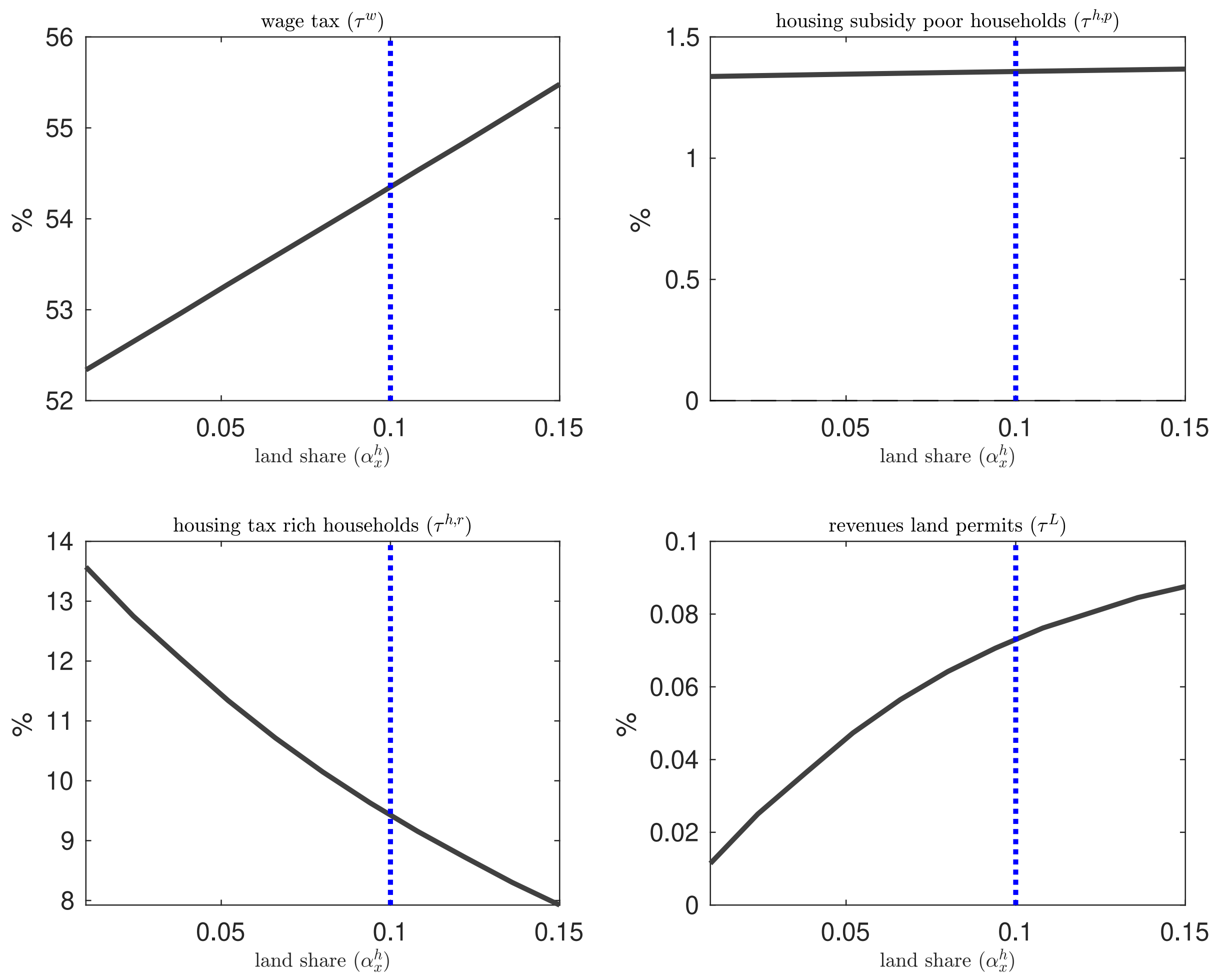
Figure OA1: U.S. Federal Government Expenditure



Notes: This figure plots the evolution of the U.S. federal government expenditure as a fraction of GDP. The blue line corresponds to the total, and the red line to the government expenditure net of transfers. Dark shaded areas denote NBER U.S. recessions. Data are quarterly from the FRED database for the period 1960:Q1-2019:Q4.

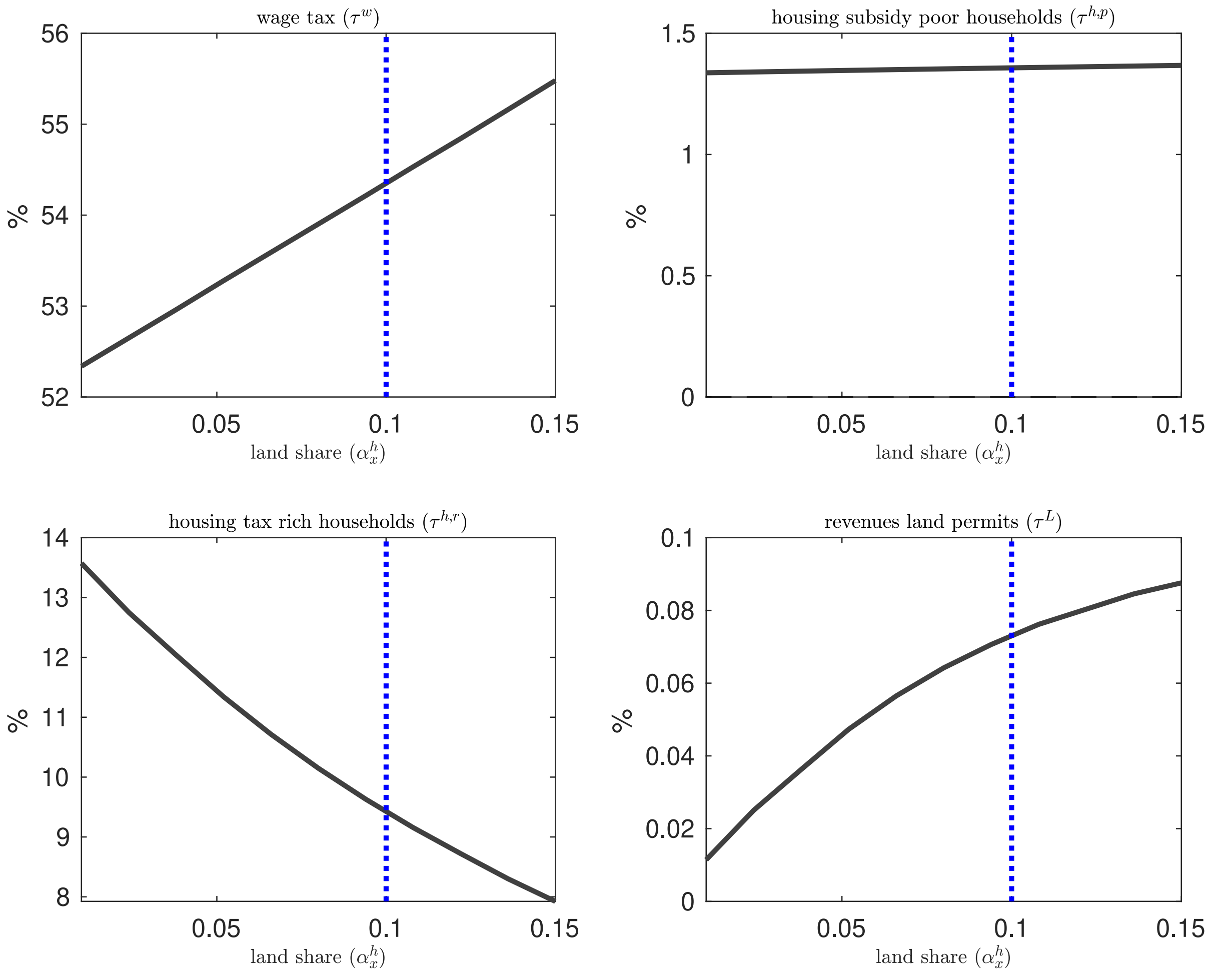
of rich households to 75%, which is approximately equal to the fraction of U.S. households around zero net wealth. In the figure we change this share from 50% to 90%.

Figure OA2: Steady States for Different Real Interest Rates



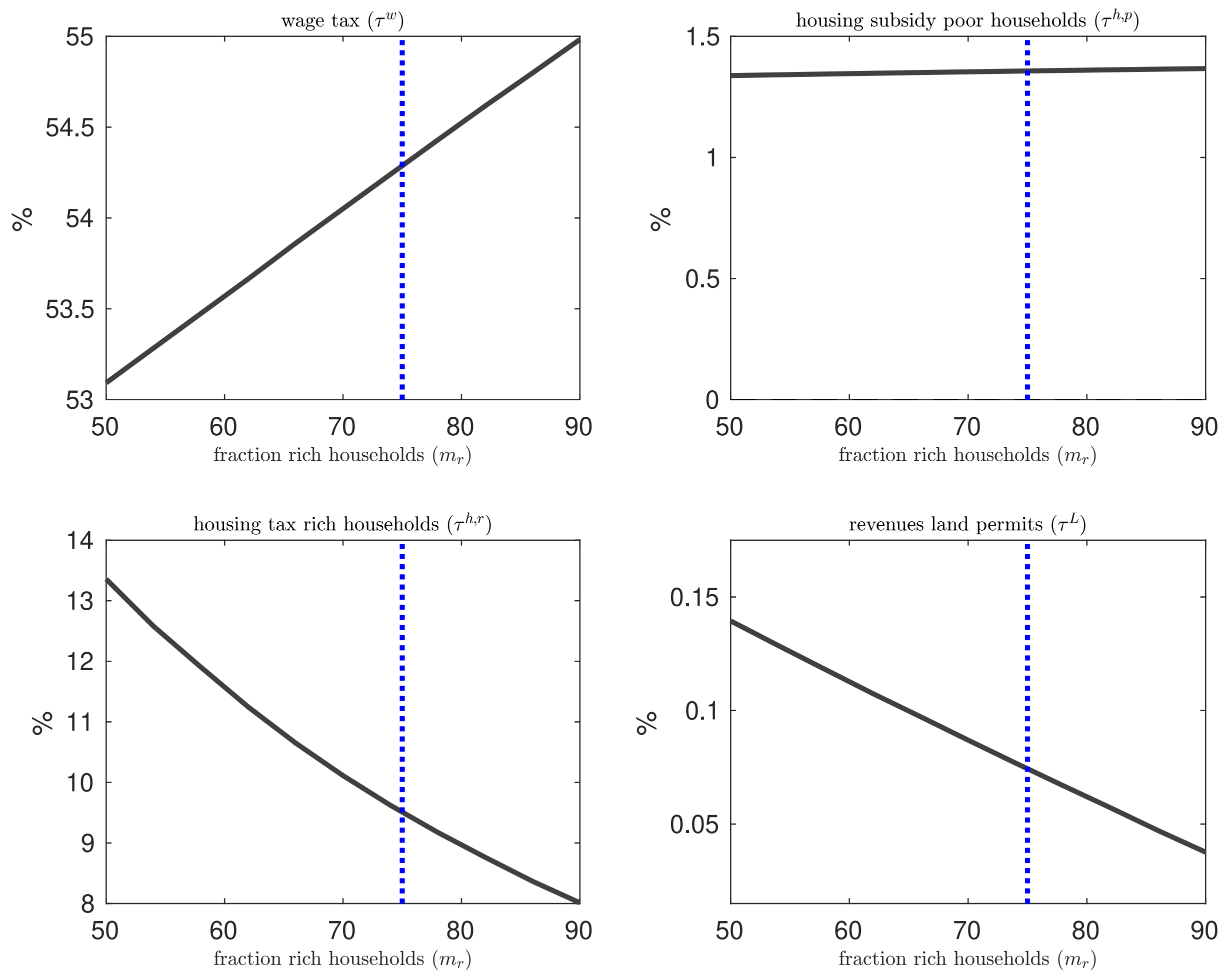
Notes: This figure plots the steady state values for the wealth-to-income ratio (v/y); net wage-to-wealth ($\hat{w}/v$); wage tax (τ^w); the housing subsidy on poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the government revenue from the sale of land permits as a fraction of government expenditure (τ^L); the housing price (q); and the housing stock (h); for different values of the real interest rate r. The dotted blue vertical line corresponds to the baseline value of r. Refer to Appendix A for details on the numerical solution of the model.

Figure OA3: Robustness: Different Land Shares



Notes: This figure plots the steady state values for the wage tax (τ^w); the housing subsidy on poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the government revenue from the sale of land permits as a fraction of government expenditure (τ^L) for different values of the parameter α_x^h, i.e., the land share in the housing sector, and for a fixed level of aggregate wealth. The dotted blue vertical line corresponds to the baseline value of α_x^h. Parameters are from Table A1 with the exception of α_x^h which is in the range $[0.01, 0.15]$. Note that we compute the weight on the labor input, in the production function of the construction sector, as the residual $1 - \alpha_k^h - \alpha_x^h$. Refer to Appendix A for details on the numerical solution of the model.

Figure OA4: Robustness: Different Rich Household Shares



Notes: This figure plots the steady state values for the wage tax (τ^w); the housing subsidy on poor households ($\tau^{h,p}$); the housing tax on rich households ($\tau^{h,r}$); the government revenue from the sale of land permits as a fraction of government expenditure (τ^L) for different values of the parameter m_r, i.e., the share of rich households. The dotted blue vertical line corresponds to the baseline value of $m_r = 1\%$. Parameters are from Table A1 with the exception of m_r which is in the range $[0.01, 0.15]$. Refer to Appendix A for details on the numerical solution of the model.