UNIVERSITÀ DEGLI STUDI DI MACERATA

CONVEGNO A.M.A.S.E.S. 2010

MACERATA XXXIV Convegno Annuale dell'Associazione per la Matematica Applicata alle Scienze Economiche e Sociali

Macerata, 1- 4 settembre 2010

Abstract relazioni presentate

nelle sessioni di

Matematica Attuariale

Logical indicators for the pension system sustainability

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15 Maggio 2010

Abstract

In the work the issue of the pension system sustainability is analyzed in a logical and mathematical framework. A new type of financial management is proposed for Defined Contribution pension scheme (see Angrisani, 2006 and Angrisani, 2008). The novelty is in the introduction of a strucutural funded component in a financial management of PAY-AS-YOU-GO type. The funded component is structurally managed both for the returns on contributions and for the evaluation of the financial sustainability. Therefore it has also the role of reducing the uncovered pension liability at sustainable levels so that the eventual negative demographic effects can be faced. We refer to this type of pension system as Partially Funded Defined Contribution (PFDC) pension system. We show that the two financial management modes, the PAYG part and the funded one, have to interact between them over time so that the sustainability can be ensured. In particular, the intergenerational equity principle will be taken into account.

The demographic structure of population underlying the pension system is not represented under specific and fictitious assumptions. Differently from many other works existing in literature, no Steady State hypothesys is used in the proposed model. So taking into account the real demographic dynamics, which can present extremely different patterns over time, the necessity to assure the certainty of pension benefits to the pension system participants is the central key point. This warranty has to be determined by means of specific and fixed rules which principally work on correlation mechanisms between the contribution rate and the demographic and economic dynamics of the pension system.

In this framework the proposed model is able to ensure the certainty of the pension system sustainability on the basis of a necessary and sufficient condition. Sustainability indicators of logical-mathematical type are derived from the solution of a differential equation which ensures the no negativity of the pension system assets over time.

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Key items: Pension system sustainability, demographic dynamics, control indicators for retirement plans.

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VARIABLE ANNUITIES:
RISK IDENTIFICATION AND RISK ASSESSMENT

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Abstract. Life annuities and pension products usually involve a number of 'guarantees', such as, e.g., minimum accumulation rates, minimum annual payments and minimum total payout. Packaging different types of guarantees is the feature of the so-called Variable Annuities. Basically, these products are unit-linked investment policies providing deferred annuity benefits. The guarantees, commonly referred to as GMxBs (namely, Guaranteed Minimum Benefits of type 'x'), include minimum benefits both in case of death and in case of survival. Following a Risk Management-oriented approach, this paper first aims at singling out all sources of risk affecting Variable Annuities ('risk identification phase'). Critical aspects arise from the interaction between financial and demographic issues. In particular, the longevity risk may have a dramatic impact on the technical equilibrium of a portfolio. Then, we deal with risk quantification ('risk assessment phase'), mostly via stochastic simulation of financial and demographic scenarios. Our main contribution is to present an integrated approach to risks in Variable Annuity products, so providing a unifying and innovative point of view.

We acknowledge financial support from CAREFIN - Bocconi Centre for Applied Research in Finance and from PRIN 2008 research project 'Retirement saving and private pension benefits: individual choices, risks borne by the providers'.
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Pricing life insurance contracts as financial options: the endowment policy as a combination of a European and a path dependent option [1]

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The presence in the life insurance market of products whose returns depend on several variables including the returns of risky assets traded in the financial markets such as stocks, market indices, or interest rates suggests the idea of formulating mathematical models to price life insurance contracts inspired by the models used in mathematical finance. For this purpose models that describe simultaneously the behaviour of the risky assets specified in the contracts and of the relevant demographic variables (i.e.: the mortality rate) are needed. Using these models the problem of pricing life insurance contracts becomes analogous to the problem of pricing derivatives in mathematical finance. This rather general idea is illustrated on a simple example: the pricing problem of a pure endowment contract.

We focus on the problem of pricing at time $t = 0$ a pure endowment contract that pays a benefit at time $t = T > 0$ if the policy holder is alive at time $t = T$. In particular we consider a contract that takes today, at time $t = 0$, a premium from the policy holder and that, if the policy holder is alive at time $t = T > 0$, gives back to the policy holder at time $t = T$ the maximum between a minimum guaranteed provision given by Ke^{rT} and the quantity $K\delta\, S/\widehat{S}_0$, where $K > 0$, $r > 0$, $\delta > 0$ are constants, and S, $\widehat{S}_0$ are respectively the prices of a risky asset specified in the contract at time $t = T$ and at time $t = 0$. That is if the policy holder is alive at time $t = T$ we associate to the contract the payoff function:

$$P_{K,\delta,\widehat{S}_0}(S,T) = \max\{K\delta\, S/\widehat{S}_0, e^{rT}K\},\ \ S > 0. \tag{1}$$

Note that K, δ, r are specified at time $t = 0$, and that δ is an adimensional quantity, r is an interest rate, K is a given (positive) amount of money. No payments are due if the policy holder is death at time $t = T$. Needless to say we are assuming that the policy holder is alive at time $t = 0$.

Pricing this contract means finding the premium that the policy holder must pay at time $t = 0$ to enter the contract. The ideas proposed and the results illustrated here about this problem are:

i) the idea of pricing insurance products as "financial options" applied to the problem of pricing a pure endowment contract. Using this idea we derive a pricing formula for the pure endowment contract that has $P_{K,\delta,\widehat{S}_0}$ given in (1) as payoff function if the policy holder is alive at the maturity time of the contract. This pricing formula is derived in the model obtained considering simultaneously the multiscale stochastic volatility model proposed in [4] to describe the dynamics of the price of the risky asset specified in the insurance contract and the mean reverting Brownian Gompertz model to describe the mortality rate [6], [7]. The pricing formula is obtained under the assumption that the mortality risk is independent of the financial risk (see [1], [3]). It is shown that in the analogy with mathematical finance a pure endowment contract can be seen as a combination of two options: a path dependent option and a European option;

[1] The research reported in this paper is partially supported by MUR - Ministero Università e Ricerca (Roma, Italy), 40%, 2007, under grant: "The impact of population ageing on financial markets, intermediaries and financial stability". The support and sponsorship of MUR are gratefully acknowledged.

ii) an efficient computational method containing an ad hoc Monte Carlo method to evaluate the pricing formula mentioned in i) whose performance improves substantially both in accuracy and computational cost the performance obtained evaluating the pricing formula mentioned in i) using a straightforward Monte Carlo approach;

iii) in the limit when the volatility of the (mean reverting) Brownian Gompertz model goes to zero an asymptotic formula for the survival probability. This formula is used to obtain a semi-explicit formula to price the pure endowment contract. In its range of validity the numerical evaluation of this semi-explicit formula greatly outperforms the computational method mentioned in ii) and the straightforward Monte Carlo approach.

These results are obtained thank to the assumption that the financial risk and the mortality risk are independent and taking advantage of the knowledge of "explicit" formulae for the transition probability density functions of the state variables of the multiscale stochastic volatility model (see [4]) and of the (mean reverting) Brownian Gompertz model (see [6] and [7]). A more complete exposition of these results is contained in [3].

Let us go into details. Let t denote a real variable interpreted as time, we consider the (real valued vector) stochastic process $(x_t, v_{1,t}, v_{2,t}, h_t)$, $t > 0$, solution of the following system of stochastic differential equations:

$$\begin{aligned} dx_t &= (r - \frac{b_1^2}{2}v_1 - \frac{b_2^2}{2}v_2)dt + b_1\sqrt{v_{1,t}}dW_t^{0,1} + b_2\sqrt{v_{2,t}}dW_t^{0,2}, \quad t > 0, && (2)\\ dv_{1,t} &= \chi_1(\theta_1 - v_{1,t})dt + \varepsilon_1\sqrt{v_{1,t}}dW_t^1, \qquad t > 0, && (3)\\ dv_{2,t} &= \chi_2(\theta_2 - v_{2,t})dt + \varepsilon_2\sqrt{v_{2,t}}dW_t^2, \qquad t > 0, && (4)\\ dh_t &= (g + \frac{1}{2}\sigma^2 + b\ln(\hat{h}_0) + bgt + b\ln(h_t))h_t dt + \sigma h_t dW_t^3,\ t > 0, && (5) \end{aligned}$$

where b_i, χ_i, ϵ_i, θ_i, $i = 1, 2$, b, σ, g, r and $\hat{h}_0$ are suitable real constants. We interpret x_t, $t > 0$, as log-return of the price S_t, $t > 0$, of the risky asset considered in the pure endowment contract, that is we have $x_t = \ln(S_t/S_0)$, $t > 0$, $v_{1,t}$, $v_{2,t}$, $t > 0$, as stochastic variances of x_t, $t > 0$, and h_t, $t > 0$, as mortality rate associated to the human life. The mortality rate h_t at time t, $t > 0$, represents the instantaneous rate of mortality of a certain population measured on an annualized basis. In the numerical experiments presented in [3] we consider real mortality tables of the Italian population to calibrate equation (5) and in the mortality tables we do not distinguish individuals by gender. Note that r is a fixed rate, the risk free rate, that, for simplicity, we have chosen equal to the rate appearing in (1), $\ln(\cdot)$ denotes the natural logarithm of $\cdot$, $W_t^{0,1}$, $W_t^{0,2}$, W_t^1, W_t^2, W_t^3, $t > 0$, are standard Wiener processes such that: $W_0^{0,1} = W_0^{0,2} = W_0^1 = W_0^2 = W_0^3 = 0$, $dW_t^{0,1}$, $dW_t^{0,2}$, dW_t^1, dW_t^2, dW_t^3, $t > 0$, are their stochastic differentials. We assume that:

$$\begin{aligned} &< dW_t^1 dW_t^2 >= 0,\ < dW_t^{0,1} dW_t^2 >= 0,\ < dW_t^{0,2} dW_t^1 >= 0,\ t > 0, && (6)\\ &< dW_t^{0,1} dW_t^1 >= \rho_{0,1}dt,\ < dW_t^{0,2} dW_t^2 >= \rho_{0,2}dt,\ t > 0, && (7)\\ &< dW_t^3 dW_t^1 >= 0,\ < dW_t^3 dW_t^2 >= 0,\ < dW_t^{0,1} dW_t^3 >= 0,\ < dW_t^{0,2} dW_t^3 >= 0,\ t > 0, && (8) \end{aligned}$$

where $< \cdot >$ denotes the expected value of $\cdot$. The quantities $\rho_{0,1}$, $\rho_{0,2} \in [-1, 1]$ are called correlation coefficients. We impose the following conditions: $b_i > 0$, $\chi_i \geq 0$, $\varepsilon_i \geq 0$, $\theta_i \geq 0$, $i = 1, 2$, $b > 0$, $\sigma > 0$, $g > 0$, $r > 0$, $\hat{h}_0 > 0$. These conditions are motivated by the meaning of equations (2)-(5). Moreover we assume that $\frac{2\chi_i\theta_i}{\varepsilon_i^2} > 1$, $i = 1, 2$. The condition $\frac{2\chi_i\theta_i}{\varepsilon_i^2} > 1$ guarantees that when $v_{i,t}$ is positive with probability one at time $t = 0$, $v_{i,t}$, solution of (3) or (4), remains positive with probability one for $t > 0$, $i = 1, 2$. The assumption that the financial risk is independent of the mortality risk is expressed by (8). Note that due to (8) model (2)-(4) can be considered independently of model (5). Equations (2)-(4) are the multiscale stochastic volatility model used to describe the financial risk [4], equation (5) is the (mean reverting) Brownian Gompertz model used to describe the mortality risk [6], [7]. The equations (2)-(5) must be equipped with the initial condition:

$$x_0 = \tilde{x}_0, v_{1,0} = \tilde{v}_{1,0}, v_{2,0} = \tilde{v}_{2,0}, h_0 = \hat{h}_0, \tag{9}$$

where $\tilde{x}_0$, $\tilde{v}_{i,0}$, $i = 1, 2$, $\hat{h}_0$ are random variables that we assume to be concentrated in a point with probability one.

Let us mention that in a human population a cohort group is defined as the subset of the population made of the individuals born in a given year. The generic cohort group will be denoted with c. Due to assumption (8) we can evaluate the value of the pure endowment contract in the model (2)-(5) as the product of two expected values, that is the expected value (see formula (11)), that gives the survival probability specified below times the expected value of the discounted payoff (1) (see formula (10)). That is, for the people belonging to the cohort group c, the pricing formula at time $t = 0$ of the pure endowment contract with maturity time $t = T > 0$ when at time $t = 0$ the price of the risky asset contained in the contract is $\widehat{S}_0$ (i.e.: when we have $S_0 = \hat{S}_0$), the stochastic variances are $\tilde{v}_{1,0}$, $\tilde{v}_{2,0}$, is:

$$V(\widehat{S}_0, \tilde{v}_{1,0}, \tilde{v}_{2,0}, 0) = E(e^{-rT} P_{K,\delta,\widehat{S}_0}) p_0(T, \hat{h}_0 \,|\, c), \widehat{S}_0 > 0, \tilde{v}_{1,0} > 0, \tilde{v}_{2,0} > 0. \tag{10}$$

where $E(\cdot)$ denotes the expected value of $\cdot$ and $p_0(T, \hat{h}_0 \,|\, c)$, $T > 0$ is, in the model (5), (9), the probability of being alive at time T ($T > 0$) conditioned to the fact of being alive at time $t = 0$ and of belonging to the cohort c. This is the survival probability that interest us. In the model (5), (9) the probability of being alive at time T given the fact of being alive at time t, $0 \leq t \leq T$ when $h_t = h'_0$ where h'_0 is a random variable concentrated in a point (denoted again with h'_0) with probability one is given by:

$$p_t(T, h'_0) = E\left(e^{-\int_t^T h_u \, du}\right), 0 \leq t \leq T. \tag{11}$$

Note that formula (11) only apparently does not specify a cohort group. In fact we can image equation (5) and the initial condition for the mortality rate contained in (9) as being specified for a cohort group. That is the different cohort groups are described by different equations (5) and initial conditions (9) that differ one from the others for the value of the parameters appearing in the equation. Let us recall that the expected value appearing in the right hand side of equation (10) relative to the financial content of the endowment contract must be computed with respect to a risk neutral measure associated with the stochastic volatility model (2)-(4) (see [4], [5] for further details). We write this expected value as a one-dimensional integral of a smooth explicitly known integrand. This last integral is evaluated using a numerical quadrature. This integral can be seen as a formula to price a European option in the multiscale stochastic volatility model (2)-(4). This is the European option announced in the title.

The second expected value appearing in (10), that is the expected value contained in $p_0(T, \hat{h}_0 | \, c)$ is the one contained in $p_t(T, h'_0)$, shown in (11), when we choose $t = 0$, $h'_0 = \hat{h}_0$ and we specify equation (5) and $\hat{h}_0$ for the cohort group c. This expected value is evaluated with a computational method based on the ideas introduced in [2]. We transform the evaluation of (11) in the evaluation of a "path dependent option" in the (mean reverting) Brownian Gompertz model and we apply an ad hoc Monte Carlo method to evaluate numerically the high dimensional integral that approximates (11). This is the path dependent option announced in the title.

In particular we apply the composite midpoint rule to approximate the integral on the interval $[t, T]$ appearing in the exponent of (11). Let h'_0 be a constant and n be a positive integer we consider the composite midpoint rule with n quadrature nodes, that is we denote with $t_i = t + (2i - 1)(T - t)/(2n)$, $i = 1, 2, \ldots, n$, the quadrature nodes, moreover for later convenience we define $t_0 = t$. The expected value appearing in (11) is approximated in the limit n goes to infinity by the following quantity:

$$p^n_t(T, h'_0) = E\left(e^{-\frac{(T-t)}{n} \sum_{i=1}^n h_{t_i}}\right), t \leq T, \, h'_0 > 0, \, n = 1, 2, \ldots. \tag{12}$$

Recall that $h_t = h'_0$, and that we have chosen $h'_0 > 0$ to be a random variable concentrated in a point (denoted with h'_0 with probability one, that is (12) can be written as:

$$p^n_t(T, h'_0) = \int_0^{+\infty} dh'_1 \int_0^{+\infty} dh'_2 \cdots \int_0^{+\infty} dh'_n \, e^{-\frac{(T-t)}{n} \sum_{i=1}^n h'_i} \, p_h(h'_0, h'_1, \ldots, h'_n), t \leq T, h'_0 > 0, n = 1, 2, \ldots, \tag{13}$$

where for $n = 1, 2, \ldots$, h'_i, $i = 0, 1, \ldots, n$ are real variables and $p_h(h'_0, h'_1, \ldots, h'_n)$ is the conditioned transition probability density function associated with the process h_t, $t > 0$, defined by (5) and given by:

$$\begin{aligned} &p_h(h'_0, h'_1, \ldots, h'_n) dh'_1 dh'_2 \cdots dh'_n = \\ &P\{h'_i < h_{t_i} < h'_i + dh'_i, \, i = 1, 2, \ldots, n \,|\, h_{t_0} = h'_0\}, h'_i > 0, i = 0, 1, \ldots, n, \end{aligned} \tag{14}$$

where $P\{\,\cdot\,\}$ denotes the probability of $\,\cdot\,$. When $t_0 = t = 0$, $h'_0 = \hat{h}_0$ and equations (5), (9) are specified for the cohort c we have that when n goes to infinity $p^n_0(T, \hat{h}_0)$ given by (14) goes to $p_0(T, \hat{h}_0 \,|\, c)$ and that,

n	t_{ASF}	t_{HMC}	t_{SMC}	N_p
50	$1.09375 \cdot 10^{-6}$	0.65625	25.203125	25000
100	$1.09375 \cdot 10^{-6}$	1.859375	54.484375	25000
200	$1.09375 \cdot 10^{-6}$	6.062500	144.3437500	25000
400	$1.09375 \cdot 10^{-6}$	21.28125	572.671875	25000
50	$1.09375 \cdot 10^{-6}$	1.3125	50.140625	50000
100	$1.09375 \cdot 10^{-6}$	3.71875	109.234375	50000
200	$1.09375 \cdot 10^{-6}$	11.953125	287.50	50000
400	$1.09375 \cdot 10^{-6}$	42.21875	1143.796875	50000

Table 1: Comparison between the execution time (measured in seconds) required to evaluate the survival probability using the asymptotic formula (t_{ASF}), the ad hoc Monte Carlo (t_{HMC}) and the straightforward Monte Carlo (t_{SMC}). The number of simulations used in the Monte Carlo methods is N_p.

in general, the quantity $p_0^n(T, \hat{h}_0)$ is a satisfactory approximation of $p_0(T, \hat{h}_0 \,|\, c)$ only for large values of n. That is a satisfactory approximation of $p_0(T, \hat{h}_0 \,|\, c)$ requires the computation of the n dimensional integral appearing in (13) for large values of n. This high dimensional integral is computed with an ad hoc Monte Carlo method analogous to the one developed in [2] to price path dependent options in the Heston model. Finally in [3] an asymptotic formula for $p_0(T, \hat{h}_0 \,|\, c)$ when σ goes to zero is derived. In its region of validity the evaluation of the asymptotic formula can substitute the evaluation of (13) for large values of n with great computational savings.

Table 1 compares the performance measured through the execution time needed to compute $p_0(T, \hat{h}_0 \,|\, c)$ using the three approaches suggested: asymptotic formula (t_{ASF}), ad hoc Monte Carlo (t_{HMC}) and straightforward Monte Carlo (t_{SMC}). We choose the parameters of the (mean reverting) Gompertz Brownian model in the region of validity of the asymptotic formula, that is, we choose $T = 10$, $\sigma = 0.02$, $g = 0.1$, $b = 0.5$ and $\hat{h}_0 = 0.05$. Table 1 shows the execution time in seconds required to compute the approximation of $p_0(T, \hat{h}_0 \,|\, c)$ with the three approaches when n is chosen to be $n = 50$, 100, 200, 400 and the number of simulations used in the Monte Carlo methods is $N_p = 25000$ or $N_p = 50000$.

Moreover Table 1 shows that the ratio t_{HMC}/t_{SMC} ranges approximately in the interval [27,50] as a function of n and is a decreasing function of n.

The website: http://www.econ.univpm.it/recchioni/finance/w12 contains some auxiliary material including some animations that helps the understanding of the content of this note. A more general reference to the work of some of the authors in mathematical finance is the website:

http://www.econ.univpm.it/recchioni/finance/ .

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Optimal insurance with counterparty default risk*

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May 12, 2010

Abstract

We study the design of optimal insurance contracts when the insurer can default on its obligations. We model default as arising endogenously from the interaction of the insurance premium, the indemnity schedule, and the evolution of the insurer's assets. We allow for different forms and degrees of dependence between the insured's wealth and the default event, to understand the joint effect of insolvency risk and background risk on efficient contracts. The results may shed light on the aggregate risk retention schedules observed in catastrophe reinsurance markets. They can also assist in the design of (re)insurance programs and in capital modeling exercises that allow for counterparty default risk.

Keywords: insurance, default risk, catastrophe risk, limited liability, incomplete markets.

*We benefited from suggestions from seminar participants at Tel Aviv University, LMU Munich, Imperial College London, Cass Business School.All errors are our own.

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XXXIV CONVEGNO AMASES

MACERATA, 1-4 SETTEMBRE 2010

ABSTRACT

SULLA GESTIONE DI UNA CASSA MALATTIA PRIVATA

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PREMESSA ED IPOTESI DI LAVORO

In questo lavoro si prende in esame la gestione di una Cassa Malattia privata in condizioni di incertezza attinenti alla morbilità ed alla sopravvivenza degli iscritti su base volontaria. Si assumono le seguenti ipotesi di lavoro:

1) Gestione "a gruppo chiuso" con l'iscrizione alla Cassa di persone "sane", o in via eccezionale "malate", distinte per sesso e per età iniziale. Si indica con i=0 lo stato di "individuo sano" e con i >0 lo stato di "individuo con patologia i-esima".
   La Cassa assicura agli iscritti una assistenza "indiretta" in caso di malattia, curata con spese a domicilio o con ricoveri ospedalieri ed eventuali interventi chirurgici, posizionando i rimborsi alla fine dell'anno in cui avviene la spesa.
2) Contributi periodici anticipati da parte degli iscritti, variabili in base ad un indice di prezzi e differenziati per sesso, per età e per stato di salute iniziali, con aggravio in caso di rischi tarati (i>0).
3) Uscita dalla collettività degli iscritti per una delle seguenti cause: i) morte, ii) decisione dell'iscritto, iii) raggiungimento dell'età massima prefissata per l'assistenza della Cassa.
4) Prestazioni della Cassa in base al seguente elenco, non prevedendosi indennizzi per danni connessi ad assenza dal lavoro o simili:
   - 4a rimborsi in base a flussi predeterminati, in caso di cure domiciliari per malattie "lievi", distintamente per grandi sottogruppi di patologie e correlativi costi specifici stimati.
   - 4b rimborsi in base a flussi predeterminati, in caso di ricovero clinico a pagamento presso strutture private per malattie "gravi" senza interventi chirurgici, distintamente per grandi sottogruppi di patologie e correlativi costi specifici stimati.
   - 4c rimborsi una-tantum predeterminati, in caso di interventi chirurgici a pagamento, distintamente per grandi sottogruppi di tipologie di intervento e correlativi costi specifici stimati.

OBBIETTIVI ED ARTICOLAZIONE DELLA RICERCA

Scopo di questa ricerca è la determinazione delle relazioni di equilibrio attuariale anche ai fini del controllo della rischiosità nella gestione di una Cassa Malattia sotto le ipotesi sopra specificate.

Con adeguate metodologie si ottengono le distribuzioni di probabilità del risultato di gestione conseguente alle contribuzioni ed alle prestazioni aleatorie, sia al livello del singolo iscritto che dell'intera collettività. In conseguenza, opportuni procedimenti consentono la determinazione delle condizioni di equilibrio della gestione e dei livelli di rischiosità. E' possibile, ma non verrà qui trattata, una successiva indagine sulla "probabilità di rovina" per la Cassa in base alla note definizioni.

Per ottenere significativi risultati nella fase applicativa dell'indagine, occorre fare uso di:

a) appropriate basi tecniche demografiche;
b) frequenze delle uscite volontarie dall'elenco degli iscritti in funzione dell'età corrente;
c) frequenze, durate e costi delle malattie lievi che comportano cure domiciliari;
d) frequenze, durate e costi delle malattie gravi che comportano ricoveri ospedalieri;
e) frequenze e costi degli interventi chirurgici e conseguenti degenze per convalescenza.

Nel lavoro viene sviluppata l'indagine sulle contribuzioni e prestazioni attese per ciascun gruppo $G_{g,s,i}$ di iscritti caratterizzati dal genere g $\{g = 1 \rightarrow$ Maschio, $g = 2 \rightarrow$ Femmina$\}$, dall'età iniziale s, espressa in anni, e dallo stato di salute i. Indicheremo le variabili con il vettore (g,s,i).

Indicando con $n_{g,s,i}$ la numerosità di $G_{g,s,i}$, i valori attuali medi (v.a.m.) delle prestazioni e delle contribuzioni relativamente a $G_{g,s,i}$ si ottengono da quelli del generico iscritto di tale gruppo moltiplicando per $n_{g,s,i}$. Sommando rispetto a (g,s,i) si ottengono i v.a.m. per l'intera collettività.

Per il calcolo operativo riguardo ad un singolo iscritto (g,s,i) il v.a.m. delle prestazioni e quello delle contribuzioni possono ottenersi mediante l'implementazione di un processo stocastico. Il v.a.m. delle contribuzioni è il valore iniziale della rendita attuariale dei contributi, soggetta a due cause di eliminazione e con rate indicizzate in base ad un opportuno indice di prezzi.

Ne segue che la condizione di equilibrio attuariale per la gestione della Cassa riguardo al gruppo $G_{g,s,i}$ è soddisfatta determinando i contributi in modo da ottenere l'azzeramento del v.a.m. delle prestazioni al netto dei contributi.

E' evidente che i contributi iniziali di equilibrio per un iscritto dipendono dalla scelta di (g,s,i), ma se per ogni $G_{g,s,i}$ essi sono tutti di equilibrio, allora cumulando si ottiene l'equilibrio della Cassa ossia l'esatta copertura delle uscite mediante le entrate, tenuto conto dei rendimenti dei margini investiti al tasso finanziario di valutazione.

E' quasi superfluo avvertire che i contributi di equilibrio dovranno subire un caricamento per sopperire alle spese amministrative della Cassa.

LA FORMULAZIONE DEL MODELLO

Lo strumento peculiare nella presente ricerca in sede applicativa è un *modello semi-Markov non omogeneo discreto con rewards* opportunamente adattato per la determinazione del valore attuale medio (v.a.m.) e del momento secondo dei valori attuali attuariali (v.a.a.) con riferimento alle uscite annue per prestazioni ed alle entrate annue per contribuzioni degli iscritti di una Cassa Malattia privata, al fine di determinare per essa le condizioni di equilibrio di gestione.

Per la predetta applicazione occorre quantificare ed introdurre nella formulazione di tale modello semi-Markov con rewards per ciascun iscritto ($\alpha\beta$, i) le seguenti grandezze:

t	=	età finale dell'intervallo di proiezione (> s)
$H_{gs,i}$	=	prob. di avere una transizione dallo stato i entro l'età t
$\chi_{gs,i}$	=	contribuzione periodica = "rate reward" di un individuo con stato iniziale i
$\psi_{gs,i}$	=	prestazione (= rimborso) = "rate reward" di permanenza nello stato i
$b_{gs,ik}(s,t)$	=	prob. di avere una transizione all'età t dallo stato i allo stato k
$\gamma_{gs,ik}$	=	"impulse reward" di transizione dallo stato i allo stato k (= rimborso)
m	=	tasso annuo di rendimento effettivo
r	=	tasso annuo di rendimento reale (deflazionato)
$INFL$	=	tasso annuo di adeguamento dei prezzi (inflazione)

con il vincolo di Fisher:

$$1+m = (1+r)(1+INFL).$$

Ovviamente le probabilità di transizione dallo stato i sono nulle se i è assorbente.

Ai fini del calcolo dei v.a.m. e degli altri momenti dei v.a.a. è equivalente attualizzare al tasso m i rewards indicizzati al tasso $INFL$ o attualizzare al tasso r i rewards indicizzati al tasso 0, ossia costanti. Per il prosieguo assumeremo senz'altro questa seconda ipotesi per $\chi_{gs,i}$, $\psi_{gs,i}$ e $\gamma_{gs,ik}$ nonché per le spese ed i caricamenti.

Gli stati del sistema costituiscono l'insieme

$$E = \{(M_{\lambda,\mu}), W, D, L\}, \quad ; \ \lambda = 1, 2 \ ; \ \mu = 0, 1, ..., h$$

dove

λ	=	indicatore del grado di malattia: 1 = "lieve", con cure domiciliari; 2 = "grave", con degenza ospedaliera
μ	=	indicatore della patologia, fra un elenco di h prefissate.
$M_{\lambda,\mu}$	=	stato di malattia di grado λ e patologia μ; ($\mu = 0 \rightarrow$ stato di "sano").
L	=	stato assorbente di iscritto "uscito per limiti di età ($s_{\max}$)"
W	=	stato assorbente di iscritto "uscito per rinuncia volontaria"
D	=	stato assorbente di iscritto "deceduto".

Conviene attribuire un indice progressivo agli stati sopra precisati, come segue:

$M_0 \rightarrow i = 0$
$M_{1,1} \rightarrow i = 1$
$M_{2,1} \rightarrow i = 2$
$M_{1,2} \rightarrow i = 3$
$M_{2,2} \rightarrow i = 4$
....
$M_{1,h} \rightarrow i = 2h-1$
$M_{2,h} \rightarrow i = 2h$
$L \rightarrow i = 2h+1$
$W \rightarrow i = 2h+2$
$D \rightarrow i = 2h+3$

I rewards sono quantificati come segue:

Rate rewards in entrata: $\chi_{gs,i}$, $\forall i$ (= ricavo annuo costante per contributi dell'iscritto (g,s,i) entrato nella collettività con lo stato i).

Per il calcolo del v.a.m. con rewards unitari si porrà: $\chi_{gs,i} = 1$.

Rate rewards in uscita: $\psi_{gs,i} = 0$; $\psi_{gs,i} = R_i$, $i > 1$ (= rimborso di costo connesso alla permanenza nello stato i)

Impulse rewards in uscita: $\gamma_{gs,ik}$ (= rimborso di costo connesso alla transizione dallo stato i allo stato k)

E' quasi superfluo osservare che dopo la transizione dallo stato i allo stato k i rate rewards in uscita valgono $\psi_{gs,k}$; inoltre risulta $k = i$ se un eventuale intervento chirurgico non ha modificato il grado di malattia e la patologia.

Per semplicità (giacché dei flussi monetari interessano soltanto i v.a.m.), l'*impulse reward* $\gamma_{gs,ik}$ si definisce come un valore medio al tempo τ, espresso da:

$$\gamma_{gs,ik} = \xi_{ik} + \pi_{gs,ik}\ \eta_{ik} . \tag{4.2}$$

dove: ξ_{ik} = costo della visita medica; $\pi_{gs,ik}$ = prob (prescrizione di intervento chirurgico); η_{ik} = costo dell'intervento chirurgico.

Con adeguate procedure si ottiene la formulazione per il calcolo del v.a.m. al tempo 0 dei contributi versati in via anticipata dagli iscritti (r,s,i), nonché del v.a.m. al tempo 0 delle prestazioni per rimborsi forniti in via posticipata agli iscritti (g,s,i) nello stato i al tempo 0. Con tali valori medi risolvendo un'equazione, si ricava il contributo annuo di equilibrio $\hat{K}_{gs,i}$ dell'iscritto (g,s,i) entrato con lo stato i, che azzera il v.a.m. del risultato di gestione.

Con terminologia attuariale, interpretando l'iscrizione alla Cassa come una copertura assicurativa, tale contributo di equilibrio è il premio equo per tale copertura.

Si ricava altresì il momento secondo del risultato di gestione, il quale in condizioni di equilibrio coincide con la varianza, che fornisce adeguate informazioni sulla rischiosità della gestione della Cassa in esame.

CALCOLO DELLE RISERVE PROSPETTIVE ED EFFETTI DEL CARICAMENTO

Per finalità di controllo del bilancio della Cassa occorre il calcolo annuale delle riserve prospettive $RP(\theta;g,s,i)$ per ogni singolo iscritto nello stato i all'ingresso (e, sommando, per l'intera Cassa), dove $\theta>0$ è il tempo di calcolo della riserva a partire dall'iscrizione, tempo cui corrisponde per l'assicurato l'età $s+\theta$. Tali riserve si ottengono con una semplice modifica delle formule che forniscono i v.a.m. dei contributi e delle prestazioni, facendo intervenire il parametro θ.

Tali riserve, calcolate senza tener conto delle spese di amministrazione ed in base al premio equo, sono da considerare riserve pure.

Peraltro per aderenza alla realtà operativa conviene tenere conto, nella gestione della Cassa, delle spese di amministrazione e conseguentemente dei premi caricati, che chiamiamo "premi di tariffa", con riferimento all'iscritto (g,s,t). Assumendo una gestione amministrativa diretta da parte della Cassa, non consideriamo spese di acquisizione e inseriamo le spese di incasso nell'insieme delle spese di amministrazione. Pertanto dal lato delle uscite occorre aggiungere ai costi tecnici per rimborsi una quota annua pro-capite per spese di amministrazione durante la permanenza dell'iscritto, quota che può considerarsi crescente per l'adeguamento dei prezzi ma che qui si assume costante a causa dell'attualizzazione al tasso reale r.

Il caricamento $\rho_{gs,i}$ del premio puro $\hat{K}_{gs,i}$ già calcolato si ottiene risolvendo una nuova relazione di equilibrio, la quale tiene conto delle spese amministrative annue pro-capite ς, che si assumono indipendenti dalle caratteristiche dell'iscritto (g,s,t).

Determinato $\rho_{gs,i}$, è agevole il calcolo della riserva prospettiva completa al tempo θ per l'iscritto (g,s,i) e poi, sommando, per l'intera Cassa.

I risultati qui ottenuti sono suscettibili di ulteriori varianti e generalizzazioni, che gli Autori si propongono di sviluppare.

HIERARCHICAL STRUCTURES IN THE AGGREGATION OF PREMIUM RISK FOR INSURANCE UNDERWRITING

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Abstract

In the valuation of the Solvency II Capital Requirement, the correct appraisal of risk dependencies acquires particular relevance. These dependencies refer to the recognition of risk diversification in the aggregation process and there are different levels of aggregation and hence different types of diversification. For instance, for a non-life company at the first level the risk components of each single line of business (e.g. premium, reserve and CAT risks) need to be combined in the overall portfolio, the second level regards the aggregation of different kind of risks as, for example, market and underwriting risk, and finally various solo legal entities could be joined together in a group.
Solvency II allows companies to capture these diversification effects in capital requirement assessment, but the identification of a proper methodology can represent a delicate issue. Indeed, while Internal Models by simulation approaches permit usually to obtain the portfolio multivariate distribution only in the independence case, generally the use of copula functions can consent to have the multivariate distribution under dependence assumptions too.
However the choice of the copula and the parameter estimation could be very problematic when only few data are available. So it could be useful to find a closed formula based on Internal Models independence results with the aim to obtain the capital requirement under dependence assumption.
A simple technique, to measure the diversification effect in capital requirement assessment, is the formula, proposed by Solvency II Quantitative Impact Studies (QIS), focused on the aggregation of capital charges, the latter equal to percentile minus average of total claims amount distribution of single line of business (LoB), using a linear correlation matrix.
On the other hand, this formula produces the correct result only for a restricted class of distributions, while it may underestimate the diversification effect.
In this paper we present an alternative method, based on the idea to adjust that formula with proper calibration factors (proposed by Sandström, (2007)) and appropriately extended with the aim to consider very skewed distribution too.
In the last part considering different non-life multi-line insurers, we compare the Capital Requirements obtained, for only premium risk, applying the aggregation formula to the results derived by elliptical copulas and Hierarchical Archimedean Copulas.

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Fair Valuation of Equity-Linked Policies under Default Risk

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Insurance Branch Category: IB11

JEL classification: G13, G22

Subject Category: IM01

Keywords: Equity-linked policies; Default risk; Fair value

Extended Abstract

During the last decades, life insurance markets have experienced a considerable number of firm defaults. Just to name a few, among the most significant, we recall Mutual Benefit Life and Executive Life in the United States (13.5 and 10.2 billion dollars, respectively) at year end 1990. In Japan, Nissan Mutual Life in 1997 and the two life insurance companies Chiyoda and Kyoei in December 2000 went bankrupt. In Europe, France's Garantie Mutuelle des Functionnaires in 1993, Equitable Life in England in 2000 and Mannheimer Leben in Germany in 2003 defaulted. More recently, during the tremendous financial crisis originating in 2007 by the collapse of the US housing bubble, the American International Group, Inc.(AIG) suffered from the cost of the financial distress and, in September 2008, the Federal Reserve Bank stepped in by organizing a bailout based on credit facilities for several billion dollars.

In order to reduce the risk of facing a financial distress, a generally held opinion supports a clear and transparent valuation of a firm's business based on the fair value principle, that is market consistent calculations of financial and non financial assets and liabilities of an insurance company. In fact, the current market value is the only correct measure of the firm financial strength properly revealing the real situation of the insurance company to the stakeholders and permitting a well-timed intervention of regulatory authorities in case of crisis.

In the United States, the fair valuation principle has been clearly stated among the generally accepted accounting principles (US GAAP). In particular, the Financial Accounting Standard No. 157 (Fair Value Measurements FAS 157) illustrates how the fair value of an insurance contract should be computed. On the contrary, in the International Financial Reporting Standards (IFRS) there is not a specific pronouncement about the fair value determination and, for insurance contracts meeting the definition of financial instruments, the International Accounting Standard 39 (IAS 39) has to be considered. Anyway, the International Accounting Standards Board issued an

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exposure draft containing guidance about the fair value measurement in the IFRS based on concepts similar to the ones already reported in FAS 157, which states: “the fair value of the liability shall reflect the nonperformance risk relating to that liability. Nonperformance risk includes but may not be limited to the reporting entitys own credit risk. The reporting entity shall consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value” (see [12] for a detailed discussion on this point).

The evaluation of insurance contracts, taking explicitly into account the insurer risk of default, was first addressed by Briys and de Varenne ([4], [5]). They apply the techniques developed by financial theorists for option pricing to establish a continuous time model for evaluating assets and liabilities of an insurance company under stochastic interest rate and default risk. They consider a stylized insurance company which sells a participating policy without mortality risk where the benefit received by the policyholder depends on a minimum guaranteed interest rate plus a share of the rate of return generated by the firm’s asset portfolio, if positive. The company is insolvent if at the contract maturity the value of the asset portfolio is less than the payment guaranteed to the policyholders. In this case, the firm goes bankrupt and the customers receive the total asset value as a rebate.

Grosen and Jorgensen [9] point out that in the Briys-de Varenne model the insolvency of the insurance company is detected only at policy maturity. To overcome this limit, they propose a model where the solvency is monitored continuously during the policy lifetime and the firm is declared bankrupt as soon as the asset value falls below a lower boundary represented by the policyholders’ initial deposit accrued at the guaranteed rate of return. The fair value of the policy is then computed by applying the evaluation techniques developed in finance to derive barrier option prices.

Bernard, Le Courtois, and Quittard-Pinon [2] generalize the Grosen-Jorgensen approach by considering participating policies in a stochastic interest rate environment. Taking into account more general bankruptcy procedures, Chen and Suchanecki [6] treat the case when the insurance company is declared insolvent if its total asset value remains below a pre-specified threshold for a certain time interval. The fair value of the policy is then computed in a Parisian barrier option framework. Ballotta, Haberman, and Wang [1] apply Monte Carlo techniques to compute the fair value of with-profit and unitized with-profit life insurance contracts under default risk.

In this paper, we consider the problem of computing the fair value of an equity-linked policy when the insurer is subject to default risk. To simplify matters, as in all the above models, we consider single premium policies embedding an interest rate guarantee without mortality risk. The evaluation framework is the one developed in modern financial theory based on the no-arbitrage principle. In this environment, an equity-linked policy is treated as a vulnerable contingent claim which expires before maturity if the firm’s total asset value reaches a pre-specified default threshold proportional to its liabilities. In the event of default, absolute priority is assumed and the policyholders have the first claim on the firm’s assets. Moreover, we suppose that the liquidation of the rebate is made at the time of default. Otherwise, if the company does not face financial distress during the policy lifetime, at maturity it is forced to pay whichever is the greater between a guaranteed amount and the value of the reference equity portfolio.

In our view, there is a key difference between the fair evaluation problem of an equity-linked policy and of a participating policy under default risk as considered in the models cited above. In the simplified case without mortality risk and postulating a constant risk-free interest rate, the problem of computing the fair value of a participating contract is univariate in that there is only one state variable, i.e., the total value of the company’s asset portfolio, governing both firm solvency and the policy benefit. On the contrary, in the case of equity-linked policies, one more state variable has to be considered because the benefit depends on the value of an equity portfolio which, in

general, has a different composition with respect to the one of the firm's assets. Consequently, the dynamics of the two entities have to be described by distinct random processes and the evaluation problem results a two-factor one.
We derive, at first, a closed-form formula to compute the fair value of an equity-linked contract in a continuous time framework. Then, we build up a discrete time model which allows the gain of greater flexibility in solving the evaluation problem when the contract embeds a surrender option. To do this, we assume that the policyholders exercise the option before maturity only if it is financially convenient, and as a result, the policy is modeled as an American-style contingent claim.

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Il Fair Value della riserva sinistri nell'assicurazione R.C.Auto in presenza dell'Indennizzo Diretto

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ABSTRACT

L’introduzione dell’Indennizzo Diretto nel ramo R.C.A. ha comportato negli ultimi anni una maggiore complessità nell’utilizzo delle metodologie statistico - attuariali per la stima della riserva sinistri, sia per la eterogeneità dei dati disponibili per le valutazioni (gestione dei sinistri ante e post 2007), sia per le diverse dinamiche sottostanti i fattori di rischio che caratterizzano e determinano il costo aziendale complessivo delle Compagnie di assicurazioni, in relazione alla gestione sinistri.

In particolare, il D.P.R. n. 254 del 18 luglio 2006, e successivamente la Convenzione tra Assicuratori per il Risarcimento Diretto (CARD) che ha recepito le norme e stabilito gli accordi, disciplinano i limiti e le modalità di applicazione del risarcimento diretto, prevedendo una nuova struttura del costo dei sinistri, basata sul meccanismo dei Forfait gestionari e debitori. Pertanto il nuovo costo aziendale risulta composto da quattro componenti (sinistri No Card, Sinistri Card, Forfait gestionari e forfait debitori), caratterizzate da evoluzioni aleatorie differenti.

Le quattro tipologie di sinistri elencate presentano, infatti, profili di sviluppo diversi: basti pensare, ad esempio, alla velocità di liquidazione dei sinistri Card, presumibilmente superiore a quella dei sinistri No Card. Tuttavia, a meno che non si effettui una suddivisione a ritroso delle serie storiche dei pagamenti ed un’analisi separata delle stesse, potrebbe verificarsi la situazione in cui in un triangolo run-off compaiano importi relativi a generazioni antecedenti all’entrata in vigore dell’indennizzo diretto, e comprensivi quindi di sinistri di tipologia diversa, che pertanto rischiano di intaccare l’attendibilità di una stima, che si basi appunto su quel triangolo.

Va inoltre sottolineato che, con l’introduzione dei ‘forfait’, il processo di riservazione di una Compagnia è influenzato dalla sinistrosità dell’intero mercato nazionale, il che rende necessaria un’analisi degli effetti di eventuali scostamenti tra la sinistrosità interna della singola compagnia e la sinistrosità media di mercato.

In questo lavoro si presentano i primi risultati di una ricerca che gli a. stanno svolgendo, al fine di individuare una metodologia per la valutazione della riserva sinistri, che sia capace di intercettare e gestire le differenze evolutive dei fenomeni sopra riportati, attraverso l’analisi

delle diverse caratteristiche delle gestioni sinistri, in modo da consentire l'applicazione dei tradizionali metodi di riservazione su dati coerenti.

Partendo da un'analisi semplificatrice, effettuata su un mercato formato da due sole compagnie, si è cercato di mettere in evidenza i differenti fenomeni che impattano sulla gestione dei sinistri, confrontando lo scenario attuale con quello presente fino all'introduzione dell'indennizzo diretto. Successivamente, si è proceduto alla generalizzazione del problema, considerando un mercato formato da un numero sufficientemente elevato di compagnie, in cui la ripartizione della totalità dei rischi non presenta particolari concentrazioni su alcune di esse.

I risultati di tale analisi preliminare sono serviti per individuare un modello che permetta di stimare la riserva sinistri utilizzando non solo dati interni alla compagnia, ma anche specifici dati di mercato, differenziati secondo diversi fattori (territoriali, macrocategorie di veicoli, tipologie di danni).

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Implementing a Solvency II non life internal model: ReReserving and Backtesting.

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Often in non-life insurance claims reserves are the largest position on the liability side of the balance sheet. Therefore, the estimation of adequate claims reserves for a portfolio consisting of several lines of business is relevant for every non-life insurance company.

In old accounting tradition italian insurance companies used to estimate nominal claims reserves for their outstanding loss liabilities. The new solvency regulations require insurance companies to move to a market-consistent valuation of their liabilities (full balance sheet approach) and to prove the adequacy every year. Under new Solvency II developments insurance companies need to calculate a risk margin to cover possible shortfalls in their liability runoff. A popular approach for the calculation of the risk margin is the cost-of-capital approach which involves the consideration of multiperiod risk measures. Because multiperiod risk measures are complex mathematical objects, various proxies are used to calculate this risk margin.

In the present paper we derive an analytic formula for the risk margin which allows the comparison of the different proxies used in practice and we develop a flexible internal model that can be used for evaluating a specific risk profile.

A case study on different liability datasets investigates the influence of the dimension on the results and gives a possible answer to some questions raised by the International Actuarial Association. In particular the so called "ReReserving" approach is compared with the classical "run-off" approach using several different stochastic methods, including the Bayesian Fisher Lange model proposed in 2008[8].

Moreover, a backtesting process compares historical results to those produced by the current model in order to validate both the reasonableness and the implementation of the assumptions.

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Pricing Ratchet equity-indexed annuities with early surrender risk in a CIR++ model*

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Extended Abstract

Equity Indexed Annuities (EIAs) are important insurance contracts in the insurance market. Ratchet EIAs allows the policyholder to partecipate in the potential appreciation of the stock market. Moreover, in order to eliminate the downside risk the insurance company tipically guarantees a minimum return.

We consider a joint evolution for the equity value with the CIR++ stochastic interest model (see [2]) which is consistent with the term structure of interest rate and permits to overcome the problem of negative interest rates of the extended Vasicek model proposed in [5].

Furthermore, as noted in [5], Ratchet EIAs pricing literature has largely ignored early surrender risk. Pricing Ratchet EIAs with early surrender risk is a challenging problem due to the deep path-dependent nature of the contingent claim.

We propose a tree lattice method for pricing Ratchet EIAs with early surrender risk in a CIR++ model. In this case, the pricing problem leads to a non-recombining tree. In order to treat the problem we use the framework of singular points tecniques (see [4], [3]). The proposed algorithm is based on appropriate no-arbitrage jump conditions at each node corresponding to each reset date. We obtain a procedure which allows to get the price in a reasonable time and the convergence of the discrete approximations to the continuous value in a simple way. We compare our algorithm with a Monte Carlo method with a larger number of simulations and discretization steps using the discretization scheme for the CIR process proposed in [1].

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*This research was supported by MIUR (Prin 2007)

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Valutazione della riserva sinistri nell'ambito dei GLM gerarchici

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Nella letteratura attuariale sono stati proposti diversi modelli stocastici per la valutazione della riserva sinistri; per un'ampia rassegna si vedano England, Verrall (2002), Wüthrich, Merz (2008). Negli ultimi anni tali modelli hanno assunto sempre maggior importanza anche per l'attività assicurativa, in seguito alle richieste delle nuove normative contabili e sui requisiti di solvibilità che, oltre alle stime dei pagamenti in sospeso, richiedono valutazioni di attendibilità delle stime.

Tra i modelli proposti ve ne sono di tipo mistura, nei quali si assume che le distribuzioni di probabilità dei pagamenti incrementali, le variabili risposta, dipendano da un vettore di parametri aleatori di rischio. Tali modelli possono essere trattati secondo l'approccio bayesiano o di credibilità (Mack (2000), de Alba (2002), Ntzoufras, Dellaportas (2002), Verrall (2004)); combinando le tecniche dei modelli lineari generalizzati (GLM) con la credibilità (Nelder, Verrall (1997), Ohlsson, Johansson (2006), Ohlsson (2008)); utilizzando i GLMM (*Generalized Linear Mixed Models*; Antonio *et al.* (2006)).

In questa nota consideriamo, per la valutazione della riserva sinistri, modelli stocastici con componenti di regressione della classe dei modelli lineari generalizzati gerarchici (*Hierarchical Generalized Linear Models,* HGLM; Lee, Nelder (1996), (2001), Lee, Nelder, Pawitan (2006)). Si tratta di particolari modelli di tipo mistura nei quali le variabili risposta condizionate ai parametri di rischio hanno distribuzioni appartenenti ad una famiglia esponenziale lineare, dipendenti ad una struttura di regressione. Anche i parametri di rischio hanno distribuzioni di una famiglia esponenziale lineare; in particolare, nei modelli coniugati, sono coniugate delle distribuzioni delle variabili risposta.

Gli HGLM estendono da un lato la classe dei GLM, in quanto introducono una particolare struttura di dipendenza tra le variabili risposta, e dall'altro la classe dei GLMM, modelli mistura nei quali i previsori lineari delle variabili risposta presentano sia effetti "fissi", sia effetti aleatori, questi ultimi con distribuzione normale.

Seguendo l'indicazione di Lee, Nelder (1996), per la stima di un HGLM si può utilizzare la *h*-verosimiglianza (*hierarchical likelihood*) che non richiede di determinare le distribuzioni marginali delle variabili risposta. Si ottengono in tal modo stimatori dei parametri di regressione e degli effetti aleatori che godono di buone proprietà statistiche. Un'estensione degli HGLM, analoga a quella che conduce dai GLM ai modelli con quasi-verosimiglianza, consente di stimare modelli semiparametrici, nei quali le distribuzioni delle variabili risposta ed anche dei parametri di rischio non sono completamente specificate.

Inoltre è possibile assumere che anche i parametri di dispersione dipendano da una struttura di regressione.

Sono dunque modelli flessibili, che consentono di scegliere in un'ampia classe le distribuzioni delle variabili risposta e dei parametri di rischio, di considerare diverse strutture di regressione e funzioni di collegamento, anche per i parametri di dispersione delle due componenti. D'altra parte sono facilmente trattabili, in quanto per l'algoritmo di stima e per le analisi di controllo dell'adattamento si può fare riferimento alle tecniche dei GLM.

Nell'ambito della valutazione della riserva sinistri, tali modelli possono essere utilizzati in presenza sia di dati aggregati, riassunti in un triangolo *run-off*, sia di dati individuali. Inoltre, tramite i parametri dai quali dipendono le distribuzioni degli effetti aleatori, con tali modelli si può tenere conto di informazioni esterne o iniziali che sono poi combinate con i dati di portafoglio.

Nella nota, abbiamo utilizzato modelli della classe degli HGLM e loro estensioni come strumento di stima e di previsione ai fini della valutazione della riserva sinistri, ipotizzando di disporre dei dati riassunti in un triangolo *run-off* e di informazioni iniziali interpretabili come speranze matematiche degli effetti aleatori. Come misura della qualità degli stimatori delle riserve di generazione e della riserva di portafoglio abbiamo considerato l'errore di previsione, il cosiddetto *root mean square error of prediction*, condizionato all'informazione portata dal triangolo *run-off*. Abbiamo determinato una stima di tale errore basandoci sui risultati asintotici degli stimatori e utilizzando alcune approssimazioni proposte in Lee, Ha (2009). L'errore di previsione così determinato tiene conto del rischio di processo e del rischio di stima, sia per i parametri di regressione che introducono la dipendenza dagli effetti fissi, sia per i paramenti che introducono gli effetti aleatori.

I risultati sono confrontati con quelli ottenuti con altri modelli proposti in letteratura: modelli di credibilità, modelli che combinano le tecniche dei modelli lineari generalizzati con la credibilità, modelli che utilizzano dati esterni (Mack (2008), Alai, Merz, Wüthrich (2009)).

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Managing longevity and disability risks in life annuities with Long Term Care

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Extended abstract

Population ageing has produced a higher number of individuals exposed to the risk of becoming disabled. Insurance companies and pension funds are devoting an increasing attention to Long Term Care (LTC) insurance in face of an increasing demand for long term care needs of individuals.
The first aim of the paper is to analyse and quantify risks involved products related to LTC. We focus on enhanced pension, a single premium cover providing an annuity enhanced when the insured becomes disabled. The typical biometric risk in LTC insurance is the risk arising from uncertainty in future disability trends as well as longevity risk. Progressive increase in lifetime duration leads to a potential increase in longevity risk for both disabled lives and healthy lives. The main consequence for annuity providers is an extension of the annuity payments period and an increase of actuarial liabilities.
An adequate management of longevity and disability risk requires life insurance companies and pension plans to model and measure them. Up to recent years, both mortality and disability have been traditionally modelled in a deterministic framework, mainly adopting adjusted projected mortality tables to model mortality and assuming adequate analytical models to model disability. The insurance companies' ability to read demographic trends has significantly improved over the last decade, but annuity providers remain largely exposed to biometric risk. As a matter of fact, even when mortality has been adequately forecasted, uncertainty in future mortality trends remains. Therefore, mortality projections must take into account the inherent uncertainty of projecting into the future.
Deterministic approach is now considered inadequate to calculate premiums and reserves and to manage this risk, and annuity providers refer now to stochastic models. Stochastic models are necessary to measure the systematic part of the biometric risk in the forecast. As a consequence, numerous works recently proposed in literature were mainly concerned with the inclusion of stochasticity into mortality models.

While the use of stochastic approaches to model mortality dynamics has been widely explored for actuarial purposes (e.g. see Milevsky and Promislow (2001), Dahl (2004), Biffis (2005), Ballotta and Haberman (2006), Dahl and Mï£¡ller (2006) and (Cairns et al., 2006, 2008)), a stochastic approach to model disability needs to be further explored. Ferri and Olivieri (2000) and Olivieri and Pitacco (2001) proposed discrete stochastic models defining different disability and mortality scenarios with weights representing subjective probabilities associated to each scenario in a portfolio of LTC insurance. Levantesi and Menzietti (2007) adopted the same approach in order to analyse biometric risk in LTC products, as well as to find the adequate solvency capital requirement and to test the effectiveness of reinsurance strategies. More recently, Olivieri and Pitacco (2009a) proposed a simplified approach to stochastic modeling in a continuous framework for disability benefits.
Our paper aims at providing a contribution to this last approach. We develop a model for risk assessment in a portfolio of life annuities with long term care benefits. These products are usually represented by a Markovian Multi-State model and are affected by both longevity and disability risks. Here, a stochastic projection model is proposed in order to represent the future evolution of mortality and disability transition intensities. Data from the Italian National Statistical Institute (ISTAT) are used to estimate the model parameters. Further, we investigate the solvency in a portfolio of enhanced pensions. To this aim a risk model based on the portfolio risk reserve is proposed and different rules to calculate solvency capital requirements for life underwriting risk are examined. Such rules are then compared with the standard formula proposed by the Solvency II project.

Keywords: Markovian multi-state model, Longevity risk, Disability risk, Solvency II, Internal models.

JEL classification: G22, J11

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Risk indicators for unfunded pension funds

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Abstract

The aim of this contribution is to investigate the sustainability of a pay-as-you-go pension fund within a stochastic framework. Financial and demographic risk sources are analysed by investigating and comparing their impact on the evolution of the fund. Melis and Trudda (2009) analyze the evolution of a "closed" pension fund financed by a PAYG system, using autoregressive processes to describe the new entrants variation rate and the Vasicek model for the global asset return. Angrisani (2009) proposes a logical mathematical model to manage a pension system with a structural funded component.

In this paper we construct stochastic risk indicators for monitoring the solvency of the fund, namely its capability to pay future obligations. The model presented is in a continuous time. A numerical application is carried out on the pension funds of the Italian Professional Orders.

Keywords. Pension Funds, Pay-As-You-Go Pension System, Demographic Risks

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Methods for the management of investment guarantees in a pension fund: identification of a sequence of optimal guarantees and price of the surrender clause.

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Traditionally the issue on which the insurance contracts management focuses is always the purely actuarial one, neglecting the other side of the coin which is the question concerning the investment of resources, that is, the premium income. This approach stems from the fact that the investment risk is not considered the primary risk factor characterizing the management of a portfolio of policies. In reality, this consideration is valid if the performance of the contract can be immunized, or hedged, by fixed-income securities, but it has no meaning in life insurance equity linked contracts, or in equity linked pension funds.

In such contracts, the predetermination of benefits is eliminated, giving more flexibility to contract through the linking of performance to a stock portfolio or investment fund. These equity linked insurance policies, or pension funds, require high insurer financial management capacity, in particular call for a separate management of these contracts that is distinct from that of other investments of the insurer.

The management of pension funds involves investment of large amounts of money by insurance companies on behalf of fund members.

In consideration of the contributions, the fund members expect a benefit at the end of the contract.

The volatile nature of the returns that an insurance or a pension fund obtains from the investment of premiums or contributions, respectively, means that companies provide a guaranteed minimum return,implementing a financial risk transfer.

Pension funds provide such guarantees to ensure to the fund members that their investment will produce a non-negative return, even in the possibility of a bad investment strategy case.

This paper presents a particular problem of discrete optimization, where we proceed to maximize the benefits expected for a given sequence of contribu-

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tions and given an overall level of guaranteed minimum.
Moreover, it presents a method for pricing of any surrender clause, which will identify the additional cost (or extra premium) paid by the member so as to have the possibility of an early exit from the pension fund, obtaining in exchange the surrender value.
Consistent with the equity-linked structure of the contract, the amount of surrender takes into account a guaranteed minimum amount payable to the member of the fund when the value of the portfolio is lower than the guarantee, at the time of the surrender request.
The stochastic evolution of the fund's portfolio is represented by a random tree and the method used to estimate the cost of surrender clause uses a backward type algorithm which is based on the Monte Carlo simulation. The problem is merely financial, because the analysis has not considered the risk of mortality. For a description of the procedure for implementation of the random tree see the text of Glasserman, [20].
The main literature reference for the pricing of guaranteed minimum benefits is in the works of Brennan and Schwartz (1976) [11], Boyle and Schwartz (1979) [10],basic works for the analysis of the equity linked funds.
In these articles the authors show how the performance of equity-linked contracts may be modeled by the option pricing theory.
In this work we describe the optimization problem to be solved when one intends to proceed to the identification of a sequence of incremental guarantees that maximize the final benefit to the pension fund member.
There are two different iterative methods, the Newton-Raphson and fixed point method, used for the calculation of contributions invested in the reference portfolio,[16].
Later we introduce the method of Lagrange multipliers for the detection of the sequence of optimal guarantees.
Moreover we proceed to the description of the method used for pricing of the surrender clause, ie premature exit from the pension fund.
Finally, there is a numerical application of the topic.

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Some stochastic claim reserving models: bootstrap simulation and measures of variability

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Estimating and monitoring of the claims reserve play a crucial role in assessing the health status of an insurance company .
The awareness that behind the run-off triangle hides a real stochastic process of generating claims requires the use of a stochastic model that is able to grasp the characteristics.The mere mechanical application of traditional techniques seems to ignore the presence of such deterministic process.
The chain ladder method continues to have a prominent role, that is why the link between it and the formal stochastic model should not be ignored. Particular emphasis is placed on stochastic models that can reproduce, under specific constraints,the same estimates of the chain ladder technique.
In recent years several authors have investigated about this link, [12], [20], [28], [3], [23], [2], [13].
Kremer (1982) [12] was the first to highlight the close link between the chain ladder method and a two-way analysis of variance model, applied to the logarithms of incremental claims paid,showing how the traditional method is characterized by an underlying mathematical structure that provides a linear regression model with the dependent variable consisting of the logarithm of payments and two explanatory variables: origin year and development year.
Renshaw e Verrall (1998) [23] presented the chain ladder technique as a generalized linear model (GLM), assuming an Over-dispersed Poisson distribution for the incremental amount of claims.
The Distribution Free chain ladder model of Mack (1993) [13] (DFCL), does not provide specific assumptions about the distribution of future payments, but simply models the first two moments.
Starting from the chain ladder technique some stochastic models, as Generalized Linear Models with Over Dispersed Poisson distribution, Gamma model and LogNormal model were applied.
In addition to statistically comparing the results of different models, bootstrap simulation was applied to residues in order to obtain information on the entire distribution of reserves.
In literature there are several examples of application of the bootstrap technique to the problem of claims reserve, as England e Verrall (1999) [8],

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Pinheiro, Andrade e Centino M. (2000) [19].
The definition of the residuals, used for the Bootstrap exercise, differs from that of the previous papers, because we used deviance residuals:

$$r^S_{D_i} = sign(y_i - \hat{\mu}_i)\sqrt{\frac{d_i}{\hat{\phi}\,(1 - h_{ii})}} \tag{1}$$

where:

$$sign(y_i - \hat{\mu}_i) = 1 \qquad se \quad y_i - \hat{\mu}_i \geq 0$$

or

$$sign(y_i - \hat{\mu}_i) = -1 \qquad se \quad y_i - \hat{\mu}_i < 0$$

As in the GLM, there is no error term, it makes sense to consider the contribution to deviance from the observation i:

$$d_i = -2(\left[y_i\hat{\theta}_i - b(\hat{\theta}_i)\right] - \left[y_i\tilde{\theta}_i - b(\tilde{\theta}_i)\right]) \tag{2}$$

The implementation of stochastic reserving models allows to get measures of variability of the claims reserves; the application of Bootstrap technique allows to test stability and goodness of fit.
An assessment of the stability of the estimation results is made possible through the analysis of prediction errors (*Root Mean Square Error Prevision*), with its decomposition in Estimation Error and Process Error.
The prediction error was also calculated with reference to estimates obtained with the Bootstrap.
In the framework of the Solvency II Project, the Committee of European Insurance and Occupational Pensions Supervisors (*CEIOPS*) has been requested by the European Commission to establish well defined solvency and supervisory standards in order to allow a convergent and harmonized application across EU of the general prudential principles in the determination of the insurance technical provisions and the required solvency capitals.
An important step in this direction is the new method proposed for computing technical provisions, which are defined by CEIOPS as the sum of two components: the best estimate component, representing the current expectation of the unpaid liabilities towards the policyholders; the risk margin component, expressing a prudential loading required to offset the liability uncertainty.
Both the best estimate and the risk margin of the OLL must be derived by well defined probabilistic models properly calibrated on the relevant claims experience. However while the definition of best estimate as the expected OLL seems to be widely accepted, the question of how the risk margin of the OLL should be exactly defined is still under discussion.
In addition to prediction errors we used, as measures of variability, confidence intervals: standard normal intervals and confidence intervals based on bootstrap percentiles.

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Loading strategies in efficient proportional reinsurance under group correlation: from single period to multiperiod optimality

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Abstract. Proportional reinsurance has played an important historical role in the theory of financial decisions. Indeed, it was with reference to a proportional reinsurance problem that B. de Finetti introduced, in his seminal paper [6], the mean-variance efficiency approach in modern financial decisions under uncertainty. This should be seen as the forerunner of the reward-risk analysis, which plays a central role in the risk management of financial institutions (banks, insurance companies, pension funds, investment funds and so on).

There is a widespread opinion (see for a recent treatment, Glineur-Walhin, [11], 2006) that closed form formulae of the whole set of proportional efficient retentions may be obtained only under no correlation, while in the correlation case there is the need to make recourse to a sequential procedure which is the counterpart of the critical line algorithm of the classical portfolio problem (see Markowitz [15], [16], [17]). The issue has been treated e.g. by Barone [1], who analyzed a specific point of the controversial issue raised by H. Markowitz [18] about the applicability of de Finetti's approach to the case of correlated risks, and by Pressacco-Serafini [20].

While this statement is true in general, there is at least one specific structure of correlation which allows to recover nice results with closed form formulae. This special structure was named group correlation by de Finetti, who treated the point in his paper and gave a quick hint to some key properties of the efficient set (see [6], p. 27-30; see also Gigante [10], 1990).

More precisely, group correlation means that the portfolio of insured risks may be divided in subgroups characterized by two properties: the correlation between each pair of policies belonging to the same group is a fixed, group specific, positive constant, whereas the correlation between pairs of risks of different groups is identically zero; moreover, the ratio between standard deviation and expected gain of each risk is a group specific constant too. In technical terms, this amounts to say that a safety loading premium charge is applied based on a standard deviation principle (Berliner [2], 1977; Gerber [9], 1974), with a loading coefficient which is the same for all risks belonging to the same group.

We remind that, in this context of group correlation, Pressacco-Serafini-Ziani ([22], 2009) recently obtained the following fundamental results regarding a single period horizon: in the mean-variance plane the set of efficient retentions is a union of parabolas, continuous and differentiable

(without kinks) also at the connection points, whose explicit expressions have been provided. In addition, closed form formulae describing the set of efficient retentions have been provided as well.

These results open the way for interesting analysis concerning the consequences of different loading strategies and correlation structures on optimal retentions. This paper is devoted to a specific analysis of this issue.

To this aim, a stylized five-group portfolio of 5.000 policies (1.000 policies for each group) has been considered. Each policy (number i, group q) is characterized by a couple of parameters: the mean $E_{i,q}$ and the standard deviation $\sigma_{i,q}$ of the corresponding risk. Keeping account of the standard deviation principle applied for premium charging, the random gain associated to each policy is characterized, denoting by l_q the group specific loading coefficient, by its expectation $m_{i,q} = l_q \cdot \sigma_{i,q}$ and its standard deviation $\sigma_{i,q}$. Then, our basic platform arises from the choice of a common, for all groups, structure of standard deviations, whose values are equally spaced in a convenient range.

After that, the overall expected gain E of the portfolio depends, linearly, on the set of loading coefficients:

$$E = \sum_{q=1}^{5} l_q \sum_{i=1}^{1.000} \sigma_{i,q} \tag{0.1}$$

while, after a specific reinsurance decision, described by the retention vector $(\mathbf{x})$, it becomes:

$$E(\mathbf{x}) = \sum_{q=1}^{5} l_q \sum_{i=1}^{1.000} x_{i,q}\sigma_{i,q} \tag{0.2}$$

In turn, the overall variance V involves also the set of group specific correlation coefficients ρ_q, so that:

$$V = \sum_{q=1}^{5} \sum_{i=1}^{1.000} \sigma_{i,q}^2 + \sum_{q=1}^{5} \left(2\rho_q \cdot \sum_{i=1}^{1.000} \sum_{j>i} \sigma_{i,q}\sigma_{j,q} \right) \tag{0.3}$$

while $V(\mathbf{x})$ becomes:

$$V(\mathbf{x}) = \sum_{q=1}^{5} \sum_{i=1}^{1.000} x_{i,q}^2 \sigma_{i,q}^2 + \sum_{q=1}^{5} \left(2\rho_q \cdot \sum_{i=1}^{1.000} \sum_{j>i} x_{i,q}x_{j,q}\sigma_{i,q}\sigma_{j,q} \right) \tag{0.4}$$

In this stylized framework, the consequences of different combinations of loading and correlation coefficients have been analyzed.

As regards correlation coefficients, besides the case of Null correlation useful for the sake of comparison, we consider three different correlation structures, which have been roughly defined, in relative sense, with labels Low, Medium and High, respectively.

As for the loading strategies, we defined at first two different loading trigger levels, labelled Big and Small, to be intended as mean along groups. On these basis, three different connections between loading strategies and correlation structures have been considered. For any given choice of correlation, we have loading strategies labelled Uniform, which means constant loading; Direct, that is increasing with correlation on a proper range; Inverse, that is decreasing with correlation, on the same range.

For all the described scenarios, we computed the set of efficient retentions and plotted the efficient frontiers on the mean-variance space. This gives an immediate flavor of the influence either of the loading levels given the correlation, or of the correlation structure given the loading.

As regards the interaction between loading and correlation for a given loading trigger, it comes out that a loading charge inversely related to the correlation level gives a more efficient frontier than the one obtained in the Direct or Uniform case. The explanation of this, at first sight, surprising result is that, the efficient retentions give a higher level of reinsurance of policies of the more risky group(s), that is those with the higher group correlation; under Inverse relation, this involves giving up quotas of policies with comparatively smaller expected gain (lower loading). In turn, this implies a corresponding disadvantage for the reinsurer(s). In the framework of group correlation, this result represents another explanation of the rationality of quota treaties approach (Bühlmann [4], 1970; Borch [3], 1974; Gerber [8], 1984; Pressacco [19], 1986; Lampaert-Walhin [14], 2005), well known in the theory of optimal reinsurance.

That's all as regards the comparisons of the overall efficient frontiers in the single period model; in the paper, we also offer a preliminary discussion of the impact of different loading and correlation structures on the optimal choices in a multiperiod problems under a mean-ruin probability framework.

Acknowledgement. We acknowledge financial support of MedioCredito FVG through the "Bonaldo Stringher" Laboratory of Finance, Department of Finance, University of Udine.

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A CRM Approach to Estimate Reserve Risk in Non-Life Insurance

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Abstract

Stochastic models for outstanding claims valuation have been recently developed with the aim to obtain a variability coefficient or the probability distribution of the reserve, useful for the assessment of the Reserve Risk capital requirement. Different methodologies, like Bootstrapping or Bayesian methods, allow to obtain the prediction error and the claims reserve distribution under some assumptions. The International Actuarial Association (IAA) proposed a different way to analyse outstanding claim reserve, focused on a paper by Meyers, Klinker and Lalonde wherein the overall reserve of a single LoB (without any distinction among different accident years) is assumed to be a compound mixed poisson process, by a similar approach used for aggregate amount of claims when premium risk is to be estimated. In the present paper we propose an extension of this approach based on the assumption that each single cell of the lower part of the triangle can be described by a Compound Poisson Process (Pure or Mixed).

This assumption leads, under independence, to obtain quite easily the exact moments of the reserve distribution only through the knowledge of the characteristics of the two main variables (number and claim size of future payments).

Furthermore, Monte Carlo methods allow to simulate outstanding claims distributions for each accident year, for the overall reserve and for the next calendar year (in case of a one-year time horizon as prescribed in Solvency II).

Model's parameters are calibrated from observed data and through a deterministic model (in the paper an average cost method is used, namely Fisher-Lange method) based on the separate estimate of number of claims to be paid and future average costs for each cell of the triangle to be estimated.

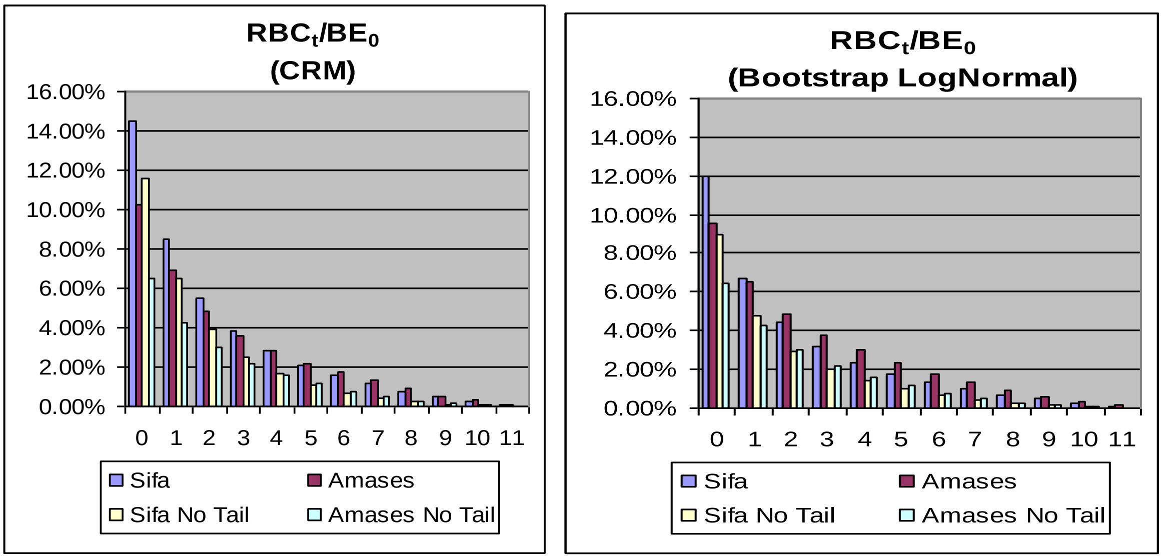
Furthermore we analyze the one-year reserve risk in the perspective adopted by Solvency II. In particular a simulation approach, now usually called "re-reserving", is here applied with the target to estimate the variability of claims development result and the percentiles, in order to quantify the capital requirement for the reserve risk. Main results will be compared using either CRM and Bootstrapping, analyzing the effect on both capital requirement for only reserve risk and risk margin.

At this regard, next Figure 1 shows the patterns of the ratios between risk based capital (RBC) for only reserve risk until the total run-off of liabilities and the initial best estimate (BE) according to different MTPL Insurers. The analysis considers data on claim experience on Motor Third Party Liability (MTPL) from two Italian insurance companies according to accounting years from 1993 to 2004. In particular the triangle SIFA is referred to a small-medium company whereas the triangle AMASES has as a reference a company roughly 10 times larger.

Each capital requirement for reserve risk at the 99.5% confidence level has been obtained as the difference between the percentile of the distribution of the insurer obligations at the end of the following year, opportunely discounted with the risk-free discount factor, and the current exit value (best estimate plus risk margin) of the claims reserve at the beginning of the year.

The distribution of the insurer obligation at the end of the year is derived by both models (CRM and Bootstrapping) and by using the YEE approximation in order to solve the circularity between the assessment of RBC and the valuation of Risk Margin with the Cost of Capital methodology.

Figure 1: Ratio between $RBC_{99.5\%}$ for reserve risk and Initial Best Estimate until the total Run-Off



It could be observed how both models have capital ratios (between capital requirement and best estimate) lower than that are actually requested by QIS4 standard formula for MTPL line (roughly three times a volatility factor equal to 12%). How expected, the smaller insurer (SIFA) has a higher RBC ratio due to the more variable and skewed claim reserve distribution.

CRM and Bootstrap lead, indeed, to different distributions of the insurer liabilities and then to different RBC. AMASES Insurer has a variability equal to 2.89% (derived by CRM model considering the total run-off distribution) lower than 4% obtained by Bootstrap, while the sampling with replacement methodology shows lower variability (4.95%) than CRM for SIFA Company. The skewness is not affected so much only when the Bootstrap method is used according to either different triangle dimensions or probabilistic assumptions (LogNormal, ODP, Normal, Negative Binomial, etc.). Furthermore it's to be emphasized how the insurers have a different pattern of the ratio between the capital requirement and the Best Estimate. The slow settlement speed of the bigger insurer, confirmed by a higher tail development factor, leads in the next years to greater ratio than SIFA (from year 2 with Bootstrap and from year 5 with CRM).

Finally the analysis will be carried on in order to evaluate the claims reserve overall distribution and the capital requirement for a multiline insurers too. In particular, we extend the model with the aim to consider correlated sub-portfolios (for example different lines of business) and we attempt to quantify the aggregation effects on the overall distribution through some case studies.

Keywords: stochastic model for claims reserve, capital requirement for reserve risk, risk theory, compound poisson process, average cost methods, multivariate claims reserves.

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TPML PREMIUM RISK MODELING WITHIN ITALIAN DIRECT REIMBURSEMENT CARD SYSTEM

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Third party motor liability insurance actuarial practice has been deeply changed by the introduction of a compulsory Direct Reimbursement (DR) scheme since February 1, 2007.

In general, DR scheme makes the non responsible party's insurer to fully indemnify its client by the total amount of any suffered claim on behalf of the responsible part insurer. The responsible part insurer will not fully compensate the other insurer, that will receive a predetermined amount, generally different (larger or smaller) by the amount effectively paid for the non responsible claim.
The effective implementation rules, drafted under the "CARD" agreement (acronym for "Convenzione Assicuratori sull'Idenizzo Diretto"), lead to three possible sub-categories of losses ("components of claim") within a single loss event: NoCard, CID and CTT. NoCard components of claim comprise loss events either with severe bodily injury (more than 9% of permanent disability) or not having been caused by the crash of two vehicle. CID components of claims represent loss events caused by the crash of two vehicle with property damage or bodily injuries. CTT represent loss events where passengers other than drivers suffer bodily injuries or property damages.

The rules regarding compensating forfeit calculation have been changed on a yearly basis since 2007. Forfeit for CID component of claim have been varied by type of claim (bodily injury or property damage), by non responsible vehicle category and geographic zone. In addition to strong investments on IT process change and claim department training, CARD scheme lead to a huge impact on actuarial modeling. Within the pricing process, special attention shall be given to negative claim amounts (that occur when received forfeit is greater than the amount of suffered claims). Consequently, the pure premium of a single risk does not depend any more by the average caused claim cost only, but by the net average cost of suffered claim cost. Actuarial literature regarding CARD is still very scarce (see e.g. [3] for a comprehensive review of existing literature and a more detailed description of the CARD system).

In addition to CARD scheme introduction, Italian insurers face the upcoming enforcement of Solvency II directives. Solvency II directives require a risk based calculation of solvency capital, that consider the joint contribution of all risks (underwriting, market, credit and operational) that the insurance bears. Solvency II standard formulas is expected to increase the Solvency Required Capital relevantly with respect to current regulatory environment, with special reference to non-life insurers.

Therefore an internal model for underwriting premium risk on a Third Party Motor Liability under CARD system have been developed (here we have assumed the convolution approach followed in [1]). The evident heterogeneity of risks within the portfolio has been taken into account as models for frequency and severity by components of claims have been developed for the most important category of vehicles. The models for frequency and severity have been calibrated by means of GAMLSS ([2]), that represent a generalization of classical GLMs and allows a joint modeling of location and dispersion parameters within ML estimation. The claim cost distribution has been modeled through a separated analysis of frictional claims (done within GAMLSS) and shock/catastrophic losses, whose analysis have been carried out by means of EVT. At the end, Monte Carlo Analysis have been carried on to provide the portfolio total loss distribution and to estimate the capital charge at 99.5%.

The empirical application of the model on a real TPML portfolio showed in that case a capital requirement build by internal model relevantly lower than the amount calculated applying Solvency II standard formula. Moreover the developed approach can be easily extended to determine the final tariff for TPML coverage.

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A trinomial-mixed model for illness insurance.

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May 15, 2010

Abstract

This paper aims to illustrate a one-period framework that can be used to calculate the risk-reserve when at the end of time horizon there are three possible states for insured. To fix idea we consider a portfolio of permanent health insurance (PHI), but the model can be adapted in other similar situations. At the beginning all insured are in the same state and at the end they can be in a state between: active, disable or died. The most critical issues in defining PHI risk come from lack of reliable experience data and uncertainty about disability and mortality rates. To this purpose, it is adopted a mixed-trinomial model where both disability's rate than mortality's rate are stochastic. This model gives a closed-formula to calculate the amount of the required capital for a given illness portfolio, based on ruin-probability in one year.

The paper is structured in this way. In Section 1 a multiple state model for PHI covers is defined, it is usefull for pricing and reserving. In Section 2 mixed-trinomial random process is constructed and moreover we illustrate the special case when mixed-factors are Gamma-distribuited. In Section 3 we defined the risk-reserve equation usefull to identify the amount of capital necessary to insurer's solvency while in Section 4 is defined the formula to calculate the ruin-probability. Finally, some results are reported and discussed in Section 5 as well as some concluding remarks.

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