Money Creation: Tax or Public Liquidity?

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PIETRO REICHLIN

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I revisit the example of non-neutral anticipated monetary expansions used in Lucas (1995) Nobel Prize Lecture, within a broader definition of monetary policy tools, such as paying a nominal return on money or using open market operations, to show that money expansions increase output by reallocating consumption across heterogenous individuals and time periods. This result survives with noninterest-bearing cash when the latter does not generate relevant distortions.

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In his nobel prize lecture, Lucas (1995) presents a simple (full information) overlapping generations model with elastic labor supply (to be called, henceforth, the *Lucas Model*) to show that anticipated monetary expansions are, in general, non-neutral. In his model, money is a pure store of value and inflation has a negative effect on output by diluting the return from working. According to Lucas' interpretation, money creation generates a distortionary tax and, as such, it is unlikely to be expansionary. The example allows him to propose imperfect information as a more promising, alternative way to explain positive, albeit temporary, effects of monetary expansions on output. In this note, I want to offer a different interpretation of the effects of monetary expansions in the class of models used in his lecture that may be useful under a more general definition of central banks liabilities and instruments.

The negative effect of money creation in the Lucas Model arises from a restriction on the policies that central banks are allowed to use, such as paying a nominal interest rate on money or, equivalently, making unlimited open market operations on government securities (so as to get full control of net public liabilities). When we remove these restrictions, money expansions stimulate output within his model. In particular, a rise in money transfers would generate a reallocation of consumption to

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Pietro Reichlin *is at Department of Economics and Finance, LUISS G. Carli and CEPR* (*E-mail:* preichlin@luiss.it).

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the old individual away from the young and a rising labor supply. One way to see this is to note that the inability of the central bank to exploit all available instruments implies that the higher inflation caused by the money transfers has a negative effect on the real interest rate and, then, no expansion in the real money stock can effectively take place in equilibrium. As a consequence, money transfers turn out to reduce old individuals' consumption and decrease labor supply. If we remove these restrictions on monetary policy, the Lucas Model provides an example where outside assets, when traded at a discount, may increase output in an economy where the market allocation of life-time consumption does not provide enough incentives for work.

In this note, I provide a reexamination of these basic results in a model economy with debt limits similar to Bewley (1986), that I call *Bewley–Lucas Model* (BLM). The equations characterizing competitive equilibria are essentially equivalent to those one can derive from a slightly generalized version of the Lucas Model.[1] However, the BLM allows for a more practical interpretation of the insurance role of money in the short run. In particular, we can understand money expansions as a way to induce more consumption smoothing in the face of limited private borrowing. These objectives appear to be particularly relevant for policymakers after the big recession. I call this a *public liquidity effect*, a term that I borrow from Woodford (1990) and Holmstrom and Tirole (1998). In particular, a public liquidity effect occurs when the government, or the Central Bank, can improve upon market allocations by providing the private sector with highly liquid public claims in exchange for less liquid private claims.

In most of the paper, I assume that all public liabilities are perfect substitute and possibly earning a positive nominal rate. This situation corresponds to the case of a Central Bank conducting monetary policy by issuing interest-bearing reserves or through open market operations with the aim to manage the net total liabilities of the public sector. I show that the Central Bank can achieve any desirable level of output by targeting the level of *public liquidity* (public liabilities plus monetary transfers) and that this policy variable has a positive impact on both output and social welfare (measured as individuals' *ex ante* expected life-time utility). In a final section, I introduce a demand for liquidity through a cash-in-advance constraint, to check whether the results obtained in the cashless model are robust to the introduction of a demand for liquidity. In fact, since consumption is a cash good, the nominal rate generates a distortion in the individuals' leisure-consumption choice, and a money expansion may exacerbate this distortion by inducing variations in the nominal rate. I consider a slightly atypical specification of the cash-in-advance constraint, such that some fraction of individuals' income can be a substitute for cash for transaction purposes. If this fraction is zero, we are back to the standard formulation (such as Stokey and Lucas 1987), if it is one, the nominal rate generates no distortion in the leisure-consumption choice. Then, I show that the results obtained in the

1. In particular, this sort of equivalence arises by assuming that the two-period lived individuals considered in the Lucas Model have a time separable and discounted utility for young and old age consumption as well as from leisure.

first part of the paper are basically unaltered provided that the fraction of income useful for transactions is sufficiently large. In this case, at any equilibrium with a positive nominal rate, money increases welfare not only because it generates more consumption smoothing (through the public liquidity effect) but also because it reduces the distortion in the allocation of consumption and leisure due to a positive nominal rate.

A reason why a reinterpretation of the Lucas Model along the lines considered in this note may be relevant is that central banks have recently moved away from traditional policies (direct control of money supply and an exclusive focus on price stabilization), cash is being progressively replaced by electronic money and other forms of exchange[2] and asset trading and liquidity provision for the purpose of financial stabilization has been an important focus of central banks operations. In other words, well-developed financial systems are likely to be characterized by small liquidity frictions and, based on recent experience, relatively large financial frictions. Hence, understanding why money growth is non-neutral over and above the distortionary effects generated by the inflation tax is a key question.

One should take the present note as a comment on Lucas' lecture in light of well-established views about the role of money. In fact, my observations are certainly not new, and are largely settled in the existing literature on monetary theory. In particular, it is well known that, except for Arrow-Debreu economies, money may affect output and incentives to work because it creates an opportunity for individuals to reallocate consumption across individuals, states, or periods of time that the market is unable to offer, or because money is a substitute for (inoperative) private insurance. Examples where outside assets help to undue the negative effects of market imperfections have been provided in Bewley (1986), Levine (1988), and Woodford (1990). In particular, the paper by Woodford (1990) shows that an increasing government debt is a way to reallocate consumption to the constrained individual away from the unconstrained. In Woodford's own words, *"increased government borrowing can benefit [borrowers and lenders], insofar as they effectively receive a highly liquid asset, government debt, in exchange for giving the government an increased claim on their future income, their own claim to which represented a highly illiquid asset"* (Woodford, 1990, p. 382). More recently, Brunnermeier and Sannikov (2015) have introduced the idea (called the *I Theory of Money*) that monetary policy can be effective (and expansionary) because it redistributes wealth across agents and affects asset values in economies characterized by financial frictions. In their model, money creation mitigates overhang problems following excessive private debt accumulation and risk exposure. In some sense, the Lucas Model falls in this class of examples, but, differently from most of them, contains no market imperfection or financial friction, except for the inability to trade money at a discount or the inability of Central Banks to regulate the size of public liquidity. In any case, it is possible to argue that the overlapping generations

2. The possibility and relevance of a scenario in which an electronic payment system would replace currency, and eliminate the advantage of clearing payments through accounts at the central bank, are suggested by King (1999) and Woodford (2003). According to Cole and Ohanian (1997), the ratio of nominal GDP to M1 has risen by a factor of about three between 1950 and 1980 in the United States.

economies and the economies characterized by financial frictions share a common feature: changes in outside assets have a non-neutral effect on allocations through redistributions of wealth across heterogeneous individuals. The fact that money serves two concurrent roles in overlapping generations models, that is, *store of value* and *means of executing intergenerational transfers*, has been recognized in Bhattacharya, Haslag, and Russell (2002). As shown by these authors, disentangling these two roles may help in understanding if and when standard prescriptions for monetary policy that are valid in economies with a single representative individual may survive in overlapping generations models.

The welfare criteria to be used to evaluate the optimal policies in the overlapping generations interpretation of the model may differ from those considered in this paper. This is particularly evident with reference to the optimality of the Friedman Rule. The latter is desirable in the BLM, as it implements a first-best allocation characterized by perfect consumption smoothing and nonbinding debt limits. Under the overlapping generations interpretation of the model, instead, the Friedman Rule equalizes consumption across generations and, within the model used in this paper (time separable and discounted utility), this is optimal only if the welfare function is the sum of individuals' utilities discounted with the subjective rate of time preference. For example, the Friedman Rule is typically nonoptimal at the Golden Rule (i.e., the stationary allocation that maximizes the representative individual's utility at young age). A large literature has provided characterizations of optimal monetary policies in overlapping generations with or without a demand for cash. Most notable examples are Weiss (1980) and Abel (1987), where the optimal policy is identified with a constant money stock. However, Haslag and Martin (2007) show that the Friedman Rule can still be optimal in overlapping generations models when mutually beneficial arrangements are allowed or if the Central Bank can make loans.

This note is organized as follows. In Section 1, I set up a reinterpretation of the model as an economy with debt limits similar to Bewley (1986). In Section 2, I consider the effects of monetary policies at stationary equilibria with and without interest-bearing money. In Section 3, I will reconsider the model with liquidity services generated by a cash-in-advance constraint. In Section 4, I conclude.

1. A MODIFIED LUCAS MODEL

In this section, I set up a model of a Bewley-economy (see Bewley 1986), that is, an economy with two types of infinitely lived individuals whose endowments are perfectly negatively correlated and alternating between a high and low value. All individuals can transfer purchasing power between periods using a pure store of value, but they cannot borrow at any period. The model is formally equivalent to a slightly more general version of the overlapping generations economy considered in Lucas (1995) and the main insights from his model can be fully recovered in my framework when noninterest-bearing money is the only asset. However, differently

from the overlapping generations model, where a changing size of outside assets induce reallocations across generations, the Bewley-economy interpretation of the model implies that these reallocations occur across lenders and borrowers and they may improve the degree of consumption smoothing, as well as modify labor supply. This mechanism allows for a more interesting interpretation of the welfare effects of changing public liabilities and it seems to be more relevant in light of the recent debate about the possibility that monetary authorities may try to mitigate the effects of financial frictions besides pursuing price stability.

There are two types of infinitely lived individuals subject to a no-borrowing constraint. The two individuals, $i = e, o$ (even, odd), have identical life-time utility function

$$U = \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t^i\right) + v\left(1 - y_t^i\right)\right),$$

where $\beta \in (0, 1)$, c^i stands for consumption, $y^i \in [0, 1]$ is labor effort and u and v satisfy the following assumption. The utility functions $u(.)$ and $v(.)$ are strictly increasing, strictly concave, continuously differentiable functions, satisfying the Inada conditions:

$$\lim_{x\to 0} u'(x) = \lim_{y\to 1} v'(1-y) = \infty, \qquad \lim_{x\to\infty} u'(x) = \lim_{y\to 0} v'(1-y) = 0.$$

The single good is produced by identical competitive firms with a technology defined by a linear production function such that, at all time $t \geq 0$, y_t units of the individuals' labor effort generates y_t units of output. The two individuals have a nonconstant labor productivity. In particular, we consider the extreme case in which individual e (o) is able to produce a unit of output for each unit of her labor time at even (odd) periods and she is totally unproductive at odd (even) periods. Labor is the only source of income in this economy. Then, by the linearity of the production function, we derive that the real wage rate is $w_t = 1$ and output is

$$y_t = \begin{cases} y_t^e & \text{for } t \text{ even,} \\ y_t^o & \text{for } t \text{ odd.} \end{cases}$$

I assume that individuals, a *fiscal authority* and a *monetary authority* (or *Central Bank*) can transfer purchasing power across periods by exchanging bonds with one-period maturity and individuals are unable to borrow.[3] The fiscal authority, at all time $t \geq 0$, imposes a time-independent real lump-sum tax, τ_t, on the employed worker (high-productivity individual) and the Central Bank makes a money transfer, H_t, to be distributed equally across individuals, that is, both individuals receive $H_t/2$ at all periods. The latter assumption insures that monetary policy is "blind" with

3. This severe limitation can be relaxed somewhat, with no substantial consequences on the main results of this paper.

respect to individuals liquidity needs, reflecting the inability to discriminate due to informational or legal constraints. This economy will be called a BLM.

We seek equilibrium configurations such that, at each time period, the low-productivity individual hits the debt limit. In particular, let (x_t^h, A_{t+1}^h) $((x_t^l, A_{t+1}^l))$ denote the level of consumption and the net nominal claims acquired at time t by the employed high-productivity (unemployed low-productivity) individual at time t. Then, the high- and low-productivity individuals' budget constraints at time t can be written as

$$\frac{1}{1+i_{t+1}} A_{t+1}^h + p_t \left(x_t^h + \tau - y_t\right) - H_t/2 = 0, \tag{1}$$

$$p_t x_t^l - H_t/2 = A_t^h, \tag{2}$$

where p_t is the price level at t and i_{t+1} the nominal interest rate on the one-period bonds maturing at time $t+1$. Since the only individual who has a leisure-consumption choice is the high-endowment individual and the low-productivity individual has a (possibly) binding debt limit, the first-order conditions characterizing the optimal consumption-leisure plan of each individual are defined by

$$u'\left(x_t^h\right) = v'(1-y_t), \tag{3}$$

$$(1+i_{t+1})p_t/p_{t+1} = u'\left(x_t^h\right)/\beta u'\left(x_{t+1}^l\right), \tag{4}$$

$$(1+i_{t+1})p_t/p_{t+1} \leq u'\left(x_t^l\right)/\beta u'\left(x_{t+1}^h\right). \tag{5}$$

The above characterize a solution to the individuals' utility maximization subject to the budget constraints and the debt limits together with the transversality condition

$$\lim_{t\to\infty} \beta^t u'\left(x_t^l\right) A_t^h/p_t = 0. \tag{6}$$

The inequality in (5) guarantees that the low-productivity individual's debt limit may be binding.

Market clearing in the good and asset market and the government (period-by-period) budget constraint provide

$$x_t^h + x_t^l = y_t, \tag{7}$$

$$A_t^h = B_t, \tag{8}$$

$$B_{t+1}/(1+i_{t+1}) = B_t - p_t\tau_t + H_t, \tag{9}$$

where B_t represents the t-period stock of nominal public liabilities. This variable may represent (interest-bearing) circulating money (i.e., checking accounts and money certificates) or government bonds yielding a common nominal rate of return i_{t+1}. Since this is a cashless economy (i.e., assets play the role of pure stores of value and generate no specific transaction services) with no financial frictions beside the debt limits, there is no specific reason why we should think about B_t as money or bonds. However, this does not prevent me from discussing about monetary policy. For concreteness, let B_t^g denote the net liabilities of the government, B_t^m the net liabilities of the Central Bank, that is, the sum of the Central Bank reserves minus assets (including government debt) and $B_t = B_t^g + B_t^m$ the consolidated net liabilities of the public sector (government and Central Bank). Assuming that the Central Bank provides "liquidity" by injecting the monetary transfer, H_t, the budget constraints of the two institutions are, respectively,

$$B_{t+1}^g/(1 + i_{t+1}) = B_t^g - p_t\tau - S_t, \tag{10}$$

$$B_{t+1}^m/(1 + i_{t+1}) = B_t^m + H_t + S_t, \tag{11}$$

where S_t represents the Central Bank nominal seignorage transferred to the fiscal authority. By consolidating the above two budget constraints, we obtain the (consolidated) public budget constraint represented by (9). I say that the Central Bank engages in *unlimited open market operations* if it has full control over the sequence $\{B_t, H_t\}_{t=0}^{\infty}$. The sequence $\{\tau_t\}_{t=0}^{\infty}$ defines a *fiscal policy* and the sequence $\{i_{t+1}, H_t, B_{t+1}\}_{t=0}^{\infty}$ a *monetary policy*. In what follows I will assume that the fiscal authority sets the tax rates independently.[4]

Observe that using the asset market clearing condition, (8), and the consolidated public sector budget constraint (9) in (1), we derive

$$x_t^h = y_t - \lambda_t, \tag{12}$$

$$x_t^l = \lambda_t, \tag{13}$$

where

$$\lambda_t = (B_t + H_t/2)/p_t \tag{14}$$

define the real value of public debt plus transfer, which I call (real) *public liquidity*. A key observation is that λ_t plays the role of a sort of public insurance: if it goes up, individuals are able to benefit from more consumption smoothing across periods of time. By equation (4), the real interest rate rises, so that borrowing becomes more expensive and, then, excess demand for borrowing falls (reducing the wedge between

4. This corresponds to the definition of a *active fiscal policy* often used in the literature.

the market rate and the borrowers' willingness to pay). At the same time, by equation (3), the fall in x^h induced by the rising λ_t, increases the marginal rate of substitution between labor and consumption, that is, individuals increase labor supply.

I now characterize a stationary equilibrium, that is, an equilibrium allocation with constant values of output, consumption, and real public liquidity. This is accomplished by imposing a constant tax rate, τ, inflation rate, μ, public liquidity, λ, net public liabilities, $b = B_t/p_t$, and real money transfers, $h = H_t/p_t$.

For given $\tau \in [0, 1)$, a *stationary equilibrium of the BLM* is an array, (y, λ, μ, i), and a sequence of prices and public liabilities, $\{p_t, B_{t+1}\}_{t=0}^{\infty}$, such that

$$u'(y - \lambda) = v'(1 - y), \tag{15}$$

$$(1 + i)/(1 + \mu) = u'(y - \lambda)/\beta u'(\lambda), \tag{16}$$

$$b(i - \mu)/(1 + i) = \tau - h, \tag{17}$$

$$p_{t+1}/p_t = B_{t+1}/B_t = (1 + \mu), \tag{18}$$

with $\lambda = b + h/2 \leq y/2$.

Equation (15) defines the individually optimal consumption-leisure allocation, (16) equates the marginal rate of substitution between present and current consumption of the unconstrained individual to the real interest rate, $(1 + i)/(1 + \mu)$, (17) defines the stationary consolidated budget constrained of the fiscal and monetary authorities, (18) defines the stationary gross inflation rate and the restriction $\lambda \leq y/2$ derives from equation (5), that is, it insures that the low-productivity individual has less consumption than the high-productivity individual, so that the former may have a binding debt limit. Observe that the latter restriction implies

$$i \leq i^f(\mu) \equiv (1 + \mu)/\beta - 1, \tag{19}$$

where $i^f(\mu)$ defines the Friedman Rule for the nominal interest rate.

By the assumptions about the utility functions, u and v, it is immediate to verify that, for all $\lambda \in [0, 1)$, there exists a value $y = \phi(\lambda)$ solving equation (15) and such that $\phi(\lambda) > \lambda$, $\phi'(\lambda) \in (0, 1)$ for all $\lambda \in (0, 1)$ and

$$\lim_{\lambda \to 0} \phi(\lambda) > 0, \qquad \lim_{\lambda \to 1} \phi(\lambda) = 1.$$

The reader may easily verify that the set of equations (15)–(18) would represent the equilibrium restrictions at steady state of a slightly more general version of the overlapping generations model in Lucas (1995), where $x^1 = y - \lambda$ and $x^2 = \lambda$

represent young and old age consumption.[5] Under this interpretation of the model, a rise in λ implies a transfer of resources from young to old age (as in the case of social security) and, through the first-order condition (15), it induces the young to work more.

A *full insurance* (First Best) equilibrium allocation is an array, $(y^f, \lambda^f, \mu^f, i^f)$ and a sequence, $\{p_t^f, B_{t+1}^f\}_{t=0}^{\infty}$, characterized by equal consumption across individuals, that is,

$$\lambda^f = y^f/2 = \phi(\lambda^f)/2, \qquad i^f = i^f(\mu^f), \qquad B_t^f/p_t^f = (2\lambda^f - \tau)/(1+\beta).$$

By equation (15), this allocation is unique and implies a positive amount of public liquidity, $\lambda^f \in (0, 1/2)$. Furthermore, the full insurance equilibrium can be achieved with non-negative public liabilities, B^f/p^f, if and only if $\lambda^f \geq \tau/2$. From now on, the latter inequality is assumed to be verified. Observe that the First-Best allocation is optimal from an *ex ante* perspective, that is, when individuals' welfare is maximized before they know their labor productivity. As an illustration valid for stationary equilibrium allocations, observe that the welfare of the high- and low-productivity individuals can be measured by

$$J^h = [u(\phi(\lambda) - \lambda) + v(1 - \phi(\lambda)) + \beta(u(\lambda) + v(1))]/(1 - \beta^2),$$
$$J^l = [u(\lambda) + v(1) + \beta(u(\phi(\lambda) - \lambda) + v(1 - \phi(\lambda))]/(1 - \beta^2).$$

Then, taking derivatives with respect to λ and exploiting the first-order conditions (15) and (16), we derive

$$\frac{\partial J^h}{\partial \lambda} = \frac{\beta u'(\lambda)}{1-\beta^2}\left(\frac{\mu - i}{1+\mu}\right), \qquad \frac{\partial J^l}{\partial \lambda} = \frac{\beta u'(\lambda)}{1-\beta^2}(i^f(\mu) - i). \tag{20}$$

By (19), $\partial J^l/\partial y \geq 0$, whereas the sign of $\partial J^h/\partial \lambda$ can only be non-negative for $i \leq \mu$, in which case a rise in λ would generate a Pareto improvement as well as increasing output. A natural selection for a measure of *ex ante* social welfare is

$$\mathcal{W} = \sum_{j=h,l} J^j/2$$

based on the idea that individuals have an *ex ante* equal probability of being borrowers or lenders. Quite trivially, in this case social welfare is increasing in λ and y, and it is maximized by setting $\lambda = \lambda^f(\mu)$ for all feasible μ. From now on, the value of $\mathcal{W}$ will be called *social welfare*. Under the overlapping generations interpretation of the equilibrium conditions, the case for $\lambda = \lambda^f$ and $x_t^h = x_t^l$ means equal consumption across generations and this allocation criterion is optimal when the welfare function

5. More specifically, Lucas (1995) considers the case of young individuals deriving no utility form consumption, a linear utility from leisure, no fiscal policy, and zero nominal rate.

is a discounted sum of all generations' utility functions with the social discount rate equal to the subjective discount rate, β.

2. MONETARY POLICY

In this section, I consider the effects of specific monetary policies in a stationary equilibrium. A natural definition of a monetary policy is a selection of a non-negative vector, $\mathcal{P}^m = (\lambda, \mu, i, b)$. Non-negativity of the nominal rate may follow from the plausible assumption that some sort of cash is ready to be used for transactions. Under this definition, the Central Bank is assumed to have access to a full range of tools, that is, paying a nominal rate on reserves and/or conducting unlimited open market operations (full control of the nominal interest-bearing net public liabilities). I call this an *unrestricted monetary policy*. Evidently, in equilibrium, some of the variables in $\mathcal{P}^m$ cannot be set independently from one another, and they may be subject to additional constraints. In particular, by the consolidated public sector budget constraint (17) and the definition of λ, the real value of the money transfer is

$$h = 2\frac{(1+i)\tau - (i-\mu)\lambda}{(1+i)+(1+\mu)}. \tag{21}$$

Notice that if $i > \mu$, that is, the equilibrium real interest rate is positive, $h \geq 0$ if and only if $\lambda \leq (1+i)/(i-\mu)$, which is always verified under the restriction $\lambda \leq 1$ and $\mu \geq -1$. Furthermore, by the consolidated budget constraint of the public sector, (17), $b > 0$ requires $\tau > h$ if $i > \mu$ and $\tau < h$ if $i < \mu$. By (21), these inequalities imply

$$\lambda > \tau/2. \tag{22}$$

I start from the case in which nominal public liabilities earn a zero nominal rate of return, that is, $i = 0$. This is the one considered in Lucas (1995), which I call the *pure currency BLM*. One way to rationalize this is to assume that the fiscal and monetary authorities cannot issue interest-bearing liabilities, and currency is the only available store of value. Evidently, in this case the definition of a monetary policy reduces to the pair (λ, μ).

PROPOSITION 1. *Provided that the inflation rate fixed by the Central Bank is small enough, there exists a stationary equilibrium of the pure currency BLM associated to a monetary policy* (λ, μ). *In this equilibrium, the Central Bank cannot set public liquidity,* λ, *independently from inflation,* μ, *to affect output. In particular, equilibrium output, social welfare, and public liquidity are decreasing in* μ.

PROOF. A formal characterization of the pure currency model is the following. For all $\mu \geq 0$, let $\lambda = l(\mu)$ be the size of public liquidity consistent with a zero nominal

rate. This is the unique value of λ in (0,1) such that the first-order conditions (15) and (16) are verified for $i = 0$, that is,

$$\frac{u'(\phi(\lambda) - \lambda)}{\beta u'(\lambda)} = \frac{1}{1+\mu}.$$

Observe that, in the pure currency model, $1/(1+\mu)$ is the real gross interest rate.

By the properties of the function ϕ, we know that $l(\mu)$ exists and it is a continuous decreasing function of μ for all $\mu \geq 0$, such that

$$\lim_{\mu \to -1} l(\mu) = 1, \qquad \lim_{\mu \to \infty} l(\mu) = 0.$$

Since $\lambda = l(\mu)$ at equilibrium, the Central Bank cannot set the level of real public liquidity independently of the inflation rate. Evidently, if $\tau > 0$ (a case not considered in the Lucas Model), $b > 0$ requires that the inflation rate is not too large, that is, $\mu \leq \mu^m$, with μ^m defined by $l(\mu^m) = \tau/2$ and the equilibrium conditions (15)–(18) reduce to the following pair of restriction on y and b:

$$y = \phi(l(\mu)), \qquad b(2+\mu) = 2l(\mu) - \tau.$$

Noticing that

$$\frac{\partial y}{\partial \mu} = \phi'(\lambda) l'(\mu) < 0, \tag{23}$$

we conclude that output is a decreasing function of μ. □

In other words, by imposing $i_{t+1} = 0$ and $B_{t+1}/B_t = 1 + \mu$ at all $t \geq 0$, the monetary authority is effectively fixing the real interest rate at $1/(1+\mu)$. By the first-order conditions for individual optimality, the real interest rate and labor supply, y, are positively related. Then, a rise in inflation, by reducing the real rate, causes a fall in output and less consumption smoothing. It is important to notice that, in this case, real public liquidity, or individuals' real net financial wealth, λ, has to adjust to a change in target inflation. A final observation is that, in this case, the full insurance allocation (i.e., the Friedman Rule), can only be achieved for $\mu = \mu^f = \beta - 1$ (permanent deflation) if $\lambda^f = l(\beta - 1) \geq \tau/2$.

Now consider an unrestricted monetary policy. The next proposition states that, in this case, the Central Bank may be able to reach two objectives simultaneously, output and inflation, by targeting the size of public liability, λ, and the rate of inflation, μ.

PROPOSITION 2. *There exists a stationary equilibrium of the BLM with unrestricted monetary policy where the Central Bank selects public liquidity, λ, and the inflation rate, μ, independently, provided that these values are large enough relative to τ. In this equilibrium, output and social welfare are increasing in λ for given μ.*

PROOF. By equations (15), (16), and (22), $i \geq 0$ and $b \geq 0$ at equilibrium if and only if

$$\psi_\mu(\lambda) \equiv (1+\mu)\frac{u'(\phi(\lambda)-\lambda)}{\beta u'(\lambda)} \geq 1, \qquad \lambda \geq \tau/2.$$

Observe that $\psi'_\mu(\lambda) > 0$, $\psi_\mu(0) = 0$, $\psi_\mu(\lambda(\mu)) = 1$, and $\lim_{\lambda\to 1}\psi_\mu(\lambda) = +\infty$. Then, $\psi_\mu(\lambda) \geq 1$ for all $\lambda \geq l(\mu)$. Then we can fix two variables, for example λ and μ, and find the remaining variables, (y, i, b), that solve (15), (16), (17) and belong to the appropriate range. It follows that the monetary authority can generate a target value of output, say $y^* \leq y^f$, by fixing inflation and public liquidity appropriately and letting i and b be set by market forces. In particular, by the arguments developed above, if the Central Bank wants to achieve y^*, the pair (λ^*, μ^*) must be such that

$$\lambda^* = \phi^{-1}(y^*) \geq \tau/2, \qquad \mu^* \geq l^{-1}(\lambda^*),$$

where ϕ^{-1} and l^{-1} are the inverse of ϕ and l, respectively. This choice generates an equilibrium pair (i^*, b^*) such that

$$i^* = (1+\mu^*)\frac{u'(y^*-\lambda^*)}{\beta u'(\lambda^*)} - 1, \qquad b^* = \frac{(1+i^*)}{(1+\mu^*)+(1+i^*)}(2\lambda^*-\tau).$$

Clearly, both output and social welfare (defined in Section 1 as the *ex ante* life-time utility of the two individuals at a stationary allocation) are increasing in λ. □

A natural way for the Central Bank to set a target output, y^*, and inflation, μ, at a nonstationary equilibrium is by following the rules:

$$B_{t+1} = (1+\mu)B_t, \tag{24}$$

$$H_t = 2(\lambda^* p_t - B_t). \tag{25}$$

In fact, by (25), we get $\lambda_t = \lambda^*$, $y_t = \phi(\lambda^*) = y^*$, and a real interest rate

$$(1+i_{t+1})p_t/p_{t+1} = r^* \equiv u'(\phi(\lambda^*)-\lambda^*)/\beta u'(\lambda^*) - 1.$$

By the public-sector budget constraint, (9), and (24), this policy provides

$$p_{t+1}/p_t = \frac{(1+\mu)b_t}{(1+r^*)(2\lambda^*-\tau-b_t)}, \tag{26}$$

$$b_{t+1} = (1+\mu)(p_t/p_{t+1})b_t, \tag{27}$$

where $b_t = B_t/p_t$. By plugging (26) into (27) and assuming $r^* > 0$, the above system of difference equations have a unique solution compatible with the transversality condition and such that:

$$p_{t+1}/p_t = 1 + \mu, \qquad b_t = \frac{(1 + r^*)}{1 + (1 + r^*)}(2\lambda^* - \tau).$$

Evidently, (24), (25) is not the only policy that may successfully implement a given output. An alternative policy could be based on a *nominal interest rate rule*. However, this policy may not be as effective and simple as the one just described. Let, for instance, the interest rate rule be

$$(1 + i_{t+1}) = \rho p_{t+1}/p_t, \tag{28}$$

with $\rho > 0$ predetermined. Then, the monetary authority is effectively targeting the real rate to ρ. By the first-order conditions, this policy generates a sequence of output levels, $\{y_t\}_{t=0}^{\infty}$, such that

$$\rho\beta u'\left(\phi^{-1}(y_{t+1})\right) = u'\left(y_t - \phi^{-1}(y_t)\right) \tag{29}$$

and a sequence of prices and money stocks such that

$$B_{t+1}/p_{t+1} = \rho\left(2\phi^{-1}(y_t) - \tau\right) - \rho B_t/p_t.$$

Whether the simple rule (28) is able to generate a desired output is open to question, since equation (29) implicitly determines a sequence of output with possibly nonmonotonic and complicated dynamics.

3. LIQUIDITY SERVICES

In this section, I show that the basic results obtained in the previous section are not fundamentally altered if we allow for a monetary asset providing specific liquidity services.

Assume that some cash must be used for transactions, with cash being represented by a noninterest-bearing public liability, to be denoted by M. In particular, I impose a slightly modified version of the standard cash-in-advance constraint such that, for all (productive and nonproductive) individuals $j = h, l$, the following inequality must be verified:

$$M_t^j \geq \max\left\{0,\, p_t\left(x_t^j - \gamma y_t^j\right) - H_t^j\right\}, \tag{30}$$

where M_t^j is the stock of cash carried over to period t and H_t^j the cash transfer from the monetary authority. Constraint (30) states that the cash carried over at time t from

period $t-1$ by individual j must be sufficient to cover the value of the consumption good bought at t less the cash transfer received from the monetary authority and some fraction, $\gamma \in [0, 1]$, of the total labor income earned in the course of the same period. In other words, labor income is, to some extent, a substitute of cash when making transactions, because some part of it serves as collateral to be pledged to the seller, or because some fraction of the wage is paid in advance. The model follows the idea that consumption is a *cash good* and leisure a *credit good* (see Stokey and Lucas 1987), so that the nominal rate of interest generates distortions in the allocation of the two goods. The case $\gamma = 0$ corresponds to the standard cash-in-advance constraint considered in Stokey and Lucas (1987), where none of the individual's current income can be used to enhance purchasing power and $\gamma = 1$ implies that the nominal rate generates no "direct" distortions in the allocation of goods. Observe that, by market clearing for money and the consumption good, this specific form of the cash-in-advance constraint implies that money velocity is $(1-\gamma)^{-1}$. Empirically, this variable hovers between 1.5 and 2 between 1960 and 2015 in the United States (according to Fred data for M2 money stock), and, then, plausible values of γ would be between 0.5 and 0.33.

As in the previous section, we assume that high-productivity individuals are subject to a real lump-sum tax $\tau_t \geq 0$, and that $H_t^j = H_t/2$ for $j = h, l$. The assumptions about individuals' labor productivity made in the previous section are maintained, so that $y_t^h = y_t$, $y_t^l = 0$ and no individual is allowed to borrow at any time period.

In what follows I am restricting attention to equilibria such that the cash-in-advance constraint (30) is binding at all $t \geq 0$ (a property always verified for $i_{t+1} > 0$). Similarly to the procedure followed in Section 1, we seek equilibrium configurations such that, at each time period, the low-productivity individual hits the debt limit. Denoting with A_{t+1}^j the end-of-period net interest-bearing asset position of the individual whose current labor income is y^j (where $y^h > 0 = y^l$), we have $A_{t+1}^h \geq 0 = A_{t+1}^l$ at all $t \geq 0$ and the cash-in-advance constraints

$$H_t/2 + M_t^l = p_t \left(x_t^h - \gamma y_t\right), \tag{31}$$

$$H_t/2 + M_t^h = p_t x_t^l. \tag{32}$$

The reader can find more details about the procedure to obtain the equilibrium characterization of the BLM with cash in Appendix A. Here I just lay out the full set of equations and unknowns defining a stationary equilibrium. In particular, the stationary equilibrium variables are those already introduced in the cashless model with the addition of real money balances, $m = M_t/p_t$, and the set of equilibrium restrictions now includes a *quantity equation* stating that the value of liquid assets must be proportional to nominal output. More formally, given a tax rate, τ, a *stationary equilibrium of the BLM with cash* is a non-negative array, $(y, \theta, h, b, m, \mu, i)$, and a

non-negative sequence, $\{p_t, B_{t+1}, M_{t+1}\}_{t=0}^{\infty}$, such that

$$u'(y-\theta) = \left(\frac{1+i}{1+\gamma i}\right) v'(1-y), \tag{33}$$

$$(1+i)/(1+\mu) = u'(y-\theta)/\beta u'(\theta), \tag{34}$$

$$b(i-\mu)/(1+i) = \tau - h + \mu m, \tag{35}$$

$$p_{t+1}/p_t = B_{t+1}/B_t = M_{t+1}/M_t = 1+\mu, \tag{36}$$

$$m + h = (1-\gamma)y, \tag{37}$$

where

$$\theta = h/2 + m - b/(1+\mu) \leq y/2 \tag{38}$$

represents the size of public liquidity for the BLM with cash and (38) insures that $x^h \geq x^l$, and it is verified for any $i \leq i^f(\mu)$, with i^f representing the nominal rate prescribed by the Friedman Rule.

Observe that public liquidity in the BLM with cash, θ, is increasing in the most liquid assets and decreasing in the previous period public debt. In fact, differently from the cashless model, consumption of the low-productivity individual falls short of her initial real claims by the amount of cash that she needs to purchase goods in the next period and she uses her interest-bearing asset holdings to buy cash.

By comparing the equilibrium restrictions for the cashless economy, (15)–(18), with those stated above, one can immediately see that there are three main novelties. First of all, the nominal rate, i, generates a distortion in the allocation of labor and consumption (equation (33)), then, the primary surplus of the public sector includes the inflation tax (equation (35)), and, finally, liquid assets verify the quantity equation (37).

Notice, also, that the First-Best allocation can only be attained for $i = 0$, $\mu = \beta - 1$, that is, the Friedman Rule with zero nominal rate. In fact, recall that efficiency requires both perfect consumption smoothing, that is, $i = i^f(\mu)$, $\theta = y/2$, and an efficient allocation of labor time $y = \phi(\theta)$, that is, $u'(y-\theta) = v'(1-y)$. However, the latter violates the first-order condition (33) for all $\gamma > 0$ and $i > 0$.

In the previous section, I have shown that, in the cashless BLM, the monetary authority is able to target a given level of output by setting two policy variables,

public liquidity, and the rate of inflation, and we have seen that equilibrium output is increasing in public liquidity. Here I show that these features of the model may still be verified in the BLM with transaction services, for a different definition of public liquidity, provided that γ is large enough (i.e., labor income is a good enough substitute for cash in transactions).

PROPOSITION 3. *For all* (θ, μ) *such that* $\theta \geq l(\mu)$, $\mu > -1$, *there exists two differentiable functions,* $y = y(\theta, \mu) \in (0, 1)$, $i = i(\theta, \mu) \geq 0$ *solving (33), (34) with partial derivatives* $(y_\theta, y_\mu, i_\theta, i_\mu)$ *such that* $y_\mu < 0$, $i_\mu > 0$, $i_\theta > 0$ *and*

$$y_\theta \geq 0 \qquad \Leftrightarrow \qquad \gamma \geq \frac{1}{1 + (1 + i)\chi(\theta, \mu)},$$

where

$$\chi(\theta, \mu) \equiv \frac{u''(y(\theta, \mu) - \theta)/u'(y(\theta, \mu) - \theta)}{u''(\theta)/u'(\theta)}.$$

PROOF. See Appendix B. □

Notice that $y(\theta, \mu)$ and $i(\theta, \mu)$ are equilibrium values provided that the equilibrium restrictions (35), (37), and (38) generate positive real stocks of money and public debt, m and b. For ease of exposition, I skip a formal proof of existence of an equilibrium.[6]

The reason why the effect of a rising public liquidity on labor effort is ambiguous in the BLM with cash is that this variable has two opposing effects on allocations. On the one hand, there is a direct positive effect on y due to an improvement in the degree of consumption smoothing. On the other, a higher θ generates a rise in the nominal rate and this, in turn, discourages labor supply due to a larger distortion in the allocation of consumption and cash goods. Observe that, if $\gamma = 0$, as in the standard cash-in-advance model, we have $y_\theta < 0$. More generally, the direct positive effect of enhancing public liquidity overcomes the indirect effect though the increasing distortions if and only if γ and/or the nominal rate are large enough.

To derive the welfare effect of increasing public liquidity, θ, I compute the partial derivatives of the stationary life-time utilities of the two types of individuals,

$$J^h = [u(y(\theta, \mu) - \theta) + v(1 - y(\theta, \mu)) + \beta(u(\theta) + v(1))]/(1 - \beta^2),$$
$$J^l = [u(\theta) + v(1) + \beta(u(y(\theta, \mu) - \theta) + v(1 - y(\theta, \mu)))]/(1 - \beta^2),$$

at equilibrium. This provides the following results:

$$\frac{\partial J^h}{\partial \theta} = \frac{\beta u'(\theta)}{1 - \beta^2}\left(\frac{(\mu - i) + (1 - \gamma) i y_\theta}{1 + \mu}\right), \tag{39}$$

6. A characterization of the restrictions required for the existence of an equilibrium is provided in Appendix B.

$$\frac{\partial J^l}{\partial \theta} = \frac{\beta u'(\theta)}{1-\beta^2}\left(\frac{(i^f(\mu)-i)+\beta(1-\gamma)iy_\theta}{1+\mu}\right). \tag{40}$$

The term $(1-\gamma)iy_\theta$ is explained by the observation that a rise (fall) in labor supply reduces (increases) the distortion to the labor-consumption allocation due to a positive nominal rate (i.e., the Central Bank monopoly rent in producing liquidity). A comparison with (20) shows that, if $y_\theta > 0$ and $i > 0$, the BLM with cash provides more ammunitions to the idea that a rise in public liquidity is welfare enhancing, relative to the cashless version of the model. In fact, since $i \leq i^f$, $\partial J^j/\partial\theta \geq 0$ for $j = h, l$ whenever $y_\theta \geq 0$ and

$$y_\theta \geq \frac{i-\mu}{(1-\gamma)i}.$$

PROPOSITION 4. *If $y_\theta \geq 0$, a rise in public liquidity at a stationary equilibrium of the BLM with cash can be Pareto improving even if the real interest rate is positive, that is, $i > \mu$ (which is excluded in the cashless model).*

Following the discussion in the previous section, the monetary authority may be able to generate some arbitrary level of output (as well as state contingent consumption) by choosing a suitable pattern for the monetary transfers. In particular, a policy achieving a target level of output is

$$H_t = 2\theta p_t + 2B_{t-1} - 2M_t, \tag{41}$$

and $p_{t+1}/p_t = M_{t+1}/M_t = B_{t+1}/B_t = 1+\mu$.

4. CONCLUSIONS

In his Nobel Prize lecture, Lucas emphasizes a "tension" between two incompatible ideas: *"that changes in money are neutral units changes, and that they induce movements in employment and production in the same direction"* (Lucas 1995, p. 248). Although these two ideas may be incompatible in economies with "perfect markets" (such as the Arrow–Debreu setup), they are not incompatible in economies with limited participation (overlapping generations economy) or tight debt limits, provided that the monetary authority is allowed to use a sufficient number of instruments, such as paying a nominal rate on money or using open market purchases of government bonds. In particular, money expansions have a positive effect on output and welfare due to a "public liquidity effect," that is, the provision of highly liquid claims in exchange for illiquid claims on private assets. When noninterest-bearing cash coexists with other type of public liabilities, the consequences of money growth on output and welfare are ambiguous, because money expansions increase the infla-

tion tax. If, however, cash is not "essential," in the sense that labor income enhances private liquidity, money expansions have a positive effect on income and welfare both because of the "public liquidity effect" considered in the cashless economy, and because the latter reduces the distortions in consumption-leisure choices due to a positive nominal rate.

APPENDIX A: EQUILIBRIUM IN THE BEWLEY–LUCAS MODEL (BLM) WITH CASH

The budget constraints and cash-in-advance constraints of the two individuals can be written as follows:

$$\frac{A_{t+1}^h}{1+i_{t+1}} + M_{t+1}^h + p_t\left(x_t^h + \tau - y_t\right) = M_t^l + H_t/2, \tag{A1}$$

$$M_{t+1}^l + p_t x_t^l = A_t^h + M_t^h + H_t/2, \tag{A2}$$

$$H_t/2 + M_t^l = p_t\left(x_t^h - \gamma y_t\right), \tag{A3}$$

$$H_t/2 + M_t^h = p_t x_t^l. \tag{A4}$$

Since I am assuming that the low-productivity individual's debt limit is binding, the first-order conditions guaranteeing that (x^j, A^j, M^j) is individually optimal for $j = 1, 2$ are defined by

$$u'\left(x_t^h\right) = \left(\frac{1+i_t}{1+\gamma i_t}\right) v'(1-y_t), \tag{A5}$$

$$\frac{p_t}{p_{t+1}}(1+i_t) = \frac{u'\left(x_t^h\right)}{\beta u'\left(x_{t+1}^l\right)} \leq \frac{u'\left(x_t^l\right)}{\beta u'\left(x_{t+1}^h\right)}. \tag{A6}$$

The above replace the analogous conditions (3), (4), and (5) for the cashless economy establishing the optimal trade-off between labor and consumption with cash-in-advance. Contrary to the case in which money does not provide transaction services, a rising nominal interest rate (i.e., the opportunity cost of holding cash) induces individuals to substitute labor for consumption since leisure is not a *cash good*.

Asset market clearing implies

$$\sum_{j=h,l} A_t^j = B_t, \tag{A7}$$

$$\sum_{j=h,l} M_t^j = M_t, \tag{A8}$$

where B_t represents the net interest-bearing liabilities of the consolidated public sector. In turn, the consolidated public budget constraint is

$$B_{t+1}/(1+i_{t+1}) + M_{t+1} = B_t + M_t + H_t - p_t\tau. \tag{A9}$$

The above may result from consolidation of the budget constraint of the fiscal authority, (10), and the following budget constraint of the monetary authority:

$$M_{t+1} - B_{t+1}^b/(1+i_{t+1}) = M_t - B_t^b + H_t + S_t, \tag{A10}$$

where B^b denotes the government securities held by the Central Bank.

Observe that equations (A2) and (A4) imply $M_{t+1}^l = A_t^h$, that is, the low-productivity individual uses noncash net assets to acquire the amount of cash that she will need to buy goods next period. Then, by asset market clearing, we derive

$$A_t^h = M_{t+1}^l = B_t, \qquad M_t^h = M_t - M_t^l = M_t - B_{t-1}$$

at all $t \geq 0$. Using the above with equations (A1) and (A3), together with and resource feasibility, we derive

$$M_t + H_t = (1-\gamma)p_t y_t, \tag{A11}$$

$$x_t^h = y_t - \theta_t/p_t, \tag{A12}$$

$$x_t^l = \theta_t/p_t, \tag{A13}$$

where

$$\theta_t \equiv (M_t + H_t/2 - B_{t-1})/p_t$$

plays the role of real public liquidity. I define an *equilibrium of the BLM with cash* as a non-negative sequence,

$$\{y_t, \theta_t, p_t, i_{t+1}, B_{t+1}, M_{t+1}\}_{t=0}^{\infty},$$

verifying equations (A5), (A6), (A9), (A11), and (A12) with x_t^h and x_t^l non-negative for all $t \geq 0$, for a given fiscal policy, $\{\tau_t\}_{t=0}^{\infty}$, and initial public liabilities, $(B_0, M_0) > 0$ and $B_{-1} > 0$.

APPENDIX B: PROOF OF PROPOSITION 3

In the previous section, I have shown that the pair of equations (33) and (34) with $i = 0$ is solved for a unique value, $(y, \theta) = (\phi(l(\mu)), l(\mu))$, with $l(\mu)$ a decreasing function of μ. Now notice that, for all $i \geq 0$, there exists a function $y = \tilde{\phi}(\theta, i)$ solving (33), with $\tilde{\phi}_\theta \in (0, 1)$, $\tilde{\phi}_i < 0$, $\tilde{\phi}(\theta, 0) = \phi(\theta)$ and $\lim_{i\to\infty} \tilde{\phi}(\theta, i) \in (0, 1)$. Now let

$$H(\theta, i, \mu) = (1 + \mu)u'(\tilde{\phi}(\theta, i) - \theta) - \beta(1 + i)u'(\theta).$$

Observe that $H(\theta, i, \mu) = 0$ implies that the vector (θ, i, μ) verifies equation (34) and $H(l(\mu), 0, \mu) = 0$. Furthermore, for all $\theta \in (0, 1)$, $H(.)$ is increasing in θ and such that

$$\lim_{i\to\infty} H(\theta, i, \mu) = -\infty.$$

Then, for i large enough and $\theta > l(\mu)$, we have

$$H(\theta, i, \mu) < 0 = H(l(\mu), 0, \mu) < H(\theta, 0, \mu).$$

By continuity, for all $\theta \geq l(\mu)$, there exists $i(\theta, \mu) \geq 0$ and $y(\theta, \mu) \equiv \tilde{\phi}(\theta, i(\theta, \mu))$ such that equations (33) and (34) are verified.

To show that $y(\theta, \mu)$ and $i(\theta, \mu)$ are part of an equilibrium, we have to show that (θ, μ) imply $y(\theta, \mu) \geq 2\theta$ (binding debt limit for the low-productivity individual) and non-negative aggregate money and public debt, that is, $m(\theta, \mu) \geq 0$, $b(\theta, \mu) \geq 0$, the latter being the values of m and b derived by plugging $y(\theta, \mu)$ and $i(\theta, \mu)$ in equations (35), (37), and (38). By solving h for the remaining variables, these three equations reduce to the following:

$$m - a_1 b = 2\theta - (1 - \gamma)y,$$
$$m - a_2 b = (2\theta - \tau)/(2 + \mu),$$

where

$$a_1 = \frac{2}{1 + \mu} > a_2 = \frac{1}{2 + \mu}\left(\frac{i - \mu}{1 + i} + \frac{2}{1 + \mu}\right).$$

Solving for m and b, we get

$$m = \frac{a_1((2\theta - \tau)/(2+\mu)) - a_2(2\theta - (1-\gamma)y)}{a_1 - a_2},$$
$$b = \frac{((2\theta - \tau)/(2+\mu)) - (2\theta - (1-\gamma)y)}{a_1 - a_2}.$$

Assuming $\tau \leq 2\theta$ and recalling that $a_1 > a_2$, we derive that $m \geq 0$, $b \geq 0$ if and only if (θ, μ) are such that

$$\frac{2\theta - \tau}{2+\mu} \geq 2\theta - (1-\gamma)y(\theta, \mu).$$

Then, (θ, μ, τ) define an equilibrium of the BLM with cash if

$$\theta \geq l(\mu), \qquad y(\theta, \mu) \geq \max\left\{2\theta, \frac{2\theta(1+\mu) + \tau}{(1-\gamma)(2+\mu)}\right\}. \tag{B1}$$

Now observe that (33) and (34) define an equation $F(y, i, \theta, \mu) = 0$ where F is a continuous function from $\mathbb{R}^4_+$ into $\mathbb{R}^2$ such that the matrix of the partial derivatives with respect to y and i is invertible. Then, by the above findings and the implicit function theorem, there exist functions $y = y(\theta, \mu)$ and $i = i(\theta, \mu)$ such that $F(y(\theta, \mu), i(\theta, \mu), \theta, \mu) = 0$. Letting

$$\eta(i) = (1+i)/(1+\gamma i), \quad \sigma_u(x) = -u''(x)/u'(x), \quad \sigma_v(y) = -v''(1-y)/v'(1-y)$$

and

$$D = \gamma\eta(i)\sigma_u(y-\theta) + \sigma_v(y),$$

we derive

$$y_\theta(\theta, \mu) = [\gamma\eta\sigma_u(y-\theta) - (1-\gamma\eta)\sigma_u(\theta)]/D,$$
$$y_\mu(\theta, \mu) = -(1-\gamma\eta)/(1+\mu)D,$$
$$i_\theta(\theta, \mu) = (1+i)[\sigma_u(\theta)(\sigma_u(y-\theta) + \sigma_v(y)) + \sigma_v(y)\sigma_u(y-\theta)]/D,$$
$$i_\mu(\theta, \mu) = (1+i)[\sigma_u(y-\theta) + \sigma_v(y)]/(1+\mu)D.$$

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